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Research Article

A Precoded OFDMA System with User Cooperation

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A new cooperative scheme for a two-user orthogonal frequency division multiple access (OFDMA) uplink communication scenario is proposed. Each user is equipped with one transmit/receive antenna. Before transmission, inter-block linear precoding is introduced to pairs of blocks. The cooperative transmission is implemented in cycles of three time slots. During each slot, a user transmits either his data, or a weighted mixture of his data and the data that he received in previous slots of the same cycle. The weights are obtained in an optimum fashion, so that a user that faces deep fading on certain subcarriers can benefit from the other user's channel, without taxing significantly the resources of that user. It is shown that the proposed scheme achieves the maximum available diversity for both users (full cooperation), or for the weak user (half cooperation) without increasing the number of antennas needed as compared to an energy-equivalent noncooperative OFDMA system that also uses inter-block precoding. Further, the proposed use of inter-block precoding allows one to exploit the cooperation induced diversity in 1.5 slots on the average; 2 slots would be needed if intra-block precoding was used instead.

1. Introduction

Multiuser Cooperation is a promising technology for improving the performance of wireless communication systems, as it has the potential to increase the data rate [1, 2], and achieve diversity order equal to the number of cooperating users [3]. Three types of cooperation have been used in the past, decode-and-forward (DF) [1, 4], amplify-and-forward (AF) [5], and coded cooperation [6]. In [4], a two-user cooperative system was considered and in that context it was shown that the AF approach performs better than the DF, with the performance gap closing as the SNR increases. Also in [4], it was shown that coded cooperation based on channel coding can in general outperform both AF and DF schemes at all SNR levels, while it is comparable to the noncooperative system at low SNR.

OFDM systems have gained popularity due to their ability to handle frequency selective fading. Various forms of cooperation in the context of OFDM systems have been considered. In [5], a hybrid forwarding scheme was proposed for cooperative relaying in OFDM-based networks that adaptively decides between AF, DF, or no relaying at all, based on the instantaneous SNR on each subcarrier. An OFDM cooperative scheme for multihop networks was proposed in [7], where in order to achieve full spatial diversity, relay selection is performed on a per-subcarrier basis instead of the entire block. Each subcarrier can determine the best relay independently at each hop, so that different subcarriers experience different paths. In [8] (Chapter 17), a general two-phase cooperative protocol for OFDM networks was studied, where in phase 1 each user transmits its own data and in phase 2 the relay decodes the source symbols that are not decoded successfully by the central node, according to feedback information sent by the central node. In order to resolve multiple users at the central code, the users can send their information in different time slots or utilize different sets of subcarriers in phase 1. It was shown in [8] that the performance of the cooperative protocol depends on the number of relays and relay selection. In [9], a multiuser OFDM network was considered where some users serve as AF relays by offering some of their subcarriers to other users. Optimal schemes of power control, subcarrier allocation, and relay selection were considered in the same paper. A DF cooperation strategy and resource-allocation algorithm for

two-user OFDMA systems was proposed in [10] and was shown to achieve the capacity region upper bound of two-user OFDMA systems.

It is well known that OFDM systems loose multipath diversity as each symbol is transmitted on one subcarrier only. Several ways have been proposed in the literature for introducing path diversity in OFDM systems. Suppose that the multipath channel is finite impulse response (FIR) with L taps. Maximum diversity gain, L, was achieved in [11, 12] via a linear receiver using redundant precoding, or oversampling at the receiver. In [13] it was shown that a single user OFDM system with nonredundant block precoding can achieve diversity gain up to L. The performance gain is exploitable using a Maximum Likelihood (ML) decoder. Reduced complexity decoding at the receiver is possible via subcarrier grouping [13], which may result in smaller than L diversity gains. Other nonredundant precoding techniques were also considered in [14–16]. A multirelay cooperative OFDM system with nonredundant precoding and AF relaying was investigated in [17]. Based on the expression of pairwise error probability (PEP), it was demonstrated that the maximum diversity order is the sum of the source-to-destination channel length and the length of the shortest channel among the relay links.

In this paper we propose a cooperative approach for a two-user OFDMA system that combines linear interblock precoding and user cooperation. The transmission occurs in cycles of three time slots each; two new precoded data blocks for each user are transmitted in each cycle. In the first slot, both users transmit their own data. In the two subsequent slots, each user transmits a weighted combination of the user's own precoded data and also data from the other user that were received in the previous slot. The weights are obtained as the solution of a constrained optimization problem that allows the user that faces a bad channel on certain subcarriers to benefit from the user that has a better channel, without taxing significantly the resources of that user. Two methods are proposed to implement this scheme: the full cooperation and the half cooperation. In the full-cooperation scheme, both users are involved in the cooperation. The base station (BS) recovers the transmitted symbols after it has collected data from both users in the three slots. In the half-cooperation scheme, only the strong user transmits cooperative information. We show that the proposed cooperative schemes combined with interblock precoding can achieve the maximum available diversity, that is, twice the length of the multipath channel. To achieve the same diversity order, a noncooperative OFDMA system that uses the same transmission energy per block pair and the same interblock precoding scheme would require at least two transmit antennas. Further, the proposed use of interblock precoding allows one to exploit the diversity induced by cooperation in 1.5 slots on the average; 2 slots would be needed if intrablock precoding was used instead.

1.1. Relation of Contribution to the Literature.
For most existing cooperative OFDM techniques [5, 7–10, 17], the users serving as relays transmit only the data of other users

during the cooperation phase. The main difference between the proposed approach and these techniques lies in the fact that each cooperating user transmits a linear combination of the user's own data and also data from the other user. Superposing user's own data and data from the other user can double the maximum diversity gain of each user.

In this paper, we propose to use interblock precoding for our proposed cooperation scheme. Inter-block precoding was previously applied to channel estimation for OFDM systems in [16] to exploit time diversity introduced by time varying channels. However, here, even if the channel is completely static, interblock precoding allows one to exploit the spatial diversity that is introduced by cooperation. In [17], intrablock precoding [13] was employed to achieve multipath diversity for multirelay cooperative OFDM system. The proposed use of interblock precoding allows one to exploit the diversity induced by cooperation in 1.5 slots on the average; 2 slots would be needed if intrablock precoding was used instead.

1.2. Paper Organization.
The paper is organized as follows. In Section 2 we describe the signal model of a multiuser OFDM system. In Section 3, we propose a full-cooperation scheme and a half-cooperation scheme for a two-user OFDMA system and provide diversity analysis. Further, we describe a modified ML decoder based on subcarrier grouping. We provide simulation results of two cooperative schemes in Section 4, and finally make some concluding remarks in Section 5.

1.3. Notation.
The small and capital letters in bold denote vectors and matrices. We denote the $N \times N$ identity matrix as $\mathbf{I}_N$ and all-zero matrix as $\mathbf{0}_N$. The statistical expectation of a random variable is denoted by $E\{\cdot\}$. The superscripts $(\cdot)^*$ and $(\cdot)^H$ denote the conjugation and Hermitian respectively. We use $\odot$ to denote element-wise multiplication.

2. Signal Model and Assumptions

Let us consider a two-user OFDMA system where users communicate with a BS. The users are assigned disjoint carriers. User 1 transmits over subcarriers in set ℓ_1 and receives over subcarriers in set ℓ_2, where $\ell_1 \bigcup \ell_2 = \{0, 1, \ldots, N - 1\}$ and $\ell_1 \bigcap \ell_2 = \{ \}$. User 2 transmits over subcarriers in set ℓ_2 and receives over subcarriers in set ℓ_1. $|\ell|$ denotes the cardinality of set ℓ. We assume that $|\ell_1| = |\ell_2| = N/2$. Let $\mathbf{s}_j^i$ denote the ith OFDM block of user j with the length $N/2$, that is transmitted over the subcarriers in ℓ_j, and r_p^i denote the corresponding signal received by user p in the ith time slot over the carriers in set ℓ_p.

The time-domain multipath channel between user j and user p is denoted by $h_{pj}(n)$, $n \in [0, L - 1]$; each channel tap is assumed to be zero-mean i.i.d. Gaussian with unit variance. The taps $h_{pj}(n)$ are assumed to be uncorrelated for different p, j pairs, and also for different discrete times n. We assume that the channel is slowly varying, that is, the channel remains constant over several OFDM blocks. The BS has perfect knowledge of the interuser and user-to-BS

channel. Let the frequency-domain channel be $H_{pj}(k) = \sum_{n=0}^{L-1} h_{pj}(n)e^{-j(2\pi/N)kn}$, $k \in [0, N-1]$. Then the received signal by user p from user j in the ith slot is given by

$$\mathbf{r}_p^i = \mathbf{H}_{pj}\mathbf{s}_j^i + \mathbf{n}_{pj}^i, \tag{1}$$

where

$$\mathbf{H}_{pj} = \text{diag}\left\{\left[H_{pj}\big(\mathcal{I}_j(1)\big), \ldots, H_{pj}\big(\mathcal{I}_j\big(\big|\mathcal{I}_j\big|\big)\big)\right]\right\} \tag{2}$$

with $\mathcal{I}_j(k)$ denoting the kth element of the set $\mathcal{I}_j$ according to some predefined ordering; $\mathbf{n}_{pj}^i$ denotes noise at user p during the transmission of the ith block from user j with the variance of its entries being σ_{pj}^i. We assume that the noise is circularly complex Gaussian with zero mean, temporally and spatially white, that is,

$$E\left\{\mathbf{n}_{pj}^i\big(\mathbf{n}_{p'j'}^{i'}\big)^H\right\} = \begin{cases} \sigma_{pj}^i\mathbf{I}_{N/2}, & i = i', \ p = p', \ j = j', \\ \mathbf{0}_{N/2}, & \text{otherwise.} \end{cases} \tag{3}$$

For simplicity we assume that for the noise variance it holds: $(\sigma_{pj}^i)^2 = (\sigma_{pj}^{i+1})^2 = \sigma_{pj}^2$.

