Distributed MPC for controlling mu-CHPs in a network

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Distributed MPC for controlling μ-CHPs in a network.

Gunn Larsen, Sebastian Trip, Nicky van Foreest and Jacquelien Scherpen

Abstract— This paper describes a dynamic price mechanism to coordinate electricity generation from micro Combined Heat and Power (μ-CHP) systems in a network of households. The control is done on household level in a completely distributed manner. Distributed Model Predictive control is applied to the network of households with μ-CHP installed. Each house has a unique demand pattern based on realistic data. Information from a few neighbors are taken into account in the local optimal control problems. Desired behavior for the network model in the distributed MPC approach is showed by simulation.

I. INTRODUCTION

The electricity grid is a highly complex system consisting of several interconnected mechanisms and interest groups. The government wants a fair, environmental friendly and reliable grid, suppliers want to earn from generation, consumers want cheap electricity to cover their comfort levels, and the network owners want efficient transport within safety limits. Environmental awareness over the last decades has realized the integration of distributed generation units. Both green energy production and local use of local generated power give an environmental benefit. If we achieve local balancing, the overall energy efficiency is improved, because network losses are avoided. In countries that have a dense gas grid, such as the Netherlands, an interesting candidate for domestic generation is a micro Combined Heat Power (μ-CHP) system. A μ-CHP unit produces heat and electricity.

Control strategies for the μ-CHP available in the literature are both heat demand and electricity demand driven. Both of which can be shown to be economically beneficial. In [1] it was shown that for a Stirling engine, gas engine and a solid oxide fuel cell that a combination of the two are more economical beneficial. The model of one μ-CHP (proton exchange membrane fuel cell) is described in [2] where Model Predictive Control (MPC) is used to control the system. A Mixed Integer Linear Program [3] is solved at each time step, to include the logics of a strictly on or off state. In [2] demand response with help of μ-CHP is studied.

Controlling such μ-CHP system to use the electricity in one household only is not significantly affecting the electricity grid. However, large scale embedding in the electricity grid and thus a change in the generation topology introduce new control challenges to an already highly complex system. The grid requires a real time balance of supply and demand,

since electricity can not be stored efficiently on large scale. In the old electricity grid situation, a few central power plants were controlled to meet the fluctuating demand. In the new situation there are many smaller units to be controlled. To achieve this control by one centralized controller may be impractical, or even impossible. The optimal control problem gets to be a very complex one when underlying uncertainties and technical specifications of all units are taken into account. Turning for distributed solutions seems to be a necessity.

Hence, we are interested in the problem of matching supply and demand locally at household level. Each household in the network performs an optimal control problem, based on local information. Prices may be communicated amongst a few neighbors in the network to improve the overall performance. With this motivation, we are looking for a dynamic price mechanism to coordinate production between houses in the network. It is essential to note that the households in this network are both producers and consumers of electricity.

Distributed optimal control via dual decomposition methods, e.g., [4], appears to be an attractive method to achieve matching of supply and demand in the electricity grid. Via the dual decomposition in fact distributed dynamic price patterns can be achieved. Due to the operational constraints of μ-CHP's, Model Predictive Control (MPC) seems to be a useful approach to solve optimal control problems subject to dynamic models, input and state constraints [5]. The extension of the distributed optimal control settings via dual decomposition methods to an MPC setting is treated in [6]. Here we study the use of the latter method for application to the (large scale) embedding of μ-CHP's in the electricity grid.

In [7] we have introduced dynamic price mechanisms, and used a more centralized version of the optimal control problem to balance a network of prosumers with μ-CHP systems. The model of the current electricity grid was compared to a fully distributed grid topology, but the results still lack the inclusion of constraints that are inherently present when using μ-CHP's, and the computations where only partly performed in a distributed sense. Here we extend those results to the application of the fully distributed MPC setting via dual decomposition and gradient iterations as presented in [6], and we include more realistic modeling considerations corresponding to the μ-CHP. Furthermore, we extend our analysis by using demand data of different types of households, and we check the scalability of the problem.

The structure of the paper is as follows; In Section II a review of the distributed MPC framework is given, in Section

III a detailed description of the network model we propose is given. Section IV presents the simulation results and Section V ends with concluding remarks.

