Far-Field Emission Pattern of a Dielectric Circular Microresonator with a Point Scatterer

Dettmann, C.P.; Morozov, G.V.; Sieber, M.; Waalkens, H.

Far-Field Emission Pattern of a Dielectric Circular Microresonator with a Point Scatterer

C. P. Dettmann, G. V. Morozov*, M. Sieber, H. Waalkens
Department of Mathematics, University of Bristol, Bristol BS8 1TW, United Kingdom
*Tel: (44-117) 331 1664, Fax: (44-117) 928 7999, e-mail: g.morozov@bristol.ac.uk

ABSTRACT
The far-field emission pattern of a two-dimensional circular microresonator with a point scatterer inside, at some distance away from the centre, is investigated theoretically. We demonstrate that the presence of the scatterer leads to significant enhancement in the directionality of the outgoing light in comparison with whispering gallery modes of a circular resonator without scatterer. Numerical results show that the highly directional modes are observed in various frequency regions depending on the position and strength of the scatterer.
Keywords: two-dimensional circular microresonator, point scatterer, directional modes.

1. INTRODUCTION

Natural low threshold characteristics of typical dielectric microdisk resonators [1, 2] make them a good alternative to the Fabry-Perot cavities in which two parallel mirrors reflect the light back and forth through the active material. Microdisk cavities naturally utilize total internal reflection of light to achieve a mirror reflectivity near unity, while in Fabry-Perot cavities such a high reflectivity requires fabrication of very expensive and relatively thick multilayered optical mirrors whose alignment and binding is another rather difficult task.

Lasers based on microdisk cavities lase on whispering gallery modes of the electromagnetic field. In such a mode, light circulates around the circumference of the microdisk, reflecting from the boundary of the disk with an angle of incidence always greater than the critical angle of total internal reflection, i.e. it remains trapped inside the microresonator. As a result, there are only small losses of radiation caused by the tunnelling effect (evanescent leakage) and by the surface roughness of the disk walls that is a fundamental limitation for currently available etching techniques. Therefore, low cavity losses, small size, and relative ease of fabrication are among main advantages of two-dimensional microdisk resonators [3, 4].

However, a serious drawback of the microdisk geometry is that the laser emission is isotropic in the plane of the disk. A promising solution for improving directionality of the light output is to deform the shape of the microdisk [5, 6]. This solution utilizes the fact that deformation causes a significant change in the light-ray dynamics so that the light eventually impinges on the disk boundary at an angle smaller than the critical one. This automatically leads to a directional refractive escape but typically spoils the threshold characteristics.

In this paper, we propose a simpler method to significantly improve directionality of the modes of conventional passive microdisk resonators. We place a point scatterer at the distance d from the disk centre into the inner region $(d < R)$ of the microdisk of the radius R. It turns out that such geometry improves the emission directionality of microdisk modes for a wide range of frequencies, especially in a visible spectral range.

2. THEORY OF MICRODISKS WITH POINT SCATTERERS

For zero axial momentum EM field, i.e. for the waves with $k_z = 0$, where z is perpendicular to the disk plane (x, y), a thin microdisk can be modelled as just a two-dimensional dielectric disk of radius R, with the effective refractive index $n_{\text{eff}}(r) = n$. This index is slightly different from the actual refractive index of the microdisk material due to the disk thickness (typically 1 to 2 wavelengths). For such a model, Maxwell's equations are reduced to two scalar Helmholtz equations corresponding to TM and TE polarizations, respectively. We should note that in polar coordinates (r, φ) each TM mode of the microdisk without scatterer is characterized by an azimuthal quantum number $m = 0, \pm 1, \pm 2, ...$ and a radial quantum number $q = 1, 2, 3, ...$.

In this paper we consider only TM modes. The electric field of such a mode (resonance) is of the form $\mathbf{E} = E_z(\mathbf{r}, k_{\text{res}})\hat{\mathbf{z}} \equiv E_z(x, y, k_{\text{res}})\hat{\mathbf{z}}$, where $k_{\text{res}} = \omega_{\text{res}}/c$ is the resonance complex wavenumber. One method for solving problems with a point scatterer is based on self-adjoint extension theory [7]. According to this theory, the resonance wave numbers k_{res} can be found from the equation

$$0 = 1 - \lambda G_{\text{reg}}(\mathbf{d}, \mathbf{d}, k_{\text{res}})\qquad\qquad(1)$$

