Preconditioning of ocean model equations

Wubs, F.; Niet, A. de; Thies, J.

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5.43 Preconditioning of ocean model equations - F. Wubs

Co-authored by:
F. Wubs [1] A. de Niet [2] J. Thies [3]

Challenge Our climate is largely determined by the global ocean flow, which is driven by wind, and gradients in temperature and salinity. Nowadays numerical models exist that are able to describe the occurring phenomena not only qualitatively but also quantitatively. For the latter, measurements are used to calibrate the parameters in the model. With such a model we want to study whether under the current conditions there exist multiple solutions in the Atlantic ocean and whether a transition can occur. Such a transition will cause a collapse of the Warm Gulf stream and dampen the increase of temperature in northern Europe due to emission of green house gasses. To study such stability questions we use continuation of steady states as a function of the forcing, for instance by increasing the amount of melt water entering the ocean. This dynamical systems approach to the study of stability of ocean flows appeared to be very fruitful [4, 5, 7]. The numerical side of this approach is that large linear systems have to be solved, for which we use Krylov subspace methods combined with preconditioning. Of course better predictions can be made if grids with high resolution are used, which poses severe demands on nowadays computers and solvers. In this contribution we will discuss a special purpose solver.

Model The ocean flow is modelled by a simplified form of the 3D Navier-Stokes equations including the Coriolis force. These are extended with an equation for temperature and one for salinity. In fact a Boussinesq form of the Navier-Stokes equations is used to model the varying density due to temperature and salinity gradients. For the vertical momentum equation the hydrostatic assumption is used. The surface is assumed to be a rigid lid. At this surface we have forcing of salinity due to evaporation and precipitation, of temperature due to solar heating and of momentum due to wind. At all other domain boundaries the normal velocity is zero and there is flux of neither heat nor salinity.

In horizontal directions, these equations are discretized on an Arakawa B-grid and in the vertical on a C-grid, which is related to the importance of the Coriolis force in horizontal directions [8].

[1] University of Groningen, Institute of Mathematics and Computing Science (IWI) P.O. Box 800, 9700 AV Groningen, The Netherlands, Email: F.W.Wubs@math.rug.nl
[2] University of Groningen, Email: A.C.de.Niet@math.rug.nl
[3] University of Groningen, Email: J.Thies@math.rug.nl

Approach To solve these equations we first restructure the system such that we can make a block incomplete LU factorization of it in which a number of smaller systems are left to be solved, some of which are standard and others are not. We discuss two which needed more attention.

The first system is a saddle-point problem with a 3D velocity field for the horizontal velocities and a 2D pressure field. The Coriolis force contributes a strong skew symmetry to the matrix, which precludes the use of standard saddle-point solvers. Where this matrix comes from is easier described in the continuous case. We split the pressure in a part that depends only on the horizontal coordinates by integrating it over the vertical and a remaining part of which the vertical integral vanishes. Furthermore by integrating the continuity equation in the vertical the vertical velocity drops out.

For the solution of this system we tried several approaches. It turned out that a solver based on artificial compressibility and a modified simpler approach performed best [2]. For the solution of the systems occurring in these approaches we use MRILU [1] and solvers from Trilinos [6].

The second non standard system is one which contains both advection-diffusion and an interaction between flow and gradients in temperature and salinity. The associated matrix occurs as a Schur complement in the factorization and will be full; therefore it is not computed, though application to a vector is possible.

For the solution of this system we also have two approaches. In the first we forget about the interaction mentioned above and just solve an advection-diffusion system. This in fact means that the block ILU preconditioner becomes a block Gauss-Seidel preconditioner. In the second approach we use just an incomplete factorization of the advection diffusion equation using MRILU for the Schur complement.

Since we have inner iterations in our preconditioning we have to use a Krylov subspace variant that can handle a varying preconditioning. For that we use FGMRES (in sequential version) and GMRESR (in parallel version). The sequential version is partially described in [3].

Parallelization For the parallelization of the continuation process we employed Trilinos. We used domain decomposition with two layers of overlap, which are needed for the discretization. The continuation is performed by LOCA and we combined the above block ILU factorization with AztecOO (Krylov subspace methods), Ifpack (incomplete sparse matrix factorizations) and ML (multilevel methods).

Gain Until one year ago we solved the system at once by MRILU and it was used many years as the workhorse. However, the drawback of MRILU for this system is that it takes too much memory and the number of iterations increased rapidly with the problem size. By using the above approach, the sequential version needed already 4 times less memory for a problem with 100,000 unknowns. On the same problem the construction of the preconditioner and the solution process is more than an order of magnitude cheaper than with MRILU. Currently we are running without difficulty problems which are 16 times as large. With the parallel version we currently can reproduce the results of the sequential version and are in a phase of finding the right mix of solvers and parameters in Trilinos for optimal performance. At the time of the conference we hope to be able to run problems with 10 million unknowns, which are needed to solve the problem we have posed ourselves (see Challenge).

During the conference we will explain the above in more detail, illustrated with computational results.

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