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*Published in:*  
9th Euro-American Workshop on Information Optics (WIO), 2010

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*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2010

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*  
Setälä, T., Hakkarainen, T., Friberg, A. T., & Hoenders, B. J. (2010). Invisibility cloaking in weak scattering. In *9th Euro-American Workshop on Information Optics (WIO), 2010* University of Groningen, The Zernike Institute for Advanced Materials.

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# Invisibility cloaking in weak scattering

(Invited Paper)

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**Abstract**—We consider invisibility cloaking of a slab object in scalar wave theory within the first-order Born approximation. We show that in the forward direction cloaking is achieved for any object slab and incident field, whereas in the backward direction cloaking is possible at least for self-imaging fields. In both cases the scattering potential of the cloak slab depends on that of the object slab. The method of object-dependent cloaking using weak slab scatterers can be a useful addition to existing cloaking methods, for instance, in atmospheric optics and biophotonics.

## I. INTRODUCTION

Cloaking of objects making them invisible or less detectable has received much attention recently due to its general interest and numerous potential applications. Several techniques for cloaking have been proposed [1]. These include coordinate transformations, reduction of scattering cross-section, use of embedded arrays of holes or dielectric particles in metal films, and utilization of anomalous resonances in certain systems. Many of the cloaking methods involve metamaterials [2] and correspond to “blind” cloaking, in which no outside electromagnetic field interacts with the object that is hidden in the cloaked domain. Another approach is cloaking “at distance”, which may use the so-called complementary media [3] and in which the object is external to the cloak. In general, all these proposal of optical invisibility cloaking are either based on strong scatterers, involve sub-wavelength particles, are not exact, are valid in a narrow spectral band, or utilize specific resonant structures. In this work, we present a different type of cloaking based on classical scattering theory with weak slab scatterers.

## II. CLOAKING CONDITION

We study a geometry in which both the object and the cloak are stratified slabs of weakly scattering material (see Fig. 1). The field propagates in the  $+z$  direction, the object is located within the interval  $[0, a]$ , and the cloak slab is positioned at  $[a_1, a_2]$ , with  $a_1 > a$ . We solve the Helmholtz equation for (scalar) light within the accuracy of the first-order Born approximation in such a system. For the cloaking to occur, it

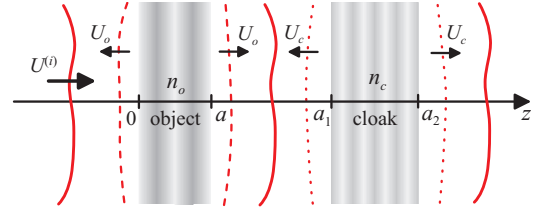


Fig. 1. Illustration of the slab geometry for cloaking with weak scatterers. The structures of the object and the cloak are independent of transverse position.

is required that [4]

$$\int_0^a F_o(z, \omega) dz = - \int_{a_1}^{a_2} F_c(z, \omega) dz \quad (1)$$

for the forward-scattered light (i.e., at  $z > a_2$ ), and

$$\int_0^a F_o(z, \omega) e^{2ik_0 mz} dz = - \int_{a_1}^{a_2} F_c(z, \omega) e^{2ik_0 mz} dz \quad (2)$$

for the backward-scattered light ( $z < 0$ ). Here  $F_o(z, \omega)$  and  $F_c(z, \omega)$  are the scattering potentials of the object and cloak slabs, respectively, the parameter  $k_0 = \omega/c$  is the free-space wave number,  $\omega$  is the angular frequency, and  $m$  denotes the (normalized)  $z$  component of light’s wave vector. The task then is to find  $F_c(z, \omega)$ ,  $a_1$ , and  $a_2$ , and the values of  $m$  that satisfy the above equations, when  $F_o(z, \omega)$  and  $a$  are known.

### A. Forward cloaking

We see that the cloaking condition in the forward direction, Eq. (1), is independent of the incident field and can be easily satisfied for any object slab. We take, for simplicity, the object and cloak slabs to be of equal thickness, i.e.,  $a_2 - a_1 = a$ , but allow  $a_1 > a$  to be arbitrary. It then readily follows that the cloaks’s scattering potential must obey

$$F_c(z + a_1, \omega) = -F_o(z, \omega), \quad (3)$$

for forward cloaking to take place. The scattering potential of the object is  $F_o(z, \omega) = k_0^2 [n_o^2(z, \omega) - 1] / 4\pi$ , where  $n_o(z, \omega)$  is the refractive index. For a weakly scattering object in vacuum surroundings we may write

$$n_o(z, \omega) = 1 + \epsilon(z, \omega) \exp[i\psi(z, \omega)], \quad (4)$$

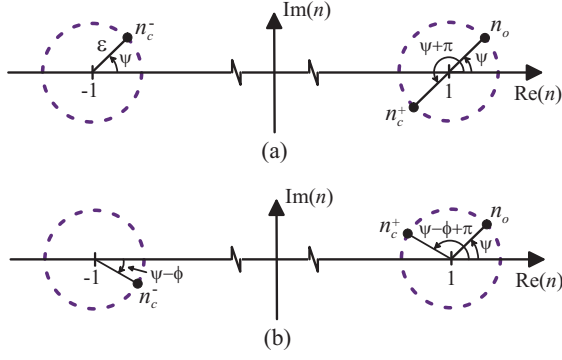


Fig. 2. Schematic representation of the refractive indices of the object,  $n_o$ , and the cloak,  $n_c^\pm$ , in complex plane: (a) forward cloaking, and (b) backward cloaking. The parameters  $\epsilon \ll 1$  and  $\psi$  characterize the object, whereas  $\phi$  is related to the illumination in backscattering.

