Weighted Robust Adaptive Filtering in Krein Space and Its Application in Active Noise Control

Jayawardhana, Bayu; Yuan, Shuqing; Xie, Lihua

[Link to publication in University of Groningen/UMCG research database](#)

Weighted Robust Adaptive Filtering in Krein Space and Its Application in Active Noise Control

Bayu Jayawardhana[1], Shuqing Yuan, Lihua Xie
School of EEE, Nanyang Technological University, Singapore 629798
1 PK655853@ntu.edu.sg

Abstract: Robust adaptive filtering ensures the minimization of the transfer function from the disturbance to the estimation error and thus, guarantees the robustness against the worst-case disturbance in the system. However, a more general approach will be given in this paper by employing frequency weighting, which offers flexibility in determining robustness sensitivity in certain frequency of interest. Using the projection in Krein space, we developed a weighted recursive H_∞ filtering that computes the stationary point of certain weighted quadratic form corresponding to an H_∞ norm of a transfer operator $T(F)$. The solution is applied to the active noise cancellation problem, where it is used in ensuring the control signal to be inside the nominal frequency range of the actuators, and avoiding the non-linearity effect caused by saturation. Experimental result shows the advantage of this weighted form. The proposed algorithm offers an alternative means in dealing with wideband noise and actuator non-linearity.

Keywords: Adaptive Filter, Active Noise Control, Robust Filter

I. Introduction

H_∞ methods have been recently introduced to the filtering problem, which resulted in a simple algorithm and yet guaranteeing the filter against a certain worst-case disturbance, see Hassibi et. al. [3]. Also in [1], the resemblance between the H_∞ filter and the popular adaptive Least Mean Squares filtering algorithm is highlighted:. It shows that the LMS and Normalized LMS is in fact H_∞ optimal with a guaranteed attenuation level of $\gamma=1$. Some related work of H_∞ filtering has also been reported by Bolzern [8]. This paper is intended to extend the H_∞ filtering to the weighted form of adaptive robust filtering. The motivation is to give flexibility in minimizing errors over the frequency range of interest. Also, the paper gives a simplification of the algorithm that leads to the weighted version of LMS algorithm. The weighting form in the LMS algorithm has been given in [13], which lacks of rigorous work.

The weighted H_∞ filter will be applied to Active Noise Control (ANC) problem in this paper, where an adaptive filter is required in generating anti-sound signal to the "secondary" source(s), which interferes destructively, 180^0 out of phase, with the noise field caused by the original "primary" source(s). The introduction of ANC will not be discussed thoroughly in this paper, and it can be found elsewhere in [7,12] and references therein.

The most popular adaptive filter for ANC is the filtered-x LMS that provides a simple algorithm and it is also a robust algorithm as inherited from the LMS algorithm. Thus, consider when the actuator(s) has nominal frequency range beyond which it will exhibit non-linearity. In this case, the linear filter may not perform well and therefore nonlinear methods are needed [6,10].

Nevertheless, if we only focus the controller for attenuating the primary noise within the nominal frequency of the actuator, then a linear filter is still viable. Using the weighted H_∞ filter introduced in this paper, the ANC experimentation was carried out to attenuate the noise within the nominal value of the actuator. As a result, when there is large noise occurring outside the nominal range, the controller does not put much effort as that within the nominal range based on the frequency weighting in the algorithm. Furthermore, several actuators with different nominal frequency ranges can be introduced to the system for attenuating wideband noise with each actuator working optimally within their nominal range.

II. Krein-space Projection

The relationship between Krein-space projection and the computation of a stationary point of quadratic forms have been given by Hassibi and Kailath, and interest readers can refer to [2]. We will present the weighted form of the robust filtering given in [2], based on the Krein space using the span of $\{Wy_i\} = \text{proj}\{y_i\}$.

