Multilateral comparisons in the ICOP project

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Multilateral Comparisons in the ICOP Project: Issues, Methods and Empirical Results

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Abstract

This paper examines the feasibility of constructing a consistent set of multilateral comparisons of manufacturing sector output and productivity within the framework of the ICOP project. A major objective of the paper is to construct truly multilateral comparisons using the existing data base of the ICOP project. This data base consists of data constructed essentially on the basis of detailed bilateral comparisons. Multilateral unit value ratios are built up from the lowest level possible (the product level). The second objective of the paper is to examine in-depth the problem of aggregation of unit value ratios. Various aggregation methods, both well-known methods and new ones, are applied and sensitivity of the results is examined. New multilateral aggregation methods are developed which take into account differences in number of matches of the underlying binary comparisons, as well as the Laspeyres-Paasche spread which is considered to be a general indicator of reliability. Finally, the paper presents empirical results derived from the application of the above procedures to data for eight countries for the 1987 benchmark year. JEL Classification: C43, O47, O57.

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1. Introduction

Since 1983, the International Comparisons of Output and Productivity (ICOP) project at the University of Groningen has been the main focal point for research on comparisons of gross domestic product from the production side of the economy.[1] The principal objective of the ICOP has been one of providing internationally comparable national income aggregates relevant for purposes of comparisons of output and productivity across countries at the level of the whole economy as well as the most important sectors of the economy including the agricultural, mining, manufacturing and service sectors of the economy. The project represents a major development since the inception of the International Comparison Project (ICP) in 1968. The ICP, in contrast, is a project aimed at providing purchasing power parities which are useful in converting the gross domestic product and its expenditure components such as the private consumption expenditure, government and investment aggregates in the economy. The ICP which is in its seventh Phase is a regular phenomenon and is undertaken by several international organizations including the OECD, EUROSTAT, Statistics Division of the United Nations and the World Bank. Results from these exercises are regularly disseminated through official publications of these organizations. Over the last two decades, the ICP and ICOP projects have been instrumental in providing data used for economic analysis and for studies on catch-up and convergence across nations. Results from these projects are currently an invaluable source of international comparable data for international organizations and researchers in governmental organizations as well as academic institutions around the world.

To date, the ICOP project has undertaken a large number of bilateral comparisons spanning several benchmark years ranging from 1975 to 1997. The main focus of the ICOP work has been the comparison of manufacturing sector output and productivity. Detailed information from country censuses are used in obtaining the basic data for comparisons in the form of unit value ratios and output and value added information (Maddison and van Ark 1994, van Ark, Monnikhof and Timmer 1999). The ICOP studies cover well over 30 countries from all the continents and account for a very

[1] See http://www.eco.rug.nl/ggdc/ for more information on the ICOP.

large proportion of the global manufacturing sector output (see van Ark (1993) on Europe, Hofman (1998) on Latin America and Pilat (1994) and Timmer (2000) on Asia). Most of the studies provide comparisons using the United States as the benchmark, but in several recent studies which include countries from the East-European region, Germany is used as the benchmark. In addition to the manufacturing sector comparisons, ICOP work also focused on the agricultural sector output and productivity covering a smaller range of countries (van Ooststroom and Maddison, 1984 and Maddison and Rao, 1996). Currently, the focus is increasingly shifted towards comparison of output and productivity in the services sectors of the economy (van Ark et al 1999, Mulder 1999)

The most common feature of all the ICOP studies is the binary nature of the comparisons. Since each comparison involves only the pair of countries under consideration, the totality of ICOP comparisons lack the internal consistency between all possible direct and indirect comparisons. This is the requirement of *transitivity*. Over the last two decades considerable research time has been devoted to the problem of finding index number formulae suitable for multilateral comparisons satisfying the property of transitivity (see Kravis et al. 1982). The lack of transitivity among ICOP comparisons to date has limited empirical analysis of productivity and convergence studies involving large sets of countries.

The issue of transitivity received attention with in the ICOP project in the last ten years. For the agricultural sector, an attempt was made in Maddison and Rao (1996) to provide consistent multilateral comparisons for a large number of countries, using data from the Food and Agriculture Organization and the Geary-Khamis method for aggregation. Outside the ICOP project, Rao (1993) provides a set of global agricultural comparisons based on multilateral methods. Pilat and Rao (1991, 1996) report results from the first ever comprehensive attempt to construct consistent multilateral comparisons on the basis of ICOP data for the manufacturing sector. Using a small set of countries, Pilat and Rao constructed multilateral comparisons using branch-level PPPs derived from several binary comparison exercises. Further the issue of additivity was also studied in considerable detail. Though the Pilat and Rao paper represents a major effort to construct transitive multilateral comparisons, their attempt was only a partial success in that the sectoral comparisons were based on non-transitive branch-level comparisons.

The principal objective of the present study is to revisit the problem of the construction of multilateral comparisons using the ICOP data base. The aim is to explore the possibility of constructing a data set consisting of internationally comparable prices and quantities for a long list of goods produced within the manufacturing sector. The second objective is to examine the feasibility of achieving transitivity for comparisons below the branch level which would be a major advance from the work reported in Pilat and Rao (1991). Another major objective of the study is to examine the feasibility of incorporating some measures of reliability of binary comparisons, as reflected by the coverage ratios of the matched products and the number of product matches, explicitly into the construction of transitive multilateral methods using recent major developments in the area of index number methods for international comparisons.

The outline of the paper is as follows. Section 2 deals with the ICOP database and the procedures followed during the course of the study in constructing a product listing and gathering the price and quantity data for items on the list. This list may serve as a starting point for future ICOP benchmarking studies. Section 3 deals with the problem of aggregation of price data below the branch level. Index number methods including the EKS, generalized EKS and CPD methods are described and applied. The resulting transitive branch level PPPs are presented. Section 4 deals exclusively with the problem of aggregation above the branch (basic heading) level. Aggregation procedures, including the Geary-Khamis, generalized EKS and generalized CPD methods are employed in the aggregation. Computational procedures required to make use of these methods are also discussed in detail. Results from all methods are presented and the problem of selection of the aggregation method is discussed. Section 5 provides a summary of international comparison results using PPPs from a few selected multilateral methods and the sensitivity of the results to the choice of the method is discussed. The paper is concluded with some comments of avenues for future research in this area.

2. Database

2.1 Description of the ICOP-database

The ICOP (International Comparisons of Output and Productivity) data base aims at providing international output and productivity comparisons using the industry-of-origin approach. In this approach industry-specific conversion factors are derived on the basis of relative producer prices. To this end, use is made of the manufacturing census. The census provides detailed information on ex-factory output values (excluding taxes and subsidies) and quantities for a large number of detailed products. By dividing outputs by quantities, unit values are derived. These unit values can be considered as an average price, averaged throughout the year for all producers and across a group of nearly similar products. Subsequently, broadly defined products with similar characteristics are matched, for example ladies' shoes, cigarettes, cheese and car tires. So far, ICOP comparisons have been made on a bilateral basis, usually taking the USA as the base country. For each matched product, the ratio of the unit values in both countries is taken. This unit value ratio (UVR) indicates the relative producer price of the matched product in the two countries. Product UVRs are used to derive an aggregate UVR for manufacturing branches[2] and total manufacturing based on a particular weighting scheme using gross value of output or value added. UVRs are weighted at base country weights (Laspeyres) and weights of the other country (Paasche) and the root of their product is taken as the final UVR (Fisher). This aggregation procedure will not be discussed in detail here. The reader is referred to Maddison and van Ark (1988), van Ark (1993) and Timmer (1996) for extensive descriptions of the ICOP methodology.

2.2 Bilateral versus multilateral

A particular feature of the ICOP-data base is its bilateral basis. This means that ICOP does not work with a pre-specified product list as is used in the International Comparison Project (ICP). Instead, in each binary comparison it works with as many products as feasible, depending on data availability. This implies that the product-list

[2] In this paper, manufacturing branches refer to a 2- or 3-digit ISIC industries within manufacturing. The total manufacturing industry is refered to as the manufacturing sector.

may be very different between different sets of binaries. This has the important advantage that country characteristicity is maintained as much as possible. On the other hand, it prohibits the direct use of multilateral methods. Multilateral comparisons are expected to satisfy an important index number property, namely base-country invariance. Within ICOP, comparisons between countries A and B can only be made through binaries with the USA (star comparisons), and therefore, the resulting comparisons are clearly not base-invariant.

Pilat and Prasada Rao (1991) made an important step to tackle this problem for comparisons of manufacturing output and productivity. They applied various multilateral indices to Fisher UVRs at the manufacturing branch level to arrive at base-invariant UVRs for total manufacturing. This was not complete satisfactorily because these Fisher UVRs at the branch level were derived in binary comparisons with the USA and hence were neither transitive nor base-invariant. Hence they were not 'truly' multilateral. To tackle the problem fundamentally, a different approach had to be taken and UVRs had to be built up by multilateral methods right from the product level.

Pilat and Prasada Rao (1991) started to do this for two major manufacturing branches (food manufacturing and chemicals, petroleum and coal products), using a set of countries for the benchmark year 1975. The set included Brazil, Mexico, Korea, Japan, UK and USA. The chosen branches are characterized by a large number of relatively homogeneous products. For each branch they drew up a list of products (containing respectively 67 and 61 products) for which data was available in at least two of the six countries. Subsequently, they applied various multilateral systems to the product level data (Geary-Khamis and Theil-Tornqvist with coverage adjustment) to generate transitive and base-invariant PPPs at the branch level. In this study we follow a similar approach to derive true multilateral manufacturing PPPs for the benchmark year 1987. The countries covered in this study are Australia, Canada, Germany, Indonesia, Japan, South Korea, Taiwan[3] and the United States. A new feature of this study compared to the original Pilat and Prasada Rao (1991) study is the attempt to derive multilateral indices for all manufacturing branches instead of only two. Using these results, we are able to derive 'more meaningful' multilateral PPPs for aggregate manufacturing. A second innovation is the application of some new weighting systems at both product and branch level aggregation as discussed in the following sections.

2.3 Preparation of the data set

For each branch, we took as a starting point the list of matched products of all seven binary comparisons. From this we derived a new list of products for which sufficient price and quantity data was available. As a rule, we chose to include only those products for which we had data in at least two other countries besides the US. [4] A number of detailed adjustments have been made which are described below.

1. For the paper branch in Australia and the electrical machinery branch in Canada we had only data for one small product. Hence we added a large product item for which there was data in these countries and the US, but not in any other country (sanitary paper in Australia and general lighting in Canada)
2. In the case that product matches in two binary comparisons appeared to be almost similar (in terms of the output value and unit value of the matched product in the US) we assumed that the same product was matched. In order to get a single output value and quantity for the US, we took the average of the US quantity and the output value across the different binaries.
3. In some cases rather detailed matches were available for a particular country, but not for the others. In that case, detailed product data was grouped. For example, 'hardwood chips' and 'softwood chips' for Australia were combined into a single item called 'wood chips', because the other countries showed only data for 'wood chips'.

