Black-hole-wave duality in string theory

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Black-hole–wave duality in string theory

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Extreme four-dimensional dilaton black holes embedded into ten-dimensional geometry are shown to be dual to the gravitational waves in string theory. The corresponding gravitational waves are the generalization of *pp*-fronted waves, called supersymmetric string waves. They are given by the Brinkmann metric and the two-form field, without a dilaton. The nondiagonal part of the metric of the dual partner of the wave together with the two-form field correspond to the vector field in the four-dimensional geometry of the charged extreme black holes.

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In this paper we continue our investigation of black holes and gravitational waves. From the point of view of general relativity those are very different geometries. However, string theory brings in a completely different concept of "equivalent" background geometries. It was understood some time ago [1] that the *pp* waves are dual to the fundamental strings [2]. The corresponding duality transformation, which is known as the σ-model duality transformation [3], has a very particular property: it changes the value of the dilaton field $e^{2\phi}$ by the g_{xx} component of the metric, where x is some direction on which all fields are independent. In our recent paper [4] we have established an analogous dual relation between more general solutions of the effective equations of the critical ($d = 10$) superstring theories. The purpose of this paper is to show that some solutions, which can be obtained by dual rotation from a particular case of supersymmetric string waves (SSW's) [5], are identified as supersymmetric extreme charged dilaton black holes upon Kaluza-Klein dimensional reduction to $d = 4$.

The first example known to us of a deep relation between gravitational waves and black-hole-type solutions was given by Gibbons [6]. He has observed that the five-dimensional *pp* waves upon Kaluza-Klein dimensional reduction to $d = 4$ are equivalent to a singular limit of the electrically charged black hole. Those black holes have the scalar field $g_{55} = \sigma$ coupling to the vector field of the form $e^{2\sqrt{3}\sigma}F^2$. The electric solutions are related via an electric-magnetic duality to monopoles. Thus this example has shown the dual relation between gravitational waves and monopoles.

We will use the σ-model duality of string theory and relate solutions of four-dimensional and ten-dimensional effective actions of string theory. We will limit ourselves by keeping only one scalar field, the fundamental dilaton. The method that we develop here may give many other interesting relations for the class of solutions, which will include more fields of string theory.

We consider the zero slope limit of the effective string action. This limit corresponds to ten-dimensional $N = 1$ supergravity. The Yang-Mills multiplet will appear in the first order of α' string corrections. The SSW's [5] in $d = 10$ are given by the Brinkmann metric [7] and the two-form

$$ds^2 = 2d\tilde{u}d\tilde{v} + 2A_M d\tilde{x}^M d\tilde{u} - \sum_{i=1}^{i=8} d\tilde{x}^i d\tilde{x}^i \ ,$$
$$B = 2A_M d\tilde{x}^M \wedge d\tilde{u} \ , \qquad A_v = 0 \ , \tag{1}$$

where $i = 1, \ldots, 8$, $M = 0, 1, \ldots, 8, 9$, and we are using the following notation for the ten-dimensional coordinates $x^M = \{\tilde{u}, \tilde{v}, \tilde{x}^i\}$. We have put the tilde over the ten-dimensional coordinates, since we will have to compare the original ten-dimensional configuration with the four-dimensional one, embedded into the ten-dimensional space. A rather nontrivial identification of coordinates describing these solutions will be required later.

The equations that $A_u(\tilde{x}^i)$ and $A_i(\tilde{x}^j)$ have to satisfy are

$$\triangle A_u = 0 \ , \qquad \triangle \partial^{[i} A^{j]} = 0 \ , \tag{2}$$

where the Laplacian is taken over the transverse directions only.

σ-model duality transformation [3] defines the changes in the metric, two-form field $B_{\mu\nu}$ and in the dilaton field $e^{2\phi}$:

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$$g'_{xx} = 1/g_{xx}\,, \qquad g'_{x\alpha} = B_{x\alpha}/g_{xx}\,,$$
$$g'_{\alpha\beta} = g_{\alpha\beta} - (g_{x\alpha}g_{x\beta} - B_{x\alpha}B_{x\beta})/g_{xx}\,,$$
$$B'_{x\alpha} = g_{x\alpha}/g_{xx}\,, \qquad B'_{\alpha\beta} = B_{\alpha\beta} + 2g_{x[\alpha}B_{\beta]x}/g_{xx}\,,$$
$$\phi' = \phi - \tfrac{1}{2}\ln|g_{xx}|\,. \qquad (3)$$

This transformation is defined for configurations with a non-null Killing vector in the x direction. String theory considers such configurations as equivalent under the condition that the x direction is compact.

