Process improvement for engineering & maintenance contractors

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Chapter 6

Process improvement incentives

6.1 Introduction

Many firm owners are looking for ways to stimulate managers to do exactly what is in the owner's interest. This is not an easy task due to the unavailability of resources such as information and time, but also due to physical distances. Since many company decisions are made in a competitive context, the behavior of rival owner-manager pairings should be taken into consideration as well when analyzing the alignment between owners and manager's interests. We study one of the manufacturing firm's main strategic decisions: how much to invest in process innovation and how to respond to competitor's investments. Process innovation is a core activity for manufacturing firms (Slack et al., 2000; Li and Rajagopalan, 2008). Its benefits range from cost reduction, to lead time reduction and quality improvement. Process innovation activities can be conducted in ad hoc improvement projects, but can also be the result of established concepts or programs such as quality management (e.g. Ittner, 1994; Sousa and Voss, 2002; Symons and Jacobs, 1995), the Capability Maturity Model (see e.g. Harter et al., 2000; Veldman and Klingenberg, 2009) and six sigma (e.g. Linderman et al., 2003). The process innovation decision is certainly non-trivial in a competitive perspective (Carrillo and Gaimon, 2000), since process innovation decisions made by one manufacturing manager can affect the decision made by the rival firm's manager, and vice versa. Therefore firm owners should think about how to stimulate process innovation in a competitive situation, while preventing them from acting in a way that deviates from the owner's main goals. Agency theory gives some guidance to this problem.

Agency theory studies principal-agent relationships. The principal (often an owner or a delegate that acts fully on behalf of the owner) gives a certain degree of decision power to an agent, who acts according to what he has been instructed to do. Clearly the key notion in agency theory is that the interests of a principal should be aligned with the interests of the agents (Eisenhardt, 1989a; Gibbons, 2005). However, even though research in this area is growing steadily (see e.g. Balasubramanian and Bhardwaj, 2004; Fershtman and Judd, 1987; Overvest and Veldman, 2008; Vickers, 1985; Vroom, 2006; Vroom and Gimeno, 2007), applications in operations management remain scarce, despite its potential contribution (Banker and Khosla, 1995). In this chapter we apply agency-theoretic ideas in an operations management setting, whereby the behavior of competing principal-agent pairs plays an important role. We model a duopoly with a non-cooperative game that explicitly captures process innovation investments and incentive contracts to stimulate these investments. Process innovation is hereby defined as a reduction of marginal costs. In the first stage a company owner (i.e. the principal) offers his manufacturing manager (i.e. the agent) an incentive contract that is a linear combination of profits and process innovation. Through the use of an innovation weight, the manufacturing manager receives a monetary incentive for his chosen process innovation level. In the second stage, the manufacturing manager decides how much to invest in process innovation and how much products to put on the market. A firm-specific cost parameter is modeled that captures the difficulty a firm has in achieving a certain process innovation level. Our main objective in the current chapter is to identify the effects these cost differences have on firms' optimal investment decisions in process innovation in duopoly, and on the height of the innovation weight.

Our analysis yields several relevant insights. Firstly, we compare the relevant decisions of a firm having a low process innovation cost parameter with the decisions made by the other firm having a high cost parameter. Secondly, we conducted a comparative statics exercise to see how both firms' Nash equilibria change in the cost parameters. Several counterintuitive results are found that add to current understanding. Thirdly, when we endogenize the decision to use a contract, we can compare the owners' expected profits in various settings (e.g. one owner uses a contract, whereas the other does not, etcetera). We show that both owners using an incentive contract for process innovation is, in itself, a Nash equilibrium. This result holds when there are differences in process innovation costs. Fourthly, we show that the process innovation variable can also be expressed as an aggregate variable, consisting of multiple types of process innovations with their associated cost parameters. These different process innovation variables can be interpreted as innovation in different processes, or different groups of homogeneous processes (with similar cost parameters within the group).

Our research contributes to existing literature in four major ways: (i) it presents an analytical model that explicitly includes the provision of incentives for process innovation. So far, very few publications do so, although much research has been done on managerial incentives and process innovation separately; (ii) we show that in equilibrium, firms always use a positive weight for process innovation; (iii) we make an in-depth comparison of firms having different costs with respect to process innovation. In many models that consider process innovation, the cost (parameter) of process innovation is treated as being equal for all firms in the market. This severely limits the relevance of those types of models, since it inhibits the comparison of firm-level equilibria and the analysis of equilibria sensitivities with respect to firm-specific model parameters; (iv) our analysis is the first to address the endogenous decision to use incentive contracts. In §6.2 we give a short overview of the current managerial incentives and process innovation literature. In §6.3 the model is described, and the Nash equilibria are obtained and analyzed. We finish the chapter with a discussion and conclusion (§6.4) and several managerial implications (§6.5).

6.2 Related literature

This chapter bridges two research areas: the field of managerial incentives and process innovation. Studies on *incentives* and *managerial incentives* using the principal-agent approach date to decades ago and have taken a central position in managerial economics. One example of a widely investigated branch of incentive research is the use of salesforce incentives (e.g. see Chen, 2005; Lal and Srinivasan, 1993; Mukhopadhyay et al., 2009).

The potential strategic effects of managerial incentives have been widely studied. Fershtman and Judd (1987) developed one of the first (game theoretic) models in this area. They find that competitive interactions in duopoly will cause owners to twist their managers away from profit maximization. Instead managers are rewarded for a combination of profits and sales. In the years that followed after this publication, several articles on the use of managerial incentives for competitive behavior appeared. Ishibashi (2001), for example, investigated the use of incentive contracts for profits and sales in situations where firms compete in prices and product quality. Vroom (2006) studied the competitive effects of organization design (i.e. centralization versus decentralization) and the role of managerial incentives. He shows that simultaneous choices regarding incentives and organization design reduce aggressiveness (i.e. managers react less strongly in terms of an increase of output when competitors' output is raised). In one of the rare empirical studies in this area, Vroom and Gimeno (2007) studied the way differences in ownership form between franchised and company-owned units affect managerial incentives and pricing in oligopolies.

In Operations Management, research on the use of managerial incentives mostly focuses on the marketing-manufacturing interface. Porteus and Whang (1991), for example, used a multiple product newsvendor model with incentives to align manufacturing decisions (i.e. stock levels) with marketing decisions (i.e. satisfying demand). Balasubramanian and Bhardwaj (2004) concluded that firm profits can be higher if objectives (in their case formulated as cost minimization versus revenue maximization) are conflicting rather than perfectly coordinated. Karabuk and Wu (2005) modeled a centralized body that allocates capacity to semiconductor manufacturing lines. Manufacturing managers are rewarded such that they reveal privately held demand information. Jerath et al. (2007) developed a model that (internally) matches the activities of marketing and operations managers through the use of contracts. They show that coordination can always be achieved by either rewarding the operations manager separately for increasing sales and reducing costs, or rewarding him (separately) for the reduction of missed sales and leftover supply. However, even though the decisions marketing and manufacturing make in this context are of significant strategic importance, the papers described here do not take the competitive context into consideration.

Process innovation, oftentimes labeled process improvement, is one of the central themes in operations management literature (Ittner and Larcker, 1997). The relevance of studying its competitive effects has not gone unnoticed. d'Aspremont and Jacquemin (1988) wrote a seminal paper on process innovation decisions in oligopoly, and explicitly address the issue of spillovers. Hauenschild (2003) extended this model by adding a stochastic element (i.e. uncertainty about the success of process innovation efforts). Bonanno and Haworth (1998) and Rosenkranz (2003) investigated the combined decision into process and product innovation and identify several conditions (e.g. market structure) that determine to which type effort is directed. Gupta and Loulou (1998) modeled several manufacturers (producing differentiated products) that have to make the combined decision of investing in process innovation and choosing channel structure, i.e. whether to distribute by themselves or through the use of an intermediary. Li and Rajagopalan (2008) developed a stochastic model of a firm's investment decisions in process improvement. They consider the timing of investment decisions and the role of knowledge accumulation. The competitive effect of process improvement is modeled with the relative quality of the process: if, for example, processes are improved but the competitor still has a better process, then the firm's cash flow will be lower than the cash flow of the competitor.

Although managerial incentives are identified as a key mechanism to stimulate process change (Carrillo and Gaimon, 2004), research in this field has been limited to date. Some articles describe the statistical relationship between practices related to incentives and innovation. Balkin et al. (2000) reported that in high-tech firms, executive short term compensation is related to innovation as measured by patents and R&D spending. This relationship, however, was not found in a control sample of low-technology firms. In an exploratory study, Ittner and Larcker (1995) described the relationships between total quality management practices and incentive systems, and the performance effects of practices exhibiting a strong relationship with incentive systems. They concluded that these performance effects seem to depend on the intensity with which formal quality programs are used. Agrell et al. (2002) published one of the few papers wherein incentives for cost reducing innovation are explicitly modeled. Their models, however, focus on the 'internal' principal-agent problem, and do not take the competitive context into consideration. Carrillo and Gaimon (2004) modeled the relationship between managerial systems (such as incentives) and the pursuit of process change. In their model, the plant manager is uncertain about the outcome of process change. When there are penalties for this uncertainty, the amount of process change can decrease as the uncertainty penalty cost increases. They suggest using appropriate managerial systems to guide the plant manager's estimate of penalty costs.

Appelbaum and Harris (1976) seem to have been the first to combine competition and a manager's utility function that is dependent on cost reduction. Their models, however, are limited in nature and do not give rise to a good comparison with related models developed later. The most important limitation is that they treat the entire industry as a single price-taking firm, thus ignoring the strategic effects firm decisions have on eachother. In addition they do not seek the optimal value of the incentive contract, but rather investigate the effects of the incentive contract on cost reducing activities and product quantity.

