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Abstract

We study an auction where two licenses to operate on a new market are sold. Winners finance their bids on a competitive debt market. Due to limited liability, the amount of debt affects their behavior on the product market. In equilibrium, consumer prices are lower than with a beauty contest, where firms obtain their licenses for free. Winning bids are lower than in a model where firms have internal funds. Higher bids cannot be financed due to credit rationing. Expected net firm profits are strictly positive, although firms are a priori identical and have the same information.

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# 1 Introduction

During the last decade, license auctions in the US and Europe sparked a huge interest, both from academics and from the general public. In the US, the FCC auctioned licenses to use the electromagnetic spectrum for personal communication services. Between July 1994 and July 1998, 16 auctions were held, where 5,893 licenses were sold. Total revenues were $22.9 billion dollars (Cramton and Schwartz, 2000; for more on the design of these auctions, see e.g. McAfee and McMillan, 1996, or McMillan 1994). Throughout Europe, licenses for "third generation" (3G) mobile telecommunication (or UMTS) took place during 2000 and 2001 (see e.g. Van Damme, 2002). These auctions, held in 9 countries, raised over $100 billion, or over 1.5% of GDP. The revenue per inhabitant differed greatly per country. (Klemperer, 2002a).

Academic economists tend to analyze such auctions using standard auction theory. This theory stresses the asymmetric information aspect of auctions. Typically, the following assumptions are made. One good is being auctioned. Bidders know their own valuation, or at least their own signal about the true value of the good being auctioned. They also know the distribution of valuations or signals of the other bidders. Given that information, bidders determine their optimal bids. There are many variations on this basic model. (For recent surveys, see Wolfstetter, 1996, or Klemperer, 1999).

Increasingly however, it is argued that the standard model may not capture all the relevant aspects of contexts like license auctions. It is argued that too much attention is given to technicalities concerning asymmetric information, and too little attention is given to the market structure and industrial organization aspects of such auctions. One particular vocal exponent of this view is Klemperer (2002b). Jehiel and Moldovanu (2000a) make a similar point. See also Jehiel and Moldovanu (2002). Hence, we need models that stress the competitive implications of the way licenses are auctioned. This paper does exactly that.

One important aspect of license auctions is that the value of a license is not something that is exogenously given. Rather, this value is also determined by the outcome of the

auction. Suppose two licenses are being sold. Ultimately, the value of a license equals the expected profits that can be made on the market where the winners of the licenses will compete. This depends on the efficiency of the firm winning a license, but also on the efficiency of the firm winning the other license. Therefore, a firm bidding on a license has to take into account not only its own information, but also who he thinks will win the other license. Jehiel and Moldovanu (2000b) study a model in which bidders do care as to who wins the auction: the type of the winner affects the payoff of the bidders that do not win. Yet, in their model, only one good is being sold, so winners of the auction do not compete with each other. In their model, the external effects of the identity of the auction winner on the others are exogenously given, and not determined endogenously in the competition stage of the model.

A second striking aspect of license auctions is that winning firms often have to take on debt to be able to finance their bid (see e.g. The Economist, 2002). From the industrial organization literature, it is known that the amount of debt a firm has, affects how it competes. Brander and Lewis (1986) show that firms that have more debt, compete more aggressively – because of limited liability. Their model has Cournot competition and uncertainty, for example about the level of marginal costs. Only after all decisions have been made, the uncertainty is resolved. The authors show that in this setup, more debt implies higher equilibrium quantities, and hence a lower equilibrium price. The effect is due to limited liability. If a firm's operating profits fall below its debt, it only pays the operating profits to the bank. When competing, firms maximize their expected operating profits minus the actual repayment to the bank. If operating profits fall below the debt level, the firm has zero payoff. This will occur if the realization of marginal cost turns out to be high. If marginal cost is low, the firm does have a strictly positive payoff. Hence, a firm focuses on strategies that yield high payoffs in good states, i.e. when marginal cost is low. Such strategies are called aggressive strategies. With higher debt, marginal cost need to be lower for a firm to have a positive payoff. Therefore, higher debt implies that firms focus on cases with lower marginal cost and apply more aggressive strategies. In the context of quantity competition this implies setting a larger quantity. This results

in a lower price. For license auctions, this implies that the amount of debt that winning firms will take on affects their expected profits in the post-auction market, and hence their bidding strategies in the auction.

In this paper, we model these issues. In our model, two licenses to operate on some new market are auctioned. The two winners of the auction will thus establish a duopoly. A number of a priori identical firms participate in the auction. The two winners finance their bids on a competitive debt market. After the auction, and after financing the bids, the two winners compete on the product market. We assume that firms have symmetric but incomplete information. There is uncertainty with respect to the marginal costs that prevail in the competition stage. When firms place higher bids, they also have to take on higher debts, which in turn affects their behavior on the product market. Firms take all these effects into account when placing their bids. We study how bidding behavior on the auction market is ultimately affected.

Our model yields some surprising insights. First, the winners of the auction have expected profits that are strictly positive. This is surprising, since a priori all firms are identical, and we don't have asymmetric information. Hence, one would expect to see profits being driven down to zero. This however is not the case, since the equilibrium of our model exhibits credit rationing. Winning bidders, even though they make positive expected profits, will not be outbid. Any higher bid would yield a debt level that implies negative net expected profits for bondholders. A higher level of debt will make firms more aggressive on the product market, reducing the probability that the debt will be repaid. Hence, a firm would be willing to bid more if only it could obtain funding, but it cannot.

Second, we find that in our model, equilibrium consumer prices are lower than they are in a case where firms obtain their license for free. One of the main concerns in the popular press is that the use of auctions will increase prices that consumers ultimately have to pay. Firms will recoup the costs of spectrum rights by simply adding a mark-up to consumer prices, the argument goes, so the higher these costs, the higher the mark-up. Of course, this argument is invalid. When prices paid at an auction are simply sunk costs, they do not influence prices that firms charge to consumers. In our model, higher fees even lead to

*lower* prices. This is due to the strategic effect of debt: as firms take on more debt, they will compete more aggressively on the product market.[1] For the same reason, consumer prices in our model are also lower than in a model where winners can finance their bids out of internal funds.

A third result we find is that winning bids are lower with debt than they are when firms have internal funds, and do not have to resort to the debt market to finance their bids. This is the case because of two reasons. First, since firms with debt compete more aggressively, equilibrium expected operating profits are decreasing in debt levels, which in turn implies that with debt, equilibrium bids will be lower. Moreover, with internal funds, firms simply continue bidding until their bids are equal to their expected operating profits. In the model with debt, they stop bidding before they reach that level, because of credit rationing.

Chowdhry and Nanda (1993) also study the strategic role of debt in an auction – but in a context entirely different from ours. They study a takeover contest, in which many raiders bid to take over a firm. In Goeree (2003), competition after the auction also affects bidding behavior. In his model, bidders use their bid to signal their private information when there is an aftermarket. Offerman and Potters (2001) present experimental evidence that a high license fee paid at the auction makes it more likely that winners will collude – so higher license fees may lead to higher consumer prices. Clayton and Ravid (2002) study the effect of the debt a firm currently has on their bidding behavior in the US FCC auctions. They find that, as debt levels increase, firms tend to reduce their bids. Note however that they study the amount of debt a firm already has when the auction takes place. We study the amount of debt a firm has to take on as a consequence of winning the auction. Moreover, in our model, the amount of debt is endogenously determined.

The setup of the paper is as follows. In the next section, we present our model, where we leave the exact mode of competition at the competition stage unspecified. We only put some weak assumptions on the equilibrium profits of the competition stage, which we use to derive our results. In section 3 we solve this model. In section 4 we summarize

[1] We also made this argument, but only intuitively, in Haan and Toolsema (2000).

our main results. In section 5 we show that three standard models of competition, the linear Hotelling, Bertrand, and Cournot models, satisfy the assumptions we put on our profit function and therefore satisfy our framework. We derive the equilibria for numerical examples based on these three competition models, and show that our results hold for all three cases. Section 6 concludes.