The signal-to-noise ratio (SNR) throughout this paper is defined as the ratio of the power of transmitted signal to the power of additive noise as

$$\text{SNR}_{pj}^i = \frac{E\left\{\big(\mathbf{s}_j^i\big)^H\mathbf{s}_j^i\right\}}{E\left\{\big(\mathbf{n}_{pj}^i\big)^H\mathbf{n}_{pj}^i\right\}}. \tag{4}$$

It is well known that a good interuser channel is a prerequisite for cooperation. In a multiuser system, the partners are selected to have a good channel between them. Therefore, throughout this paper we assume that the interuser channels are sufficiently good.

We will next discuss a scenario where both users transmit and receive simultaneously using the same antenna, that is, in full duplex mode. Since there could be practical difficulties in such scenario, we will later discuss an approach where time division multiplexing is used to achieve full duplex operation. As that approach does not change the following analysis nor the conclusions drawn in this paper, for simplicity, we continue to present our methods assuming full duplex operation.

3. The Precoded Cooperation Scheme

First, the users perform *interblock precoding* on pairs of successive data blocks before they enter the OFDM system. As it will be shown in a subsequent subsection, the purpose of the precoding is to exploit the multipath diversity and spatial diversity that is introduced by the cooperative retransmissions.

Let us express $\mathbf{W}_j$ be the unitary precoding matrix for user j as

$$\mathbf{W}_j = \begin{bmatrix} \mathbf{W}_{j1} \\ \mathbf{W}_{j2} \end{bmatrix}, \tag{5}$$

where $\mathbf{W}_{j1}$ ($N/2 \times N$) contains the first half of the rows of $\mathbf{W}_j$ while $\mathbf{W}_{j2}$ ($N/2 \times N$) contains the other half. On denoting the uncoded blocks of user j by $\mathbf{d}_j = [(\mathbf{d}_j^i)^T, (\mathbf{d}_j^{i+1})^T]^T$, the precoded blocks are

$$\mathbf{s}_j^i = \mathbf{W}_{j1}\mathbf{d}_j, \quad \mathbf{s}_j^{i+1} = \mathbf{W}_{j2}\mathbf{d}_j, \quad j = 1, 2. \tag{6}$$

Second, each user transmits two precoded data blocks in a cycle of 3 slots. Two cooperative transmission schemes are considered for a three-slot cycle, namely, the full-cooperation scheme and the half-cooperation scheme.

3.1. The Full-Cooperation Scheme. In this scheme, the two users superimpose their own data to the data received from the other user. Two blocks from each user, that is, $\mathbf{s}_1^i$ and $\mathbf{s}_1^{i+1}$, $i = 1, 2$ are transmitted and recovered in three time slots as follows.

Slot i. Both users transmit their own data, $\mathbf{s}_1^i$ and $\mathbf{s}_2^i$, respectively. These are received as $\mathbf{H}_{21}\mathbf{s}_1^i + \mathbf{n}_{21}^i$ and $\mathbf{H}_{12}\mathbf{s}_2^i + \mathbf{n}_{12}^i$, respectively, by the other user.

Slot $i + 1$. The users transmit a weighted combination of their own data $\mathbf{s}_1^{i+1}$ ($\mathbf{s}_2^{i+1}$) and the signal that they received during the previous slot after it has been scaled by α (β) and mapped from the incoming carriers to outgoing carriers. The amount of power allocated for cooperation by users 1 and 2 is proportional to α^2 and β^2, respectively. The selection of those weights is formulated as an optimization problem in Section 3.5. The transmitted signals of both users, that is, $\mathbf{t}_1^{i+1}$ and $\mathbf{t}_2^{i+1}$ are given by

$$\begin{aligned} \mathbf{t}_1^{i+1} &= \mathbf{s}_1^{i+1} + \alpha\big(\mathbf{H}_{12}\mathbf{s}_2^i + \mathbf{n}_{12}^i\big), \\ \mathbf{t}_2^{i+1} &= \mathbf{s}_2^{i+1} + \beta\big(\mathbf{H}_{21}\mathbf{s}_1^i + \mathbf{n}_{21}^i\big). \end{aligned} \tag{7}$$

Slot $i + 2$. Both users again transmit $\mathbf{R}_1'\mathbf{s}_1^i$ and $\mathbf{R}_2'\mathbf{s}_2^i$ as their own data, plus the signal that they received during slot $i + 1$. Note that there is a component of $\mathbf{s}_1^i$ ($\mathbf{s}_2^i$) in the received signals by users 1 (2). In order to eliminate that component, the precoding for that block is modified as

$$\mathbf{R}_1' = \mathbf{R}_2' = \mathbf{I}_{N/2} - \alpha\beta\mathbf{H}_{12}\mathbf{H}_{21}, \tag{8}$$

$\alpha\beta\mathbf{H}_{12}\mathbf{H}_{21}$ can be obtained at each user by correlating the signal that was received in the $(i+1)$th slot with the signal that was transmitted in the ith time slot. Therefore, the transmitted signals $\mathbf{t}_1^{i+2}$ and $\mathbf{t}_2^{i+2}$ can be expressed as

$$\begin{aligned} \mathbf{t}_1^{i+2} &= \mathbf{s}_1^i + \alpha\mathbf{H}_{12}\mathbf{s}_2^{i+1} + \alpha\beta\mathbf{H}_{12}\mathbf{n}_{21}^i + \alpha\mathbf{n}_{12}^{i+1}, \\ \mathbf{t}_2^{i+2} &= \mathbf{s}_2^i + \beta\mathbf{H}_{21}\mathbf{s}_1^{i+1} + \alpha\beta\mathbf{H}_{21}\mathbf{n}_{12}^i + \beta\mathbf{n}_{21}^{i+1}. \end{aligned} \tag{9}$$

In the $(i + 3)$th slot, the cycle is repeated with two new data blocks. Table 2 shows the transmit signals of each user during three slots.

The signals received at the BS during slots $i, i+1, i+2$ over ℓ_1 are:

$$\mathbf{y}_1^i = \mathbf{H}_{01}\mathbf{s}_1^i + \mathbf{n}_{01}^i, \tag{10}$$

$$\mathbf{y}_1^{i+1} = \mathbf{H}_{01}\mathbf{s}_1^{i+1} + \alpha\mathbf{H}_{01}\mathbf{H}_{12}\mathbf{s}_2^i + \alpha\mathbf{H}_{01}\mathbf{n}_{12}^i + \mathbf{n}_{01}^{i+1}, \tag{11}$$

$$\mathbf{y}_1^{i+2} = \mathbf{H}_{01}\mathbf{s}_1^i + \alpha\mathbf{H}_{01}\mathbf{H}_{12}\mathbf{s}_2^{i+1} + \alpha\beta\mathbf{H}_{01}\mathbf{H}_{12}\mathbf{n}_{21}^i,$$
$$+ \alpha\mathbf{H}_{01}\mathbf{n}_{12}^{i+1} + \mathbf{n}_{01}^{i+2}. \tag{12}$$

Similarly, the received signals over carriers in ℓ_2 during slots $i, i+1, i+2$ are:

$$\mathbf{y}_2^i = \mathbf{H}_{02}\mathbf{s}_2^i + \mathbf{n}_{02}^i, \tag{13}$$

$$\mathbf{y}_2^{i+1} = \mathbf{H}_{02}\mathbf{s}_2^{i+1} + \beta\mathbf{H}_{02}\mathbf{H}_{21}\mathbf{s}_1^i + \beta\mathbf{H}_{02}\mathbf{n}_{21}^i + \mathbf{n}_{02}^{i+1}, \tag{14}$$

$$\mathbf{y}_2^{i+2} = \mathbf{H}_{02}\mathbf{s}_2^i + \beta\mathbf{H}_{02}\mathbf{H}_{21}\mathbf{s}_1^{i+1} + \alpha\beta\mathbf{H}_{02}\mathbf{H}_{21}\mathbf{n}_{12}^i$$
$$+ \beta\mathbf{H}_{02}\mathbf{n}_{21}^{i+1} + \mathbf{n}_{02}^{i+2}. \tag{15}$$

Based on (10), (12), and (14), let us form the matrix equation:

$$\mathbf{y_1} = \begin{bmatrix} \mathbf{y}_1^i \\ \mathbf{y}_1^{i+2} \\ \mathbf{y}_2^{i+1} \end{bmatrix} = \mathbf{H}_1 \begin{bmatrix} \mathbf{s}_1^i \\ \mathbf{s}_2^{i+1} \end{bmatrix} + \mathbf{n}_1, \tag{16}$$

where

$$\mathbf{H}_1 = \begin{bmatrix} \mathbf{H}_{01} & \mathbf{0}_{N/2} \\ \mathbf{H}_{01} & \alpha\mathbf{H}_{01}\mathbf{H}_{12} \\ \beta\mathbf{H}_{02}\mathbf{H}_{21} & \mathbf{H}_{02} \end{bmatrix},$$

$$\mathbf{n}_1 = \begin{bmatrix} \mathbf{n}_{01}^i \\ \alpha\beta\mathbf{H}_{01}\mathbf{H}_{12}\mathbf{n}_{21}^i + \alpha\mathbf{H}_{01}\mathbf{n}_{12}^{i+1} + \mathbf{n}_{01}^{i+2} \\ \beta\mathbf{H}_{02}\mathbf{n}_{21}^i + \mathbf{n}_{02}^{i+1} \end{bmatrix}. \tag{17}$$