Although the strategic relevance of managerial incentives for process innovation is clear, it appears that very little analytical models on this theme exist to date. In a recent publication, Kopel and Riegler (2006) revised the model of Zhang and Zhang (1997). In the model, managerial incentives are given for sales and profit in a duopoly with spillovers. Process innovation is treated as one of the main decision variables. However, process innovation was only rewarded indirectly; through the profit function. Overvest and Veldman (2008) made the first model wherein an agent's pay is directly related to process innovation. They concluded that in equilibrium, an agent is always rewarded for process innovation, and that the degree to which process innovation is rewarded diminishes as the amount of competitors in the market increases. With the exception of that publication, no analytical contributions on the direct use of managerial incentives on process innovation exist. The major difference between the model presented in this chapter and the models of Zhang and Zhang (1997) and Kopel and Riegler (2006) is that in the current model the manufacturing manager's incentive contract is a function of process innovation. The incentive contract can therefore be seen as an actual employment contract, describing the weight that should be given to the manufacturing manager's process innovation undertakings. An owner who offers such an incentive contract commits to high levels of process innovation (in the literature, such contracts are oftentimes called strategic commitment devices). In addition, we explicitly model firm-specific process innovation cost parameters, which allows us to analyze the effects of cost differences on relevant firm-level equilibria. To keep the analysis tractable we do not model spillovers. To the best of our knowledge, these issues have not been considered before.

6.3 The model

6.3.1 Research purpose and modeling procedure

We construct and analyze a game-theoretic model to address the effect process innovation cost differences between firms have on (i) optimal investment decisions in process innovation and (ii) the height of the incentive contract. We also investigate what the effects of these differences are on firm profits, and whether or not firm owners decide to use an incentive contract at all. In the current chapter we define the process innovation level as all the efforts leading to efficiency improvements within a company's production process for a certain (group of) products, which are ultimately leading to marginal cost reductions of that company's (group of) products. Such an interpretation of process innovation is very common in manufacturing firms (Carrillo and Gaimon, 2002). An important, related debate to the question what constitutes process innovation, addresses the distinction between learning by doing and learning before doing. The basic premise of learning by doing is that learning is a continuous activity that cannot be detached from the process this learning stems from (Hatch and Mowery, 1998; Von Hippel and Tyre, 1995). Learning before doing, on the other hand, is an activity in which deliberate investments are made before the process has started to take place (e.g. Ethiraj et al., 2005; Hayes et al., 2005; Lederer and Rhee, 1995). As in Fine and Porteus (1989) we limit ourselves in the current chapter to learning before doing.

In the first stage, an incentive contract is offered to the manufacturing

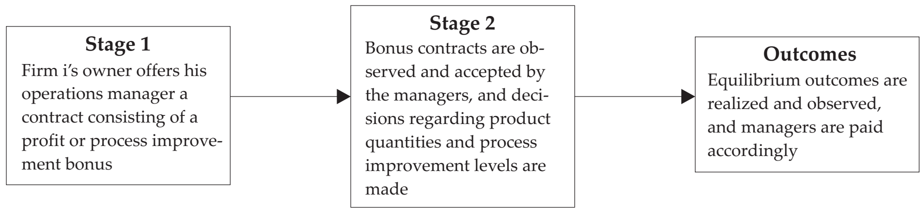

Figure 6.1. Timing and stages within the game.

manager by the company owner. The incentive contract is a linear combination of profits and process innovation. We assume that this contract is accepted. In the second stage, the manufacturing manager makes decisions on the process innovation level to choose, and the product quantity to put on the market (we assume that the quantity produced is equal to the quantity sold; therefore 'quantity' can be read in both ways). After these stages, profits and process innovation levels are observed and the manufacturing manager is paid accordingly. In both stages rival firm behavior is monitored and the optimal reaction to the rival firm's potential decisions is considered. The timing and the stages of the model are depicted in figure 6.1.

6.3.2 The basic model

The two-stage game theoretic model we develop is based on Fershtman and Judd (1987), d'Aspremont and Jacquemin (1988) and Overvest and Veldman (2008). We investigate a duopoly in which two firms compete in a Cournot market of homogeneous goods. We choose a duopoly because it gives us the maximum degree of insight into how firms react to one another's choice. Cournot models are characterized by an inverse demand function: the higher the total product quantity that is being put on the market, the lower the market price. This immediately clarifies the strategic nature of such models: the higher the product quantity supplied by one firm, the lower the price for the other. In operations management, quantity setting Cournot models are very common (Anupindi and Jiang, 2008). See also Goyal and Netessine (2007), Hughes and Thevaranjan (1995), Lus and Muriel (2009), Waller and Christy (1992), Xiao et al. (2007), Yang and Zhou (2006), Zhang (2002) and Zhu and Weyant (2003).

The two firms are indexed by i and j; $i, j = 1, 2$, and $i \neq j$. We assume that the two firms are profit maximizers, and are risk neutral. The firms produce q_i units, with $Q = q_i + q_j$ being the total output in the market.

They face a deterministic inverse demand function that is characterized by $p = a - bQ$, where $p = p_i = p_j$ is the unit price for product, a is the intercept of the demand function, and b is the sensitivity of the price with respect to Q. We assume that $a, b > 0$.

The process innovation level for a firm is denoted by x_i. Using the positive constant c, which is similar for both firms, we can write the marginal cost of production as $c - x_i$. As in d'Aspremont and Jacquemin (1988) x_i reduces marginal costs, allowing firms to produce more. Since negative marginal costs are unrealistic, we require $c > x_i$. The total cost of undertaking process innovation is $\frac{1}{2}\gamma_i x_i^2$, where γ_i is the firm-specific process innovation cost parameter. The quadratic expression of total process innovation cost, which is frequently assumed in industrial organization and operations management literature (e.g. d'Aspremont and Jacquemin, 1988; Hughes and Thevaranjan, 1995; Tseng, 2004), indicates that there are diminishing returns to process innovation investments (also see Adner and Levinthal, 2001). The firm-specific γ_i can be easily interpreted as the difficulty firms have in process innovation. We only investigate active competitive situations, which implies that $q_i, x_i > 0$. Firm profit π_i equals total sales R_i minus total costs C_i, where $R_i = pq_i$ and $C_i = (c - x_i)q_i + \frac{1}{2}\gamma_i x_i^2$. Written in extended form,

$$\pi_i = (a - b(q_i + q_j) - c + x_i)q_i - \frac{1}{2}\gamma_i x_i^2, \qquad i, j = 1, 2; i \neq j. \tag{6.1}$$

From this expression it should be clear that we require $c < a$.

We can apply two transformations to our model. First, we can normalize the variables and profit functions. Using a subscript n to denote a normalized variable or parameter, define $\gamma_i = \frac{\gamma_{i,n}}{b}$, $q_i = \left(\frac{a-c}{b}\right) q_{i,n}$, $x_i = (a - c)x_{i,n}$ and $\pi_i = \frac{(a-c)^2}{b}\Pi_{i,n}$. The normalized profit function $\Pi_{i,n}$ can now be written as

$$\Pi_{i,n} = (1 - q_{i,n} - q_{j,n} + x_{i,n})q_{i,n} - \frac{1}{2}\gamma_{i,n} x_{i,n}^2, \qquad i, j = 1, 2; i \neq j. \tag{6.2}$$

Through normalization, we have a model that has the market size of 1, and solutions of the normalized variables that depend only on $\gamma_{1,n}$ and $\gamma_{2,n}$. Note that $\frac{(a-c)^2}{b}$, $\frac{a-c}{b}$ and $a - c$ are positive constants. Since we require $c > x_i$, we also have $x_{i,n} < \frac{c}{a-c}$. In the remainder of this chapter we will analyze the normalized models and drop the subscript n, unless explicitly stated otherwise.

Second, we can express the process innovation level as an aggregate variable. Using a single process innovation variable and cost parameter suggests that we are dealing with a uniform (or single) process. A natural extension is to include multiple innovation variables; each with its associated cost parameter. Such an extension is important since the manufacturing process most often consists of several sub-processes (e.g. steps in the assembly process) in which process improvement 'difficulty' (viz. the cost parameter) differs from sub-process to sub-process. An obvious approach to deal with multiple process innovation variables would be to include these variables separately in the profit function, and calculate the equilibria for each of them. However, as in Gaalman (1978), we can also view x_i as an aggregate variable. Define $x_{i,k}$ as firm i's process innovation in the k^{th} process, then for K processes the aggregate variable can be defined as $x_i = \sum_{k=1}^{K} x_{i,k}$. All the K processes of firm i have an associated cost parameter $\theta_{i,k}$, $k = 1, \ldots, K$. As we show in the appendix, the aggregate process innovation cost parameter γ_i can be defined as follows: $\gamma_i = 1/\sum_{k=1}^{K} \frac{1}{\theta_{i,k}}$. When we have found the optimal aggregate x_i^*, the individual optima $x_{i,k}^*$ satisfy

$$x_{i,k}^* = \left(\frac{\gamma_i}{\theta_{i,k}}\right) x_i^*, \qquad i = 1, 2. \tag{6.3}$$

In order to analyze the effects of using process innovation incentive contracts and cost differences, we will first establish a baseline case in which no incentive contract is used.

