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Published in:

Proceedings of the Eleventh International Symposium on Mathematical Theory of Networks and Systems

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

1993

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Maschke, B. M., & Schaft, A. J. V. D. (1993). System-theoretic properties of port-controlled Hamiltonian systems. In *Proceedings of the Eleventh International Symposium on Mathematical Theory of Networks and Systems* (pp. 349-352). University of Groningen, Johann Bernoulli Institute for Mathematics and Computer Science.

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System-theoretic properties of port-controlled Hamiltonian systems

B.M. Maschke * A.J. van der Schaft †

In our previous paper [1] it has been shown how by using a generalized bond graph formalism the dynamics of non-resistive physical systems (belonging to different domains, i.e., electrical, mechanical, hydraulical, etc.) can be given an *intrinsic* Hamiltonian formulation of dimension equal to the order of the physical system. Here “Hamiltonian” has to be understood in the generalized sense of defining Hamiltonian equations of motion with respect to a *general* Poisson bracket (not necessarily of maximal rank). The Poisson bracket is fully determined by the network structure of the physical system (called “junction structure” in bond graph terminology), while the Hamiltonian equals the total internally stored energy. A striking example is the direct Hamiltonian formulation of (nonlinear) *LC*-circuits [2]. Subsequently in [3] the interaction of non-resistive physical systems with their environment has been formalized by including external ports in the network model, naturally leading to two conjugated sets of external variables: the inputs u represented as generalized flow sources, and the outputs y which are the conjugated efforts.

This leads to an interesting class of physical control systems, called *port-controlled Hamiltonian systems* in [3], formally defined as follows. The state space M (the space of energy variables) is a *Poisson manifold*, i.e. is endowed with a *Poisson bracket*. Recall [1], [2], [3] that a Poisson bracket on M is a bilinear map from $C^\infty(M) \times C^\infty(M)$ into $C^\infty(M)$ ($C^\infty(M)$ being the smooth real functions on M), denoted as

$$(F, G) \mapsto \{F, G\} \in C^\infty(M), \quad F, G \in C^\infty(M) \quad (1)$$

which satisfies for every $F, G, H \in C^\infty(M)$ the following properties

$$\{F, G\} = -\{G, F\} \quad (\text{skew-symmetry}) \quad (2)$$

$$\{F, G \cdot H\} = \{F, G\} \cdot H + G \cdot \{F, H\} \quad (\text{Leibniz-rule}) \quad (3)$$

$$\{F, \{G, H\}\} + \{G, \{H, F\}\} + \{H, \{F, G\}\} = 0 \quad (\text{Jacobi-identity}) \quad (4)$$

Then for every $H \in C^\infty(M)$ we can define, at any $x \in M$, the mapping $X_H(x) : C^\infty(M) \rightarrow \mathbf{R}$ as $X_H(x)(F) = \{F, H\}(x)$, $F \in C^\infty(M)$. It follows from the Leibniz rule (3) that X_H

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is a smooth vectorfield on M , called the *Hamiltonian vectorfield* corresponding to the *Hamiltonian* H , and the Poisson bracket $\{, \}$. In local coordinates $x = (x_1, \dots, x_n)$ for M the Hamiltonian dynamics $\dot{x} = X_H(x)$ take the form

$$\begin{bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_n \end{bmatrix} = J(x) \begin{bmatrix} \frac{\partial H}{\partial x_1}(x) \\ \vdots \\ \frac{\partial H}{\partial x_n}(x) \end{bmatrix} \quad (5)$$

where the skew-symmetric *structure matrix* $J(x)$ is given as

$$J(x) = [J_{ij}(x)]_{i,j=1,\dots,n}, J_{ij}(x) = \{x_i, x_j\} \quad (6)$$

A *port-controlled Hamiltonian system* on the Poisson manifold M is now given as

$$\dot{x} = X_H(x) + \sum_{j=1}^m g_j(x) u_j \quad (7)$$

$$y_j = \langle dH(x), g_j(x) \rangle, \quad j = 1, \dots, m \quad (8)$$

where $H : M \rightarrow \mathbf{R}$ is the *internally stored energy*, the inputs $u \in \mathbf{R}^m$ are the external *flows* (due to external sources), and the outputs $y \in \mathbf{R}^m$ are the conjugated efforts. The input vectorfields $g_j(x)$ model the interaction of the system with the external sources (modulated transformers in bond-graph terminology.) One immediately obtains the characteristic property $\frac{d}{dt}H = \sum_{j=1}^m u_j y_j$, expressing the fact that the increase in internal energy of a port-controlled Hamiltonian system equals the energy supplied at the ports.

Example The *LC*-circuit of Figure 1 is described as the port-controlled Hamiltonian system

$$\begin{pmatrix} \dot{Q} \\ \dot{\varphi}_1 \\ \dot{\varphi}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} Q/C \\ \varphi_1/L_1 \\ \varphi_2/L_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} v, \quad (9)$$

$\dot{x} \qquad \qquad J(x) \qquad \qquad dH(x) \qquad \qquad g(x)$

where $J(x)$ is the structure matrix of a Poisson bracket on $M = \mathbf{R}^3$. Note that the output $y = \varphi_1/L_1$ is the current through the first inductor L_1 .

