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A NONLINEAR CHARACTERISTIC IMPEDANCE TRACKING CONTROLLER FOR THE CCM-DCM BOOST CONVERTER

Dimitri Jeltsema, Jacquelien M.A. Scherpen, and J. Ben Klaassens

Fac. ITS, Dept. of Electrical Eng., Delft University of Technology,
Control Systems Engineering Group
P.O. Box 5031, 2600 GA Delft, The Netherlands.
E-mail: d.jeltsema/j.m.a.scherpen/j.b.klaassens@its.tudelft.nl

ABSTRACT

A robustly stable nonlinear control algorithm for the DC-to-DC single switch Boost converter is proposed. The scheme does not require current sensors but only measurements of the capacitor voltage. The design procedure is based on the well-known passivity-based control (PBC) methodology. With this technique damping is injected by 'placing' artificial resistors in the circuit such that the characteristic impedance can be matched dynamically. In contrast to the conventional damping assignment philosophy, the design presented in this paper does not need the extension with an adaptive mechanism to ensure robustness against unmodeled changes in the load resistance. The controller is tested for both continuous (CCM) and discontinuous conduction (DCM) mode.

1. INTRODUCTION

For many years the problem of regulating the output voltage and input current of the Boost type power converter has attracted many researchers from both the field of system and control theory and power electronics. Besides its wide range of applications, this circuit describes in form and function a major family of power electronic converters. It is a nonlinear and non-minimum phase switching device, which introduces severe restrictions on the achievable closed-loop bandwidth. Many publications on this subject start with deriving the average dynamics which subsequently are linearized in order to obtain a small signal model. This model is then to be used for analysis and controller design. As a result, the control will often be composed of an inner and an outer loop realized by PI, PID or lead-lag type filters. Due to the nonlinear behavior of the conversion ratio, the resonance frequency is varying with the desired output voltage. This makes it sometimes hard to tune the controllers as to ensure robust performance, especially in the presence of large setpoint changes, disturbances or errors that cause circuit operation to deviate from the nominal point of operation. Furthermore, many linear control schemes require full state measurements.

Recently, various attempts have been made to overcome the aforementioned problems. One of these attempts is the passivity-based controller (PBC) design methodology for switch-mode power converters, which has proven to be an interesting alternative for other, mostly linear, control techniques. This technique stems from classical Euler-Lagrange dynamics theory and the closely related field of Robotics. The application to single-switch DC-to-DC power converters was first proposed in (H. Sira-Ramírez *et al.*, 1997) and is generalized to larger networks, like the coupled-inductor Ćuk converter and three-phase rectifiers, in e.g. (Jeltsema *et al.*, 2001; Ortega *et al.*, 1998; Scherpen *et al.*, 2000). In these works it is shown that a PBC design method is applicable to the average pulse-width modulated (PWM) models of switch regulated power converters, provided that such (idealized) models correspond to systems derivable from the classical Euler-Lagrange dynamics theory.

One of the major advantages of underscoring the physical structure, e.g., energy and interconnection, of these circuits is that the nonlinear phenomena and features are explicitly incorporated in the model, and thus in the corresponding PBC. This allows an interpretation in similar physical terms of the controlled closed-loop system, which has recently led us to the notion of injecting virtual resistors into the circuit (Jeltsema *et al.*, 2001). From a circuit-theoretic point of view, the controller produces a computed duty ratio function which forces the closed-loop dynamics to act as if there are resistors connected in series and/or parallel to the real circuit elements. In this way the characteristic impedance(s) of the filter elements can be matched by the controller. As a result, power is not reflected and resonance problems, especially during start-up or setpoint changes, are minimized. In this paper we extend the developments of (Jeltsema *et al.*, 2001) to converter structures having a nonlinear conversion ratio. We will also show that, although the design procedure is based on continuous average PWM models, the PBC algorithms can also be used for converters operating in discontinuous current mode.

The remainder of this paper is organized as follows. In

Section 2 the model used for the controller design is introduced. Section 3 presents our main results regarding the controller design and tuning (for sake of brevity, the proofs regarding the stability and robustness are presented in the Appendix). After that, we present an extensive simulation study where the proposed controller is tested on a switched-mode mathematical model of the Boost converter. Finally, we end with some conclusions.

