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Published in:
Proceedings of the 8th Mechatronics Forum International Conference

Publication date:
2002

Please refer to this paper as:
D. Jeltsema, R. Ortega, J. Clemente-Gallardo, J.M.A. Scherpen, J.B. Klaassens, "Brayton-Moser equations and new passivity properties for nonlinear electro-mechanical systems", Proc. 8th Mechatronics Forum International Conference, University of Twente, The Netherlands, June 2002, pp 979-988.

BRAYTON-MOSER EQUATIONS AND NEW PASSIVITY PROPERTIES FOR NONLINEAR ELECTRO-MECHANICAL SYSTEMS[‡]

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Abstract *– This paper presents an alternative framework for a practically relevant class of nonlinear electro-mechanical systems. The formalism is based on a generalization of Brayton and Moser's mixed-potential function. Instead of focusing on the usual energy-balance, the models are constructed using the power flowing through the system. The main objective is to put forth the mixed-potential function as a new building block for modeling, analysis and controller design purposes for electro-mechanical systems.*

1 Introduction

It is well-known that the Port-Hamiltonian (van der Schaft, 2000) equations form a very suitable and natural framework to describe the dynamics of a broad class of nonlinear electrical, mechanical and electro-mechanical systems. In this paper we present a dual formulation of the dynamics of nonlinear electro-mechanical systems in terms of the co-energy (power) variables. The method uses the classical Brayton-Moser (Brayton and Moser, 1964) equations based on the notion of kinetic, magnetic and electric co-energy and the definition of a mixed-potential function. Originally, this framework stems from the early sixties and seems to be very little known in the systems and control community. In the new setting the mixed-potential function exists of power preserving potentials, mechanical content, electro-magnetic content and electrical co-content. The main advantage of a well-defined dual formulation is that essential and important properties can be translated from one framework to another. One of these useful properties is that the mixed-potential function can be used as a starting point to derive a new family of storage functions. Instead of using the total stored energy as a storage function, as with Port-Hamiltonian systems, we use the mixed-potential function. The results of this paper form a starting point to overcome the dissipation obstacle in electro-mechanical systems that cannot be stabilized by the energy-balancing technique as recently proposed in (Ortega, van der Schaft, Mareels and Maschke, 2001). At a more general level, our objective in this paper is to put forth the mixed-potential function as a new building block for analysis and controller design for electro-mechanical systems and bring under the attention again the Brayton-Moser equations as a fair alternative to the Lagrangian and Port-Hamiltonian frameworks.

[‡]Sponsored by the Control Training Site (CTS), Marie Curie Fellowships.

For reasons of space and clarity, detailed proofs are omitted and the developments are restricted to a practically relevant class of linear and nonlinear mechanical, electrical and electro-mechanical systems.

Notation: By $T_x(x,\cdot)$ we denote the derivative with respect to x, i.e,

$$T_x(x,\cdot) = \frac{\partial T}{\partial x}(x,\cdot).$$

Consequently, $T_{xx}(x,\cdot) = \frac{\partial^2 T}{\partial x^2}(x,\cdot)$, etc.. By $\hat{y}(x)$ we define the vector of the constitutive relations for the k (x–controlled) elements of a certain type (resistance, capacitance, etc.), $\hat{y}(x) = (\hat{y}_1(x), \ldots, \hat{y}_k(x))^\top$.

2 Brayton-Moser's Equations

In the early sixties, Brayton and Moser (Brayton and Moser, 1964) have shown that the dynamical behavior of a very general class of complete[1] nonlinear electrical circuits, with ρ capacitors and σ inductors is governed by the following set of differential equations

$$\begin{aligned} \tfrac{d}{dt} H_u^*(u,i) &= P_u(u,i) \\ -\tfrac{d}{dt} H_i^*(u,i) &= P_i(u,i), \end{aligned} \tag{1}$$