where $\mathbf{d}$ is the position of the point scatterer, λ is its coupling strength, and $G(\mathbf{d},\mathbf{d},k_{res})$ is the regularized Green's function of the operator $\left(\Delta + k_{res}^2 n^2(r)\right)$ at the point $(\mathbf{d},\mathbf{d})$. This function is obtained from the unregularized Green's function by subtracting the term $\ln\left(k_0 |\mathbf{r}-\mathbf{d}|\right)/2\pi$ with k_0 being an arbitrary constant. The unregularized function satisfies the standard equation

$$\left[\Delta + k_{res}^2 n^2(r)\right] G(\mathbf{r},\mathbf{d},k_{res}) = \delta(\mathbf{r}-\mathbf{d}), \qquad n(r) = \begin{cases} n, & r < R \\ 1, & r > R \end{cases} \tag{2}$$

with appropriate boundary conditions. In particular, those conditions for open disk resonators require $G(\mathbf{r},\mathbf{d},k_{res})$ to be finite at $r = 0$, to be outgoing wave for $r \to \infty$, and to be smooth at $r = R$. Removing the singularity of the unregularized Green's function at the point $(\mathbf{d},\mathbf{d})$ and introducing the new coupling parameter a, which is defined by $2\pi/\lambda \equiv -\ln(k_0 a)$, we obtain that equation (1) takes the form

$$0 = \frac{i\pi}{2} - \ln\left(\frac{k_{res} na}{2}\right) - \gamma + \frac{i\pi}{2} \sum_{m=-\infty}^{m=+\infty} \frac{H_m(k_{res} nR)H_m'(k_{res} R) - n H_m'(k_{res} nR)H_m(k_{res} R)}{n H_m(k_{res} nR)J_m'(k_{res} nR) - H_m'(k_{res} R)J_m(k_{res} nR)} J_m^2(k_{res} nd) \tag{3}$$

where J_m, H_m are Bessel and Hankel functions of the first kind and $\gamma \cong 0.5772$ is the Euler-Mascheroni constant. The electric field of the corresponding resonance mode is given by

$$E_z(\mathbf{r},k_{res}) = N G(\mathbf{r},\mathbf{d},k_{res}) \tag{4}$$

where $G(\mathbf{r},\mathbf{d},k_{res})$ is the unregularized Green's function from the equation (2), N is a normalization factor.

3. FAR-FIELD EMISSION

In order to quantify the far-field directionality of the electric field, we consider its asymptotic behaviour for $r \to \infty$ which has the form

$$E_z(\mathbf{r},k_{res}) = E_z(r,\varphi,k_{res}) \propto \frac{\exp(ik_{res} r)}{\sqrt{r}} f(\varphi) \tag{5}$$

To characterize the directionality we compute the normalized variance of the far-field intensity

$$\Delta_{|f|^2} = \int_0^{2\pi} |f(\varphi)|^4 d\varphi \Bigg/ \left(\int_0^{2\pi} |f(\varphi)|^2 d\varphi\right)^2 - 1 \tag{6}$$

