Modelling the trade off between period lenght and stages in a period batch control system

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MODELLING THE TRADE OFF BETWEEN PERIOD LENGTH AND STAGES IN A PERIOD BATCH CONTROL SYSTEM[1]

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SOM theme A: Intra-firm coordination and change

Abstract

Period Batch Control (PBC) is a production planning system that has strongly been propagated as a simple and effective instrument in obtaining the benefits of Group Technology, such as short throughput times and low work in progress. In order to obtain these benefits, PBC decomposes the manufacturing system in N stages and gives each stage the same amount of time P to complete the required operations. At the end of a period with length P the work is transferred to the next stage, and new work arrives from the preceding stage. One of the problems faced with when designing a PBC system is that there is little support from literature in the selection of a suitable period length for the stages. In this paper we address the problem of determining the period length P and the number of stages (and hence PBC periods) N, assuming the total manufacturing lead time T = N*P is held constant. We present an overview of factors that have to be taken into account when determining suitable values for N and P and formulate a mathematical model to gain insight in the inherent trade offs.

Keywords Group Technology; Period Batch Control; Cellular manufacturing

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[1] This is an extended version of the paper 'On the determination of the period length in a period batch control system', J. Riezebos, proceedings 32nd Matador Conference, Manchester, 10-11 July 1997.

1. Introduction

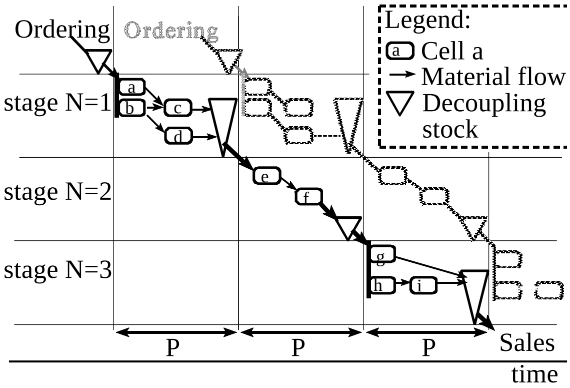

Figure 1.Stages and Periods in PBC

Batch manufacturing firms face the need to produce both efficiently and flexible in small batches and with short throughput times. One way to achieve this is to redesign their production system according to group technology principles. This results in a decomposition of the production system in stages that consist of various manufacturing cells. As group technology aims at producing several subsequent operations in a single cell, a cell has to be equipped with the technology and skills to perform these operations.

Period Batch Control (PBC) is a production planning system that has been proposed for application within group technology ([1],[2]). Essential for PBC is the periodicity of the system. The total manufacturing throughput time T is the same for all products. T is divided into a number N of equally lengthened periods (length P), so T=N*P. Each stage in the manufacturing system has exactly one period available to complete the required operations, hence N equals the number of stages in the manufacturing system. Figure 1 illustrates this system for N=3.

Figure 1 shows that the cells in each stage produce within one period the next period requirements of the cells in the succeeding stage. The time required for performing these activities may not exceed the length of the period. In this way PBC makes it possible to synchronize the activities that are performed in the various stages, as the system ‘guarantees’ that all activities in a stage are finished at the end of the period. Synchronization results in a smooth flow through the system and a rather transparent production plan. This transparency

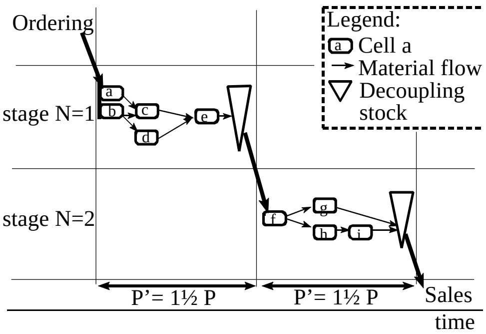

Figure 2: N=2

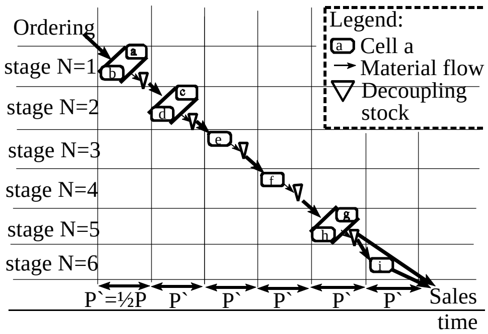

Figure 3: N=6

supports a good communication with the sales department and helps also in the identification of serious loading problems in the cells. A consequence of this periodicity is that all activities which have to be performed in the same period have the same release and due date, i.e. period start and finish.