Theorem 1
Given Krein-space state equations
$$x_{n+1} = F_n x_n + G_n u_n$$
$$y_n = H_n x_n + v_n \qquad\qquad 0 \le n \le N$$
$$y_{n,p} = Wy_n$$
with $\left\langle \begin{bmatrix} u_n \\ Wv_n \\ x_0 \end{bmatrix}, \begin{bmatrix} u_m \\ Wv_m \\ x_0 \end{bmatrix} \right\rangle_\kappa = \begin{bmatrix} Q_n\delta_{nm} & 0 & 0 \\ 0 & R_n\delta_{nm} & 0 \\ 0 & 0 & \Pi_0 \end{bmatrix}$

The innovations can be computed by
$$e_n = Wy_n - WH_n\hat{x}_n \qquad\qquad 0 \le n \le N$$
$$\hat{x}_{n+1} = F_n\hat{x}_n + K_{p,n}(Wy_n - WH_n\hat{x}_n)$$
$$K_{p,n} = F_n P_n H_n^* W^* R_{e,n}^{-1}$$

where $R_{e,n} = <e_n, e_n>_\kappa = R_n + WH_n P_n H_n^* W^*$, and the P_n can be recursively computed via the following Riccati equations:

$$P_0 = \Pi_0$$

$$P_{n+1} = F_n P_n F_n^* - K_{p,n} R_{e,n} K_{p,n}^* + G_n Q_n G_n^*$$

Formulae above recursively compute the stationary point of the following second-order form

$$J_n = x_0^* \Pi_0^{-1} x_0 + \sum_{j=0}^n u_j^* Q_j^{-1} u_j$$

$$+ \sum_{j=0}^n (Wy_j - WH_j x_j)^* R_j^{-1} (Wy_j - WH_j x_j)$$

Based on this formulation, we can present the frequency weighted robust filtering in the following section.

III. Frequency Weighted H_∞ Filter

Consider the following frequency weighted cost function

$$\|T_f\|_\infty^2 = \sup \frac{\|W(z).e_f(z)\|_2^2}{x_0^*(z)\Pi_0^{-1}x_0(z) + \|u(z)\|_2^2 + \|W(z).v(z)\|_2^2}$$

and $\|T_p\|_\infty^2 = \sup \dfrac{\|W(z).e_p(z)\|_2^2}{x_0^*(z)\Pi_0^{-1}x_0(z) + \|u(z)\|_2^2 + \|W(z).v(z)\|_2^2}$

where $W(z)$ is a stable frequency weighting function and notation w in the T_f and T_p represents the weighted version of the transfer function.

Lemma 1. Given a scalar $\gamma_f > 0$, then $\|T_f\|_\infty \le \gamma_f$ iff there exists $\hat{z}_{k|k}$ (for all $k \le i$) such that, for all nonzero complex vectors x_0 and for all nonzero causal sequences $\{u_j, v_j\}_{j=0}^i$, the scalar second order form

$$J_{f,i} = x_0^* \Pi_0^{-1} x_0 + \sum_{k=0}^i u_k^* u_k$$

$$+ \sum_{k=0}^i (WY_k - WH_k^\theta X_k)^* (WY_k - WH_k^\theta X_k) -$$

$$- \gamma_f^{-2} \sum_{k=0}^i (W\hat{Z}_{k|k} - WL_k^\theta X_k)^* (W\hat{Z}_{k|k} - WL_k^\theta X_k)$$

satisfies $J_{f,i} > 0$. We can rewrite J_i as follows:

$$J_{f,i} = x_0^* \Pi_0^{-1} x_0 + \sum_{k=0}^i u_k^* u_k$$

$$+ \sum_{k=0}^i \left(W_j \begin{bmatrix} Y_k \\ \hat{Z}_{k|k} \end{bmatrix} - W_j \begin{bmatrix} H_k^\theta \\ L_k^\theta \end{bmatrix} X_k \right)^* R_j^{-1} \left(W_j \begin{bmatrix} Y_k \\ \hat{Z}_{k|k} \end{bmatrix} - W_j \begin{bmatrix} H_k^\theta \\ L_k^\theta \end{bmatrix} X_k \right)$$

$$(1)$$

where $R_j = (I \oplus -\gamma_f^2 I)$ and $W_j = W \oplus W$, Y_k, H_k^θ, and X_k are the augmented state from the standard filtering state space that is formulated by