2. LARGE SIGNAL AVERAGED PWM MODEL

Throughout the paper we will use the nonlinear averaged PWM equations for our control design and analysis purposes. The idealized average PWM equations of the DC-to-DC Boost converter with a single uni-directional switch, depicted in Figure 1, are given by

$$\begin{aligned} L\dot{z}_1 &= E + \Delta E - (1-\mu)z_2 \\ C\dot{z}_2 &= (1-\mu)z_1 - \frac{P(z_2) + \Delta P(z_2)}{z_2}, \end{aligned} \qquad (1)$$

where $z_1 \in \mathbb{R}^+$ is the average inductor current[1], $z_2 \in \mathbb{R}^+$ is the average capacitor voltage and μ is the continuous control signal limited to the closed set $\mu \in U := [0,1]$. Furthermore, $P(z_2) \geq 0$ denotes the nominal average power dissipated in the load, $\Delta P(z_2)$, with $P(z_2) + \Delta P(z_2) \geq 0$, reflects the load uncertainty. The constant nominal value of the voltage source is represented by E and ΔE denotes an unknown (time-varying) disturbance satisfying $|\Delta E| < E$. The control objective is to regulate the output voltage z_2 towards some constant desired equilibrium value $z_2^* > E$, regardless of any disturbances generated by ΔE and $\Delta P(z_2)$. An additional objective is to suppress the inrush (start-up) current as much as possible.

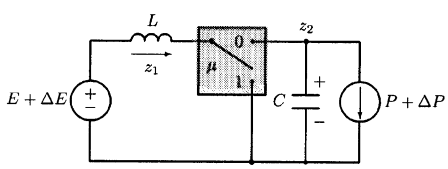

Figure 1. Idealized Boost circuit.

3. CONTROLLER DESIGN

Following the PBC methodology as proposed in (H. Sira-Ramírez *et al.*, 1997), our control objective will be achieved by forcing the closed-loop system to be passive with respect to a desired closed-loop error storage function and by adding damping as to ensure asymptotic stability. This is tantamount to defining the closed-loop error dynamics as

$$\begin{aligned} L\dot{\tilde{z}}_1 + (1-\mu)\tilde{z}_2 + \frac{P_{1_d}(\tilde{z}_1)}{\tilde{z}_1} &= \psi_1 \\ C\dot{\tilde{z}}_2 - (1-\mu)\tilde{z}_1 + \frac{P_{2_d}(\tilde{z}_2)}{\tilde{z}_2} &= \psi_2, \end{aligned} \qquad (2)$$

where $\tilde{z}_i := z_i - \xi_i$, $i = 1, 2$, and ξ_i is a desired auxiliary state to be defined using ψ_i (notice that the load perturbation term is neglected at this point because it is unknown). Furthermore, the required damping is added by choosing some desired dissipated power $P_{i_d}(\tilde{z}_i) \geq 0$. At this stage, one usually takes $P_{1_d}(\tilde{z}_1) := R_s^i \tilde{z}_1^2$ for some $R_s^i \geq 0$ and $P_{2_d}(\tilde{z}_2) := P(\tilde{z}_2)$, see e.g. (Ortega *et al.*, 1998; H. Sira-Ramírez *et al.*, 1997). The next step is then to derive the controller dynamics by letting ψ_i be a function of the auxiliary states ξ_1, ξ_2, i.e., $\psi_i(\xi_1, \xi_2)$, and by forcing $\psi_i = 0$ in order to fulfill the objective and to solve for μ. As is discussed in (Jeltsema *et al.*, 2001), this particular choice of damping assignment is interpretable as *series damping injection*, i.e., the closed-loop dynamics act as if there are virtual resistors connected in series with the inductors in the circuit. Unfortunately, this is certainly not the best choice because

1. this will lead to an indirectly controlled closed-loop system that is highly sensitive to load uncertainties;
2. an expensive current sensor is needed for current feedback.