where $u = (u_1, \ldots, u_\rho)^\top \in \mathbb{R}^\rho$ and $i = (i_1, \ldots, i_\sigma)^\top \in \mathbb{R}^\sigma$ are the voltages across the capacitors and the currents through the inductors, respectively. The scalar function $H^* : \mathbb{R}^{\rho+\sigma} \to \mathbb{R}$ denotes the sum of the electric and magnetic co-energy stored in the circuit. If we assume that the inductors do not depend on the capacitor voltages and the capacitor do not depend on the inductor currents, then the total stored co-energy is given by $H^*(u,i) = \int_0^u \hat{q}(u')du' + \int_0^i \hat{\varphi}(i')di'$, where $q = \hat{q}(u)$ and $\varphi = \hat{\varphi}(i)$ are the capacitor charges and the inductor fluxes, respectively. By $P : \mathbb{R}^{\rho+\sigma} \to \mathbb{R}$ we denote the mixed-potential function, which consists of the difference of the content and the co-content plus a term which coincides with the definition of a Dirac structure, as used for the Port-Hamiltonian formalism. The content is defined as the difference between the current potentials of the current-controlled voltage sources and resistances in the circuit, which has been introduced by W. Millar and C. Cherry in the early-fifties (see (Brayton and Moser, 1964) and the references therein). A similar definition holds for the co-content, which is the difference between the voltage potentials of the voltage-controlled current sources and resistances (conductances). Let $\hat{r}(i)$, $\hat{g}(u)$, $\hat{e}(i)$ and $\hat{b}(u)$ denote the constitutive relations of the resistances (r), conductances (g) and, (possibly controlled) voltage and current sources (e and b), respectively, then the mixed-potential function is defined as

$$P(u,i) = \underbrace{\int_0^u \{\hat{b}(u') - \hat{g}(u')\}du'}_{\text{electrical co-content}} - \underbrace{\int_0^i \{\hat{e}(i') - \hat{r}(i')\}di'}_{\text{electro-magnetic content}} + \underbrace{u^\top \psi^\top i}_{\text{interconnection}}. \tag{2}$$

Here ψ is reflecting the power-preserving interconnection structure of the circuit, which is similar to the interconnection matrix used in Port-Hamiltonian systems, see e.g. (Jeltsema and Scherpen, 2002; van der Schaft, 2000). For that reason, the last term of the

[1] A circuit is complete if the set of inductor currents and capacitor voltages can be chosen independently such that the Kirchhoff's laws are satisfied and such that each branch in the circuit is determined by at least one element from the set of currents or voltages.

right-hand side of (2) is denoted as the conserved power. At this point, we remark that (1) together with the total stored co-energy as defined above establish a 'canonical' set of Brayton-Moser (BM) equations. For any general form of $H^*(u,i)$ other than the one defined here, we may refer to (1) as the generalized BM equations. In the canonical case $H^*_{ui} = H^*_{iu} = 0$ and therefore (1) can be rewritten as $-Q(w)\dot{w} = P_w(w)$, with $w = (u_1, \ldots, u_\rho, i_1, \ldots, i_\sigma)^\top$ and $Q(w) = \mathrm{diag}\{-H^*_{uu}, H^*_{ii}\}(w)$. We may interpret $Q(w)$ as a pseudo Riemannian metric on the state space $\mathbb{R}^\rho \times \mathbb{R}^\sigma$ (van der Schaft, 1984). In general $H^*_{uu}(w) > 0$ and $H^*_{ii}(w) > 0$, so the metric is indefinite. We come back to this later on. In the sequel we shall denote the electro-magnetic content by $J(i)$ and the electrical co-content by $G(u)$. In the next section we will try to translate the BM equations to the framework of mechanical systems.

3 Brayton-Moser Description of Mechanical Systems

In this section we want to rewrite Lagrange's equations of motion for mechanical systems in a similar form as the (BM) equations of the previous section. In the construction we do not want to elaborate on the existence of a mixed-potential function of mechanical type, but we will focus on a topological construction of such function in order to obtain a BM type description for mechanical systems.

