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# A necessary condition and a backstepping observer for nonlinear fault detection

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## Abstract

A formulation of the fundamental problem of residual generation for fault detection in nonlinear systems based on the concept of output invariance is given. Consequences are derived in the geometric context. A necessary condition, which recovers the same result in the case of linear systems, and a normal form suitable for the solution of the problem are obtained. The recently introduced backstepping observer is then used as a residual generator.

## 1 Introduction

The theory of fault detection and isolation (FDI) deals with the conditions under which a filter exists and is able to "deduce", with reasonable accuracy, if a fault has appeared. Isolation, in particular, means that, among possibly many faults, the device has to specify which fault is really occurring. This task is usually performed in the presence of contemporary faults, disturbances and uncertainties. The detection and isolation of faults is carried out by endowing the filter with outputs signals, called residual signals - one for each possible fault - which become nonzero if the corresponding fault arises. A possible formulation of the problem is to require the residuals to be completely decoupled by signals which differ from the fault. That is, after a transient period, in which the initial conditions act on the residual, the residual itself can be considered practically zero if the corresponding fault has not appeared yet.

The problem of detecting and isolating faults for linear systems (by requiring the complete decoupling of the residual from all what is different from the fault) was formulated and solved in [18]. In particular, the Fundamental Problem of Residual Generation (FPRG) and the Extended Problem of Residual Generation (EPRG) were introduced ([18]). The solutions of these two problems show how to construct an FDI filter (or *residual generator*) by exploiting traditional concepts from linear control theory, such as unknown input observers

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([2], [3], [4]), eigenvalue assignments through output injection, (conditioned) invariant subspaces. This (geometric) approach to FDI is considerable because it leads to state precise, concise and constructive necessary and sufficient conditions for the problem resolution. Attempts to generalize this approach to more general classes of systems have been carried out. See e.g. [10] for a survey, the references therein, and [20]. In [5], [8], some geometric conditions are proved to be sufficient for the existence of a linear up to input/output injection FDI filter. A detailed exposition of (other) geometric solutions to the problem of FDI for nonlinear input-affine systems can be found in [6], [7], [9], to which the interested reader is referred for the proofs of some of the results given here.

One of the aims of this paper is to investigate some necessary conditions for the problem of FDI for nonlinear input-affine systems. They can be derived from the formulation of the problem stated using the concept of output invariance. In particular this leads to see how naturally the concept of conditioned invariant distributions ([14]) arises in the nonlinear FPRG, in the same way as conditioned invariant subspaces ([1]) arise in the linear FPRG. The arguments are similar to those used in [17] for the problem of output tracking with disturbance rejection. Moreover, we delineate under which conditions the recently introduced backstepping approach to observer design ([15]) can be used as an FDI filter. So, in Section 2 we give geometric characterizations of the nonlinear version of the FPRG which closely resemble the linear analogue when expressed in geometric terms; in Section 3, these characterizations are used to derive necessary conditions and to comment some assumptions; in Section 4 the backstepping observer approach is applied to the problem; conclusions are drawn in Section 5.

## 2 Output invariance and fault detection

Consider the *analytic* systems (we will relax such assumption in regularity hypothesis - see Remark 2):

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^M g_i(x)u_i \\ y_j &= h_j(x), \quad j = 1, \dots, p \end{aligned} \quad (1)$$

where the input functions may represent control, disturbance and fault signals. The  $j$ -th output  $y_j(t; x^0; u_1, \dots, u_M)$  corresponding to an initial condition  $x^0$  and to the set of input functions  $u_1, \dots, u_M$  is *unaffected* by (or *invariant* under)  $u_i$  if, for any initial condition  $x^0$  in  $U$  ( $U$  being the open set of  $\mathbb{R}^n$  - or, more abstractly, the state space manifold, - on which the system is defined) and any collection of *admissible* input functions  $u_1, \dots, u_{i-1}, u_{i+1}, \dots, u_M$ , there holds:

$$\begin{aligned} y_j(t; x^0; u_1, \dots, u_{i-1}, u_a, u_{i+1}, \dots, u_M) = \\ y_j(t; x^0; u_1, \dots, u_{i-1}, u_b, u_{i+1}, \dots, u_M) \end{aligned}$$

for all  $t \geq 0$  and any pair  $u_a, u_b$  (for a detailed treatment of *output invariance* see [13], Paragraphs 3.1-3.3). We say that  $y_j$  is *affected* by  $u_i$  if it is *not* unaffected by  $u_i$ .