An important problem we face when designing a PBC system is that literature gives little support in the selection of the number of stages and a suitable period length for these stages. The length of the manufacturing throughput time T is often exogenously determined, especially in case the firm makes to order. However, if T is given, there are still some alternatives in determining N and P (see fig 2+3).

The main question in this paper is what factors have to be taken into account when determining suitable values for the period length and the number of periods in a PBC system. The paper is organized as follows: In section 2 we present the results from a literature review. In this section we conclude that existing literature often implicitly assumes that the total manufacturing lead time T changes when N or P is varied (T=N*P). The implications of a change in P or N are therefore not considered on its own, but directly related to its consequences for the total manufacturing lead time T.

In this paper we assume that the size of T is held constant when varying N and P. Section 3 describes factors that have to be taken into account when determining N. Section 4 does the same for P. Section 5 presents an overview of relevant factors in the trade off between a high N combined with a small P, and a small N combined with a high P, holding T constant. We develop a simple mathematical model to show implications of the selected values for N and P. The paper ends with our conclusions.

2. Literature review

Literature on PBC can be distinguished in papers that describe the principles of the system and its applicability in specific situations (e.g., [1], [2], [7], [11]) and papers that analyse the performance of the system in comparison with other planning systems (e.g., [6], 10]).

The first type of paper presents some guidelines for the use of PBC (e.g., [4] and [7]):

1 there must be enough capacity to complete all the parts/products ordered each period;

2 it must be possible to complete each batch of parts/products in one period; and

3 effective capacity (which is reduced by setup activities) has to be acceptably high.

The determination of the parameters of the PBC system (including N and P) does not directly follow from these guidelines. We have to look more specific to this literature to detect how these parameters can be determined. First, we will describe the order of determining N and P. Next, we describe the effect of varying N and finally we describe the effect of varying P.

Order of determining N and P

In the guidelines presented above, Burbidge assumes the production activities that have to be performed in the period are already known when the length of the period has still to be determined. This means, for example, that cells have been designed and part families have been defined and allocated to these cells, and that information is available on the size of the batches. In his work on Production Flow Analysis (PFA) he states that already during PFA's simplification phase by factory flow analysis the decision about the division of the material flow system in several stages should be made (page 53 of [3]). Burbidge therefore first determines the number and contents of the stages and afterwards the period length P.

Zelenovic and Tesic [11] choose first the 'operating period' P and use for this decision information from the bill of materials, production programme and the production system itself. The operational groups are formed afterwards. Note that the sequence of the processes that are to be performed in the production system is known when the decision on the period length is taken, but not the division in stages and cells. This is contrary to New[7], who assumes that the stages are given and therefore describes how a suitable value for P can be found.

Literature that analyses the performance of PBC, such as Kaku and Krajewski [6] and Yang and Jacobs[10], uses a fixed number of stages (both papers used N=2) and varies the length of P in the experimental design.

The preliminary conclusion from this survey is that most authors first determine the number of stages N and afterwards a suitable period length P. We will apply the same sequence in discussing the contribution of literature on determining both parameters.

Effect of varying N

Burbidge is the only author who gives attention to the effect of varying N. He mentions that, in general, he has tended to accept as stages the existing processing stages found in traditional factories, which are normally bounded by stores (see [4]). Subcontracting operations, such as painting and anodising, have been treated as an extra stage.

In [3] he states that the number of stages has a significant effect on both the investment in stocks and costs, as each period that is added to the standard ordering scheme increases the throughput time, increases the work in progress, and reduces flexibility to follow market changes. According to Burbidge, N depends mainly on the number of *unavoidable* processing stages. Furthermore, N can also be affected by the processing throughput time, for example if the assembly lead time is long compared with the time required for the production of parts. In that case it may be necessary to allow two periods for assembly. The main disadvantage is that increasing the number of stages directly affects the total throughput time T.

Note that all effects of varying N mentioned so far can be assigned to the main effect of the increase in T.

Effect of varying P

Burbidge [1] says on the choice of period length: ‘The problem when choosing the programming and ordering period, is to balance the gains with a short period, such as a reduced investment in work in progress and an increase in the flexibility to follow market changes, against the losses which may be caused by an increase in the number of set-ups. Ideally, the period would be used both in assembly and component processing groups’.