Similarly, based on (13), (15), and (11), let us form the matrix equation:

$$\mathbf{y_2} = \begin{bmatrix} \mathbf{y}_2^i \\ \mathbf{y}_2^{i+2} \\ \mathbf{y}_1^{i+1} \end{bmatrix} = \mathbf{H}_2 \begin{bmatrix} \mathbf{s}_2^i \\ \mathbf{s}_1^{i+1} \end{bmatrix} + \mathbf{n}_2, \tag{18}$$

where

$$\mathbf{H}_2 = \begin{bmatrix} \mathbf{H}_{02} & \mathbf{0}_{N/2} \\ \mathbf{H}_{02} & \beta\mathbf{H}_{02}\mathbf{H}_{21} \\ \alpha\mathbf{H}_{01}\mathbf{H}_{12} & \mathbf{H}_{01} \end{bmatrix},$$

$$\mathbf{n}_2 = \begin{bmatrix} \mathbf{n}_{02}^i \\ \alpha\beta\mathbf{H}_{02}\mathbf{H}_{21}\mathbf{n}_{12}^i + \beta\mathbf{H}_{02}\mathbf{n}_{21}^{i+1} + \mathbf{n}_{02}^{i+2} \\ \alpha\mathbf{H}_{01}\mathbf{n}_{12}^i + \mathbf{n}_{01}^{i+1} \end{bmatrix}. \tag{19}$$

By observing (16) and (18), and keeping in mind that $\mathbf{s}_j^i$ and $\mathbf{s}_j^{i+1}$ are functions of both $\mathbf{d}_j^i$ and $\mathbf{d}_j^{i+1}$, $j = 1, 2$,

one can see that cooperation has effectively created two transmission paths for the information blocks. This effect is analogous to employing two transmitters. We should note that interblock precoding was used in [16] to exploit time diversity introduced by time varying channels. Here, even if the channel is completely static, interblock precoding allows us to exploit spatial diversity introduced by cooperation. The proposed scheme with interblock precoding requires on the average 1.5 slots for each data block to achieve the double diversity induced by cooperation. Without interblock precoding, two slots would be required.

Combining (16) and (18), the following MIMO problem can be formulated at the receiver:

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} = \mathbf{H}\mathbf{d} + \mathbf{n}, \tag{20}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \begin{bmatrix} \mathbf{W}_{11} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_{22} \end{bmatrix} \\ \mathbf{H}_2 \begin{bmatrix} \mathbf{0} & \mathbf{W}_{21} \\ \mathbf{W}_{12} & \mathbf{0} \end{bmatrix} \end{bmatrix} \tag{21}$$

$$\mathbf{d} = \begin{bmatrix} \mathbf{d}_1 \\ \mathbf{d}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{d}_1^i \\ \mathbf{d}_1^{i+1} \\ \mathbf{d}_2^i \\ \mathbf{d}_2^{i+1} \end{bmatrix}, \qquad \mathbf{n} = \begin{bmatrix} \mathbf{n}_1 \\ \mathbf{n}_2 \end{bmatrix}.$$

Assuming knowledge of $\mathbf{H}$, recovery of $\mathbf{d}$ based on $\mathbf{y}$ is discussed in Section 3.4.

3.1.1. Transmission Energy Adjustment. Let $\tilde{\sigma}_i^2, \sigma_i^2$ be the power of one data block transmitted by user i without and with cooperation, respectively. For simplicity let us take $\tilde{\sigma}_1 = \tilde{\sigma}_2 = \tilde{\sigma}$ and $\sigma_1 = \sigma_2 = \sigma$. In the cooperative OFDM scheme, the transmission of $\mathbf{s}_j^i$ and $\mathbf{s}_j^{i+1}$ requires three slots, as opposed to the two slots required in the no-cooperation scheme. To maintain the energy used by the two schemes for the transmission of a block pair at the same level, we need to adjust the transmission power. In the noncooperative case, the transmission of 2 blocks requires energy equal to $P_{\text{noco}} = 2\tilde{\sigma}_1^2|\ell_1| + 2\tilde{\sigma}_2^2|\ell_2| = 2\tilde{\sigma}^2 N$. Under cooperation, the energy spent by user 1 and user 2 to transmit 3 blocks is

$$\Gamma_1 = 3\sigma^2|\ell_1| + 2\alpha^2\left(\text{trace}\left(|\mathbf{H}_{12}|^2\right)\sigma^2 + \sigma_{12}^2|\ell_1|\right)$$
$$+ \alpha^2\beta^2\sigma_{21}^2\text{trace}\left(|\mathbf{H}_{12}|^2\right),$$

$$\Gamma_2 = 3\sigma^2|\ell_2| + 2\beta^2\left(\text{trace}\left(|\mathbf{H}_{21}|^2\right)\sigma^2 + \sigma_{21}^2|\ell_2|\right)$$
$$+ \alpha^2\beta^2\sigma_{12}^2\text{trace}\left(|\mathbf{H}_{21}|^2\right). \tag{22}$$

To ensure that the energy spent is the same in cooperative and noncooperative cases it should hold: $P_{\text{noco}} = \Gamma_1 + \Gamma_2$.

Since the channel taps are assumed to be zero-mean unit-variance Gaussian random variables, the magnitudes $|H_{ij}(k)|$ are i.i.d. Rayleigh distributed, that is, $|H_{ij}(k)| \sim$ Rayleigh$(1/2)$. Let $\overline{\sigma^2}$ be the average σ^2 over the interuser channel coefficients. It holds

$$4\widetilde{\sigma}^2 = \overline{\sigma}^2 \left(6 + 2\alpha^2 + 2\beta^2\right) + 2\alpha^2\sigma_{12}^2 + 2\beta^2\sigma_{21}^2 + \alpha^2\beta^2\left(\sigma_{21}^2 + \sigma_{12}^2\right)$$

$$\Longrightarrow \overline{\sigma}^2 = \frac{4\widetilde{\sigma}^2 - 2\alpha^2\sigma_{12}^2 - 2\beta^2\sigma_{21}^2 - \alpha^2\beta^2\left(\sigma_{21}^2 + \sigma_{12}^2\right)}{6 + 2\alpha^2 + 2\beta^2}. \tag{23}$$

When σ_{21}, σ_{12} are sufficiently small (23) can be approximated as $\overline{\sigma}^2 \approx 2\widetilde{\sigma}^2/(3 + \alpha^2 + \beta^2)$.

3.1.2. Diversity Analysis. It is shown in [13] that for a single user OFDM system, the maximum diversity gain achievable with one transmit antenna is equal to the number of independent fading paths of the channel. Diversity is related to the bit error rate performance [18] and is usually increased by adding more transmitters and receivers. In this section, we follow a similar procedure as in [13] to study the diversity gain achieved by (20) and show that (20) achieves the full spatial diversity available, that is, $2L$ without adding more transmitters.

The probability of $\hat{\mathbf{d}}$ being detected when $\mathbf{d}$ is transmitted is

$$P\left(\mathbf{d} \longrightarrow \hat{\mathbf{d}} \mid \mathbf{H}_{01}, \mathbf{H}_{02}, \mathbf{H}_{12}, \mathbf{H}_{21}\right) \le \exp\left(-\frac{d^2(\mathbf{y}, \hat{\mathbf{y}})}{4\sigma_n^2}\right), \tag{24}$$

where $d^2(\mathbf{y}, \mathbf{y}) = \|\mathbf{y} - \hat{\mathbf{y}}\|^2$, $\mathbf{y} = \mathbf{R}_n^{-1/2}\mathbf{H}\mathbf{d}$, $\hat{\mathbf{y}} = \mathbf{R}_n^{-1/2}\mathbf{H}\hat{\mathbf{d}}$ and σ_n^2 is the noise variance. Then we have

$$d^2(\mathbf{y}, \hat{\mathbf{y}}) = \|\mathbf{y} - \hat{\mathbf{y}}\|^2 = \left\|\mathbf{R}_n^{-1/2}\mathbf{H}[\mathbf{e}_1^T, \mathbf{e}_2^T]^T\right\|^2$$

$$= \left[\widetilde{\mathbf{e}}_1^H\mathbf{H}_1^H, \widetilde{\mathbf{e}}_2^H\mathbf{H}_2^H\right]\mathbf{R}_n^{-1}\begin{bmatrix}\mathbf{H}_1\widetilde{\mathbf{e}}_1 \\ \mathbf{H}_2\widetilde{\mathbf{e}}_2\end{bmatrix}, \tag{25}$$

where $\mathbf{e}_i = \hat{\mathbf{d}}_i - \mathbf{d}_i$ and

$$\widetilde{\mathbf{e}}_1 = \begin{bmatrix}\mathbf{W}_{11}\mathbf{e}_1 \\ \mathbf{W}_{22}\mathbf{e}_2\end{bmatrix}, \qquad \widetilde{\mathbf{e}}_2 = \begin{bmatrix}\mathbf{W}_{21}\mathbf{e}_2 \\ \mathbf{W}_{12}\mathbf{e}_1\end{bmatrix}. \tag{26}$$

Let us define

$$\mathbf{h} = [h_{01}(0), \dots, h_{01}(L-1), h_{02}(0), \dots, h_{02}(L-1)]^T,$$

$$\mathbf{R}_h = E\{\mathbf{h}\mathbf{h}^H\}, \qquad \widetilde{\mathbf{h}} = \mathbf{R}_h^{-1/2}\mathbf{h} \tag{27}$$

and partition $\widetilde{\mathbf{e}}_i$ into two $(N/2) \times 1$ vectors $\widetilde{\mathbf{e}}_{i1}$ and $\widetilde{\mathbf{e}}_{i2}$. Then, (25) can be further rewritten as

$$d^2(\mathbf{y}, \hat{\mathbf{y}}) = \widetilde{\mathbf{h}}^H \underbrace{\mathbf{R}_h^{1/2}\mathbf{G}\mathbf{R}_h^{1/2}}_{\widetilde{\mathbf{G}}}\widetilde{\mathbf{h}} = \widetilde{\mathbf{h}}^H\widetilde{\mathbf{G}}\widetilde{\mathbf{h}}, \tag{28}$$

where

$$\mathbf{G} = \begin{bmatrix}\mathbf{F}_1^H & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_2^H\end{bmatrix}\begin{bmatrix}\mathbf{G}_1^H\mathbf{G}_2^H\end{bmatrix}\mathbf{R}_n^{-1}\begin{bmatrix}\mathbf{G}_1 \\ \mathbf{G}_2\end{bmatrix}\begin{bmatrix}\mathbf{F}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_2\end{bmatrix},$$

$$\mathbf{G}_1 = \begin{bmatrix}\mathbf{E}_{11} & \mathbf{0} \\ \mathbf{E}_{11} + \alpha\mathbf{E}_{12}\mathbf{H}_{12} & \mathbf{0} \\ \mathbf{0} & \beta\mathbf{E}_{11}\mathbf{H}_{21} + \mathbf{E}_{12}\end{bmatrix},$$

$$\mathbf{G}_2 = \begin{bmatrix}\mathbf{0} & \mathbf{E}_{21} \\ \mathbf{0} & \mathbf{E}_{21} + \beta\mathbf{E}_{22}\mathbf{H}_{21} \\ \alpha\mathbf{E}_{21}\mathbf{H}_{12} + \mathbf{E}_{22} & \mathbf{0}\end{bmatrix}, \tag{29}$$

$$\mathbf{E}_{ij} = \text{diag}\{\widetilde{\mathbf{e}}_{ij}\},$$

and $\mathbf{F}_1$ and $\mathbf{F}_2$ are submatrices of the N-point DFT matrix corresponding to $\mathcal{l}_1$ and $\mathcal{l}_2$.