3.1 The Lagrangian description

It is well-known that a rather general class of nonlinear mechanical systems defined on a differentiable manifold $\mathcal{M}$, with local coordinates $x = (x_1, \ldots, x_m)^\top$ and m degrees of freedom, admit a Lagrangian description on the tangent bundle $\mathcal{TM}$. In local coordinates the Euler-Lagrange equations are given by

$$\tfrac{d}{dt} L_v(x,v) - L_x(x,v) = 0, \tag{3}$$

where the corresponding generalized velocities are denoted as $v = (v_1, \ldots, v_m)^\top$ and the scalar function $L(x,v)$ denotes the Lagrangian which is defined as the difference between kinetic co-energy $T^*(x,v)$ and potential energy $V(x)$, i.e., $L(x,v) = T^*(x,v) - V(x)$. In this paper we restrict our developments to mechanical systems where the Lagrangian is of the form $L(x,v) = \frac{1}{2}\sum_{j,k} M_{jk}(x) v_j v_k - V(x)$, where $M_{jk}(x)$ refers to the (j,k)-th element of $M(x)$, with $M(x)$ a positive definite symmetric $m \times m$ matrix called the interia or generalized mass matrix and hence defines locally a Riemannian metric on $\mathcal{M}$. In view of the BM setting, to be treated in the next subsection, we may rewrite the first term of the left-hand side of (3) as follows $\dot{T}^*_v(x,v) = T^*_{vv}(x,v)\dot{v} + T^*_{vx}(x,v)v$. For Lagrangians of the form considered herein, with $T^*_{vv}(x,v) = M(x)$, the expressions above can be summarized by defining $C(x,v)v = T^*_{vx}(x,v) - T^*_x(x,v)$ and rewriting (3) as

$$M(x)\dot{v} + C(x,v)v + V_x(x) = 0. \tag{4}$$

The kj-th element of $C(x,v)$ is univocally defined from the elements of $M(x)$ introducing Christoffel symbols of the first kind (van der Schaft, 2000) such that $\dot{M}(x) = C(x,v) + C^\top(x,v)$.

Next, we like to include the effect of a set of external and dissipative forces on the system. Let the externally supplied forces be given by $\tau = (\tau_1, \ldots, \tau_m)^\top$, where $\dim\{\tau\} \leq m$ (i.e., we can consider underactuated as well as fully actuated systems).

For the dissipation we consider the usual description in terms of the Rayleigh dissipation defined in local coordinates as

$$R(v) = \int_0^v \hat{\delta}(v')dv',$$

where $\hat{\delta}(v)$ represents the vector of functions describing the characteristics of the mechanical dissipation depending on the velocities. In order to be consistent with the notation of Section 2 we may refer to the difference of the supplied and dissipated velocity potentials as the total 'mechanical content' denoted by $D(x,v)$, i.e., $D(x,v) = v^\top \tau - v^\top V_x(x) - R(v)$. Notice that we consider the conserved forces $V_x(x)$ as external forces. Hence, the complete expression for mechanical system with dissipation and external controls becomes

$$M(x)\dot{v} + C(x,v)v = D_v(v). \tag{5}$$

We are now ready to define a BM description by introducing a mixed-potential function of mechanical type. In the remaining of the document, we will assume that the mechanical system is defined on $\mathbb{R}^m$ and hence the approach can be considered to be global.

3.2 Mechanical Content and Mixed-Potential

Next, our purpose is to write the equations obtained in the previous subsection in a form which formulates the equations of motion of mechanical systems into the BM framework. We have to search for the suitable function $P \in C^\infty(\mathbb{R}^m) \subseteq C^\infty(\mathcal{TM})$ which allows us to write (5) in a BM type fashion, i.e,

$$M(x)\dot{v} = P_v(x,v), \tag{6}$$

which corresponds to the canonical BM equations as they were originally defined. The latter suggests that we should just proceed by integrating the right-hand side of (5) with respect to the velocities v in order to obtain the power conserved, supplied and dissipated in the system. However, the problem that arises here is that in general the metric M does depend on x instead of v, where in the electrical domain often the metric depends on i and u only. Consider the power-balance with respect to $T^*(x,v)$:

$$\begin{aligned}\dot{T}^*(x,v) &= v^\top M(x)\dot{v} + \tfrac{1}{2}v^\top \dot{M}(x)v \\ &= v^\top D_v(v) + \tfrac{1}{2}v^\top \big[\dot{M}(x) - 2C(x,v)\big]v.\end{aligned}$$