A straightforward application of the σ-model duality transformations given in (3) on the SSW solution given in Eq. (1) leads to the following new supersymmetric solution of the zero slope limit equations of motion:

$$ds^2 = 2e^{2\phi}\{d\tilde{u}d\tilde{v} + A_i d\tilde{u}d\tilde{x}^i\} - \sum_{i=1}^{i=8} d\tilde{x}^i d\tilde{x}^i\,,$$
$$B = -2e^{2\phi}\{A_u d\tilde{u}\wedge d\tilde{v} + A_i d\tilde{u}\wedge d\tilde{x}^i\}\,,$$
$$e^{-2\phi} = 1 - A_u\,, \qquad (4)$$

where as before, the functions $A_M = \{A_u = A_u(\tilde{x}^j), A_v = 0, A_i = A_i(\tilde{x}^j)\}$ satisfy Eqs. (2). We called this solution generalized fundamental strings [4], since at that time we had in mind only a subclass of these solutions, which depend on the eight-dimensional transverse coordinates and have the interpretation of a macroscopic string. However, if we assume that the functions A_M do not depend on all eight transverse coordinates, but only on some of them, the name given to this class of solutions is not appropriate anymore. To avoid possible confusion, we would call the generic solutions given in Eq. (4) the dual partner of the SSW.[1]

We can make the following particular choice of the vector function A_M. First of all these functions will depend only on three of the transverse coordinates, $\tilde{x}^1, \tilde{x}^2, \tilde{x}^3$, which will eventually correspond to our three-dimensional space. Second, we choose one of A_i, e.g., A_4 to be related to A_u:

$$A_u = -\frac{\mu}{\rho}, \qquad A_4 = \xi A_u\,,$$

$$A_1 = A_2 = A_3 = A_5 = \cdots = A_8 = 0 \qquad (5)$$

where $\rho^2 = \sum_{i=1}^{i=3}\tilde{x}^i\tilde{x}^i \equiv \mathbf{x}^2$ and μ is a constant. We will specify the constant ξ later. Note that Eqs. (2) are solved outside[2] $\rho = 0$. We get

$$ds^2 = 2e^{2\phi}\{d\tilde{u}d\tilde{v} + \xi(1 - e^{-2\phi})d\tilde{x}^4 d\tilde{u}\} - \sum_{i=1}^{i=8} d\tilde{x}^i d\tilde{x}^i\,,$$
$$B = -2e^{2\phi}(1 - e^{-2\phi})\{d\tilde{u}\wedge d\tilde{v} + \xi d\tilde{u}\wedge d\tilde{x}^4\}\,,$$
$$e^{-2\phi} = 1 + \frac{\mu}{\rho}\,. \qquad (6)$$

We perform the coordinate change

$$\hat{x} = \tilde{x}^4 + \xi\tilde{u}\,, \qquad \hat{v} = \tilde{v} + \xi\tilde{x}^4\,. \qquad (7)$$

We also shift B on a constant value, since equations of motion depend on $H = dB$ only.

The dual wave solution (6) takes the form

$$ds^2 = 2e^{2\phi}d\tilde{u}d\hat{v} + \xi^2 d\tilde{u}^2 - d\hat{x}^2 - \sum_{i=1}^{i=3} d\tilde{x}^i d\tilde{x}^i - \sum_{i=5}^{i=8} d\tilde{x}^i d\tilde{x}^i\,,$$
$$B = -2e^{2\phi}d\tilde{u}\wedge d\hat{v}\,,$$
$$e^{-2\phi} = 1 + \frac{\mu}{\rho}\,. \qquad (8)$$

When $\xi^2 = -1$, we have

$$ds^2 = 2e^{2\phi}d\hat{v}d\tilde{u} - d\tilde{u}^2 - d\hat{x}^2 - \sum_{i=1}^{i=3} d\tilde{x}^i d\tilde{x}^i - \sum_{i=5}^{i=8} d\tilde{x}^i d\tilde{x}^i\,,$$
$$B = -2e^{2\phi}d\tilde{u}\wedge d\hat{v}\,,$$
$$e^{-2\phi} = 1 + \frac{\mu}{\rho}\,. \qquad (9)$$