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[3] The Taiwanese census is available for 1986 and not for 1987. Hence Taiwanese quantities refer to the year 1986. Taiwanese unit values have been updated to 1987 using product price indices for the US (see Timmer 1998).

[4] Note that we based all our information on the original binaries with the US. Hence, we did not include items for which data might be available in two or more other countries but not in the US. Pilat and Prasada Rao (1991) did include some small items for which this was the case.

4. In a number of cases, important products (in terms of value) were only available at an aggregate group level. This is the opposite situation from the previous one. We did not want to loose these important products and hence we made some additional assumptions. We decomposed the aggregate product into lower-level product categories using the price and quantity ratios for the product categories from the US census. This ensures that the unit value ratios between the US and the other country for the lower-level product categories are the same as the original UVR for the product group. For example shoes in Indonesia and Korea were subdivided into men's and women's shoes, and in Taiwan they were subdivided into men's, women's, children's and athletic shoes. Other decompositions included tires (for Canada and Korea), steel sheets (Taiwan and Germany), vacuum cleaners and lime (Japan), loudspeakers and rough wood (Indonesia) and aluminum sheets (Germany).

Table 2.1 Number of products for which data is available in multilateral data set, manufacturing branches , 1987

	USA	Austra-lia	Canada	Ger-many	Indo-nesia	Japan	South Korea	Taiwan
Food, beverages and tobacco	52	28	33	29	22	17	29	11
Textile mill products	20	11	6	9	8	12	7	4
Wearing apparel	24	16	16	16	11	5	5	11
Leather products	11	5	6	6	6	4	9	7
Wood products	8	3	4	4	7	1	4	5
Paper, printing & publishing	10	2	4	5	6	5	2	6
Chemical products	45	13	28	13	18	27	35	12
Rubber and plastic products	7	1	5	2	3	5	5	2
Non-metal. mineral products	10	6	5	7	3	9	5	5
Basic & fabr. metal products	30	7	8	21	9	16	20	8
Mach. & transport equipment	13	5	4	4	5	7	9	2
Electrical mach. and equip.	25	7	2	15	9	16	12	14
Total manufacturing	256	103	121	131	107	124	142	87
Number of matches in original binary comparisons	-	178	200	271	214	193	190	119

Source: Based on matching tables from binary comparisons with the USA. Australia from Pilat et al (1993), Canada from De Jong (1996), Germany from ICOP/LCRA estimates (1996), Indonesia from Szirmai (1994), Japan and South Korea from Pilat (1994) and Taiwan from Timmer (1998).

As a result of these procedures we ended up with a list of 256 manufacturing products for which we have data on prices and quantities for at least three countries (see Appendix Table 3 for full list). Table 2.1 shows the number of products per branch and per country for which data has been included in our multilateral data set. It was initially feared that much of the information used in the original binaries would be lost, especially in branches with many heterogeneous products, such as machinery. This appeared not to be the case as shown in Table 2.1. Even in the machinery branch quite a number of products were included. To get an idea about the number of product matches which have been lost, we included in the last row of Table 2.1 the number of matches which have been made in the original binary comparisons with the USA.

More important than the number of matches however, is the manufacturing output share which is covered by these matches. This gives an indication about the representativeness of the data used. The first two columns in Table 2.2 give for each country the percentage of manufacturing output which is covered by the products which are included in our list and for which data on values and quantities is available. For example, the last row in the table shows that the 256 products for which US data is available and which are included in our list cover 28 per cent of total manufacturing output produced in the US.

he other four columns show how much data has been lost with respect to the original binary comparisons with the USA. Although quite a number of product matches were lost compared to the original binaries as shown in Table 2.1, the most important products were retained in our list. This is indicated by the rather high coverage ratio of the new list, compared to the coverage ratios in the original binaries (see last four columns in Table 2.2).

This result is surprising given the fact that in each binary those products were matched which appeared to match in that particular comparison. In practice however, because the binary comparisons were done one after each other by different researchers, use was made of experience collected in previous work, for example with respect to particular groups of products in the US census which are easy, or hard, to match. As a result, in each binary a large number of common matches have been made which made it worthwhile to carry out the multilateralization exercise in this study.

Table 2.2 Comparison of coverage ratio of products in multilateral data set and in original binary comparisons for 1987, total manufacturing

	Number of products for which data is available	Coverage of manufac-turing output	Coverage ratio USA	Coverage ratio USA	Covera ge ratio other country	Coverage ratio other country
			New data set	Original binary	New data set	Original binary
Australia	103	17.5	12.3	15.1	17.5	23.1
Canada	121	26.6	19.4	21.6	26.6	27.8
Germany	131	19.6	19.6	24.8	19.6	24.4
Indonesia	107	52.6	15.6	19.6	52.6	60.7
Japan	124	17.5	17.6	19.9	17.5	20.0
South Korea	142	30.8	17.7	21.0	30.8	36.7
Taiwan	87	19.0	11.4	15.3	19.0	26.4
USA	256	28.0				

Source: see Table 2.1

3. Aggregation below the Branch Level

Within the ICOP methodology, item-level prices are aggregated to total manufacturing UVRs in a stepwise procedure. Here we use a two-stage procedure in which item-level prices first are aggregated to branches and second from branch to total manufacturing.[5] The present section deals exclusively with aggregation procedures relevant for the first stage of the ICOP work. The next section deals with aggregation at the branch level. The first stage is somewhat similar to the first step involved in the International Comparison Project (ICP) where item level prices are aggregated in order to compute PPPs at basic heading levels (a level of aggregation above which it is possible to assign weights in the form of expenditures or expenditure shares). Where necessary, essential differences between the approach followed in this study and the ICP are highlighted.

[5] In this respect the present study provides a departure from the standard ICOP work, as well as the Pilat and Rao (1991) study on multilateral comparisons within the ICOP framework. See Appendix I for an outline of the standard ICOP-methodology.

3.1 Notation and Preliminaries

In this section we describe the notation used in sections 3 and 4 of this paper and present basic data.[6]

Data

Let p_{ij}^{b} and q_{ij}^{b} represent the unit value and the production quantity of the i-th matched product in j-th country (i=1,2,...,N and j=1,..,M). Superscript b refers to branch b, b varying in general from 1 to B but in the present case we have a total of 12 branches in the manufacturing sector comparisons. We note that prices and quantities are positive whenever they are observed in a certain country. So it is possible that the table of prices and quantities may have many blank entries. Table 3.1 shows the price data used in the study of the food, beverages and tobacco branch.

The steps involved in the compilation of the matrix given in Table 3.1 are fully explained in Section 2. The main point to note here is that prices are recorded for all the commodities in the US, but only for a subset in the case of other countries. From Table 3.1 it is also clear that binary price comparisons between countries can be made only on the basis of price (and quantity) data for commodities that are common to both countries. It can be seen from the table that some comparisons may be based only on a handful of commodities and in some instances it may not be possible to obtain a direct price comparisons between a pair of countries because no common products are identified in the basic data.[7]

The price data in Table 3.1 also indicates that comparisons between certain countries are weaker or less reliable than some others. This can be seen by the number of common items for which prices are available in both countries, with the

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[6] The notation used may be at slight variance with established ICOP notation commonly used in ICOP working papers and research publications. This is mainly due to the multilateral nature of the present study.

[7] This was the case for a number of comparisons involving Australia. In those cases we estimated the Laspeyres and Paasche indices by using the indirect comparison via the USA. This was done for Canada-Australia in the metal and paper branch, Australia -Indonesia in rubber and plastics, Japan-Australia in Wood and Paper, Korea- Australia and Taiwan-Australia in the paper branch.

Table 3.1 Prices in the food branch in eight countries, in nat. currencies, 1987

	USA	Australia	Canada	Ger-	Indone-	Japan	Korea	Taiwan
1 Bacon	2.52		3.42				3.575	
2 Beef tallow	0.31					85.0	358	
3 Beer	0.63	0.74	1.17	1.35	1.093	296.5	279	14.5
4 Butter	3.15	2.23	5.24	7.33	3.127	1.183.2	3.055	
5 Candy not containing	2.72				2.163		2.081	
6 Canned meat	2.58	3.35		4.49		617.3	3.878	
7 Cattle feeds (incl. dairy	0.16	0.19		0.36				
8 Cheese	2.96			5.19		659.6	6.007	
9 Chewing gum	5.27		12.22	8.47			2.644	
10 Chocolate	4.46		5.21	9.12	3.785		2.342	137.4
11 Cigarettes	0.03	0.02	0.02		26	3.0	20	0.4
12 Cocoa butter	4.75			12.51	4.871			
13 Complete Chicken feed	0.16	0.24	0.26	0.43				
14 Concentrated milk	0.84	0.89	1.66				1.932	
15 Dog food and cat food	0.70		0.86	1.52				
16 Dry whole milk	2.46	1.87		5.24				
17 Fluid milk	0.42	0.40	0.74	0.74	440		523	36.4
18 Frankfurter	2.44		3.38		2.206			
19 Gin	1.59			2.44		349.1	2.261	
20 Glucose syrup	0.0002	0.00045					0	
21 Grape wines 14% or less	0.85	1.36	2.28	3.97				
22 Ham	3.38		3.23				4.623	
23 Ice cream	0.83	0.99	1.47		2.630		770	62.1
24 Ice milk	0.62	1.37			390			
25 Instant coffee	16.31	17.38					10.223	
26 Jams	1.57	1.87		2.94				
27 Malt	0.18	0.27		0.72				
28 Margarine	1.14		1.41		556	277.4	837	
29 Milk powder	1.86				4.369	537.9	3.113	161.4
30 Molasses	0.05	0.05		0.19	65			
31 Natural cheese	2.94	2.18	4.33					
32 Non-fat dry milk	1.76	1.32		4.03				
33 Pig feeds	0.21	0.24	0.26	0.39				
34 Redried tobacco	4.56		6.49		2.699		2.969	
35 Refined sugar	0.54		0.60	1.33	475	197.5	303	18.1
36 Rice milled	0.24				360	317.5	702	10.8
37 Roasted coffee	5.19	12.07	7.47		3.247		4.624	
38 Rum	1.63			3.43			2.082	
39 Sausages	2.89		3.73	8.26			1.830	
40 Semolina	0.19	0.41	0.46					
41 Shortening oils	0.64	1.11				176.4		
42 Soy bean oil	0.39		0.58			101.6	530	
43 Soybean Meal	0.21		0.33					9.9
44 Starches	0.18	0.54	0.44			74.7	348	
45 Tea	6.62		7.29			1.666.2		110.8
46 Wheat flour	0.19	0.35	0.42	0.63	261	102.6	208	16.6
47 Whiskey	2.88		4.92	6.03		1.455.4	9.929	
48 Yoghurt	1.39	1.41	3.15	2.16	2.002			
49 Young chickens	1.15	2.44	2.33	3.35	4.462			
50 Beef	2.54		3.44	4.16	3.616			
51 Cocoa powder	2.24		3.37	4.10	3.517			
52 Turkeys	1.37	3.00	2.62	4.74				

Source: Rao and Timmer (2000), Appendix Table III number ranging from zero to N. An attempt is made in this paper to incorporate this information into the construction of multilateral index numbers.