$$X_{k+1} = \begin{bmatrix} x_{k+1} \\ \xi_{k+1} \end{bmatrix} = \begin{bmatrix} F_k^{NxN} & 0^{NxNL} \\ B^{NLxN} & A^{NLxNL} \end{bmatrix} \begin{bmatrix} x_k \\ \xi_k \end{bmatrix} + \begin{bmatrix} G_k \\ 0 \end{bmatrix} u_k;$$

$$Y_k = H_k^\theta X_k + V_k; \qquad\qquad (2)$$

with $F_k^\theta = \begin{bmatrix} F_k^{NxN} & 0^{NxNL} \\ B^{NLxN} & A^{NLxNL} \end{bmatrix}$; $G_k^\theta = \begin{bmatrix} G_k \\ 0 \end{bmatrix}$;

$$H_k^\theta = H_k \oplus H_{k-1} \oplus ... \oplus H_{k-L};$$

$$B^{NLxN} = \begin{bmatrix} I^{NxN} \\ 0^{N(L-1)xN} \end{bmatrix};$$

$$A^{NLxNL} = \begin{bmatrix} 0^{NxN} & 0^{NxN} & \cdots & 0^{NxN} \\ I^{NxN} & \ddots & & \vdots \\ \vdots & & 0^{NxN} & 0^{NxN} \\ 0^{NxN} & \cdots & I^{NxN} & 0^{NxN} \end{bmatrix};$$

and $\{F_k, G_k, H_k\}$ are the representation for the standard adaptive filter, N is the length of the state (x_k) and L is the length of frequency weighted filter.

The second order form of (1) is similar to that in the Theorem 1, and the stationary point of this form can be found by first introducing the auxiliary Krein state-space as follows:

$$X_{i+1} = F_i^\theta X_i + G_i^\theta u_i$$

$$\begin{bmatrix} Y_i \\ \hat{Z}_{i|i} \end{bmatrix} = \begin{bmatrix} H_i^\theta \\ L_i^\theta \end{bmatrix} X_i + V_i \qquad (3)$$

$$\begin{bmatrix} y_{i,p} \\ \hat{z}_{i|i,p} \end{bmatrix} = W_j \begin{bmatrix} Y_i \\ \hat{Z}_{i|i} \end{bmatrix}$$

where the disturbances $\{x_0, u_i, v_i\}$ are assumed to be elements in a Krein space κ with

$< u_i, u_j >_\kappa = I\delta_{ij}$, $\qquad < x_0, x_0 >_\kappa = \Pi_0$, $\qquad$ and

$< WV_i, WV_j >_\kappa = R_j$; $\quad R_j = (I \oplus -\gamma_f^2 I)$; $\quad W_j = W \oplus W$;

and F_k^θ, G_k^θ, H_k^θ as described above in (2); $L_k^\theta = L_k \oplus L_{k-1} \oplus ... \oplus L_{k-L}$.

Then, the corresponding Theorem 1 can be applied to compute the stationary point of above $J_{f,i}$ and gives result in the aposteriori weighted robust filter below.

Aposteriori Weighted Robust Filter. For a given $\gamma_f > 0$, if the $\{F_j\}_{j=0}^i$ are nonsingular, then the second-order form (1) satisfies $J_{f,i} > 0$ iff for all $j = 0, \ldots, i$, we have

$$P_j^{-1} + H_j^{\theta*} W_j^* W_j H_j^\theta - \gamma_f^{-2} L_j^{\theta*} W_j^* W_j L_j^\theta > 0,$$

where $P_0 = \Pi_0$ and P_j satisfies the Riccati recursion

$$P_{j+1} = F_j^\theta P_j F_j^{\theta*} + G_j^\theta G_j^{\theta*}$$

$$- F_j^\theta P_j [H_j^{\theta*} \quad L_j^{\theta*}] W_j^* R_{e,j}^{-1} W_j \begin{bmatrix} H_j^\theta \\ L_j^\theta \end{bmatrix} P_j F_j^{\theta*}$$

with $R_{e,j} = R_j + W_j \begin{bmatrix} H_j^\theta \\ L_j^\theta \end{bmatrix} P_j [H_j^{\theta*} \quad L_j^{\theta*}] W_j^*$

and $R_j = (I \oplus -\gamma_f^2 I)$; $W_j = W \oplus W$; (4)