We can identify this particular dual partner of the SSW solution with the uplifted dilaton black hole if we make the following identification of coordinates:

$$t = \hat{v} = \tilde{v} + \xi\tilde{x}^4\,,$$
$$x^4 = \tilde{u}\,,$$
$$x^9 = \hat{x} = \tilde{x}^4 + \xi\tilde{u}\,,$$
$$x^{1,2,3,5,\ldots,8} = \tilde{x}^{1,2,3,5,\ldots,8}\,. \qquad (10)$$

Our dual wave becomes

$$ds^2 = 2e^{2\phi}dt dx^4 - \sum_4^9 dx^i dx^i - d\mathbf{x}^2\,,$$
$$B = -2e^{2\phi}dx^4\wedge dt\,,$$
$$e^{-2\phi} = 1 + \frac{\mu}{\rho}\,. \qquad (11)$$

This is an extreme electrically charged four-dimensional black hole, which is embedded into ten-dimensional geometry in stringy frame, as we will explain in the next section.

The embedding of the four-dimensional bosonic solutions of the effective superstring action into ten-dimensional geometry is not unique, in general.[3] There are different ways to identify the vector field of the charged black hole in 4 with the nondiagonal com-

[1]Indeed, we are going to show that the dual partner of the SSW corresponds, in particular, to the lifted black holes, when the functions A_M depend only on the coordinates of the three-dimensional space. Therefore, we want to stress that the dual partner of the SSW includes more general configurations than strings.

[2]In order to solve Eqs. (2) everywhere, it is understood that a source term at $\rho = 0$, representing an unknown object, perhaps a six-brane, has to be added to these equations. We hope that this point can be worked out in an analogy with the combined action for the macroscopic fundamental string, where the source term comes from the σ-model action; see Eqs. (3,1)–(3,3) in [2].

[3]We are grateful to E. Witten for attracting our attention to this problem.

ponent of the metric in the extra dimensions as well as with the two-form field. Also the identification of the four-dimensional dilaton with the fundamental ten-dimensional dilaton and/or with some components of the metric in the extra dimension is possible.

However, the identification of the four-dimensional solution with the ten-dimensional one becomes unique under the condition that the supersymmetric embedding for both solutions is identified. Dimensional reduction of $N = 1$ supergravity down to $d = 4$ has been studied by Chamseddine [8] in canonical geometry. We are working in stringy metric and also in slightly different notation. In a subsequent publication we will present a detailed derivation of the compactification of the bosonic part of the effective action of the ten-dimensional string theory, which is consistent with supersymmetry [9]. Here we are interested in the relation between the extreme charged dilaton black holes, which have unbroken supersymmetry [11] when embedded into $d = 4$, $N = 4$ supergravity[4] and the corresponding ten-dimensional supersymmetric configuration.

We start with the zero slope limit of the effective ten-dimensional superstring action. The bosonic part of the action is

$$S = \frac{1}{2}\int d^{10}x e^{-2\phi}\sqrt{-g}\,[-R + 4(\partial\phi)^2 - \tfrac{3}{4}H^2]\,, \tag{12}$$

where the ten-dimensional fields are the metric, the axion and the dilaton.

We want to make connection with the bosonic part of the $N = 4$, $d = 4$ action. In this particular case we are interested in compactifying six spacelike coordinates. All fields are assumed to be independent of six compactified dimensions. According to Chamseddine [8] dimensional reduction of $N = 1$, $d = 10$ supergravity to $d = 4$ gives $N = 4$ supergravity coupled to six matter multiplets. We are interested here only in dimensional reduction to $N = 4$ supergravity without matter multiplets.

Let us first reduce from $d = 10$ to $d = 5$ by trivial dimensional reduction, when we do not keep the nondiagonal components of the metric and two-form field. We denote the ten-dimensional fields by un upper index $^{(10)}$ and the 5-dimensional fields by a caret. The ten-dimensional indices are capital letters $M, N = 0, \ldots, 9$, the five-dimensional indices will carry a caret $\hat{\mu}, \hat{\nu} = 0, \ldots, 4$, and the compactified dimensions will be denoted by capital I's and J's, $I, J = 5, \ldots, 9$. We take the $d = 10$ fields to be related to the $d = 5$ ones by

$$\begin{aligned}
g^{(10)}_{\hat{\mu}\hat{\nu}} &= \hat{g}_{\hat{\mu}\hat{\nu}}\,,\\
g^{(10)}_{I\hat{\nu}} &= 0\,,\\
g^{(10)}_{IJ} &= \eta_{IJ} = -\delta_{IJ}\,,\\
B^{(10)}_{\hat{\mu}\hat{\nu}} &= \hat{B}_{\hat{\mu}\hat{\nu}}\,,\\
B^{(10)}_{I\hat{\nu}} &= 0\,,\\
B^{(10)}_{IJ} &= 0\,,\\
\phi^{(10)} &= \hat{\phi}\,.
\end{aligned} \tag{13}$$