II. REVIEW

We aim to model a network of households with μ-CHP in a distributed Model Predictive Control (MPC) setting. We use MPC to handle constraints and predictions models, and we use the distributed control setting so that local controllers only have to take into account local information.

Here we give a quick review of a distributed MPC scheme for convex problems, as described in [6]. The technique is based on dual decomposition and sub-gradient iterations.

Since the 1960s dual decomposition methods for finding the optimal control trajectories have been developed. Later these decomposition methods were also developed for dynamical large-scale optimization problems. The original problem is replaced by several smaller subproblems. Each subproblem is isolated, except through a small interface depending on the structure of the original problem. In [8] it has been shown that dynamic price mechanisms result from the dual decomposition method for distributed optimization of feedback systems. In [6] the method is combined with Model Predictive Control (MPC).

Consider a distributed system given by state equations

$$x^{k+1} = Ax^k + Bu^k + w^k, \qquad (1)$$

where k is the discrete time variable, x^k is the to-be-controlled vector of n users, u^k contains the m control inputs, and w^k contains the n disturbances. Information matrix A is an $n \times n$ matrix that specifies the topology of the network, and B is an $n \times m$ input matrix.

Let i be a user in the network. The state, input and disturbance take its values in

$$
\begin{aligned}
x_i^k &\in X_i, \quad i = 1, ..., n, \quad \forall k \in \mathbb{Z} \\
u_i^k &\in U_i, \quad i = 1, ..., m, \quad \forall k \in \mathbb{Z} \\
w_i^k &\in W_i, \quad i = 1, ..., n, \quad \forall k \in \mathbb{Z}
\end{aligned}
\qquad (2)
$$

where the sets X_i, U_i, W_i are constraining sets. The disturbances w_i^k are assumed bounded $|w_i^k| \leq w_{max}$. Models with this boundness assumption and dual decomposition is presented in [9].

There is a local cost $l_i(x_i^k, u_i^k)$ associated with each user i at every time step k, where $l_i \geq 0$ and with $l_i(0,0) = 0$. This cost is assumed to be independent in time. The objective is to find the sequence $\{u^k\}_{k=0}^{\infty}$ given initial values x^0 that minimize value function

$$V^{\infty}(x^0, u^0, ..., u^{\infty}) = \sum_{k=0}^{\infty} \sum_{i=1}^{n} l_i(x_i^k, u_i^k), \qquad (3)$$

for the system (1) with (2).

When the above problem is not possible to solve, because of constraints and because we want to incorporate new measurements of w^k on each time step k, we formulate the problem in a MPC setting.

To use over the MPC horizon N, we introduce a new discrete time variable $\tau = 0, ..., N$ starting at time k, and replace the minimization of $V^{\infty}(x^0, u^0, ..., u^{\infty})$ with the minimization of

$$V^N(x^k, \hat{u}^0, ..., \hat{u}^N) = \sum_{\tau=0}^{N} \sum_{i=1}^{n} l_i(\hat{x}_i^{\tau}, \hat{u}_i^{\tau}), \qquad (4)$$

for the system (5c) given the real state x^k at time k. The hat notation is to distinguish variables used in the finite horizon problem and variables used for the real system (1). Equation (1) is used to predict $\hat{x}^{\tau}$. According to the receding horizon principle of MPC, after obtaining the finite optimal control sequence, only the first control input is implemented in (1), $u^k = \hat{u}^{\tau=0}$.

The *Centralized MPC* problem recalculated at every k is given by

$$V_{opt}^N = \min_{\hat{u}^{\tau}} V^N(x^k, \hat{u}^0, ..., \hat{u}^N), \qquad (5a)$$

s.t.

$$\hat{x}^{\tau=0} = x^k, \qquad (5b)$$

$$\hat{x}^{\tau+1} = A\hat{x}^{\tau} + Bu^{\tau} + \hat{w}^{\tau}, \quad \tau = 0, ..., N, \qquad (5c)$$

$$\hat{x}^{\tau} \in X = X_1 \times ... \times X_n, \quad \tau = 0, ..., N, \qquad (5d)$$

$$\hat{u}^{\tau} \in U = U_1 \times ... \times U_n, \quad \tau = 0, ..., N, \qquad (5e)$$

where $\hat{x}^{\tau}, \hat{w}^{\tau}$ represents the predicted states and disturbances of $x^{k+\tau}$ and $w^{k+\tau}$.