Price Index Numbers

In this study we focus mainly on the construction of price index numbers. Quantity indices can be derived indirectly using the value ratios. Let I_{jk} (j,k = 1,..,M) represent the price index number for country k with country j as the base. Since prices in these countries are expressed in national currencies, I_{jk} can be interpreted as a measure of the purchasing power parity between currency k and j and denoted by PPP_{jk}. If PPPs are all expressed with respect to a base currency (currency of a numeraire or reference country), we may simply denote the parities by PPP_j (j=1,2,..M). In such cases it is important to indicate the numeraire currency, in our case US$.

The matrix of all pairwise comparisons can be written as

$$I_{MxM} = \begin{bmatrix} I_{11} & I_{12} & . & . & I_{1M} \\ I_{21} & I_{22} & . & . & I_{2M} \\ . & . & . & . & . \\ . & . & . & . & . \\ I_{M1} & . & . & . & I_{MM} \end{bmatrix} \qquad (3.1)$$

We note that $I_{jj} = 1$ for all j and if the index satisfies country reversal test

$$I_{jk} \times I_{kj} = 1 \text{ for all j and k}$$

then

$$I_{kj} = 1 / I_{jk}$$

in the above matrix.

The problem is one of combining the price and quantity data to construct a matrix of price comparisons. For this purpose it is possible to use a range of standard index number formulae. In this paper we focus in particularly on those methods that satisfy the "transitivity" property.

Transitivity

An index number formula I_{jk} is said to satisfy the transitivity property if and only if for all choices of j,k and l (j,k,l = 1,2,..,M), the index satisfies

$$I_{jk} = I_{jl} \times I_{lk} \tag{3.2}$$

Equation (3.2) requires that the formula should be such that the application of the formula to make a direct comparison I_{jk} should result in the same measure as an indirect comparison between j and k through a link country l. Note that the transitivity property ensures internal consistency of the index numbers in the matrix given in (3.1). As will be noted below, many of the standard index number formulae do not satisfy this requirement.

A further point of relevance is stated in the following result:

Result: an index number formula I satisfies transitivity property in (3.2) if and only if there exist M positive real numbers $\Pi_1, \Pi_2, ..., \Pi_M$, such that

$$I_{kj} = \frac{\Pi_k}{\Pi_j} \tag{3.3}$$

for all j and k. Proof of this statement is straightforward (see Rao and Banerjee, 1984). This result is quite important since it shows that when transitivity property is satisfied, all we need to measure are M real numbers $\Pi_1, \Pi_2, ..., \Pi_M$, and then all the necessary indices in (3.1) can be calculated using these M numbers, thus reducing the dimensions of the problem involved. The numbers in (3.3) can be given a simple interpretation, with Π_j representing the general price level in country j.

3.2 Multilateral methods

In this section we describe those multilateral methods that have been used in the study to compute PPPs at the branch level. This part is somewhat similar to the ICP work where item-level prices are aggregated to get PPPs for the basic headings. There is one major difference between our work and the ICP: in our case there is quantity (and value) data for products that are matched during the ICOP binary comparisons work. This information can be used in the aggregation procedure

In this section we consider the country-product-dummy method proposed, and used, in the context of ICP (see Summers 1973 and Kravis et all. 1982) as well as the Elteto-Koves-Szulc (EKS) method (see Hill, T.P. 1982 and Kravis et all. 1982 and Rao and Lee 2000) and some new variants of the EKS method proposed in this study. A more detailed discussion of multilateral methods for aggregation above the branch level is presented in Section 4.

Country-Product-Dummy (CPD) Method

The CPD method represents a simple regression approach to explain levels of prices of commodities in different countries. The method postulates that the observed price of a commodity, say i-th commodity in j-th country, p_{ij}, is the product of three components: the purchasing power parity or the general price level in a country relative to other countries (denoted by π_j^*); the price level of the i-th commodity relative to other commodities (denoted by η_i^*) and a random disturbance term v_{ij}. The model underlying the CPD method can be stated as:

$$p_{ij} = \pi_j^* \cdot \eta_i^* \cdot v_{ij}$$

or in a logarithmic form and rewriting:

$$\begin{aligned} \ln p_{ij} &= \ln \pi_j^* + \ln \eta_i^* + \ln v_{ij} \\ &= \pi_j + \eta_i + u_{ij} \end{aligned} \tag{3.4}$$

Further explanation of the model and a numerical illustration can be found in Maddison and Rao (1996). In order to estimate π_j (j=1,..M) and η_i (i=1,..n), it is possible to apply ordinary least squares to the following model:

$$\ln p_{ij} = \pi_1 D_1 + \pi_2 D_2 + ... + \pi_M D_M + \eta_1 D_1^* + \eta_2 D_2^* + ... + \eta_n D_n^* + u_{ij} \tag{3.5}$$

where D_j's and D^*_i's are respectively country and commodity dummy variables with the property that

D_j = 1 if price observation p_{ij} belongs to country j
0 otherwise

and $D_i^* = 1$ if price observation p_{ij} refers to i-th commodity
$\quad\quad 0$ otherwise

From the model it is obvious that irrespective how big the data set we have, it is impossible to estimate all the parameters due to the presence of perfect multicollinearity. So it is customary to estimate all the parameters after imposing a restriction. Usually one of the parameters is set to zero. In our application of the CPD method we set $\pi_1 = 0$, or equivalently $\pi_1^* = 1$. Since country 1 in our list is the United States, all the PPPs and commodity specific effects (η_i) are all estimated using US dollar as the numeraire currency.

Table 3.2 presents the PPPs for each of the manufacturing branches in this study using the CPD-method. A major disadvantage of the unweighted CPD-method is that it ignores the available information on quantities. Hence it gives equal weight to all products. As within ICOP, quantity information is also available, this information should be included as well. One way to do this, is to use the EKS-method.

Table 3.2 PPPs for different manufacturing branches using Country-Product-Dummy Method (US dollar = numeraire)

	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
Food, bev. and tobacco	1.00	1.27	1.59	2.27	1186.8	321.2	1112.2	41.5
Textile mill products	1.00	1.80	1.64	2.11	922.5	173.6	833.8	24.9
Wearing apparel	1.00	1.43	1.36	3.26	352.4	221.5	1142.5	17.7
Leather products	1.00	1.19	1.29	2.70	425.5	189.0	531.6	15.0
Wood products	1.00	1.73	1.76	3.27	984.7	532.6	1276.8	55.4
Paper, print & publishing	1.00	1.56	1.31	2.07	1357.5	175.7	714.5	34.4
Chemical products	1.00	1.74	1.50	2.37	1642.6	223.9	899.9	42.0
Rubber and plastic pro.	1.00	1.21	1.25	2.24	978.5	106.2	691.2	29.5
Non-metallic mineral pro.	1.00	1.40	1.36	1.16	643.6	181.9	547.9	17.8
Basic & fabr. metal pro.	1.00	1.88	1.40	2.18	960.9	188.4	779.4	35.2
Mach. & transp. equipment	1.00	1.21	1.64	2.00	1840.6	139.8	506.8	36.7
Electrical machinery	1.00	1.66	1.29	2.77	449.5	173.4	430.2	15.9

Source: Based on data from Rao and Timmer (2000), Appendix III.

Elteto-Koves-Szulc (EKS) Method

The EKS method, proposed by Elteto and Koves (1964) and Szulc (1964)[8], is designed to construct transitive multilateral comparisons from a matrix of binary/pairwise comparisons derived using a formula which does not satisfy the transitivity property. The EKS method in its original form uses the binary Fisher PPPs (F_{jk}: j,k=1,..M) as the starting point.

The computational form for the EKS index is given by

$$EKS_{jk} = \prod_{l=1}^{M} \left[F_{jl} \cdot F_{lk} \right]^{1/M} \tag{3.6}$$

The formula defines the EKS index as an unweighted geometric average of the linked (or chained) comparisons between countries j and k using each of the countries in the comparisons as a link.

The EKS method in (3.6) produces comparisons which are transitive. In addition these indices also satisfy the important least squares property that indices in (3.6) deviate the least from the pairwise Fisher binary comparisons.[9] This property is in line with the property of characteristicity espoused in Drechsler (1973). Since Fisher index is considered to be ideal and possesses a number of desirable properties, the EKS method has a certain appeal since it preserves the Fisher indices to the extent possible, while constructing multilateral index numbers. However, a major problem with the EKS formula is that it gives equal weights to all linked comparisons [F_{jl} . F_{lk}], effectively assuming that they are of equal reliability. Following Rao (1999), it can be argued that in practice it possible to show that some link comparisons are intrinsically more reliable than others. For example in the present study, we find that some pairwise Fisher indices are based on price data for many commodities while in other cases comparisons are based on prices for only one or two items. It is desirable to take this information into account when constructing the EKS multilateral indices.

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[8] It is now well recognised that Gini proposed this method in 1924. We will continue to refer to this as the EKS-method as it is the case with most publications of international organisations.

[9] A formal proof of this is given in Rao and Baneerjee (1984).

We outline the method described in Rao (1999) and apply the new method to consider different measures of reliability.

Generalized EKS Method

In order to generalize the EKS method to incorporate weights to various linked comparisons involved in equation (3.6), it is necessary to look at the EKS method from a different angle. Suppose we wish to derive a set of index numbers I_{jk} which are transitive and minimize the log-distance from the Fisher indices, then we

$$\text{minimize} \quad \sum_j \sum_k (\ln I_{jk} - \ln F_{jk})^2$$
$$\text{subject to} \quad I_{jk} = I_{jl} \cdot I_{lk} \qquad \forall j,k,l$$

Using the result stated in Section 3.1 on transitive index numbers, the above problem can be restated as one finding Π_1, Π_2,..., Π_M, which minimizes

$$\sum_j \sum_k (\Pi_k - \Pi_j - \ln F_{jk})^2 \qquad (3.7)$$

Then the required index I_{jk} is defined as the ratio $\exp(\hat{\Pi}_k)/\exp(\hat{\Pi}_j)$ where (^) shows that these are solutions to the minimization problem. After some simple algebraic manipulation it can be shown that the EKS index is related to the solution above as:

$$EKS_{jk} = \frac{\exp(\hat{\Pi}_k)}{\exp(\hat{\Pi}_j)} = \exp(\hat{\Pi}_k - \hat{\Pi}_j)$$

Considering further equation (3.7), it is evident that $\hat{\Pi}$'s are the ordinary least squares estimators of Π's (which are the best linear unbiased estimators) in the following model specification

$$\ln F_{jk} = \Pi_k - \Pi_j + u_{jk}$$
$$\text{with} \quad E(u_{jk}) = 0 \quad \text{and} \quad v(u_{jk}) = \sigma^2 \qquad (3.8)$$

Given the model specification in (3.8), it is possible to discriminate between different pairs of countries using some indicators of reliability. This can be achieves using the following model

$$\ln F_{jk} = \Pi_k - \Pi_j + u_{jk}$$
$$\text{with} \quad E(u_{jk}) = 0 \quad \text{and} \quad v(u_{jk}) = \frac{\sigma^2}{w_{jk}} \tag{3.9}$$

where w_{jk} is a measure of reliability. If w_{jk} is large we consider that particular Fisher index, F_{jk}, to be reliable. Modified EKS indices can be obtained by applying generalized least squares or ordinary least squares to (3.9)

$$\sqrt{w_{jk}} \ln F_{jk} = \sqrt{w_{jk}}\Pi_k - \sqrt{w_{jk}}\Pi_j + u^*_{jk}$$
$$\text{with} \quad E(u^*_{jk}) = 0 \quad \text{and} \quad v(u^*_{jk}) = \sigma^2 \qquad \forall j,k = 1,..M,\ j \neq k \tag{3.10}$$

Weighting Schemes for the Generalized EKS System

Given the general structure underlying the process of according weights to different linked comparisons, it is necessary to specify the matrix weights to make the method operational. In this study we consider two sets of weights for aggregation below the branch level. These are described below.

