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PION EXCHANGE CURRENTS IN ELASTIC ELECTRON DEUTERON SCATTERING

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The effects of pion-exchange pari and recoil currents on the electromagnetic form factors of the deuteron are calculated. Both exchange currents give significant contributions to the charge form factor and, despite some cancellation, their net effect is appreciable for $q^2 \geqslant 10\ \mathrm{fm}^{-2}$.

There has been considerable interest in the question of meson exchange-current contributions to the electromagnetic (e.m.) form factors of the two- and three-nucleon systems following the recent observation that at large momentum transfer two-body (exchange) currents may give large, and in some cases dominant, contributions to the electron scattering cross section [1–5]. While it has always been realized that exchange-current contributions to magnetic form factors (or, more generally, magnetic multipole transition rates) may be large, it has usually been assumed that at low momentum transfer charge form factors (or electric multipole transition rates) are well-described by the single nucleon (impulse) approximation model. The reason for this assumption is the well-known Siegert theorem which implies that the low electric multipole transition rates are insensitive to meson exchange currents [6, 7].

Nevertheless, in a recent letter Kloet and Tjon [5] observed that the pair current process of fig. 1a gives a large contribution to the charge form factor of ^{3}He at fairly low momentum transfers. There are, however, other exchange currents of one pion range which cannot be dismissed *a priori* as small. Thus, we consider the effect on the deuteron e.m. form factor of the two exchange currents which involve only nucleons and single pions in the intermediate states. These two processes are represented as Feynman diagrams in fig. 1. The diagram in fig. 1a is the

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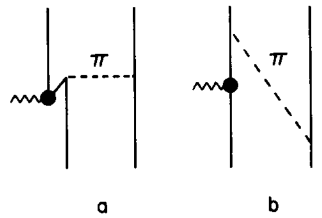


Fig. 1. One-pion-exchange currents in the deuteron. a) pair current, b) recoil current

"pair" current and the diagram in fig. 1b is the "recoil" current. (We use the nomenclature of ref. [8]). The Feynman diagram in fig. 1b is a time-ordered diagram. Both processes contribute to the charge form factors of the $A = 3$ nuclei as well. Although Kloet and Tjon considered only the pair current process, there is no reason to consider these processes separately. We find considerable cancellation between them in the deuteron, and such cancellation may well also occur in the $A = 3$ systems.

There are many other exchange current processes contributing to the deuteron e.m. form factors, but they all have shorter range and cannot be calculated unambiguously [3]. An estimate of two such processes ($\rho\pi$ and $\sigma\omega$) is given in ref. [3]. That estimate suggests that those processes are of little significance in the region $q^2 \lesssim 20\,\mathrm{fm}^{-2}$. In this region the charge form factor is expected to have a minimum, and it is therefore a region of interest.

The impulse approximation expressions for the deuteron e.m. form factors are well known and can be found, for example, in ref. [9]. We shall not list them here.

The pair current process in fig. 1a can be calculated either in pseudoscalar or pseudovector πN coupling theory. Only the leading term in the expression for the pair current operator, often referred to as the Kroll-Ruderman or "seagull" term, is the same in both theories but has isovector character and does not contribute to the deuteron e.m. form factors. We shall here use pseudoscalar coupling theory which is unambiguous in contrast to pseudovector theory, in which a form factor ambiguity exists [10]. The isoscalar part of the pair current operator as calculated with pseudoscalar coupling theory has the form:

$$j_0^{\mathrm{pair}} = \frac{g^2}{4m^3\omega^2} G_M^S(q^2)(\tau^1\cdot\tau^2\,\vec{\sigma}^1\cdot\vec{q}\;\vec{\sigma}^2\cdot\vec{\ell}), \qquad \vec{j}^{\,\mathrm{pair}} = \mathrm{i}\,\frac{g^2}{16m^4\omega^2} G_M^S(q^2)\,\tau^1\cdot\tau^2\,(\vec{\ell}\times\vec{q})(\vec{\sigma}^2\cdot\vec{\ell}). \tag{1}$$

Here g is the πN pseudoscalar coupling constant ($g^2/4\pi = 14.5$), m the nucleon mass, $\vec{q}$ the momentum of the virtual photon and $\vec{\ell}$ the momentum of the exchanged pion. G_M^S is the isoscalar nucleon magnetic form factor and $\omega = \sqrt{\ell^2 + \mu^2}$ with μ the pion mass.

In arriving at these expressions (1) we have consistently dropped terms of order $(q/2m)^2$ compared to 1 and neglected all non-local terms in the current operator. These expressions thus should not be used at very large momentum transfer but are probably adequate for $q^2 \lesssim 25\ \mathrm{fm}^{-2}$.

