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On K.M.S. evolutions and liouville in quantum statistical mechanics.
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INTRODUCTION AND SUMMARY

Statistical mechanics is the study of macroscopic properties of
large systems starting from the microscopic properties. A system in
a box is inadequate as a model for large systems since it presupposes
the unphysical fiction of a completely isolated system. For instance
a system in a box neither exhibits phase transitions (cf. [1] theory
of Yang and Lee)nor approach to equilibrium (cf. [1] Poincaré cycle),
both well known phenomena for physical systems. To reproduce the
thermodynamic behaviour one performs the so-called thermodynamic limit,
i.e. one uses the "Gibbs Ansatz" to calculate the expectation values
of observables and takes the limit V (volume), N (number of particles)
$\rightarrow \infty$, keeping N/V fixed.

For the precise description of the above mentioned phenomena one
has to consider as a model a thermodynamic system, i.e. a system
consisting of an infinite number of particles in an infinite volume
with finite density. Such a description is most adequately given in
an algebraic setting, thereby assuming that the observables generate
a C^*-algebra $\mathfrak{U}$ and the dynamics is described by a one-parameter
group of *-automorphisms $\{\alpha_t\}$ of $\mathfrak{U}$. The expectation values of the
observables, in particular correlation functions, are given by positive
linear functionals (i.e. states) on $\mathfrak{U}$.

We restrict ourselves to equilibrium quantum statistical mechanics.
It can be argued that equilibrium states of thermodynamic systems
satisfy the so-called K.M.S. condition, which is a direct generaliza-
tion of the "Gibbs Ansatz" (cf. [2]), with respect to $\{\alpha_t\}$ at inverse
temperature $\beta > 0$.

An important problem is to mathematically describe the dynamics
that gives rise to a thermodynamical equilibrium state. It is therefore
of interest to investigate which automorphism groups are *evolutions*,
i.e. admit a K.M.S. state. For this one has, among other things,to
consider the properties of the infinitesimal generator of the auto-
morphism group $\{\alpha_t\}$. Generally speaking the infinitesimal generator

is an *unbounded* derivation. (In chapter I we present a short introduction to this subject, whereas in chapter III we specialize to separable U.H.F. algebras. A more complete survey can be found in [3].)

For the finite system, where the algebra of observables consists of all the bounded operators on a suitable infinite dimensional separable complex Hilbertspace, one knows which automorphism groups, characterized by a selfadjoint operator H ("Hamiltonian"), admit of K.M.S. states. In this case the automorphism group admits of a K.M.S. state iff there is a $\beta > 0$ such that $e^{-\beta H}$ is a trace class operator. A K.M.S. state admitted by such a dynamics is the well known Gibbs state and this state is, in accordance with the Yang-Lee theory, the only one.

For a thermodynamic system there is no Hamiltonian. We have to look for other means to distinguish automorphism groups that admit of a K.M.S. state from automorphism groups that do not admit of a K.M.S. state. One proves that only strongly continuous automorphism groups are candidates for evolutions in case of a separable algebra of observables.

We will treat this problem for two models, namely quantum lattice systems and Fermi systems. For these models the C^*-algebra of observables is of a particular, technically simple, nature. It is a so-called separable U.H.F. algebra. It has a local structure built in that permits of a natural discussion of thermodynamic systems starting from finite systems. This local structure gives rise to the so-called approximately inner automorphism groups, i.e. automorphism groups that can be approximated by a sequence of uniformly continuous one-parameter groups of inner *-automorphisms. In particular the physically important class of dynamics obtained as a limit of finite volume dynamics is approximately inner.

Closely connected with any strongly continuous automorphism group $\{\alpha_t\}$ of a separable U.H.F. algebra is a set of K.M.S. states (*not with respect to* $\{\alpha_t\}$!). This set has a cluster point ω. (Its properties are investigated in chapter IV). It is known [4] that, if $\{\alpha_t\}$ is approximately inner, ω satisfies the K.M.S. condition with respect to $\{\alpha_t\}$. It has been conjectured [4] that every strongly continuous one-parameter group of *-automorphisms of a separable U.H.F. algebra is approximately

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inner. Obviously the truth of this conjecture gives a unique character-
ization of evolutions. An example of an approximately inner group of
automorphisms is provided by the usual quantum lattice dynamics (cf.
section 3.3).