Because $\mathbf{R}_n$ is generally invertible, it is reasonable to assume that $\widetilde{\mathbf{G}}$ has full rank. Conditioned on the interuser channels $\mathbf{H}_{12}$ and $\mathbf{H}_{21}$, the pairwise error probability is [19]

$$P\left(\mathbf{d} \longrightarrow \hat{\mathbf{d}} \mid \mathbf{H}_{12}, \mathbf{H}_{21}\right) \le \prod_{i=1}^{2L}\frac{1}{1 + (1/4\sigma_n^2)\lambda_i\left(\widetilde{\mathbf{G}}\right)}, \tag{30}$$

where $\lambda_i(\cdot)$ denotes the ith eigenvalue of a matrix in the decreasing order.

It can be seen that for high SNR the decay of the error probability is of the order of $2L$. We should emphasize that interblock precoding is essential in achieving this diversity. Intuitively, using interblock precoding, the data within a block and between blocks can share all the available channels equally, and thus the receiver can obtain the maximum number of copies of those data. This can also been seen analytically as follows. Without interblock precoding, that is, $\mathbf{W} = \mathbf{I}$, the signal model in (20) becomes

$$\mathbf{y} = \begin{bmatrix}\mathbf{y}_1 \\ \mathbf{y}_2\end{bmatrix} = \begin{bmatrix}\mathbf{H}_1 \\ \mathbf{H}_2\end{bmatrix}\begin{bmatrix}\mathbf{d}_1^i \\ \mathbf{d}_2^{i+1} \\ \mathbf{d}_2^i \\ \mathbf{d}_1^{i+1}\end{bmatrix} + \mathbf{n}. \tag{31}$$

Since both $\mathbf{H}_1$ and $\mathbf{H}_2$ can be partitioned into six $(N/2) \times (N/2)$ diagonal matrices as seen in (17), (19), the ML decoding algorithm is performed on a pair of $(d_1^i(k), d_2^{i+1}(k))$ and $(d_2^i(k), d_1^{i+1}(k))$ per subcarrier:

$$\mathbf{y}_1^k = \begin{bmatrix}y_1^i(k) \\ y_1^{i+2}(k) \\ y_2^{i+1}(k)\end{bmatrix} = \mathbf{H}_1^k\underbrace{\begin{bmatrix}d_1^i(k) \\ d_2^{i+1}(k)\end{bmatrix}}_{\mathbf{c}_1^k} + \mathbf{n}_1^k,$$

$$\mathbf{y}_2^k = \begin{bmatrix}y_2^i(k) \\ y_2^{i+2}(k) \\ y_1^{i+1}(k)\end{bmatrix} = \mathbf{H}_2^k\underbrace{\begin{bmatrix}d_2^i(k) \\ d_1^{i+1}(k)\end{bmatrix}}_{\mathbf{c}_2^k} + \mathbf{n}_2^k, \tag{32}$$

where

$$\mathbf{H}_1^k = \begin{bmatrix} H_{01}(k) & 0 \\ H_{01}(k) & \alpha H_{01}(k)H_{12}(k) \\ \beta H_{02}(k)H_{21}(k) & H_{02}(k) \end{bmatrix},$$

$$\mathbf{H}_2^k = \begin{bmatrix} H_{02}(k) & 0 \\ H_{02}(k) & \beta H_{02}(k)H_{21}(k) \\ \alpha H_{01}(k)H_{12}(k) & H_{01}(k) \end{bmatrix},$$

$$\mathbf{n}_1^k = \begin{bmatrix} n_{01}^i(k) \\ \alpha\beta H_{01}(k)H_{12}(k)n_{21}^i(k) + \mathfrak{A} \\ \beta H_{02}(k)n_{21}^i(k) + n_{02}^{i+1}(k) \end{bmatrix},$$

$$\mathbf{n}_2^k = \begin{bmatrix} n_{02}^i(k) \\ \alpha\beta H_{02}(k)H_{21}(k)n_{12}^i(k) + \mathfrak{B} \\ \alpha H_{01}(k)n_{12}^i(k) + n_{01}^{i+1}(k) \end{bmatrix},$$

(33)

where $\mathfrak{A}$ denotes $\alpha H_{01}(k)n_{12}^{i+1}(k) + n_{01}^{i+2}(k)$ and $\mathfrak{B}$ denotes $\beta H_{02}(k)n_{21}^{i+1}(k) + n_{02}^{i+2}(k)$. Repeating the diversity analysis as above, we get

$$P\left(\mathbf{c}_j^k \longrightarrow \hat{\mathbf{c}}_j^k \mid H_{01}(k), H_{02}(k), H_{12}(k), H_{21}(k)\right)$$

$$\leq \exp\left(-\frac{\left\|\mathbf{y}_j^k - \hat{\mathbf{y}}_j^k\right\|^2}{4\sigma_n^2}\right), \quad j = 1, 2.$$

(34)

Similar to (28), we have

$$\left\|\mathbf{y}_j^k - \hat{\mathbf{y}}_j^k\right\|^2$$

$$= \left\|\underbrace{\left(\mathbf{R}_j^k\right)^{-1/2}\mathbf{H}_j^k(\hat{\mathbf{c}}_j^k - \mathbf{c}_j^k)}_{\mathbf{e}_j^k}\right\|^2$$

(35)

$$= \tilde{\mathbf{h}}^H \mathbf{R}_h^{1/2} \begin{bmatrix} \mathbf{f}_k^* & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_k^* \end{bmatrix} \left(\mathbf{G}_j^k\right)^H \underbrace{\left(\mathbf{R}_j^k\right)^{-1}}_{} \mathbf{G}_j^k \underbrace{\begin{bmatrix} \mathbf{f}_k^T & \mathbf{0} \\ \mathbf{0} & \mathbf{f}_k^T \end{bmatrix}}_{\mathbf{F}_k} \mathbf{R}_h^{1/2}\tilde{\mathbf{h}},$$

$$\underbrace{}_{\mathbf{Z}_j^k}$$

where $\mathbf{R}_j^k = E\{\mathbf{n}_j^k(\mathbf{n}_j^k)^H\}$,

$$\mathbf{G}_1^k = \begin{bmatrix} e_1^k(1) & 0 \\ e_1^k(1) + \alpha H_{12}(k)e_1^k(2) & 0 \\ 0 & \beta H_{21}(k)e_1^k(1) + e_1^k(2) \end{bmatrix},$$

$$\mathbf{G}_2^k = \begin{bmatrix} 0 & e_2^k(1) \\ 0 & e_1^k(1) + \beta H_{21}(k)e_2^k(2) \\ \alpha H_{12}(k)e_2^k(1) + e_2^k(2) & 0 \end{bmatrix}.$$

(36)

and $\mathbf{f}^k$ contains the first $2L$ entries of the column in the N-point DFT matrix corresponding to the kth subcarrier in $\mathcal{l}_j$. Since the rank of $\mathbf{F}_k$ is two, the maximum rank of $\mathbf{Z}_j^k$ is two. Thus, without interblock precoding, the maximum diversity gain that the cooperation scheme could achieve would be two.

To achieve the full diversity for both users we need $\alpha, \beta \neq 0$. If we choose $\alpha \neq 0, \beta = 0$ as an example, user 1 cannot achieve the maximum diversity $2L$. However, if the channel of one user is very bad, this user should terminate cooperation to maintain its own signal power at a certain level, that is, set α or β to zero. Unlike pure transmit diversity, where we always have a good (wired) channel between the transmitters, cooperation can exhibit the same performance only when the interuser channel is good. In OFDM, where we have multiple carriers, some carriers will enjoy the full diversity gain by cooperation while some carriers will not.

3.2. The Half-Cooperation Scheme. In the full cooperation scheme, both users are involved in the cooperation. In order to keep the total energy consumed by the full-cooperation scheme equal to that of the no-cooperation scheme, we have to reduce the power assigned to each data block. Therefore, the maximum diversity gain is doubled at the price of SNR. It is expected that, at low SNR, the full-cooperation scheme will yield worse performance in terms of BER than the no-cooperation scheme. One might wonder whether the performance at low SNR can be improved by sacrificing diversity to some extent. Next, we investigate another scheme in which only the strongest of the two users cooperates. In particular, user 1 serves as a relay for user 2, while user 2 does not help user 1. Unlike the full-cooperation scheme, users send their information separately to the BS. Again, three slots are required for two users to transmit two blocks of data as follows.

Slot i. Both users transmit their own data $\mathbf{s}_1^i$ and $\mathbf{s}_2^i$, respectively. User 1 receives $\mathbf{H}_{12}\mathbf{s}_2^i + \mathbf{n}_{12}^i$ from user 2.