We can give now the formulation of the NLFPRG. Considered the system

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{\mu} g_i(x)u_i + \ell(x)m + \sum_{i=1}^s p_i(x)w_i \\ y_j &= h_j(x), \quad j = 1, \dots, p \end{aligned} \quad (2)$$

defined on a neighborhood  $U$  of the origin, with  $f(0) = 0$ ,  $h(0) = 0$ , and  $m$  a *scalar fault* signal, find, if possible, a filter

$$\begin{aligned} \dot{\hat{x}} &= \hat{f}(y, \hat{x}) + \sum_{i=1}^{\mu} \hat{g}_i(y, \hat{x})u_i \\ r &= \hat{h}(y, \hat{x}) \end{aligned} \quad (3)$$

$\hat{x} \in \mathbb{R}^{\nu}$ ,  $r \in \mathbb{R}^{\tilde{p}}$ ,  $1 \leq \tilde{p} \leq p$ , with  $\hat{f}(0,0) = 0$ ,  $\hat{h}(0,0) = 0$ , such that, when considered the *cascaded system* (with obvious significance of the symbols)

$$\begin{aligned} \dot{x}^e &= f^e(x^e) + \sum_{i=1}^{\mu} \hat{g}_i^e(x^e)u_i + \ell^e(x^e)m + \\ &\quad \sum_{i=1}^s p_i^e(x^e)w_i \\ r &= h^e(x^e) \end{aligned} \quad (4)$$

defined on  $U^e$ , a neighborhood of  $(x, \hat{x}) = (0,0)$ , the following properties hold: (i) if  $m = 0$ , then  $r$  is unaffected by  $u_i, w_j, \forall i, j$ ; (ii)  $r$  is affected by  $m$ ; (iii)  $\lim_{t \rightarrow \infty} \|r(t; x^0, \hat{x}^0; u_1, \dots, u_{\mu}, 0, w_1, \dots, w_s)\| = 0$  for any initial condition  $x^0, \hat{x}^0$  in a suitable set containing the origin, and any set of admissible inputs (note that the convergence to zero of the residual is required in absence of fault -  $m = 0$ ).

For the time being we refer to the NLFPRG without the stability requirement (iii).

A similar formulation for the class of state-affine systems was given in [11].

**Remark 1** It is not difficult to notice that a filter which solves the NLFPRG in particular assures that at least for an initial condition  $(x(0), \hat{x}(0)) \in U^e$ , a certain collection  $(u_1, \dots, u_{\mu}, w_1, \dots, w_s)$ , a certain time

$\bar{t} \geq 0$ , (at least) an index  $j \in \{1, \dots, \tilde{p}\}$  and a certain fault signal  $\bar{m}$ , its  $j$ -th residual satisfies

$$\begin{aligned} r_j(\bar{t}; x^0, \hat{x}^0; u_1, \dots, u_{\mu}, \bar{m}, w_1, \dots, w_s) \neq \\ r_j(\bar{t}; x^0, \hat{x}^0; u_1, \dots, u_{\mu}, 0, w_1, \dots, w_s). \end{aligned}$$

Requiring  $r$  to be unaffected by the control and disturbance signals only when  $m = 0$  is not a restrictive condition. Indeed, when a fault has occurred what is important is that the residual is evidently nonzero. Regarding the possibility for the  $u_i$ 's,  $w_j$ 's to vanish the effect of  $m$  on  $r$ , we are assured this cannot happen by the requirement (ii). Such a condition, in fact, guarantees that the fault influences the residual, at least in the sense specified above, whatever the signals  $u_1, \dots, u_{\mu}, w_1, \dots, w_s$  are. Note also that, for linear systems, the NLFPRG reduces *exactly* (see [6], [7]) to the FPRG as introduced in [18].