One way to overcome the first problem is by extending the control with an adaptive mechanism (Ortega *et al.*, 1998). The drawback of this solution is that the controller will become more expensive from a computational point of view. Another drawback is that a nice physical interpretation of the closed-loop dynamics in terms of physical elements, like resistors, inductors, capacitors and transformers, is lost.

We are now ready to present the first part of our main result for which the proof is established in the Appendix. Consider the error dynamics (2). Suppose we set the damping terms as

$$P_{1_d}(\tilde{z}_1) := 0, \text{ and } P_{2_d}(\tilde{z}_2) := P(\tilde{z}_2) + P_p^i(\tilde{z}_2),$$

where $P_p^i(\tilde{z}_2) = G_p^i \tilde{z}_2^2 \geq 0$ denotes the virtual power generated by the injected parallel damping[2]. Following the PBC design methodology we then have that

$$\psi_1 = E - L\dot{\xi}_1 - (1-\mu)\xi_2 \qquad (3)$$

$$\psi_2 = -C\dot{\xi}_2 + (1-\mu)\xi_1 - \frac{P(\xi_2)}{\xi_2} + \frac{P_p^i(\tilde{z}_2)}{\tilde{z}_2}. \qquad (4)$$

[1] Because the converter is assumed to have an uni-directional switch, the currents and voltages are restricted to, $z_i \geq 0$, $i = 1, 2$, i.e., the positive real set including the origin denoted by $\mathbb{R}^+$.

[2] Voltage-controlled parallel resistors are usually denoted as conductances, e.g., in the linear case the conductance G denotes the inverse resistance $G = R^{-1}$ $[\Omega^{-1}]$.

At this point one is tempted to fix $\xi_2 = z_2^*$ and then let $\psi_i = 0$ as to solve for the control μ. As is known from (H. Sira-Ramírez *et al.*, 1997) this will not result in a feasible controller due to the non-minimum phase nature of z_2 (for more details, see (H. Sira-Ramírez *et al.*, 1997)). However, since after setting $\psi_i = 0$, there are three variables left and only two equations to be satisfied we might as well let $\xi_1 = z_1^*$. Hence, from (3) we obtain

$$\mu = 1 - \frac{E}{\xi_2}, \ \xi_2 > 0, \tag{5}$$

and let ξ_2 be the solution of the nonlinear differential equation (4) with $\psi_2 = 0$, i.e.,

$$C\dot{\xi}_2 + (G + G_p^i)\xi_2 - G\frac{(z_2^*)^2}{E\xi_2} = G_p^i z_2, \tag{6}$$

with $\xi_2(0) > 0$, and where, without loss of generality, we have assumed that the nominal load conductance is linear and thus that $P(\xi_2) = G\xi_2^2$. Notice that the only signal used for feedback is the average capacitor voltage z_2, while we have aimed at indirect regulation of the capacitor voltage via regulation of

$$\xi_1 = z_1^* = G\frac{(z_2^*)^2}{E}.$$

The average error dynamics are then given by

$$L\dot{\tilde{z}}_1 = -(1-\mu)\tilde{z}_2 \tag{7}$$
$$C\dot{\tilde{z}}_2 = (1-\mu)\tilde{z}_1 - (G + G_p^i)\tilde{z}_2. \tag{8}$$

We conclude the first part with the remark that the concept as presented above can be considered as parallel damping injection, i.e., with the controller (5)–(6) we have connected a virtual parallel resistor to the output capacitor of the Boost converter (for a detailed explanation of this concept, see (Jeltsema *et al.*, 2001)). Moreover, this concept enables us to control a non-minimum phase system based on measurements of the non-minimum phase output(s) only.

The question that arises is how to adjust the damping parameter G_p^i as to ensure a predefined desired transient behavior of the closed-loop dynamics. To answer this question, consider the ideal average error dynamics of the Boost converter (7)–(8). A circuit-theoretic interpretation of (7)–(8) can be represented by the circuit as shown in Figure 2. In the figure the conversion ratio of the boost converter is modeled by an ideal transformer conducting both AC and DC currents, known as the Middlebrook-Ćuk transformer (Middlebrook and Ćuk, 1976). The turns ratio of the transformer is equal to the duty ratio $n := 1 - \mu$.