# 2 The model

We have a three-stage model. In the first stage, the *auction stage*, $N > 2$ bidders compete in a sealed-bid license auction, where winning bidders pay their own bid. Alternatively, we may assume a simultaneous ascending auction of the type that is often used in license auctions (see e.g. Milgrom, 2000). For our results, this is not important. Firms are ex ante identical. The two highest bidders obtain a license to operate in a new market. Without loss of generality, the highest bidder will be denoted firm 1 and the second highest bidder will be denoted firm 2. Their bids are denoted $b_1$ and $b_2$. In the case of ties, winners will be decided by coin toss. In stage 2, the *debt stage*, the two winning firms finance their bids. They do so by issuing bonds on a perfectly competitive bond market. We will refer to the buyers of these bonds as bondholders. Firm $i$ obtains an amount $b_i$ to pay for its bid, against the promise of repaying $d_i$ at the end of the game. We assume that if it is not the case that both firms are able to secure financing in the debt stage, then the auction will be declared void, and a new auction will be held. In stage 3, the *competition stage*, the two winning firms compete on the output market. That is, they set strategic variables $s_1$ and $s_2$, which may be either prices or quantities. When setting these, the firms still face uncertainty with respect to market conditions. This assumption is necessary to have a strategic effect of debt. We model this as uncertainty about the marginal costs firms will incur once operating. Marginal costs $c$ are constant and equal for all firms, but unknown in advance. They are drawn from a continuously differentiable, strictly positive probability density function $f(\cdot)$ with domain $[\underline{c}, \bar{c}]$. This is common knowledge. After setting their strategic variables, uncertainty about marginal costs is resolved, consumers make their

decisions and, if possible, debts are repaid. For simplicity, we assume no discounting.

In the remainder of this section, we give a detailed description of the three main stages of the model: the auction stage, the debt stage, and the competition stage. We describe these stages in the same order as we solve the model: by backward induction, starting with the last stage. We end this section by giving a formal definition of the equilibrium in our model.

**Competition stage** In this stage, the two firms that have won the auction in stage 1, and managed to obtain funding, compete on the output market. They do so by simultaneously setting a strategic variable. Let $s_i \geq 0$ denote the value of the strategic variable set by firm $i$, which may refer to either price or quantity. Expected profits of each firm will depend on debt levels that are determined in the debt stages, and the strategic variables set by each firm. We denote the expected net profits of firm $i$, so its profit after repayment of debt, as $\Pi_i^F(s_i, s_j, d_i, d_j)$, where $d_i$ denotes the amount of debt firm $i$ has taken on in the debt stage. Subscripts refer to the identity of the firm, the superscript $F$ indicates that we are considering firms. We look for a Nash equilibrium in the competition subgame, hence

$$s_i^*(d_i, d_j) = \arg\max_{s_i} \Pi_i^F(s_i, s_j^*, d_i, d_j),$$

for $i = 1, 2$; $j \neq i$. We assume existence and uniqueness of this equilibrium.

Suppose that marginal costs happen to be $c$. Note that this value is unknown when firms set their strategic variable. The true value of $c$ will only be revealed after these are set. For such a given value $c$, we define the operating profits of firm $i$ as its gross profits, before its debt is repaid. These operating profits depend on $c$ and on the strategic variables the firms set. Hence we denote them as $\pi_i(s_i, s_j, c)$. Obviously, operating profits equal revenues minus costs, and we can write $\pi_i(s_i, s_j, c) = R_i(s_i, s_j) - cq_i(s_i, s_j)$, with $R_i$ operating revenue of firm $i$, and $q_i$ the quantity sold by firm $i$. When the strategic variable is quantity, we obviously have $q_i = s_i$. When the strategic variable is price, $q_i$ will depend on both prices via the demand function. In equilibrium, when values $s_i^*$ and $s_j^*$ are set, we

can denote operating profits of firm $i$ when marginal costs are $c$, as

$$\begin{aligned}\pi_i^*(d_i, d_j, c) &\equiv \pi_i\left(s_i^*(d_i, d_j), s_j^*(d_j, d_i), c\right) \\ &= R_i^*\left(s_i^*(d_i, d_j), s_j^*(d_j, d_i)\right) - cq_i^*(s_i^*(d_i, d_j), s_j^*(d_j, d_i)). \qquad (1)\end{aligned}$$

From this, it is easy to see that $\pi_i^*(d_i, d_j, c)$ is decreasing in $c$. We make the following additional assumptions:

**Assumption 1** *$\pi_i^*(d_i, d_j, c)$ is strictly decreasing in $d_j$ for any $c$.*

**Assumption 2** *$\pi_i^*(d, d, c)$ is strictly decreasing in $d$ for any $c$.*

Assumption 1 states that the equilibrium operating revenues of a firm are decreasing in the amount of debt that the competitor holds. More debt makes the competitor more aggressive, which hurts firm $i$. Assumption 2 states that when both firms have the same level of debt $d$, and when that common debt level increases, then operating revenues of both firms decrease. In section 5, we show that these assumptions are satisfied in a model with linear demand and either Hotelling competition, Bertrand competition, or Cournot competition with differentiated products.

**Debt stage** In the debt stage, the two firms that have won the auction have to find bondholders who are willing to lend them the money to pay for their bids. When a winning firm has submitted a bid $b_i \geq 0$, a debt contract can be represented by $(b_i, d_i)$: firm $i$ receives $b_i$ now, in return for the promise to repay $d_i$ at the end of the game. We assume limited liability. A firm cannot make a repayment that is higher than its operating profit in the competition stage. If the operating profits fall short, the firm goes bankrupt, and the bondholders only receive the firm's operating profits, if any. Naturally, this is taken into account when the debt contract is determined. Firms can make a take-it-or-leave-it offer to bondholders.

Consider the expected gross profits to the bondholders. Suppose that firm $i$ has debt $d_i$. It can just repay its debt when $\pi_i^*(d_i, d_j, c) = d_i$. In that case, gross operating profits

are just sufficient to cover the promised repayment $d_i$. Denote the value of $c$ for which this equality is satisfied as $\hat{c}_i$. The firm's operating profits are zero when $\pi_i^*(d_i, d_j, c) = 0$. Denote the value of $c$ for which this equality is satisfied as $\check{c}_i$, provided this value is smaller than $\bar{c}$. Otherwise, we have $\check{c}_i = \bar{c}$. If $c \leq \hat{c}_i$, firm $i$ is able to fully repay its debt. Debtholders then receive $d_i$. The probability that this occurs is denoted $\Pr(c \leq \hat{c}_i)$. With $c \in (\hat{c}_i, \check{c}_i)$, the firm makes positive operating profits, but these are insufficient to repay the debt $d_i$. In that case, all operating profits will be paid to the bondholders, and the firm's net profits are zero. If $c > \check{c}_i$, operating profits are negative, and bondholders receive nothing. The firm's net profits in this case are also zero. Note that, for given $d_i$ and $d_j$, the equilibrium of the competition stage is uniquely determined by assumption. We can therefore write $\hat{c}_i$ and $\check{c}_i$ as a function of these debt levels: $\hat{c}_i(d_i, d_j)$ and $\check{c}_i(d_i, d_j)$. The expected repayment to bondholders $R_i^B(d_i, d_j)$ can thus be written as

$$R_i^B(d_i, d_j) = \Pr(c \leq \hat{c}_i(d_i, d_j))d_i + \int_{\hat{c}_i(d_i,d_j)}^{\check{c}_i(d_i,d_j)} \pi_i^*(d_i, d_j, c) f(c) dc. \tag{2}$$

Here, the superscript $B$ refers to bondholders, and the subscript $i$ refers to the fact that these bondholders lend to firm $i$. The expected net profits to bondholders are then given by $R_i^B(d_i, d_j) - b_i$.

We can now also give a formal expression for the expected net profits of firm $i$: it will earn operating profits minus its debt, as long as $c < \hat{c}_i$. Otherwise, its profits are zero. Hence

$$\Pi_i^F(s_i, s_j, d_i, d_j) = \int_{\underline{c}}^{\hat{c}_i(d_i,d_j)} (\pi_i(s_i, s_j, c) - d_i) f(c) dc. \tag{3}$$

We assume

**Assumption 3** *$R_i^B(d, d)$ is strictly concave in $d$, and strictly positive for some $d > 0$.*

Thus, we assume that if firms have the same debt level, then the expected profits of bondholders are quasi-concave in that common debt level. Again, in section 5, we show that the assumption is satisfied for the linear Hotelling, Cournot and Bertrand models. Note that firms are a priori identical. Therefore, $R_1^B(d, d) = R_2^B(d, d)$, so we

can drop the subscript. We will refer to the unique maximizer of $R^B(d,d)$ as $d^*$. Thus $d^* \equiv \arg\max_d R^B(d,d)$. Obviously, $R^B(d,d)$ is strictly increasing in $d$ for $0 \leq d < d^*$.