It is known from [14] (see, for instance, [13], Lemma 3.3.1) that, as a consequence of the Fliess' functional expansion formula, condition (i) is equivalent to

$$L_{g_i^e} L_{\tau_1^e} \dots L_{\tau_{r^e-1}^e} h_j^e = 0, \quad i = 1, \dots, \mu \quad (6)$$

$$L_{p_i^e} L_{\tau_1^e} \dots L_{\tau_{r^e-1}^e} h_j^e = 0, \quad i = 1, \dots, s \quad (7)$$

for each  $j = 1, \dots, \tilde{p}$ , each sequence  $\{\tau_1^e, \dots, \tau_{r^e-1}^e\}$  in the set of vector fields  $\{f^e, g_1^e, \dots, g_{\mu}^e, p_1^e, \dots, p_s^e\}$  and each integer  $r^e \geq 1$  (if  $r^e = 1$ , the sequence is to be intended void). For the same reason, the requirement (ii) is equivalent to the existence of (at least) an index  $j \in \{1, \dots, \tilde{p}\}$  such that

$$L_{\ell^e} L_{\tau_1^e} \dots L_{\tau_{s^e-1}^e} h_j^e \neq 0 \quad (8)$$

where now  $s^e \geq 1$  is the minimum integer for which the previous condition is satisfied (if  $s^e = 1$  the previous disjunction coincide with  $L_{\ell^e} h_j^e \neq 0$ ). By virtue of condition (i) and the minimality of  $s^e$ , the vector fields of the sequence belong to a more restricted family, and in particular (6),(7),(8) are equivalent to (6),(7) and

$$L_{\ell^e} L_{f^e}^{k-1} h_j^e = 0, \quad 1 \leq k < s^e \quad (9)$$

$$L_{\ell^e} L_{f^e}^{s^e-1} h_j^e \neq 0 \quad (10)$$

for some index  $j \in \{1, \dots, \tilde{p}\}$  and a positive integer  $s^e$ . So we can conclude:

**Theorem 1** *The NLFPRG is solvable if and only if (6),(7), (9),(10) are satisfied.*

Let us introduce the *Observation Space* for (4)-(5) in absence of fault ( $m = 0$ ),  $\mathcal{O}_e$ , as the smallest vector subspace of all smooth functions containing  $h_1^e, \dots, h_{\tilde{p}}^e$  and closed under differentiation along the vector fields  $f^e, g_1^e, \dots, g_{\mu}^e, p_1^e, \dots, p_s^e$ . We also consider the associated codistribution

$$\Omega_{\mathcal{O}_e} := \text{span}\{d\lambda, \lambda \in \mathcal{O}_e\}$$

and its annihilator  $\Omega_{\mathcal{O},e}^\perp$ .

As a consequence of the relation between the coefficients of the Fliess' functional expansion formula and the functions which define the observation space (see [13], Remark 3.1.2), it is easy to realize that the requirement (i) is equivalent to  $\text{span}\{G^e, P^e\} \subset \Omega_{\mathcal{O},e}^\perp$ , with  $\text{span}\{G^e, P^e\} := \text{span}\{g_1^e, \dots, g_\mu^e, p_1^e, \dots, p_s^e\}$ . For the same reason, (ii), expressed as in (8), is equivalent to  $\ell^e \notin \Omega_{\mathcal{O},e}^\perp$ . Consequently, (i), (ii) are equivalent to:

$$\begin{aligned} \text{span}\{G^e, P^e\} &\subset \Omega_{\mathcal{O},e}^\perp \\ \ell^e &\notin \Omega_{\mathcal{O},e}^\perp \end{aligned} \quad (11)$$