Suppose now that μ is constant, say $\mu = \mu^*$. In that case we can equate the error control-to-output voltage relation

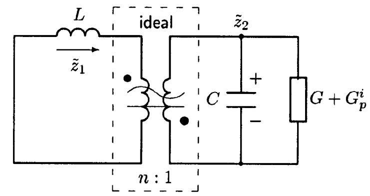

Figure 2. Average closed-loop error dynamics modeled as an Middlebrook-Ćuk transformer.

as a linear differential equation of the form

$$\ddot{\tilde{z}}_2 + \underbrace{\frac{G + G_p^i}{C}}_{2\beta\omega_0}\dot{\tilde{z}}_2 + \underbrace{\frac{(1-\mu^*)^2}{LC}}_{\omega_0^2}\tilde{z}_2 = 0. \tag{9}$$

From classical control theory we know that in order to have a perfect match between L, C and $G + G_p^i$, i.e., perfect damping and zero-overshoot, β has to satisfy $\beta = 1$ (see, e.g., (Philips and Harbor, 1991)). This is accomplished if and only if

$$\left(\frac{G + G_p^i}{C}\right)^2 = 4\frac{(1-\mu^*)^2}{LC},$$

and thus that

$$G_p^i(\mu^*) = \begin{cases} 0 & \text{for } G^{-1} = Z_c(\mu^*) \\ \dfrac{1 - GZ_c(\mu^*)}{Z_c(\mu^*)} & \text{for } G^{-1} \neq Z_c(\mu^*), \end{cases}$$

where

$$Z_c(\mu^*) = \frac{1}{2}\sqrt{\frac{L}{(1-\mu^*)^2 C}}$$

denotes the characteristic impedance of the circuit. This means that for every equilibrium (z_1^*, z_2^*, μ^*) we have an unique value for the characteristic impedance Z_c and thus for the injected damping G_p^i. Then, if μ is changing suddenly from one equilibrium to another we can try to estimate, or in other words, 'track' the characteristic impedance by plugging the calculated μ of (5)–(6) back into the equation for G_p^i, i.e,

$$G_p^i(\mu) = \frac{1 - GZ_c(\mu)}{Z_c(\mu)}. \tag{10}$$

Hence, after substitution of (10) we conclude our main result. The proofs concerning the stability of the proposed controller are presented in Appendix A.

Remark: The design of the PBC algorithm is carried out for the ideal average dynamic PWM model of the Boost converter. If the PWM frequency is chosen sufficiently high this model will capture the essential dynamic behavior, and, as a result, the controller is also well defined. A

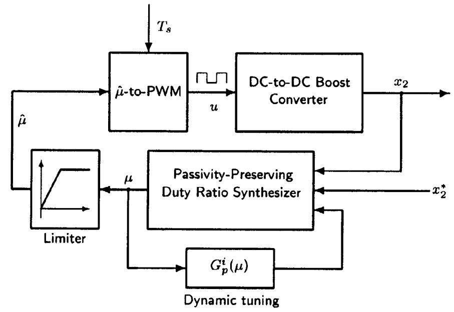

Figure 3. Feedback scheme for the parallel damping PBC controlled Boost converter.

representation of the parallel damping design philosophy is given in Figure 3. Here, $F_s = T_s^{-1}$ denotes the PWM switching frequency, u is the discrete switch control function living in the discrete set $\{0, 1\}$ and x_2 represents the 'real' capacitor voltage. Furthermore, impedance matching is accomplished by the dynamically tuned parameter $G_p^i(\mu)$ as defined in (10) and $\hat{\mu}$ denotes the calculated duty ratio limited to the closed set $[0, 1]$.

4. SIMULATION EXPERIMENTS

In this section the controller (5)–(6) together with the tuning parameter (10) is tested through computer simulations[3] using SIMULINK for a Boost converter with the discrete values for the switch. The only signal used for feedback is the 'real' capacitor voltage x_2. The behavior of the closed-loop system is compared to the following criteria

i) Transient and steady state response to step changes in the desired output voltage reference.

ii) Attenuation of step changes in the unmodeled load uncertainties and step changes in the voltage source.