Hence, by using the fact that due to the form of the kinetic co-energy the matrix $\dot{M}(x) - 2C(x,v)$ is skew-symmetric, the power-balance becomes $\dot{T}^*(x,v) = v^\top D_v(v)$, which implies that the forces $C(x,v)v$ are workless and can therefore not be contained in $P(x,v)$. Instead of (6) we may look for a $P \in C^\infty(\mathbb{R}^m)$ satisfying

$$\tfrac{d}{dt}T_v^*(x,v) = P_v(x,v), \tag{7}$$

which corresponds to the generalized form (1). Hence, in a similar fashion as in Section 2, the mixed-potential function of mechanical type is constructed as

$$P(x,v) = \underbrace{D(v)}_{\text{content}} + \underbrace{\int_0^v T_x^*(x,v')dv'}_{\text{'geometrical'}}. \tag{8}$$

Notice that if the metric is constant, i.e, does not depend on x, then the system is described by

$$M\dot{v} = P_v(x, v). \tag{9}$$

The mixed-potential $P(x, v)$ is now reduced to $P(x, v) = D(x, v)$, which is just the mechanical content of the system.

Remark 1. For general manifolds, the above derivations are just local constructions. However, it is a very well-known property of classical mechanics that the equations of the motion are given by the following coordinate-free expression (see (Abraham and Marsden, 1978)):

$$\nabla_v v + \mathrm{grad}V = 0 \tag{10}$$

Here the first term captures the force of 'geometrical origin'. This force will in general be different from zero as soon as the metric tensor is not constant. Assuming that the curvature corresponding to the connection is non zero, we can claim that this terms reflects the non-trivial structure of the manifold: all the points are not the same for the system, even from the geometrical point of view. We want to remark here that the Brayton-Moser description can also be established in a coordinate-free setting. The interested reader is referred to (Clemente-Gallardo et al.) for a detailed geometrical treatment.

4 Towards an Unified Co-Energy Description

In this section the results of previous sections will be generalized to a practically relevant class of nonlinear electro-mechanical systems. We restrict our developments to mechanical and electrical system interconnected by either an electric field or a magnetic field coupling. A sufficient condition to obtain a canonical set BM equations for the class of electro-mechanical systems considered herein is that the mechanical mass-matrix is constant and that the the coupling between the electrical and the mechanical part is represented in the co-energy. This assumption will be relaxed in (Clemente-Gallardo et al.). The theoretical developments are facilitated by an illustrative example using a levitated ball system will be treated at the end of the section.

4.1 Electric Field Coupling

In order to obtain a combined dynamical description of an electro-mechanical system, with possibly several electrical or magnetical interconnections, we only need to combine the previous developments in an appropriate way. To this end, consider a system consisting of a mechanical subsystem Σ_m and an electrical subsystem consisting of the interconnection of Σ_ρ and Σ_σ. Suppose that there exists an interconnection between Σ_m and Σ_ρ due to an electric field coupling parameterized by the generalized coordinates $x = (x_1, \ldots, x_m)^\top \in \mathbb{R}^n$, with $n = m+\rho+\sigma$. Then, the interconnected system Σ is completely determined by the following set of canonical BM equations

$$\Sigma \ : \ \Phi H^*_{ww}(x, w)\dot{w} = P_w(x, w), \tag{11}$$

where the vector $w \in \mathbb{R}^n$ represents the generalized mechanical velocities, the voltages across the capacitances and the currents through the inductances, respectively, i.e., $w = (v_1, \ldots, v_m, u_1, \ldots, u_\rho, i_1, \ldots, i_\sigma)^\top$, and $\Phi = \mathrm{diag}\{I_{m\times m}, -I_{\rho\times\rho}, I_{\sigma\times\sigma}\}$. The