Let us now recall that the distribution  $\Omega_{\mathcal{O},e}^\perp$  is invariant under  $\{f^e, g_1^e, \dots, g_\mu^e, p_1^e, \dots, p_s^e\}$  and is contained in  $\ker dh^e$  (by definition, since  $\Omega_{\mathcal{O},e}$  contains  $\text{span}\{dh^e\}$ ). Moreover, let  $\langle f^e, g_1^e, \dots, g_\mu^e, p_1^e, \dots, p_s^e | \text{span}\{dh^e\} \rangle$  be the smallest codistribution which is invariant under  $\{f^e, g_1^e, \dots, g_\mu^e, p_1^e, \dots, p_s^e\}$  and contains  $\text{span}\{dh^e\}$ . It is known (see [13], Lemma 1.9.1) that such a codistribution exists and is smooth. Then set:

$$Q^e := \langle f^e, g_1^e, \dots, g_\mu^e, p_1^e, \dots, p_s^e | \text{span}\{dh^e\} \rangle^\perp.$$

We know ([13], Lemma 1.6.3) that  $Q^e$  is a distribution invariant under  $\{f^e, g_1^e, \dots, g_\mu^e, p_1^e, \dots, p_s^e\}$ , it is contained in  $\ker dh^e$  and is the largest distribution with such two properties. As a consequence,  $\Omega_{\mathcal{O},e}^\perp \subset Q^e$ . Moreover, recalling that (see [13], Remark 2.3.3 and Lemma 1.9.5) around any regular point  $\Omega_{\mathcal{O},e}^\perp = Q^e$  and  $Q^e$  is involutive, it is possible to state the following:

**Theorem 2** *If the point  $(x, \hat{x}) = (0, 0)$  is a regular point of  $\Omega_{\mathcal{O},e}$ , locally around  $(x, \hat{x}) = (0, 0)$ , (11) hold if and only if*

$$\begin{aligned} \text{span}\{G^e, P^e\} &\subset Q^e \\ \ell^e &\notin Q^e \end{aligned} \quad (12)$$

As noted in [6], it is more convenient to focus the attention on the problem of determining a filter which satisfies (12) and the stability requirement (iii). So, following the terminology of [6], we shall henceforth consider this stronger version of the problem and refer to it as the *regular* version of the NLFPRG.

As a consequence of what stated above, in view of the results in [16] and [13], Paragraph 1.7, the system obtained cascading the filter which solves the NLFPRG to the process admits a particular normal form.

**Corollary 1** *The regular NLFPRG is solvable if and only if there exist a diffeomorphism  $\hat{x}^e = \Phi(x^e)$  which transforms the cascaded system into:*

$$\begin{aligned} \dot{\hat{x}}_1^e &= \hat{f}_1^e(\hat{x}_1^e) + \hat{\ell}_1^e(\hat{x}_1^e, \hat{x}_2^e)m \\ \dot{\hat{x}}_2^e &= \hat{f}_2^e(\hat{x}_1^e, \hat{x}_2^e) + \sum_{i=1}^{\mu} \hat{g}_{2,i}^e(\hat{x}_1^e, \hat{x}_2^e)u_i + \hat{\ell}_2^e(\hat{x}_1^e, \hat{x}_2^e)m + \end{aligned}$$

$$\begin{aligned} &\sum_{i=1}^s \hat{p}_{2,i}^e(\hat{x}_1^e, \hat{x}_2^e)w_i \\ r &= \hat{h}_1^e(\hat{x}_1^e) \end{aligned}$$

with  $\hat{\ell}_1^e$  nonzero on the domain of definition of the diffeomorphism and the system  $(\hat{h}_1^e, \hat{f}_1^e)$  observable<sup>1</sup>, and a set of initial conditions  $\hat{x}_1^e(0)$  such that  $\lim_{t \rightarrow \infty} \|\tau(t; \hat{x}_1^e(0); 0)\| = 0$ .