At this point we would like to stress that so far we did not really use the Jacobi-identity (4); in fact the whole definition of port-controlled Hamiltonian system goes through for brackets (1) *not* satisfying (4). Furthermore, from a (bond-graph) modelling point of view it is not a priori clear why the Jacobi-identity *should* be necessarily satisfied (although it *is* in many examples!). On the other hand, the satisfaction of the Jacobi-identity is equivalent to the existence of so-called *canonical* coordinates. In particular, if $J(x)$ has *maximal rank*

$n = 2k$, then by the Jacobi-identity there exist coordinates $q_1, \dots, q_k, p_1, \dots, p_k$ in which the Hamiltonian dynamics (6) reduces to the *standard* Hamiltonian equations

$$\dot{q}_i = \frac{\partial H}{\partial p_i}(q, p), \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}(q, p), \quad i = 1, \dots, k \quad (10)$$

Note that if we assume the dynamics (7) to be Hamiltonian for *all* input values u , then the input vectorfields g_j are necessarily Hamiltonian, i.e., of the form X_{H_j} for some functions $H_j : M \rightarrow \mathbf{R}, j = 1, \dots, m$. In the case of a Poisson bracket of maximal rank n this means that we are back to the Hamiltonian control systems studied e.g. in [4], [5]. However this assumption is quite restrictive, as can be already seen from the example given above. (Since $g(x)$ in (9) is not in the image of $J(x)$ it cannot be a Hamiltonian vectorfield!)

On the other hand, one may make the *weaker* assumption that the input vectorfields are *Poisson bracket preserving*, i.e. they satisfy

$$L_{g_j}\{F, G\} = \{L_{g_j}F, G\} + \{F, L_{g_j}G\}, \quad \text{for all } F, G \in C^\infty(M) \quad (11)$$

for $j = 1, \dots, m$, as trivially holds in the above example. The property that the Poisson bracket of the port-controlled Hamiltonian system (7) satisfies the Jacobi-identity (4) together with the property that the input-vectorfields are bracket-preserving as in (11) may be succinctly expressed by requiring that the Poisson bracket is preserved by the dynamics (7) for *every* choice of internal energy H and *every* input value u !

In the previous paper [3] we have given a rather complete treatment of realizability, controllability and observability properties of *linear* port-controlled Hamiltonian systems. In the present note we will announce a few theorems on nonlinear port-controlled Hamiltonian systems. Because of space limitations details and proofs will be given elsewhere.

The first theorem expresses the fact that for observable port-controlled Hamiltonian systems the Poisson bracket is uniquely determined by the input-output behavior.

Theorem 1 Consider two port-controlled Hamiltonian systems

$$\begin{aligned} \dot{x}_i &= X_{H^i}(x_i) + \sum_{j=1}^m g_j^i(x_i)u_j^i, \quad x_i \in M_i, \\ \Sigma_i : \\ g_j^i &= \langle dH^i(x_i), g_j^i(x_i) \rangle, \quad i = 1, 2 \end{aligned} \quad (12)$$

where M_1 and M_2 are Poisson manifolds with Poisson brackets $\{, \}_1$ and $\{, \}_2$, respectively. Assume that g_j^i satisfy the property (11), $j = 1, \dots, m$, for $i = 1, 2$. Assume that Σ_2 is observable in the sense that the observation space O^2 distinguishes points in M_2 and that $\dim dO^2(x_2) = \dim M_2$, for each $x_2 \in M$. Suppose now that every state x_1 of Σ_1 is indistinguishable from some state of Σ_2 . Then there exists a unique smooth mapping $\varphi : M_1 \rightarrow M_2$, mapping Σ_1 into Σ_2 , i.e.

$$\varphi_* X_{H^1} = X_{H^2}, \varphi_* g_j^1 = g_j^2, \quad \langle dH^2, g_j^2 \rangle \circ \varphi = \langle dH^1, g_j^1 \rangle, \quad j = 1, \dots, m \quad (13)$$

which is also Poisson bracket preserving, i.e.

$$\{F \circ \varphi, G \circ \varphi\}_2 = \{F, G\}_1 \circ \varphi, \quad \text{for all } F, G \in C^\infty(M_2) \quad (14)$$

Theorem 2 Consider the port-controlled Hamiltonian system (7), (8). Assume that g_j satisfy (11). Denote its observation space by O , and its strong accessibility algebra by C_o . Then in the obvious notation

$$X_O = [X_H, C_o] \quad (15)$$

In case the structure matrix $J(x)$ has maximal rank this implies that locally observable port-controlled Hamiltonian systems are necessarily strongly accessible.

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Figure 1