6.3.3 Equilibrium outcomes in the baseline case

We obtain the equilibrium outcomes by simultaneously solving the first-order necessary conditions of both firms with respect to product quantity and process innovation level (i.e. $\partial\Pi_i/\partial q_i = 0$, $\partial\Pi_i/\partial x_i = 0$, $i = 1, 2$) (see Gibbons, 1992). This yields the following Nash equilibria (we denote the equilibria with an asterisk):

Proposition 1. *(i) When no incentive contract is used, and firms 1 and 2 choose product quantities and process innovation levels simultaneously, their optimal product quantities and process innovation levels are*

$$q_i^* = \frac{\gamma_i(\gamma_j - 1)}{\gamma_i(3\gamma_j - 2) - 2\gamma_j + 1}, \tag{6.4}$$

$$x_i^* = \frac{\gamma_j - 1}{\gamma_i(3\gamma_j - 2) - 2\gamma_j + 1}, \tag{6.5}$$

(ii) Optimal profit is

$$\Pi_i^* = \frac{\gamma_i(2\gamma_i - 1)(\gamma_j - 1)^2}{2(\gamma_i(3\gamma_j - 2) - 2\gamma_j + 1)^2} \tag{6.6}$$

for $i, j = 1, 2; i \neq j$.

As we show in the appendix the sufficient second-order conditions hold when $\gamma_1 \geqslant \frac{1}{2}$ and $\gamma_2 \geqslant \frac{1}{2}$. It is straightforward to see that process innovation levels, product quantities and profits are all positive when $\gamma_1 > 1$ and $\gamma_2 > 1$. It can easily be verified that the equilibrium outcomes decrease in a firm's own process innovation cost parameter, and increase in the rival firm's cost parameter. This is summarized in the following proposition:

Proposition 2. *(i) In equilibrium, a firm's product quantity, process innovation level and profit decreases in its process innovation cost parameter:* $\partial q_i^*/\partial\gamma_i < 0, \partial x_i^*/\partial\gamma_i < 0, \partial\Pi_i^*/\partial\gamma_i < 0$*,* $i = 1, 2$*;*
(ii) In equilibrium, a firm's product quantity, process innovation level and profit increases in its rival's process innovation cost parameter: $\partial q_i^*/\partial\gamma_j > 0, \partial x_i^*/\partial\gamma_j > 0, \partial\Pi_i^*/\partial\gamma_j > 0$*,* $i, j = 1, 2; i \neq j$*.*

The proposition indicates that, for a firm, an increase in a process innovation cost parameter makes process innovation more expensive. As a result, the process innovation level decreases and less product quantities can be put on the market. Knowing that a firm's product quantity and process innovation level decrease, the rival firm will increase its product quantity and process innovation level.

6.3.4 Equilibrium outcomes in the case with an incentive contract

We now turn to the case where both firm owners deviate from an instruction to purely maximize firm profits. Owners now offer the manufacturing manager an incentive contract in which process innovation is directly rewarded. We introduce an *innovation weight*, denoted as λ_i, which can be used to express the monetary value the manufacturing manager receives in return for his process innovation investments (the innovation weight λ_i can also be normalized. In this case $\lambda_i = \left(\frac{a-c}{b}\right)\lambda_{i,n}$. As earlier, we drop the subscript n in the remainder of this chapter). We model the manufacturing manager's incentive contract as his pay S_i, being a linear combination of profit and the process innovation level. The function he maximizes is

$$S_i = \Pi_i + \lambda_i x_i, \qquad i = 1, 2. \tag{6.7}$$

It is important to note that we do not put any restrictions on the value of λ_i, allowing it to take on negative values (which would mean that the manufacturing manager is punished for undertaking process innovation). As is noted in earlier work (e.g. Fershtman and Judd, 1987; Vroom, 2006), in reality this function will not be the manager's actual pay. His pay will actually be in a form equivalent to $A_i + B_i S_i$. However, A_i and B_i are constants which are independent of the decisions regarding product quantity and process innovation level, so that the equilibrium outcomes would be similar. Substituting (6.2) into (6.7) we have

$$S_i = (1 - q_i - q_j + x_i)q_i - \frac{1}{2}\gamma_i x_i^2 + \lambda_i x_i, \qquad i, j = 1, 2; i \neq j. \tag{6.8}$$

The standard solution concept to the two-stage model is the sub-game perfect Nash equilibrium (Fudenberg and Tirole, 1992). The profit, product quantity, process innovation and the innovation weight equilibria are obtained through the well-known backwards induction procedure (e.g. Gibbons, 1992; Mas-Colell et al., 1995). This implies in the current chapter that we start by finding the optimal product quantities and process innovation levels that are chosen in the second stage, conditional on λ_1 and λ_2. We do so by simultaneously solving the first-order necessary conditions $\partial S_i/\partial q_i = 0$, $\partial S_i/\partial x_i = 0$, $i = 1, 2$. To illustrate the role of the innovation weight, it is useful to introduce the concept of the reaction function (see e.g. Cachon and Netessine, 2005). Consider for example the optimal product quantity choice. Using the solutions for a firm's first-order necessary conditions, we can express the optimal product quantity choice of firm 1 as a function of the product quantity choice of firm 2 and vice versa. This yields both firms' reaction functions:

$$q_i = \frac{\gamma_i(1 - q_j) + \lambda_i}{2\gamma_i - 1}, \qquad i, j = 1, 2; i \neq j. \tag{6.9}$$

Equation (6.9) illustrates that an increase in the rival firm's product quantity reduces the optimal product quantity for a firm when $\gamma_1 > \frac{1}{2}$ and $\gamma_2 > \frac{1}{2}$. The innovation weight λ_i positively influences the optimal product quantity, which is due to its (implicit) effect on process innovation. The intersection of the reactions functions of both firms yields the second-stage Nash equilibria in product quantities. See figure 6.2 for an illustration. The same can be done for the Nash equilibria in process innovation levels. The Nash equilibria we find in the second stage are essentially functions of λ_1, λ_2 and both firms' process innovation cost parameters:

$$q_i^* = \frac{(2\gamma_j - 1)\lambda_i - \gamma_i\lambda_j + \gamma_i(\gamma_j - 1)}{\gamma_i(3\gamma_j - 2) - 2\gamma_j + 1}, \qquad i, j = 1, 2; i \neq j, \tag{6.10}$$

$$x_i^* = \frac{(3\gamma_j - 2)\lambda_i - \lambda_j + \gamma_j - 1}{\gamma_i(3\gamma_j - 2) - 2\gamma_j + 1}, \qquad i, j = 1, 2; i \neq j. \tag{6.11}$$

Note that these outcomes are similar to the outcomes presented in proposition 1 if $\lambda_1 = 0$ and $\lambda_2 = 0$. In the first stage owner 1 and 2 optimize their profit functions with respect to λ_1 and λ_2, respectively. In order to find the

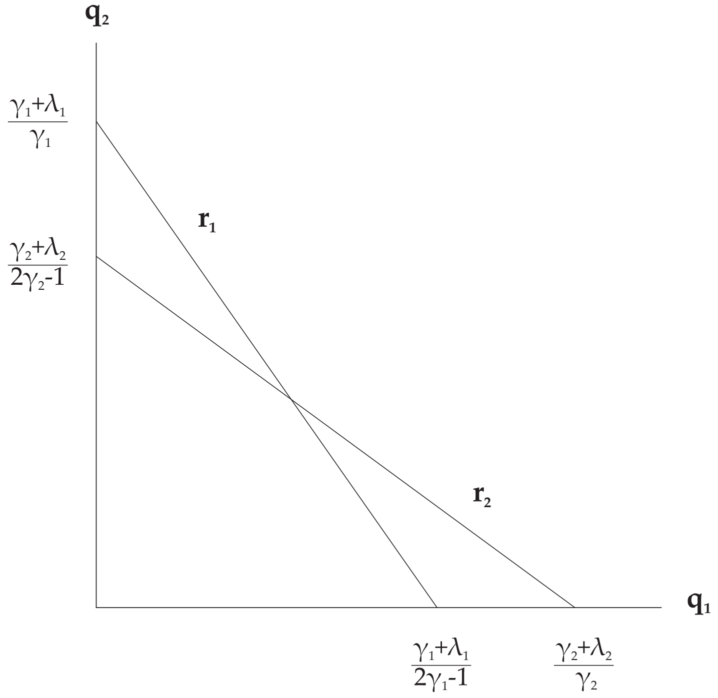

Figure 6.2. Reaction functions r_1 and r_2 with respect to chosen product quantities, where r_i, $i = 1, 2$, refers to the reaction function of player i, which is a function of the quantities of player j.

innovation weight Nash equilibria λ_1^* and λ_2^*, we substitute the second stage outcomes into both firms' profit functions, and simultaneously solve the first-order necessary conditions $\partial\Pi_i/\partial\lambda_i = 0$, $i = 1, 2$, for λ_1 and λ_2. Finally, the second stage outcomes q_i^* and x_i^* can be found by substituting λ_1^* and λ_2^* into (6.10) and (6.11) and the profit function (6.2). Proposition 3 states the subgame perfect Nash equilibria (a derivation of the equilibria for $n \geqslant 3$ firms using matrix algebra can be obtained from the authors upon request).