Weights based on Number of Matches

The first set of weights are defined using the number of items that are common to a given pair of countries. A comparison between two countries for a given branch is considered to be more reliable if it is based on more matches. Let n_{jk} be the number of common products between j and k and n* the total number of items in the branch (according to our pre-specified list described in section 3.1), then we specify:

$$w_{jk} = \frac{n_{jk}}{n^*} \qquad \forall j,k,\ j \neq k$$
$$w_{jk} = 0 \qquad j = k$$

We put a zero on the diagonal as the Fisher index will be 1 by definition and hence log Fisher will be 0.[10] Table 3.3 provides a matrix of weights for the food, beverages and tobacco branch. It shows that, for example, for the Canada-USA binary comparison prices of 33 products were used out of a total of 52 items (33/52 = 0.635), where as for Germany-Taiwan prices of only 5 products have been used. consequently, the Canada-US comparisons gets a heavier weight in the EKS formula. Note that the table is of course symmetric.

Table 3.3 Weights based on number of product matches for all binary comparisons, food, beverages and tobacco branch, 1987

	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
USA	0.000	0.538	0.635	0.558	0.423	0.327	0.558	0.212
Australia	0.538	0.000	0.327	0.327	0.212	0.135	0.231	0.096
Canada	0.635	0.327	0.000	0.346	0.308	0.192	0.365	0.173
Germany	0.558	0.327	0.346	0.000	0.231	0.154	0.250	0.096
Indonesia	0.423	0.212	0.308	0.231	0.000	0.154	0.269	0.173
Japan	0.327	0.135	0.192	0.154	0.154	0.000	0.288	0.135
Korea	0.558	0.231	0.365	0.250	0.269	0.288	0.000	0.173
Taiwan	0.212	0.096	0.173	0.096	0.173	0.135	0.173	0.000

Source: See Table 2.1

Weights based on Hill's Distance Function

We have also considered an alternative measure of reliability which is based on the spread between Laspeyres and Paasche index numbers. Beginning from the work of Bortkiewicz (1924), it is generally accepted that the Laspeyres-Paasche spread reflects variability in the price and quantity ratios as well as the strength of the correlation between the price and quantity ratios over time or across countries. Van Ark, Monnikhof and Timmer (1999) provide a decomposition of the spread into the different components along these lines for many binary ICOP comparisons. Hill (1999) provides a formal measure of reliability based on this spread and discusses

[10] For pairs of countries for which no common commodities could be found, Laspeyres and Paasche indices were derived through a link involving the US. Consequently, a weight of 0 was assigned.

various properties of this measure. The distance between two countries j and k (d_{jk}) is measured for all j and k by

$$d_{jk} = \left| \ln\left(\frac{L_{jk}}{P_{jk}} \right) \right|$$

where L_{jk} and P_{jk} may refer to price index numbers or to quantity index numbers. Since a large value of d_{jk} represents a larger spread between the Laspeyres and Paasche indices, we postulate that the weights needed for our weighted EKS method are inversely proportional to the distance function. Thus, for all j and k ($j \neq k$)

$$w_{jk} = \frac{1}{d_{jk}}$$

If only one item was matched, the weight is assigned a value of zero.

The following table shows the distance matrix used in our study for the food, beverages and tobacco branch. The corresponding Laspeyres-Paasche spreads were given in Appendix Table II. The table shows that, for example, binaries of Canada, Germany and Japan with the USA get a much higher weight than the comparisons Korea and Indonesia with the USA. However, it is also shown that the weight for the Australia-Korea binary is by far the largest of all. This is because the Paasche-Laspeyres spread is very close to 1, although 12 matches have been made. Due to the definition of the distance given above, this table is symmetric as well.

Table 3.4 Weights based on Hill's distance function for all binary comparisons, food, beverages and tobacco branch, 1987

	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
USA	0.00	9.37	54.29	43.35	2.36	79.65	3.65	4.20
Australia	9.37	0.00	11.61	26.92	11.93	4.70	1496.83	2.68
Canada	54.29	11.61	0.00	118.66	28.66	12.18	6.29	7.42
Germany	43.35	26.92	118.66	0.00	1.92	17.05	3.74	14.33
Indonesia	2.36	11.93	28.66	1.92	0.00	7.65	7.56	9.65
Japan	79.65	4.70	12.18	17.05	7.65	0.00	6.16	10.27
Korea	3.65	1496.83	6.29	3.74	7.56	6.16	0.00	5.30
Taiwan	4.20	2.68	7.42	14.33	9.65	10.27	5.30	0.00

Source: Tables 3.2 and 3.3

Other weigthing measures might be used, such as measures for the similarity in the production structure of countries. This is an area for further research.

3.3 Branch Level PPPs

Various methods described in the preceding sections were applied in the task of calculating transitive multilateral PPPs for each of the manufacturing branches. The following table presents the results for the food, beverages and tobacco branch using various methods. Results for the remaining branches are included in the appendix.

Table 3.5 PPPs using various methods for food, beverages and tobacco manufacturing branch in 7 countries, 1987 (in national currency per US$)

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.20	1.06	1.09	1.07	1.27
Canada	1.47	1.48	1.49	1.48	1.59
Germany	2.20	2.13	2.17	2.06	2.27
Indonesia	1215.9	1295.0	1285.2	1338.3	1186.8
Japan	307.2	327.1	321.4	321.0	321.2
South Korea	904.7	913.0	917.1	903.5	1112.2
Taiwan	35.4	35.8	35.6	35.2	41.5

Source: Product data from Table 3.1. Fisher using (II.3), CPD using (3.3), EKS using (3.6), weighted EKS using (3.10) with weights from Tables 3.3 and 3.4.

The standard ICOP PPPs refer to the Fisher PPPs which are not transitive. Pilat and Rao (1991) used the Fisher PPPs as an input into aggregation at a higher level. The table presents a choice of four alternative methods which are transitive. The transitive unweighted EKS PPPs are rather different from the binary Fisher as follows from a comparison of columns 1 and 2 in the table. The weighted EKS based on the number of matches is rather close to the unweighted EKS. One would expect that for countries with a large number of items for which prices are available, the weighted EKS would be closer to the binary Fisher than the unweighted EKS. This expectation is not always borne out. For example Germany has a high number of priced items (see Table 2.1) and the product weighted EKS is closer to the Fisher than the unweighted one as expected. However, Taiwan has a low number of product matches, but nevertheless the weighted EKS is closer to the original binary Fisher than the unweighted EKS. Similarly, the Hill's distance weighted EKS generates some surprising result for Germany as the PPP is pulled even further away from the original Fisher, which is not expected looking at the weights in Table 3.4.

We consider the use of the unweighted CPD to be inappropriate since it ignores the available quantity information. Comparing the results of the unweighted CPD in the last column with the other columns, it is clear that the unweighted CPD generates quite different results although there is no uniform bias. Except for Indonesia and Japan, it delivers a PPP well above the PPPs delivered by other methods. This method is included at this stage only to highlight the deficiency attached to the standard ICP methodology where no weights are used for purposes of aggregation below the basic heading level.

From a theoretical perspective the choice for a transitive multilateral index is between the two sets of weighted EKS parities. There is no a priori reason to prefer one specification against the other. Ideally we would have liked to incorporate both measures of reliability into a single model. Work is currently in progress on this issue. In this study we use both of these parities as inputs into the next level of aggregation. It is also possible to take a weighted or unweighted geometric average of columns 3 and 4 in Table 3.5 as a compromise, where weights may reflect researchers' subjective ranking of these two specifications.

4. Aggregation Above the Branch Level

In this section we outline the aggregation procedures used in computing purchasing power parities for the manufacturing sector as a whole. Since aggregation at this level was considered in detail by Pilat and Rao (1991, 1996) and also by Maddison and Rao (1996) in the context of agriculture sector comparisons, we focus our attention on the most recent developments in this area. Thus the treatment here complements earlier ICOP publications, and it is advised that this section is used in conjunction with material included in the three papers cited above.

While technically the problem of aggregation is the same whether it is below or above the manufacturing branch level, the main difference is in the type of data available for this purpose. At branch level we have price data, in the form of PPPs derived through aggregation below branch level, and data on the total value of manufacturing output in the sector. The quantity information is implicit in the data. This means that we have a table of price and quantity information with no missing entries. Table 4.1. shows the price (unweighted EKS method) and quantity data (gross value of output) for the 12 major branches of manufacturing. We note that the column of prices for the United States is equal to one for all branches.

In terms of selecting an index number for purposes of aggregating branch level data, we look for methods that satisfy transitivity as well as the additivity or matrix consistency property (see Pilat and Rao 1991, pp.15-16). The only aggregation method which satisfies the additivity constraint is the Geary-Khamis method. In this part of the multilateralisation of ICOP work we consider three principal aggregation methods: the Geary-Khamis, weighted EKS and the weighted CPD method. These are described below.

4.1 Geary-Khamis method

The Geary-Khamis (G-K) method derives its name from its principal proponents Geary (1958) and Khamis (1970). The G-K method, unlike the standard index numbers, defines the purchasing power parities of currencies PPP_j (j=1,..M), and also a set of international average prices P_i (i=1,..N), one for each commodity, or in this case branch, in terms of observed price and quantity data.

Using the notation in section 3.1, equations that define the PPP_j's and P_i's can be written as below. International price, P_i, of i-th commodity is defined as

$$P_i = \frac{\sum_{j=1}^{M} \frac{p_{ij} q_{ij}}{PPP_j}}{\sum_{j=1}^{M} q_{ij}} \tag{4.1}$$

Thus the international price of i-th commodity is defined by first calculating the total value of output of i-th commodity across all countries which are in national currency units, converted into a common currency unit using the purchasing power parities. This total value, now expressed in a common currency unit, is then divided by the total output of this commodity across all countries. This definition of international average price is consistent with standard national accounts and statistical practices used in defining national average price from regional price data.