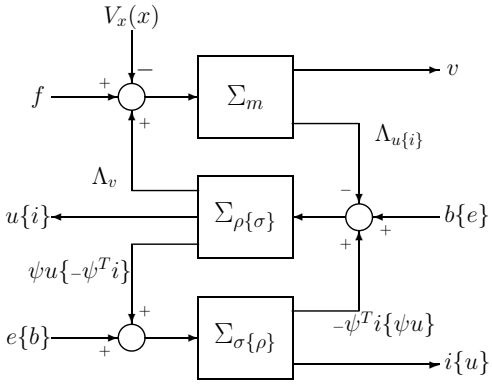

Figure 1: Electrically or magnetically coupled electro-mechanical system: the coupling terms are defined as $\Lambda = \int_0^v H_x^*(x, w')dv'$.

scalar function $H^* : \mathbb{R}^n \to \mathbb{R}$ denotes the sum of the kinetic, electric and magnetic co-energy. The mixed-potential function $P : \mathbb{R}^n \to \mathbb{R}$ is defined as

$$P(x,w) \;=\; \underbrace{D(x,v)}_{\text{mech. part}} + \underbrace{J(i) - G(u) + i^\top \psi u}_{\text{electrical part}} + \underbrace{\int_0^v H_x^*(x,w')dv'}_{\text{coupling part}}. \qquad (12)$$

Recall that $D(x, v)$, $J(i)$ and $G(u)$ represent the mechanical content, the electrical content and and the electrical co-content, respectively. Furthermore, the coupling forces stemming from electrical and electro-mechanical origin are captured by the term $\int_0^v H_x^*(x, w')dv'$. Notice that under the assumption that there only exists a coupling due to an electric field, $H_{ix}^*(x, w)$ should be equal to zero for all i, x.

4.2 Magnetic Field Coupling

Suppose now that there exists an interconnection between Σ_m and Σ_σ due to a magnetic field coupling parameterized by the generalized configuration variables x. Then, in a similar fashion as in the previous case, the interconnected system Σ is completely determined by the following differential equations

$$\Sigma \;:\; \Phi H_{ww}^*(x,w)\dot{w} = P_w(x,w). \qquad (13)$$

Here w is a before and $\Phi = \mathrm{diag}\{I_{m\times m}, I_{\rho\times\rho}, -I_{\sigma\times\sigma}\}$. The mixed-potential function $P : \mathbb{R}^n \to \mathbb{R}$ is now given by

$$P(x,w) \;=\; D(v) - J(i) + G(u) - i^\top \psi u + \int_0^v H_x^*(x,w')dv'. \qquad (14)$$

Compare the latter with (12) (notice the signs). Again, we notice that under the assumption that there only exists a coupling due to a magnetic field, $H_{ux}^*(x, w)$ should be equal to zero for all u, x. It is interesting to observe that if we write (11) or (13) in the form $-Q(x, w)\dot{w} = P_w(x, w)$, with $Q(x, w) = -\Phi H_{ww}^*(x, w)$, we may, as for electrical circuits, interpret $Q(x, w)$ as a pseudo Riemannian metric parameterized by x.

A schematic interpretation of an electrically (resp. magnetically) coupled electro-mechanical system is depicted in Figure 1. The notation $\{\cdot\}$ refers to the magnetic field coupling case. Notice that throughout the developments we have treated the conservative forces as external signals acting on the system. These forces are parameterized by x, where x is in general the solution of $\dot{x} = v$. This means that in every point on $\mathbb{R}^n$ we have a BM type description of the dynamics. We conclude the section with a simple example.

Example 1. Consider a levitated ball system (Ortega et al., 2001). Let the total co-energy and the potential energy stemming from the gravitational force be defined by

$$H^*(x, v, i, u) = \frac{m}{2}v^2 + \frac{l(x)}{2}i^2 + \frac{c}{2}u^2$$

and $V(x) = m\tilde{g}x$, respectively, with m the mass of the ball, $\tilde{g}$ the gravitational constant, c the parasitic capacitance of the coil and $l(x) = l_o + \frac{\alpha}{\beta - x}$ the inductance of the coil. Furthermore, let r be the coil resistance, b the source current, and g the internal source conductance. Using the developments of Section 4.2, we find with $0 \leq x < \beta$

$$P(x, v, i, u) = -m\tilde{g}v + \frac{r}{2}i^2 - bu + \frac{g}{2}u^2 + iu + \int_0^v \frac{\alpha}{2(\beta - x)^2}i^2 dv'.$$