Proposition 3. *(i) When both firm owners use an incentive contract to stimulate their manufacturing managers, then in equilibrium the innovation weight can be expressed as:*

$$\lambda_i^* = \gamma_i\gamma_j(\gamma_i(3\gamma_j - 4) - 2\gamma_j + 2)/\Phi, \tag{6.12}$$

(ii) Both manufacturing managers choose the following product quantities and process innovation levels:

$$q_i^* = \gamma_i(3\gamma_j - 2)(\gamma_i(3\gamma_j - 4) - 2\gamma_j + 2)/\Phi, \tag{6.13}$$

$$x_i^* = 2(2\gamma_j - 1)(\gamma_i(3\gamma_j - 4) - 2\gamma_j + 2)/\Phi, \tag{6.14}$$

and (iii) earn the following profits:

$$\Pi_i^* = \gamma_i(\gamma_i(2 - 3\gamma_j)^2 - 2(1 - 2\gamma_j)^2)(\gamma_i(3\gamma_j - 4) - 2\gamma_j + 2)^2/\Phi^2, \tag{6.15}$$

where $\Phi = \gamma_i^2(3\gamma_j - 2)(9\gamma_j - 8) - 2\gamma_i(3\gamma_j - 2)(7\gamma_j - 4) + 4(1 - 2\gamma_j)^2$ *and* $i, j = 1, 2; i \neq j$.

It can be verified that Φ is a symmetric polynomial function of γ_1 and γ_2. It can also be noted that $q_i^* = \frac{3\gamma_j - 2}{\gamma_j}\lambda_i^*$ and $x_i^* = \frac{4\gamma_j - 2}{\gamma_i\gamma_j}\lambda_i^*$ for $i, j = 1, 2; i \neq j$. In the appendix the sufficient second-order conditions to ensure maximization are given for both stages. Using the geometrical properties of the reactions functions, it can be proved that the Nash equilibria found are unique (also see Cachon and Netessine, 2005); the reaction functions in both stages are linear for the decision of one firm with respect to the decision of the other firm, implying that they can intersect only once. To make sure product quantities and process innovation levels are strictly positive for both firms we need to inspect the roots of the numerators and the denominator (i.e. Φ) of the equilibria (eqs. 6.13-6.14). The roots of the *common part* of the numerators of q_i and x_i yield the conditions

$$\gamma_j > \frac{4\gamma_i - 2}{3\gamma_i - 2}, \qquad i, j = 1, 2; i \neq j, \tag{6.16}$$

which are sufficient to ensure positivity of all the numerators and Φ for $\gamma_1 > \frac{2}{3}$ and $\gamma_2 > \frac{2}{3}$. These conditions also ensure that the sufficient second-order conditions of both stages are satisfied.

The two conditions form a convex set in the Euclidian γ_1, γ_2 plane. Due to symmetry we only consider the case for which $\gamma_1 > \gamma_2$ from this point forward. Combining this with the conditions in (6.16) gives the convex set Γ, see figure 6.3. Note that Γ restricts γ_1 to $\left(\frac{3+\sqrt{3}}{3}\right) < \gamma_1 < \infty$ and γ_2 to $\frac{4}{3} < \gamma_2 < \infty$. We are now in the position to analyze the implications of differences in firms' process innovation cost parameter. Proposition 4 describes the key results.

Proposition 4. *In* Γ*, in equilibrium, (i) both firm owners offer their manufacturing managers an incentive contract that gives a positive weight to process innovation (i.e.* $\lambda_1^* > 0$*,* $\lambda_2^* > 0$*),*
(ii) and both firms earn positive profits (i.e. $\Pi_1^* > 0$*,* $\Pi_2^* > 0$*).*
(iii) Furthermore, compared to firm 1, firm 2 produces more, conducts more process innovation, gives a higher weight to process innovation and earns a higher profit (i.e. $q_2^* > q_1^*$*,* $x_2^* > x_1^*$*,* $\lambda_2^* > \lambda_1^*$ *and* $\Pi_2^* > \Pi_1^*$*).*

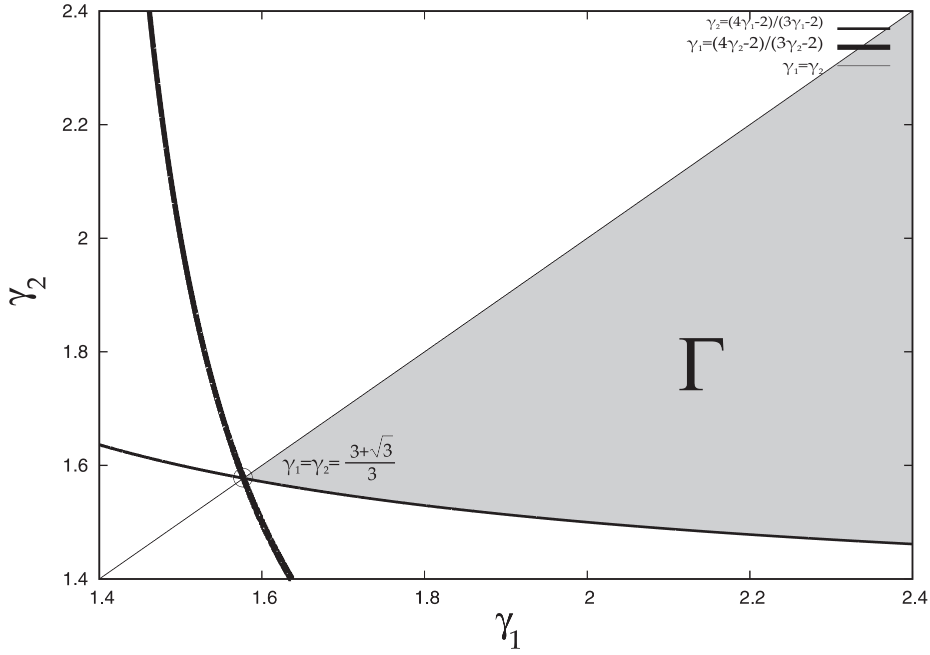

Figure 6.3. The convex set Γ bounded by the conditions in (6.16).

The proofs of (i) and (ii) are straightforward and are therefore omitted. With respect to (iii), it can be shown that firm 2's product quantity, process innovation level and innovation weight are higher when the condition $\gamma_2 > \frac{2\gamma_1-1}{3\gamma_1-2}$ applies. In Γ this condition is always satisfied. Proving that firm 2 earns higher profits than its rival is somewhat more involved and is done in the appendix.

The proposition states that a firm owner would offer his manufacturing manager an incentive contract that directly rewards process innovation undertakings, showing that the incentive contract acts as a strategic commitment device for process innovation. The proposition also states that firm 1, which is the firm with the higher process innovation cost parameter, conducts less process innovation and, as a result, puts less product quantities on the market. Whereas this result may confirm intuition, a more remarkable result is that the manufacturing manager in firm 1 is rewarded *less* for process innovation, implying that his owner wants to prevent him from innovating too much. An alternative response to a higher process innovation cost parameter of firm 1 could have been to stimulate process innovation *more* (implying a *higher* λ_1^*). Our results clearly show that this is not the case.

6.3.5 Strategic form analysis of incentive contracts

The decision on the use of incentive contracts is made ex ante by the owners, who are primarily interested in profit maximization. However, until now we have made the exogenous assumption that both owners offer their manufacturing manager an incentive contract for process innovation, and found that the incentive weight for process innovation (i.e. λ_i) is always positive. Let us consider the case where the usage of an incentive contract is *endogenously* determined in order to see whether there is an optimal decision for firm owners, and what constitutes that decision.

A firm owner can independently decide to instruct his manufacturing manager to maximize firm profits. We denote this strategy by P. Alternatively he can offer his manufacturing manager an incentive contract with an innovation weight λ_i, $i = 1, 2$, which is denoted by S. Combining these decisions a 2×2 matrix can be made, giving a strategic form expression of expected firm profits, see figure 6.4. We added a superscript to differentiate between the profits in the four sub-games: Π_i^{*P} is the outcome for owner i when both owners choose strategy P, and similarly for Π_i^{*S} and strategy S, $i = 1, 2$. Furthermore Π_i^{*PS} is the outcome for owner i when owner 1 chooses strategy P while owner 2 chooses strategy S, and similarly for Π_i^{*SP} when the chosen strategies are the reverse , $i = 1, 2$.

We also denote the four sub-games as (P, P), (P, S), (S, P) and (S, S), where, for example, (P, S) refers to the sub-game in which owner 1 chooses strategy P and owner 2 chooses strategy S. Note that firm profits Π_i^{*P} and Π_i^{*S}, $i = 1, 2$ are the results given in section 6.3.3 and 6.3.4, respectively. Through a comparison of expected profits we can verify that the pure-strategy Nash equilibrium in the 2×2 non-cooperative game is (S, S): if an owner chooses strategy P, the best response of the rival owner will be strategy S. Furthermore if an owner chooses strategy S, the other owner will respond with S as well (provided, of course, that the positivity conditions of each strategy combination are satisfied). In that line of reasoning, the optimal choice for both owners (and thus the pure-strategy Nash equilibrium of the game) is to *always* use an incentive contract, i.e. (S, S). We summarize in the following proposition:

Proposition 5. *The pure-strategy Nash equilibrium in the game given in figure 6.4 is (S, S), which means that both owners decide to use an incentive contract in which their manufacturing manager is directly rewarded for process innovation using an innovation weight λ_i, given $\gamma_1 > \gamma_2$ and satisfied positivity conditions for q_i and x_i, $i = 1, 2$.*

The intuition behind this result is that an incentive contract acts as a true commitment device, stimulating firms to innovate more in processes, resulting

	P - *Owner 2 instructs his manager to maximize profits*	**S -** *Owner 2 offers an incentive contract including an innovation weight λ_2*
P - *Owner 2 instructs his manager to maximize profits*	Π_1^{*P}, Π_2^{*P}	Π_1^{*PS}, Π_2^{*PS}
S - *Owner 2 offers an incentive contract including an innovation weight λ_2*	Π_1^{*SP}, Π_2^{*SP}	Π_1^{*S}, Π_2^{*S}

Figure 6.4. Strategic form expression of expected normalized profits.

in higher product quantities and profits. Since both firms will not allow the rival firm to take away market share, an optimal response is to use always use an incentive contract, even though the expected profits for both firms are higher in quadrant 1 for all $\gamma_i \in \Gamma$, $i = 1, 2$. In that sense, the game is a true prisoners' dilemma.