Next we need to obtain a distributed formulation of (5) using dual decomposition and sub-gradient iterations. The first step is to decouple state equations (1) using dual decomposition. We see that the right hand side in (1) depends on neighboring states through the information matrix A. Each user introduces a local variable v^k representing the guess of expected influence from connected users. Additional equality constraints are introduced, since a guess about neighbors' action should agree with the neighbor's reality. The system is now given by the decoupled state equations

$$x^{k+1} = A_D x^k + Bu^k + v^k + w^k \qquad (6)$$

$$\text{s.t.} \quad v^k = A_o x^k \qquad (7)$$

where $A_D = \text{diag}(A)$ and $A_o = A - A_D$.

The constraints (7) are relaxed by introducing Lagrangian multipliers λ^k to cost function (5a), which are interpreted as prices [4]. We obtain an *Almost distributed MPC* formulation of (5). The solution is the same as for the centralized MPC under convexity assumptions, i.e. $l_1, ..., l_n$ are convex [6].

$$V_{opt}^N = \max_{\lambda} \min_{\hat{u}^{\tau}, \hat{x}^{\tau}, \hat{v}^{\tau}} \sum_{\tau=0}^{N} l(\hat{x}^{\tau}, \hat{u}^{\tau}) + (\lambda^{\tau})^T (\hat{v}^{\tau} - A_o \hat{x}^{\tau}) =$$

$$\max_{\lambda} \sum_{i} \min_{\hat{u}_i^{\tau}, \hat{x}_i^{\tau}, \hat{v}_i^{\tau}} \sum_{\tau=0}^{N} l_i(\hat{x}_i^{\tau}, \hat{u}_i^{\tau}) + \lambda_i^{\tau} \hat{v}_i^{\tau} - (\sum_{j \neq i} \lambda_j^{\tau} A_{ji}) \hat{x}_i^{\tau}$$

s.t. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (8)$

$$\hat{x}_i^{\tau=0} = x_i^k, \quad i = 1, ..., n,$$

$$\hat{x}_i^{\tau+1} = A_{ii}\hat{x}^{\tau} + B_i \hat{u}^{\tau} + \hat{v}_i^{\tau} + \hat{w}_i^{\tau}, i = 1, .., n, \tau = 0, .., N,$$

$$\hat{x}_i^{\tau} \in X_i, \quad i = 1, ..., n, \quad \tau = 0, ..., N,$$

$$\hat{u}_i^{\tau} \in U_i. \quad i = 1, ..., n, \quad \tau = 0, ..., N,$$

where $l(\cdot) = \sum_{i=1}^n l_i(\cdot)$. The inner minimization problem is now fully distributed. Only price information from connected users are needed. However, as the problem is stated above, global coordination to obtain the right prices in the outer maximization problem is still required. Therefore, we added the word "Almost" in the method.

In order to make the problem fully distributed, in [6] gradient iterations are included. We call the new value function in (8) $V^N(x^k, \hat{u}^0, ..., \hat{u}^N, \lambda^0, ..., \lambda^N)$. We observe that $V^N(x^k, \hat{u}^0, ..., \hat{u}^N, \lambda^0, ..., \lambda^N)$ is concave in λ, even if the original problem is not convex [10]. The optimal price sequence can be found with the means of gradient iterations. When $\nabla_\lambda V^N(x^k, \hat{u}^0, ..., \hat{u}^N, \lambda^0, ..., \lambda^N) = 0$ the constraints from (7) are met.

Prices are updated according to

$$\lambda_{i,r+1}^\tau = \lambda_{i,r}^\tau + \gamma_{i,r}[v_{i,r}^\tau - \sum_{j \neq i} A_{ij} x_{j,r}^\tau], \quad \tau = 0, ..., N \quad (9)$$

In this way the price updates are also completely distributed, only depending on neighboring users. Gradient iterations (9) are performed over subscripts r, and $\gamma_{i,r}$ chosen such that we converge to the optimum. A *Completely distributed MPC* is obtained.