Slot i + 1. Both users transmit their own data $\mathbf{s}_1^{i+1}$ and $\mathbf{s}_2^{i+1}$, respectively. At the time of transmission, user 1 receives $\mathbf{H}_{12}\mathbf{s}_2^{i+1} + \mathbf{n}_{12}^{i+1}$ from user 2.

Slot i + 2. User 2 terminates transmission. User 1 transmits the signal that he received in the previous two slots over $\mathcal{l}_1$ and $\mathcal{l}_2$.

In the $(i + 3)$th slot, the cycle is repeated with two new data blocks. Table 3 shows the transmit signals of each user during the three slots. The received signals at the BS containing user 1's data and user 2's data are, respectively, equal

$$\mathbf{y}_1 = \begin{bmatrix} \mathbf{H}_{01} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{01} \end{bmatrix}\mathbf{W}_1\begin{bmatrix} \mathbf{d}_1^i \\ \mathbf{d}_1^{i+1} \end{bmatrix} + \begin{bmatrix} \mathbf{n}_{01}^i \\ \mathbf{n}_{01}^{i+1} \end{bmatrix}, \quad (37)$$

$$\mathbf{y}_2 = \begin{bmatrix} \mathbf{H}_{02} & \mathbf{0} \\ \alpha\mathbf{H}_{01}\mathbf{H}_{12} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{02} \\ \mathbf{0} & \beta\mathbf{H}_{01}\mathbf{H}_{12} \end{bmatrix} \mathbf{W}_2 \begin{bmatrix} \mathbf{d}_2^i \\ \mathbf{d}_2^{i+1} \end{bmatrix}$$

$$+ \begin{bmatrix} \mathbf{n}_{02}^i \\ \alpha\mathbf{H}_{01}\mathbf{H}_{12} + \mathbf{n}_{01}^{i+2} \\ \mathbf{n}_{02}^{i+1} \\ \beta\mathbf{H}_{01}\mathbf{H}_{12} + \widetilde{\mathbf{n}}_{01}^{i+2} \end{bmatrix}, \tag{38}$$

where $\widetilde{\mathbf{n}}_{01}^{i+2}$ represents the addictive Gaussian noise on the user-to-BS channel for user 1 in the $(i + 2)$th slot over $\mathcal{l}_2$.

3.2.1. Transmission Energy Adjustment. Under cooperation, the energy spent by user 1 and user 2 to transmit 3 blocks is:

$$\Gamma_1 = 2\sigma^2|\mathcal{l}_1| + (\alpha^2 + \beta^2)\left(\text{trace}\left(|\mathbf{H}_{12}|^2\right)\sigma^2 + \sigma_{12}^2|\mathcal{l}_2|\right),$$

$$\Gamma_2 = 2\sigma^2|\mathcal{l}_2|. \tag{39}$$

Similar to the full-cooperation scheme, the average signal power over the interuser channel coefficients $\widetilde{\sigma}^2$ is given by

$$4\widetilde{\sigma}^2 = \overline{\sigma}^2(4 + \alpha^2 + \beta^2) + (\alpha^2 + \beta^2)\sigma_{12}^2$$

$$\Longrightarrow \overline{\sigma}^2 = \frac{4\widetilde{\sigma}^2 - (\alpha^2 + \beta^2)\sigma_{12}^2}{4 + \alpha^2 + \beta^2}. \tag{40}$$

When $\sigma_{21}, \sigma_{12} \ll \sigma_{01}, \sigma_{02}$, (40) can be approximated as $\overline{\sigma}^2 \approx \widetilde{\sigma}^2/(1 + (\alpha^2 + \beta^2)/4)$. In the full-cooperation scheme, $\overline{\sigma}^2 \approx 2\widetilde{\sigma}^2/(3 + \alpha^2 + \beta^2) < \overline{\sigma}^2 \approx \widetilde{\sigma}^2/(1 + (\alpha^2 + \beta^2)/4)$. Therefore, the half-cooperation scheme can save more transmission power for each data block. It is expected that when SNR is relatively low, the half-cooperation scheme can yield better performance than the full-cooperation scheme.

3.2.2. Diversity Analysis. Similar to the scenarios discussed in [13], the maximum diversity gain of user 1 in (37) is L. From the analysis of Section 3.1.2, the maximum diversity gain of user 2 in (38) is $2L$. This indicates that user 1 has to sacrifice its performances for the sake of user 2. On the other hand, the full-cooperation scheme is a win-win situation for both users when the SNR is relatively high.

Table 1 summarizes the maximum diversity gain of the full-cooperation scheme (FC), the full-cooperation scheme without precoding (FC-no precoding), the half-cooperation scheme (HC), the no-cooperation scheme with precoding (NC) and the no-cooperation scheme without precoding (NC-no precoding).

3.3. Time Division Duplexing. The cooperation scheme described above is strongly dependent on the users being able to both receive and transmit simultaneously. However, in a practical situation this might be difficult. Nevertheless it is possible to effectively achieve full duplex operation by time division duplexing.

In the original scheme both users transmit during the entire duration of time slot i (N symbols plus the cyclic prefix). However, we can allocate half a time slot for each user to the data vectors $\mathbf{s}_1^i$ and $\mathbf{s}_2^i$. During time slot i, user 1 will first transmit $N/2$ data symbols plus the cyclic prefix. Next, user 2 will transmit his own $N/2$ data symbols plus the cyclic prefix. During each transmission, all the other users will be in receive mode. Therefore, there is no difference between this time division approach (half duplex) and the full duplex one, and the analysis and conclusions hold in this case too.

3.4. Symbol Recovery. The maximum diversity can be best exploited using ML decoding. In general, ML decoding has prohibitively high complexity especially when the number of subcarriers N is large. Here, following the main idea of [13], we implement ML by optimal subcarrier grouping. The set of all subcarriers is divided into K equally spaced groups. Each group contains $J = N/K$ subcarriers. In order to achieve the maximum multipath diversity, it should hold that $J \geq 2L$ (see Section 4 for diversity analysis). To reduce the complexity further, we let two users exchange their subcarriers to transmit data in the second slot, that is, in slot $i + 1$, user 1 transmits over $\mathcal{l}_2$ and receives over $\mathcal{l}_1$, while user 2 transmits over $\mathcal{l}_1$ and receives over $\mathcal{l}_2$. By this way, the minimum J can be reduced to L to achieve diversity of the order of $2L$. For simplicity, we assume that $K = N/L$ is an integer. The sets of subcarriers for two users $\mathcal{l}_1$ and $\mathcal{l}_2$ are divided into K groups, each group containing $J = L$ equally spaced subcarriers. Let us define

$$\mathbf{p}_k = \left[k, k + K, \ldots, k + \left(\frac{L}{2} - 1\right)K\right],$$

$$\mathbf{y}_j^{i,k} = \left[\mathbf{y}_j^i(p_k(1)), \ldots, \mathbf{y}_j^i\left(p_k\left(\frac{L}{2}\right)\right)\right]^T,$$

$$\mathbf{d}_j^{i,k} = \left[\mathbf{d}_j^i(p_k(1)), \ldots, \mathbf{d}_j^i\left(p_k\left(\frac{L}{2}\right)\right)\right]^T,$$

$$\mathbf{n}_{pj}^{i,k} = \left[\mathbf{n}_{pj}^i(p_k(1)), \ldots, \mathbf{n}_{pj}^i\left(p_k\left(\frac{L}{2}\right)\right)\right]^T,$$

$$\mathbf{H}_{pj}^k = \text{diag}\left\{\left[H_{pj}(\mathcal{l}_1(p_k(1))), \ldots, H_{pj}\left(\mathcal{l}_1\left(p_k\left(\frac{L}{2}\right)\right)\right)\right]\right\},$$

$$\mathbf{H}_{pj}^{k'} = \text{diag}\left\{\left[H_{pj}(\mathcal{l}_2(p_k(1))), \ldots, H_{pj}\left(\mathcal{l}_2\left(p_k\left(\frac{L}{2}\right)\right)\right)\right]\right\},$$

$$k = 1, 2, \ldots, K, \tag{41}$$

where $\mathbf{p}_k$ denotes the subcarrier pattern for the kth group; $\mathbf{d}_j^{i,k}$ represents the transmitted signal of the jth user in the ith slot over the kth group of subcarriers; $\mathbf{y}_j^{i,k}$ denotes the received signal at BS in the ith slot over the kth group of subcarriers in $\mathcal{l}_j$; $\mathbf{n}_{pj}^{i,k}$ denotes the noise at user p over the kth group of subcarriers during the transmission of the data block from user j in the ith slot (The 0th user represents the BS); $\mathbf{H}_{pj}^k$ and $\mathbf{H}_{pj}^{k'}$ are the fading coefficients of the channel from the jth user to the pth user over the kth group of subcarriers in $\mathcal{l}_1$ and $\mathcal{l}_2$, respectively. If we take

TABLE 1: Maximum diversity gain for the various schemes.

	FC	FC-no precoding	HC	NC	NC-no precoding
Maximum diversity gain of user 1	$2L$	2	L	L	1
Maximum diversity gain of user 2	$2L$	2	$2L$	L	1

TABLE 2: Transmitted signal for the full-cooperation scheme.

	User 1	User 2
Slot i	$\mathbf{s}_1^i$	$\mathbf{s}_2^i$
Slot $i+1$	$\mathbf{s}_1^{i+1} + \alpha(\mathbf{H}_{12}\mathbf{s}_2^i + \mathbf{n}_{12}^i)$	$\mathbf{s}_2^{i+1} + \beta(\mathbf{H}_{21}\mathbf{s}_1^i + \mathbf{n}_{21}^i)$
Slot $i+2$	$\mathbf{s}_1^i + \alpha\mathbf{H}_{12}\mathbf{s}_2^{i+1} + \alpha\beta\mathbf{H}_{12}\mathbf{n}_{21}^i + \alpha\mathbf{n}_{12}^{i+1}$	$\mathbf{s}_2^i + \beta\mathbf{H}_{21}\mathbf{s}_1^{i+1} + \alpha\beta\mathbf{H}_{21}\mathbf{n}_{12}^i + \beta\mathbf{n}_{21}^{i+1}$

TABLE 3: Transmit Signal for the half-cooperation scheme.