In the debt stage firms 1 and 2 now simultaneously set $d_1$ and $d_2$ such that their expected profits are maximized, given what will occur in the competition stage. For a Nash equilibrium in the debt stage we thus need

$$\begin{aligned} d_i^*(b_i,b_j) &= \arg\max_{d_i} \int_{\underline{c}}^{\hat{c}_i(d_i,d_j^*)} \left(\pi_i^*\left(d_i,d_j^*\right) - d_i\right) f(c)dc, \\ &\text{s.t.} \begin{cases} R_i^B(d_i,d_j^*) \geq b_i, \\ R_j^B(d_j^*,d_i) \geq b_j. \end{cases} \end{aligned}$$

for $i = 1,2$ and $j \neq i$. The condition stresses that a winning bidder has to assure that both firms can find financing: the expected repayment to both bondholders has to be at least equal to the amount of money they provided. If this cannot be satisfied, the auction will be declared void. We will denote the Nash equilibrium profits that follow from this stage as $\Pi_i^{F^*}(b_i,b_j)$. Formally

$$\Pi_i^{F^*}(b_i,b_j) = \Pi_i^F(s_i^*, s_j^*, d_i^*(b_i,b_j), d_j^*(b_j,b_i)),$$

where, for tractability, we have dropped the arguments $(d_i^*, d_j^*)$ of $s_i^*$, which in turn also depend on $b_i$ and $b_j$.

**Auction stage** In the auction stage, $N > 2$ identical firms submit a bid to obtain a license. We assume that if it is not the case that both winners are able to secure financing in the debt stage, then the auction will be declared void, and a new auction will be held. Note therefore that also a firm that is able to secure financing will lose its license when its competitor is not able to do so. This is to rule out cases in which one firm submits a bid that is so high that the bond market is only willing to finance it if the other firm does not obtain financing. In that case, by submitting a very high bid, a firm may be able to effectively shut any competitor out of the market and obtain a monopoly. As a technical condition, we assume that, if an auction has to be held again, the original winners both receive some fine $\varepsilon$, which can be infinitely small. This is to rule out equilibria in which

all firms always submit very high bids, and the auction is never resolved. The fine $\varepsilon$ can even consist of the extra costs involved for a firm to participate in a second auction.

Formally, suppose that at the auction stage, firm $k$ submits a bid $B_k$, $k \in \{1, \ldots, N\}$. Expected net profits of firm $k$ at the auction stage can then be denoted $\Pi_k^{Fa}(B_1, \ldots B_N)$. The highest bid is $b_1 \equiv \max\{B_1, \ldots, B_N\}$, the second highest bid $b_2 \equiv \max\left\{\{B_1, \ldots, B_N\} \backslash b_1\right\}$. For the case ties occur, we define $T$ as the number of firms that have submitted the same bid as the second-highest bidder: $T \equiv \#\{k | B_k = b_2\}$. Note that the highest bidder may also be among these. Given the vector of bids, the probability of obtaining a license for firm $k$ now equals

$$P_k(B_1, \ldots, B_N) = \begin{cases} 0 & \text{if} \quad B_k < b_2 \\ \frac{1}{T} & \text{if} \quad b_1 > b_2 = B_k \\ \frac{2}{T} & \text{if} \quad b_1 = b_2 = B_k \\ 1 & \text{if} \quad B_k = b_1 > b_2 \end{cases}$$

and we have

$$\Pi_k^{Fa}(B_1, \ldots, B_N) = P_k(B_1, \ldots, B_N) \cdot \Pi_i^{F^*}(s_i^*, s_j^*, d_i^*(B_k, B_{-k})), d_j^*(B_{-k}, B_k)),$$

with $B_{-k} = \max\left\{\{B_1, \ldots B_N\} \backslash B_k\right\}$.

**Equilibrium concept** Putting together all the elements of the three subgames, we can now define the subgame perfect Nash equilibrium of the full game as follows

**Definition 1** *The Subgame Perfect Nash Equilibrium of the game described above consists of bids $(B_1^*, \ldots B_N^*)$ for all bidders, and debt levels $(d_1^*, d_2^*)$ and strategic variables $(s_1^*, s_2^*)$ for the two highest bidders, such that we have*

1. *Equilibrium at the competition stage:*

$$s_i^* = \arg\max_{s_i} \Pi_i^F(s_i, s_j^*, d_i^*, d_j^*) = \arg\max_{s_i} \int_{\underline{c}}^{\hat{c}_i(d_i^*, d_j^*)} \left(\pi_i\left(s_i, s_j^*, c\right) - d_i^*\right) f(c) dc,$$

*for $i = 1, 2$ and $j \neq i$;*

*2. Equilibrium at the debt stage:*

$$d_i^* = \arg\max_{d_i} \int_{\underline{c}}^{\hat{c}_i(d_i,d_j^*)} \left(\pi_i^*\left(d_i, d_j^*\right) - d_i\right) f(c)dc, \tag{4}$$

$$s.t. \begin{cases} R_i^B(d_i, d_j^*) \geq b_i, \\ R_j^B(d_j^*, d_i) \geq b_j. \end{cases} \tag{5}$$

*for $i = 1, 2$ and $j \neq i$;*

*3. Equilibrium at the auction stage:*

$$B_k^* = \arg\max_{B_k} \Pi_k^{Fa}(B_1^*, \ldots, B_{k-1}^*, B_k, B_{k+1}^*, \ldots, B_N^*)$$

*for all $k \in \{1, \ldots .N\}$,*

*with all functions and variables defined in the main text.*

# 3 Solving the model

We now derive the equilibrium of the model described in the previous section. We first present two lemma's on $R_i^B(d_i, d_j)$.

**Lemma 1** *$R_i^B(d_i, d_j)$ is strictly decreasing in $d_j$ for any given $d_i$, $i, j = 1, 2$, $i \neq j$.*

**Proof.** Using Leibniz's rule, from (2) the partial derivative of $R_i^B(d_i, d_j)$ with respect to $d_j$ is given by

$$\begin{aligned} \frac{\partial}{\partial d_j} R_i^B(d_i, d_j) = & \frac{\partial \Pr(c \leq \hat{c}_i(d_i, d_j))}{\partial d_j} d_i \\ & + \pi_i^*(d_i, d_j, \check{c}_i(d_i, d_j)) f(\check{c}_i(d_i, d_j)) \frac{\partial \check{c}_i(d_i, d_j)}{\partial d_j} \\ & - \pi_i^*(d_i, d_j, \hat{c}_i(d_i, d_j)) f(\hat{c}_i(d_i, d_j)) \frac{\partial \hat{c}_i(d_i, d_j)}{\partial d_j} \\ & + \int_{\hat{c}_i(d_i,d_j)}^{\check{c}_i(d_i,d_j)} \frac{\partial \pi_i^*(d_i, d_j, c)}{\partial d_j} f(c)dc \end{aligned}$$

From the definition of $\check{c}_i(d_i, d_j)$, the second term in this expression equals zero. Further, since $\pi_i^*(d_i, d_j, \hat{c}_i(d_i, d_j)) = d_i$ and because $f(c)$ is the probability density function of $c$, the

first and third terms cancel out. Thus, only the fourth term remains. This term is negative because of assumption 1. This proves the lemma. ■

Thus, as the debt of firm $j$ increases, the expected repayment to the bondholders of firm $i$ decreases. Intuitively, this can be seen as follows. From assumption 1, we have that for every value of $c$, operating profits of firm $i$ decrease as the debt level of firm $j$ increases. Therefore, the probability that $i$ will be able to repay its debt decreases. The profits that bondholders can capture when the firm cannot fully repay its debt, also decrease.

We also have

**Lemma 2** *$R_i^B(d_i, d) < R_i^B(d, d)$ for all $d_i < d \leq d^*$.*

**Proof.** Since $d_i < d \leq d^*$, we have from assumption 3 that $R_i^B(d, d) > R_i^B(d_i, d_i)$. From lemma 1, we have $R_i^B(d_i, d_i) > R_i^B(d_i, d)$. Hence $R_i^B(d, d) > R_i^B(d_i, d)$. ■

Now we turn to the equilibrium of the auction stage. Recall that $d^* \equiv \arg\max_d R^B(d, d) > 0$ and define $b^* \equiv R^B(d^*, d^*) = \max_d R^B(d, d)$.

**Theorem 1** *The unique symmetric equilibrium of our model is*

1. $B_k = b^* = \max_d R^B(d, d)$, for $k = 1, \ldots, N$,
2. $d_i^* = \arg\max_d R^B(d, d)$, for $i = 1, 2$,
3. $s_i^* = \arg\max_{s_i} \Pi_i^F(s_i, s_j^*, d_i^*, d_j^*)$, for $i = 1, 2$, $j \neq i$.