In order for the completely distributed formulation to converge to problem (8), the inner minimization problem of (8) and gradient iterations (9) need to be solved iteratively. The convergence might need many iterations. A stopping criterion that guarantees a sub optimal bound is therefore given in [6].

Even though (9) converges for non convex problems, the combined problem of gradient iterations and minimization of the inner problem might not converge if the problem is not convex. If input set U is discrete we might end up in an alternating solution, as we will see later.

If X_i, U_i are convex sets, and the cost functions $\sum_{i=1}^n l_i(x_i^k, u_i^k)$ are convex, we are guaranteed that the centralized MPC problem (5) and the decentralized MPC problem (8) provide the same solution. Thus, the completely distributed MPC gives the optimal solution, if we iterate to convergence.

III. SYSTEM DESCRIPTION

In this section we describe our model of a network of households where each house is both producer and consumer of electricity. Such households are often refereed to as "prosumers" in the literature. For the network of prosumers it is a common goal to balance production and consumption of electricity. We seek to describe a mechanism balancing the amount of electricity withdrawn and added to the electricity grid on household level.

One strategy to achieve balance in the network is to match production and consumption only inside each house. However, it is clear that if neighbors cooperate and share some information with each other, the balancing could be done more efficiently. Imagine house A has a high demand, but no generation opportunity. House B is using their μ-CHP system to cover heat demand, but has excess electricity

production. House B could earn money by selling electricity to neighbor A, while the electricity is still locally produced and consumed in the network. Motivated by this, we wish to have a communication structure in the network.

We achieve this communication through dynamic coupling between the houses' notion of imbalance, x^k, at each node

$$x^{k+1} = Ax^k + Bu^k + w^k. \quad (10)$$

Here the imbalance x^k is the difference between production and demand, u^k is change in production, i.e. $u^k = p^{k+1} - p^k$ where p^k is the production at time k, and w^k is change in demand, i.e. $w^k = d^{k+1} - d^k$ where d^k is the demand at time k. At time $k = 0$, the imbalance is initiated $x^0 = d_i^0 - p_i^0$. As time evolves, x_i^k becomes a combination of imbalances on several nodes. For this reason we call x^k the notion of imbalance at each node. The change in demand is bounded $|w^k| \leq w_{\max}$, since the network connections are secured by the network. Underlying uncertainties, demand patterns and technical specifications on the μ-CHP are allowed to vary from household to household.

Each household only controls its own μ-CHP. Thus, every column and row of B contain maximally one non-zero element. If there is no generator present in household i then $B_{ii} = 0$. The topology of the network is given by the A matrix.

A. Information matrix A

The information matrix A specifies the direction and weight of communication in the network. Imbalance information (or prices) are only communicated to neighbors connected with an edge. As an example; we have a directed graph $D = (H_n, E_n)$, with n households. The household set is given by $H_n = \{1, ..., n\}$, and $E_n \subseteq H_n \times H_n$ denotes the edge set. There is an edge in the graph $((i, j) \in E_n)$ whenever information is communicated directly from household i to household j. Figure 1 displays a graph where $n = 5$, and the arrows represent the edges. The graph structure is given by the information matrix A, where $A_{ij} \neq 0$ if and only if $(i, j) \in E_n$. The columns of A should sum up to one $\sum_j A_{ij} = 1$, to ensure that the total amount of imbalance $\sum_i x_i^k$ does not increase or decrease with time due to other effects than control inputs u^k and changes in demand w^k. We have chosen the weights in A nonnegative. An entry A_{ij} equal to zero means that the there is no information shared between the corresponding households. The higher the value of A_{ij} is, the larger the part of the imbalance from this household j is regulated in the local optimization problem of household i.
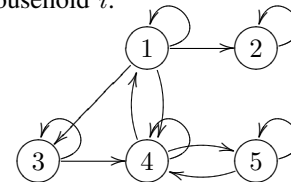

Fig. 1. A graph with five users. The arrow from household i to household j indicates the direction of information flow. Self-loops come from diagonal elements of A.

B. The local cost

We want to obtain a perfect balance between electricity production and electricity consumption in the network. In other words we aim to regulate the states x^k to the origin. In addition we want to achieve this as cheap as possible in u^k. We choose a quadratic cost function, to ease computation and assure feasibility. An explicit expression for l_i in eq (4) is

$$l_i(x_i^k, u_i^k) = R_i(x_i^k)^2 + Q_i(u_i^k)^2 \qquad (11)$$

with weigths $R_i > 0$, $Q_i \geq 0$.