New [7] notes that the total processing time of products in a stage (the ‘machining content of components’) determines the need for a larger period length (6 weeks instead of 2 or 4). The shorter the period length, the more flexible the production system will be, and the less uncertain the forecast of the sales in period N+1, as this sales period is reached earlier in time.

Whybark [9] emphasizes the positive effects of a shorter period length, and mentions a quicker customer response, a higher potential market share, a higher percentage orders that can be made strictly to order, and a lower cycle stock. On the other hand, a higher period length will result in higher manufacturing efficiency, fewer setups, lower manufacturing costs, and larger purchase quantities.

Yang and Jacobs [10] found that a higher P results in a decreasing mean order tardiness, and less variety in order tardiness. Process dependability therefore increases as P increases. However, this is accompanied with an increase in the work in progress and the various stocks in the system. They therefore conclude that increasing the period length does not seem like a good alternative to improving delivery performance.

From this survey we conclude that most effects of a smaller P that are mentioned sofar can be explained by the resulting reduction in the total manufacturing lead time T. The same was true for the effect of a reduction in N. This raises the question what effects of varying N and P remain significant if the total manufacturing lead time T is held constant. In other words, what are the advantages if we vary N with P=T/N, T fixed? We will answer this question in section 5. First, we identify important factors when determining N and P.

3. On determining N

Is there any problem if the number of periods in a PBC system (N) is determined through the detection of the number of ‘unavoidable processing stages’ in the production system, as is done by Burbidge? The main problem is that the number of processing stages does not give any insight in the complexity of the coordination problem in a stage. A production planning system should handle this complexity issue in an adequate way. By taking the technologically based division of the production system as design criterion for the production planning system, no attention is given to the possibility of the cells within the stages to handle the remaining planning and control problem. We will illustrate this argument first for sequential coordination between cells in a stage (see [8]), and next for coordination within a cell.

Sequential coordination between cells in subsequent stages is properly handled by PBC, as PBC synchronizes the flow from period to period through the cyclic scheduling procedure. However, if the stage is defined using the types of processes applied (stage=processing stage), this can result in the presence of multiple cells that deliver to each other within the same stage. PBC does not accommodate for the required sequential coordination between these cells. Therefore, an informal system has to cope with the remaining coordination within and between the cells in a stage, as described in [2]. This can result in serious problems. If the coordination of flows between the cells conflicts with a proper sequencing of products within the cell, the possibility of using close-scheduling deteriorates quickly (close-scheduling, i.e. transfer batch << process batch, is propagated in [2] for products that require many operations in a stage). Finally, with an informal system problems occur if processes require the contents of the batches to change between the cells in a stage.

PBC does not support the coordination problem within a cell. This type of coordination is often assumed to be rather easily solvable. However, the number of stages in PBC influences the complexity of the coordination problem within the cell. Increasing the number of stages can lead to less variety in the number of operations per product within a stage, and this results in standardization and transparency. The coordination complexity is also influenced by the production system used within the cell (flow shop or job shop). It is therefore too easy to suppose that the intracell coordination can easily be accomplished for by the informal planning system (see [5]). If this were true, a PBC system with N=1 would be optimal.

4. On determining P

All required operations for a product in each stage j have to be performed within one period of length P. The number of operations for a product in stage j and therefore the division of all operations over these stages depends on N. Generally, some slack time is available for the group of operations of a product in stage j. In this paragraph we develop a mathematical model that helps to determine P.

A general restriction on P can be described as:

(1) $P \geq \sum_{i=1..n_{jh}} \{s_i + q_h * p_i\}$ $\forall$ stages j=1..N and $\forall$ products h,

where n_{jh} is the number of operations for product h that have to be performed in stage j, s_i is the setup time required for operation i, q_h is the batch size of product h, p_i is the processing time of operation i.

Note that if $(P- \sum_{i=1..njh} \{s_i + q_h*p_i\}) < 0$, a feasible schedule can still exist. However, such a schedule requires close-scheduling: subsequent operations at a product in a stage are performed in parallel. If only the first part of a batch is finished at machine 1, this part is already transferred to the second machine for processing. Meanwhile, the second part of the batch is processed in the first machine, etcetera. Close-scheduling can be applied if the distance between the machines is small, the parts can easily be transported, and the machine operators are willing to cooperate in finishing the complete batch as soon as possible. If cellular manufacturing is applied, these conditions generally hold, making close-scheduling possible. However, the willingness of operators to cooperate generally decreases if the number of products that require close-scheduling in a stage increases.

