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Risk allocation in a public–private catastrophe insurance system: an actuarial analysis of deductibles, stop-loss, and premiums

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Key words
Coverage premium ratio; deductibles; flood risk; reinsurance; risk aversion; stop-loss; tail value at risk.

Abstract

A public–private (PP) partnership could be a viable arrangement for providing insurance coverage for catastrophe events, such as floods and earthquakes. The objective of this paper is to obtain insights into efficient and practical allocations of risk in a PP insurance system. In particular, this study examines how the deductible and stop-loss levels (retentions) for, respectively, the insured and the insurer, relate to the corresponding maximum required coverage and premium amounts under the 99.9% tail value at risk (TVaR) damage constraint. A practical example of flood insurance in the Netherlands is studied in which the (re)insurance could be provided either by a risk-averse (private) or a risk-neutral (public) agency, which could result in large differences in premiums.

Introduction

Recent extreme weather events, such as Hurricane Katrina on August 2005 and the Tsunami in Japan in 2011, have raised questions about the responsibilities of the public and private sectors with respect to providing adequate financial compensation for natural disaster losses (Niehaus, 2002; Gollier, 2005; Cummins, 2006). In the wake of these events, many insurers were forced to disclose large losses in their annual reports, which in the United States led them to stop offering insurance or to increase premiums significantly for insurance coverage for catastrophe risk (Jaffee *et al.*, 2008). In an attempt to make catastrophe insurance coverage available at an affordable price, most of the existing catastrophe insurance systems, such as those for floods, have been developed as a public–private (PP) insurance scheme, with some sort of involvement of the government. The government always regulates insurance markets. In addition to setting the regulatory framework, the government actively underwrites risks in a PP insurance system by providing compensation for extreme damage through public reinsurance or a state guarantee (Seldon, 1997; Ermolieva and Ermoliev, 2005). The possible roles and responsibilities of the government in either a public, private, or PP insurance arrangement for catastrophe risk are summarised in Paudel *et al.* (2012).

In order to design a well-functioning, either public or PP, insurance system, it is essential to understand and assess potential risks, and how these risks should be shared between different public and private stakeholders (Niehaus, 2002; Grossi and Kunreuther, 2005). Such a risk allocation should be based on the financial capacity of each of the stakeholders while taking their commercial and social interests into account. In particular, we take as a starting point that insurance has to be affordable for the insured and commercially interesting to underwrite for insurers. For the adequate functioning of a PP insurance system, consumers should be aware of the risks that they face and of what it costs to insure such risk in order to decide whether or not to purchase insurance. In particular, we assume here that in deciding on the amounts of insurance to purchase, consumers minimise their costs (premiums to be paid) with respect to a constraint on the amount of risk that they are willing to carry themselves. For private insurers, it is essential that they are able to assess the extent of the risk involved in their participation in the PP insurance system so that they can set premiums that are sufficient to cover potential damage. To ensure the stability of underwriting results, private insurers generally cede a part of the risk to a reinsurer, which comes at a cost. We assume that the primary insurers aim to minimise the (re)insurance premiums they have to pay, thus minimising their level of indemnity, while limiting their own risk to a predefined level. But, minimising reinsurance coverage may imply that the insurer has to carry additional risk. Therefore, the insurer has to strike a balance between the (re)insurance coverage and expected risk exposure, in such a way that expected worst-case losses can be limited. Finally, governments, who usually function as a last resort for catastrophe damage in a PP insurance system, should facilitate the adequate functioning of a system while taking into account their own public responsibilities and the financial consequences that providing coverage for catastrophe events entails for public budgets (e.g. Paudel *et al*., 2012). We assume that the government aims to ensure affordability of catastrophe insurance by taking on the role of a public reinsurer that charges premiums close to the expected value of flood damage (the actuarially fair premium).

The premium to be paid can vary significantly between risk-averse (RA) and risk-neutral (RN) agencies (Bernard and Tian, 2009), which is why we account for this distinction in this study. Because the primary insurance and reinsurance companies are assumed to be RA, they demand an extra surcharge on the premium compared with an RN agency, such as the government (Kunreuther *et al*., 2013). This surcharge reflects a compensation that insurers require for covering highly uncertain risks and for the high capital costs that insurers incur for being able to cover the extremely large losses that catastrophes can cause. The government has different interests in the management of catastrophe risk, such as ensuring the affordability of the disaster insurance. In a PP insurance system, it is assumed that the government acts as a reinsurer of last resort and can be regarded as a RN insurance agency. The reason why the government provides the (re)insurance at an RN rate is that it does not require an extra surcharge on the (re)insurance premiums, which helps to make the insurance affordable to the insured. In other words, a public reinsurer like the government does not require a risk aversion surcharge on the premium because it can borrow money easily in the capital market at lower costs than commercial insurance companies in case a catastrophe triggers many large claims that exceed reserves.

This article derives the required maximum (re)insurance coverage amounts for the insured and the insurer for an efficient and practically feasible range of the deductibles and stop-losses in the Netherlands. In the light of these results, the advantages of a PP partnership in insuring flood damage will be discussed. This is an interesting case study because flood risk is generally excluded from property insurance, and it has been proposed to introduce a PP insurance system to cover the low-probability high-impact flood risks in the Dutch river delta (Botzen and van den Bergh, 2008; Aerts and Botzen, 2011). The method implemented in this paper has a similar objective to the studies by Raviv (1979), Gollier (1996), Froot (2001), and Huang (2006), namely optimising the insured's and the insurers' final wealth. However, the main difference with the existing studies is that this paper focuses on the practical application of the method, while the other studies provide a theoretical design of optimal insurance policies under value at risk (VaR)-loss constraints (Raviv, 1979; Gollier, 1996; Froot, 2001; Huang, 2006). A numerical approach to a Pareto efficiency allocation model is used to study the insured's choice for a *deductible*[1] with regard to the required maximum insurance coverage (RMIC), which meets the 99.9% tail VaR (TVaR) damage constraint. A similar analysis is conducted for the insurer's choice for a *stop-loss*[2] level, with regard to the required maximum reinsurance coverage (RMRC) and associated reinsurance premium. Analytical solutions to the optimisation, as proposed by the above-mentioned studies, are difficult to realise in practice. Therefore, we opt for a numerical approach that has practical relevance. Relevant levels for the RMIC and the RMRC, which meet the TVaR damage condition, are estimated within the plausible ranges of deductibles and stop-losses that are consistent with practical experience in the insurance markets.

Methodology and data

Basic concept

The low-lying areas in the Netherlands are divided into 53 dyke-ring areas. The analysis in this paper is conducted for each individual dyke-ring area. Every dyke-ring area has its own closed flood protection system of dykes, dams, and sluices that protect it from floods caused by rivers and the sea. A dyke-ring is an individual administrative unit under the Water Embankment Act of 1995, which guarantees a particular level of protection against flood risk for each dyke-ring area (Aerts and Botzen, 2011). For instance, a dyke-ring with a safety standard of 1/1250 can withstand a flood with a severity that occurs on average once in 1250 years. In line with Seifert *et al.* (2013), we define here a flood as the inundation of land that is normally dry, which is caused by high water levels in rivers or high levels of sea water resulting from storm surge (Seifert *et al*., 2013).

Let the random variable X stand for the flood damage for a dyke-ring area, with outcomes contained in a finite interval $[0,T]$. X is assumed to possess a continuous density function $f(x)$ with a (finite and) positive mean $E(X)$. An insurance system with two (no reinsurance) or three layers (with reinsurance) is examined, in which the insured, the insurer, and the reinsurer, or the government can participate. This insurance system is defined as a private insurance if private primary and reinsurance companies underwrite the risk, while we define it as a PP insurance system if the government acts as a reinsurer. Figure 1 describes the conceptual three-layer model. In this model, the insured and the insurer

[1]The deductible is the amount of expenses that must be paid out of pocket by a policyholder (insured) before an insurer will pay any expenses.