Then, after substitution of the latter into (13) yields

$$\begin{aligned} m\frac{dv}{dt} &= \frac{\alpha}{2(\beta - x)^2}i^2 - m\tilde{g} \\ c\frac{du}{dt} &= b - gu - i \\ l(x)\frac{di}{dt} &= u - \frac{\alpha}{(\beta - x)^2}vi - ri. \end{aligned}$$

It should be remarked that we have gained more information about the structure here. In particular, the coupling force $\frac{\alpha}{(\beta - x)^2}vi$ is explicitly appearing in the co-energy model, while if we would use the Hamiltonian framework (van der Schaft, 2000) this property is hidden in the Hamiltonian. Notice that

$$\int_0^v \frac{\alpha}{2(\beta - x)^2}i^2 dv'$$

plays a role similar to the term iu stemming from $u^\top \psi i$, with $\psi = 1$, i.e, both define the interconnections between the different subsystems. □

5 Stability and Passivity in the Co-Energy Framework

The motivation behind the previous developments can be summarized as follows. First, In the context of electrical circuits it is shown in (Jeltsema and Scherpen, 2002) that the BM equations (1) bear a marked similarity in structure to the Port-Hamiltonian equations, see e.g., (van der Schaft, 2000). The most trivial duality between the two frameworks is that Port-Hamiltonian systems assume the circuit elements to be flux and charge controlled, while the BM equations impose the restriction that the elements are current and voltage controlled. One reason to work with Port-Hamiltonian systems is that the equations are formulated in natural physical variables. In case of conservative

circuits this can be considered as a reasonable argument. However, the inclusion of static elements, like sources and resistors seems not so natural in this framework. In principal, the constitutive relations of voltage sources, current sources and resistive elements are rather considered in terms of currents or voltages (Ohm's law), instead of fluxes or charges. It seems then more natural to use the BM equations. In the context of feedback controller design for electro-mechanical systems, an additional advantage of using the BM equations for the electrical part of the system is that the dynamics are directly expressed in measurable quantities. Similar arguments hold for the mechanical part of the system, where it is more common to measure velocity instead of momenta. The second motivation concerns some interesting properties of the mixed-potential and is best illustrated by a simple example.

Example 2. Consider a mechanical system with m degrees of freedom and generalized configuration variables $x \in \mathbb{R}^m$, with as total stored kinetic co-energy $T^*(v) = \frac{1}{2}v^\top M v$, $v = \dot{x}$, $M = M^\top > 0$, Rayleigh dissipation $R(v) > 0$ and external forces τ. In the new setting the equations of motion are described by

$$-Q\dot{v} = P_v(v) \tag{15}$$

where $Q = T^*_{vv}(v) = M$ and $P(v) = R(v) - v^\top \tau$ denotes the mixed-potential for the system. Consider then the time-derivative of $P(v)$, i.e.,

$$\dot{P}(v) = -\dot{v}^\top P_v(v) = -\dot{v}^\top Q \dot{v}. \tag{16}$$

Clearly, $P(v)$ decreases along the solutions of (15) except at the equilibria, where $P_v(v) = 0$. This example suggests that, following ideas stemming from Lyapunov's stability theory, one could derive, under the assumption that $P(v) \geq 0$, that every solution of (15) tends to the equilibrium $v = 0$ as $t \to \infty$. Trivially, this is ensured when $\tau = 0$. Hence, P can be used as an alternative candidate Lyapunov function. □