If this is the case, then one possible H_∞ filter with level γ_f is given by

$$\hat{Z}_{j|j} = L_j^\theta \hat{X}_{j|j}, \text{ where } \hat{X}_{-1|-1} = 0,$$

$$\hat{X}_{j+1|j+1} = F_j^\theta \hat{X}_{j|j} + K_{1,j}(WY_{j+1} - WH_{j+1}^\theta F_j^\theta \hat{X}_{j|j})$$

$$K_{1,j} = P_{j+1} H_{j+1}^{\theta*} W^* (I + WH_{j+1}^\theta P_{j+1} H_{j+1}^{\theta*} W^*)^{-1} \quad (5)$$

As remarked in [3], the above filter is also one among many possible filters with level γ_j, but for brevity and simplicity, we shall use the above filter in the rest part of the paper. All filters that guarantee $J_{f,i} > 0$ can be derived using the similar method as described by Theorem 1 in [3]. The above filter is also known as the central filter as it corresponds to the elimination of nonlinear causal contractive mapping S in the parameterization of all filters that satisfy $J_{f,i} > 0$.

Normalized Frequency Weighted LMS
Theorem 2. Consider the filtering state-space model as follows

$$w_{n+1} = w_n$$
$$d_n = h_n w_n + v_n \quad (6)$$

with $w_0 = \hat{w}_{-1|-1}$ and we want to minimize the weighted H_∞ norm of the transfer function operator $T_f(F)$ from the weighted disturbances $\mu^{-1/2}(w - \hat{w})$ and $\{WV_j\}_{j=0}^\infty$ to the weighted filtered error $\{WZ_{j|j} - Wh_j^\theta \hat{w}_{j|j}\}_{j=0}^\infty$, with W is the frequency weighted filter with length of L and $h_j^\theta = [h_j \quad h_{j-1} \quad \cdots \quad h_{j-L+1}]^T$.

If $\{Wh_j^\theta\}$ is exciting, then the minimum H_∞ norm is $\gamma_{f,opt} = 1$ with the optimal weighted aposteriori H_∞ filter given by

$$\hat{Z}_{j|j} = h_j^\theta \hat{w}_{j|j} \text{ and}$$

$$\hat{w}_{j+1|j+1} = \hat{w}_{j|j} + \frac{\mu h_{j+1}^{\theta*} W^*}{I + \mu Wh_j^\theta h_j^{\theta*} W^*}(WD_{j+1} - Wh_{j+1}^\theta \hat{w}_{j|j})$$

with $D_j = [d_j^T \quad d_{j-1}^T \quad \cdots \quad d_{j-L+1}^T]^T$

The above estimation is actually the normalized frequency weighted LMS algorithm, which for a given initial weight $\hat{w}_{-1|-1}$, guarantees attenuation level $\gamma=1$.

Proof. It can be easily shown by applying the weighted H_∞ aposteriori filter given in (3, 4, 5) with $F_i=I$, $G_i=0$, $H_i=h_i$ and $L_i=h_i$. Notice that if $F_i=I$, then the augmented state space of X_j in (2) and (3) can be simplified to the following filtering state space

$$w_{i+1} = w_i$$

$$\begin{bmatrix} Y_i \\ \hat{Z}_{i|i} \end{bmatrix} = \begin{bmatrix} h_i^\theta \\ h_i^\theta \end{bmatrix} w_i + V_i$$

$$\begin{bmatrix} y_{i,p} \\ \hat{z}_{i|i,p} \end{bmatrix} = W_i \begin{bmatrix} Y_i \\ \hat{Z}_{i|i} \end{bmatrix}$$

with w_i the state vector; $h_j^\theta = [h_j \quad h_{j-1} \quad \cdots \quad h_{j-L+1}]^T$ and $W_i = W \oplus W$ is the frequency weighting vector.