**Proof.** The proof proceeds in four steps. We will establish that we cannot have an equilibrium in which one of the winning bids is lower than $b^*$ (step 3). Then, we show that no firm has an incentive to defect to a higher bid when all firms bid $b^*$ (step 4). Taken together, these steps imply that in the unique symmetric equilibrium both winning bids equal $b^*$, provided that expected firm profits are non-negative in that case. Before establishing these steps, we show that firms can indeed obtain funding in this candidate equilibrium, and that the only symmetric debt level that yields nonnegative net profits to the bondholders for $b = b^*$ is $d^*$ (step 1). Then, we show that firms' profits are indeed

strictly positive in this equilibrium. (step 2). Therefore, any firm submitting a bid $b < b^*$ can improve by bidding $b = b^*$ and having a probability of $2/N$ to obtain the equilibrium firm profits. Together, this proves the theorem.

1. *The bond market is willing to finance any pair of bids* $(b_1, b_2)$ *for which* $b_1 \leq b^*$ *and* $b_2 \leq b^*$. — We only have to show that for such $(b_1, b_2)$ there exists a pair $(d_1, d_2)$ such that (5) is satisfied. But such debt levels do exist: using assumption 3 and $R^B(0,0) = 0$, bondholders make nonnegative net profits when setting $d_1 = d_2 = d^*$.

2. *If* $d_1 = d_2 = d^*$, *the firms earn strictly positive profits.* — If both firms' debts equal $d^*$, the net profits to the firm $\Pi^F(s^*, s^*, d^*, d^*)$ will be strictly positive if $\hat{c}(d^*, d^*) > \underline{c}$. We will argue below that we must have $\hat{c}(d^*, d^*) > \underline{c}$. First note that $\hat{c}(d, d)$ is strictly decreasing in $d$, from assumption 2. Now consider the bondholders lending to firm $i$, who earn

$$R^B(d,d) = \Pr(c \leq \hat{c}(d,d))d + \int_{\hat{c}(d,d)}^{\check{c}(d,d)} \pi^*(d,d,c) f(c) dc$$

if $d_1 = d_2 = d$. Using Leibniz's rule, the derivative of $R^B(d,d)$ with respect to $d$ can be written as

$$\begin{aligned}\frac{\partial R^B(d,d)}{\partial d} = \frac{\partial \Pr(c \leq \hat{c}(d,d))}{\partial d} d + \Pr(c \leq \hat{c}(d,d)) \\ + \pi^*(d,d,\check{c}(d,d)) f(\check{c}(d,d)) \frac{\partial \check{c}(d,d)}{\partial d} \\ - \pi^*(d,d,\hat{c}(d,d)) f(\hat{c}(d,d)) \frac{\partial \hat{c}(d,d)}{\partial d} \\ + \int_{\hat{c}(d,d)}^{\check{c}(d,d)} \frac{\partial \pi^*(d,d,c)}{\partial d} f(c) dc.\end{aligned}$$

From the definition of $\check{c}(d,d)$, the third term in this expression equals zero. Further, since $\pi^*(d,d,\hat{c}(d,d)) = d$ and because $f(c)$ is the probability density function of $c$, the first and fourth terms cancel out. Now, substituting $\hat{c}(d,d) = \underline{c}$, we have

$$\left.\frac{\partial R^B(d,d)}{\partial d}\right|_{\hat{c}(d,d)=\underline{c}} = \int_{\underline{c}}^{\check{c}(d,d)} \frac{\partial \pi^*(d,d,c)}{\partial d} f(c) dc < 0.$$

Thus, for $d^* = \arg\max_d R^B(d,d)$ we must have $\hat{c}(d^*, d^*) > \underline{c}$.

3. *In the auction stage, we cannot have an equilibrium* $(b_1, b_2)$ *such that* $b_1 < b^*$ *and* $b_2 \leq b^*$. — Consider one of the other bidders at the auction, say firm 3. Such a firm can outbid firm 1 by bidding $B_3 = b_1 + \varepsilon$, with $\varepsilon$ such that $B_3 \leq b^*$. From the previous step, we have that firm 3 can find financing for its bid. Also, it can make strictly positive profits when doing so. Hence, the original situation is not an equilibrium.

4. *Suppose that* $b_1 = b_2 = b^*$. *Then, when a firm bids some* $b > b^*$, *it is not possible for both firms to obtain financing.* — At $(b_1, b_2) = (b^*, b^*)$, we saw that there is a solution of the debt stage where both firms obtain $d = d^*$ and bondholders receive zero expected profits. Now consider the case in which $b_1 = b^*$ and $b_2 > b^*$. The conditions (5) imply that to be able to have financing for both firms in the new situation, we need to find a $(d_1, d_2)$ such that

$$R_1^B(d_1, d_2) \geq R^B(d^*, d^*) = b^*, \tag{6}$$

and

$$R_2^B(d_1, d_2) > R^B(d^*, d^*) = b^*. \tag{7}$$

The argument proceeds in the following steps:

   (a) *There is no such* $(d_1, d_2)$ *with either* $d_1 = d^*$ *or* $d_2 = d^*$. — Suppose $d_2 = d^*$. If $d_1 < d^*$, we have $R_1^B(d_1, d_2) < R_1^B(d^*, d^*)$ from lemma 2, contradicting (6). If $d_1 > d^*$, we have $R_2^B(d_1, d_2) < R_2^B(d^*, d^*)$ from lemma 1, contradicting (7). If $d_1 = d^*$, both (6) and (7) trivially hold with equality, thus ruling out the possibility $d_1 = d^*$. With the exact same arguments, we cannot have $d_2 = d^*$.

   (b) *There is no such* $(d_1, d_2)$ *with* $d_1 > d^*$ *and* $d_2 > d^*$. — Suppose $d_1 \geq d_2 > d^*$. From lemma 1, $R_2^B(d_1, d_2) \leq R_2^B(d_2, d_2)$. From assumption 3, $R_2^B(d_2, d_2) < R_2^B(d^*, d^*)$. Hence $R_2^B(d_1, d_2) < R_2^B(d^*, d^*)$, contradicting (7). With the same argument, the case $d_2 \geq d_1 > d^*$ contradicts (6).

   (c) *There is no such* $(d_1, d_2)$ *with* $d_1 < d^*$ *and* $d_2 < d^*$. — Suppose $d_1 \leq d_2 < d^*$. From lemma 2, $R_1^B(d_1, d_2) \leq R_1^B(d_2, d_2)$. From assumption 3, $R_1^B(d_2, d_2) <$

$R_1^B(d^*, d^*)$. Hence $R_1^B(d_1, d_2) < R_1^B(d^*, d^*)$, contradicting (6). With the same argument, the case $d_2 \leq d_1 < d^*$ contradicts (7).

(d) *There is no such* $(d_1, d_2)$ *with either* $d_1 < d^* < d_2$ *or* $d_2 < d^* < d_1$. — Consider the first possibility. From lemma 1, $R_1^B(d_1, d_2) < R_1^B(d_1, d^*)$. From lemma 2, $R_1^B(d_1, d^*) < R_1^B(d^*, d^*)$. Hence $R_1^B(d_1, d_2) < R_1^B(d^*, d^*)$, contradicting (6). With the same argument, the case $d_2 < d^* < d_1$ contradicts (7). This establishes the result.

■

Hence, we have a unique equilibrium. In that equilibrium, all firms submit the same bids, and firms that win the auction choose the same debt level. That debt level is the common debt level that maximizes expected repayment to the bondholders. Yet, bondholders profits are driven to zero. At the auction stage, firms will increase their bids as long as they are still able to obtain financing, that is, up to the point where expected profits of bondholders are zero.

Intuitively, this can be understood as follows. Suppose that in equilibrium, we have debt levels $d_1$ and $d_2$ that do *not* maximize bondholders' expected repayments. This cannot be an equilibrium. Two other firms are now able to offer a higher expected repayment to bondholders, which implies that they can also place a higher bid at the auction stage. These firms are also willing to place such a higher bid, since expected firm profits are strictly positive. Hence, in equilibrium, the expected repayment to bondholders is necessarily maximized. The equilibrium bids at the auction stage equal those maximum expected repayments, because of perfect competition in the debt market.