To implement equation (4.1), it is necessary to define the parities, PPP_j. The G-K method defines these parities as below. For each j

$$PPP_j = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i \, q_{ij}} \qquad (4.2)$$

Equation (4.2) is in the form of a Paasche index where PPP_j is defined as the ratio of total value of production derived using national prices (p_{ij}) and international prices (P_j). Essentially, PPP_j in (4.2) measures the level of prices in country j relative to the international average price.

It is easy to see that the G-K method consists of a system of (M+N) linear homogeneous equations in as many unknowns (PPP's and P_i's). Rao (1971) and Khamis (1972) have shown that under very mild conditions on price and quantity data, this system of equations has a unique solution for the parities and international prices when one of the unknowns is fixed at an arbitrarily chosen level. In practice one of the PPP_j's, say the first one, is set at unity. This means that all the PPP_j's express parities with respect to the first country currency and the international prices are expressed in the currency unit of the first country. In the present study we use US dollar as the reference or numeraire currency.

Computational scheme

The system of equations (4.1) and (4.2) can be solved using matrix inversion routines or through a simple iterative scheme. The computational scheme starts with an initial set of values for the parities. The most common starting point is to set $PPP_j = 1$ for all j. Using the initial set of PPP_j's and equation (4.1), we can compute international prices P_i for each commodity. These prices are then used in equation (4.2) to compute PPP_j's in iteration 1. We normalize these PPPs to make, say $PPP_1 = 1$. If the normalized PPP_j's are different (at a defined level of accuracy like up to 4 or 5 decimal points), then this process is repeated until the values converge.

The analytical properties of the G-K method, in particular the existence of a unique positive solution, guarantees that this iterative scheme converges, and converges to the same value irrespective of the starting point. Maddison and Rao (1996) provide a more detailed account of the iterative scheme and also a numerical illustration of the scheme. The convergence of the scheme is usually very fast. In the present application convergence was achieved in four iterations.

One of the attractive properties of the G-K method is that it satisfies additivity. Rewriting equation (4.2), we find

$$\frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{PPP_j} = \sum_{i=1}^{N} P_i \, q_{ij} \qquad (4.3)$$

The left-hand side of (4.3) is the national value aggregate converted into a common currency unit using PPP_j's. The right-hand side of the equation is the total output value of country j valued at international average prices. Thus the volume comparisons across countries can be constructed using either of the approaches, but the resulting comparisons are the same.

An additional advantage of the G-K method is that it is possible to make international comparisons of sub-aggregates. For example, if we wish to collapse 12 manufacturing branches into 5 major branches, the G-K method facilitates this quite easily through the international prices.

The G-K method, because of all the nice properties discussed above, has been the main aggregation procedure used in all the phases of ICP until now. Kravis et al. (1982) provide an excellent discussion about the choice of the methodology. However, in the more recent times, the OECD and EUROSTAT comparisons of GDP are being compiled using the EKS method. This shift towards the use of EKS system is mostly due to the "characteristicity" property associated with the EKS method.

4.2 Generalized CPD method

The CPD method, discussed in section 3.3, has never been considered as an aggregation procedure for international comparisons even though it has potentially the same kind of results as the G-K method. The regression estimation of the CPD model provides PPPs as well as international prices in the form of η_i* (or exp(η_i)). The principal reason for any lack of such applications is that it does not make use of any quantity or value data. Thus until recently, the CPD method has remained as an aggregation procedure below the basic heading level (where no quantity information is present) and also as a method for filling holes in price information (Summers 1973).

However, Rao (1996) has generalized the CPD method to incorporate quantity and value data directly into the CPD method. Rao has also shown that the resulting PPPs and international prices are identical to those resulting from the Rao (1990) method for international comparisons. The generalized CPD method suggests that estimation of equation (3.8)

$$\ln p_{ij} = \pi_1 D_1 + \pi_2 D_2 + ... + \pi_M D_M + \eta_1 D_1^* + \eta_2 D_2^* + ... + \eta_n D_n^* + u_{ij}$$

is conducted after weighting each observation according to its value share. This is equivalent to the application of ordinary least squares after transforming the equation premultiplied by (4.4)

$$\sqrt{w_{ij}} \ln p_{ij} = \pi_1 \sqrt{w_{ij}} D_1 + \pi_2 \sqrt{w_{ij}} D_2 + ... + \pi_M \sqrt{w_{ij}} D_M + \eta_1 \sqrt{w_{ij}} D_1^* + ... + \eta_n \sqrt{w_{ij}} D_n^* + v_{ij}$$

Table 4.1 Basic data for aggregation above branch level: EKS PPPs and gross value of output at branch level, 1987

EKS PPP	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
Food, beverages and tobacco	1	1.06	1.48	2.13	1,295.02	327.09	913.03	35.85
Textile mill products	1	1.54	1.80	2.24	900.99	172.01	708.68	25.50
Wearing apparel	1	1.49	1.38	3.13	448.81	224.43	869.20	16.73
Leather products	1	1.35	1.30	3.06	557.47	225.10	631.89	17.74
Wood products	1	1.88	1.37	2.96	1,372.02	603.77	1,175.81	52.05
Paper, printing & publishing	1	1.74	1.39	2.11	1,402.15	198.50	743.83	35.69
Chemical products	1	1.56	1.27	2.20	1,628.80	269.21	976.49	39.38
Rubber and plastic products	1	1.22	1.26	2.25	974.47	100.72	709.54	30.19
Non-metallic mineral products	1	1.62	1.33	1.61	929.21	183.59	477.74	21.94
Basic & fabricated metal products	1	1.71	1.48	2.17	1,147.25	188.58	760.32	34.28
Machinery & transport equipment	1	1.17	1.14	1.95	1,773.73	96.07	432.24	40.83
Electrical machinery and equipment	1	1.56	1.36	2.51	956.15	156.88	393.88	18.85
Exchange rate (per US$)	1	1.43	1.33	1.80	1,644.00	144.64	823.00	31.87

Gross value of output (in million nat cur)	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan[a]
Food, beverages and tobacco	350,483	24,850	51,166	151,956	8,741,033	27,220,693	11,892,010	280,921
Textile mill products	62,786	4,429	6,918	35,325	3,830,262	7,809,897	11,184,137	327,252
Wearing apparel	64,243	2,600	6,961	22,440	594,389	4,073,963	4,212,300	99,726
Leather products	9,082	1,323	1,490	7,988	160,648	1,106,901	2,722,430	65,025
Wood products	107,209	6,117	19,543	35,517	3,772,627	7,492,598	1,636,839	102,046
Paper, printing & publishing	245,184	10,552	35,678	58,417	1,317,438	17,511,615	4,743,123	127,760
Chemical products	359,960	11,311	41,266	246,076	11,476,641	25109518	16,515,621	462,716
Rubber and plastic products	86,634	4,624	9,177	57,236	2,647,691	11,404,822	6,428,204	331,787
Non-metallic mineral products	61,477	5,433	8,375	41,289	1,392,558	8,990,701	4,243,142	98,849
Basic & fabricated metal products	267,614	22,940	37,550	171,198	3,620,139	33,444,172	14,183,415	416,740
Machinery & transport equipment	550,606	15,549	71,121	373,404	2,021,373	63,191,960	15,775,422	283,731
Electrical machinery and equipment	171,286	6,377	18,221	195,007	794,225	35,860,090	16,730,669	526,595
Other manufacturing	139,337	1,927	6,585	25,943	148942.734	8,120,581	3,637,868	150,955
Total manufacturing	2,475,901	118,032	314,050	1,421,796	40,517,966	251337511	113,905,180	3,274,102

Note: [a] Data refers to 1986.

Sources: see Table 2.1 for gross value of output EKS PPPs from Appendix.

where $w_{ij} = \dfrac{p_{ij} q_{ij}}{\sum_{i=1}^{N} p_{ij} q_{ij}}$ is the value share of i-th commodity in j-th branch.

Equation (4.4) has the property that the estimated π_i's and η_i's track observed prices (in logarithmic form) of more important commodities more closely than the original model, importance measured using the expenditure, or output, share

The generalized CPD has the same type of output as the G-K system, but is capable of incorporating extraneous information about the price structures. For example Rao and Stefano (2000) postulate a spatially autocorrelated structure for the disturbances in (4.4), which incorporates additional information that price structures in geographically contiguous countries and those with strong trade links have similarities in the price structure.

The computational scheme to compute estimates of π_j's and η_i's is very similar to that outlined in section 3.3. It is possible to show from the normal equations that these estimates coincide with PPPs and P_i's from the Rao system. However, the Rao method does not possess the additivity property like the G-K model. Pilat and Rao (1991) report results which show that Rao-method fails additivity by very narrow margins. So if additivity is dropped as a requirement, the generalized CPD method has the potential to perform quite well and to replace the G-K method.

4.3 Weighted EKS method

At this second stage of aggregation we consider both the EKS method and the weighted EKS version. In applying the weighted EKS method we use two sets of weighting matrices. The first matrix is identical to that defined using the Hill (1999) distance based on the Laspeyres-Paasche spread as described in section 3.3. Now these indices are calculated using the price-quantity data at the branch level, rather than the product level (see Table 4.1). The second matrix considered for weighting purposes is the matrix of coverage ratios (see Table 2.2). For each country j, the coverage ratios c_j, is defined as the ratio of the matched output (output for which price information is available) to the total manufacturing sector. Similarly for each pair of countries j and k, we define the coverage ratio c_{jk} as the average of the coverage ratios in countries j and k based on the products matched between countries j and k. The coverage ratios range from 0 to 1 and higher ratios imply greater reliability of the comparison. Hence they have a higher weight in the weighted EKS procedure. Table 4.2 shows the coverage ratios calculated from the product table in the appendix and Table 4.1.

Table 4.2 Average coverage ratios for binary comparisons, total manufacturing, 1987.

	USA	Australia	Canada	Ger-many	Indon-esia	Japan	Korea	Taiwan
USA	0.280	0.149	0.230	0.196	0.341	0.176	0.243	0.152
Australia	0.149	0.175	0.113	0.117	0.134	0.090	0.122	0.085
Canada	0.230	0.113	0.266	0.163	0.274	0.157	0.194	0.127
Germany	0.196	0.117	0.163	0.196	0.198	0.131	0.147	0.116
Indonesia	0.341	0.134	0.274	0.198	0.526	0.213	0.264	0.208
Japan	0.176	0.090	0.157	0.131	0.213	0.175	0.165	0.122
Korea	0.243	0.122	0.194	0.147	0.264	0.165	0.308	0.153
Taiwan	0.152	0.085	0.127	0.116	0.208	0.122	0.153	0.190

Source: product value data derived from appendix table. Branch gross output from Table 4.1.

The coverage ratios in Table 4.2, constructed using our multilateral data set, are lower than the coverage ratios reported in the respective ICOP binary comparisons. This is mainly because we had to drop quite a number of matches which appeared only in one comparison (see section 2 for a description). However, we are pleasantly surprised that the coverage ratios have not dropped dramatically in the process of compiling price-quantity data for the present exercise. Computational procedures for implementing the weighted EKS are the same as those reported in section 3.3.