	User 1	User 2
Slot i	$\mathbf{s}_1^i$	$\mathbf{s}_2^i$
Slot $i+1$	$\mathbf{s}_1^{i+1}$	$\mathbf{s}_2^{i+1}$
	User 1	User 1
Slot $i+2$	$\alpha(\mathbf{H}_{12}\mathbf{s}_2^i + \mathbf{n}_{12}^i)$	$\beta(\mathbf{H}_{12}\mathbf{s}_2^{i+1} + \mathbf{n}_{12}^{i+1})$

the full-cooperation scheme as an example, the model for the received signal by grouping subcarriers can be reduced to

$$
\mathbf{y}_k =
\begin{bmatrix} \mathbf{y}_1^{i,k} \\ \mathbf{y}_1^{i+2,k} \\ \mathbf{y}_2^{i+1,k} \\ \mathbf{y}_2^{i,k} \\ \mathbf{y}_2^{i+2,k} \\ \mathbf{y}_1^{i+1,k} \end{bmatrix}
=
\underbrace{\begin{bmatrix} \begin{bmatrix} \mathbf{H}_{01}^k & \mathbf{0} \\ \mathbf{H}_{01}^k & \alpha\mathbf{H}_{01}^k\mathbf{H}_{12}^k \\ \beta\mathbf{H}_{02}^k\mathbf{H}_{21}^k & \mathbf{H}_{02}^k \end{bmatrix} & \begin{bmatrix} \widetilde{\mathbf{W}}_{11} & \mathbf{0} \\ \mathbf{0} & \widetilde{\mathbf{W}}_{22} \end{bmatrix} \\ \begin{bmatrix} \mathbf{H}_{02}^{k'} & \mathbf{0} \\ \mathbf{H}_{02}^{k'} & \beta\mathbf{H}_{02}^{k'}\mathbf{H}_{21}^{k'} \\ \alpha\mathbf{H}_{01}^{k'}\mathbf{H}_{12}^{k'} & \mathbf{H}_{01}^{k'} \end{bmatrix} & \begin{bmatrix} \mathbf{0} & \widetilde{\mathbf{W}}_{21} \\ \widetilde{\mathbf{W}}_{12} & \mathbf{0} \end{bmatrix} \end{bmatrix}}_{\mathbf{H}_k}
$$

$$
\times \underbrace{\begin{bmatrix} \mathbf{d}_1^{i,k} \\ \mathbf{d}_1^{i+1,k} \\ \mathbf{d}_2^{i,k} \\ \mathbf{d}_2^{i+1,k} \end{bmatrix}}_{\mathbf{d}_k} + \underbrace{\begin{bmatrix} \mathbf{n}_2^k \\ \mathbf{n}_1^k \end{bmatrix}}_{\mathbf{n}_k} = \mathbf{H}_k\mathbf{d}_k + \mathbf{n}_k,
\tag{42}
$$

where

$$
\mathbf{n}_1^k = \begin{bmatrix} \mathbf{n}_{01}^{i,k} \\ \alpha\beta\mathbf{H}_{01}^k\mathbf{H}_{12}^k\mathbf{n}_{21}^{i,k} + \alpha\mathbf{H}_{01}^k\mathbf{n}_{12}^{i+1,k} + \mathbf{n}_{01}^{i+2,k} \\ \beta\mathbf{H}_{02}^k\mathbf{n}_{21}^{i,k} + \mathbf{n}_{02}^{i+1,k} \end{bmatrix}
$$

$$
\mathbf{n}_2^k = \begin{bmatrix} \mathbf{n}_{02}^{i,k} \\ \alpha\beta\mathbf{H}_{02}^{k'}\mathbf{H}_{21}^{k'}\mathbf{n}_{12}^{i,k} + \beta\mathbf{H}_{02}^{k'}\mathbf{n}_{21}^{i+1,k} + \mathbf{n}_{02}^{i+2,k} \\ \alpha\mathbf{H}_{01}^{k'}\mathbf{n}_{12}^{i,k} + \mathbf{n}_{01}^{i+1,k} \end{bmatrix}.
\tag{43}
$$

By optimal subcarrier grouping, we only perform the precoding on a group of subcarriers.

Let $\mathbf{U}_L$ be an $L \times L$ unitary Vandermonde matrix defined as in [13]. The precoding matrixes $\mathbf{W}_1$ and $\mathbf{W}_2$ in (6) can be simplified as $\mathbf{W} = \mathbf{U}_L \odot \mathbf{I}_{N/L}$.

3.5. Optimal Allocation of Power during Allocation.
In this section, we discuss the optimization of power allocation parameter α and β based on the model of (42). We assume that user 1 is the strong user, that is, user 1 has less subcarriers in deep fade as compared to user 2 (weak user). During the cooperation, user 1 will assist user 2, while at the same time, will also receive some help.

Let us define $\mathbf{x} = [\alpha^2, \beta^2]^T$ to be the vector of the parameters to be determined. The objective function is defined in terms of the SINR of user 1 and user 2. In order to derive the SINR for the users, we need to separate the data of each user in the model of (42) as

$$
\mathbf{y}_k = \mathbf{H}_k^1 \begin{bmatrix} \mathbf{d}_1^{i,k} \\ \mathbf{d}_1^{i+1,k} \end{bmatrix} + \mathbf{H}_k^2 \begin{bmatrix} \mathbf{d}_2^{i,k} \\ \mathbf{d}_2^{i+1,k} \end{bmatrix} + \mathbf{n}_k,
\tag{44}
$$

where $\mathbf{H}_k^1$ and $\mathbf{H}_k^2$ contain the first half and the second half of the columns of $\mathbf{H}_k$, respectively. On letting $\mathbf{s}_j(k)$ denote the SINR of the jth user at the kth subcarrier, we have

$$
s_1(k) = \frac{\mathrm{Tr}\left\{E\left\{\mathbf{H}_k^1\left(\mathbf{H}_k^1\right)^H\right\}\right\}}{\mathrm{Tr}\left\{E\left\{\mathbf{H}_k^2\left(\mathbf{H}_k^2\right)^H + \mathbf{n}_k\mathbf{n}_k^H\right\}\right\}},
$$

$$
s_2(k) = \frac{\mathrm{Tr}\left\{E\left\{\mathbf{H}_k^2\left(\mathbf{H}_k^2\right)^H\right\}\right\}}{\mathrm{Tr}\left\{E\left\{\mathbf{H}_k^1\left(\mathbf{H}_k^1\right)^H + \mathbf{n}_k\mathbf{n}_k^H\right\}\right\}}
\tag{45}
$$

where the numerators of $\mathbf{s}_j(k)$, $j = 1, 2$ are linear functions of $\mathbf{x}$ and the denominators are the polynomials of $\mathbf{x}$.

The optimization problem is formulated as follows. We wish to maximize the SINR on the worst subcarriers of the weak user, subject to the constraint that the SINR on all
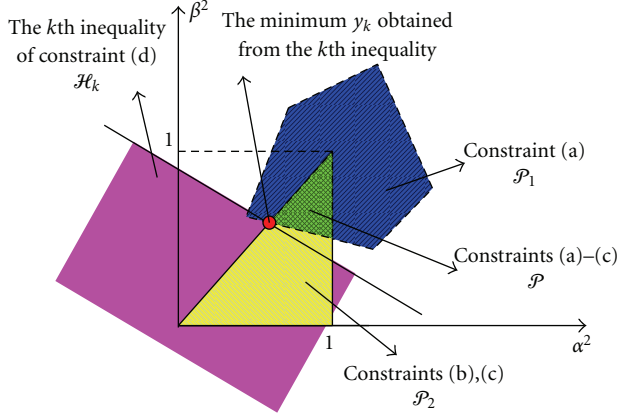
FIGURE 1: Geometric interpretation of the solution of the optimization problem of (48).

subcarriers of the strong user is above some threshold η, that is,

$$\max_{\mathbf{x}=[\alpha^2,\beta^2]^T} \min_k \quad s_2(k)$$

$$\text{s.t.} \quad \text{(a)} \ s_1(k) \geq \eta, \quad k = 0,\ldots,K-1;$$
$$\text{(b)} \ [0,0]^T \leq \mathbf{x} \leq [1,1]^T; \tag{46}$$
$$\text{(c)} \ [-1,1]\mathbf{x} \leq 0,$$

where the last constraint means that user 2 never spends more energy than user 1 when helping user 1. The threshold depends on the applications that user 1 needs to transmit, and is here assumed given. The lower the threshold, the more help user 1 will provide. The advantage for user 1 is that if the user has subcarriers on which the SINR is less than η, the situation on those subcarriers will improve.