Using the expressions (1) we may derive the following expressions for the e.m. form factors due to the pair current‡.

$$F_C^{\mathrm{pair}}(q^2) = -\tfrac{1}{2}(g^2/4\pi)(\mu/m)\,G_M^S(q^2)\,(q/m)^2 I_1^P(q^2), \qquad F_Q^{\mathrm{pair}}(q^2) = 6(g^2/4\pi)(\mu/m)\,G_M^S(q^2) I_2^P(q^2)/m^2,$$
$$F_M^{\mathrm{pair}}(q^2) = \tfrac{1}{4}(g^2/4\pi)(\mu/m)^3\,G_M^S(q^2) I_3^P(q^2). \tag{2}$$

The structure functions I^P above are defined as

$$I_1^P = \frac{\mu}{q}\int_0^\infty \mathrm{d}r\,\bar{Y}_1(\mu r) j_1(gr/2)\,[u^2 + 4\sqrt{2}uw - w^2],$$

$$I_2^P = \frac{\mu}{q}\int_0^\infty \mathrm{d}r\,\bar{Y}_1(\mu r)\,\{j_1(gr/2)\,[u^2 - uw/\sqrt{2} + 5w^2/4] - 27(\mu/q) j_2(qr/2)\,w^2/2\mu r\},$$

$$I_3^P = \int \mathrm{d}r\,\{j_0(qr/2)\,[u^2 Y_0(\mu r) + \sqrt{2}uw\,Y_2(\mu r) + w^2(Y_0(\mu r) - Y_2(\mu r)/2)]$$
$$+ j_2(qr/2)\,[u^2 Y_2(\mu r) + \tfrac{1}{2}\sqrt{2}uw(Y_0(\mu r) + Y_2(\mu r)) - \tfrac{1}{4}w^2(Y_0(\mu r) - 3Y_2(\mu r))]\}. \tag{3}$$

Here $u(r)$ and $w(r)$ are the usual reduced wave functions for the S- and D-wave components of the deuteron. We

‡ The electric and magnetic form factors G_E and G_M appearing in the standard expression for the differential cross section are related to the form factors F by $G_E^2 = F_C^2 + q^4 F_Q^2/18$, $G_M = \sqrt{2/3}\,q\,F_M/2m$.

have used the notation [8]

$$Y_0(x) = e^{-x}/x, \quad \bar{Y}_1(x) = (1 + 1/x)\, Y_0(x)\,, \qquad Y_2(x) = (3/x^2 + 3/x + 1)\, Y_0(x). \tag{4}$$

Let us now turn to the question of the recoil current diagram. With pseudoscalar πN coupling theory the isoscalar part of the current operator associated with the diagram in fig. 1b has the form

$$J_0^{\rm rec} = \frac{g^2}{m^2\omega^3}\, G_E^S(q^2)\, \tau^1\cdot\tau^2\, (\vec{\sigma}^1\cdot\vec{\ell})(\vec{\sigma}^2\cdot\vec{\ell}), \qquad \vec{J}^{\rm rec} = i\, \frac{g^2}{16m^3\omega^3}\, G_M^S(q^2)\, \tau^1\cdot\tau^2\, (\vec{\ell}\times\vec{q})(\vec{\sigma}^2\cdot\vec{\ell}). \tag{5}$$

In arriving at (5) we have made approximations similar to those made in the derivation of the pair current operator. In (5) $G_M^S(q^2)$ is the isoscalar nucleon electric form factor.

Using (5) one may derive the following expressions for the contributions to the deuteron form factors due to recoil current:

$$F_C^{\rm rec}(q^2) = \tfrac{1}{4}(\mu/m)^2 G_E^S(q^2)(g^2/4\pi) I_1^R(q^2), \qquad F_Q^{\rm rec}(q^2) = 3(\mu/m)^3\, G_E^S(q^2)(g^2/4\pi) I_2^R(q^2)/q^2,$$
$$F_M^{\rm rec}(q^2) = \tfrac{1}{4}(\mu/m)^2\, G_M^S(q^2)(g^2/4\pi) I_3^R(q^2). \tag{6}$$

The structure functions I^R are defined as

$$I_1^R = \frac{2}{\pi}\int_0^\infty dr\, j_0(qr/2)\, [u^2\bar{K}_0(\mu r) + 4\sqrt{2}\,uwK_2(\mu r) + w^2(\bar{K}_0(\mu r) - 2K_2(\mu r))]\,,$$
$$I_2^R = \frac{2}{\pi}\int_0^\infty dr\, j_2(qr/2)\, [u^2K_2(\mu r) + \tfrac{1}{2}\sqrt{2}\,uw(\bar{K}_0(\mu r) - 2K_2(\mu r)) + w^2(6K_2(\mu r) - \bar{K}_0(\mu r))/4]\,,$$
$$I_3^R = \frac{2}{\pi}\int_0^\infty dr\, \{j_0(qr/2)\, [u^2\bar{K}_0(\mu r) + \sqrt{2}\,uw\,K_2(\mu r) + w^2(\bar{K}_0(\mu r) - K_2(\mu r)/2)]$$
$$+ j_2(qr/2)\, [u^2K_2(\mu r) + \tfrac{1}{2}\sqrt{2}\,uw(\bar{K}_0(\mu r) + K_2(\mu r)) - \tfrac{1}{4}w^2(\bar{K}_0(\mu r) - 3K_2(\mu r))]\} \tag{7}$$