We get

$$S = \frac{1}{2}\int d^5x e^{-2\hat{\phi}}\sqrt{-\hat{g}}[-\hat{R} + 4(\partial\hat{\phi})^2 - \tfrac{3}{4}\hat{H}^2]\,. \tag{14}$$

As a second step we reduce from $d = 5$ to $d = 4$, keeping the nondiagonal components of the metric and two-form field. Since we are interested also in supersymmetry, we will work with the five-beins at this stage. The four-dimensional indices do not carry a caret. We parametrize the five-beins as follows:

$$(\hat{e}_{\hat{\mu}}{}^{\hat{a}}) = \begin{pmatrix} e_\mu{}^a & A_\mu \\ 0 & 1 \end{pmatrix}, \qquad (\hat{e}_{\hat{a}}{}^{\hat{\mu}}) = \begin{pmatrix} e_a{}^\mu & -A_a \\ 0 & 1 \end{pmatrix}, \tag{15}$$

where $A_a = e_a{}^\mu A_\mu$. With this parametrization, the five-dimensional fields decompose as

$$\begin{aligned}
\hat{g}_{44} &= \hat{\eta}_{44} = -1\,,\\
\hat{g}_{4\mu} &= -A_\mu\,,\\
\hat{g}_{\mu\nu} &= g_{\mu\nu} - A_\mu A_\nu\,,\\
\hat{B}_{4\mu} &= B_\mu\,,\\
\hat{B}_{\mu\nu} &= B_{\mu\nu} + A_{[\mu}B_{\nu]}\,,\\
\hat{\phi} &= \phi\,,
\end{aligned} \tag{16}$$

where $\{g_{\mu\nu}, B_{\mu\nu}, \phi, A_\mu, B_\mu\}$ are the four-dimensional fields.

The four-dimensional action for the four-dimensional fields becomes

$$S = \frac{1}{2}\int d^4x e^{-2\phi}\sqrt{-g}[-R + 4(\partial\phi)^2 - \tfrac{3}{4}H^2 + \tfrac{1}{4}F^2(A) + \tfrac{1}{4}F^2(B)]\,, \tag{17}$$

where

$$\begin{aligned}
F_{\mu\nu}(A) &= 2\partial_{[\mu}A_{\nu]}\,,\\
F_{\mu\nu}(B) &= 2\partial_{[\mu}B_{\nu]}\,,\\
H_{\mu\nu\rho} &= \partial_{[\mu}B_{\nu\rho]} + \tfrac{1}{2}\{A_{[\mu}F_{\nu\rho]}(B) + B_{[\mu}F_{\nu\rho]}(A)\}\,.
\end{aligned} \tag{18}$$

Now, we study the dimensional reduction of the gravitino. We are specifically interested in the supersymmetry transformation rule of the gravitino in $d = 4$ supergravity without matter. This leads to the identification of the matter vector fields D_μ and the supergravity vector fields V_μ:

$$\begin{aligned}
D_\mu &= \frac{1}{2}(A_\mu - B_\mu)\,,\\
V_\mu &= \frac{1}{2}(A_\mu + B_\mu)\,,
\end{aligned} \tag{19}$$

respectively. Now we want to truncate the theory keeping only the supergravity vector field V_μ. We have then

$$V_\mu = A_\mu = B_\mu\,, \qquad D_\mu = 0\,. \tag{20}$$

[4]We do not know, at present, whether the embedding of these black holes into other theories, including the Abelian part of a Yang-Mills multiplet, will also correspond to some unbroken supersymmetries.