6.3.6 Comparing process innovation levels

A final step in the analysis of the equilibria would be the comparison of the process innovation level given in (6.5) -the process innovation level in the baseline case- with the process innovation level given in (6.14), which is the case with an incentive contract. Denoting the outcome in (6.5) temporarily with $x^*_{i,0}$ and the outcome in (6.14) with $x^*_{i,\lambda}$, we can show that the inequalities $x^*_{i,\lambda} > x^*_{i,0}, i = 1, 2$ lead to one relevant condition in Γ, namely

$$\gamma_2 > \frac{7}{6} + \frac{1}{6}\sqrt{\frac{27\gamma_1 - 26}{3\gamma_1 - 2}}, \tag{6.17}$$

which is derived from the solutions of the equality $x^*_{1,\lambda} = x^*_{1,0}$. The right-hand side of the inequality in (6.17) has an upper limit $\gamma_2 = \frac{5}{3}$ as $\gamma_1 \to \infty$. This implies that for firm 1, the use of an incentive contract leads to more process innovation for nearly all relevant parameter settings. Furthermore, since no solutions to $x^*_{2,\lambda} = x^*_{2,0}$ exist in Γ, the manufacturing manager of firm 2 will always innovate more when he is offered an incentive contract. In the following sub-section we investigate the effects of the cost parameters on firm equilibria.

6.3.7 Comparative statics

If we would have considered only the case where $\gamma_1 = \gamma_2 = \gamma$, then a change in this parameter would imply a simultaneous and similar change for both firms, as is done in Overvest and Veldman (2008), for example. An investigation of the effect of cost parameter differences (here: $\gamma_1 > \gamma_2$) would yield more insight into the exact behavior of the outcomes of the game. Obviously we only consider the set Γ.

All the partial derivatives can be expressed in closed form, and when set to zero, all the solutions are closed form. An analysis of the derivatives leads us to the results given in table 6.1. Using this table, three main observations can be made:

- First, both players' product quantity and process innovation equilibria change in the same fashion as in the case without an incentive contract (see section 6.3.3), that is, a rival's increase in the process innovation cost parameter is beneficial for a firm and an increase in a firm's own process innovation cost negatively influences equilibrium outcomes;

- Second, the innovation weight equilibria decrease in the process innovation cost parameters. The intuition behind this result is that with an incentive contract, both firms commit to higher process innovation levels compared to the 'standard' one stage Cournot game. When the optimal innovation weight for one firm decreases due to an increase in the process innovation cost parameter, his optimal process innovation level will converge to the more 'natural' standard Cournot outcomes. This creates an opportunity for the rival firm to converge to the standard outcomes as well, thus permitting him to put less weight on process innovation through the use of an innovation weight;

- Third, the derivatives with respect to γ_2 are monotonic everywhere in Γ (with the exception of $\partial\lambda^*_1/\partial\gamma_2$). However, as is shown in the notes of the table, the derivatives with respect to γ_1 are not monotonic everywhere: the derivatives switch sign in some areas in Γ. This latter result can be illustrated further using the reaction functions. Reconsider, for example, firm 1's reaction function in product quantities, given in (6.9). The reaction function also holds for the equilibrium outcomes, implying that a firm's equilibrium product quantity is influenced directly through

Table 6.1. Comparative statics. Note that an upward arrow ↑ (downward arrow ↓) denotes an increase (decrease).

	γ_1 ↑	γ_j ↑	Note
q_1^*	↓ ↑$_{(a)}$	 ↑	[a] applies when $\gamma_1 < \frac{2((1-2\gamma_2)^2(3\gamma_2-4)+\sqrt{\zeta_1})}{(3\gamma_2-2)(12\gamma_2^2-23\gamma_2+8)}$, where $\zeta_1 = -\gamma_2(1-2\gamma_2)^2(3\gamma_2^2-6\gamma_2+2)$, $\forall\ \gamma_2 \in (\frac{23+\sqrt{145}}{24}, \frac{3+\sqrt{3}}{3})$.
q_2^*	↑ ↓$_{(b)}$	 ↓	[b] applies when $\gamma_1 > \frac{2((2\gamma_2-1)(3\gamma_2-5)-\sqrt{\zeta_2})}{(18\gamma_2^2-39\gamma_2+16)}$, where $\zeta_2 = (2\gamma_2-1)(3\gamma_2^2-6\gamma_2+2)/(3\gamma_2-2)$, $\forall\ \gamma_2 \in (\frac{3+\sqrt{3}}{3}, \frac{13+\sqrt{41}}{12})$.
x_1^*	↓ ↑$_{(c)}$	 ↑	[c] applies when $\gamma_1 < \frac{2((\gamma_2-1)(9\gamma_2-8)+\sqrt{\zeta_3})}{(3\gamma_2-4)(9\gamma_2-8)}$, where $\zeta_3 = -\gamma_j(9\gamma_2-8)(3\gamma_2^2-6\gamma_2+2)/(3\gamma_2-2)$, $\forall\ \gamma_2 \in (\frac{4}{3}, \frac{3+\sqrt{3}}{3})$.
x_2^*	↑ ↓$_{(d)}$	 ↓	[d] applies when $\gamma_1 > \frac{2((\gamma_2-4)(2\gamma_2-1)-\sqrt{\zeta_4})}{(\gamma_2-2)(15\gamma_2-8)}$, where $\zeta_4 = \gamma_2(2\gamma_2-1)(3\gamma_2^2-6\gamma_2+2)/(3\gamma_2-2)$, $\forall\ \gamma_2 \in (\frac{3+\sqrt{3}}{3}, 2)$.
λ_1^*	↓ ↑$_{(a)}$	↓ ↑$_{(e)}$	[e] applies when $\gamma_2 < \frac{2((1-2\gamma_1)^2(3\gamma_1-2)+\sqrt{\zeta_5})}{\gamma_1(9\gamma_1^2-9\gamma_1+2)}$, where $\zeta_5 = (2\gamma_1-1)^3(3\gamma_1-2)(3\gamma_1^2-6\gamma_1+2)$, $\forall\ \gamma_1 \in (2+\sqrt{2}, \infty)$.
λ_2^*	↓ ↑$_{(f)}$	 ↓	[f] applies when $\gamma_1 < \frac{2((1-2\gamma_2)^2(3\gamma_2-2)+\sqrt{\zeta_6})}{\gamma_j(9\gamma_2^2-9\gamma_2+2)}$, where $\zeta_6 = (2\gamma_2-1)^3(3\gamma_2-2)(3\gamma_2^2-6\gamma_2+2)$, $\forall\ \gamma_2 \in (\frac{3+\sqrt{3}}{3}, 2+\sqrt{2})$.

the firm 2's product quantity and the height of the innovation weight (and indirectly through the process innovation level). Differentiating this expression with respect to γ_1 yields

$$\frac{\partial q_1^*}{\partial \gamma_1} = \frac{1}{(2\gamma_1 - 1)^2}(q_2^* - 2\lambda_1^* - 1) + \frac{1}{2\gamma_1 - 1}\left[-\gamma_1 \frac{\partial q_2^*}{\partial \gamma_1} + \frac{\partial \lambda_1^*}{\partial \gamma_1}\right]. \quad (6.18)$$

It can be shown that the first component of the right-hand side in (6.18), i.e. $(2\gamma_1 - 1)^{-2}(q_2^* - 2\lambda_1^* - 1)$, is negative everywhere in Γ. Thus the sign of the entire derivative $\partial q_1^*/\partial \gamma_1$ depends only on the sign of the component between square brackets. It can be shown that the entire derivative is positive if and only if $\partial \lambda_1^*/\partial \gamma_1$ is positive (as a comparison, the derivative of the reaction function in the baseline case can be found by omitting the two terms in (6.18) that involve a λ). Analysis of the other second-stage equilibria derivatives, i.e. q_2, x_1 and x_2, lead to similar structures: a derivative (for example, $\partial x_1^*/\partial \gamma_2$) is determined by the way the rival's variable responds with respect to that parameter (in the case of this example $\partial x_2^*/\partial \gamma_2$) *and* the way the innovation weight is influenced (in the same case: $\partial \lambda_1^*/\partial \gamma_2$).

We summarize in the following proposition (proofs can be found in the appendix).

Proposition 6. *In Γ, for sufficiently large process innovation cost parameters (see table 6.1),*
(i) the product quantity and process innovation level of firm i are monotonically increasing in the rival's process innovation cost parameter γ_j, and monotonically decreasing in its own process innovation cost parameter γ_i, $i, j = 1, 2; i \neq j$;
(ii) the weight λ that is given to process innovation for firm i is monotonically decreasing in both the rival's process innovation cost parameter γ_j and its own process innovation cost parameter γ_i, $i, j = 1, 2; i \neq j$.