The following expression results for P in case close-scheduling is allowed (we assume that each operation requires a different machine and there is only one machine available for an operation):

(2) $P \geq \max_{i=1..njh} \{r_i + q_h*p_i+\sum_{t=i+1..njh} p_t \}$ $\forall$ stages j=1..N and $\forall$ products h,

where r_i is the earliest starting time of operation i: $r_i = \max_{k=1..i} \{s_k+\sum_{t=k..(i-1)} p_t\}$

if r_{i-1} exists, this expression is equal to: $r_i = \max [s_i, r_{i-1}+p_{i-1}]$.

To this formulation we can add a restriction with respect to the maximum capacity of each machine in a period. Let machine k perform at most one operation of each product h in stage j. This operation we call k(h). The following expression results:

(3) $P \geq \sum_h \{s_{k(h)} + q_h *p_{k(h)}\}$ $\forall$ machines k

We restrict the number of products that might be close-scheduled in a period to C. Let:

I_j^h = 0 if close scheduling of product h in stage j is not required,

= 1 if h has to be close-scheduled in stage j and this is possible within period length P,

= ∞ otherwise (i.e. h cannot be scheduled within period length P of stage j)

The problem of determining P is now stated as:

Select P such that (3) , (4) and (5) hold:

(4) $P \geq (1-I_j^h) * \sum_{i=1..njh} \{s_i + q_h*p_i\} + I_j^h *\max_{i=1..njh} \{r_i + q_h*p_i+\sum_{t=i+1..njh} p_t\}$

(5) $\sum_h I_j^h \leq C$

$\forall$ stages j=1..N and $\forall$ products h.

This formulation of the problem does not guarantee that the close-scheduling effort in a stage is equally distributed among the cells in this stage. If all close-scheduling effort is concentrated in one cell, this cell will face long waiting times for the close-scheduled products, and will therefore not be able to approximate the estimated throughput times. A direct computation of the work load for the various processes in a cell would give more precise information on the expected waiting time and the possibility of close scheduling if these processes are involved. This is very useful if cells share the use of some processes (shared resources).

An important characteristic of the PBC system is the cyclic nature of it. At the start of each period a new amount of work arrives in the cells within the stage and at the end of the period all work has to be finished. This significantly affects the net capacity of processes that are preceded or succeeded by another process in the same stage. If the length of the period decreases and the precedence relation remains the same within the period, these processes have to wait more often on the preceding process or have to stop more frequently, to let the succeeding process finish its work within the period. The time required for these start-up or finish activities is independent of P, so, the preceding or succeeding processes are faced with a decrease in net capacity. We call this the start/finish effect.

A related effect is known as the set-up time effect. This effect is mentioned in most literature on PBC(e.g., [2], [7]). The period length determines the number of production cycles of each product per year. If the demand of product h per time unit (D_h) is equally distributed over the periods, then $q_h = D_h * P$, P expressed in the same time unit. If P decreases, the batch size $q_h=D_h*P$ also decreases, but the process has still to be set up for all products h, so the total required setup time per period remains constant. The net capacity therefore decreases, resulting in less slack for this process.

5. Trade off between N P and N P

Given that T = N*P is held constant, Table 1 presents relevant factors in choosing the relative size of N and P. We use the basic input/output model to describe the positive effects of the relative size of N and P on system input, output, process and control.

Table 1: Positive effects of a choice for either (N small, P large) or (N large, P small), T=N*P constant

Factor	**N small, P large**	**N large, P small**
Input	Increased mix flexibility	Less material in process through just-in-time delivery
Process	Fewer problems with long processing times Less start/finish losses Less set-up time losses More attractive work packages per process	Decreased technology and skills variety Higher utilization of bottlenecks Less variety in number of operations per stage Less close-scheduling effort

Control	Less programming efforts	Easier subcontracting Easier coordination of shared resources Easier sequential coordination between cells Better progress control Better synchronization
Output	Less forecasting effort More levelled demand variations per period	Less finished stock

The frequency of input in the system decreases in case N becomes smaller. Figure 4 shows that if both the period length is doubled and the number of periods halved, then the material that was formerly required in the second period has now to be available at the start of the first period, and so on. This early shipment causes an increase in the investment in stocks, but makes it also possible to improve the selection of work for a process within a period. The latter makes it easier to balance the load within a stage and hence allows an increase of the mix flexibility.

