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Smedinga, Rein

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The reflection operator in discrete event systems

Rein Smedinga
department of computing science
University of Groningen
p.o.box 800
9700 AV Groningen, the Netherlands
tel. +31 50 633937
E-mail: rein@cs.rug.nl

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Abstract

THE reflection operator [T.V91] can be defined on discrete event systems to yield a structure outside the normal scope of such systems, but nevertheless very useful in design problems.

Here we define the operator for a discrete event system (DES), being a generalization of trace structures [Sne85]. We construct controllers using the reflection operator for which we, temporarily, go beyond the normal scope of a DES. These controllers can be computed effectively, using corresponding (finite) graphs.

Control of a DES

IN this chapter we define a control problem for discrete event systems and give a solution using the reflection operator.

1.1 Introduction

We define a discrete event system (DES) to be a triple [Sme90]

$$P = \langle \mathbf{a}P, \mathbf{b}P, \mathbf{t}P \rangle$$

with:

$$\begin{aligned} \mathbf{a}P & \quad \text{the alphabet (some finite set of symbols, the events)} \\ \mathbf{b}P & \subseteq (\mathbf{a}P)^* \quad \text{the behaviour set} \\ \mathbf{t}P & \subseteq (\mathbf{a}P)^* \quad \text{the task set} \end{aligned}$$

We assume the behaviour set of a DES P to be prefix-closed:

$$\mathbf{b}P = \mathbf{pref}(\mathbf{b}P)$$

(if some string of events is a behaviour of P then so should be all prefixes of that string). The task set denotes the completed behaviour of P . Of course, each completed task should also be a behaviour:

$$\mathbf{t}P \subseteq \mathbf{b}P$$

For any $x \in \mathbf{t}P$ we assume that, after x , P may stop without performing another event, while after $x \in \mathbf{b}P \setminus \mathbf{t}P$ the system will eventually perform another event or it deadlocks.

Two DESs will be given a special name:

$$\begin{aligned} \mathbf{empty}(A) & = \langle A, \emptyset, \emptyset \rangle \\ \mathbf{skip}(A) & = \langle A, \{\epsilon\}, \{\epsilon\} \rangle \end{aligned}$$

We will also need systems P in which the properties $\mathbf{b}P = \mathbf{pref}(\mathbf{b}P)$ and $\mathbf{t}P \subseteq \mathbf{b}P$ are not met. We call these systems generalized discrete event systems (GDESs). Such a system may have for example a behaviour set that is by no means prefix-closed or a task that is no behaviour! A GDES goes beyond our scope of a discrete event system. Nevertheless, it will play a crucial role in the remainder of this paper.

For GDESs P and R we define the interaction by [Sme90]:

$$\begin{aligned} &\langle \mathbf{a}P \cup \mathbf{a}R, \\ &\{x : x \in (\mathbf{a}P \cup \mathbf{a}R)^* \wedge x[\mathbf{a}P \in \mathbf{b}P \wedge x[\mathbf{a}R \in \mathbf{b}R : x]\}, \\ &\{x : x \in (\mathbf{a}P \cup \mathbf{a}R)^* \wedge x[\mathbf{a}P \in \mathbf{t}P \wedge x[\mathbf{a}R \in \mathbf{t}R : x]\} \end{aligned}$$

where \lceil stands for alphabet restriction. The interaction of P and R will be denoted by $P \parallel R$ (pronounce “ P parallel R ”). It can be viewed as a simultaneous weaving of the trace structures $\langle \mathbf{a}P, \mathbf{b}P \rangle$ with $\langle \mathbf{a}R, \mathbf{b}R \rangle$ and $\langle \mathbf{a}P, \mathbf{t}P \rangle$ with $\langle \mathbf{a}R, \mathbf{t}R \rangle$ [Sne85]. Because the weaving of prefix-closed structures is again prefix-closed and weaving is \subseteq -monotonic, we immediately have the property that the interaction of two DESs is again a DES.

We assume the following partial ordering on GDESs:

$$P \subseteq R \equiv \mathbf{a}P = \mathbf{a}R \wedge \mathbf{b}P \subseteq \mathbf{b}R \wedge \mathbf{t}P \subseteq \mathbf{t}R$$

We say P is a *subsystem* of R . Given some alphabet A we can find a greatest system, namely $\langle A, A^*, A^* \rangle$ and a smallest one, namely $\mathbf{empty}(A)$. Moreover, each set of systems P_i with equal alphabet has a greatest upper bound, namely the union of all those systems: $\langle \mathbf{a}P, \bigcup_i \mathbf{b}P_i, \bigcup_i \mathbf{t}P_i \rangle$.

For interactions of systems the common events can be seen as internal events. Such events need no longer be visible outside the interaction. Therefore, we introduce a second interaction operator that deletes the common events:

$$P \parallel\!\!\! \parallel R = (P \parallel R) \lceil (\mathbf{a}P \div \mathbf{a}R)$$

The operator $\parallel\!\!\! \parallel$ (pronounce “blend”) is like the blend in trace theory [Sne85]. It is monotonic with respect to \subseteq and associative if no event occurs in more than two of the alphabets.