The corresponding Riccati equation of (4) is simplified into:
$$P_{j+1} = P_j$$

$$- P_j [h_j^{\theta*} \quad h_j^{\theta*}] W_j^* (R_j + W_j \begin{bmatrix} h_j^\theta \\ h_j^\theta \end{bmatrix} P_j [h_j^{\theta*} \quad h_j^{\theta*}] W_j^*)^{-1} W_j \begin{bmatrix} h_j^\theta \\ h_j^\theta \end{bmatrix} P_j$$

with $R_j = (I \oplus -\gamma^2 I)$;

Using the matrix inversion lemma, the above equation becomes

$$P_{j+1}^{-1} = P_j^{-1} + [h_j^{\theta*} \quad h_j^{\theta*}] \begin{bmatrix} W^* W & 0 \\ 0 & -\gamma^{-2} W^* W \end{bmatrix} \begin{bmatrix} h_j^\theta \\ h_j^\theta \end{bmatrix}$$

$$= P_j^{-1} + (1 - \gamma^{-2}) h_j^{\theta*} W^* W h_j^\theta \quad (7)$$

and starting with $P_0^{-1} = \mu^{-1} I$, we have

$$P_{j+1}^{-1} = \mu^{-1} I + (1 - \gamma^{-2}) \sum_{i=0}^j h_i^{\theta*} W^* W h_i^\theta \quad (8)$$

If Wh_i^θ is exciting, then P_{j+1}^{-1} is positive definite iff $\gamma_f \geq 1$. If we employ the $\gamma = 1$ to the weighted H_∞ aposteriori filter,

then we got the Normalized frequency weighting LMS algorithm as given in Theorem 2.

Lemma 2. Given a scalar $\gamma_p > 0$, then $\|T_p\|_\infty \leq \gamma_p$ iff there exists $\hat{z}_k$ (for all $k \leq i$) such that, for all nonzero complex vectors x_0 and for all nonzero causal sequences $\{u_j, v_j\}_{j=0}^i$, the scalar second order form:

$$J_{p,i} = x_0^* \Pi_0^{-1} x_0 + \sum_{k=0}^i u_k^* u_k$$

$$+ \sum_{k=0}^i (WY_k - WH_k^\theta X_k)^* (WY_k - WH_k^\theta X_k) \qquad (9)$$

$$- \gamma_p^{-2} \sum_{k=0}^i (W\hat{Z}_k - WL_k^\theta X_k)^* (W\hat{Z}_k - WL_k^\theta X_k)$$

satisfies $J_{p,i} > 0$;

Following the Lemma 2, an apriori H_∞ predictor of level γ_p will exist if there are $\hat{z}_k$ that guarantee $J_{p,i} > 0$. Consequently, we have the following auxiliary Krein state-space and use the corresponding Theorem 1 in order to find the stationary point of above $J_{p,i}$.

$$X_{i+1} = F_i^\theta X_i + G_i^\theta u_i$$

$$\begin{bmatrix} Y_i \\ \hat{Z}_i \end{bmatrix} = \begin{bmatrix} H_i^\theta \\ L_i^\theta \end{bmatrix} X_i + V_i \qquad (10)$$

$$\begin{bmatrix} y_{i,p} \\ \hat{z}_{i,p} \end{bmatrix} = W_j \begin{bmatrix} Y_i \\ \hat{Z}_i \end{bmatrix}$$

where the disturbances $\{x_0, u_i, v_i\}$ are assumed to be elements in a Krein space κ with

$< u_i, u_j >_\kappa = I\delta_{ij}$, $< x_0, x_0 >_\kappa = \Pi_0$, and

$< WV_i, WV_j >_\kappa = R_j$; $R_j = (I \oplus -\gamma_j^2 I)$; $W_j = W \oplus W$;

and F_k^θ, G_k^θ, H_k^θ as described in (2);
$L_k^\theta = L_k \oplus L_{k-1} \oplus ... \oplus L_{k-L}$.

Apriori Weighted Robust Filter. For a given $\gamma_p > 0$, if the $\{F_j\}_{j=0}^i$ are nonsingular, then the second-order form satisfies $J_{p,i} > 0$ iff for all $j = 0,...,i+1$, we have

$\tilde{P}_j^{-1} = P_j^{-1} - \gamma_j^{-2} L_j^{\theta*} W^* W L_j^\theta > 0$, where P_j is the same as in the aposteriori filter.

