The Sticky Geometry of the Cosmic Web

Hidding, Johan; Weygaert, Rien van de; Vegter, Gert; Jones, Bernard J.T.; Teillaud, Monique

Published in:
EPRINTS-BOOK-TITLE

Publication date:
2012

The Sticky Geometry of the Cosmic Web

Johan Hidding
Kapteyn Institute,
Groningen University
Groningen, the Netherlands
hidding@astro.rug.nl

Rien van de Weygaert
Kapteyn Institute,
Groningen University
Groningen, the Netherlands
weygaert@astro.rug.nl

Gert Vegter
Johan Bernoulli Institute
Groningen University
Groningen, the Netherlands
g.vegter@rug.nl

Bernard J.T. Jones
Kapteyn Institute,
Groningen University
Groningen, the Netherlands
bernard@astrag.demon.co.uk

Monique Teillaud
INRIA
Sophia Antipolis, France
monique.teillaud@inria.fr

ABSTRACT

In this video we highlight the application of Computational Geometry to our understanding of the formation and dynamics of the Cosmic Web. The emergence of this intricate and pervasive weblike structure of the Universe on Megaparsec[1] scales can be approximated by a well-known equation from fluid mechanics, the Burgers' equation. The solution to this equation can be obtained from a geometrical formalism. We have extended and improved this method by invoking weighted Delaunay and Voronoi tessellations. The duality between these tessellations finds a remarkable and profound reflection in the description of physical systems in Eulerian and Lagrangian terms.

The resulting Adhesion formalism provides deep insight into the dynamics and topology of the Cosmic Web. It uncovers a direct connection between the conditions in the very early Universe and the complex spatial patterns that emerged out of these under the influence of gravity.

Categories and Subject Descriptors

J.2 [**Physical Sciences and Engineering**]: Astronomy; F.2.2 [**Analysis of Algorithms and Problem Complexity**]: Nonnumerical Algorithms and Problems—*Geometrical problems and computation*

Keywords

Cosmology, Large Scale Structure of the Universe, Numerical Methods, Voronoi Diagram, Delaunay Triangulation

1. THE COSMIC WEB

The Cosmic Web is the fundamental spatial organization of matter on scales of a few up to a hundred Megaparsec. Galaxies, intergalactic gas and dark matter exist in a wispy weblike arrangement of dense compact clusters, elongated filaments, and sheetlike walls, amidst large near-empty void regions. The filaments are the transport channels along which matter and galaxies flow into massive high-density cluster located at the nodes of the web. The weblike network is shaped by the tidal force field accompanying the inhomogeneous matter distribution.

Structure in the Universe has risen out of tiny primordial (Gaussian) density and velocity perturbations by means of gravitational instability [8, 11]. The large-scale anisotropic force field induces anisotropic gravitational collapse, resulting in the emergence of elongated or flattened matter configurations. According to Zeldovich, the collapse of a primordial cloud (dark) matter passes through successive stages, first assuming a flattened sheetlike configuration as it collapses along its shortest axis. This is followed by a rapid evolution towards an elongated filament as the medium axis collapses and, if collapse continues along the longest axis, may ultimately produce a dense and compact cluster or halo [9]. The hierarchical setting of these processes, occurring simultaneously over a wide range of scales complicates the picture considerably.

2. ZELDOVICH & ADHESION MODELS

The simplest model that describes the emergence of structure and complex patterns in the Universe is the Zeldovich Approximation (ZA) [11, 9]. In essence, it describes a ballistic flow, driven by a constant (gravitational) potential. The resulting Eulerian position $x(t)$ at some cosmic epoch t is specified by the expression,

$$x(t) = q + D(t)\ u_0(q),$$

where q is the initial "Lagrangian" position of a particle, $D(t)$ the time-dependent structure growth factor and $u_0 = -\boldsymbol{\nabla}_q \Phi_0$ its velocity. The nature of this approximation may be appreciated by the corresponding source-free equation of motion,

$$\frac{\partial u}{\partial D} + (u \cdot \boldsymbol{\nabla}_x)u = 0.$$

The use of ZA is ubiquitous in cosmology. One major application is its key role in setting up initial conditions in cosmological N-body simulations. Of importance here is its nonlinear extension in terms of Adhesion Model.

[1]The main measure of length in astronomy is the parsec. Technically a parsec is the distance at which we would see the distance Earth-Sun at an angle of 1 arcsec on the sky. One Megaparsec is 3.26 million light-years; this is approximately the distance between our Milkyway and the nearest other galaxy: Andromeda.

SCG'12, June 17–20, 2012, Chapel Hill, North Carolina, USA.
ACM 978-1-4503-1299-8/12/06.

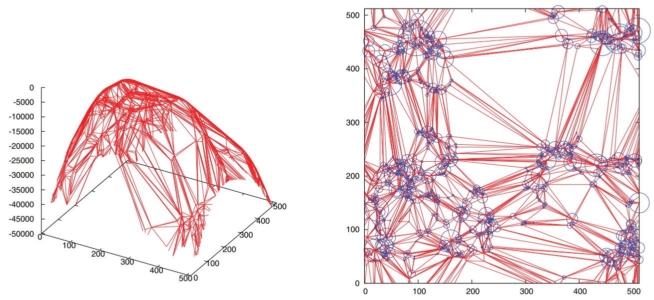

Figure 1: Convex hull of the modified potential and the resulting weighted Delaunay triangulation.