Here we have introduced the notation

$$\bar{K}_0(x) = K_0(x) - K_1(x)/x. \tag{8}$$

The functions $K_\nu(x)$ are modified Bessel functions.

It is important to note that $F_c^{\rm rec}(0) \neq 0$ so that this recoil process leads to a patently spurious change in the deuteron charge. This difficulty is overcome by a proper renormalization of the deuteron wave function which allows for mesonic degrees of freedom [8]. Clearly, recoil and renormalization effects must be considered together. In a calculation of one-pion exchange current effects it is consistent to consider only the renormalization correction to the one-body current operator due to single-pion production and absorption [8]. The effect of the wave function renormalization is to add to the e.m. form factors a contribution

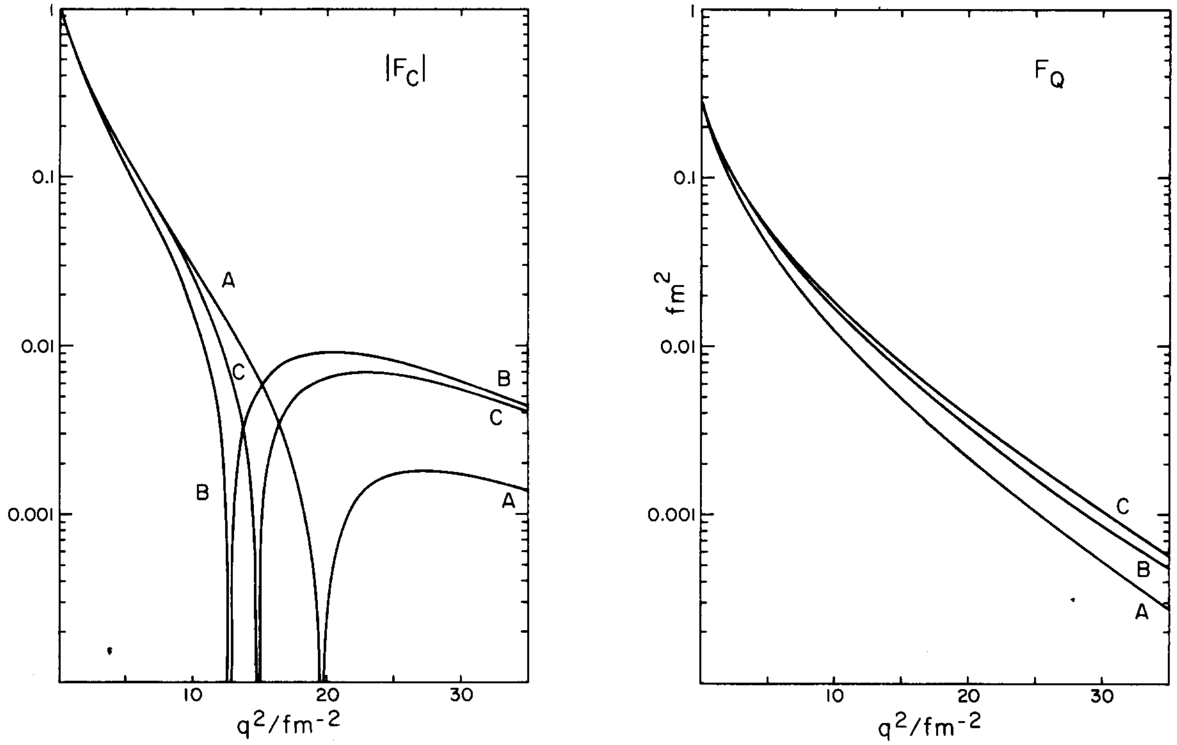
$$F_X^{\rm ren}(q^2) = (Z-1)\, F_X^{\rm imp}(q^2), \qquad X = C, Q, M. \tag{9}$$

Here $F_X^{\rm imp}$ is the relevant impulse approximation form factor and Z is the matrix element of the single pion renormalization operator, which has the expression

$$\tilde{Z} = 1 - \frac{g^2}{8m^2}\int \frac{d^3\ell}{(2\pi)^3}\, (\vec{\sigma}^1\cdot\vec{\ell})(\vec{\sigma}^2\cdot\vec{\ell})\, \frac{\exp(i\,\vec{\ell}\cdot\vec{r})}{\omega^3}\, \tau^1\cdot\tau^2. \tag{10}$$