The design parameter of the Boost converter are chosen as follows: $E = 10$V, $L = 10\mu$H, $C = 50\mu$H, $F_s = 50$kHz and $R = G^{-1} = 5\Omega$. The initial conditions are set to $x_1(0) = x_2(0) = 0$ and $\xi_2(0) = 1$.

In Figure 4 (top) the typical open-loop response to a step change in the desired capacitor voltage x_2^* from 0 to 37.5V is depicted. As can be seen, the behavior is quite oscillatory and the inrush current shows a high peak of about four times its desired value ($x_1^* = 28.125$A). The plot at the bottom of Figure 4 shows the closed-loop behavior of the controlled circuit. It is seen that the controller manages to rapidly force the current and voltage trajectories to their desired values within a 2% accuracy. An even better accuracy can be obtained by filtering the capacitor voltage by a low-pass filter before applying it to the controller (not shown here). It is interesting to remark that despite the fact that we only measure x_2, both the inductor current and capacitor voltage do not show any overshoot during the start-up. This can be explained by the fact that at every time instant the circuit impedance is matched by the controller parameter $G_p^i(\mu)$. In this way there are no power reflections between the load and the source (think, for example, of a lossless transmission line that is characteristically terminated by its characteristic resistance). The tuning parameter together with the controlled characteristic impedance is shown in Figure 5.

[3]During the preparation of this paper we have tried to implement the controller of the previous section in a real-time environment. Unfortunately, due to hardware problems at that time we where not able to present the results herein within the deadline of submission. Future experimental results will be reported elsewhere.

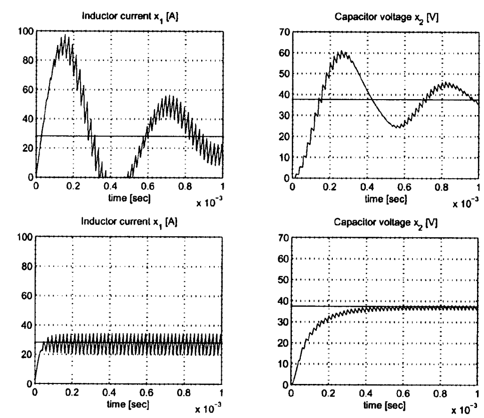

Figure 4. Start-up response: (top) Open-loop; (bottom) Closed-loop.

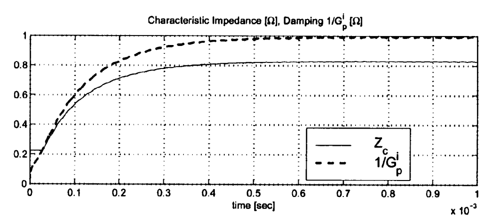

Figure 5. Start-up response: Controlled characteristic impedance $Z_c(\mu)$ and damping parameter $G_p^i(\mu)$.

Furthermore, Figure 6 shows the closed-loop response to an unmodeled step change in the load. We notice that the converter operation mode switches from CCM to DCM, while the capacitor voltage is rapidly restored to its desired value. In Figure 7 we show the response for a step change in the applied voltage. Again, the controller restores the desired capacitor voltage and readjust the inductor current as to maintain constant power. The last two figures, Figure 8 and 9, show the responses for different setpoints. We notice that for a known load the circuit is characteristically terminated and thus the trajectories do not show any overshoot or oscillations anywhere.

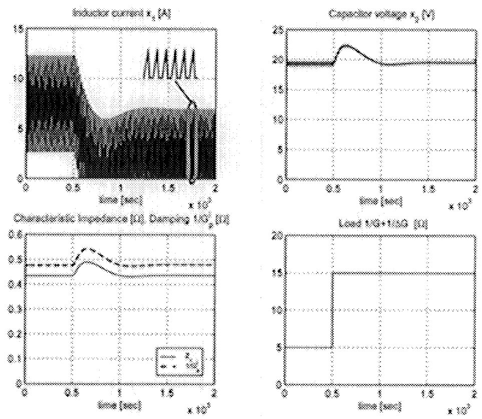

Figure 6. Closed-loop response for a load perturbation.