4.4 Manufacturing Sector PPPs and International Prices

In this section we briefly present the purchasing power parities computed using various formulae. Results presented here are based on the EKS, the weighted EKS with weights based on the Hill distance function and the coverage ratio, the Geary-Khamis and the generalized CPD-Rao methods of aggregation. These methods are used for aggregation above branch level. We note that each of the methods could be applied with the price and quantity data derived at the branch level from various aggregation procedures below branch level as described in section 3. A complete set of results is presented in Appendix Table IV. To keep the presentation simple, we present only a subset of purchasing power parities for the manufacturing sector as a whole derived from a range of combinations of aggregation procedures below and above the branch level. We only present PPPs which are transitive and make fully use of the available price and quantity data. The results are given in Table 4.3. The rows in this table give the aggregation method used at above branch level, whereas the columns refer to the methods used below branch level (EKS and two variations of weighted EKS). Note that all PPPs are normalised to the PPP derived with using EKS as the aggregation procedure for below as well as above branch level. This PPP is set to 1 for each country in order to provide easier comparisons across countries. Also given is the difference between the maximum and minimum of each row and column to indicate the spread in outcomes when different aggregation procedures are used.

Looking at Table 4.3, a number of observations can be made. First, the sensitivity of the PPPs to the choice of a particular aggregation formulae is lowest in developed countries and highest in developing countries such as Indonesia, South Korea and Taiwan. For example, the difference between the highest and lowest PPP for Germany (out of the 15 possible combinations of methods) is 4.9 per cent, while 14.5 per cent in the case of Taiwan. This is due both to the effect of the rather different structure of Taiwanese manufacturing, but also to the fact that reliability of the Taiwanese data, in terms of numbers of products matched and percentage of output covered, is rather low compared to the other countries. Second, looking at the results for the EKS fomulae, it appears clearly that the choice of the aggregation formulae below branch level is more important than the choice of a particular method above branch level. Whereas the difference between the various unweighted and weighted EKS procedures below branch level create considerable differences (up to 7 per cent in the case of Taiwan), the choice of a particular EKS scheme for aggregation above branch level results in only a minor differences (below 2 per cent in all cases).

Table 4.3 : Purchasing Power Parities using different transitive multilateral index formulae below and above branch level, Total Manufacturing, 1987
(Expressed relative to PPPs derived using EKS below and EKS above basic heading level)

	Australia				Canada			
Method below / Method above	EKS	WEKS (Hill)	WEKS (match)	*Max - Min*	EKS	WEKS (Hill)	WEKS (match)	*Max - Min*
EKS	1.000	1.018	1.012	*0.018*	1.000	1.024	1.006	*0.024*
WEKS(Hill)	1.000	1.019	1.011	*0.019*	0.997	1.022	1.005	*0.024*
WEKS(Cov ratio)	1.003	1.022	1.016	*0.019*	1.001	1.024	1.007	*0.023*
Weighted CPD	0.992	1.033	0.986	*0.047*	1.006	1.056	0.995	*0.061*
Geary-Khamis	0.965	0.979	0.978	*0.014*	0.996	1.017	1.004	*0.020*
Max - Min	*0.038*	*0.054*	*0.038*	*0.068*	*0.010*	*0.040*	*0.012*	*0.061*

	Germany				Indonesia			
Method below / Method above	EKS	WEKS (Hill)	WEKS (match)	*Max - Min*	EKS	WEKS (Hill)	WEKS (match)	*Max - Min*
EKS	1.000	0.995	1.001	*0.007*	1.000	1.015	1.013	*0.015*
WEKS(Hill)	0.987	0.984	0.991	*0.006*	1.000	1.010	1.010	*0.011*
WEKS(Cov ratio)	0.999	0.995	1.002	*0.007*	1.005	1.021	1.018	*0.016*
Weighted CPD	1.012	1.033	0.992	*0.042*	0.950	1.003	0.948	*0.056*
Geary-Khamis	1.003	0.991	1.004	*0.013*	0.937	0.959	0.953	*0.022*
Max - Min	*0.025*	*0.049*	*0.013*	*0.049*	*0.068*	*0.062*	*0.071*	*0.084*

	Japan				South Korea			
Method below / Method above	EKS	WEKS (Hill)	WEKS (match)	*Max - Min*	EKS	WEKS (Hill)	WEKS (match)	*Max - Min*
EKS	1.000	0.998	0.997	*0.003*	1.000	0.985	1.016	*0.031*
WEKS(Hill)	0.996	0.996	0.997	*0.000*	0.989	0.974	1.006	*0.032*
WEKS(Cov ratio)	1.005	1.002	1.001	*0.003*	1.007	0.993	1.025	*0.033*
Weighted CPD	0.973	1.002	0.956	*0.046*	0.981	0.994	0.980	*0.014*
Geary-Khamis	0.917	0.925	0.921	*0.008*	0.947	0.938	0.959	*0.021*
Max - Min	*0.087*	*0.077*	*0.081*	*0.087*	*0.060*	*0.056*	*0.067*	*0.088*

	Taiwan			
Method below / Method above	EKS	WEKS (Hill)	WEKS (match)	*Max - Min*
EKS	1.000	0.930	0.979	*0.070*
WEKS(Hill)	1.005	0.941	0.988	*0.063*
WEKS(Cov ratio)	1.008	0.938	0.988	*0.070*
Weighted CPD	0.940	0.908	0.908	*0.032*
Geary-Khamis	0.900	0.863	0.886	*0.038*
Max - Min	*0.107*	*0.078*	*0.101*	*0.145*

Source: See Table 2.1 and 4.1.

Third, PPPs for countries like Indonesia and Taiwan based on the Geary-Khamis and the generalized CPD method are well below those derived using EKS or weighted EKS procedures. In almost all cases, PPPs from the generalized CPD are above the PPPs from the GK method. This result suggests that use of PPPs from the GK method is likely to overstate the gross value added of countries like Indonesia, South Korea and Taiwan, and therefore likely to have their productivity levels overstated.

An issue which has not attracted much attention in the literature so far, is the calculation of international prices. Both the GK and generalized CPD method generate international prices for manufacturing branches. In Table 4.4 we present these prices. Each column refers to the aggregation method used below the basic heading level, the most preferred are the weighted EKS PPPs based on Hill and matched products as these are transitive and take into account reliability measures.

Table 4.4: International Prices for Manufacturing Branches using weighted CPD and G-K Method

	Fisher	EKS	WEKS (Hill)	WEKS (match)
CPD-method				
Food, beverages and tobacco	1.10	1.09	1.06	1.11
Textile mill products	0.98	0.97	0.99	0.99
Wearing apparel	0.99	0.99	0.98	0.99
Leather products	0.99	0.99	0.98	0.99
Wood products	1.10	1.11	1.08	1.10
Paper, printing & publishing	1.05	1.05	1.03	1.05
Chemical products	1.30	1.26	1.15	1.34
Rubber and plastic products	0.96	0.96	0.98	0.96
Non-metallic mineral products	0.97	0.97	0.96	0.97
Basic & fabricated metal products	1.10	1.09	1.11	1.11
Machinery & transport equipment	0.77	0.76	0.73	0.78
Electrical machinery and equipment	0.86	0.85	0.84	0.85
Total manufacturing	1.00	1.00	1.00	1.00

Geary-Khamis				
Food, beverages and tobacco	1.14	1.14	1.13	1.14
Textile mill products	1.02	1.01	1.05	1.02
Wearing apparel	1.05	1.07	1.05	1.07
Leather products	1.03	1.04	1.02	1.04
Wood products	1.24	1.26	1.24	1.24
Paper, printing & publishing	1.03	1.05	1.03	1.04
Chemical products	1.11	1.11	1.09	1.12
Rubber and plastic products	0.85	0.83	0.85	0.82
Non-metallic mineral products	0.96	0.96	0.96	0.96
Basic & fabricated metal products	1.05	1.05	1.10	1.04
Machinery & transport equipment	0.80	0.78	0.80	0.79
Electrical machinery and equipment	0.92	0.94	0.92	0.93
Total manufacturing	1.00	1.00	1.00	1.00

Source: See Table 2.1 and 4.1.

International prices above unity for any given branch suggests that, on average, prices in that branch are above the level for the whole manufacturing sector. Both methods indicate that food products, wood products and chemicals are relatively expensive, while electrical machinery, and especially machinery and transport equipment is relatively cheap. The relative price structures shown in Table 4.4 reflect the international average prices derived using weighted CPD or GK methods, these relativities are maintained irrespective of which currency is used as the numeraire currency.

A point to note here is the differences in these international prices from the two aggregation methods. Compared to the GK-method, the generalized CPD seems to show lower price levels for branches like the food, beverages and tobacco, wood products and wearing apparel but higher for chemical products and basic and fabricated metal products. If additivity is a property that is considered important, at this stage it is recommended that the Geary-Khamis international prices be used.

5. Summary and Conclusions

In this study we have considered the problem of constructing consistent (transitive) multilateral comparisons using the existing ICOP database. In contrast to the earlier study undertaken by Pilat and Rao (1991), considerable emphasis is placed on the construction of transitive multilateral comparisons below the branch level. Several features of the price and quantity data below the basic heading level make it an interesting exercise. In this study, we proposed and used methods that are considerably superior to those used in the ICP since we make use of the quantity data available below the basic heading level.

Since several aggregation methods are available, and each method leads to a different set of PPPs for the branch under consideration, it is necessary to choose the most appropriate method and the PPPs resulting from it. From a theoretical perspective, our preference is for the use of a weighted EKS method in the place of the standard EKS method below the basic heading, or in this case manufacturing branch, level. The price-quantity data compiled for the multilateral exercise suggests that pairwise comparisons between countries are made on the basis of different number of matches and, therefore the binary comparisons differ in their reliability. The weighted EKS takes into account this information. Within the weighted EKS method, we have two further choices available, one based on the Hill's distance measure derived on the basis of the Laspeyres-Paasche spread and the other based solely on the number of matches. At this stage, we have not been able to incorporate them simultaneously in deriving our weighted EKS indices. It would require some *a priori* weighting when introduced into the covariance structure of the disturbances involved in Section 3.3. Given this, our own preference at the present stage is to use a geometric average of the PPPs resulting from the two weighted EKS systems and use those PPPs as basic input into aggregation above the basic heading level.

For purposes of aggregation above the basic heading level, the two competing methods are the Geary-Khamis method and the weighted EKS method with two versions again based on the Hill's distance measure and another based on the coverage ratios. The GK method has the attractive property of additivity. The weighted EKS does not satisfy this requirement. At this stage, we have no reason to discriminate between the PPPs from the two versions of the weighted EKS method. As shown in this paper, the generalized CPD appears to hold promise but further work is still in progress on this method. Therefore we suggest the use of a simple geometric mean of these two sets of weighted EKS PPPs as the preferred method of computing transitive multilateral PPPs.