# 4 Implications

In this section we derive the main results of the model. In particular, we analyze equilibrium profits, and the consumer price level and the fees paid at the auction in the equilibrium of our model. We compare prices and fees to those in alternative setups, i.e. compare the

results from an auction with debt to those with a beauty contest or a standard auction (without external financing).

The first result is a corollary from the (proof of the) theorem presented in the previous section.

**Corollary 1** *Expected net profits of participants in the auction are strictly positive.*

This is despite the fact that there are more than 2 bidders, that all have the same information. Therefore, when bidders still make positive expected profits, one would expect the losing bidders to outbid them. That, however, is not the case in the equilibrium of our model, since we have credit rationing. Suppose one bidder would submit some bid larger than $b^*$. To finance such a bid, the debt this firm has to take on is necessarily higher than $d^*$. But more debt makes a firm more aggressive on the product market, which implies that the expected repayment to bondholders is lower. Debtholders thus have to provide more money, but face a lower expected repayment. Since expected profits of bondholders are zero with the contract $(b^*, d^*)$, this implies that they are negative for any $(b, d)$ with $b > b^*$ and $d > d^*$. Hence, any bidder outbidding $b^*$ will not be able to find financing, and equilibrium profits of firms are strictly positive.

To see why firm's expected equilibrium profits are strictly positive, note the following. Due to limited liability, firm profits can never be negative. Hence, when expected profits are zero, they have to be zero for every possible realization of marginal costs $c$. In such a situation, bondholders capture all the operating profits of the firm. But a lower debt level increases those operating profits, and hence the expected repayment. Thus, zero expected profits for firms cannot maximize the expected repayment to debtholders, which implies that it cannot be an equilibrium.

This can more easily be seen from Figure **??**. This figure describes a firm's gross profits, which are decreasing in the firm's debt level $d$. From gross profits, the amount $R^B$ flows to the bondholders. The firm is left with the remainder, i.e. the difference between gross profits and $R^B$. This difference is called $\Pi^F$ (the shaded area in the figure). Clearly, both $R^B$ and $\Pi^F$ can never exceed gross profits. But expected net firm profits $\Pi^F$ can equal

zero, if the debt level is sufficiently high. They do so when all (gross) profits flow to the bondholders (for any realization of $c$). Since gross profits are decreasing, $R^B$ is strictly concave, and $R^B$ cannot exceed the gross profits, this must occur at a tangency point of the two curves, and this tangency must be to the right of the maximum of $R^B$. This implies that at the maximum, which describes the equilibrium of our model, expected net firm profits $\Pi^F$ must exceed zero.

Note that expected net firm profits $\Pi^F$ equal zero for values of $d$ greater or equal to the one satisfying $\hat{c} = \underline{c}$. Note that if $d$ exceeds this value, the firm will earn zero profits anyway, and the firm's incentives are distorted. Therefore, for these values of $d$, the gross profit curve as well as the $R^B$ curve are not well defined. This is illustrated by drawing dashed lines instead of solid lines in the figure.

INSERT FIGURE 1 ABOUT HERE

Now we turn to consumer prices. We will first show that consumer prices are decreasing in the amount of debt of the firms competing on the market.

**Proposition 1** *In the competition subgame, consumer prices are decreasing in the debt levels.*

**Proof.** Totally differentiating the first-order-condition (FOC) of firm $i$ in the competition stage,

$$\frac{\partial \Pi_i^F}{\partial s_i} = 0,$$

yields

$$\frac{\partial^2 \Pi_i^F}{\partial s_i^2} ds_i + \frac{\partial^2 \Pi_i^F}{\partial s_i \partial s_j} ds_j + \frac{\partial^2 \Pi_i^F}{\partial s_i \partial d_i} dd_i = 0.$$

A similar equality can be given for firm $j$. This system of two equations can be solved using Cramer's rule to give

$$\frac{ds_i}{dd_i} = -\frac{\frac{\partial^2 \Pi_i^F}{\partial s_i \partial d_i} \frac{\partial^2 \Pi_j^F}{\partial s_j^2}}{H},$$

where

$$H \equiv \frac{\partial^2 \Pi_i^F}{\partial s_i^2}\frac{\partial^2 \Pi_j^F}{\partial s_j^2} - \frac{\partial^2 \Pi_i^F}{\partial s_i \partial s_j}\frac{\partial^2 \Pi_j^F}{\partial s_j \partial s_i}.$$

We assumed uniqueness and stability of the equilibrium of the competition stage. That requires

$$\left|\frac{\partial^2 \Pi_i^F}{\partial s_i \partial s_j}\right| < \left|\frac{\partial^2 \Pi_i^F}{\partial s_i^2}\right|$$

(see e.g. Tirole, 1988, p. 226), and a similar inequality for firm $j$. Thus, we have $H > 0$. Furthermore, from the second-order-conditions (SOCs), we must have $\partial^2 \Pi_i^F / \partial s_i^2 < 0$, and the same for firm $j$. The sign of $\frac{ds_i}{dd_i}$ is therefore the same as the sign of $\partial^2 \Pi_i^F / \partial s_i \partial d_i$.

In order to sign this expression, consider

$$\frac{\partial \Pi_i^F}{\partial s_i} = \int_{\underline{c}}^{\hat{c}_i(d_i,d_j)} \frac{\partial \pi_i}{\partial s_i}(s_i, s_j, c)\, f(c) dc = 0 \tag{8}$$

(from (3)). Taking the derivative with respect to $d_i$, we find

$$\frac{\partial^2 \Pi_i^F}{\partial s_i \partial d_i} = \frac{\partial \pi_i}{\partial s_i}(s_i, s_j, \hat{c}_i(d_i, d_j))\, f(\hat{c}_i(d_i, d_j)) \frac{\partial \hat{c}_i(d_i, d_j)}{\partial d_i}.$$

Clearly, $f(\hat{c}_i(d_i, d_j)) > 0$. Totally differentiating the expression that defines $\hat{c}_i$, $\pi_i^*(s_i, s_j, \hat{c}_i(d_i, d_j)) - d_i = 0$, yields

$$\frac{\partial \pi_i^*}{\partial c}(s_i, s_j, \hat{c}_i(d_i, d_j))\, d\hat{c}_i - dd_i = 0,$$

so $\partial \hat{c}_i(d_i, d_j)/\partial d_i = 1/\frac{\partial \pi_i^*}{\partial c}(s_i, s_j, \hat{c}_i(d_i, d_j)) < 0$. Finally, consider the sign of $\frac{\partial \pi_i}{\partial s_i}(s_i, s_j, \hat{c}_i(d_i, d_j))$. For this, we need the following result:

$$\frac{\partial^2 \Pi_i^F}{\partial s_i \partial c} \begin{cases} < 0 \text{ if } s_i = q_i \\ > 0 \text{ if } s_i = p_i \end{cases} \tag{9}$$

This can easily be seen from (1) by taking the derivative with respect to $c$ first, and then with respect to $s_i$. From (9), $\frac{\partial \pi_i}{\partial s_i}(s_i, s_j, \hat{c}_i(d_i, d_j))$ is decreasing in $c$ in case of quantity competition ($s_i = q_i$) and increasing in $c$ in case of price competition ($s_i = p_i$). Also, from the FOC (8, the integral over the interval $[\underline{c}, \hat{c}_i(d_i, d_j)]$ of this expression equals zero. Evaluated in the upper bound $\hat{c}_i(d_i, d_j)$, the expression must therefore be negative in case of quantity competition ($s_i = q_i$), but positive in case of price competition ($s_i = p_i$). This implies

$$\frac{ds_i}{dd_i} \begin{cases} < 0 \text{ if } s_i = q_i \\ > 0 \text{ if } s_i = p_i \end{cases}.$$

Combining this with downward sloping demand, we conclude that firm $i$'s price must be decreasing in the firm's debt level. ■

Intuitively, higher debt leads firms to focus on more beneficial situations, that is, situations with lower marginal cost. This is because if marginal cost is very high, the firms end up with zero anyway, due to limited liability. The higher the debt level of the firm, the larger the range of marginal cost for which the firm earns zero. So, higher debt implies a focus on lower marginal cost on average. When marginal cost is lower, firms set higher quantities (with quantity competition) or lower prices (with price competition). Thus, higher debt implies more aggressive competition, i.e. lower prices (Brander and Lewis, 1986).

Several interesting corollaries of this proposition can easily be derived. First, consider the case of a beauty contest. For sake of comparability, we continue to assume that the firms bidding at the auction have zero internal funds.