**Remark 2** So far, we have assumed the analyticity of the process and of the appended system. It can be noticed, however, (see, e.g., [19], Proposition 4.23) that the regularity assumption assures that the same conclusions can be drawn if process and filter are *smooth*.

**Remark 3** In [5], [8] a linearization up to I/O injection approach is used to tackle the regular NLFPRG. The class of nonlinear systems for which a filter exists which solves the problem is such that, in suitable coordinates, the plant-filter cascade can be written as:

$$\begin{aligned} \dot{e} &= Ae + \hat{\ell}_1^e(e, \hat{x}_2^e)m \\ \dot{\hat{x}}_2^e &= \hat{f}_2^e(e, \hat{x}_2^e) + \sum_{i=1}^{\mu} \hat{g}_{2,i}^e(e, \hat{x}_2^e)u_i + \hat{\ell}_2^e(e, \hat{x}_2^e)m + \\ &\sum_{i=1}^s \hat{p}_{2,i}^e(e, \hat{x}_2^e)w_i \\ r &= Ce \end{aligned}$$

with  $\hat{\ell}_1^e$  nonzero on the domain of definition of the system, the pair  $(C, A)$  observable and  $\sigma(A) \subset \mathcal{C}^-$ .

In Section 4, we shall consider a slightly different formulation of the NLFPRG, which can be obtained by relaxing in (i) the requirement  $m = 0$ . We refer to this formulation as the *strong* NLFPRG. As before, we shall adopt the regular version of the problem. Indeed, substantially with the same arguments, assuming additionally the regularity of the codistribution  $\bar{\Omega}_{\mathcal{O},e}$  associated to the smallest subspace of functions containing  $h_1^e, \dots, h_p^e$  and closed under differentiation along the vector fields  $f^e, g_1^e, \dots, g_\mu^e, \ell^e, p_1^e, \dots, p_s^e$ , and the nonsingularity and involutivity of the largest distribution  $\bar{Q}^e$  invariant under  $\{f^e, \ell^e\}$  and contained in  $\ker dh^e$ , it can be proved that the strong NLFPRG (without the stability requirement) is solvable if and only if

$$\text{span}\{G^e, P^e\} \subset \bar{Q}^e \quad (13)$$

$$\ell^e \notin \bar{Q}^e. \quad (14)$$

As before, finding the filter which satisfies these stronger conditions will be referred to the regular

<sup>1</sup>In the sense of [12] (see also [13], Theorem 1.9.7).

strong NLFPRG. The latter is solvable if and only if the system admits the normal form

$$\begin{aligned}\dot{\hat{x}}_1^e &= \hat{f}_1^e(\hat{x}_1^e) + \hat{\ell}_1^e(\hat{x}_1^e)m \\ \dot{\hat{x}}_2^e &= \hat{f}_2^e(\hat{x}_1^e, \hat{x}_2^e)m + \sum_{i=1}^{\mu} \hat{g}_{2,i}^e(\hat{x}_1^e, \hat{x}_2^e)u_i + \hat{\ell}_2^e(\hat{x}_1^e, \hat{x}_2^e)m + \\ &\quad \sum_{i=1}^s \hat{p}_{2,i}^e(\hat{x}_1^e, \hat{x}_2^e)w_i \\ r &= \hat{h}_1^e(\hat{x}_1^e)\end{aligned}$$

with  $\hat{\ell}_1^e$  nonzero on the domain of definition of the diffeomorphism and the subsystem  $(\hat{h}_1^e, \hat{f}_1^e, \hat{\ell}_1^e)$  observable, and there exists a set of initial conditions  $\hat{x}_1^e(0)$  such that  $\lim_{t \rightarrow \infty} \|r(t; \hat{x}_1^e(0); 0)\| = 0$ . That is, requiring (i) without  $m = 0$ , the disturbance and the control inputs can never affect  $r$ . This is due to the fact that the fault-residual behavior is described by the  $\hat{x}_1^e$ -subsystem, which is isolated from the control and disturbance signals. On the contrary, in the previous case, in presence of fault, the residual  $r$  could be affected by  $(u, w)$  through the state variables  $\hat{x}_2^e$  which appears in the vector field  $\hat{\ell}_1^e$ .