C. The Prices

Lagrange multipliers have the intuition of being prices that optimizers would pay to loosen constraints. This intuition serves well here, where we use prices to decentralize a centralized optimum.

It is interesting to note that each household i has a unique price λ_i, which in general differs from prices elsewhere in the network. The price changes locally depending on the local imbalance x_i^k. When the price rises at one node, the prices at connected nodes also rise slowly with time as long as there is still imbalance that is not compensated for. When local production is possible, fluctuations in price will not travel far in the network.

D. The μ-CHP

In this paper we are interested in the essential characteristics of an operating μ-CHP unit. We want to implement constraints of such system in the distributed MPC setting described in section II. Therefore we abstract out from the details of a specific μ-CHP unit that is present in our lab.

We set a minimum on time T_{on} that the device has to be on after it is turned on. This represents the start up phase, when the μ-CHP should not be turned off. We also consider a minimum off time T_{off}, as some of the μ-CHPs need to cool off before starting up again. The production p^k, is between zero when it is off and at maximum p_{max} when it is on.

Ultimately we want the power output to either zero or within a range $p_{min} \leq p^k \leq p_{max}$, which makes the input set U non convex. Since we run into difficulties due to the non convexity and logics connected to the constraints, we describe two minimization problems.

The problems are solved at each household i given prices λ from neighbor houses. The first problem is convex, which makes it easy to solve. The second one is more close to reality, but requires more computation effort, and sometimes the solution do not converge. We want to see if devices with the above characteristics can be controlled in a network of households, to schedule a balance between electricity production and consumption. Our interest is also to see whether the first problem is useful compared to the second, and to look at how the prices at the different nodes behaves in the two settings.

First we formulate *Problem (12)* which is a convex problem. This has an advantage that we can guarantee convergence [10], but this will not capture a strictly on-off

behavior of the μ-CHP since production p^k can take on any value in an interval.

The constraints that will be considered are a minimum off time T_{off} after shut down, a minimum on time T_{off} after start up, and a maximum and minimum production $0 \leq p^k \leq p_{max}$.

We initiate predictions $\hat{x}_i^{\tau=0}, \hat{w}_i^{\tau=0}, \hat{p}_i^{\tau=0}$ with the measured values at time step k. Prices λ_i are found by gradient iterations (9).

$$V_{\text{opt},i}^N = \min_{\hat{u}_i^\tau, \hat{x}_i^\tau, \hat{v}_i^\tau} \sum_{\tau=0}^{N} l_i(\hat{x}_i^\tau, \hat{u}_i^\tau) + \lambda_i^\tau v_i^\tau - (\sum_{j \neq i} \lambda_j^\tau A_{ji})\hat{x}_i^\tau,$$

$$\text{s.t.} \qquad (12)$$

$$\hat{x}_i^{\tau=0} = x_i^k, \ \hat{w}_i^{\tau=0} = w_i^k, \ \hat{p}_i^{\tau=0} = p_i^k,$$
$$\hat{w}_i^\tau = 0, \ \tau = 1, ..., N,$$
$$\hat{x}_i^{\tau+1} = A_{ii}\hat{x}_i^\tau + B_i\hat{u}_i^\tau + \hat{v}_i^\tau + \hat{w}_i^\tau, \ \tau = 0, ..., N,$$
$$\hat{p}_i^{\tau+1} = \hat{p}_i^\tau + \hat{u}_i^\tau, \ \tau = 0, ..., N,$$
$$\hat{p}_i^\tau = 0, \ \tau = 0, ..., T_{i,\text{off}} - t_{i,\text{off}}^k,$$
$$\hat{p}_i^\tau \neq 0, \ \tau = 0, ..., T_{i,\text{on}} - t_{i,\text{on}}^k,$$
$$0 \leq \hat{p}_i^\tau \leq p_{i,\text{max}},$$
$$t_{i,\text{off}}^{\tau+1} = t_{i,\text{off}}^\tau + 1, \ t_{i,\text{on}}^{\tau+1} = t_{i,\text{on}}^\tau + 1.$$

where l_i is given in (11). The variables t_{off}, t_{on} behaves a counter to ensure that the μ-CHP is off/on for the required number of time steps T_{off}, T_{on} after shut down/ramp up. The counter t_{off} is set to zero when the μ-CHP is turned off, which means that both $p^{k+1} = 0$ and $p^k \neq 0$. Similarly, t_{on} is set to zero when $p^{k+1} \neq 0$ and $p^k = 0$.