5. CONCLUSIONS

In this paper we have presented a simple controller for the DC-to-DC Boost converter which takes into account the nonlinear conversion ratio of the converter. Furthermore, we have once more advocated the use of parallel damping as to avoid current measurements and adaptive strategies. An additional advantage of this scheme is that a non-minimum phase circuit can be controlled based on measurements of its non-minimum phase output only. The characteristic impedance matching criterion can be argued to be a little conservative. For large converter structures it will in many cases also lead to cumbersome equations and difficult controllers. Less conservative and simpler tuning criteria are subject to future research and will be reported elsewhere.

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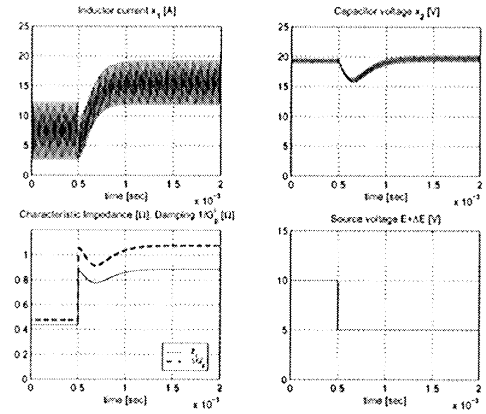

Figure 7. Closed-loop response for a source perturbation.

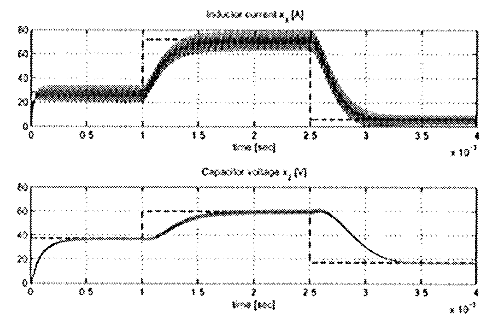

Figure 8. Closed-loop response for different set-points.

A. APPENDIX

Internal Stability: Here we proof that (5)–(6) is a suitable controller for the stabilization task with respect to the in-

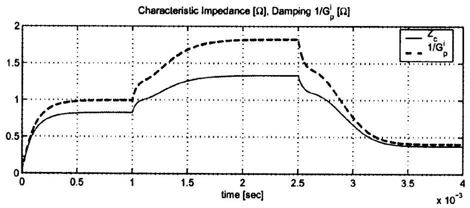

Figure 9. Controlled characteristic impedance for different set-points.

ternal stability, i.e., although we only measure the non-minimum phase output variable z_2, the zero-dynamics of the controller remain stable. For that, we proceed by eliminating ξ_2 from the equations (5) and (6). Using (5), $\xi_2 = \frac{E}{1-\mu}$. Substitution of the latter into (6) yields after some algebraic manipulations,

$$\dot{\mu} = G\frac{(z_2^*)^2}{CE^2}(1-\mu)^3 + G_p^i(\mu)\frac{z_2}{CE}(1-\mu)^2 - \frac{G+G_p^i(\mu)}{C}(1-\mu). \quad (11)$$

Notice that the latter is also a possible realization of the control law. The zero-dynamics associated with (11) are obtained by letting z_2 coincide with its respective desired value, that is $z_2 = z_2^*$. It is easily checked that the physically relevant equilibrium points of (11) are given by $\mu = 1$ and $\mu = \mu^* = 1 - E/z_2^*$ for all $z_2^* \geq E$. The phase-plane diagram of (11) shown if Figure 10 shows that the $\mu = \mu^*$ is a locally stable equilibrium point, while $\mu = 1$ is unstable. This corresponds to the fact that if $\mu = 1$ for too long, the current through the inductor increases until the converter blows up. We conclude that the controller, although based on measuring the non-minimum phase output voltage, is feasible for all μ in the range $0 \leq \mu < 1$.

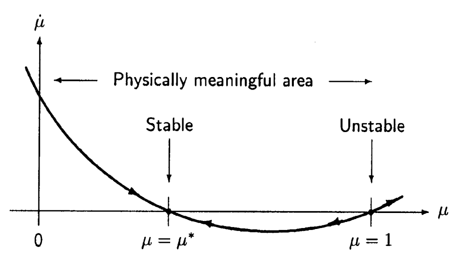

Figure 10. Zero-dynamics for the parallel damping PBC.