The truncated action is[5]

$$S = \frac{1}{2}\int d^4x e^{-2\phi}\sqrt{-g}[-R + 4(\partial\phi)^2 - \tfrac{3}{4}H^2 + \tfrac{1}{2}F^2(V)]\,, \tag{21}$$

where

$$F_{\mu\nu}(V) = 2\partial_{[\mu}V_{\nu]}\,,$$
$$H_{\mu\nu\rho} = \partial_{[\mu}B_{\nu\rho]} + V_{[\mu}F_{\nu\rho]}(V)\,. \tag{22}$$

The embedding of the four-dimensional fields in this action in $d = 10$ is

$$\begin{aligned} g^{(10)}_{\mu\nu} &= g_{\mu\nu} - V_\mu V_\nu\,,\\ g^{(10)}_{4\nu} &= -V_\nu\,,\\ g^{(10)}_{44} &= -1\,,\\ g^{(10)}_{IJ} &= \eta_{IJ} = -\delta_{IJ}\,,\\ B^{(10)}_{\mu\nu} &= B_{\mu\nu}\,,\\ B^{(10)}_{4\nu} &= V_\nu\,,\\ \phi^{(10)} &= \phi\,. \end{aligned} \tag{23}$$

These formulas can be used to uplift any U(1) four-dimensional field configurations, including dilaton and axion, to a ten-dimensional field configurations in a way consistent with supersymmetry.

The conclusion of this supersymmetric dimensional reduction is the following.

(i) The dilaton of the supersymmetric four-dimensional extreme black holes is identified as the fundamental dilaton of string theory (and not as one of the modulus fields).

(ii) Dimensional reduction of $d = 10$ supergravity to $d = 4$ gives $N = 4$ supergravity without six matter multiplets under condition that $g_{4\mu} = -B_{4\mu}$. Therefore the vector field of the four-dimensional configuration is actually a nondiagonal component of the metric in the extra dimension as well as the two-form field. This works in our case since we have, according to (11),

$$g^{(10)}_{4t} = -B^{(10)}_{4t} = -V_t = e^{2\phi}\,. \tag{24}$$

We will use the above formulas to uplift the dilaton black hole with one vector field. The electrically charged extreme $4d$ black hole is given by[6] [10]

$$\begin{aligned} ds^2_{\rm str} &= e^{4\phi}dt^2 - d\mathbf{x}^2\,,\\ V &= -e^{2\phi}dt\,,\\ B &= 0\,,\\ e^{-2\phi} &= 1 + \frac{2M}{\rho}\,. \end{aligned} \tag{25}$$

The uplifted configuration according to Eq. (23) is

$$\begin{aligned} ds^2 &= 2e^{2\phi}dt dx^4 - d\mathbf{x}^2 - (dx^4)^2 - dx^I dx^I\,,\\ B^{(10)} &\equiv B^{(10)}_{MN}dx^M \wedge dx^N = -2e^{2\phi}dx^4\wedge dt\,,\\ \phi^{(10)} &= \phi\,. \end{aligned} \tag{26}$$

Let us choose the parameter μ in the dual partner to the wave, given in Eq. (9) equal to the double mass of the black hole:

$$\mu = 2M\,. \tag{27}$$

This makes the uplifted black hole (26) identical to the dual partner to the wave, given in Eq. (9).

For a better understanding of the black-hole-wave relation it is useful to do the following. By adding and subtracting from the metric the term $e^{4\phi}dt^2$ we can rewrite the dual wave in $d = 10$, given in Eq. (11)

$$\begin{aligned} ds^2 &= e^{4\phi}dt^2 - d\mathbf{x}^2\\ &\quad -(dx^4 - e^{2\phi}dt)^2 - dx^I dx^I\,,\\ B &= -2e^{2\phi}dx^4\wedge dt\,,\\ e^{-2\phi} &= 1 + \frac{2M}{\rho}\,. \end{aligned} \tag{28}$$

Now it is easy to recognize in the first two terms in the metric the four-dimensional metric and in the third term the nondiagonal component of the ten-dimensional metric, which together with the nondiagonal component of the two-form plays the role of the vector field in the four-dimensional geometry.

The case $\xi^2 = -1$, which gives the four-dimensional black hole in Minkowski space with the signature $(1,3)$ times the compact six-dimensional space with the signature $(0,6)$ corresponds to a complex ten-dimensional wave in the space with the signature $(1,9)$:

$$\begin{aligned} ds^2 &= 2d\tilde{u}d\tilde{v} - \frac{4M}{\rho}\,d\tilde{u}(d\tilde{u} - i d\tilde{x}^4) - \sum_{i=1}^{i=8} d\tilde{x}^i d\tilde{x}^i\,,\\ B &= -i\,\frac{4M}{\rho}\,d\tilde{u}\wedge d\tilde{x}^4\,. \end{aligned} \tag{29}$$

By performing a rotation $i\tilde{x}^4 = \tilde{\tau}$ one can get

$$\begin{aligned} ds^2 &= 2d\tilde{u}d\tilde{v} - \frac{4M}{\rho}\,d\tilde{u}(d\tilde{u} - d\tilde{\tau}) + d\tilde{\tau}^2 - \sum_{i=1}^{i=7} d\tilde{x}^i d\tilde{x}^i\,,\\ B &= -\frac{4M}{\rho}\,d\tilde{u}\wedge d\tilde{\tau}\,. \end{aligned} \tag{30}$$

This makes the wave real but with the signature of the space $(2,8)$.