**Corollary 2** *A beauty contest leads to higher consumer prices than an auction does.*

**Proof.** This result immediately follows from the above proposition, using the fact that with a beauty contest, $d = 0$, whereas in the equilibrium of our model, $d = d^* > 0$. ■

In the equilibrium of our model the equilibrium debt level $d^*$ is strictly positive, whereas with a beauty contest licenses are given away for free and $d = 0$. Since firms compete more aggressively when they hold debt, consumer prices are lower when licenses are auctioned - and firms are forced to use debt - than when they are given away for free in a beauty contest.

Now suppose that we replace our auction stage with a mechanism in which government sets a take-it-or-leave-it fee $b$. Among all firms that are willing to pay that fee, government then randomly assigns the two licenses, and winning firms have to take on debt to finance $b$. Again, we continue to assume that the firms have zero internal funds. Note that a beauty contest is a special case of this mechanism, with $b = 0$. We now have the following.

**Corollary 3** *When the government sets some take-it-or-leave-it fee $b \leq b^*$ in the first stage, rather than having an auction, then consumer prices are decreasing in $b$.*

**Proof.** This follows from the above proposition if $dd/db > 0$. It can easily be seen that this is indeed the case. Recall that the firms present take-it-or-leave-it debt contracts to the bondholders. The firms can therefore maximize their net return $\Pi_i^F$. The bondholders will accept any contract on or below the curve $R^B(d,d)$. Since expected gross operating profits, denoted by $G$, are decreasing in $d$ by assumption, $\Pi_i^F = G - R^B(d,d)$ is maximized for the smallest $d$ satisfying $b = R^B(d,d)$, that is, the debt contract will always be on the increasing part of the curve $R^B(d,d)$. Thus, we have $dd/db > 0$. (See also Figure **??**.) ■

Note that our results depend crucially on the strictly positive debt levels that result from the stage in which the licenses are awarded. For comparison, we now discuss what happens when firms do have sufficient internal funds. We assume that firms have internal funds that are sufficiently large such that operating losses can never exceed these funds. That is, if operating profits fall below zero, this is a true loss to the firm.

**Corollary 4** *When firms have access to sufficient internal funds, license fees paid at an auction will be higher than when firms have to resort to external finance. Consumer prices in the former case will be at the same level as they are with a beauty contest. Firms make zero expected profits.*

**Proof.** In our model, equilibrium license fees are

$$b^* = \max_d \left\{ \Pr(c \leq \hat{c}(d,d))d + \int_{\hat{c}_i(d,d)}^{\check{c}_i(d,d)} \pi_i^*(d,d,c) f(c) dc \right\},$$

where the maximum is obtained for some $d^* > 0$. With internal funds, the license fees paid in equilibrium are (according to standard auction theory)

$$b^{int} = \int_{\underline{c}}^{\check{c}_i(0,0)} \pi_i^*(0,0,c) f(c) dc.$$

From Assumption 2 we have

$$\pi_i^*(0,0,c) > \pi_i^*(d^*,d^*,c).$$

Also, totally differentiating the expression that determines $\check{c}_i$ yields

$$\frac{\partial \check{c}_i(d_i,d_j)}{\partial d_i} = -\frac{\partial \pi_i^*/\partial c}{\partial \pi_i^*/\partial d_i}(s_i, s_j, \check{c}_i(d_i,d_j)) < 0.$$

From this, it can be seen that for each realization of $c$, the contribution to $b^*$ falls below that to $b^{int}$, and we conclude that $b^* < b^{int}$. This proves the first statement in the proposition. The third result follows directly from the set-up of the model: as all firms are identical, in a standard auction, all profits will be competed away. Since firms have no debt and bids in a standard auction are only sunk costs, the competition stage is not affected by the auction stage, which implies the second statement. ■

Clearly, with internal funds there is no strategic effect of debt, and the same price level results as with a beauty contest. In the auction the firms bid up to their expected profits $E(\pi^*)$ from competing on the output market. Without internal funds, we have $b^* = \max_d R^B(d,d)$. In order to compare these bid levels, first note that the strategic effect of debt (assumptions 1 and 2) implies that operating profits $\pi$ will be lower with external funds for any given realization of $c$. Now consider the components of $E(\pi^*)$ and $R^B(d,d)$. For small $c$, we integrate $d$ for the case with external funds, but $\pi > d$ for internal funds. For intermediate values of $c$, we integrate $\pi$ in both cases, but this term is higher for internal funds than it is for external funds. For somewhat larger $c$, firms get $\pi$ with internal funds but zero with external funds, and for very high $c$, they get zero in either case. Summing up, bids must be higher with internal funds than they are with external funds.

The intuition is straightforward. The strategic effect of debt means more intense competition with (more) debt, and therefore lower expected profits. Furthermore, in our model, the firms do not bid up to the level where they have zero expected profits. Instead, the bidding stops at the point where the bond market is just willing to finance the bids. Thus, with external finance firms will pay lower fees in the auction than they do with internal finance. Also, with internal finance, there is no strategic effect of debt and therefore prices are the same as with a beauty contest.

# 5 Examples

In this section, we give some numerical examples. We consider a simple Hotelling, Bertrand, and Cournot game and derive the equilibrium of our model for those cases. We show that the assumptions we made to derive our results are satisfied for these three modes of competition. For each model, we also derive the equilibrium on the auction, debt, and product markets for the case where licenses are given away for free in a beauty contest, and for the case in which an auction is held, but winning firms can finance their bids from internal funds. We illustrate the main results derived in the previous sections by comparing the results in subsection 5.4.

## 5.1 Hotelling

Assume that the products of the two winning firms are not seen as perfect substitutes. This can be modelled using a Hotelling line of length one, on which one firm is located at 0 and the other at 1. In this subsection, we index the winning firms by their location, so $i = 0, 1$. A mass of consumers, normalized to 1, is uniformly distributed on the line. Note that we can interpret location as taste rather than physical location. For simplicity, we normalize transportation costs per distance unit to 1. For a consumer located at $x$, the costs of purchasing from the firm located in 0 is given by $p_0 + x$, while the costs of purchasing from the firm in 1 is $p_1 + (1 - x)$, with $p_i$ the price charged by the firm located at $i$, $i = 0, 1$. The willingness to pay is $v$ for every consumer, with $v$ high enough so the market is always covered. Marginal costs $c$ will be drawn from a uniform distribution on $[0, 2]$, hence $f(c) = 1/2$ and $[\underline{c}, \bar{c}] = [0, 2]$.

The indifferent consumer $z$ is given by $z = \frac{1}{2}(1 + p_1 - p_0)$. Operating profits thus equal

$$\pi_i(p_i, p_j, c) = \frac{1}{2}(1 + p_j - p_i)(p_i - c). \tag{10}$$

$i, j = 0, 1$, $i \neq j$. Using these, we have

$$\hat{c}_i = p_i - \frac{2d_i}{1 + p_j - p_i}$$

and $\check{c}_i = p_i$. In stage 3, firm $i$s expected net operating profits equal

$$\Pi_i^F = \int_0^{\hat{c}_i} \left( \frac{1}{2}\left(1 + p_j - p_i\right)\left(p_i - c\right) - d_i \right) \frac{1}{2} dc.$$

Plugging in $\hat{c}_i$ and $\pi_i(p_i, p_j, c)$,

$$\Pi_i^F(p_i, p_j, d_i, d_j) = \frac{\left(p_i\left(1 + p_j - p_i\right) - 2d_i\right)^2}{8\left(1 + p_j - p_i\right)}. \tag{11}$$

Note that in equilibrium the debt a firm takes on can never be higher than the maximum operating profits it can make (substituting $c = 0$ and $d = 0$), hence in equilibrium we have $d < \frac{1}{2}$. Taking the first-order condition of (11) yields four possible solutions for $p_i$:

$$\begin{aligned}
p_i^1 &= \frac{1}{2}\left(1 + p_j\right) + \frac{1}{2}\sqrt{\left(1 + p_j\right)^2 - 8d_i}, \\
p_i^2 &= \frac{1}{2}\left(1 + p_j\right) - \frac{1}{2}\sqrt{\left(1 + p_j\right)^2 - 8d_i}, \\
p_i^3 &= \frac{5}{6}\left(1 + p_j\right) + \frac{1}{6}\sqrt{\left(1 + p_j\right)^2 + 24d_i}, \\
p_i^4 &= \frac{5}{6}\left(1 + p_j\right) - \frac{1}{6}\sqrt{\left(1 + p_j\right)^2 + 24d_i}.
\end{aligned} \tag{12}$$

Yet, plugging either $p_i^1$ and $p_i^2$ back into the numerator of (11) yields zero profits, which implies that these roots are not feasible. Note also that $1 + p_j - p_i^3 = \frac{1}{6}\left(1 + p_j\right) - \frac{1}{6}\sqrt{\left(1 + p_j\right)^2 + 24d_i}$, which implies that when using $p_i^3$ the denominator of (11) becomes negative, which implies negative profits. Therefore, $p_i^4$ is the only relevant solution. It is not possible to find a clean analytical solution for equilibrium prices for general values of $d_i$ and $d_j$. Suppose both firms have the same level of debt $d$. We can then solve for equilibrium prices to find

$$p^*(d, d) = 2 - 2d. \tag{13}$$

It can be shown that the second-order conditions for the firms' problem at the competition stage are satisfied for any $d < \frac{1}{2}$.