Since a real μ-CHP can not produce infinitely small amounts of electricity, we have chosen a threshold $p_{min} > 0$ in the implementation. If a solution u^k is found such that the production is brought below this value, it is interpreted as the μ-CHP is shut down. In this case u^k is not implemented to the system. Note that inside a minimization step the controller does not know that the μ-CHP will shut down at some point in the horizon. This is here imposed outside the minimization problem. We can use a QP-solver to solve the minimization problem.

Second we formulate *Problem (16)*. In contrast to problem (12) we here introduce a mechanism that resets t_{off}, t_{on} inside the minimization. In addition the production is now only allowed to be zero or within a range $p_{min} \leq p^k \leq p_{max}$. Hence, we add the notion of shut down/start up to the optimization problem. Since p does not take values in a convex set, this means that U is also not convex.

The constraints that will be considered are a minimum off time T_{off}, a minimum on time T_{on}, and strictly on-off behavior.

We use a Mixed Integer Quadratic Program (MIQP) to solve the problem. This type of program that deals with binary variables, is described in [3]. Similar to [2], we introduce a binary variable, r^k, which indicates whether the μ-CHP is running or not.

$$r^k = \begin{cases} 1 & \text{if the } \mu\text{-CHP is on,} \\ 0 & \text{if the } \mu\text{-CHP is off.} \end{cases} \qquad (13)$$

For correct operation we also need to know if the μ-CHP is turned on or off at a given time step. To keep track of this action we introduce action variables a^k.

$$a^k = \begin{cases} -1 & \text{turn off } \mu\text{-CHP,} \\ 0 & \text{stay as it is,} \\ 1 & \text{turn on } \mu\text{-CHP.} \end{cases} \quad (14)$$

The equation describing the relation between the run state of the μ-CHP and the action taken at the given time-step is given by

$$r^{k+1} - r^k = a^k. \quad (15)$$

This should ensure proper operation.

Again prices are given by (9). The minimization to be solved at each household node is

$$V_{\text{opt},i}^N = \min_{\hat{u}_i^\tau, \hat{x}_i^\tau, \hat{v}_i^\tau} \sum_{\tau=0}^{N} l_i(\hat{x}_i^\tau, \hat{u}_i^\tau) + \lambda_i^\tau \hat{v}_i^\tau - (\sum_{j \neq i} \lambda_j^\tau A_{ji}) \hat{x}_i^\tau,$$

$$\text{s.t.} \quad (16)$$

$$\hat{x}_i^{\tau=0} = x_i^k, \ \hat{w}_i^{\tau=0} = w_i^k, \ \hat{p}_i^{\tau=0} = p_i^k,$$

$$\hat{w}_i^\tau = 0, \ \tau = 1, ..., N,$$

$$\hat{x}_i^{\tau+1} = A_{ii}\hat{x}^\tau + B_i u^\tau + \hat{v}_i^\tau + \hat{w}_i^\tau, \ \tau = 0, ..., N,$$

$$\hat{a}_i^\tau \in \{-1, 0, 1\}, \ \tau = 0, ..., N,$$

$$\hat{r}_i^\tau = 0, \ \tau = 0...T_{i,\text{off}} - t_{i,\text{off}}^\tau,$$

$$\hat{r}_i^\tau = 1, \ \tau = 0...T_{i,\text{on}} - t_{i,\text{on}}^\tau,$$

$$\hat{r}_i^{\tau+1} - \hat{r}_i^\tau = \hat{a}_i^\tau, \ \tau = 0, ..., N,$$

$$\hat{p}_i^{\tau+1} = \hat{p}_i^\tau + \hat{u}_i^\tau, \ \tau = 1, ..., N,$$

$$\hat{r}_i^\tau \cdot p_{i,\text{min}} \leq \hat{p}_i^\tau \leq \hat{r}_i^\tau \cdot p_{i,\text{max}}, \ \tau = 0, ..., N,$$