We will prove (proposition 4.3) that all the quasi-free evolutions
of the Clifford algebra (a particular U.H.F. algebra appropriate for
the description of Fermion systems) are approximately inner [*]).
Furthermore we extend the above result to a large class of automorphisms,
connected in some way with the quasi-free evolution. In particular we
show (proposition 4.8) that certain global perturbations of these auto-
morphism groups again give approximately inner automorphism groups.

The approximately inner property of $\{\alpha_t\}$ is sufficient to ensure
that the cluster point ω is a suitable K.M.S. state. Whether this property
is necessary is not clear. It is quite possible that all strongly
continuous automorphism groups of a separable U.H.F. algebra admit a
K.M.S. state, without all being approximately inner. To investigate
this it is natural to consider in more detail the clusterpoint ω,
mentioned above. We prove (proposition 4.12) that ω satisfies some
interesting inequalities, which may be useful in solving this problem.
For instance we show (proposition 4.13) that a state on a general C^*-
algebra which satisfies these inequalities is separating. Hence this
state satisfies the K.M.S. condition on the level of the von Neumann
algebra.

The observables of continuous quantum systems, as opposed to quantum
lattice systems, are described by the so-called quasi-local algebra.
The temporal behaviour of correlation functions is determined by the
spectral properties of the so-called *Liouville operator*. In the last
chapter we will study, also for general continuous quantum systems,
the spectral properties of the Liouville operator, which is defined as
follows: a state ω that satisfies the K.M.S. condition with respect to

[*]) Professor Sakai brought to our attention that R. McGovern obtained
related results [8].

an automorphism group $\{\alpha_t\}$ is α_t-invariant and gives rise to a G.N.S. representation in which $\{\alpha_t\}$ is implemented by a strongly continuous group of unitary operators. The infinitesimal generator H_ω of this unitary group is called a Liouville operator. It is the direct generalization of the well known Liouville operator of ordinary classical statistical mechanics.

With the help of the notion of Arveson spectrum (cf. [5]), to be defined in chapter I, we show that the spectrum of a Liouville operator is symmetric around the origin. This is a well known result from the Tomita-Takesaki theory. Our proof, however, shows in a very simple way the role of the separating character of the state.

Let ω_1 and ω_2 be K.M.S. states with respect to an evolution $\{\alpha_t\}$ of a C^*-algebra $\mathfrak{A}$ at inverse temperatures β_1 and β_2 (possibly $\beta_1 = \beta_2$). Denote the Liouville operators by H_{ω_1} and H_{ω_2}. We show that in many cases the spectra of these operators coincide as sets. For the non-separable quasi-local algebra $\mathfrak{A}$ this is true if $\{\alpha_t\}$ satisfies some *regularity* conditions. For a separable C^*-algebra, for instance the separable U.H.F. algebra, the spectra of the Liouville operators belonging to the same automorphism group are always the same (cf. [6]). Moreover for the free Bose gas, where the algebra of observables is neither separable nor quasi-local, this is also true, due to the fact that $\{\alpha_t\}$ acts in an asymptotically abelian way.

Suppose we have a non-primary K.M.S. state ω at an inverse temperature β that admits of a decomposition into extremal K.M.S. states ω_γ, i.e. $\omega = \int d\mu_\omega(\gamma) \omega_\gamma$, where ω_γ represent the pure phases. Without any further assumption one can prove that a discrete point in the spectrum of H_ω appears as discrete point ("survives the decomposition") in the spectra of H_{ω_γ} for γ in some set with non-zero μ-measure [7].

For the particular case that all the G.N.S. representation spaces are separable we show that a real number λ is a discrete point of $\sigma(H_\omega)$ *iff* it is a discrete point of $\sigma(H_{\omega_\gamma})$ for γ in some set with non-zero μ-measure. ($\sigma(H_\omega)$ is the spectrum of H_ω).

Similar statements are made with respect to generators of unitary groups that implement other groups of automorphisms, such as space translations.

In section 5.3 and 5.5 we apply the foregoing to the two-dimensional Ising model and the ideal Bose gas.