Two final results wrap up this comparative statics section. First, we can analyze the derivatives of firm profits with respect to the process innovation cost parameters. Since these derivatives are rather complex and monotonicity is hard to prove, we have to resort to numerical analysis. Recall that the profit expressions are normalized, making the sign of the derivatives (in the normalized cost parameter γ) independent on the parameters a, b and c. Numerical samples all show that both firms' profits monotonically increase in the rival's process innovation cost parameter, and monotonically decrease in their own process innovation cost parameter. This implies that the shown non-monotonicity of the derivatives of q and x do not incur any sign changes

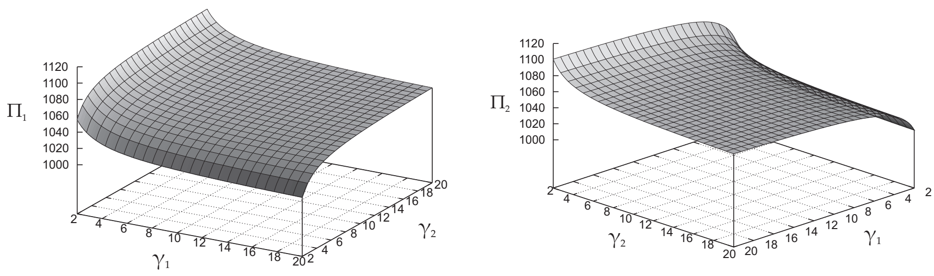

Figure 6.5. Firm profits ∗1000 when $a = 10000$, $b = 10$, $c = 250$.

in the derivatives of Π. An illustration is given in figure 6.5. Second, we earlier defined the process innovation cost parameter γ as an aggregate parameter, consisting of separate cost parameters for each process innovation $x_{i,k}$. It is straightforward to see that γ_i increases in $\theta_{i,k}$ and decreases in K. The implication for the comparative statics results of proposition 6 are given in the following corollaries.

Corollary 1. *When the relevant equilibria are increasing (decreasing) in γ_i, they are increasing (decreasing) in $\theta_{i,k}$ for a given K, $i = 1, 2$.*

Corollary 2. *When the relevant equilibria are increasing (decreasing) in γ_i, they are decreasing (increasing) in K, $i = 1, 2$.*

6.4 Discussion and conclusion

In the current chapter we extend an existing game-theoretic model for managerial incentives for process innovation in a duopolistic setting. We show that the incentive contract is always positive in the area where process innovation levels and product quantities are also positive. The firm with the lowest process innovation cost parameter innovates more, supplies higher product quantities on the market, receives a higher proportion of his process innovation level (i.e. his contract value is higher), and earns a higher profit. When we endogenize the decision to use an incentive contract, we observe that both firm owners will always offer their manufacturing managers such a contract.

Another significant contribution of our models is the insight into how equilibria change with a change in process innovation cost parameters. For a sufficiently large process innovation cost parameter (see table 6.1), the process innovation level and product quantity decrease for a firm in that firm's cost parameter, and increase in the rival firm's innovation cost parameter. However, one remarkable result is that both firms' innovation weights are decreasing in both firms' process innovation cost parameter, suggesting that the firms are innovating too much. When the incentive contract equilibrium for one firm decreases in a cost parameter, the best response of the other firm is to reduce the incentive contract value as well.

The analysis we provide has some limitations. One important assumption in the current framework is that cost reductions are measurable, and that these cost reductions can be traced back to certain actions taken by managers. Particularly in a highly innovative environment, this assumption would not always hold (see e.g. Loch and Tapper (2002) and Melnyk et al. (2004) for a discussion on performance measurement in operations management). Moreover, Christen (2005) showed that the acquisition of information is a strategic choice, and that cost uncertainty can actually act "like a 'fog' that lessens the destructive effect of price competition when products are close substitutes, and thus increase expected profits (Christen, 2005, p.668)".

An interesting extension of our framework would be the inclusion of other effects of process innovation. In the current framework process innovation is modeled as a cost reducing activity. Process innovation, however, can also lead to quality improvements and lead time reduction, which could change the competitive effects of innovation considerably. Moreover while process innovation can have a positive effect on one performance dimension, it can negatively influence another (Carrillo and Gaimon, 2002). As Repenning and Sterman (2002) showed, investments in process innovation do not always pay off. A relevant extension of the models presented here would therefore be the inclusion of a stochastic effect that incorporates uncertainty in unforeseeable events during production (as in Sommer and Loch (2009)) and the risk of failure (or being disruptive, as in Li and Rajagopalan (2008)). Nevertheless, even without these extensions we believe that the current models give substantial insight into the important role of managerial incentives and process innovation in competitive settings.

6.5 Managerial implications

Our models show that manufacturing managers who are explicitly rewarded for process innovation supply higher product quantities to the market and undertake more process innovation. Moreover, rewarding manufacturing managers for process innovation in a competitive duopolistic setting is optimal for both firm owners: if both firm owners acknowledge the use of an incentive contract as a relevant strategic variable, then that incentive contract will be used as a competitive weapon. Consumers benefit from such competitive interactions; market prices will decrease with the total product quantities put on the market.

The models are a typical representation of principal-agent problems. The manufacturing managers act as observers of the relevant market and process innovation cost parameters, and act according to what is in his and the owner's interest. When process innovation cost parameters are concerned, the high-cost producer will always be worse off: he produces less, innovates less, and the innovation weight will be lower. As a result, profits and the manufacturing manager's pay will be lower compared to the competitor. One important piece of insight for firm owners is provided by the comparative statics analysis. It gives the conditions under which the equilibria decrease or increase as process innovation cost parameters change. Such insights are relevant since firm owners are typically uncertain about market conditions and firms' cost functions, hence they are uncertain about manufacturing managers' decisions and the subsequent outcomes (in fact, this uncertainty is the main reason why firm owners design smart mechanisms to align their interests with the interests of manufacturing managers). The comparative statics can be used by firm owners to obtain some understanding about the outcomes if a process innovation cost parameter is not according to initial expectations.

The analysis provided is not necessarily limited to a single manufacturing manager's pay. He can, for example, instruct different teams in the manufacturing plant to optimize his utility function, and let their bonus be directly related to their contribution to the total level of process innovation. The aggregation variable described in this chapter can be the appropriate division mechanism for that.

Appendix

Derivation of the aggregate model

As in d'Aspremont and Jacquemin (1988) we could model the profit function with a single process innovation variable as

$$\Pi_i = (1 - q_i - q_j + x_i)q_i - \frac{1}{2}\theta_i x_i^2, \qquad i,j = 1,2; i \neq j. \tag{6.19}$$

The generalization of (6.19) for K process innovation types would become

$$\Pi_i = (1 - q_i - q_j + \sum_{k=1}^{K} x_{i,k})q_i - \frac{1}{2}\sum_{k=1}^{K}\theta_{i,k} x_{i,k}^2, \qquad i,j = 1,2; i \neq j. \tag{6.20}$$

As we will show in sub-section 6.3.3, one of the steps of solving the game-theoretic model is taking the first-order necessary condition $\frac{\partial \Pi_i}{\partial x_{i,k}} = 0$, $i = 1,2$. We will use this condition in order to redefine the profit function. The first-order necessary condition for an optimum with respect to $x_{i,k}$ can be defined as

$$\frac{\partial \Pi_i}{\partial x_{i,k}} = q_i - \theta_{i,k} x_{i,k} = 0, \qquad i = 1,2.$$

Solving this equation gives

$$x_{i,k} = \frac{q_i}{\theta_{i,k}}, \qquad i = 1,2. \tag{6.21}$$

We can define the sum of the process innovation variables as $x_i = \sum_{k=1}^{K} x_{i,k}$. Using (6.21) the following should hold at the optimum:

$$x_i = \sum_{k=1}^{K} \frac{q_i}{\theta_{i,k}} = q_i \sum_{k=1}^{K} \frac{1}{\theta_{i,k}}, \qquad i = 1,2. \tag{6.22}$$

Using (6.22) we can restate (6.21) as

$$x_{i,k} = \left(\frac{1}{\theta_{i,k}} / \sum_{k=1}^{K} \frac{1}{\theta_{i,k}}\right) x_i, \qquad i = 1,2. \tag{6.23}$$

From this expression we observe that the individual process innovation values are a weighted fraction of the aggregate process innovation value. Define $\gamma_i = \sum_{k=1}^{K} \frac{1}{\theta_{i,k}}$, then (6.20) can be written as (6.2). Note that in sub-section 6.3.4 the goal function would have become

$$S_i = (1 - q_i - q_j + \sum_{k=1}^{K} x_{i,k})q_i - \frac{1}{2}\sum_{k=1}^{K}\theta_{i,k} x_{i,k}^2 + \lambda_i \sum_{k=1}^{K} x_{i,k}, \qquad i,j = 1,2; i \neq j.$$

(6.24)

It should be clear that the transformation also applies to the model given in section 6.3.4.

Second-order conditions in the baseline case

To verify whether the sufficient second-order condition for profit maximization holds we inspect the Hessian of the two firms:

$$H = \begin{pmatrix} \frac{\partial^2 \Pi_i}{\partial q_i^2} & \frac{\partial^2 \Pi_i}{\partial q_i \partial x_i} \\ \frac{\partial^2 \Pi_i}{\partial x_i \partial q_i} & \frac{\partial^2 \Pi_i}{\partial x_i^2} \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -\gamma_i \end{pmatrix}, i = 1, 2.$$

Negative semi-definiteness for reaching a global maximum requires $\frac{\partial^2 \Pi_i}{\partial q_i^2} \leqslant 0$, $\frac{\partial^2 \Pi_i}{\partial x_i^2} \leqslant 0$ and the determinant of the entire matrix to be $\geqslant 0$, which holds when $\gamma_i \geqslant \frac{1}{2}$, $i = 1, 2$.