Many positive effects for the organization and utilization of the process in case P increases have already been mentioned in section 4. Generally, an increase in P makes a higher utilization of the various processes possible. However, we found two anomalous effects of an increase in P.

The first effect is the possibly higher utilization of a bottleneck in case P decreases. Figure 5 illustrates this effect. The upper side of this figure shows a situation with 2 stages. The bottleneck is in the second stage. Close-scheduling is applied to finish the second operation within the period length P. The maximum capacity is 4 products per period P.

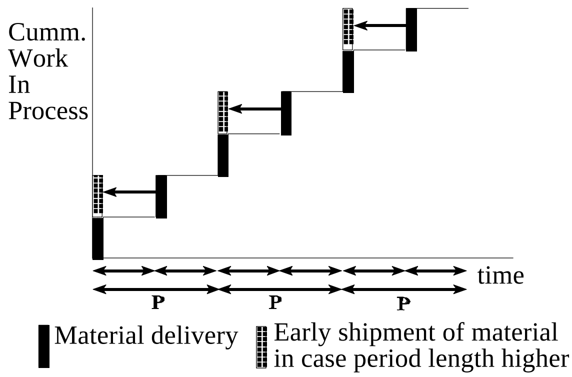

Figure 4: Increase of WIP if P and N

The lower side of figure 5 shows the combination of the two stages to a new stage with a doubled period length 2P. The batch size has to increase to 8 products to realize the same output. The combination of the bottleneck with a preceding process causes a longer waiting time before the bottleneck can start, and hence it is not possible to finish the 8 products within the doubled period length.

The second remarkable effect that we found concerns the possibility of a decrease in close-scheduling requirements if P decreases. This effect can easily be proven using the model that we presented in section 4. Let I_{12} denote the close-scheduling requirement in the new stage 12 that is a combination of stages 1 and 2. Note that the period length is doubled in stage 12. The following results are obtained:

(6) $P(I_{12}=0 \mid I_1=I_2=0) > 0$

(7) $P(I_{12}=1 \mid I_1=I_2=0) > 0$

(8) $P(I_{12}=\infty \mid I_1=I_2=0) > 0$

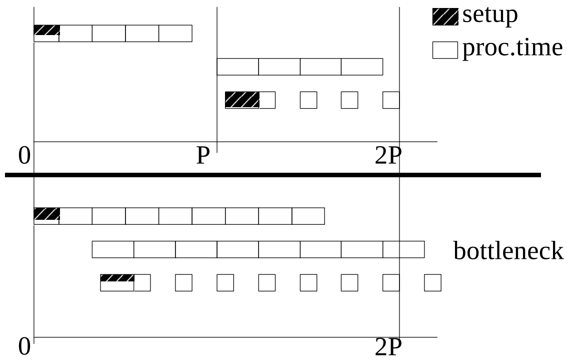

Figure 5: Higher bottleneck utilization if P , N

In words this means that there is a probability that close-scheduling is required and that it even may not be possible to close-schedule the product in the doubled period length, although it was not necessary to close-schedule the product in the two distinguished periods. The proof of these is straightforward and presented in the appendix.

A related result is (9): $P(I_{12}=0 \mid I_1=I_2=1) = 0$,

which says that the contrary effect (no close scheduling required in the longer period, while it was necessary in the two separate stages) does not hold (proof also in the appendix).

The control and coordination effort increases as a consequence of the increase in close-scheduling requirements if P increases. The informal planning system within a stage has to cope with this coordination, as PBC does not support the planning within a stage. If there are subcontracting activities that can only take place after some processes in a stage have been finished, the coordination of these activities becomes more problematic. The same holds for the use of shared resources. Isolating such activities or resources in a single stage makes their coordination easier. The focus of the coordination changes to coordination between the stages, and PBC accommodates this sequential coordination through its synchronization mechanism. The increased number of stages makes it necessary to organize more frequently a programme meeting for determination of the production programme in the subsequent period. This requires more time from management, more information gathering and forecasting effort, but it leads also to a better progress control.

Finally, the output of the system is less sensitive for variations in demand if the period length is larger. However, a smaller period leads to less investment in finished stock or to more frequent delivery to the customer.