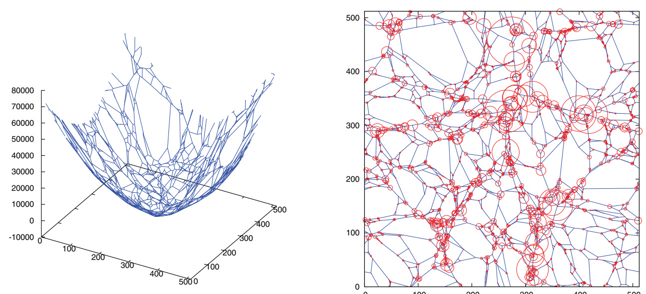

Figure 2: Convex conjugate of the modified potential and the resulting weighted Voronoi diagram.

The ZA breaks down as soon as self gravity of the forming structures becomes important. To 'simulate' the effects of self gravity, Gurbatov et al. [2, 3] included an artificial viscosity. This results in the Burgers' equation,

$$\frac{\partial u}{\partial D} + (u \cdot \boldsymbol{\nabla}_x)u = \nu \boldsymbol{\nabla}_x^2 u,$$

a well known PDE from fluid mechanics. This equation has an exact analytical solution [6],

$$u(x, D) = \frac{\int dq \; \frac{x-q}{D} \; \exp[G/(2\nu)]}{\int dq \exp[G/(2\nu)]},$$

where $G(x, q, D) = \Phi_0(q) - (x - q)^2/(2D)$. In the limit of $\nu \to 0$, the solution is

$$\Phi(x, D) = \max_q \left[\Phi_0(q) - \frac{(x-q)^2}{2D} \right].$$

This leads to a geometric interpretation of the Adhesion Model. The solution follows from the evaluation of the convex hull of the velocity potential modified by a quadratic term [10]. We found that the solution can also be found by computing the weighted Voronoi diagram [7] of a mesh weighted with the velocity potential. It is of special importance that this computation is done on a periodic space. This is essential to treat boundaries correctly.

3. ADHESION & TESSELLATIONS

The duality between the weighted Voronoi diagram and the weighted Delaunay triangulation has a deep connection with the concepts of Eulerian and Lagrangian coordinates in fluid mechanics. The Voronoi diagram represents volume. A vertex of the Delaunay triangulation represents a mass-element. Its dual Voronoi cell is the volume this mass-element occupies. As time moves on, the particle is expanding in a struggle for dominance over its neighbours, a struggle that is lost once the vertex becomes redundant (in the sense that it no longer contributes to the Delaunay triangulation). Physically the particle is then trapped in a larger structure. In general particles first stream into walls (objects of co-dimension 1), moving down into objects of increasing co-dimension. [4, 5].

4. THIS VIDEO

We use the Computational Geometry Algorithms Library [1] for our key geometric computations: weighted Delaunay triangulations in both 2D and 3D (at this point, the periodic triangulation package of CGAL only offers 3D periodic non-weighted Delaunay triangulations).

This video takes us from the initial conditions, through both the Zeldovich and Adhesion models, to a display of how the topology of the Cosmic Web depends directly on the starting point.

5. REFERENCES

[1] CGAL, *Computational Geometry Algorithms Library* http://www.cgal.org.

[2] S. N. Gurbatov and A. I. Saichev. Probability distribution and spectra of potential hydrodynamic turbulence. *Radiophysics and Quantum Electronics*, 27:303–313, Apr. 1984.

[3] S. N. Gurbatov, A. I. Saichev, and S. F. Shandarin. The large-scale structure of the universe in the frame of the model equation of non-linear diffusion. *MNRAS*, 236:385–402, Jan. 1989.

[4] J. Hidding. Adhesion: a sticky way of understanding large scale structure. Master's thesis, Groningen University, 2010.

[5] J. Hidding, R. van de Weygaert, G. Vegter, and B. Jones. The geometry of the adhesion generated web (to be subm.). *MNRAS*, 2012.

[6] E. Hopf. The partial differential equation $u_t + uu_x = \mu_{xx}$. *Communications on Pure and Applied Mathematics*, 3(3):201–230, 1950.

[7] A. Okabe, B. Boots, and K. Sugihara. *Spatial tessellations: concepts and applications of Voronoi diagrams*. John Wiley & Sons, Inc., New York, NY, USA, 2000.

[8] P. J. E. Peebles. *Large-Scale Structure of the Universe*. Princeton University Press, 1980.

[9] S. F. Shandarin and Y. B. Zeldovich. The large-scale structure of the universe: Turbulence, intermittency, structures in a self-gravitating medium. *Reviews of Modern Physics*, 61:185–220, Apr. 1989.

[10] M. Vergassola, B. Dubrulle, U. Frisch, and A. Noullez. Burgers' equation, Devil's staircases and the mass distribution for large-scale structures. *Astronomy & Astrophysics*, 289:325–356, Sept. 1994.

[11] Y. B. Zeldovich. Gravitational instability: An approximate theory for large density perturbations. *Astronomy & Astrophysics*, 5:84–89, Mar. 1970.