In Table 5.1 we provide the output results derived using the GK and weighted EKS PPPs averaged over the two specifications for the weighting schemes available. These results are compared with the original binary comparisons. On the basis of this table and the results presented in this paper, three general observations can be made. First, with respect to the basic item-level data, it has been shown that it is possible to derive a common set of item prices and quantities from the various binary ICOP studies. This set can be used in a multilateralisation framework. As we only included products for which we had data on prices and quantities in at least three countries, parts of the original data set could not be used. In columns 2 and 3 of Table 5.1 we compare the effect of the smaller data set on the Fisher binary for total manufacturing. This effect appears to be small.

Second, we found that the choice of a particular aggregation method below the branch level is more important than the choice of a particular aggregation method above branch level. Results are more sensitive to the choice of the former than to the choice of the latter (see Table 4.3). Third, below branch level, additivity cannot be imposed. It has been argued that the weighted EKS is superior to the unweighted EKS as it includes extra information on the reliability of the various underlying binary comparisons. Above branch level weighted EKS can be used, or GK when additivity is required. As shown in the last column of Table 5.1, the GK method has the potential to generate results which may result in an understatement of the PPPs for some of the developing countries.

Table 5.1 Comparison of gross value added using alternative PPPs, total manufacturing, 1987

	Exchange rate (nat cur per US$)	Fisher Binary PPP Original data set	Fisher Binary PPP New data set [a]	Preferred EKS Multilateral PPP[b]	Geary Khamis Multilateral PPP[b]
Australia	1.43	1.49	1.47	1.43	1.37
Canada	1.33	1.33	1.36	1.35	1.35
Germany [d]	1.80	2.21	2.13	2.20	2.21
Indonesia [e]	1644	1200	1306	1262	1189
Japan	144.6	181.5	187.9	184.7	170.6
South Korea	823.0	699.6	679.0	676.5	642.3
Taiwan	31.87	29.60	30.23	30.57	27.78
USA	1.00	1.00	1.00	1.00	1.00

*Note*s: [a] Fisher binary PPP based on new data set (see section 2), reweighted at branch level with gross value added.

[b] Below branch level, a geometric mean of Hill's distance and weighted EKS using number of matched products (see Appendix Table 1) is used for Geary Khamis and weighted EKS above branch level. Preferred EKS multilateral is defined as geometric mean of weighted EKS using Hill's distance and weighted EKS using percentage covered above branch level.

Source: Exchange rate, original Fisher binary PPP and gross value added for Australia from Pilat et all (1993), Canada from De Jong (1996), Germany from van Ark (1993), Indonesia and Taiwan from Timmer (2000), South Korea and Japan from Pilat (1994).

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Appendix I ICOP-methodology

The ICOP methodology for aggregating item-level prices, usually referred to as the unit value ratios (UVRs), involves a three-stage aggregation process. Item-level prices are aggregated into sample industry PPPs, and further into branch PPPs and finally a PPP for total manufacturing. At the first stage, the UVRs belonging to a particular industry or group are aggregated resulting in purchasing power parities (PPPs) for the sample industries to which these items belong. The Fisher index number formula is used in aggregating item-level price data. The nature and the number of sample industries used in any particular binary comparison typically depend upon the coverage ratios associated with products that are matched in a given comparison exercise. A rule of thumb used in most past ICOP comparisons is that if the matched items cover twenty-five percent or more of the output in an industry, PPP for the industry (referred to as the sample industry) is calculated. In the second stage, the PPPs for the sample industries are aggregated further to yield PPPs for major manufacturing branches using Fisher index number formula along with weights derived from the gross value added in each of the sample industry. The final stage of the ICOP methodology essentially aggregates branch level PPPs and a single manufacturing sector PPP is derived, again using value added weights and the Fisher formula. These stages are involved in all of the ICOP comparisons to date. Maddison and van Ark (1988) provides an excellent summary and details of the procedures involved. Szirmai et al (1995) also presents the ICOP methodology in a more formal style along with a numerical example outlining the steps involved.

There are three major issues that can be raised about the multi-stage procedure used in the ICOP studies. The first and foremost concerns the binary nature of the ICOP comparisons, that is the procedures outlined above are employed with only a pair of countries at a time. The process of matching products and the subsequent aggregation using the Fisher formula imply that the binary comparisons made under the ICOP scheme do not satisfy the transitivity property and, therefore, are unsuitable for multilateral comparisons involving several countries. As stated in the introduction to this paper, this aspect of the ICOP comparisons forms the core of the present study.

The second issue, a relatively minor one, relates to the arbitrary nature of the cut-off coverage used in identifying a sample industry. The present study, therefore, dispenses with the concept of sample industry and uses a two-stage approach which involves aggregation to the manufacturing branch level and then on to the sector as a whole.

The third issue concerns the use of value added weights in aggregating branch level PPPs. The use of value added weights has traditionally been justified on the grounds that it is consistent with the use of the PPPs in a single deflation procedure to convert gross value added in different countries into a common currency unit. Following Timmer (2000), it can be argued that from an analytical view point single deflation procedure implies the use of an output price index, or PPP, for purposes of deflating both the gross value of outputs and the value of intermediate inputs used in the production process. Thus the single deflation procedure requires that the output PPP, or price index, is computed properly, using output prices and weights, and the resulting index is used for deflating various aggregates. Consistent with this notion of single deflation, the present study uses output quantities as weights in computing various index number formulae.

Appendix II Binary Methods within ICOP

In this appendix we briefly describe index number formulae used in binary ICOP comparisons. These are the Laspeyres, Paasche and Fisher index numbers.

Laspeyres index

The Laspeyres index between a pair of countries j and k, denoted by L_{jk}, is obtained using the quantity weights of the base country j. Thus

$$L_{jk} = \frac{\sum_{i=1}^{n} p_{ik} q_{ij}}{\sum_{i=1}^{n} p_{ij} q_{ij}} \qquad \text{(II.1)}$$

The index is the ratio of the value aggregates derived by valuing country j quantities at its own prices (p_{ij}) and at the prices of the "other" country (p_{ik}). We note that (II.1) can only be defined on the basis of price data for commodities that are common in both countries. Table II.1 presents the Laspeyres PPPs for the food manufacturing branch.

Table II.1 Laspeyres PPP for food manufacturing branch for all binary comparisons, 1987 (in national currency per unit of base country currency)

	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
USA	1.000	1.264	1.486	2.222	1502.68	309.125	1037.59	39.849
Australia	0.880	1.000	1.492	1.971	1394.17	321.684	841.047	45.883
Canada	0.686	0.731	1.000	1.393	918.570	228.989	663.451	27.297
Germany	0.461	0.527	0.724	1.000	647.252	176.430	433.279	19.439
Indonesia	0.001	0.001	0.001	0.003	1.000	0.214	0.775	0.025
Japan	0.003	0.004	0.005	0.005	4.096	1.000	3.474	0.094
Korea	0.001	0.001	0.002	0.003	1.472	0.339	1.000	0.043
Taiwan	0.032	0.032	0.042	0.055	36.746	11.721	27.880	1.000

Source: see Table 3.1

Paasche index

The Paasche index, denoted by P_{jk}, is defined using the quantities of country k and is defined by

$$P_{jk} = \frac{\sum_{i=1}^{n} p_{ik} q_{ik}}{\sum_{i=1}^{n} p_{ij} q_{ik}} \qquad \text{(II.2)}$$

The Paasche index uses quantities in country k as weights. The results for the food manufacturing branch are given in Table II.2.

We note from the formulae in (II.1) and (II.2) a certain asymmetry in the use of quantity information in defining these indices. This shows up in the difference between the Laspeyres and Paasche indices. In Table II.3 we give the spread between the Laspeyres and Paasche PPP for each binary comparison. For example for

Indonesia/USA the Laspeyres PPP is 1503 Rps/US$, while the Paasche is 984 Rps/US$. Hence the Paasche-Laspeyres spread is 0.65.

Table II.2 Paasche PPP for food manufacturing branch for all binary comparisons, 1987 (in national currency per unit of base country currency)

	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
USA	1.000	1.136	1.459	2.171	983.857	305.268	788.749	31.407
Australia	0.791	1.000	1.369	1.899	1282.048	260.061	841.609	31.608
Canada	0.673	0.670	1.000	1.381	887.068	210.943	565.985	23.857
Germany	0.450	0.507	0.718	1.000	384.804	187.089	331.611	18.129
Indonesia	0.001	0.001	0.001	0.002	1.000	0.244	0.679	0.027
Japan	0.003	0.003	0.004	0.006	4.668	1.000	2.953	0.085
Korea	0.001	0.001	0.002	0.002	1.290	0.288	1.000	0.036
Taiwan	0.025	0.022	0.037	0.051	40.757	10.634	23.085	1.000

Source: see Table 3.1

Fisher index

The Fisher index, F_{jk}, is defined as the geometric average of the Laspeyres and Paasche indices, given by

$$F_{jk} = \sqrt{L_{jk} \cdot P_{jk}} \qquad \text{(II.3)}$$

The Fisher index is the formula used in all the ICOP binary comparisons to date. The Fisher index satisfies many desirable statistical, as well as economic-theoretic, properties. Diewert (1976, 1992) examines these properties and describes the Fisher index (along with the Tornqvist index) to be "exact" and "superlative". In addition, the Fisher index is also known to be an "ideal" index since it satisfies time and factor reversal tests (see Allen 1975). However, the Fisher index is of limited use for purposes of multilateral comparisons since it fails to satisfy the transitivity property. Fisher PPPs for food manufacturing are shown in Table II.4.