A more standard form of the above problem is

$$\min_{\mathbf{x}=[\alpha^2,\beta^2]^T} \max_k \quad \frac{1}{\mathbf{s}_2(k)}$$

$$\text{s.t.} \quad \text{(a)} \ \mathbf{s}_1(k) \geq \eta, \quad k = 0,\ldots,K-1;$$
$$\text{(b)} \ [0,0]^T \leq \mathbf{x} \leq [1,1]^T; \tag{47}$$
$$\text{(c)} \ [-1,1]\mathbf{x} \leq 0.$$

or equivalently,

$$\min_{\mathbf{x},y} \quad y$$

$$\text{s.t.} \quad \text{(a)} \ \mathbf{s}_1(k) \geq \eta, \quad k = 0,\ldots,K-1;$$
$$\text{(b)} \ 0 \leq \mathbf{x} \leq 1; \tag{48}$$
$$\text{(c)} \ [-1,1]\mathbf{x} \leq 0;$$
$$\text{(d)} \ \frac{1}{\mathbf{s}_2(k)} \leq y, \quad k = 0,\ldots,K-1.$$

Since the denominators of $s_j(k)$, $j = 1,2$ are polynomials of $\mathbf{x}$, finding the solution of the problem of (48)

is not easy. We will proceed by making some simplifying assumptions. Let us assume that the interuser channels are quite good and that the noise at the user end is very small so that σ_{12}^2 and σ_{21}^2 are negligible as compared to σ_{01}^2 and σ_{02}^2. Since the coefficients of the high orders of $\mathbf{x}$ are linear combinations of σ_{12}^2 and σ_{21}^2, the high orders of $\mathbf{x}$ can be ignored. Therefore, the denominators of $s_j(k)$, $j = 1,2$ can be approximated as a linear function in $\mathbf{x}$. Let $1/s_2(k)$ be represented by $(a_k\alpha^2 + b_k\beta^2 + c_k)/(\tilde{a}_k\alpha^2 + \tilde{b}_k\beta^2 + \tilde{c}_k)$. Finding the solution of (48) is based on the following observations.

(1) The constraints (a)–(c) are linear so they give rise to the feasible set shown by a polyhedron $\mathcal{P}$ in Figure 1. The irregular pentagon $\mathcal{P}_1$ and a triangle $\mathcal{P}_2$ are formed by constraints (a) and (b)-(c), respectively.

(2) With a fixed y, the kth inequality in the constraint (d) yields a halfspace $\mathcal{H}_k$:

$$\frac{a_k\alpha^2 + b_k\beta^2 + c_k}{\tilde{a}_k\alpha^2 + \tilde{b}_k\beta^2 + \tilde{c}_k} \leq y \tag{49}$$

$$\left(b_k - \tilde{b}_k y\right)\beta^2 \leq (\tilde{a}_k y - a_k)\alpha^2 + (\tilde{c}_k y - c_k),$$

where $a_k, b_k, \tilde{a}_k, \tilde{b}_k > 0$.

(3) When y takes the minimum value 0, $\beta^2 \leq -(a_k/b_k)\alpha^2 - (c_k/b_k)$, $k = 0,\ldots,K-1$ and thus the feasible set is empty. As y increases, the dimension of the feasible set increases. There are several different scenarios.

(a) If $0 < y < (b_k/\tilde{b}_k)$,

$$\beta^2 \leq \underbrace{\left(-\frac{\tilde{a}_k}{\tilde{b}_k} + \frac{\left(\tilde{a}_k/\tilde{b}_k\right)b_k - a_k}{b_k - \tilde{b}_k y}\right)\alpha^2}_{u(y)}$$

$$+ \underbrace{\left(-\frac{\tilde{c}_k}{\tilde{b}_k} + \frac{\left(\tilde{c}_k/\tilde{b}_k\right)b_k - c_k}{b_k - \tilde{b}_k y}\right)}_{v(y)}. \tag{50}$$

(i) If $((\tilde{a}_k/\tilde{b}_k)b_k - a_k \geq 0 \ \& \ (\tilde{c}_k/\tilde{b}_k)b_k - c_k > 0)$ or $((\tilde{a}_k/\tilde{b}_k)b_k - a_k > 0 \ \& \ (\tilde{c}_k/\tilde{b}_k)b_k - c_k = 0)$, the halfspace $\mathcal{H}_k$ approaches $\mathcal{P}$ as y is increasing in $(0, b_k/\tilde{b}_k)$, and finally $\mathcal{H}_k$ intersects with $\mathcal{P}$. Let y_k denote the minimum y that the kth inequality of the constraint (d) yields. Then this y_k is achieved when $\mathcal{H}_k$ touches a vertex of $\mathcal{P}$.

(ii) If $(\tilde{a}_k/\tilde{b}_k)b_k - a_k \leq 0 \ \&(\tilde{c}_k/\tilde{b}_k)b_k - c_k \leq 0$, $v(y)$ and $u(y)$ are decreasing functions of y. Therefore, the halfspace $\mathcal{H}_k$ and $\mathcal{P}$ do not intersect within $(0, b_k/\tilde{b}_k)$.

(iii) If $((\tilde{a}_k/\tilde{b}_k)b_k - a_k < 0 \ \&(\tilde{c}_k/\tilde{b}_k)b_k - c_k > 0)$, or $((\tilde{a}_k/\tilde{b}_k)b_k - a_k > 0 \ \& (\tilde{c}_k/\tilde{b}_k)b_k - c_k < 0)$, y_k is achieved on a vertex of $\mathcal{P}$ if $y_k \in (0, b_k/\tilde{b}_k)$.

(b) If y_k does not exist when $0 < y < b_k/\tilde{b}_k$, we have to consider the scenario in which $y \geq b_k/\tilde{b}_k$. First, we consider $y = b_k/\tilde{b}_k$ and so $0 \leq ((\tilde{a}_k/\tilde{b}_k)b_k - a_k)\alpha^2 + ((\tilde{c}_k/\tilde{b}_k)b_k - c_k)$.

  (i) If $(\tilde{a}_k/\tilde{b}_k)b_k - a_k \geq 0$ & $(\tilde{c}_k/\tilde{b}_k)b_k - c_k \geq 0$, the intersection of $\mathcal{H}_k$ with $\mathcal{P}$ is $\mathcal{P}$ itself. Thus, $y_k = b_k/\tilde{b}_k$.

  (ii) If $((\tilde{a}_k/\tilde{b}_k)b_k - a_k < 0$ & $(\tilde{c}_k/\tilde{b}_k)b_k - c_k > 0)$ or $((\tilde{a}_k/\tilde{b}_k)b_k - a_k > 0$ & $(\tilde{c}_k/\tilde{b}_k)b_k - c_k < 0)$, we have

$$y_k = \begin{cases} \dfrac{b_k}{\tilde{b}_k}, & \text{if } \exists \alpha \in \mathcal{P} \text{ s.t.} \\[2mm] & 0 \leq \left(\dfrac{\tilde{a}_k}{\tilde{b}_k}b_k - a_k\right)\alpha + \left(\dfrac{\tilde{c}_k}{\tilde{b}_k}b_k - c_k\right) \\[2mm] > \dfrac{b_k}{\tilde{b}_k} & \text{and refer to (c), otherwise.} \end{cases}$$

$$(51)$$

  (iii) If $((\tilde{a}_k/\tilde{b}_k)b_k - a_k \leq 0$ & $(\tilde{c}_k/\tilde{b}_k)b_k - c_k < 0)$ or $(\tilde{a}_k/\tilde{b}_k)b_k - a_k < 0$ & $(\tilde{c}_k/\tilde{b}_k)b_k - c_k \leq 0$, $y_k > (b_k/\tilde{b}_k)$ then refer to (c).

(c) If y_k does not exist when $0 < y \leq b_k/\tilde{b}_k$, we have to consider the scenarios in which $y > b_k/\tilde{b}_k$ and so $\beta \geq u(y) + v(y)$. When $(\tilde{a}_k/\tilde{b}_k)b_k - a_k \leq 0$ & $(\tilde{c}_k/\tilde{b}_k)b_k - c_k \leq 0$, we can always find a feasible y_k.

In conclusion, we know

  (i) If $(\tilde{a}_k/\tilde{b}_k)b_k - a_k = 0$ & $(\tilde{c}_k/\tilde{b}_k)b_k - c_k = 0$, any elements in the set $\mathcal{P}$ can give rise to the minimum y, $y_k = (b_k/\tilde{b}_k)$. However, this case happens with small probability.

  (ii) Otherwise, y_k always falls on a vertex of $\mathcal{P}$.

Since K inequalities of constraint (d) need to be satisfied simultaneously, the optimal $\hat{\mathbf{x}} = [\hat{\alpha}^2, \hat{\beta}^2]^T$ is a vertex of $\mathcal{P}$ satisfying

$$\left(b_j - \tilde{b}_j y_j\right)\hat{\beta}^2 = \left(\tilde{a}_j y_j - a_j\right)\hat{\alpha}^2 + \left(\tilde{c}_j y_j - c_j\right), \qquad (52)$$

where y_j is the maximum value of the set $\{y_k, k = 0, \dots, K-1\}$ obtained from the K inequalities. Therefore, we can determine the optimal power allocation parameters $\mathbf{x}$ with the aid of the geometric interpretation of (48). The minimum y is the maximum value of $\{z_k \mid (b_k - \tilde{b}_k z_k)\hat{\beta}^2 = (\tilde{a}_k z_k - a_k)\hat{\alpha}^2 + (\tilde{c}_k z_k - c_k), k = 0, \dots, K-1\}$.

4. Simulation Results

In this section, we provide simulation result to illustrate the performance of the proposed full-cooperation (FC) and half-cooperation (HC) schemes. To illustrate the advantages of cooperation in addition to precoding, we compare the two
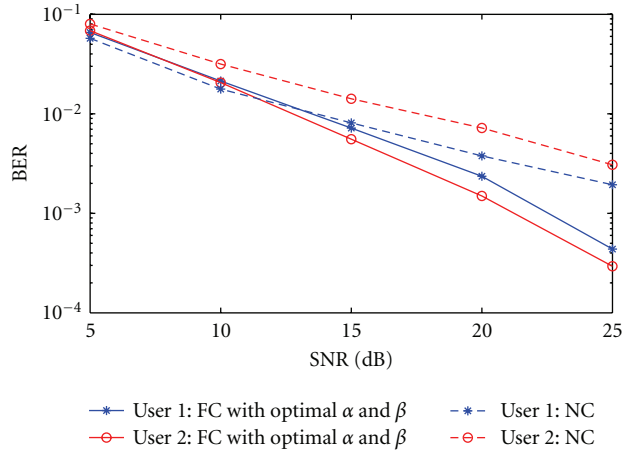

FIGURE 2: BER performance of each user with FC and NC, where user 1 has $N1 = 1$ and user 2 has $N2 = 2$ subcarriers in deep fade.

proposed approaches to a noncooperative scheme (NC) that uses the same interblock precoding strategy and is equivalent in terms of power consumption.