Unfortunately, as is shown before, for general electrical and electro-mechanical systems Q is in general indefinite and therefore P in its present from is not suitable to determine stability or passivity. One way to overcome this problem is by looking for another pair $\{Q^\star, P^\star\}$ in place of $\{Q, P\}$ such that the form of (11) or (13) is preserved, i.e,

$$-Q^\star(w)\dot{w} = P^\star_w(w), \tag{17}$$

and such that $Q^\star(w) > 0$ and $P^\star(w) \geq 0$ for all w. As proposed in (Brayton and Moser, 1964), general pairs $\{Q^\star, P^\star\}$ can be obtained under the condition that

$$Q^\star Q^{-1} P_w = P^\star_w. \tag{18}$$

The procedure is basically as follows. If $\{Q^1, P^1\}$ and $\{Q^2, P^2\}$ are two pairs describing (11) or (13), then so are $\{\alpha Q^1 + \beta Q^2, \alpha P^1 + \beta P^2\}$, with $\alpha, \beta \in \mathbb{R}$. This obviously gives us considerable freedom in constructing other pairs. In order to find one nontrivial pair other than $\{Q, P\}$ we know from (Brayton and Moser, 1964) that if K is any constant symmetric matrix, then the pair $Q^\star = P_{ww} K Q$ and $P^\star = \frac{1}{2} P_w^\top K P_w$ is one possible choice[2]. Hence, more general pairs are obtained by superposition, i.e.,

$$\begin{aligned} J^\star &= \lambda Q + P_{ww} K Q \\ P^\star &= \lambda P + \tfrac{1}{2} P_w^\top K P_w, \end{aligned} \tag{19}$$

[2]This is easily seen by noting that $P^\star_w = P_{ww} K P_w$ and thus $Q^\star Q^{-1} P_w = P_{ww} K Q Q^{-1} P_w = P^\star_w$.

with $\lambda \in \mathbb{R}$. Having made these observations, we shall now provide an alternative framework to obtain some new passivity properties where $P^\star$ serves as a candidate storage function. The key motivation for our developments are the new passivity properties as proposed in (Ortega and Shi, 2002). In this paper the authors propose to use the dissipative content or co-content to establish passivity of nonlinear RL or RC circuits, respectively. These properties are useful for stabilization purposes, like energy-balancing. Here we want to extend these propositions to a more general class of systems. To do this we proceed as follows. In order to accommodate the new mixed-potential function $P^\star$ in the desired form, we need to extract the power supply sources. Assume that the sources are independent, i.e., not controlled, and let $f = (\tau_1 - V_{x_1}(x), \ldots, \tau_m - V_{x_m}(x), b_1 \ldots, b_\rho, e_1, \ldots, e_\sigma)^\top$, then (11) or (13) can be written as

$$-Q(x,w)\dot{w} = \tilde{P}_w(w) + \Phi f, \tag{20}$$

where $\tilde{P}(w)$ denote the mixed-potentials given in (12) or (14) with the supplied powers $w^\top \Phi f$ extracted from the equations. A new set of equations describing (20) is then obtained by performing the transformations (19), i.e.,

$$-J^\star \dot{w} = \lambda \tilde{P}_w + P_{ww} K \tilde{P}_w + \lambda \Phi f + P_{ww} K \Phi f. \tag{21}$$

Hence, by observing that $P_{ww}(w) = \tilde{P}_{ww}(w)$, $\tilde{P}^\star(w)$ is obtained by integrating the first equation of the right-hand side of (21) with respect to w yielding $\tilde{P}^\star(w) = \lambda \tilde{P}(w) + \frac{1}{2}\tilde{P}_w(w) K \tilde{P}_w(w)$ and $f^\star(x,w) = \lambda \Phi f + \tilde{P}_{ww}(w) K \Phi f$. We are now ready to postulate the following

Proposition 1. *Assume that there exists a constant symmetric matrix K and a $\lambda \in \mathbb{R}$ such that $Q^\star(x,w) > 0$ and $\tilde{P}^\star(w) \geq 0$, then (11) or (13) satisfy the inequality*

$$\int_0^t -\dot{w}(t') f^\star(t') dt' \geq \tilde{P}^\star[w(t)] - \tilde{P}^\star[w(0)]. \tag{22}$$

Consequenlty, the systems define a passive port with port variables $(\dot{w}, f^\star)$ and storage function $\tilde{P}^\star(w)$.