where  $0 \leq \epsilon(z, \omega) \ll 1$  and  $\psi(z, \omega)$  is a real quantity. Hence the refractive index is close to unity, with a small absorptive part. It then follows from Eq. (3) that the cloak's refractive index may assume two possible values [4]

$$n_c^\pm(z + a_1, \omega) = \pm 1 \mp \epsilon(z, \omega) \exp[i\psi(z, \omega)], \quad (5)$$

with the upper or lower signs taken. These solutions, together with the object refractive index, are schematically illustrated in Fig. 2(a). It is observed that one of these solutions,  $n_c^+(z + a_1, \omega)$ , is positively refractive and weakly amplifying, while the other solution,  $n_c^-(z + a_1, \omega)$ , is negatively refractive and slightly absorbing. Thus, an ordinary weakly scattering slab object can be cloaked, in the forward direction, by a similar slab of either slightly amplifying (active) normal material or weakly absorbing (passive) metamaterial. In these cases the cloaking is perfect (within the first-order Born scattering), it occurs for an incident field of any spatial and spectral distribution, and it is independent of the cloak position  $a_1 > a$ .

### B. Backward cloaking

Comparison of Eqs. (1) and (2) indicates that the condition for the cloaking of a slab object in the backward direction is much more stringent than in the forward direction. It can be shown [4] that Eq. (2) can be satisfied only for discrete values of  $m$ . A given value of  $m$  corresponds to a conical illumination such that the longitudinal ( $z$ ) component of the incident wave vectors is  $k_z = mk_0$ , with  $0 \leq m \leq 1$ . For object and cloak slabs of equal thickness  $a < a_1$ , Eq. (2) then implies that cloaking in the backscattering direction takes place if

$$F_c(z + a_1, \omega) = -F_o(z, \omega) \exp[-i\phi(\omega)], \quad (6)$$

where  $\phi(\omega) + \alpha 2\pi = 2k_0 m a_1$  is a real quantity independent of  $m$ , and  $\alpha$  is an integer. Hence, allowed values of  $m$  are

$$m_\alpha = \frac{\pi}{k_0 a_1} \left( \alpha + \frac{\phi(\omega)}{2\pi} \right), \quad (7)$$

provided  $0 \leq m_\alpha \leq 1$ . For instance, if the illumination and the system parameters are such that  $\phi(\omega) = \pi$ , the scattering potential of the cloak is  $F_c(z + a_1, \omega) = F_o(z, \omega)$ . This shows

that, in reflection, an exact replica of the object slab also works as a cloak.

For a weakly scattering object characterized by Eq. (4), the backward cloaking requirement of Eq. (6) is satisfied by two different cloaks specified by

$$n_c^\pm(z + a_1, \omega) = \pm 1 \mp \epsilon(z, \omega) \exp\{i[\psi(z, \omega) - \phi(\omega)]\}, \quad (8)$$

again with either the upper or the lower signs assumed. The refractive indices are schematically illustrated in Fig. 2(b). Hence cloaking in reflection can be achieved by positively or negatively refracting materials, which are either lossy or amplifying (or both), depending on the object slab [through parameters  $\epsilon(z, \omega)$  and  $\psi(z, \omega)$ ] and the incident field [via the phase  $\phi(\omega)$ ]. It is important to note, however, that backward cloaking does not necessarily require the use of negative-index or amplifying media; for fields incident from directions specified by Eq. (7), a dielectric slab object can be cloaked by an ordinary absorbing medium.

The values of  $m_\alpha$  in Eq. (7) correspond to plane waves with wave vectors on a conical surface of angle  $\theta_c = \arccos(m_\alpha)$ . Such a field is a nondiffracting Bessel Beam [5]. The adjacent  $m_\alpha$  are equally spaced with separation  $\Delta m_\alpha = \lambda/2a_1$ . Hence, moving the cloak farther away from the object (increasing  $a_1$ ) increases the number of allowed  $m_\alpha$  values and makes them denser. A superposition of Bessel beams is, in general, not a propagation-invariant, but rather a self-imaging field [4], [5]. The self-imaging (Talbot) distance is  $z_T = 2a_1$ . Thus, within the first-order Born scattering, cloaking of a slab object in reflection ( $z \leq 0$ ) is obtained for self-imaging fields by placing a cloak slab specified by Eq. (8) at  $a_1 = Nz_T/2$ , where  $N$  is an integer.

## III. CONCLUSION

In forward scattering within the first Born approximation a stratified slab object can be cloaked by an appropriate cloak slab for an incident field of any spatial and spectral structure, whereas in reflection cloaking is achievable at least for self-imaging fields. For certain illuminations [ $\phi(\omega) = \pi$ ] cloaking is perfect in both half-spaces. The results do not contradict the theory of nonscattering scatterers or the optical theorem.

## ACKNOWLEDGMENT

Work supported by the Academy of Finland (grant 128331).

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