[2]In the actuarial field, there are different types of stop-losses. In this paper, the term 'stop-loss' indicates a specific amount of loss, which needs to be paid by the primary insurer before the reinsurer will pay any expenses.

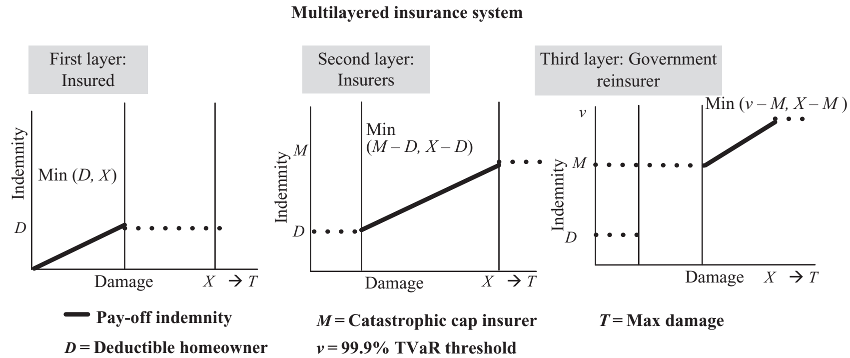

Figure 1 A conceptual model of risk allocation in a three-layer insurance system.

can choose their retention level (deductible and stop-loss) within a range that is consistent with practical experience, while they minimise the corresponding insurance (reinsurance) coverage that they have to purchase. In this paper, the term insured refers to a homeowner within a (dyke-ring) area, and, in the similar way, the term 'insurer' refers to an insurance company.

The black solid lines, in Figure 1, represent the part of the total damage per event X that could be covered by each of the stakeholders, as is stated on the top of each graph. D and M are, respectively, the deductible of the insured and the stop-loss of the insurer, and T is the maximum damage.[3] The insured covers the first part of the damage, $min(D, X - D)$, the insurer covers the middle part, $min(M - D, X - D)_{X>D}$, and the governments or reinsurers cover the last part, $min(v, X - M)_{X>M}$. It is assumed that the damage outliers that lie above the 99.9% TVaR threshold (v) (see *TVaR estimate*) are not insured because these are generally uninsurable or too expensive to insure.

A mathematical expression of Figure 1 of the allocation of flood risk within a three-layer insurance system can be given as:

$$\text{Layer } 1 = \begin{cases} 0, & \text{If} \quad X = 0 \\ X, & \text{If} \quad 0 < X < D \\ D, & \text{If} \quad X \geq D \end{cases} \tag{1}$$

$$\text{Layer } 2 = \begin{cases} 0, & \text{If} \quad X < D \\ X - D & \text{If} \quad D < X < M \\ M - D, & \text{If} \quad X \geq M \end{cases} \tag{2}$$

$$\text{Layer } 3 = \begin{cases} 0, & \text{If} \quad X < M \\ X, & \text{If} \quad M < X < v \\ v, & \text{If} \quad X \geq v \end{cases} \tag{3}$$

where, D is the deductible amount to be paid by an insured; $M - D$ is the damage covered by an insurer with stop-loss amount M; $v - M$ is the damage covered by the reinsurer or the government, with T as the maximum damage; and $0 \leq D \leq M \leq v \leq T$, and $0 < X \leq T$. An insured with a deductible level D purchases flood insurance from insurers in exchange for a total average premium[4] payment equal to π and receives an RMIC equal to $I(X)$, with $I(X) = \lambda \bar{I}$. The $\bar{I}$ is the expected value of coverage, and λ is the coverage to premium ratio (CP-ratio), with $\lambda \geq 0$. The coverage can be provided by the insurer alone (two layers) or by the insurer and reinsurer (government) together (three layers). If no administrative costs are taken into consideration, it is assumed that the total average premium will be equal to the expected insurance coverage amount, . The CP-ratio, λ, shows the relationship between the required maximum coverage and the corresponding (re)insurance premiums under the predefined TVaR damage constraint for the associated (stop-loss) deductible levels. The CP-ratio provides a rough indication of how the insurance coverage relates to the corresponding average premium for the varying deductibles and stop-losses within a given range. For example, if the stop-loss, M, for the insurer approaches the maximum damage, then the CP-ratio might also increase because the RMRC declines more slowly than the corresponding average reinsurance premium (see results). The RMRC declines more slowly because the potential maximum damage remains high, although the expected damage (and expected premium) amount would be much lower. Furthermore, it provides useful information on whether an insurance product is priced fairly from an actuarial perspective (premiums are close to the expected value of covered losses, i.e. the risk) and what losses are relatively cheap or expensive to insure. For example, for a consumer, a high CP-ratio indicates that the consumer can purchase a relatively high coverage for a low price.

[3]We have assumed that the total damage due to a flood can never be higher than the total exposed property value located in a specific dyke-ring area. Moreover, a minimum amount has been assumed per dyke-ring because if there is a flood there will always be a certain amount of damage.

[4]In practice, part of the premium consists of the administrative costs of providing the insurance. For the sake of simplicity, we have omitted this cost category from our estimates (Cummins *et al.*, 2001).

Table 1 Coverage and premium flows for the insured and the insurers

	(Re)insurance coverage		(Re)insurance premium	
Stakeholder	Max. own risk	Max. ceded part	To receive	To pay
Insured	D	$\lambda\bar{I}$	N.A.	π
Insurers	$M - D$	$\lambda\bar{I} - M$	$(1 - \delta)\pi$	$\delta\pi$

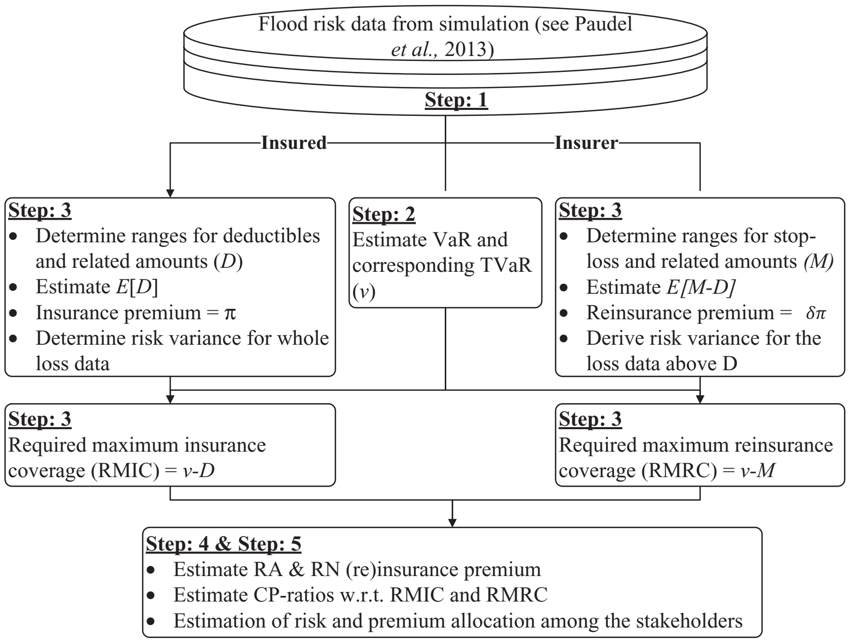

Figure 2 A simplified conceptual design of the pseudo-code implemented in numerical computing software MATLAB (MathWorks, Inc., Natick, MA, USA).