$$t_{i,\text{off}}^{\tau+1} = \begin{cases} t_{i,\text{off}}^\tau + 1 & \text{when } \hat{r}_i^\tau = 0, \\ t_{i,\text{off}}^\tau & \text{when } \hat{r}_i^\tau = 1, \end{cases}$$

$$t_{i,\text{on}}^{\tau+1} = \begin{cases} t_{i,\text{on}}^\tau & \text{when } \hat{r}_i^\tau = 0, \\ t_{i,\text{on}}^\tau + 1 & \text{when } \hat{r}_i^\tau = 1, \end{cases}$$

where l_i is given in (11). The counter $t_{i,\text{off}}$ is set to zero when action $\hat{a}_i^\tau = -1$ is taken, i.e. when the μ-CHP is turned off. When $\hat{a}_i^\tau = 1$ and the μ-CHP is turned on, $t_{i,\text{on}}$ is set to zero. Now the on/off constraints are not set outside the problem, but is updated inside the MPC optimization. Another difference with Problem 12 is that when we do find a solution, we are guaranteed that it is a feasible solution. However, when U_i is not convex we will sometimes not converge in the iterations between gradient step for prices and minimization to find control input. We may fluctuate between two solutions. In the implementation we stop the iteration when the input fluctuate, the last input before fluctuation is implemented.

IV. SIMULATION RESULT

We perform simulations with realistic demand patterns [11] generated by Energy research Center of the Netherlands (ECN). Five different type of households are taken into consideration; a single person in an apartment, two persons in a semi-detached house, two persons in a sixties terraced house, three persons in a modernized sixties terraced house, and finally a five persons house in a detached house. The simulations use patterns that represent half a day in November month, when we can assume that the heat demand is

high in the houses, so that the heat production is not wasted. The resolution is one minute.

The prediction we use for the change in demand pattern $\hat{w}^{\tau=0}$ is based on the real measurement w^k. We assume that the demand stays the same over the MPC horizon such that $\hat{w}^\tau = 0$ for $\tau > 0$. In the simulation displayed in figures 2 - 4 we use a circular topology, given by information matrix

$$A = \begin{bmatrix} 0.6 & 0.2 & 0 & 0 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0.6 & 0.2 \\ 0.2 & 0 & 0 & 0.2 & 0.6 \end{bmatrix} \quad (17)$$

Each user weights their own imbalance with 0.6 and two neighbor imbalances with 0.2.

For the simulations we use a prediction horizon of $N = 8$, minimum production $p_{\text{min}} = 0.3$ kW, maximum production $p_{max} = 1$ kW, minimum time off $T_{\text{off}} = 15$ min, and minimum on time $T_{\text{off}} = 15$ min.

Fig. 2 shows the net imbalance, demand and production for the network of five distinct household types solving problem with both a QP and MIQP solver. Both solvers are from GuRoBi version 4.5. The figure suggests that the performance of the overall network is comparable in the two formulations. The production curve p plotted in red (QP) and magenta (MIQP) follows the blue demand curve d well, and so the total imbalance x in the network is kept close to zero like we stated in the objective.

The distribution of which house produces at what time, differs from the two methods. This can be seen in Fig. 3. In the QP setting the μ-CHP is turned off in three time slots k = 200,...,400 , k = 600,...,630 and k = 690,...,710. However, in the MIQP setting it is only turned off once k = 600,...,650. When the μ-CHP is on, the production is modulated in the range 0.3 kW till 1 kW.

Fig. 4 shows the price, integrated λ, for each household. The prices increase rapidly in the beginning of the simulation. After $k = 300$ minutes it flattens and fluctuates between values 30000 and 50000 in this figure. The purple line is the price pattern corresponding to MIQP in Fig. 3. We notice that the price rises when the μ-CHP is turned off. When the μ-CHP is switched on the price immediately decreases. Since the price rises when electricity shortage rises this stimulates the device to be turned on when needed.