Note that the reaction functions (12) are increasing in the rival's price. Suppose $d_1$ increases. Then, the reaction curve of firm 1 shifts inwards, while that of firm 0 is unaffected. Equilibrium prices of both firms then decrease. Yet, the equilibrium price of firm 1 decreases by more than that of firm 0, since reaction curves have a slope that is necessarily

smaller than 1. But that implies that both price and market share of firm 0 decrease - the latter being the case since the total size of the market is fixed, and the price of firm 1 decreases by more than that of firm 0. Thus, equilibrium operating profits of firm 0 are decreasing in the debt level of firm 1, so assumption 1 is satisfied.

Plugging (13) back into (10), we have

$$\pi_i^*(d,d,c) = 1 - d - \frac{1}{2}c.$$

This implies that $\pi_i^*(d,d,c)$ is decreasing in $d$, and assumption 2 is satisfied.

With a common debt level $d$, we have $\hat{c}_0 = \hat{c}_1 = 2 - 4d$ and $\check{c}_0 = \check{c}_1 = 2 - 2d$. Hence,

$$\begin{aligned} R^B(d,d) &= \frac{1}{2}(2-4d)d + \int_{2-4d}^{2-2d} \frac{1}{2}\left(1 - d - \frac{1}{2}c\right) dc \\ &= d - \frac{3}{2}d^2. \end{aligned}$$

Thus, $R^B(d,d)$ is strictly concave in $d$ on the relevant interval, and $d^* \equiv \arg\max_d R^B(d,d) = \frac{1}{3} > 0$. Therefore, assumption 3 is also satisfied, which implies that we can apply theorem 1. In this case, the unique equilibrium thus has $d^* = \frac{1}{3}$, $b^* = R^B(d^*,d^*) = \frac{1}{6}$, and $p^* = p^*(\frac{1}{3},\frac{1}{3}) = \frac{4}{3}$. Firms earn strictly positive expected profits, which equal $\frac{1}{18} \simeq 0.055556$.

## 5.2 Bertrand

Now consider a model of Bertrand competition among two firms, $i = 1,2$, producing heterogeneous goods. Inverse demand for firm $i$'s product is given by

$$p_i = 1 - q_i - \theta q_j,$$

for $i,j = 1,2$, $i \neq j$, where $q_i$ is the quantity sold by firm $i$ and $\theta \in (0,1)$ is a parameter measuring the degree of product heterogeneity. With $\theta$ close to 1 we have relatively homogeneous goods. When $\theta$ is close to 0 the products are strongly differentiated. Demand can be written in direct form as

$$q_i = \frac{1}{1+\theta}\left(1 + \frac{\theta}{1-\theta}p_j - \frac{1}{1-\theta}p_i\right).$$

Assume that marginal costs $c$ will be drawn from a uniform distribution on $[0,1]$, hence $f(c)=1$ on $[\underline{c},\bar{c}]=[0,1]$. Operating profits of firm $i$ are given by

$$\pi_i(p_i,p_j,c)=\frac{1}{1+\theta}\left(1+\frac{\theta}{1-\theta}p_j-\frac{1}{1-\theta}p_i\right)(p_i-c). \tag{14}$$

From this expression,

$$\hat{c}_i=p_i-\frac{(1-\theta^2)\,d_i}{1-\theta+\theta p_j-p_i}$$

and $\check{c}_i=p_i$. Using this, we find

$$\Pi_i^F=\frac{\left(p_i\left(1-\theta+\theta p_j-p_i\right)-\left(1-\theta^2\right)d_i\right)^2}{2\left(1-\theta^2\right)\left(1-\theta+\theta p_j-p_i\right)}.$$

Again, taking the first-order condition yields four possible solutions, and using the same strategy as in the case of Hotelling competition, we can eliminate three. The relevant solution is

$$p_i=\frac{5}{6}\left(1-\theta+\theta p_j\right)+\frac{1}{6}\sqrt{\left(1-\theta+\theta p_j\right)^2+12\left(1-\theta^2\right)d_i}. \tag{15}$$

As before, it is not possible to find a clean analytical solution for equilibrium prices. Consider the case in which both firms face the same debt $d$. We then have

$$p^*(d,d)=\frac{1}{6-4\theta}\left(5-4\theta-\sqrt{\left(1+4d\left(1+\theta\right)\left(3-2\theta\right)\right)}\right). \tag{16}$$

Again, the reaction functions are increasing, and an increase in $d_j$ shifts inwards the reaction curve for firm $j$. This decreases both $p_i$ and $p_j$, but $p_j$ decreases by more. Note that the slope of the reaction function (15) is smaller than $\theta$, so the change in $p_j$ is larger in absolute value than $\frac{1}{\theta}$ times the change in $p_i$. Using (14) this implies that $\pi_i^*\left(d_i,d_j,c\right)$ decreases. This implies that $\pi_i^*\left(d_i,d_j,c\right)$ is strictly decreasing in $d_j$, and assumption 1 is satisfied. Further,

$$\pi^*(d,d,c)=\frac{2d\left(1+\theta\right)\left(-3+2\theta\right)+\left(2-2\theta-3c+2c\theta\right)\left(1+\sqrt{\left(1+12d+4d\theta-8d\theta^2\right)}\right)}{2\left(3-2\theta\right)^2\left(1+\theta\right)}.$$

It can be verified that for $\theta$ not too small, this is strictly decreasing in $d$. As an example, consider $\theta=\frac{1}{2}$; then

$$\pi^*(d,d,c)=\frac{1}{24}\left(1+\sqrt{\left(1+12d\right)}\right)\left(3-\sqrt{\left(1+12d\right)}-4c\right)$$

is strictly decreasing in $d$. This confirms assumption 2 for $\theta = \frac{1}{2}$.

We also have:

$$R^B(d,d) = \frac{2(1-\theta)}{(3-2\theta)}d - \frac{(5+3\theta-2\theta^2)\,d^2}{1+\sqrt{(1+4d(1+\theta)(3-2\theta))}}$$

It can be verified that $R^B(d,d)$ is strictly concave in $d$, hence assumption 3 is satisfied. With $\theta = \frac{1}{2}$, we have $d^* = \frac{1}{54} + \frac{1}{27}\sqrt{7} \simeq 0.11651$, $b^* = R^B(d^*,d^*) = \frac{25\sqrt{7}-1}{324\left(5+\sqrt{7}\right)} \simeq 0.026297$, and $p^* = \frac{7}{12} - \frac{1}{12}\sqrt{7} \simeq 0.36285$. Firms' expected net profits are $\Pi^F(d^*,d^*) = \frac{1}{144}\frac{\left(\sqrt{7}-4\right)^2}{5+\sqrt{7}} \simeq 1.6658 \times 10^{-3} > 0$.