We consider an OFDM system with $N = 16$ subcarriers and 4QAM signals. We use the model of [20] to generate channels consisting of two equal power taps with normalized Doppler shift equal to 0.001. The channel is virtually static in order to eliminate temporal diversity due to by channel variation and thus highlight diversity due to cooperation and multipath. The SNR of the interuser channel is fixed at 30 dB. For the interblock precoding, we use 2×2 unitary matrices and group carriers into blocks of two. Two users exchange their subcarriers as described in Section 3.4.

In our simulations, we assign unit power to each OFDM symbol for the noncooperative scheme. The power of each OFDM block in the proposed cooperative schemes is determined by (23) and (40). This guarantees that cooperative and noncooperative schemes consume the same energy during a cycle of three slots. In the following figures and discussion the term SNR refers to the SNR for the noncooperative scheme, that is, the reciprocal of the noise power. We assume $\sigma_{01}^2 = \sigma_{02}^2$. We force user 1 and user 2 to have $N1$ and $N2$ deep-fading subcarriers, respectively. The variance of nondeep-fading subcarriers is set to 1 while the variance of subcarriers in deep fade is set to 0.001. We consider three cases where $\{(N1 = 1, N2 = 2), (N1 = 1, N2 = 3), (N1 = 2, N2 = 3)\}$.

Figures 2, 3, and 4 compare the BER performances of each user for FC and NC in three cases described above for SNR01 = SNR02. Since our proposed approach of optimizing α and β holds only when $\sigma_{01}^2, \sigma_{01}^2 \gg \sigma_{21}^2, \sigma_{21}^2$, we consider the scenarios of relatively small SNR01 and SNR02. Let the threshold in (48) for the SINR of user 2 over all the subcarriers be $\eta = [0.10, 0.10, 0.15, 0.20, 0.20]$ corresponding to SNR01 = SNR02 = $[5, 10, 15, 20, 25]$ dB, respectively. In each channel realization, we update the optimal α and β with knowledge of channel coefficients and noise variance. The procedure to determine the optimal α and β based on the analysis in Section 3.5 is sketched as follows:
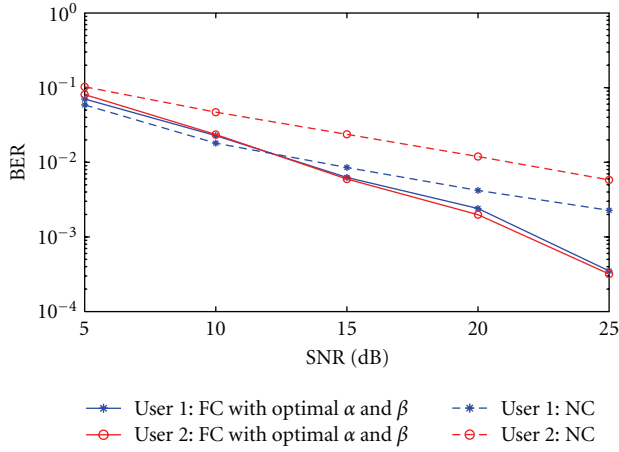
FIGURE 3: BER performance of each user with FC and NC, where user 1 has $N1 = 1$ and user 2 has $N2 = 3$ subcarriers in deep fade.
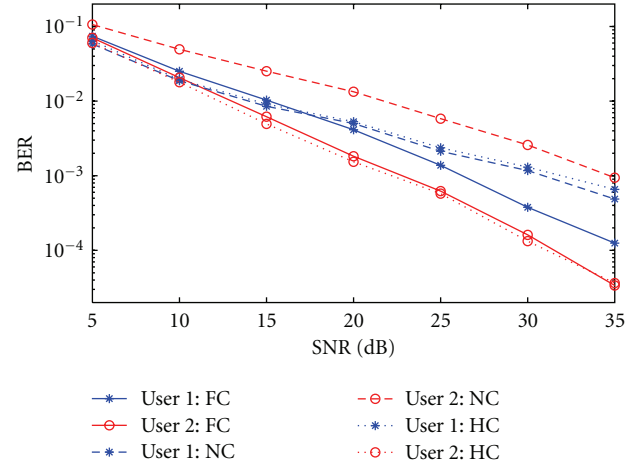

FIGURE 6: BER performance of each user with FC, HC and NC, where user 1 has $N1 = 1$ and user 2 has $N2 = 3$ subcarriers in deep fade.
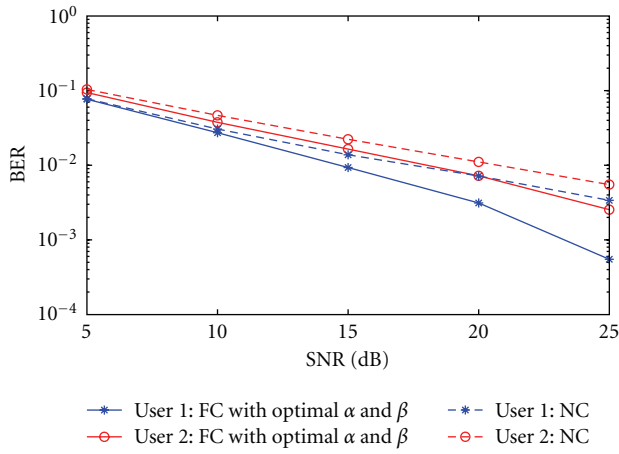

FIGURE 4: BER performance of each user with FC and NC, where user 1 has $N1 = 2$ and user 2 has $N2 = 3$ subcarriers in deep fade.
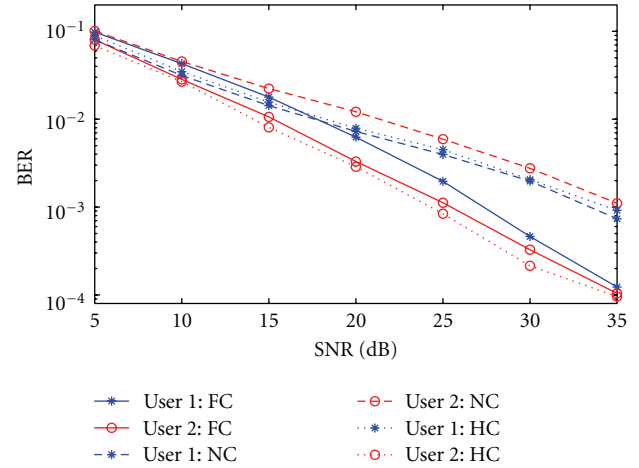

FIGURE 7: BER performance of each user with FC, HC and NC, where user 1 has $N1 = 2$ and user 2 has $N2 = 3$ subcarriers in deep fade.
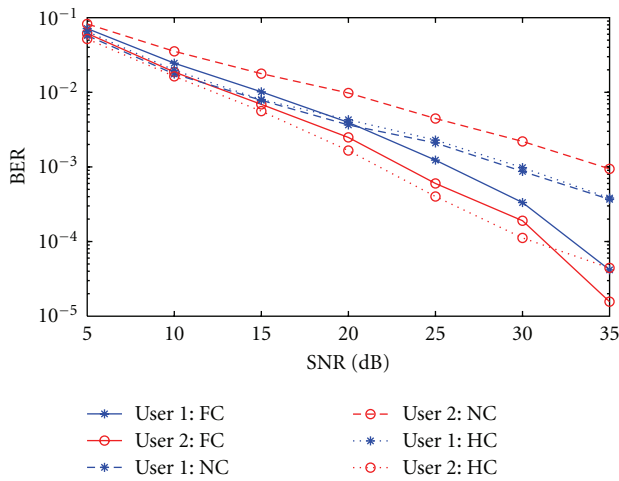
(1) We first find the vertices of the feasible sets of α^2 and β^2 satisfying the constraints (a)–(c) of (48);

(2) We determine the vertex that gives rise to the the minimum y_k for the kth constraint of (d) in (48), and record the value of y_k;

(3) The optimal solution y of (48) is the maximum element of the set $\{y_k, k = 0, \ldots, K - 1\}$. Based on that maximum value for y the optimal α and β are found via (52).

Figures 2–4 show that FC can significantly improve the performances of both users at higher SNR.

Figures 5, 6, and 7 show the BER performance of each user for the FC, HC and NC for SNR01 = SNR02 = 5 dB~ 35 dB. Both α and β are fixed to 0.5 for HC, while for FC it is taken $\alpha = 0.6, \beta = 0.3$. One can see that HC can significantly improve the performances of user 2 with

FIGURE 5: BER performance of each user with FC, HC and NC, where user 1 has $N1 = 1$ and user 2 has $N2 = 2$ subcarriers in deep fade.

a negligible penalty on the other user's performance as compared to NC. At low SNR, HC performs slightly better than FC with regards to user 2's performances. When the two users encounter relatively high SNR, the FC scheme can improve the performance of both users. When the antennas are not able to switch from one scheme to another, the FC scheme is always a wise choice regardless of the environment.

5. Conclusion

In this paper, we have proposed and compared two precoded schemes with user cooperation for two-user OFDMA systems. By analyzing the pairwise error probability of the proposed system, we have shown that the full-cooperation scheme can double the diversity available to both users without requiring additional transmitters. Therefore, the full-cooperation scheme can improve the BER performance of both users when the SNR of the users towards the receiver is relatively high so that the fading dominates the performance. On the other hand, when the SNR of two users is low, the half-cooperation scheme can achieve slightly better performance than the full-cooperation scheme. Furthermore, the use of interblock precoding, as compared to intrablock precoding, reduces the number of time slots required by the cooperative OFDM system to achieve the maximum diversity induced by cooperation. The extension of the proposed scheme to the multiuser case is not trivial; it involves selecting the users to cooperate with each other, or modifying the proposed scheme to render the cooperation of more than two users feasible. Such extension will be part of future work.

Acknowledgments

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