1.2 The reflection operator

From [T.V90] and [T.V91] we have the following definition of the reflection of some GDES:

Definition 1.1 *The reflection of a GDES P is defined by*

$$\sim P = \langle \mathbf{a}P, (\mathbf{a}P)^* \setminus \mathbf{b}P, (\mathbf{a}P)^* \setminus \mathbf{t}P \rangle$$

□

Notice that, if P is a DES, $\sim P$ is not.¹ This is why we need GDESs as well.

Property 1.2

- (a) $P \subseteq R \equiv \sim R \subseteq \sim P$
- (b) $\sim \sim P = P$
- (c) $\sim \mathbf{skip}(\emptyset) = \mathbf{empty}(\emptyset)$
- (d) $P \parallel \sim P = \mathbf{empty}(\mathbf{a}P)$
- (e) $P \parallel\!\!\! \parallel \sim P = \mathbf{empty}(\emptyset)$

□

¹Except for $\mathbf{empty}(\emptyset)$ and $\mathbf{skip}(\emptyset)$, see property 1.2.

1.3 A control problem

Assume systems P , L_{min} , and L_{max} are given with $L_{min} \subseteq L_{max}$. Our control problem is finding a system R such that

$$L_{min} \subseteq P \parallel R \subseteq L_{max}$$

In this formulation L_{min} and L_{max} describe minimal and maximal wanted behaviours of the interaction. Mostly, L_{min} describes the minimal acceptable behaviour and L_{max} the legal or admissible behaviour. Sometimes (see [LL91]), L_{min} is used to denote the desired behaviour and L_{max} to denote the tolerated behaviour in case the desired behaviour cannot be reached. Notice that $\mathbf{a}L_{min} = \mathbf{a}L_{max}$ and R should be such that $\mathbf{a}R = \mathbf{a}P \div \mathbf{a}L_{min}$.

Earlier versions of this control problem (formulated using trace structures instead of DESs) can be found in [Sme89]. The control problem can also be viewed as a design problem: P is a first design guess, R is needed to complete the design.

We claim that the following GDES leads to a solution of the control problem.

$$F(P, L) = \sim(P \parallel \sim L)$$

First, we need two properties concerning the reflection.

Property 1.3

$$P \subseteq R \equiv P \parallel \sim R = \mathbf{empty}(\emptyset)$$

proof: [T.V90]

$$\begin{aligned} & P \subseteq R \\ \equiv & \text{ [definition of } \subseteq \text{]} \\ & \mathbf{a}P = \mathbf{a}R \wedge \mathbf{b}P \subseteq \mathbf{b}R \wedge \mathbf{t}P \subseteq \mathbf{t}R \\ \equiv & \text{ [set theory]} \\ & \mathbf{a}P = \mathbf{a}R \wedge \mathbf{b}P \cap ((\mathbf{a}R)^* \setminus \mathbf{b}R) = \emptyset \wedge \mathbf{t}P \cap ((\mathbf{a}R)^* \setminus \mathbf{t}R) = \emptyset \\ \equiv & \text{ [definition of } \sim \text{]} \\ & \mathbf{a}P = \mathbf{a}(\sim R) \wedge \mathbf{b}P \cap \mathbf{b}(\sim R) = \emptyset \wedge \mathbf{t}P \cap \mathbf{t}(\sim R) = \emptyset \\ \equiv & \text{ [definition of } \parallel \text{]} \\ & P \parallel \sim R = \langle \emptyset, \emptyset, \emptyset \rangle \end{aligned}$$

□

Property 1.4

$$P \parallel R \subseteq S \equiv P \subseteq F(R, S)$$

proof: [T.V90]

$$\begin{aligned} & P \parallel R \subseteq S \\ \equiv & \text{ [property 1.3 and definition of } \subseteq \text{]} \\ & (P \parallel R) \parallel \sim S = \mathbf{empty}(\emptyset) \wedge \mathbf{a}P \div \mathbf{a}R = \mathbf{a}S \\ \equiv & \text{ [} \mathbf{a}P \cap \mathbf{a}R \cap \mathbf{a}S = \emptyset \Rightarrow \parallel \text{ is associative]} \\ & P \parallel (R \parallel \sim S) = \mathbf{empty}(\emptyset) \wedge \mathbf{a}P \div \mathbf{a}R = \mathbf{a}S \\ \equiv & \text{ [property 1.2 (b) and set theory]} \\ & P \parallel \sim \sim (R \parallel \sim S) = \mathbf{empty}(\emptyset) \wedge \mathbf{a}P = \mathbf{a}R \div \mathbf{a}S \\ \equiv & \text{ [property 1.3 and definition of } \subseteq \text{]} \\ & P \subseteq \sim (R \parallel \sim S) \end{aligned}$$