One possible apriori H_∞ filter with level γ_p is given by

$$\hat{Z}_j = L_j^\theta \hat{X}_j, \text{ where } \hat{x}_0 = 0,$$

$$\hat{X}_{j+1} = F_j^\theta \hat{X}_j + K_{1,j}(WY_j - WH_j^\theta \hat{X}_j)$$

and $\quad K_{1,j} = \tilde{P}_j H_j^{\theta*} W_j^* (I + WH_j^\theta \tilde{P}_j H_j^{\theta*} W^*)^{-1}$

As in the aposteriori case, the apriori filter $\hat{Z}_j = L_j^\theta \hat{X}_j$ is also one among many possible filters and all filters that guarantee $J_{p,i} > 0$ can be derived using the similar method as in [3].

The Frequency Weighted LMS

Theorem 3. Consider the same filtering state space equation as in (6), and we want to minimize the weighted H_∞ norm of the transfer function operator $T_p(F)$ from the weighted disturbances $\mu^{-1/2}(w - \hat{w})$ and $\{WV_j\}_{j=0}^\infty$ to the weighted prediction error $\{WZ_j - Wh_j^\theta \hat{w}_{j|j-1}\}_{j=0}^\infty$, $\hat{w}_{-1} = 0$ with W is the frequency weighting filter with length of L and $h_j^\theta = [h_j \quad h_{j-1} \quad \cdots \quad h_{j-L+1}]^T$.

If $\{Wh_j^\theta\}$ is exciting and $0 < \mu < \inf \dfrac{1}{Wh_j h_j^* W^*}$ (11)

then the optimal H_∞ norm is $\gamma_{p,opt} = 1$, with the optimal frequency weighted H_∞ apriori filter given by

$$\hat{Z}_j = h_j^\theta \hat{w}_{j-1}$$

and $\quad \hat{w}_j = \hat{w}_{j-1} + \mu h_j^{\theta*} W^* (WD_j - Wh_j^\theta \hat{w}_{j-1})$.

where $D_j = [d_j^T \quad d_{j-1}^T \quad \cdots \quad d_{j-L+1}^T]^T$

Proof. It is obtained by the similar fashion as in the frequency weighted normalized LMS. This equation implies that if μ is properly chosen, the frequency weighted LMS guarantees the attenuation level $\gamma = 1$.

If the weighted input data $\{Wh_j^\theta\}$ is not excited, then it is easy to derive that $\gamma_{f,opt} < 1$ and $\gamma_{p,opt} < 1$, and in this case, the frequency weighted LMS and the frequency weighted normalized LMS now become suboptimal frequency weighted H_∞ filter.

IV. Experimentation Setup

In the ANC experimentation, an industrial duct was employed with the primary acoustic disturbance came from the fan motor noise induced in the upstream of the duct as depicted in Fig. 1, where it also illustrated the positions of the reference microphone, the actuators and the error microphones.

As previously stated, that the frequency weighted robust filter is used in ensuring the control signal to be inside the nominal frequency range. In the experiment, the actuators used in dealing with the fan noise consist of one

sub-woofer and one standard speaker, and each has different frequency response characteristic as depicted in Fig. 2.

We can see that the nominal frequency for the sub-woofer is 50Hz, where for the standard speaker, it is approximately 100Hz. In other words, the sub woofer can excite low frequency sound better than the standard speaker.
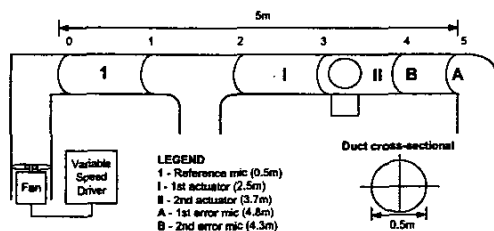