Insurance companies have the possibility to transfer a part of the RMIC amount (RMRC) $\gamma\bar{I}$ to a third party, like reinsurers or the government, which are either RA or RN. The reinsurance coverage ratio γ, with $0 < \gamma \leq 1$, indicates the coverage that is allocated to the reinsurer, and $\gamma\bar{I} \leq \bar{I}$. This transfer is made in exchange for a reinsurance premium $\delta\pi$, with $0 < \delta < 1$ and $\delta\pi \leq \pi$. Insurers may want to transfer risk in order to reduce the probability of having to pay large reimbursement amounts that result from insurance claims. This implies that part of the total coverage, namely $(1-\gamma)\bar{I}$, is covered by insurers themselves in exchange for a portion of the premium that equals $(1 - \delta)\pi$. Table 1 summarises the coverage and premium flows for the insured and the insurers (for more details, see *RMIC and RMRC estimates* and *Premium estimates*).

The following main steps in the approach can be distinguished (as shown in Figure 2):

Step 1: Flood risk estimations for each of the 53 low-lying areas ('dyke-ring areas') in the Netherlands are derived from Paudel *et al.* (2013) (see *Flood risk estimates*);

Step 2: The VaR and the corresponding TVaR levels are estimated from the loss data obtained in step 1 for both the insured and the insurers (see *TVaR estimates*);

Step 3: Solve the objective functions numerically to obtain RMIC and RMRC, under the corresponding TVaR damage conditions, for the predefined ranges of deductibles and stop-losses that are consistent with practice (see *RMIC and RMRC estimates*).

Step 4: Estimate the RA and RN (re)insurance premiums for different levels of deductibles and stop-losses (see *Premium estimates*).

Step 5: Determine the relation between the premiums and the corresponding coverage amounts, for both two- and three-layer insurance systems in terms of CP-ratios (see *Relation between coverage and premium*).

Figure 2 depicts the corresponding conceptual pseudo-code design for the methodological implementation as it is described in the following sections.

Flood risk estimates (step 1)

The flood risk estimates for the Netherlands used in this paper are derived from Paudel *et al.* (2013), who estimate loss probability curves for each of the 53 dyke-ring areas (see step 1 in Figure 2). To deal with data scarcity, Paudel *et al.* (2013) apply Bayesian Inference by combining information from the two main Dutch studies about flood risk in these dyke-ring areas, namely the Aandacht voor Veiligheid (Aerts *et al.*, 2008; Aerts and Botzen, 2011) and the Veiligheid Nederland in Kaart projects (Wouters, 2005; Aerts *et al.*, 2008). This method accounts for the high uncertainty of flood damage by applying representative fat-tailed probability distributions, which are suitable for modelling catastrophe risk (Paudel *et al.*, 2013).

TVaR estimates (step 2)

In a three-layer insurance system, we are interested in the optimal insurance portfolio for the first two stakeholders, namely the insured and the insurer. These portfolios are obtained by minimising the coverage function, which meets the corresponding 99.9% TVaR constraint for flood damage. This reflects an underlying assumption that the insured aims to minimise the premium (which minimises automatically insurance coverage) that he/she has to pay for flood insurance given a constraint of the amount of own risk that the insured is willing to carry. The insurer aims to minimise the reinsurance premium (which minimises automatically reinsurance coverage) given a constraint on the amount of risk that the insurer is willing/able to expose itself to. The TVaR in this paper represents the maximum damage to be insured and is defined as the expected damage in the worst α% of the cases. There are two main reasons for choosing TVaR. First, compared with other risk measures (i.e. VaR and maximum statistics), it catches the losses located on the right tail of the loss density curve, including the outliers, more adequately. Second, the stochastic outliers are very large and highly uncertain (i.e. the maximum damage), which may provide a distorted view of reality when these are used in their original form. TVaR overcomes this problem because it essentially includes the information about outliers in the expected value and does not depend only on one value, like VaR. Besides this, the TVaR is also a more appealing risk measure because it is what is called a 'coherent risk measure' that meets requirements of monotonicity, homogeneity, and subadditivity, which are important in a statistical assessment, especially if the input data are scarce and not homogeneous (Artzner *et al.*, 1999; Dhaene *et al.*, 2008).

A general expression of TVaR (v), with a confidence level $\alpha \in (0,1)$, for random losses (X) with $E[X] < \infty$ and distribution function F_X can be given with (McNeil and Frey, 2000; Chiragiev and Landsman, 2007):

$$v = E\left[X \mid X \le VaR_\alpha(X)\right] \tag{4}$$

where $VaR_\alpha(X)$ is the VaR function of F_X at the α level. In our case, α is set equal to 0.01%.

RMIC and RMRC estimates (step 3)

The objective of the stakeholders in the first two layers is to minimise the coverage amount, which leads to a lower premium, while limiting the own risk (deductible and stop-loss) with a predefined level of certainty. Figure 3 shows a conceptual cumulative loss function, which depicts the own risk for each of the three stakeholders.

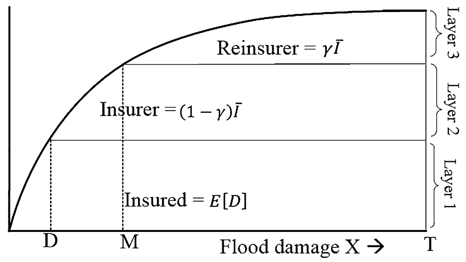

Figure 3 A conceptual model of a cumulative loss function with three layers of own-risk for the insured, the insurer, and the reinsurer.

Relation between deductibles and insurance coverage for homeowners

We follow the approach, with a major modification, taken by Wang *et al.* (2005), who model the consumer decision about how much insurance coverage to purchase as a minimisation of insurance costs, subject to a constraint that the consumer's own risk does not exceed a certain level.[5] The insured aims to minimise the amount of coverage, which automatically minimises the total premium to be paid, under a desirable level of the deductible that meets the 99.9% TVaR damage constraint. It should be noted that this approach assumes that the maximum coverage cannot exceed the TVaR, which implies that the consumer is not insured for extremely large damage above TVaR because such damage is unlikely to occur in practice and is expensive to insure. The total insurance coverage to be provided by insurance under the minimised deductible level is (Wang *et al.*, 2005):

$$Minimize\ \bar{I} = \int_D^v (x-D) f(x)dx,\ for\ D < x \le T \tag{5}$$

$$Subject\ to\ P\left\{\min(X, D) \le v - \lambda\bar{I}\right\} \ge 1-\alpha \tag{6}$$

where $v \ge I(X), D \approx v - \lambda\bar{I}$ gives the approximate level of the deductible (exposed amount of own risk for the insured)

[5]This approach deviates from the expected utility theory, which is often used in economics to model individual decision making under risk (Von Neumann and Morgenstern, 1944), but in practice is often not a good description for individual behaviour, especially towards extreme risks (Starmer, 2000). Our approach assumes that individuals are RA and insure for most losses, except very unlikely and extremely large losses for which insurance is relatively expensive. Moreover, individuals are assumed to be willing to take on a certain level of own risk by means of a deductible if this saves on insurance costs. Both of these characteristics can be observed in actual insurance purchase behaviour.

for the insured with respect to the RMIC coverage that is necessary to meet the given TVaR damage condition for the insured (see Appendix 1). Eqn (5) states that the insured minimises the necessary required coverage amount under the condition given by Eqn (6). Eqn (6) means that in 99.9% cases of flood events, the deductible $[min(X,D)]$ amount should not surpass the difference between the TVaR and RMIC.[6] The TVaR[7] [Eqn (4)] can be estimated with:

$$v = \frac{1}{(1-\alpha)} \int_{\alpha}^{1} VaR_u(X)\,du \tag{7}$$

where $VaR_u(X) = F_X^{-1}(\alpha)$, with $\alpha = 0.01$.