Proof. We start by differentiating $\tilde{P}^\star(w)$ along the solutions of (21), i.e.,

$$\tfrac{d}{dt}\tilde{P}^\star(w) = -\dot{w}^\top Q^\star(x,w)\dot{w} - \dot{w}^\top f^\star. \tag{23}$$

Since $Q^\star(x,w) > 0$ by assumption, then $\dot{w}^\top Q^\star(x,w)\dot{w}$ is positive definite and equal to zero if and only if $\dot{w} = 0$. Following Lyapunov ideas, we may conclude that $\tilde{P}^\star(w)$ is a decreasing function except at the equilibria $\dot{w} = 0$. The proof is completed by integrating (23) from 0 to t. ■

As pointed out before, a possible application for the new passivity properties is the problem of stabilization of the equilibria of electro-mechanical systems which can not be stabilized by the energy-balancing due to the dissipation obstacle identified in (Ortega et al., 2001). The rationale of the approach is to express the left-hand side of (22) as a function of w, say $-F^\star(w)$, and to search for a solution of the PDE given by

$$\hat{f}^\star(w) = -F^\star_w(w), \tag{24}$$

yielding a desired closed-loop storage function $P^\star_{\rm cl} = \tilde{P}^\star(w) + F^\star(w)$. Notice that we are actually trying to shape the power instead of the energy. For that reason it seems natural to refer to the latter as 'power-balancing'. For reasons of space we do not elaborate further on this topic herein. Applications of the theory will be reported elsewhere.

6 Final Remarks

In the first part of the paper we have presented a novel power-based framework for the modeling of a class of electro-mechanical systems. For the class of systems which can be modeled in this this framework it is sufficient that the coupling terms appear in the co-energy function. However, it is unclear how to derive, if possible, a more general statement about the class of admissible systems in the BM framework. The second part is concerned with the definition of some new passivity properties based on the mixed-potential function of the systems. These properties might become useful to, e.g., overcome the dissipation obstacle in the energy-balancing methodology. It should be pointed out that the developments regarding the modeling part herein differ from the setting proposed in (Kugi, 2001) for two reasons: (i) in (Kugi, 2001) the mechanical part is described by the Euler-Lagrange equations, while the electrical part is covered by some kind of generalized Brayton-Moser description, (ii) here the powers stemming from both the mechanical and electrical domain are included in one mixed-potential function explicitly revealing the couplings between the different subsystems, while in the afore mentioned reference each electrical system has its own potential functions containing only powers stemming from the electrical parts of the system. The main advantage of defining one potential function for the complete system is that we can exploit the stability properties proposed in (Brayton and Moser, 1964) and extend the ideas of (Ortega and Shi, 2002) to the electro-mechanical case.

References

Abraham, R. and Marsden, J. E. (1978). *Foundations of Mechanics*, Benjamin / Cummings, 2nd edition.

Brayton, R. K. and Moser, J. K. (1964). A theory of nonlinear networks I and II, *Quart. Appl. Math.* **12**(1 and 2): 1–33 and 81–104.

Clemente-Gallardo, J., Jeltsema, D., Scherpen, J. M. A. and Ortega, R. (2002). Coordinate-free description of Brayton and Moser's equations, *In progress*.

Jeltsema, D. and Scherpen, J. M. A. (2002). On nonlinear RLC networks: Port-controlled Hamiltonian systems dualize the Brayton-Moser equations, *To appear in proc. 15th IFAC World Congress, Barcelona, Spain, July 2002.*

Kugi, A. (2001). *Nonlinear Control Based on Physical Models*, Springer-Verlag.

Ortega, R. and Shi, B. E. (2002). A note on passivity of nonlinear RL and RC circuits, *To appear in proc. 15th IFAC World Congress, Barcelona, Spain, July 2002.*

Ortega, R., van der Schaft, A. J., Mareels, I. and Maschke, B. M. (2001). Putting energy back in control, *IEEE Control Syst. Magazine* **21**(2): 18–33.

van der Schaft, A. J. (1984). *System Theoretic Description of Physical Systems*, CWI (Centrum voor Wiskunde en Informatica) Tract 3.

van der Schaft, A. J. (2000). *$\mathcal{L}_2$-Gain and Passivity Techniques in Nonlinear Control*, Springer-Verlag London Limited.