Fig. 1. Experimental duct configuration

Therefore, during the multichannel experimentation, the subwoofer dealt with the low frequency noise below 200Hz, while the standard speaker handled the higher frequency noises ($\geq$160Hz). The frequency weightings used in experiments are based on the nominal frequency of each of the speakers as seen above, which is a low pass filter with cut off frequency of 200Hz and a high pass filter with cut off frequency of160Hz.
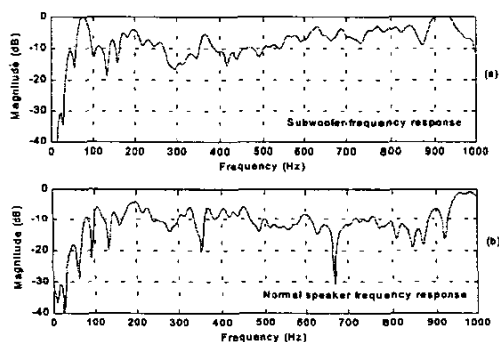

Fig. 2. Estimated secondary path frequency response. (a) Secondary path employing sub-woofer; (b) Secondary path employing standard speaker

The control block diagram, employing filtered-x approach, is illustrated in Fig. 3 [7,12]. It is the common approach in dealing with the secondary path effect, the transfer function from the control signal to the summing junction in the error microphone(s). Other approaches for ANC control are not considered in the paper, but readers can refer to [5,7,12,13] for different methods.

For single channel ANC experimentation, we employed the standard speaker as the actuator, one reference microphone and one error microphone, placed in the end of the duct.
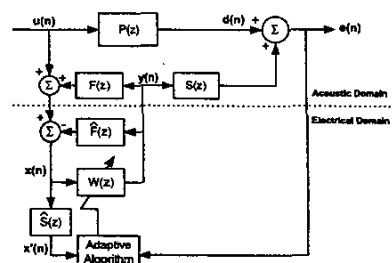

Fig. 3. Filtered-x approach block diagram for ANC with feedback neutralization filter.

V. Experimental Result

Evaluating from the coherence function of reference signal to the error signal, it shows that the possible attenuation that can be achieved is within the frequencies around 40-160Hz, 230-330Hz and 370-450Hz (Fig. 4). And from the fan noise spectra, we learnt that the dominant noise is in the frequency below 100Hz. This fact told us that the standard speaker alone was not sufficient. It is our objective to utilize the standard speaker, using the weighted filtering algorithm, to put more effort inside its nominal range during noise attenuation.
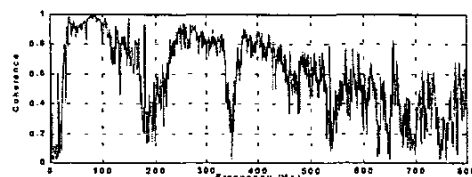

Fig. 4. Coherence function of the fan noise between the reference with error mic at 4.8m

The result of the single channel ANC, utilizing the standard speaker, is given in Fig. 5. From the Fig. 5b, we see that non-weighted algorithm performs poorly for frequency above 120Hz. Both non-weighted robust filter tried to reduce the worst-case disturbance (in this case between 40-160Hz), which apprehended to be outside the working area of the actuator. It did reduce the low frequency noise, but with trade-off in widening the noise and saturating the control signal. And also human perceptions are more perceptive to frequency higher than 100Hz, thus it is effortless to attenuate the low frequency with less hearing effect to the human. From Fig. 5a, the weighted algorithms give about 7-10dB attenuation in the frequencies above 200Hz (as expected by the coherence function in Fig. 4) and there is no noise spreading occurring as it is in the non-weighting case.

In the extension to the multichannel ANC, the sub woofer was employed in handling the low frequency noise. In tandem, both weighted and non-weighted algorithms achieved a satisfactory result (Fig. 6).
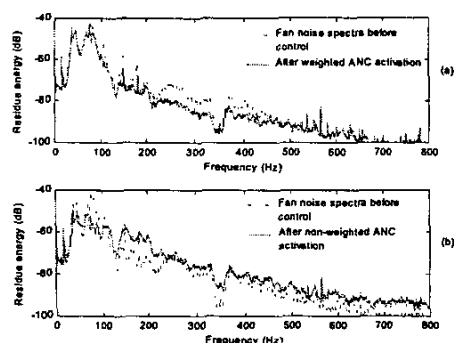
Fig. 5. Experimental result on single channel ANC (a) experiment using weighted Normalized LMS; (b) experiment using standard Normalized LMS

If a careful observation is taken, we can see that the weighted algorithm still yielded better attenuation than that of the non-weighted one. Table 1 provides the total energy reduction achieved using multichannel configuration for weighted and non-weighted algorithm.