Formally, the average deductible amount for the insured can be given with:

$$E[D] = \int_{0,x \le D}^{D} (x-D) f(x)\,dx + \int_{D,x>D}^{v} D f(x)\,dx \tag{8}$$

Relation between stop-loss and reinsurance coverage

An insurer with a stop-loss level M, and a retention level D for the insured, aims to optimise its portfolio by minimising reinsurance coverage under the predefined TVaR (v) damage condition defined at the certainty level $1 - \alpha$. In other words, the insurer aims to minimise reinsurance costs subject to a (regulatory) constraint on retained risk. We get the desirable M by:

$$Minimize\ \gamma \bar{I} = \int_{M}^{v} (x-M) f(x)\,dx,\ for\ M < x \le T \tag{9}$$

$$subject\ to\ P\{\min(M-D, X-D)_{D<X} \le v - D - \lambda\gamma\bar{I}\} \ge 1-\alpha \tag{10}$$

where $M \approx v - \lambda\gamma\bar{I}$, and $\lambda\gamma\bar{I}$ is the RMRC (see Appendix 1). $v - \lambda\gamma\bar{I}$ gives the desirable level of the stop-loss (exposed amount of own risk for the insurer) with respect to the maximum reinsurance provided to the insurers. The average coverage amount to be provided by an insurer is equal to:

$$(1-\gamma)\bar{I} = E[M-D] = \int_{D,D<x \le M}^{M} (x-D) f(x)\,dx + \int_{M,M \le x<T}^{v} (M-D) f(x)\,dx \tag{11}$$

[6]The difference between the maximum damage (T) and TVaR (v) is not insured.

[7]In this formula, the copula effect has not been included. Because it is assumed in this paper that the maximum damage cannot exceed the total economic value within a dyke-ring area, Eqn (7) provides an accurate estimation for TVaR.

Premium estimation and relation with coverage (step 4)

The amount of premium in each layer depends on the type of (re)insurer, whether it is an RA or an RN agency. The average reinsurance coverage can be provided by the government or a commercial reinsurance company, and is equal to the difference between the insurer's stop-loss M and the v, as Eqn (9) shows. In general, catastrophe risks are difficult to diversify fully in the market, and insurers and reinsurers tend to be highly RA because catastrophe losses are highly correlated (Gerber and Pafumi, 1997; Froot, 2001; Kunreuther, 2002; Gollier, 2005). A government acts as a last resort for catastrophe risk and has different interests and responsibilities compared with the commercial (re)insurance companies and the insured. In this respect, it is common to assume that the government is an RN agency. The government usually covers extreme risk that is located on the right-hand side of the loss distribution. This leads to a more affordable total premium to be paid by property owners and makes the insurance system more feasible in a sense that consumers are more likely to buy coverage. Moreover, if the government acts as a reinsurer in a PP flood insurance system, then this is also attractive for primary insurers that can buy reinsurance at lower costs than would be possible if reinsurance is provided by private reinsurance companies.

Premiums are calculated for two categories of insurer risk attitudes: (1) both private insurers and reinsurers are RA; and (2) an RN government acts as a reinsurer, and the private insurer is also RN. The flood insurance premium to be paid by a homeowner to a RA insurer, within a specific dyke-ring area, can be given with (Kaas *et al.*, 2004; Kunreuther *et al.*, 2011):

$$\pi = \frac{E[X-D] + \sigma^2_{0 \le X \le v} * r}{NH * RP} \tag{12}$$

where $\sigma^2_{0 \le X \le v}$ is the risk variance estimate for flood losses between $inf(X)$ and v; r is the risk aversion coefficient for the insurer; NH denotes the number of houses within a dyke-ring area; and RP indicates the return period of a flood event for a specific dyke-ring.

If an insurer is RN, the total premium is equal to:

$$\pi = \frac{E[X-D]}{NH * RP} \tag{13}$$

The portion of the premium to be withheld by an RA insurer to cover part $(M - D)$ of flood damage, with the approximated risk variance $\sigma^2_{D \le X \le M}$ (from the damage between D and M) equal to:

$$(1-\delta)\pi = \frac{E[X-D]_{D<X \le M} + \sigma^2_{D \le X \le M} * r}{NH * RP} \tag{14}$$

and an RN insurer requires a premium equal to:

$$(1-\delta)\pi = \frac{E[X-D]_{D<X\leq M}}{NH*RP} \quad (15)$$

The portion of the premium to be received by RA reinsurers to cover part $(v-M)$ of flood damage, with the approximated risk variance $\sigma^2_{M\leq X\leq v}$ (from the loss data between M and v) equal to:

$$\delta\pi = \frac{E[v-M]_{M<X\leq v} + \sigma^2_{M\leq X\leq v} * r}{NH*RP} \quad (16)$$

If an RN government participates in a PP insurance system as a reinsurer, then Eqn (16) becomes:

$$\delta\pi = \frac{E[v-M]}{NH*RP} \quad (17)$$

The standard deviation of flood damage σ varies per coverage layer. The risk aversion coefficient rate r is derived from an exponential utility function $u(w) = -\alpha e^{-\alpha w}$, where w represents the insured amount, and α is rate of risk aversion[8] for the insurer and the reinsurer towards catastrophe risk, which in this case is equal to r. Estimates of σ and $r = -u(xw)'' / u(w)'$(which is based on α equal to 0.0005) are taken from Paudel *et al.* (2013). This level of r implies that a moderate degree of insurer's and reinsurer's risk aversion is included as a premium mark-up, which is quite common for non-life insurance products. A larger degree of risk aversion may be more appropriate if coverage is provided for heavy-tailed catastrophe losses, like floods. Therefore, the premiums for private insurance coverage in this paper could be seen as conservative estimates.

Relation between coverage and premiums (step 5)

The CP-ratios provide an indication of how the RMIC and RMRC relate to the corresponding premiums for the given ranges of deductibles and stop-losses. As discussed by Paudel *et al.* (2012), practical experience shows that deductible levels for catastrophe insurance systems vary between 1% and 15%, while the level of the stop-loss for insurers varies between 50% and 90% of total damage. A slightly broader range of 1–20% for deductibles and 40–94% for the stop-losses has been applied to the numerical estimations of the coverage functions and the CP-ratios in this paper.[9]

[8]In practice, the risk-aversion rate to catastrophe risk can differ between insurers and reinsurers. However, owing to a lack of sound scientific evidence about how these rates differ, the same risk-aversion rate is used for both agencies in this paper.

[9]In practice, the deductible and stop-loss levels may depend on the specific risk appetite of the insured and the insurer and their own financial capacity to cover damage. Nevertheless, we regard the chosen ranges as a plausible practical approximation of deductible and stop-loss levels.

Table 2 Ranges of the coverage to premium ratios (CP-ratios) for risk-averse (RA) and risk-neutral (RN) primary insurers estimated at the 1% and 20% deductible levels, for the dyke-ring areas 7, 14, and 36

	CP-ratio RA		CP-ratio RN	
Dyke-ring	1%	20%	1%	20%
7	2.76	2.86	3.18	3.36
14	1.36	1.47	3.18	3.37
36	2.51	2.56	3.19	3.37

Results

Results for dyke-ring areas 7, 14, and 36

Because of space limitations, the detailed results are provided here only for these three representative dyke-ring areas: the Noordoostpolder (7), Zuid-Holland (14), and Land van Heusden/de Maaskant (36). These three dyke-ring areas share similar geographical features and a common flood probability with the three main classes of dyke-ring areas in the Netherlands, namely intertidal areas, coastal areas, and areas vulnerable to river flooding.[10] The second subsection provides the global results for all 53 dyke-ring areas.