Here $\omega = \sqrt{\ell^2 + \mu^2}$. For the deuteron the matrix element Z is

$$Z = 1 - \tfrac{1}{4}(g^2/4\pi)(\mu/m)^2 I_1^R(0). \tag{11}$$



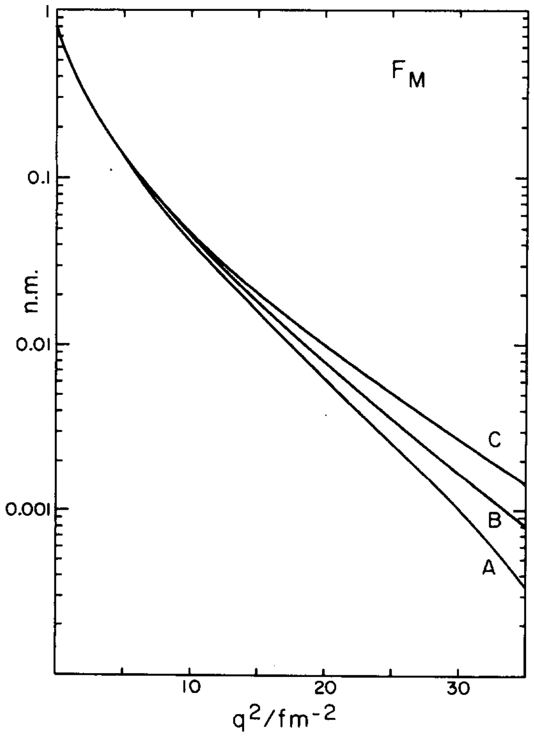


Fig. 2. The electromagnetic form factors of the deuteron. The curves A are the impulse approximation result, curves B the result when the pair current is included and curves C the result when both the pair and recoil + renormalization effects are taken into account.

It is easy to see that $F_c^{ren}(q^2)$ exactly cancels with $F_c^{rec}(q^2)$ at $q^2 = 0$ only. In view of this cancellation the neglect of terms of order $(q/2m)^2$ compared to 1 in the derivation of eqs. (5) may not be as well justified as in the pair current case.

With these results we have computed the deuteron e.m. form factors for the deuteron wave function of the Reid soft-core potential [11]. For the nucleon e.m. form factors G_E^S and G_M^S we used the five-parameter dipole fit of ref. [12]. The results are shown in figs. 2. The impulse approximation result is described by the curves A. Curves B show the result of including the pair current process only, and the curves C show the result when pair, recoil and renormalization effects are included. These exchange current effects modify the theoretical values for the static deuteron properties by a 0.17 fm^2 decrease in the deuteron mean square charge radius, a 0.018 fm^2 increase in the electric quadrupole moment, and a 0.05 n.m. decrease in the deuteron magnetic moment. The resulting value for the quadrupole moment, 0.289 fm^2, remains in essential agreement with the empirical value of 0.2875 fm^2 [13]. An explanation of the difference between the empirical value for the magnetic moment of 0.8574 n.m. and the present theoretical value of 0.8118 n.m. may require inclusion of other exchange current processes and effects of isobar admixtures in the deuteron wave function [15].

The results show that while the two exchange-current processes only moderately influence the quadrupole (F_Q) and magnetic form factors (F_M), their effect on the charge form factor (F_c) (and consequently tensor polarization [14]) is appreciable even for $q^2 < 20$ fm^{-2}. The effect of the pair current alone is to shift the minimum in the form factor down from ≈ 19 fm^{-2} to ≈ 13 fm^{-2}. This result is similar to that of Kloet and Tjon [5] for the charge form factor of ^{3}He. There is however some cancellation between the pair and recoil + renormalization currents so that the combined exchange current effect only shifts the minimum to ≈ 15 fm^{-2}. The combined exchange currents also increase the secondary maximum in F_c by more than a factor of 2.

Our conclusions are thus (1) there is considerable cancellation between the various exchange current processes of one pion range, and such processes should be considered simultaneously, (2) the pion exchange currents are at least as important at low momentum transfer as the ρ–π and σ–ω currents considered in ref. [3], and (3) the presence of pion exchange currents would complicate determination of the short-range part of the deuteron wave function from G_E and the tensor polarization as suggested in ref. [14]. In this regard we note that the wave function dependence of exchange contributions should be roughly comparable to that of the impulse approximation at roughly one-half of the value of momentum transfer under consideration (i.e., exchange current contributions are relatively insensitive to wave functions).

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