Table 1. Total Energy Reduction achieved by Multichannel ANC in Error Mic in position 4.8m

Algorithm	Total Energy Reduction (dB)
Fx-Normalized LMS	9.9338
Frequency Weighted Fx-Normalized LMS	10.2975

We can notice that a distribution of actuators to be working on each nominal range of frequency is better than none, and this is verified by the experimental result.
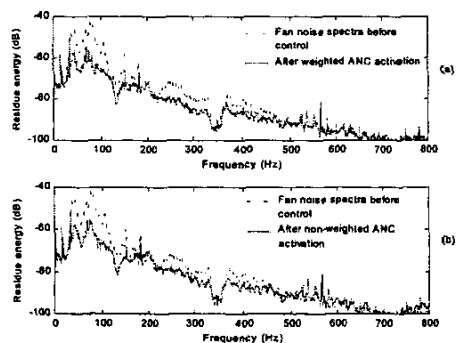

Fig. 6. Experimental result on multi channel ANC using 1-2-2 configuration (a) experiment using weighted Normalized LMS; (b) experiment using standard Normalized LMS.

VI. Conclusion

The motivations for the frequency weighted H_∞ filtering have been presented in this paper. And the application of weighted algorithm to the ANC problem is also given which shows an alternative way in dealing with actuators saturation non-linearity inherited from the actuator nominal frequency range.

Reference

[1]   B. Hassibi, A. H. Sayed, T. Kailath, "H_∞ Optimality of the LMS Algorithm", IEEE Trans. on Signal Processing, Vol. 44, No. 2, February 1996

[2]   B. Hassibi, A. H. Sayed, T. Kailath, "Recursive Linear Estimation in Krein Spaces – Part I: Theory", IEEE Trans. on Automatic Control, vol. 41, pp. 18-33, 1996

[3]   B. Hassibi, A. H. Sayed, T. Kailath, "Recursive Linear Estimation in Krein Spaces – Part II: Applications", IEEE Trans. on Automatic Control, vol. 41, pp. 34-49, 1996

[4]   B. Hassibi, T. Kailath, "H_∞ Bounds for the Recursive-Least-Squares Algorithm", in Proc. IEEE Conf. Decision Control, Orlando, FL, pp. 3927-3929, December 1994

[5]   B. Sayyarrodsari, J. P. How, B. Hassibi, A. Carrier, "An H_∞-Optimal Alternative to the FxLMS Algorithm", Proc. American Control Conf., Philadelphia, Pennsylvania, pp. 1116-1121, June 1998

[6]   M. Bouchard, B. Paillard, C. T. L. Dinh, "Improved Training of Neural Networks for the Nonlinear Active Control of Sound and Vibration", IEEE Trans. on Neural Networks, Vol. 10, pp. 391-401, March 1999

[7]   P. A. Nelson, S. J. Elliot, "Active Control of Sound", London, U.K.:Academic, 1992

[8]   P. Bolzern, P. Colaneri, G. De Nicolao, "H_∞ Robustness of Adaptive Filters Against Measurement Noise and Parameter Drift", Automatica, Vol. 35, pp. 1509-1520, 1999

[9]   P. Bolzern, P. Colaneri, G. De Nicolao, U. Shaked, "Guaranteed H_∞ Robustness Bounds for Wiener Filtering and Prediction", Int. Journal of Robust and Nonlinear Control, Vol. 12, pp. 41-56, 2002

[10] S. D. Snyder, N. Tanaka, "Active Control of Vibration Using a Neural Network", IEEE Trans. on Neural Networks, Vol. 6, No. 4, pp. 819-828, July 1995

[11] S. Haykin, "Adaptive Filter Theory", 2nd Ed. Englewood Cliffs, NJ: Prentice Hall, 1991

[12] S. M. Kuo, D. R. Morgan, "Active Noise Control Systems: Algorithms and DSP Implementations", New York: Wiley, 1996

[13] S. M. Kuo, J. Tsai, "Residual Noise Shaping Technique for Active Noise Control Systems", Journal of the Acoustical Society of America, Vol. 95, pp. 1665-1668, March 1994