CP-ratio

Table 2 provides an overview of estimated CP-ratios λ for the total insurance premiums that are collected from homeowners by either a RA or a RN insurer [Eqn (13)]. These CP-ratios are calculated as $\lambda = I(X) / \pi$ for primary insurance and deductible level (D), and indicate the ratio of the total optimal RMIC to the total premiums to be paid by the insured, without making any distinction between the insurer and the reinsurer. An examination of the CP-ratios is of interest because it provides an indication of the attractiveness of the insurance for consumers; a high CP-ratio indicates that consumers get a high coverage value for their premiums paid. As can be expected, the CP-ratios are considerably lower if the insurer is RA because the RA premiums include a surcharge for the insurer's risk aversion that is dependent on the risk variance, while such a surcharge is not included in the RN premiums [see Eqns (12–15)]. This

[10]Dyke-ring Noordoostpolder is representative of the majority of dyke-ring areas that have a flood probability of about 1/4000 per year. Dyke-ring Zuid-Holland (along with Noord-Holland) is one of the dyke-rings with the lowest flood probability in the Netherlands of about 1/10 000 per year. This dyke-ring is located along the densely populated coastline and has a high concentration of property values. Dyke-ring 36, Land van Heusden/De Maaskant, shares similar features with the majority of the river dyke-ring areas, which have a flood probability of about 1/1250 per year (Bouwer *et al.*, 2010).

Table 3 Coverage to premium ratios (CP-ratios) for reinsurance coverage corresponding to the 40% and 94% stop-loss levels for the dyke-ring areas 7, 14, and 36

	CP-ratios corresponding with the 40% and 94% levels of stop-loss			
	RA		*RN*	
Dyke-ring	Stop-loss: 40%	Stop-loss: 94%	Stop-loss: 40%	Stop-loss: 94%
7	3.07	12.87	3.71	19.15
14	1.37	7.23	3.69	14.79
36	2.9	9.89	3.72	18.62

RA, risk averse; RN, risk neutral.

implies that the homeowners within a dyke-ring area with a low CP-ratio (e.g. dyke ring 14) usually pay a higher insurance premium relative to the required coverage amount, which is obviously undesirable. For example, the premiums paid in dyke-ring 14 are relatively high because of the large potential damage and higher risk variance. In general, a dyke-ring with a high damage and high variance has a low CP-ratio.

Table 3 provides an overview of the estimated CP-ratios of the RMRC amount to the reinsurance premium for the two boundaries of the range of stop-losses for an RA and an RN reinsurer. These CP-ratios are calculated as $\lambda = \gamma\bar{I}/\delta\pi$ for reinsurance (coverage provided by the reinsurer to the insurer) and indicate the ratio of the optimal RMRC to the total reinsurance premiums to be paid. The CP-ratios corresponding to the 40% stop-loss level are notably lower compared with those estimated at the 94% stop-loss level. The CP-ratios in Table 3 for the 94% stop-loss level are very different from those in Table 2. This is because the RMRC (99.9% TVaR minus the related retention amount) decreases more slowly than the corresponding average reinsurance premiums with the increasing stop-losses. This is mainly caused by the fact that the potential maximum coverage amount for the losses located on the right tail of the density function remains high, although the expected amount of damage (and thus the average reinsurance premiums) would be much lower. The CP-ratios slowly increase in the loss range between 40% and 80%, while the risk variance remains stable in that range. Therefore, an insurer could benefit most from a high CP-ratio and choose a stop-loss level between 80% and 85% of the damage threshold, while the losses with a very high risk variance can be avoided and passed on to the reinsurer (see also part b in Figure 4). However, if an insurer is RN, it can also choose to provide a full coverage himself and not purchase reinsurance, but this has the disadvantage that it will be confronted with highly uncertain large losses. Dyke-ring 14 has a CP-ratio of 1.37 for a 40% stop-loss, which is the lowest amount compared with other dyke-ring areas. This indicates that purchasing insurance from an RA insurer in dyke-ring 14 is most expensive.

Relationship among the RMIC, the deductible, and the insurance premium

The trade-off between the deductibles and the corresponding premiums and RMIC amount is depicted in Figure 5. Part a in Figure 5 shows the effect of increasing the deductible amounts on the RA (dotted line) and RN premiums (dashed line) for the three dyke-ring areas. In general, premiums decrease if deductible levels are higher. As can be observed, the RN premiums are always lower, and the gap between the RA and the RN premiums increases, although slowly, with increasing deductible levels. This is the case because the risk variance for losses above level D increases with increasing deductible amounts, which results in a higher RA premium compared with the RN premium. The premium surcharge of an RA insurer compared with an RN insurer is the highest in dyke-ring area 14 (130–136%), while this surcharge is much lower in dyke-ring areas 7 (16–22%) and 36 (28–35%). Part b in Figure 5 depicts, by means of CP-ratio, the relationship between the increasing deductible levels and its impact on the RA and RN premiums and the associated coverage amount. Higher deductible levels are in general associated with higher C/P ratios. As can be observed, the difference between the RA and RN C/P ratios is not always linear. Although this is hard to see, part b in Figure 5 shows that the RA premiums decrease less if the deductible increases at low deductible levels, while this increase is stronger around the 15% deductible data point. This indicates that high deductibles around the 15% level are attractive for consumers, which is consistent with practice (Paudel *et al.*, 2012). The overall results for the 53 dyke-ring areas show that increasing the deductible level above the 15% level results in only moderate premium savings. Therefore, the 15% deductible level is chosen here as one of the desirable trade-off points between risk and premiums. In general, the average deductible amount ($E[D]$) at 15% of the damage level, implies that policyholders pay approximately 24% of the actual total damage from their own pocket.

Relationship among the RMRC, stop-loss, and reinsurance premium

Part a in Figure 4 depicts the impact of varying levels of the stop-losses on the RA reinsurance premiums and the corresponding RMRC. Part b in Figure 4 shows the CP-ratios of the RA and RN reinsurance premiums. The CP-ratios of the RA premiums are always lower than those of the RN premiums, as was observed in Figure 5. The CP-ratio increases rapidly above the 80% damage quintile, which indicates that the degree of uncertainty of losses in that quintile is high. In order

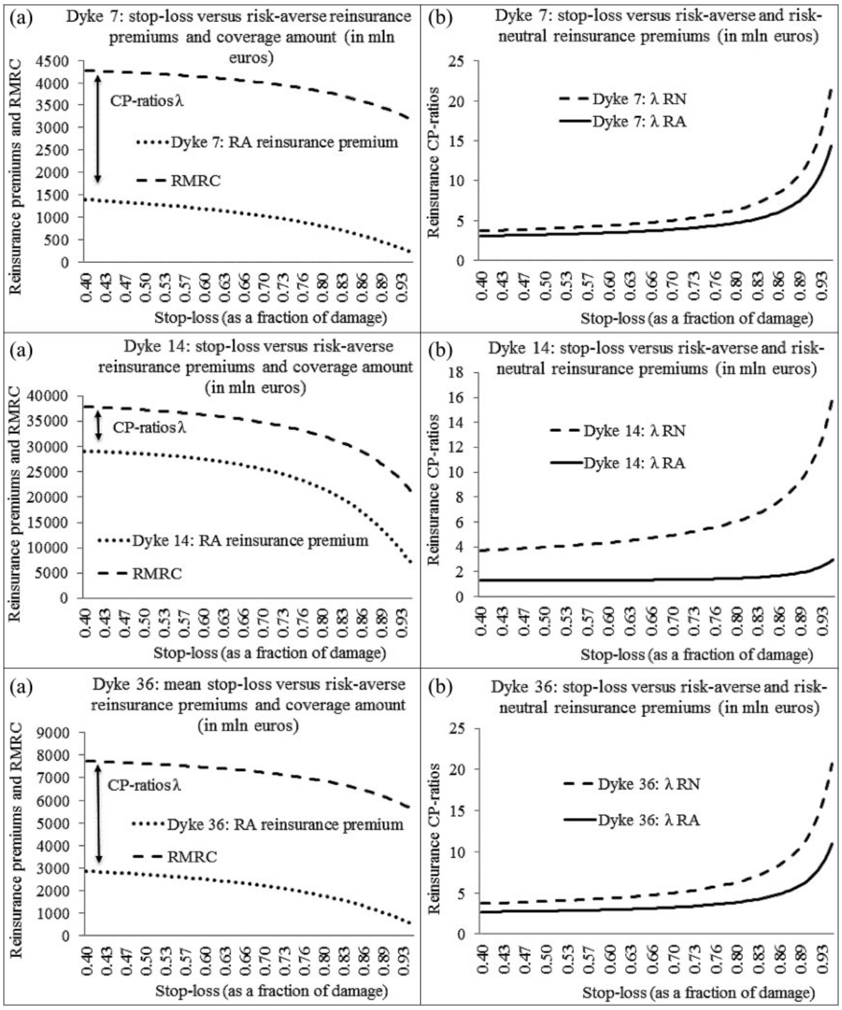

Figure 4 (a) Stop-loss versus risk-averse (RA) reinsurance premiums and required maximum reinsurance coverage (RMRC) amounts, and (b) the coverage to premium ratio (CP-ratio) in terms of RA and risk-neutral (RN) premium amounts.