Overall Stability: We will now complete the proof that the equilibrium point $(z_1, z_2, \mu) = (z_1^*, z_2^*, \mu^*)$ of the boost converter in closed-loop with the controller (5)–(6) is locally asymptotically stable. We first take G_p^i constant satisfying $G + G_p^i > 0$. To this end, we analyze the closed-loop error dynamics (7)–(8) and check if $\tilde{z} \to 0$ as $t \to \infty$. The corresponding co-energy with respect to the error dynamics is $H(\tilde{z}) = \frac{1}{2}L\tilde{z}_1^2 + \frac{1}{2}C\tilde{z}_2^2$. We then proceed by taking the time derivative of the total co-energy $H(\tilde{z})$, i.e.,

$$\dot{H}(\tilde{z}) = -(G + G_p^i)\tilde{z}_2^2, \quad (12)$$

which implies that $\tilde{z}_2 \to 0$, i.e., $z_2 \to \xi_2$. Asymptotic stability of the overall system is guaranteed if we can show that $\tilde{z}_1$ also approaches its equilibrium point $\tilde{z}_1 = 0$ and $z_2, \xi_2 \to z_2^*$. From the error dynamics we know that $(1-\mu)\tilde{z}_1 = (G + G_p^i(\mu))\tilde{z}_2 \to 0$ as $t \to \infty$, and thus it remains to be shown that $\mu \to \mu^*$, $0 \leq \mu^* < 1$, as $t \to \infty$. To this end, we introduce the following auxiliary variable

$$\lambda = \frac{\xi_2^2}{2} - \frac{(z_2^*)^2}{2} = \frac{1}{2}\left(\frac{E}{1-\mu}\right)^2 - \frac{(z_2^*)^2}{2}, \quad (13)$$

which is well defined for μ in the vicinity of the stable equilibrium point $\mu = \mu^*$ for all $0 \leq \mu^* < 1$. It is straightforward to show that λ satisfies the following linear differential equation

$$\dot{\lambda} = -2\frac{G}{C}\lambda + \frac{G_p^i}{C}\tilde{z}_2. \quad (14)$$

Because the Jacobian of the system (7)–(8) is bounded and Lipschitz, we may conclude that $\tilde{z}_2 \to 0$ exponentially fast, and thus that $\lambda \to 0$ and $\mu \to \mu^*$ as well. For the case that $G_p^i = G_p^i(\mu)$ it is sufficient to check that $G + G_p^i(\mu) > 0$. This is satisfied as long as $Z_c(\mu) > 0$, which is always the case for every $L, C > 0$. The differential equation for λ now satisfies

$$\dot{\lambda} = -2\frac{G}{C}\lambda + \frac{2}{\xi_2}\sqrt{\frac{E^2}{CL}}\tilde{z}_2,\ \xi_2(0) > 0, \quad (15)$$

which again implies that if $\tilde{z}_2 \to 0$ exponentially fast, $\xi_2 \to z_2^*$ and $\lambda \to 0$, and consequently, $\mu \to \mu^*$. This concludes the proof.

Robustness: The dynamics of the perturbed boost converter in closed-loop with the latter controller are represented by

$$\begin{aligned} L\dot{z}_1 &= E + \Delta E - \frac{E+\Delta E}{\xi_2}z_2 \\ C\dot{z}_2 &= \frac{E+\Delta E}{\xi_2}z_1 - \frac{G\Delta G}{G+\Delta G}z_2 \\ C\dot{\xi}_2 &= G\frac{(z_2^*)^2}{(E+\Delta E)\xi_2} - (G+G_p^i)\xi_2 + G_p^i z_2. \end{aligned}$$

Recall that ΔG represents the unmodeled load uncertainty and ΔE represents the source disturbances. At the equilibrium point the left-hand sides of the closed-loop equations is equal to zero. It is then easily checked that $\xi_{2*} = z_2^*$ independently of G, ΔG and G_p^i. Similar arguments hold for source voltage disturbances under the condition that $E + \Delta E$ is assumed to be known, i.e., both E plus ΔE are measured and fed into the controller.