Second-order conditions in the case with an incentive contract

The sufficient second-order condition for profit maximization in the first stage is

$$\frac{\partial^2 \Pi_i}{\partial \lambda_i^2} = \frac{-\gamma_i(2 - 3\gamma_j)^2 + 2(1 - 2\gamma_j)^2}{(1 - 2\gamma_j + \gamma_i(3\gamma_j - 2))^2}, \qquad i, j = 1, 2; i \neq j. \tag{6.25}$$

This second-order condition is strictly negative if

$$\gamma_i > \frac{2(1 - 2\gamma_j)^2}{(2 - 3\gamma_j)^2}, \qquad i, j = 1, 2; i \neq j.$$

To ensure maximization in the second stage, we evaluate the Hessian

$$H = \begin{pmatrix} \frac{\partial^2 S_i}{\partial q_i^2} & \frac{\partial^2 S_i}{\partial q_i \partial x_i} \\ \frac{\partial^2 S_i}{\partial x_i \partial q_i} & \frac{\partial^2 S_i}{\partial x_i^2} \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -\gamma_i \end{pmatrix}, i = 1, 2.$$

Negative semi-definiteness for reaching a global maximum requires $\frac{\partial^2 S_i}{\partial q_i^2} \leqslant 0$ and $\frac{\partial^2 S_i}{\partial x_i^2} \leqslant 0$. Further, the determinant is $\geqslant 0$ when $\gamma_i \geqslant \frac{1}{2}$, $i = 1, 2$. In Γ this condition holds.

	P	**S**
P	$\Pi_1^{*P} = \gamma_1(2\gamma_1 - 1)(\gamma_2 - 1)^2/\Phi_1^P$, $\Pi_2^{*P} = \gamma_2(2\gamma_2 - 1)(\gamma_1 - 1)^2/\Phi_2^P$	$\Pi_1^{*PS} = \gamma_1(2\gamma_1 - 1)\times$ $(2 - 2\gamma_2 + \gamma_1(3\gamma_2 - 4))^2/\Phi_1^{PS}$, $\Pi_2^{*PS} = \gamma_2(\gamma_1 - 1)^2/\Phi_2^{PS}$
S	$\Pi_1^{*SP} = \gamma_1(\gamma_2 - 1)^2/\Phi_1^{SP}$, $\Pi_2^{*SP} = \gamma_2(2\gamma_2 - 1)\times$ $(2 - 2\gamma_1 + \gamma_2(3\gamma_1 - 4))^2/\Phi_2^{SP}$	$\Pi_1^{*S} = \gamma_1(\gamma_1(2 - 3\gamma_2)^2 - 2(1 - 2\gamma_2)^2)\times$ $(\gamma_1(3\gamma_2 - 4) - 2\gamma_2 + 2)^2/\Phi$, $\Pi_2^{*S} = \gamma_2(\gamma_2(2 - 3\gamma_1)^2 - 2(1 - 2\gamma_1)^2)\times$ $(\gamma_2(3\gamma_1 - 4) - 2\gamma_1 + 2)^2/\Phi$

Note: $\Phi_1^P = \Phi_2^P = 2(1 - 2\gamma_2 + \gamma_1(3\gamma_2 - 2))^2$,
$\Phi_1^{PS} = (\gamma_2(2 - 3\gamma_1)^2 - 2(1 - \gamma_1)^2)^2$,
$\Phi_2^{PS} = \gamma_2(2 - 3\gamma_1)^2 - 2(1 - \gamma_1)^2$,
$\Phi_1^{SP} = \gamma_1(2 - 3\gamma_2)^2 - 2(1 - \gamma_2)^2$,
$\Phi_2^{SP} = (\gamma_1(2 - 3\gamma_2)^2 - 2(1 - \gamma_2)^2)^2$ and
$\Phi = (\gamma_1^2(3\gamma_2 - 2)(9\gamma_2 - 8) - 2\gamma_1(3\gamma_2 - 2)(7\gamma_2 - 4) + 4(1 - 2\gamma_2))^2$.

Figure 6.6. Normalized payoff matrix.

Proof of proposition 5

Proof. Profits within the four sub-games are given in figure 1.6. Showing that (S, S) is the only pure strategy Nash equilibrium (given $\gamma_1 > \gamma_2$) can be done through the elimination of implausible strategies. We will first show that owner 1 will always choose strategy S, by demonstrating that $\Pi_1^{*SP} > \Pi_1^{*P}$ and $\Pi_1^{*S} > \Pi_1^{*PS}$. In order to compare profits we first have to verify positivity of the process innovation and product quantity variables in all four sub-games. We assume active equilibria and analyze the open area of the $\gamma_1 - \gamma_2$ plane (that is potentially bounded by the positivity conditions). Again we will use the appropriate subscripts to distinguish the outcomes in the different sub-games. Note that process innovation levels and product quantities in the (P, P) sub-game are given in (6.4) and (6.5), and the relevant equilibria in the (S, S) sub-game are given in (6.13) and (6.14). The outcomes in the (P, S) sub-game are as follows:

$$q_1^{*PS} = \frac{\gamma_1(3\gamma_1\gamma_2 - 4\gamma_1 - 2\gamma_2 + 2)}{\gamma_2(2 - 3\gamma_1)^2 - 2(1 - \gamma_1)^2}, \tag{6.26}$$

$$x_1^{*PS} = \frac{3\gamma_1\gamma_2 - 4\gamma_1 - 2\gamma_2 + 2}{\gamma_2(2 - 3\gamma_1)^2 - 2(1 - \gamma_1)^2}, \tag{6.27}$$

$$q_2^{*PS} = \frac{\gamma_2(\gamma_1 - 1)(3\gamma_2 - 2)}{\gamma_2(2 - 3\gamma_1)^2 - 2(1 - \gamma_1)^2}, \tag{6.28}$$

$$x_2^{*PS} = \frac{2(\gamma_1 - 1)(3\gamma_2 - 2)}{\gamma_2(2 - 3\gamma_1)^2 - 2(1 - \gamma_1)^2}. \tag{6.29}$$

The outcomes in the (S, P) sub-game can be found by swapping all the subscripts of the left-hand side and right-hand side in (6.26) - (6.29).

It is easy to see that in the (P, P) sub-game $q_1^{*P} > 0$, $x_1^{*P} > 0$, $q_2^{*P} > 0$ and $x_2^{*P} > 0$ if $\gamma_1 > 1$ and $\gamma_2 > 1$. Also in the (S, P) sub-game, $q_1^{*SP} > 0$, $x_1^{*SP} > 0$ if $\gamma_1 > 1$ and $\gamma_2 > 1$. However, $q_2^{*SP} > 0$, $x_2^{*SP} > 0$ if $\gamma_2 > \frac{2\gamma_1 - 2}{3\gamma_1 - 4}$, which is convex in the interval $\gamma_1 \in (\frac{4}{3}, \frac{3+\sqrt{3}}{3})$. The intersection of $\frac{2\gamma_1-2}{3\gamma_1-4}$ with the diagonal yields the well-known intersection point $(\frac{3+\sqrt{3}}{3}, \frac{3+\sqrt{3}}{3})$, whereas equating $\frac{2\gamma_1-2}{3\gamma_1-4}$ with 1 (i.e. the lower boundary of γ_2) yields $\gamma_1 = 2$. Thus the minimum value for γ_2 in the interval $\gamma_1 \in (\frac{3+\sqrt{3}}{3}, 2)$ is $\frac{2\gamma_1-2}{3\gamma_1-4}$; the minimum value for γ_2 in the interval $\gamma_1 \in (2, \infty)$ is 1.

Equating profit functions Π_1^{P*} and Π_1^{*SP} yields the solutions $\gamma_1 = 0$, $\gamma_2 = 0$ and $\gamma_2 = 1$. Now it is easy to verify that $\Pi_1^{*SP} > \Pi_1^{P*}$ if the positivity conditions in the (P, P) and (S, P) sub-games are met. The positivity conditions in the (P, S) and (S, S) sub-games are similar: both firms' process innovation and product quantity equilibria are strictly positive if $\gamma_1 > \frac{3+\sqrt{3}}{3}$, $\gamma_2 > \frac{4\gamma_1-2}{3\gamma_1-2}$ and $\gamma_2 > \frac{4}{3}$. Equating Π_1^{*PS} and Π_1^{*S} yields the solutions $\gamma_2 = 0$, $\gamma_2 = \frac{4\gamma_1-2}{3\gamma_1-2}$ and $\gamma_2 = \frac{2((3\gamma_1-2)^3 \pm \sqrt{\zeta_7})}{(2-3\gamma_1)^2(9\gamma_1-8)}$, where $\zeta_7 = \gamma_1(2 - 3\gamma_1)^2(2 + \gamma_1(\gamma_1(9\gamma_1 - 8) - 2))$. Using simple algebra it can be demonstrated that $\frac{4\gamma_1-2}{3\gamma_1-2} > \frac{2((3\gamma_1-2)^3 \pm \sqrt{\zeta_7})}{(2-3\gamma_1)^2(9\gamma_1-8)}$. Thus $\Pi_1^{*S} > \Pi_1^{*PS}$ when the positivity conditions are satisfied. Since we have already found that $\Pi_1^{*SP} > \Pi_1^{P*}$, it is proved that P is an implausible strategy for firm owner 1, and that firm owner 1 will always choose strategy S.