### 3 Necessary conditions for fault detection

The characterization of the regular NLFPRG given in the last section are used here to derive a necessary condition for its solvability. The results are proved using arguments as those described in [21] for linear systems. In [17], the author, independently from [21], used the nonlinear version of these arguments to face the problem of nonlinear output tracking with disturbance rejection. It is showed here that the results of [17] can be adapted to prove a weaker necessary condition for the solvability of the regular NLFPRG. In [18], the authors express the conditions in terms of *unobservability* subspaces (see [21]). These subspaces belong to the larger class of *conditioned invariant* subspaces (see [1]). Conditioned invariant distributions were introduced in [14]. The necessary conditions for the regular NLFPRG solvability are given in terms of conditioned invariant distributions. So, let us recall the concept of conditioned invariant distribution as introduced in [14] (see also [17]).

**Definition 1**  $\Delta$  is a locally conditioned invariant distribution for the system (2) (with  $m = 0$ ,  $w = 0$ ) if there hold

$$\begin{aligned}[f, \Delta \cap \ker dh] &\subset \Delta \\ [g_j, \Delta \cap \ker dh] &\subset \Delta, \quad j = 1, \dots, \mu.\end{aligned}$$

The main result of this section is now stated:

**Theorem 3** Suppose a solution to the regular NLFPRG exists. Then there exists a locally conditioned invariant distribution  $\Delta$  containing  $\text{span}\{P\}$ , with  $\text{span}\{P\} := \text{span}\{p_1, \dots, p_s\}$ , which satisfies the property  $\ell \notin \Delta$ . If, additionally, the filter has a residual function of the form  $h^e(x^e) = h_0^e(x) - h_1^e(\hat{x}) := \hat{h}_0^e(y) - h_1^e(\hat{x})$ , then  $\Delta$  is also contained in  $\ker dh_0^e$ .

**Remark 4** In the linear case, the residual function is of the form  $r = J\hat{x} + Hy$  and it is easily verified that a locally conditioned invariant distribution is a conditioned invariant subspace. This means that if the FPRG is solvable in the linear case, then a conditioned invariant subspace  $\mathcal{S}$  exists which contains  $\text{Im}P$ , is contained in  $\ker(HC)$  and satisfies  $L \notin \mathcal{S}$ . In fact, this is the necessary condition for the linear FPRG (cfr. [18]), where actually the conditioned invariant subspace is in particular an unobservability subspace, and therefore the condition also suffices to imply the existence of an *observable* disturbance-decoupled subsystem. See [6], [7] for details on the relationship between certain conditioned invariant distributions and a suitable disturbance-decoupled subsystem which has a nonlinear observability property.

It is well-known (see, e.g., [14], page 343) that finding an arbitrary conditioned invariant distribution is a much more difficult task than finding the smallest conditioned invariant distribution containing a given distribution, in this case  $\text{span}\{P\}$ . Such a distribution, which we denote with  $\Delta_*$ , exists, can be computed (the algorithm is given in [14], page 341) and is involutive. It is then clearly true that:

**Corollary 2** If the regular NLFPRG is solvable, then the smallest locally conditioned invariant distribution  $\Delta_*$  containing  $\text{span}\{P\}$  is such that  $\ell \notin \Delta_*$ .

Despite of the condition given in Theorem 3, that given in the previous corollary provides an effective way to check if the problem is solvable (any system admits many conditioned invariant distributions containing  $\text{span}\{P\}$ , whereas only one  $\Delta_*$  exists). Nevertheless, the condition  $\ell \notin \Delta_*$  may be too conservative. In fact, in the case of linear systems, this condition is not sufficient and a more specific kind of conditioned invariant subspaces must be taken into account in order to obtain a sufficient condition (see also Remark 4).