To check the scalability of the problem we compared the QP-simulation time for $n = \{5, 50, 250\}$ households, both for centralized and distributed computations. For the distributed case we look at the average number of gradient iterations per node, while for the centralized case we look at computation time. If we normalize the values to the five houses case, the distributed values are $\{1, 1.01, 1.67\}$ and the centralized values are $\{1, 5.83, 27.27\}$.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

By modeling an electrical micro grid in the framework of section II, we have seen that the μ-CHP can be controlled in a distributed manner using MPC and only communicating to a few neighbors. We proposed one network model with
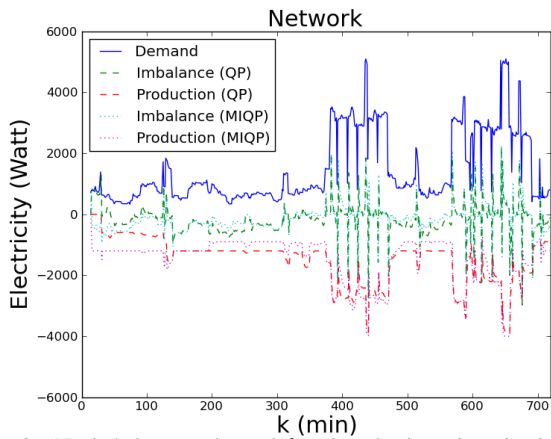
Fig. 2. Net imbalance x, demand d, and production p in a circular network of five households. One household of each type.
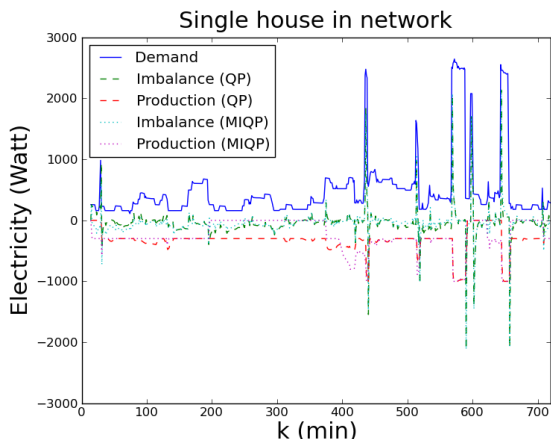

Fig. 3. Node number 5 where the demand pattern comes from a household of a married couple with three children.
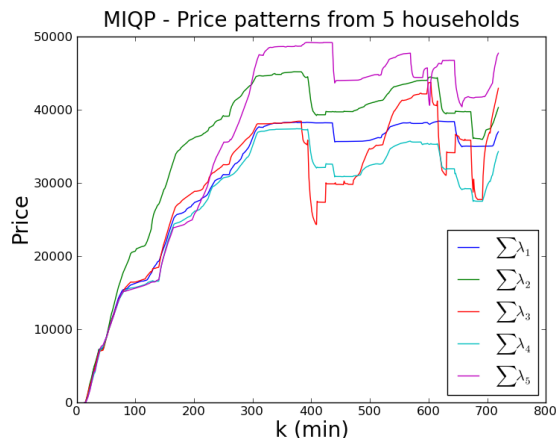

Fig. 4. The integrated change in prices λ_i for each household type in the network.

two different μ-CHP models, each requiring two different solving techniques. The first one having the advantage that convex problems are fast to solve, and the second one having the advantage that the problem is more realistic. On a network scale the two approaches behave similar, however considering each node there are differences.

A unique price pattern is generated for each node. The prices stimulate local production for local demand when a local generator is available. This is exactly what we wanted.

When the sub-gradient iterations with the inner minimization problem is stopped before it has converged, the solution is suboptimal compared to the centralized problem. Sub-optimality bounds are given in [6].

B. Future Works

The μ-CHP is constrained by heat demand in the household. This coupling was not taken into consideration in this work, where we were mainly interested in how devices with some typical behavior could be modeled in the distributed MPC framework explained in section II. However, the MPC framework enables to incorporate a new set of constraints. We have the patterns for energy use for tap water and room heating, and will model heat storage of the μ-CHP. This will give an even more realistic behavior.

Demand was taken as an external signal in the model presented in section III. However, it is clear that when the household is aware of its price pattern also demand will be affected. We aim at including a dynamic model for the part of the demand that is flexible inside a household.

VI. ACKNOWLEDGMENTS

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