to avoid carrying these losses with a large uncertainty and to arrive at a good trade-off between the reinsurance premiums and the risk variance, the insurer might choose a stop-loss between 80% and 85%. This has led to the choice of an 84% stop-loss amount as one of the appropriate trade-off points between the RMRC and the corresponding reinsurance premiums, viewed from an insurer's perspective. Based on the average insurance and reinsurance premiums, the insurer covers approximately 57% of the damage while 43% is covered by the reinsurer, excluding the deductible (see Table 4).

Results for all 53 dyke-ring areas

Premium and coverage estimates per homeowner within a specific dyke-ring area

Table 5 illustrates the RN and RA insurance premiums to be paid by the individual homeowner within a specific dyke-ring area and the corresponding total RMIC[11] amount per homeowner. Columns 1 and 2, respectively, show the dyke-ring numbers and the number of houses within each corresponding dyke-ring area; column 3 is the estimated 15% average deductible amounts ($E[D]$); columns 4 and 5 are, respectively, RA and RN insurance premiums; and column 6 shows the corresponding annual expected value of required maximum coverage amounts (i.e. the maximum required flood coverage × the average yearly flood probability). The last row indicates per column the average amount per homeowner collectively for all 53 dyke-ring areas.

The results in Table 5 are shown for a deductible of 15% per homeowner within a specific dyke-ring area. These amounts are slightly higher than the corresponding 15% damage quintiles because these include the constant deductible

[11]These are the required maximum coverage amounts to be provided collectively by the insurer and the reinsurer per homeowner.

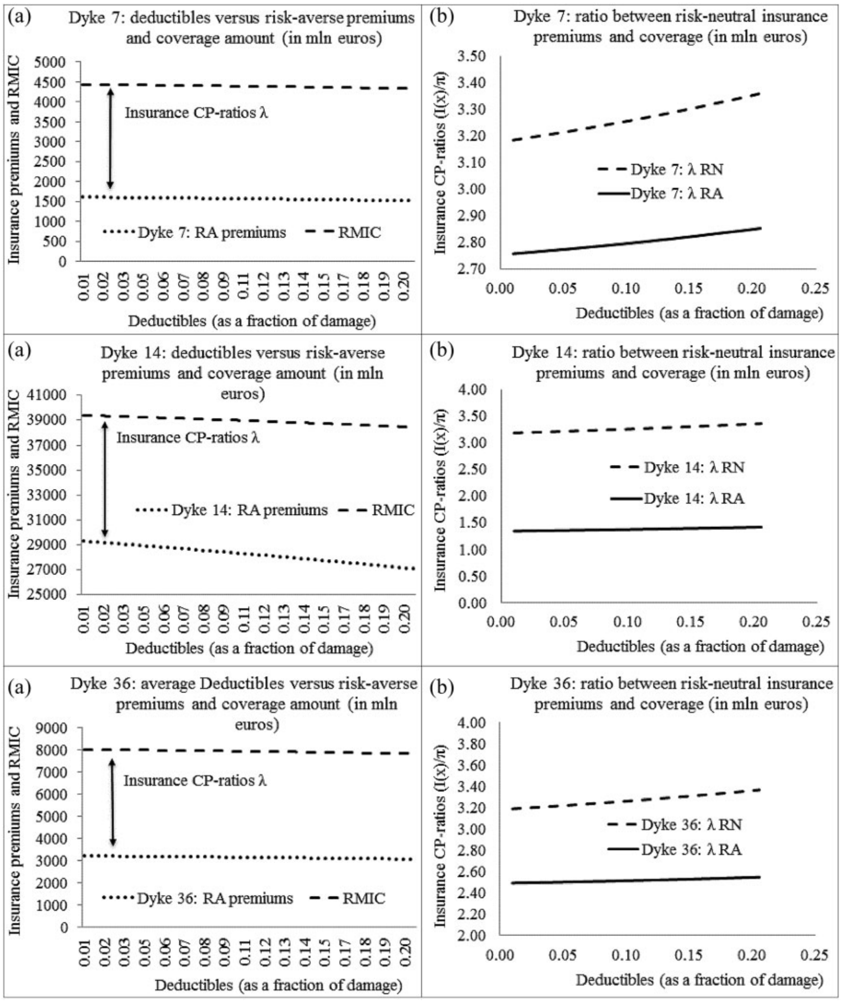

Figure 5 Effect of the average deductible amounts on (a) risk-averse (RA) and risk-neutral (RN) insurance premiums (b) and the coverage to premium ratio (CP-ratio) for primary insurance for the dyke-ring areas 7, 14, and 36.

Table 4 The allocation of expected coverage of flood damage between the insurer and the reinsurer based on 84% stop-loss and 15% deductible for the dyke-ring area 7, 14, and 36

	Coverage allocation based on 84% stop-loss and 15% deductible	
Dyke-ring nr.	Insurer $1-\gamma$	Reinsurer γ
7	0.57	0.43
14	0.58	0.42
36	0.57	0.43

amount that should be paid if damage exceeds the quintile [see Eqn (8) and the insured's part of the losses in Figure 3]. A higher deductible amount leads to a lower premium and RMIC amount, which also means that the probability that the insurance and the reinsurance coverage come into effect is lower. For instance, for the main dyke-ring area 14, the annual expected value of the deductible is €1.97. The premium charged to purchase the annual expected RMIC of €12.83, for an individual homeowner in dyke-ring 14, equals either €6.43 or €3.88, depending on whether the insurer is RA or RN. It should be realised that the RMIC is an annual expected amount, which in case of an average flood probability of 1/10 000 for dyke-ring 14 corresponds to a maximum coverage of €128 300 per flood event. Dyke-ring area 16 has the highest RN premium (€337.80), while dyke-ring area 6 has the lowest RN premium (€0.18). A total of 19 dyke-ring areas have an average RN premium per homeowner that is higher than €21.00, which equals the average RN premium. Some dyke-ring areas have considerably higher premiums and necessary coverage amounts because they have a low number of houses among which the flood risk is shared and/or a high risk potential that leads to a high risk variance.