6. Conclusions

This paper has treated the important trade off between the length of a period P and the number of periods N in a period batch control system. Literature gives attention to the possibility of shortening the total manufacturing lead time T through decreasing N and P. In this paper we have stressed that there is also an important trade off between N and P if the total manufacturing lead time is not affected.

The complexity of the coordination of the manufacturing system has to be taken into account when choosing N. A simple mathematical model helps to determine the minimum length of a period. This model is mainly used to illustrate some problems with various sizes of P, such as the start/finish effect, the setup time effect, and the need for close-scheduling.

Finally, the trade off between N and P is treated through the presentation of a table of benefits if one chooses for either a PBC system with small N and high P or large N and small P. Some anomalous effects could be illustrated using the mathematical model.

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Appendix

Suppose we have a product of which q units have to be produced subsequently in a number of stages. In stage 1 'a' operations are required, in stage 2 'b' operations. If close-scheduling was not required in stage j, this is indicated by the variable I_j being 0. Now suppose the period length P is doubled to 2P and the stages 1 and 2 are conjuncted to a new stage 12. In this new stage a total of 'a+b' operations have to be performed. Additionally, the number of products to be made in a period increases to 2q.

The question is now what will happen to the variable I_{12}, e.g. the close-scheduling requirement in the new stage 12. The following results are obtained:

(6) $P(I_{12}=0 \mid I_1=I_2=0) > 0$

(7) $P(I_{12}=1 \mid I_1=I_2=0) > 0$

(8) $P(I_{12}=\infty \mid I_1=I_2=0) > 0$

(9) $P(I_{12}=0 \mid I_1=I_2=1) = 0$

Let $I_1=I_2=0$ (no close-scheduling needed).

So: $\sum_{i=1..a} \{s_i + q*p_i\} \leq P$ and

$\sum_{i=a+1..a+b} \{s_i + q*p_i\} \leq P$

$I_{12}=0 \iff q*\sum_{i=1..a+b} p_i \leq 2P-\sum_{i=1..a+b}\{s_i+q*p_i\}$

It can easily be seen that for specific values of q, s_i, and p_i this inequality will not hold, making close-scheduling necessary.

In some cases close-scheduling will even not be possible within 2P:

For $P(I_{12}=\infty \mid I_1=I_2=0)>0$ we have to proof :

q, a, b, s_i, p_i (i=1..a+b) such that:

(10) $\sum_{i=1..a} (s_i+q*p_i) \le P$

(11) $\sum_{i=a+1..a+b} (s_i+q*p_i) \le P$

(12) $\max_{i=1..a+b}\{r_i+2*q*p_i+\sum_{t=i+1..a+b} p_t\} > 2P$.

Figure 6 shows an example that proofs this.

The proof of (9) that $P(I_{12}=0 \mid I_1 = I_2 = 1) = 0$ is straightforward: Given that:

(13) $\sum_{i=1..a} \{s_i + q*p_i\} \ge P$

(14) $\max_{i=1..a} \{r_i + q*p_i + \sum_{t=i+1..a} p_t\} \le P$

(15) $\sum_{i=1..b} \{s_i + q*p_i\} \ge P$

(16) $\max_{i=1..b} \{r_i + q*p_i + \sum_{t=i+1..b} p_t\} \le P$

we have to proof:

(17) $\sum_{i=1..a+b} \{s_i + 2*q*p_i\} > 2P$

which is equivalent with $P(I_{12}>0 \mid I_1=I_2=1) = 1$

Proof of (17) follows directly from:

$\sum_{i=1..a+b}\{s_i+2*q*p_i\} > \sum_{i=1..a} \{s_i+q*p_i\} + \sum_{i=1..b}\{s_i+q*p_i\} \ge P+P$.

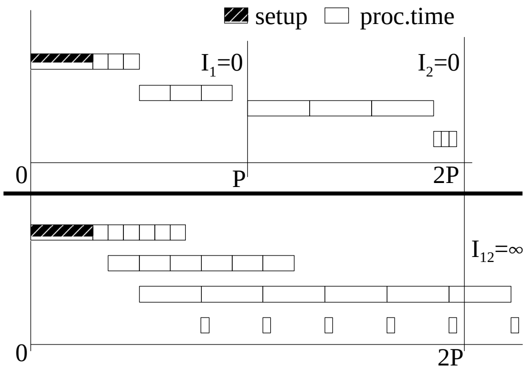

Figure 6: $P(I_{12}=\infty \mid I_1=I_2=0) > 0$