## 5.3 Cournot

Finally, we consider quantity competition among the two firms, $i = 1, 2$. Suppose inverse demand is

$$p = 1 - q_1 - q_2.$$

Again, we assume $f(c) = 1$ on [0,1]. Operating profits then are

$$\pi_i(p_i, p_j, c) = (1 - q_i - q_j - c)\,q_i \tag{17}$$

$i, j = 1, 2$, $i \neq j$. From this expression,

$$\hat{c}_i = 1 - q_i - q_j - \frac{d_i}{q_i}$$

and $\check{c}_i = 1 - q_i - q_j$. Using this, we find

$$\Pi_i^F = \frac{1}{2q_i}\left(q_i\left(1 - q_i - q_j\right) - d_i\right)^2.$$

Again, taking the first-order condition yields for possible solutions, three of which can be eliminated. The relevant solution is

$$q_i = \frac{1}{6} - \frac{1}{6}q_j + \frac{1}{6}\sqrt{\left(1 - 2q_j + q_j^2 + 12d_i\right)}. \tag{18}$$

When firms face the same debt $d$, we have

$$q^*(d,d) = \frac{1}{8} + \frac{1}{8}\sqrt{(1+16d)}. \tag{19}$$

Reaction functions (18) are now decreasing, and an increase in $d_j$ shifts the reaction curve for firm $j$ outwards. This increases $q_j$ but decreases $q_i$. The slope of the reaction function (18) exceeds $-1$, so the change in $q_j$ is larger than that in $q_i$ (in absolute value). Using (17) this implies that $\pi_i^*(d_i, d_j, c)$ decreases with $d_j$, so assumption 1 is satisfied. Further,

$$\pi^*(d,d,c) = \frac{1}{32}\left(3 - \sqrt{(1+16d)} - 4c\right)\left(1 + \sqrt{(1+16d)}\right).$$

This expression is strictly decreasing in $d$, confirming assumption 2. Also, we can derive

$$R^B(d,d) = \frac{1}{2}d\left(1 - \frac{16d}{1+\sqrt{(1+16d)}}\right),$$

which is strictly concave in $d$. Further, $d^* = \arg\max_d R^B(d,d) = \frac{1}{72} + \frac{1}{36}\sqrt{7} \simeq 0.087382 > 0$.

In the equilibrium of the game with Cournot competition in the third stage, we have $b^* = R^B(d^*, d^*) = \frac{25\sqrt{7}-1}{432\left(5+\sqrt{7}\right)} \simeq 0.019723$; $d^* = \frac{1}{72} + \frac{1}{36}\sqrt{7} \simeq 0.087382$; and $p^* = \frac{7}{12} - \frac{1}{12}\sqrt{7} \simeq 0.36285$. Firms' expected net profits are $\Pi^F(d^*, d^*) = \frac{1}{192}\frac{\left(\sqrt{7}-4\right)^2}{5+\sqrt{7}} \simeq 1.2493 \times 10^{-3} > 0$.

## 5.4 Summary of examples

In the three examples above, we derived the equilibrium of the game described in section 2 where in the third stage firms compete according to the Hotelling, Bertrand, and Cournot models. Now, we compare the results in different setups.

Recall that in our model, the two firms competing on the output market have obtained a license at a sealed-bid auction, and they have used debt to finance their winning bids. Alternatively, we could consider a beauty contest, in which licenses are simply assigned to two firms, and no fee is involved. In that case, firms have no debt. We also assume that there is no limited liability in this case. That is, for a given price level, if the realization of marginal cost turns out to be high, firms may have strictly negative profits. By maximizing expected operating profits

$$E(\pi) = \int_{\underline{c}}^{\bar{c}} \pi_i(s_i, s_j, c) f(c) dc,$$

we find that the Hotelling model results in $p^* = 2$. Both the Bertrand model with $\theta = \frac{1}{2}$ and the Cournot model yield $p^* = \frac{2}{3}$.

However, also with a beauty contest, firm profits may turn out to be negative, when firms behave aggressively and $c$ turns out to be high. Even though the firm does not have to take on debt, it still faces the possibility of having negative operating profits. Therefore, we also study the case of a beauty contest with limited liability. In that case, firms maximize

$$E(\pi) = \int_{\underline{c}}^{\check{c}(0,0)} \pi_i(s_i, s_j, c) f(c) dc.$$

An alternative setup is a license auction where firms do have internal funds to pay their license fees. This is the case usually studied in auction theory. Again we assume that there is no limited liability, or alternatively, that firms have sufficient internal funds. In this setup, $d = 0$ as well, and equilibrium prices will be the same as with a beauty contest. However, in equilibrium, firms do have to pay a license fee now, equal to their winning bids in the auction. As predicted by standard auction theory, the firms will bid up to their expected profits from competing on the output market. For the Hotelling model, using $p^* = 2$, we have $b^* = E(\pi^*) = \frac{1}{2}$. For the Bertrand model with $\theta = \frac{1}{2}$, $b^* = \frac{1}{27}$, and for the Cournot model we find $b^* = \frac{1}{36}$.

Table 1 summarizes these results, illustrating that the main results (in particular, the corollaries) presented in the previous sections indeed hold in our three examples.

# 6 Discussion and conclusion

We have shown that license auctions when winning bids are financed through debt lead to different outcomes than standard auction theory predicts. At least in our framework there may be a negative relation between consumer prices and the fees paid. Thus, higher fees may imply lower prices for consumers. Further, we argued that when firms use external funds to finance licenses, both the fees and the resulting consumer price are lower than with internal funds. These results are driven by the strategic effect of debt, or more precisely, the strategic effect of limited liability. If firms pay higher fees to obtain their license, they need more debt and compete more aggressively by setting lower prices. Also, we have shown that with debt financing, the winners of the auction have strictly positive expected

| Setup | $b^*$ | $d^*$ | $p^*$ |
|---|---|---|---|
| Hotelling | | | |
| - beauty contest | - | - | 2 |
| - with internal funds | - | - | 2 |
| - auction with internal funds | 1/2 | - | 2 |
| - auction with debt | 1/6 | 1/3 | 4/3 |
| | | | |
| Bertrand ($\theta = \frac{1}{2}$) | | | |
| - beauty contest | - | - | 2/3 |
| - with internal funds | - | - | 1/2 |
| - auction with internal funds | $1/27 \simeq 0.037$ | - | 2/3 |
| - auction with debt | $\frac{25\sqrt{7}-1}{324\left(5+\sqrt{7}\right)}$ | $\frac{1}{54}+\frac{1}{27}\sqrt{7}$ | $\frac{7}{12}-\frac{1}{12}\sqrt{7}$ |
| | $\simeq 0.026$ | $\simeq 0.117$ | $\simeq 0.363$ |
| Cournot | | | |
| - beauty contest | - | - | 2/3 |
| with internal funds | - | - | 1/2 |
| - auction with internal funds | $1/36 \simeq 0.028$ | - | 2/3 |
| - auction with debt | $\frac{25\sqrt{7}-1}{432\left(5+\sqrt{7}\right)}$ | $\frac{1}{72}+\frac{1}{36}\sqrt{7}$ | $\frac{7}{12}-\frac{1}{12}\sqrt{7}$ |
| | $\simeq 0.020$ | $\simeq 0.087$ | $\simeq 0.363$ |

Table 1: Equilibrium bids, debts, and prices when firms compete on the output market according to the Hotelling, Bertrand, or Cournot model and licenses are allocated using a beauty contest or an auction.

profits. They are simply not able to bid up to the point where their bids equal the expected payoff from competition, because of credit rationing.

These results suggest that in deciding whether or not to auction, and in auction design, it is important to realize how winners will finance their bids. When external finance is used, results from standard auction theory, implicitly based on internal finance, do not necessarily apply. Winning bids and consumer prices are lower, and expected net profits for winners are strictly positive. However, it is not straightforward to see what this implies for social welfare. Prices are lower in equilibrium, but the probability that firms go bankrupt increases.

In our model, we made a number of simplifying assumptions, for example with respect to the uncertainty that winning bidders face on the product market. We assumed that this uncertainty concerns the level of marginal cost. One may argue that uncertainty is more

likely to concern fixed costs rather than marginal costs. However, our model can easily be adapted to address this type of uncertainty, without affecting the qualitative result. We only need that the uncertainty about fixed costs is not resolved when firms set their strategic variables in the competition stage. For our results, it is necessary that more debt makes firms more aggressive. Alternatively, there may be uncertainty about demand. This, however, could change the results of our model. Showalter (1995) shows that in this case, and with price competition, the strategic effect of debt implies that higher debt leads to higher prices rather than lower prices.

Our model can be extended in a number of ways. One straightforward extension concerns the number of licenses that is being sold. In our model, $N > 2$ potential entrants compete for 2 licenses. Alternatively, they could compete for $n$ licenses, with $2 \leq n \leq N$. On the output market, $n$ firms would then compete. In this setup, there would still be a strategic effect of debt. However, an increase in $n$ will imply a decrease in the bids because of lower expected operating profits, and thus in the debt level. Thus, an increase in $n$ weakens the strategic effects of debt which may decrease firms' equilibrium profits.

It would also be interesting to look at asymmetries in the amount of internal funds that firms have, for example by looking at a case in which some firms are able to fully finance their bids on through internal funds, whereas other firms have to finance their entire bid on the credit market. One possible interpretation of such a scenario is that incumbent firms have their own funds, whereas potential entrants need debt financing.

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Figure 1.