Table II.3 Paasche-Laspeyres PPP Ratio for food manufacturing branch for all binary comparisons, 1987

	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
USA	1.00	0.90	0.98	0.98	0.65	0.99	0.76	0.79
Australia	0.90	1.00	0.92	0.96	0.92	0.81	1.00	0.69
Canada	0.98	0.92	1.00	0.99	0.97	0.92	0.85	0.87
Germany	0.98	0.96	0.99	1.00	0.59	1.06	0.77	0.93
Indonesia	0.65	0.92	0.97	0.59	1.00	1.14	0.88	1.11
Japan	0.99	0.81	0.92	1.06	1.14	1.00	0.85	0.91
Korea	0.76	1.00	0.85	0.77	0.88	0.85	1.00	0.83
Taiwan	0.79	0.69	0.87	0.93	1.11	0.91	0.83	1.00

Source: Tables II.1 and II.2

Table II.4 Fisher PPP for food, beverages and tobacco manufacturing branch for all binary comparisons, 1987 (in national currency per national currency)

	USA	Australia	Canada	Germany	Indonesia	Japan	Korea	Taiwan
USA	1.000	1.198	1.472	2.197	1215.900	307.200	904.700	35.380
Australia	0.834	1.000	1.429	1.935	1336.900	289.200	841.300	38.080
Canada	0.679	0.700	1.000	1.387	902.700	219.800	612.800	25.520
Germany	0.455	0.517	0.721	1.000	499.100	181.700	379.100	18.770
Indonesia	0.001	0.001	0.001	0.002	1.000	0.229	0.726	0.026
Japan	0.003	0.004	0.005	0.006	4.373	1.000	3.203	0.090
Korea	0.001	0.001	0.002	0.003	1.378	0.312	1.000	0.039
Taiwan	0.028	0.026	0.039	0.053	38.700	11.165	25.369	1.000

Source: Tables II.1 and II.2

Appendix Table III PPPs using various methods for manufacturing branches in 7 countries, 1987 (in national currency per US$)

(a) Food, beverages and tobacco

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.20	1.06	1.09	1.07	1.27
Canada	1.47	1.48	1.49	1.48	1.59
Germany	2.20	2.13	2.17	2.06	2.27
Indonesia	1215.90	1295.02	1285.24	1338.34	1186.76
Japan	307.19	327.09	321.39	320.98	321.20
South Korea	904.65	913.03	917.11	903.54	1112.20
Taiwan	35.38	35.85	35.61	35.24	41.54

(b) Textile products

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.68	1.54	1.56	1.84	1.80
Canada	1.53	1.80	1.71	1.91	1.64
Germany	2.42	2.24	2.30	2.38	2.11
Indonesia	827.37	900.99	915.82	1020.53	922.47
Japan	173.73	172.01	173.49	181.88	173.58
South Korea	747.40	708.68	729.44	796.13	833.78
Taiwan	25.97	25.50	25.85	23.30	24.86

C Wearing Apparel

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.52	1.49	1.47	1.56	1.43
Canada	1.36	1.38	1.36	1.36	1.36
Germany	3.09	3.13	3.13	3.42	3.26
Indonesia	428.69	448.81	449.42	418.20	352.39
Japan	198.71	224.43	219.48	201.41	221.54
South Korea	998.14	869.20	889.43	808.59	1142.52
Taiwan	17.30	16.73	17.19	15.41	17.74

D. Leather products

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.39	1.35	1.38	1.14	1.19
Canada	1.28	1.30	1.31	1.26	1.29
Germany	2.80	3.06	3.10	3.02	2.70
Indonesia	579.02	557.47	565.69	521.08	425.47
Japan	205.14	225.10	223.95	218.94	189.04
South Korea	696.64	631.89	645.72	651.15	531.61
Taiwan	18.36	17.74	17.73	17.09	15.05

E. Wood products

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.93	1.88	1.87	1.93	1.73
Canada	1.38	1.37	1.36	1.34	1.76
Germany	3.09	2.96	3.01	3.07	3.27
Indonesia	1283.34	1372.02	1351.48	1331.80	984.69
Japan	491.04	603.77	517.77	492.43	532.63
South Korea	1240.59	1175.81	1228.75	1218.27	1276.75
Taiwan	50.85	52.05	49.75	47.56	55.42

F. Paper products

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.80	1.74	1.64	1.82	1.56
Canada	1.44	1.39	1.41	1.46	1.31
Germany	2.15	2.11	2.11	2.08	2.07
Indonesia	1517.12	1402.15	1405.38	1355.05	1357.50
Japan	182.10	198.50	195.29	181.57	175.71
South Korea	721.65	743.83	752.61	737.77	714.51
Taiwan	33.95	35.69	36.19	34.61	34.39

G. Chemicals

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.33	1.56	1.61	1.53	1.74
Canada	1.30	1.27	1.29	1.32	1.50
Germany	2.05	2.20	2.24	2.15	2.37
Indonesia	1909.66	1628.80	1766.38	1588.05	1642.56
Japan	293.47	269.21	274.99	253.19	223.87
South Korea	1151.34	976.49	1072.87	835.63	899.88
Taiwan	32.13	39.38	38.53	32.52	41.95

H. Rubber and plastic products

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.26	1.22	1.19	1.48	1.21
Canada	1.28	1.26	1.26	1.28	1.25
Germany	2.32	2.25	2.23	2.31	2.24
Indonesia	957.22	974.47	990.23	1124.93	978.54
Japan	107.16	100.72	100.25	104.41	106.22
South Korea	610.68	709.54	677.37	745.35	691.24
Taiwan	31.13	30.19	30.00	30.86	29.51

I. Non-metallic mineral products

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.50	1.62	1.57	1.56	1.40
Canada	1.34	1.33	1.33	1.31	1.36
Germany	1.57	1.61	1.60	1.61	1.16
Indonesia	1036.61	929.21	921.72	964.17	643.60
Japan	188.77	183.59	186.13	183.50	181.92
South Korea	465.80	477.74	473.89	478.34	547.89
Taiwan	21.42	21.94	21.69	24.76	17.83

J. Basic and fabricated metal

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.84	1.71	1.75	1.76	1.88
Canada	1.49	1.48	1.49	1.46	1.40
Germany	2.14	2.17	2.16	2.24	2.18
Indonesia	1137.49	1147.25	1145.63	1214.16	960.94
Japan	190.83	188.58	184.38	219.07	188.41
South Korea	756.82	760.32	764.13	824.38	779.38
Taiwan	32.04	34.28	33.86	33.99	35.17

K. Machinery and transport equipment

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	1.13	1.17	1.17	1.12	1.21
Canada	1.17	1.14	1.16	1.18	1.64
Germany	1.97	1.95	1.97	1.99	2.00
Indonesia	1900.96	1773.73	1817.70	1802.94	1840.64
Japan	99.00	96.07	98.63	99.19	139.82
South Korea	429.58	432.24	431.66	420.79	506.83
Taiwan	36.82	40.83	38.49	33.14	36.70

L. Electrical machinery

	Fisher	EKS	Weighted EKS Number of matches	Weighted EKS Hill's distance	Unweighted CPD
USA	1.00	1.00	1.00	1.00	1.00
Australia	2.07	1.56	1.63	1.66	1.66
Canada	1.49	1.36	1.35	1.49	1.29
Germany	2.04	2.51	2.35	2.24	2.77
Indonesia	796.25	956.15	871.68	868.38	449.48
Japan	158.21	156.88	157.12	153.59	173.44
South Korea	414.47	393.88	389.44	384.98	430.18
Taiwan	18.17	18.85	17.85	19.77	15.87

Source: Product data from Table 3.1. Fisher using (II.3), CPD using (3.3), EKS using (3.4), weighted EKS using (3.10).

Appendix Table IV Purchasing Power Parities for the Manufacturing Sector using various aggregation methods below and above branch level, 1987 (national currencies per US dollar)

Method above branch level	Method below branch level				
Australia/USA	Fisher	CPD	EKS	WEKS(Hill)	WEKS(match)
Laspeyres	1.470	1.497	1.429	1.456	1.439
Paasche	1.449	1.480	1.390	1.414	1.405
Fisher	1.460	1.488	1.409	1.435	1.422
EKS	1.470	1.495	1.404	1.430	1.421
WEKS(Hill)	1.457	1.492	1.405	1.431	1.419
WEKS(Cov ratio)	1.474	1.502	1.409	1.435	1.426
Weighted CPD	1.443	1.342	1.393	1.450	1.384
Geary-Khamis	1.417	1.464	1.355	1.375	1.373
Canada/USA	Fisher	CPD	EKS	WEKS(Hill)	WEKS(match)
Laspeyres	1.358	1.498	1.336	1.368	1.345
Paasche	1.350	1.490	1.325	1.355	1.337
Fisher	1.354	1.494	1.331	1.362	1.341
EKS	1.356	1.482	1.335	1.367	1.344
WEKS(Hill)	1.355	1.492	1.332	1.364	1.342
WEKS(Cov ratio)	1.356	1.486	1.336	1.368	1.345
Weighted CPD	1.354	1.348	1.343	1.410	1.329
Geary-Khamis	1.352	1.485	1.330	1.357	1.341
Germany/USA	Fisher	CPD	EKS	WEKS(Hill)	WEKS(match)
Laspeyres	2.159	2.267	2.190	2.187	2.201
Paasche	2.086	2.205	2.154	2.137	2.157
Fisher	2.122	2.236	2.172	2.162	2.179
EKS	2.155	2.280	2.216	2.204	2.219
WEKS(Hill)	2.139	2.253	2.188	2.182	2.196
WEKS(Cov ratio)	2.157	2.281	2.215	2.204	2.220
Weighted CPD	2.144	2.046	2.242	2.290	2.198
Geary-Khamis	2.147	2.259	2.223	2.197	2.225

Indonesia/USA	Fisher	CPD	EKS	WEKS(Hill)	WEKS(match)
Laspeyres	1,430	1,285	1,374	1,384	1,398
Paasche	1,236	1,095	1,235	1,267	1,254
Fisher	1,330	1,186	1,303	1,324	1,324
EKS	1,262	1,111	1,243	1,263	1,259
WEKS(Hill)	1,253	1,099	1,243	1,256	1,256
WEKS(Cov ratio)	1,270	1,121	1,250	1,270	1,266
Weighted CPD	1,193	982	1,181	1,248	1,178
Geary-Khamis	1,165	1,058	1,166	1,192	1,185

Japan/USA	Fisher	CPD	EKS	WEKS(Hill)	WEKS(match)
Laspeyres	209.6	213.0	215.0	208.5	210.9
Paasche	156.8	177.2	154.8	157.4	155.8
Fisher	181.3	194.3	182.4	181.2	181.3
EKS	184.1	197.0	184.7	184.4	184.1
WEKS(Hill)	185.0	196.7	184.0	184.1	184.1
WEKS(Cov ratio)	185.1	198.1	185.6	185.1	185.0
Weighted CPD	178.5	173.7	179.8	185.1	176.6
Geary-Khamis	170.6	186.0	169.5	171.0	170.1

South Korea/USA	Fisher	CPD	EKS	WEKS(Hill)	WEKS(match)
Laspeyres	749.1	775.2	721.5	705.8	740.1
Paasche	633.9	670.5	618.7	613.3	623.7
Fisher	689.1	721.0	668.1	657.9	679.4
EKS	696.8	737.2	677.0	667.0	688.1
WEKS(Hill)	688.2	727.8	669.6	659.4	681.2
WEKS(Cov ratio)	703.8	744.6	682.0	671.9	694.1
Weighted CPD	680.0	652.2	664.1	672.8	663.6
Geary-Khamis	659.9	695.2	641.4	634.7	649.0

Taiwan/USA	Fisher	CPD	EKS	WEKS(Hill)	WEKS(match)
Laspeyres	32.792	35.464	35.419	32.173	34.535
Paasche	27.160	26.922	28.359	27.263	27.784
Fisher	29.844	30.899	31.693	29.617	30.976
EKS	29.904	31.301	31.732	29.518	31.059
WEKS(Hill)	30.159	31.774	31.876	29.864	31.337
WEKS(Cov ratio)	30.216	31.742	31.979	29.760	31.337
Weighted CPD	28.058	26.883	29.821	28.809	28.811
Geary-Khamis	27.362	27.828	28.574	27.375	28.121

Source: see Tables 2.1 and 4.1.