So far we have only treated the detection of the faults. As already observed in [18] for the linear case, it is straightforward (see [7]) to formulate the Nonlinear Extended Problem of Residual Generation (NLEPRG, see [18] for the case of linear systems), which takes into account the contemporary presence of multiple faults, and extend the above conditions also to the problem

of isolating the faults. The result is here omitted for reasons of space.

#### 4 Backstepping observer for fault detection

Let us first recall the backstepping observer of [15]:

**Theorem 4** For a system in the observable form

$$\begin{aligned} \dot{x}_1 &= x_2 \\ &\vdots \\ \dot{x}_{s^e-1} &= x_{s^e} \\ \dot{x}_{s^e} &= f_{s^e}(x) \\ y &= x_1 \end{aligned} \quad (15)$$

there exists an observer

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \mu_1(\hat{x})(y - \hat{x}_1) \\ &\vdots \\ \dot{\hat{x}}_{s^e-1} &= \hat{x}_{s^e} + \mu_{s^e-1}(\hat{x})(y - \hat{x}_1) \\ \dot{\hat{x}}_{s^e} &= f_{s^e}(\hat{x}) + \mu_{s^e}(\hat{x})(y - \hat{x}_1) \end{aligned} \quad (16)$$

such that for any compact set  $K$ , positively invariant for (18), there exist constants  $M > 0$ ,  $\epsilon > 0$ ,  $\gamma > 0$  such that if  $x(0) \in K$  and  $\|e(0)\| < \epsilon$  then  $\|e(t)\| < M\|e(0)\|\exp(-\gamma t)$ .

**Remark 5** While (16) has the structure of a high-gain observer, it is *not* a high-gain observer since its gain functions are not of high-gain type. The main reasons why the former is here adopted instead of the latter is that the (nonconstant) gains of the backstepping observer are iteratively constructed. This seems particularly suited in view of the extension of the present approach to the case of uncertain systems. The use of high-gain observers has been pursued to tackle a different problem of stabilization of the residual signal in [9], [7].

Throughout the section we make the following assumption, which is easily understood in view of Theorem 1.

**Assumption 1** For the process (2), a filter (3) exists so that the vector fields of the cascaded system satisfies the following set of conditions:

$$\begin{aligned} L_{g_i^e} L_{\tau_1^e} \dots L_{\tau_{r^e-1}^e} h_j^e &= 0, \quad i = 1, \dots, \mu \\ L_{p_i^e} L_{\tau_1^e} \dots L_{\tau_{r^e-1}^e} h_j^e &= 0, \quad i = 1, \dots, s \end{aligned}$$

for each  $j = 1, \dots, \tilde{p}$ , each sequence  $\{\tau_1^e, \dots, \tau_{r^e-1}^e\}$  in the set of vector fields  $\{f^e, g_1^e, \dots, g_\mu^e, p_1^e, \dots, p_s^e\}$  and each integer  $r^e \geq 1$  (if  $r^e = 1$ , the sequence is to be intended void) and

$$\begin{aligned} L_{\ell^e} L_{f^e}^{k-1} h_j^e &= 0, \quad 1 \leq k < s^e \\ L_{\ell^e} L_{f^e}^{s^e-1} h_j^e &\neq 0 \end{aligned}$$

for some index  $j \in \{1, \dots, \tilde{p}\}$  and a positive integer  $s^e$ .