Table 5 The expected value of the deductible ($E[D]$), the annual expected value of required maximum insurance coverage (RMIC) per homeowner, and the corresponding risk-averse (RA) and risk-neutral (RN) insurance premiums to be paid by the individual homeowner within a specific dyke-ring area (in euros per year)

Dyke-ring number	Number of houses	Deductible ($E[D]$)	Premium π		RMIC at 99.9% TVaR
	Per dyke-ring area	At 15% damage	*RA* $E[X - D]$	*RN* $E[X - D]$	$I(X)$
6	468 014	0.09	0.30	0.18	0.60
13	412 013	1.54	5.03	3.02	9.99
14	1 659 248	1.97	6.45	3.88	12.83
44	292 938	3.34	10.90	6.55	21.68
47	37 179	3.57	11.67	7.01	23.20
32	48 501	3.72	12.16	7.31	24.18
34	160 741	3.91	12.77	7.67	25.39
17	165 235	4.40	14.38	8.64	28.58
8	99 069	5.09	16.63	9.99	33.06
36	165 555	6.60	21.56	12.96	42.87
7	22 234	6.82	22.29	13.39	44.31
18	2054	7.78	25.44	15.29	50.57
19	5696	7.86	25.70	15.44	51.09
46	3227	7.96	26.02	15.64	51.74
21	32 152	8.78	28.70	17.25	57.05
45	103 282	8.98	29.34	17.63	58.33
11	18 610	10.30	33.66	20.23	66.91
35	37 524	10.44	34.13	20.52	67.87
4	214	10.45	34.16	20.53	67.92
49	7836	15.13	49.46	29.73	98.33
15	79 164	15.49	50.63	30.43	100.65
48	59 881	18.66	60.97	36.65	121.23
1	494	20.41	66.72	40.10	132.65
52	42 040	20.98	68.57	41.21	136.32
12	8274	21.00	68.62	41.25	136.44
10	11 128	21.61	70.62	42.45	140.41
9	33 556	22.80	74.50	44.78	148.12
41	109 400	22.94	74.98	45.06	149.07
25	18 064	23.30	76.14	45.76	151.38
28	3353	23.79	77.76	46.74	154.60
5	5331	24.20	79.08	47.53	157.23
2	1345	25.04	81.82	49.18	162.68
42	5611	26.15	85.48	51.38	169.95
51	4532	27.79	90.83	54.59	180.58
3	801	28.00	91.49	54.99	181.91
27	9060	28.75	93.94	56.46	186.78
22	47 243	30.51	99.70	59.92	198.22
24	18 287	31.49	102.92	61.86	204.63
29	49 060	32.64	106.68	64.12	212.10
26	14 655	34.13	111.56	67.05	221.80
30	29 532	35.88	117.26	70.48	233.14
50	18 320	36.75	120.11	72.19	238.80
39	169	45.38	148.30	89.13	294.85
33	26	46.12	150.72	90.59	299.66
38	16 781	48.12	157.27	94.53	312.69
20	62 823	51.49	168.29	101.15	334.59
40	458	55.81	182.41	109.63	362.66
53	86 300	57.99	189.52	113.91	376.81
43	120 526	60.59	198.01	119.01	393.69
37	12	63.95	209.00	125.61	415.53
31	7087	67.76	221.44	133.09	440.27
23	115	83.36	272.42	163.74	541.63
16	82 340	103.36	337.80	203.03	671.61
Average		**10.13**	**35.00**	**21.00**	**69.80**

TVaR, tail value at risk.

Estimates of both the total premiums retained by primary insurers, and reinsurers and the corresponding required coverage

Table 6 shows the estimates of the allocation of the RA and RN premiums per policy between primary insurers and reinsurers, and the annual expected value of the corresponding maximum coverage amounts based on the 15% deductible and 84% stop-loss levels. The results are presented in ascending order by the stop-loss levels shown in column 2. Column 3 shows the RA insurance premium retained by the insurer, and column 4 the RA reinsurance premium retained by the reinsurer. Columns 5 and 6 show, respectively, the RN insurance premiums retained by the insurer and the reinsurer. The last column represents the total required coverage amounts to be covered by the reinsurer. The last row provides the average amounts per homeowner for all 53 dyke-ring areas altogether and makes no distinction between the individual dyke-ring areas. The insurer passes a part of premium to the reinsurer, which is the amount shown in the 'reinsurer' columns in Table 6, and in exchange receives the corresponding RMRC in the event of a flood. The columns 'insurer' in Table 6 show the part of the premium that is retained by the primary insurers, which is the difference between the premiums that they receive from the policyholders and the premiums that they pay for reinsurance. Table 6 shows that the ratio between the RA and the RN primary insurance premium is much lower compared with this ratio for the reinsurance part. This indicates that the rare losses with large variance that are covered by reinsurance are more expensive to insure compared with the small losses with a low variance.

On average, flood losses exceed the stop-loss level with a probability of 84%, which implies that these losses above the stop-loss are covered by reinsurance. With an RMRC amount of €0.51 per homeowner on an annual expected value basis, dyke-ring 6 is the cheapest area to purchase flood insurance, while, with an annual expected value of required reinsurance coverage of €568.26, dyke-ring 16 is the most expensive. There are 19 dyke-ring areas with an RMRC that is higher than the overall average amount of €58.33. In line with our expectations, the RA (re)insurance premiums are always higher than the RN amounts (see also Figure 4, part b). The average risk-aversion surcharges on the insurance and reinsurance premiums (i.e. RA/RN) are, respectively, 20% and 148%. This implies that the reinsurance is relatively expensive compared with the primary insurance part. This is consistent with the practice that the RA reinsurers demand an extra surcharge on premiums because of the high uncertainty of large reinsured losses. This may imply that a system involving an RN insurer and reinsurer would lead to a lower deductible level for the insured, while a system involving a RN reinsurer, such as the government, might result in lower stop-losses. The significant difference between the premiums and the necessary reinsurance coverage amounts per homeowner for individual dyke-ring areas is caused either by high expected flood damage relative to a low number of houses per dyke-ring area or by a high risk variance, or a combination of both. For instance, a dyke-ring area with a high number of houses will have a relatively low average premium per homeowner, even if the average flood loss is very high (i.e. dyke-ring area 14). Appendix 1 shows the same results as in Table 6 in terms of total amounts per dyke-ring, instead of averages per homeowner. From these results, it follows that the total RA insurance and reinsurance premiums are approximately 187% of the RN premiums. The associated average allocation of insurance coverage among the insured, the insurer, and the reinsurer is 24%, 43%, and 33%, respectively.

Discussion

The estimates of flood insurance premiums and the associated coverage amounts presented in the tables and graphs shown in the results will be discussed in this section with respect to the following main aspects: the trade-off between necessary (re)insurance coverage and premiums; the differences between the RA and the RN premiums; the allocation of risk and premiums among stakeholders; and the main implications of the results for the insured, the insurer, and the reinsurer.

The trade-off between the RMIC and the RMRC, and the corresponding insurance and reinsurance premiums

In practice, the choice of a deductible and stop-loss level may depend on the financial capacity and risk appetite of the insured and the insurer, which will differ between individuals and insurance companies. In our application, we have derived general efficient and practically feasible deductible and stop-loss levels by examining the trade-offs between the premiums to be paid in relation to the required coverage and risk retention, as shown by the estimated CP-ratios. As can be observed from Figures 3 and 5, the derivation of a deductible level for the insured is less clear-cut than choosing a stop-loss level for insurers, although the results still provide useful guidelines for arriving at an approximation.

The RMIC hardly changes with respect to the corresponding range of deductibles (between 0% and 20% of losses, see part b in Figure 5) because the loss variance remains low and stable in this range. By purchasing insurance, a homeowner aims to protect his property against a large amount of damage that can exceed his financial capacity, which implies that the homeowner prefers a low deductible level (Paudel *et al*., 2012). A high deductible leads to a low premium (see Figure 5), but at the same time the related deductible

Table 6 An overview of the annual expected value of required maximum reinsurance coverage (RMRC), and the allocation of premiums between insurers and reinsurers, based on an 84% stop-loss and a 15% deductible (per homeowner in euros per year)