We continue by showing that firm owner will always choose strategy S as well. As was shown above, the positivity conditions in the (P, P) sub-game are satisfied if $\gamma_1 > 1$ and $\gamma_2 > 1$. In the (P, S) sub-game, $q_1^{*PS} > 0$, $x_1^{*PS} > 0$, $q_2^{*PS} > 0$ and $x_2^{*PS} > 0$ if $\gamma_1 > \frac{3+\sqrt{3}}{3}$, $\gamma_2 > \frac{4\gamma_1-2}{3\gamma_1-2}$ and $\gamma_2 > \frac{4}{3}$ (the latter constraint is found by taking the limit $\lim_{\gamma_1 \to \infty} \frac{4\gamma_1-2}{3\gamma_1-2} = \frac{4}{3}$).

Equating Π_2^{*P} and Π_2^{*PS} yields the solutions $\gamma_1 = 0$, $\gamma_2 = 0$ and $\gamma_1 = 1$. It can be verified that $\Pi_2^{*PS} > \Pi_2^{*P}$ if $\gamma_1 > 1$ and $\gamma_2 > 1$.

As we mentioned, the relevant equilibria in the (S, P) sub-game are strictly positive for firm 1 if $\gamma_1 > \frac{3+\sqrt{3}}{3}$, and the minimum value for γ_2 is determined by the relevant intervals of γ_1: in the interval $\gamma_1 \in (\frac{3+\sqrt{3}}{3}, 2)$ is the minimum value for γ_2 is $\frac{2\gamma_1-2}{3\gamma_1-4}$. The minimum value for γ_2 in the interval

$\gamma_1 \in (2, \infty)$ is 1. In the (S, S) sub-game, the positivity conditions are satisfied if $\gamma_1 > \frac{3+\sqrt{3}}{3}$, $\gamma_2 > \frac{4\gamma_1-2}{3\gamma_1-2}$ (or $\gamma_1 > \frac{2\gamma_2-2}{3\gamma_2-4}$) and $\gamma_2 > \frac{4}{3}$. Equating Π_2^{*SP} and Π_2^{*S} yields $\gamma_1 = 0$, $\gamma_1 = \frac{4\gamma_2-2}{3\gamma_2-2}$ (or: $\gamma_2 = \frac{2\gamma_1-2}{3\gamma_1-4}$) and $\gamma_1 = \frac{2((3\gamma_2-2)^3 \pm \sqrt{\zeta_8})}{(2-3\gamma_2)^2(9\gamma_2-8)}$, where $\zeta_8 = \gamma_2(2-3\gamma_2)^2(2+\gamma_2(\gamma_2(9\gamma_2-8)-2))$. Since $\frac{2\gamma_2-2}{3\gamma_2-4} > \frac{4\gamma_2-2}{3\gamma_2-2}$ and $\frac{2\gamma_2-2}{3\gamma_2-4} > \frac{2((3\gamma_2-2)^3 \pm \sqrt{\zeta_8})}{(2-3\gamma_2)^2(9\gamma_2-8)}$, $\Pi_2^{*S} > \Pi_2^{*SP}$ if the positivity conditions for the (S, S) sub-game are satisfied. Thus the only plausible strategy for firm owner 2 is S. Since we have already found that firm owner 1 will also choose strategy S, (S, S) is the only pure strategy Nash equilibrium. This completes our proof. □

Proof of proposition 4(iii)

Proof. We can use the direct expressions of the profit levels to prove that $\Pi_2^* > \Pi_1^*$ in Γ. Subtracting Π_1^* from Π_2^* yields a fourth-degree polynomial in both γ_1 and γ_2. Since $\gamma_1 = \gamma_2$ is a solution for the equality $\Pi_1^* = \Pi_2^*$, the expression $(\Pi_2^* - \Pi_1^*)$ can be divided by $(\gamma_2 - \gamma_1)$, which results in an analyzable cubic polynomial in γ_1 and γ_2:

$$\frac{\Pi_2^* - \Pi_1^*}{\gamma_2 - \gamma_1} = \frac{16\gamma_1^3(2-3\gamma_2)^2(\gamma_2-1) - 2\gamma_1^2(7\gamma_2-4)(24\gamma_2^2-35\gamma_2+12)}{\Phi^2} + \frac{16\gamma_1(1-2\gamma_2)^2(4\gamma_2-3) - 8(2\gamma_2-1)^3}{\Phi^2}. \tag{6.30}$$

where Φ is the well-known denominator in the Nash equilibria (note that we have already defined that $\Phi > 0,\ \forall\ \gamma_1, \gamma_2 \in \Gamma$). Cubic polynomials have three solutions, of which either one or three solution(s) is (are) real. Three real solutions exist if the discriminant (which, for a cubic polynomial $ax^3 + bx^2 + cx + d$, is given by $b^2c^2 - 4ac^3 - 4b^3d - 27a^2d^2 + 18abcd$) is larger than zero. When we express the right-hand side of (6.30) as a function of γ_1, the discriminant is defined as

$$256\gamma_2^4(2\gamma_2-1)^3(192\gamma_2^4 - 696\gamma_2^3 + 941\gamma_2^2 - 536\gamma_2 + 108).$$

Since $\gamma_2 > \frac{4}{3}$ in Γ we can introduce a parameter ϕ that satisfies $\gamma_2 = \phi + \frac{4}{3}$. Upon substitution we find that the discriminant becomes

$$\left(\frac{1}{59049}\right)(256(3\phi+4)^4(6\phi+5)^3(5184\phi^4+8856\phi^3+5535\phi^2+2208\phi+628)),$$

which is clearly $> 0,\ \forall\ \phi > 0$, implying that the discriminant is strictly positive in Γ. This proves that the right-hand side of (6.30) has three real solutions (note that from Descartes' rule of signs it is easy to see that the roots of the numerator of (6.30) in γ_1 are positive for $\gamma_2 > \frac{4}{3}$) . It can be shown that the largest of the three solutions, namely

$$\gamma_2 = \frac{\delta_1}{\delta_2} + \frac{\delta_3 + (48\sqrt{3}\sqrt{\delta_4} + \delta_5)^{\frac{2}{3}}}{\delta_1 + (48\sqrt{3}\sqrt{\delta_4} + \delta_5)^{\frac{1}{3}}}, \tag{6.31}$$

where
$\delta_1 = 168\gamma_1^3 - 341\gamma_1^2 + 224\gamma_1 - 48$,
$\delta_2 = 24(3\gamma_1 - 2)^2(\gamma_1 - 1)$,
$\delta_3 = \gamma_1^2(8\gamma_1 - 5)(72\gamma_1^3 - 165\gamma_1^2 + 128\gamma_1 - 32)$,
$\delta_4 = -(3\gamma_1 - 2)^4(2\gamma_1 - 1)^3(\gamma_1 - 1)^2(192\gamma_1^4 - 696\gamma^3 + 941\gamma_1^2 - 536\gamma_1 + 108)$,
$\delta_5 = 13824\gamma_1^9 - 91584\gamma_1^8 + 244152\gamma_1^7 - 343277\gamma_1^6 + 277440\gamma_1^5 - 129840\gamma_1^4 + 2768\gamma_1^3 - 3456\gamma_1^2$,
is increasing and concave with $\gamma_2 \to \frac{2}{3}$ as $\gamma_1 \to \infty$. This solution is clearly outside Γ so that the other two solutions are also outside Γ. It can now be checked that everywhere in Γ, $\Pi_2 > \Pi_1$. □

Proof of proposition 6

Proof. We only consider cases in Γ. All the partial derivatives of q_i^*, x_i^*, and λ_i^*, $i = 1, 2$, with respect to γ_1 or γ_2 have a quadratic numerator in either γ_1 or γ_2 and a denominator Φ^2 (which is positive in Γ). Consider the following derivative (i.e. note (a) in table 6.1):

$$\begin{aligned}\frac{\partial \lambda_1^*}{\partial \gamma_1} &= \frac{-2\gamma_2(\gamma_1^2(3\gamma_2 - 2)(12\gamma_2^2 - 23\gamma_2 + 8) - 4\gamma_1(1 - 2\gamma_2)^2(3\gamma_2 - 4))}{\Phi^2} \\ &\quad - \frac{8\gamma_2(1 - 2\gamma_2)^2(\gamma_2 - 1)}{\Phi^2}.\end{aligned} \tag{6.32}$$

When we set the numerator to zero, we get the solutions $\gamma_2 = 0$,

$$\gamma_1 = \frac{2((1 - 2\gamma_2)^2(3\gamma_2 - 4) + \sqrt{\zeta_1})}{(3\gamma_2 - 2)(12\gamma_2^2 - 23\gamma_2 + 8)} \tag{6.33}$$

and

$$\gamma_1 = \frac{2((1 - 2\gamma_2)^2(3\gamma_2 - 4) - \sqrt{\zeta_1})}{(3\gamma_2 - 2)(12\gamma_2^2 - 23\gamma_2 + 8)}, \tag{6.34}$$

where $\zeta_1 = -\gamma_2(1 - 2\gamma_2)^2(3\gamma_2^2 - 6\gamma_2 + 2)$. Solution (6.33) is continuous and convex in Γ with the well-known intersection point $(\frac{3-\sqrt{3}}{3}, \frac{3+\sqrt{3}}{3})$ and a minimum value $\gamma_2 \to 1.460$ when $\gamma_1 \to \infty$ (note that the value $\gamma_2 = 1.460$ is derived from the second component in the denominator of (6.33) and also that $\zeta_1 < 0$ when $\gamma_2 > \left(\frac{3+\sqrt{3}}{3}\right)$, making the solutions shift from real to complex).

Similar analyses can be made for the other derivatives; therefore they are omitted here. Table 6.1 gives all the relevant conditions and intervals for the different derivatives. □