It is clear that, in this way, we are assuming that we can construct a filter whose residual is decoupled from all the input signals except for the fault. In the sequel we will see that this assumption is substantially enough to assure that the backstepping observer of [15] can also work as a fault detection filter. Additionally, let us assume, for the extended system, that  $x^e = 0$  is a regularity point of  $\tilde{\Omega}_{\mathcal{O},e}$  and the nonsingularity of the smallest codistribution invariant under  $\{f^e, \ell^e\}$  and containing  $\text{span}\{dh^e\}$ ; let  $\bar{Q}^e$  the annihilator of such a codistribution which results to be nonsingular and involutive. We know from what discussed in Section 2 that the assumption and the regularity hypotheses just introduced are equivalent to the properties

$$\text{span}\{G^e, P^e\} \subset \bar{Q}^e, \quad \ell^e \notin \bar{Q}^e.$$

The following result is a standard fact (see [13]) and relates the two previous sets of conditions:

**Lemma 1** The differentials of the set  $\{dh_j^e, L_{f^e} dh_j^e, \dots, L_{f^e}^{s^e-1} dh_j^e\}$  are linearly independent. Moreover,

$$\bar{Q}^e \subset \bigcap_{k=1}^{s^e} \ker(dL_{f^e}^k - 1h_j^e). \quad (17)$$

In particular, the first part can be found in [13], Lemma 4.1.1, while the inclusion is easily understood by thinking of the definition of the observation space and noting that, in the regularity hypotheses introduced, the codistribution associated with the observation space is  $(\bar{Q}^e)^\perp$ . A diffeomorphism then exists which transforms the extended system into:

$$\begin{aligned} \dot{z}_1^1 &= z_2^1 \\ &\vdots \\ \dot{z}_{s^e-1}^1 &= z_{s^e}^1 \\ \dot{z}_{s^e}^1 &= f_{s^e}^1(z^1, z^2) + \ell_{s^e}^1(z^1, z^2)m \\ \dot{z}^2 &= f^2(z^1, z^2) + \ell^2(z^1, z^2)m \\ \dot{z}^3 &= f^3(z) + \ell^3(z)m + p^2(z)w \\ r &= z_1^1 \end{aligned}$$

with  $\ell_{s^e}^1$  a nonzero function on the domain on which the system is defined.

**Remark 6** If  $s^e = (\dim \bar{Q}^e)^\perp$ , the filter obtained from a copy of the  $z^1$ -subsystem with a linear correction term designed in order to assign the spectrum of the linearized estimate error system through output injection works as a fault detection filter. However, the filter designed through the backstepping procedure is, so far as  $m = 0$ , an "exact" observer in the sense that it is not based on the linear approximation of the system.

The backstepping observer introduced above, endowed with a residual function, assures the local exponential decay to zero of the absolute value of the residual signal in absence of fault, and the finiteness of the relative degree between the fault and the residual. Specifically, in the above hypotheses, we have:

**Theorem 5** *Let  $s^e = (\dim \bar{Q}^e)^\perp$ . If for the  $z^1$ -subsystem a backstepping observer (16) exists, chosen the residual function  $\bar{r} = r - z_1^1$ , there holds that, for any initial condition  $z_1^1(0)$  in a suitable set<sup>2</sup> and any  $\hat{x}^1(0)$  sufficiently close to  $z_1^1(0)$ , the residual converges to zero in absence of fault. Moreover, the residual is unaffected by  $w, u$  and affected by  $m$ .*

The proof is a straightforward consequence of the hypotheses. We only want to remark that the decoupling of the  $z^1$ -subsystem from the disturbance and the control input so far as  $m = 0$  implies that  $r$  is unaffected by  $w, u$ . Similarly, straightforward calculations show that the system composed by the process and the filter, with output  $r$ , has a finite relative degree between the fault and the residual.

**Remark 7** The hypothesis  $s^e = (\dim \bar{Q}^e)^\perp$  is only done for the sake of simplicity. The same conclusion can be drawn if the  $z^1$ -subsystem can be imbedded into a system locally (strongly) observable. See [15], for details.

## 5 Conclusions

The problem of residual generation for nonlinear fault detection and isolation has been discussed by exploiting the concept of output invariance. Characterizations of the problem have been derived in geometric terms. These characterizations have been shown to be useful in deriving necessary conditions for the problem solvability, which partly generalizes the linear conditions. Finally, conditions under which the recently introduced backstepping observer can serve as a fault detection filter as well have been analyzed.

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<sup>2</sup>Which is contained in the positively invariant set.

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