Thus we may conclude that string theory considers as dual partners the extreme $4d$ electrically charged dilaton black hole embedded into ten-dimensional geometry, as given in Eqs. (28) or (26), and the Brinkmann-type ten-dimensional wave (29) and (30).

If we choose the $\xi^2 = 1$ case we get the stringy equivalence between a Brinkmann-type ten-dimensional wave,

$$\begin{aligned} ds^2 &= 2d\tilde{u}d\tilde{v} - \frac{4M}{\rho}\,d\tilde{u}(d\tilde{u} - d\tilde{x}^4) - \sum_{i=1}^{i=8} d\tilde{x}^i d\tilde{x}^i\,,\\ B &= -\frac{4M}{\rho}\,d\tilde{u}\wedge d\tilde{x}^4\,, \end{aligned} \tag{31}$$

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[5]This action, which came from the ten-dimensional theory is slightly different from the corresponding four-dimensional action in our previous papers, e.g., in [11], due to the difference in notation. The detailed explanation of this difference will be given in [9].

[6]There is a difference of a $1/\sqrt{2}$ factor in the vector field with respect to the one given in [11].

and a lifted Euclidean four-dimensional electrically charged dilaton black hole with the signature $(0,4)$ and six-dimensional space with the signature $(1,5)$:

$$\begin{aligned} ds^2 &= -e^{4\phi}dt^2 - d\mathbf{x}^2 + (dx^4 + e^{2\phi}dt)^2 - dx^I dx^I \ , \\ B &= -2e^{2\phi}dx^4 \wedge dt \ , \\ e^{-2\phi} &= 1 + \frac{2M}{\rho} \ . \end{aligned} \tag{32}$$

With such a choice of signature, the gravitational wave does not have imaginary components. However, the fact that the metric as well as the two-form field of the gravitational wave in $d = 10$ have an imaginary component to be dual to the lifted black hole in Minkowski space is strange. Note that this is necessary only if one insists that the $d = 10$ space as well as the $d = 4$ space are both Minkowski spaces. One can avoid imaginary components by allowing the changes in the signature of the space-time when performing duality and dimensional reduction as explained above. Still this remains a puzzle.

Could we actually consider the dual relation between waves and lifted black holes as something more than pure algebraic curiosity? We believe that the answer to this question is "yes." The dual relation displayed above was established at the zero slope limit of the effective action of the superstring theory. The issue of α' corrections in string theory has been studied extensively for the waves [5], [4]. The *pp* waves have the best known properties of absence of such quantum corrections [12]. The SSW are known to have the special property of the absence of α' corrections under the condition that non-Abelian Yang-Mills fields are added to the configuration, which at the zero slope limit $\alpha' = 0$ consists only of the metric and the two-form [5]. It was explained in [4] that the importance of σ-model duality between supersymmetric configurations is in the fact that the structure of α' corrections is under control for the dual solution if it was under control for the original solution. In this way we have found that the nice properties of the *pp* waves [12] are carried over to the fundamental string solutions. The present investigation shows that the electrically charged extreme black hole embedded into ten-dimensional geometry may require to be supplemented by some non-Abelian Yang-Mills configuration to avoid the possible α' corrections. In this respect we would like to stress that the study of the properties of quantum corrections established via duality may become a powerful mechanism of the investigation of quantum theory despite the strange imaginary factors in the waves, which are dual partners of the uplifted black holes.

At the very minimal level one can consider the method developed above, which consists of stringy duality combined with Kaluza-Klein dimensional reduction, as a solution generating method. This method has the advantage of generating new supersymmetric solutions from the original ones. If we did not know that extreme four-dimensional black holes are supersymmetric, we would discover this via the supersymmetric properties of ten-dimensional gravitational waves. We hope to derive more general four-dimensional supersymmetric solutions starting from our ten-dimensional supersymmetric waves and to explore generic relation between supersymmetry and duality [9].

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