From this definition it follows that $\Delta_{|f|^2} = 0$ and $\Delta_{|f|^2} = 0.5$ for the resonant TM modes of the microdisk without scatterer which have $m = 0$ and $m \neq 0$ respectively. To illustrate the emission directionality of the microdisk with a scatterer we consider a GaAs microdisk of effective refractive index $n = 3$ and radius $R = 1\,\mu m$ [8] with a point scatterer placed at the distance $d = 0.495\,\mu m$. The complex wave numbers of corresponding resonant modes, k_{res}, can be found from the equation (3) if we vary the coupling strength a from 0 to ∞. We are interested in the modes that have both high directionality and low threshold characteristics (high Q-factors). In near infrared range of frequencies one possibility is the mode $k_{res} R = 2.0571 - 0.0164 i$, corresponding to the coupling strength $a \approx 0.754$. It has $\Delta_{|f|^2} = 2.12$ and $Q = 251$. In Fig. 1 we compare the function $|f(\varphi)|^2$ for this mode with two resonant TM modes of the microdisk without scatterer. In a spectral range of green light, the resonant mode $k_{res} R = 12.5513 - 0.0016 i$, corresponding to $a \approx 0.003$, has both extremely high directionality $\Delta_{|f|^2} = 4.71$ and a very high factor $Q = 15700$. In Fig. 2 we compare the function $|f(\varphi)|^2$ for this mode with the resonant mode $m = 21$, $q = 4$ [the closest one in terms of $\mathrm{Re}(kR)$] of the microdisk without scatterer.
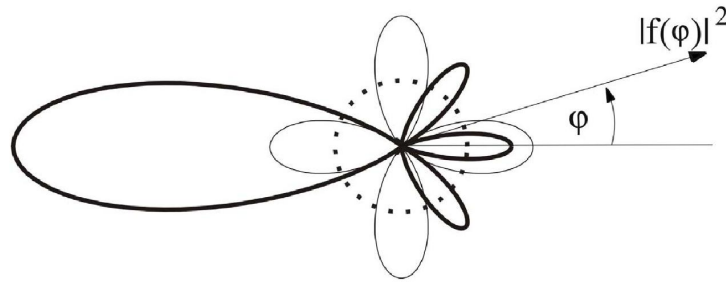
Figure 1. Polar plot of the far-field intensity $\left|f(\varphi)\right|^2$ for the two TM modes of the disk without scatterer: $m = 0, q = 1$ (dashed circle), $m = 2, q = 2$ (thin solid line), and for the high directional TM mode $k_{res}R = 2.0571 - 0.0164i$ of the disk with the scatterer (thick solid line).

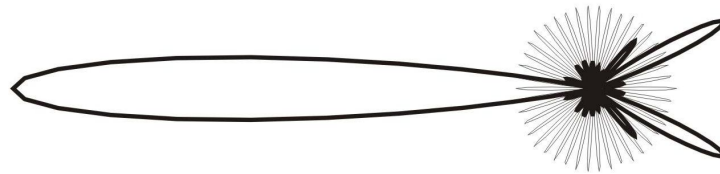*Figure 2. Polar plot of the far-field intensity $\left|f(\varphi)\right|^2$ for the TM mode, $m = 21$, $q = 4$, of the disk without scatterer (thin solid line) and for the high directional TM mode $k_{res}R = 12.5513 - 0.0016i$ of the disk with the scatterer (thick solid line).*

4. CONCLUSIONS

In summary, we demonstrated the existence of highly directional TM-modes in the emission spectrum of a two-dimensional passive microdisk cavity with a point scatterer. These modes can appear even for the scatterer with a very weak coupling constant which promises the feasibility of an experimental realization of such cavities. It would be interesting and potentially very useful to get a deeper insight into the output directionality by relating the resonant modes to the underlying ray dynamics in the semiclassical limit.

REFERENCES

[1]   S. L. McCall, A. F. J. Levi, R. E. Slusher, S. J. Pearton, and R. A. Logan, Whispering-gallery mode microdisk lasers, *Appl. Phys. Lett.*, vol. 60, pp. 289-291, January 1992.

[2]   R. E. Slusher, A. F. J. Levi, U. Mohideen, S. L. McCall, S. J. Pearton, and R. A. Logan: Threshold characteristics of semiconductor microdisk lasers, *Appl. Phys. Lett.*, vol. 63, pp. 1310-1312, September 1993.

[3]   K. J. Vahala: Optical microcavities, *Nature*, vol. 424, pp. 839 -845, August 2003.

[4]   V. S. Ilchenko and A. B. Matsko: Optical resonators with whispering gallery modes – part II: applications, *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 12. pp. 15-22, January/February 2006.

[5]   J. U. Nöckel and A.D. Stone: Ray and wave chaos in asymmetric resonant optical cavities, *Nature*, vol. 385, pp. 45-47, January 1997.

[6]   C. Gmachl, F. Capasso, E. E. Narimanov, J. U. Nöckel, A. D. Stone, J. Faist, D. L. Sivco, and A. Y. Cho: High-power directional emission from microlasers with chaotic resonators, *Science*, vol. 280, pp. 1556-1563, June 1998.

[7]   T. Shigehara: Conditions for the appearance of wave chaos in quantum singular systems with a pointlike scatterer, *Phys. Rev. E*, vol. 50, pp. 4357-4370, December 1994.

[8]   E. Peter, P. Senellart, D. Martrou, A. Lemaître, J. Hours, J. M. Gérald and J. Bloch: Exciton-photon strong-coupling regime for a single quantum dot embedded in a microcavity, *Phys. Rev. Lett.*, vol. 95, 067401, August 2005.