Dyke-ring number	Stop-loss (**M**)	Premium RA π		Premium RN π		RMRC $\lambda\gamma\bar{I}$
	84%	Insurer $(1-\delta\pi)$	Reinsurer $(\delta\pi)$	Insurer $(1-\delta\pi)$	Reinsurer $(\delta\pi\delta\pi)$	TVaR at 99%
6	0.29	0.11	0.17	0.09	0.07	0.51
13	4.77	1.79	2.78	1.49	1.12	8.46
14	6.12	2.30	3.57	1.91	1.44	10.85
44	10.35	3.89	6.03	3.24	2.43	18.34
47	11.07	4.16	6.45	3.46	2.60	19.63
32	11.54	4.34	6.73	3.61	2.71	20.46
34	12.12	4.55	7.06	3.79	2.84	21.48
17	13.64	5.12	7.95	4.27	3.20	24.18
8	15.78	5.93	9.20	4.93	3.70	27.97
36	20.46	7.69	11.93	6.40	4.80	36.28
7	21.15	7.94	12.33	6.61	4.96	37.49
18	24.14	9.07	14.07	7.55	5.66	42.79
19	24.39	9.16	14.22	7.63	5.72	43.23
46	24.70	9.28	14.40	7.72	5.79	43.78
21	27.23	10.23	15.88	8.52	6.39	48.27
45	27.84	10.46	16.23	8.71	6.53	49.35
11	31.94	12.00	18.62	9.99	7.49	56.62
35	32.39	12.17	18.88	10.13	7.60	57.42
4	32.42	12.18	18.90	10.14	7.61	57.47
49	46.94	17.63	27.36	14.68	11.01	83.20
15	48.04	18.05	28.01	15.02	11.27	85.16
48	57.86	21.73	33.73	18.09	13.57	102.57
1	63.31	23.78	36.91	19.80	14.85	112.23
52	65.07	24.44	37.93	20.35	15.26	115.34
12	65.12	24.46	37.96	20.36	15.28	115.44
10	67.02	25.17	39.07	20.96	15.72	118.81
9	70.70	26.56	41.22	22.11	16.59	125.33
41	71.15	26.73	41.48	22.25	16.69	126.13
25	72.26	27.14	42.12	22.59	16.95	128.09
28	73.79	27.72	43.02	23.07	17.31	130.81
5	75.05	28.19	43.75	23.47	17.61	133.04
2	77.65	29.17	45.27	24.28	18.22	137.64
42	81.12	30.47	47.29	25.37	19.03	143.80
51	86.19	32.38	50.25	26.95	20.22	152.79
3	86.83	32.61	50.62	27.15	20.37	153.91
27	89.15	33.49	51.97	27.88	20.91	158.04
22	94.61	35.54	55.15	29.58	22.19	167.71
24	97.67	36.69	56.94	30.54	22.91	173.14
29	101.24	38.03	59.02	31.66	23.75	179.46
26	105.87	39.77	61.72	33.10	24.84	187.67
30	111.28	41.80	64.87	34.80	26.10	197.26
50	113.98	42.81	66.45	35.64	26.74	202.05
39	140.74	52.86	82.04	44.01	33.02	249.48
33	143.03	53.72	83.38	44.72	33.55	253.54
38	149.25	56.06	87.01	46.67	35.01	264.57
20	159.70	59.99	93.10	49.94	37.47	283.10
40	173.10	65.02	100.91	54.13	40.61	306.85
53	179.86	67.55	104.85	56.24	42.19	318.82
43	187.91	70.58	109.55	58.76	44.08	333.11
37	198.34	74.50	115.62	62.02	46.53	351.58
31	210.14	78.93	122.51	65.71	49.30	372.51
23	258.53	97.10	150.71	80.84	60.65	458.28
16	320.57	120.41	186.88	100.24	75.20	568.26
Average	**32.90**	**12.36**	**19.18**	**10.29**	**7.72**	**58.33**

RA, risk averse; RN, risk neutral; TVaR, tail value at risk.

amount that has to be covered by the policyholder in the event of a flood becomes higher. Increasing the deductible implies that the insured has to carry the larger losses located on the right part the flood damage density function, which may exceed the financial capacity of the insured. The 15% deductible level is chosen here as one of the efficient trade-off points between risk and premiums.

The trade-off that the insurer faces is as follows. If an insurer chooses to transfer a significant portion of the insured risk to the reinsurer, then the insurer needs to transfer a large part of the received premium to this reinsurer as well. However, if the reinsurance coverage purchased is too low, then the insurer is required to hold large financial reserves that should be sufficient to cover large losses because otherwise the insurer may be confronted with potentially large losses that exceed the insurer's financial capacity, which can result in insolvency. Therefore, the insurer needs to strike a balance between the reinsurance premiums to be paid and the RMRC. The CP-ratios for the reinsurance part shown in Figure 4 increase rapidly above the 80% loss threshold because of the high RMRC that will be needed to cover highly uncertain losses located on the right tail of the loss distribution, while the corresponding reinsurance premium remains relatively low if the stop-loss increases. In our application, the 84th percentile is chosen as an efficient level for the stop-loss because insurers profit by retaining premiums for the coverage below this level, while losses above this level are highly uncertain and extremely large and are therefore attractive to pass on to reinsurers. In particular, the risk variance of losses above this threshold increases rapidly as the increase in CP-ratios indicates, which suggests that it is attractive for the insurer to purchase reinsurance for these high and uncertain losses. Although a higher stop-loss will lead to lower reinsurance premiums, at a certain point these benefits will not outweigh the increase in the high uncertainty of potential extreme damage faced by the insurer that may exceed the insurer's financial capacity.

Main differences between the RA and the RN premiums

As expected, the estimated RA and RN (re)insurance premiums for both the insured and the insurer are very different because of the (re)insurer's risk aversion to losses with a high variance. This difference is more evident for large losses, which is a clear indication that the extreme losses are more expensive to insure. As the risk aversion rate for the insurer and reinsurer are assumed to be similar and constant, it can be concluded that the extra surcharge for RA (re)insurance premiums is mainly caused by a high variance of flood loss covered by reinsurance. This difference is greatest for those dyke-ring areas, like dyke-ring 14, with large flood losses and risk variance. The reinsurers usually require a high premium surcharge for very large losses, which is not included in this study (Zajdenweber, 1996). Therefore, the very high CP-ratios shown in the results provide only an indication that the large losses are either difficult to insure or only insurable in exchange for high premiums, or in an extreme case may even be uninsurable. But, from an economic perspective, insurance for large infrequent losses is an effective way for individuals to hedge risks. The results here show that in an application to catastrophe risks, a PP insurance arrangement may be a feasible solution for providing such coverage, in a sense that it results in substantially lower premiums than when it is provided through a private catastrophe insurance scheme.

Allocation of risk and premiums among the stakeholders

Although in practice flood deductibles are often imposed by insurers, here we use an estimate of a desirable level based on the practical experience. With the deductible of 15% of damage, the insured will have to cover nearly the first 38% of total flood damage, while the remaining 62% of flood damage is covered by the insurer and the reinsurer together. The part of the flood risk that is paid by the insured seems to be relatively high compared with the coverage amounts to be provided by the insurer and the reinsurer. This 38% includes all losses that fall below the 15th loss percentile plus the constant value equal to the deductible amount that an insured is required to pay out of his pocket if losses exceed this threshold level [see layer 1 in Figure 3 and Eqn (8)]. An advantage of this relatively high deductible level is that it reduces the premium to be paid by the insured. Moreover, a high deductible level serves the purpose of providing incentives to policyholders to take action to prevent damage once a flood is imminent, such as moving valuables to higher floors (Botzen and van den Bergh, 2008). Table 4 shows that reinsurance covers approximately 42–43% of the total insured amount while this amount for the primary insurance layer lies between 57% and 58% of the damage. Comparing these with the corresponding insurance and reinsurance premiums (see Table 6), it can be concluded that the reinsurer receives a relatively high portion of the premiums in order to compensate the reinsurer for covering the extreme part of risk. This is in accordance with practice, where the premiums are relatively higher for large losses that are very uncertain compared with relatively small and more certain losses (Ericson and Doyle, 2004).

Implications for the insured, the insurers, and the reinsurers

The calculations in this paper show that insurance for catastrophe risk is a complex product to price because of the extremely low flood frequency that entails large uncertainties. In particular, losses located on the far right tail of the loss density function with a large dispersion are difficult to estimate with some degree of certainty. Given these difficulties, it is understandable that commercial insurance and reinsurance companies are reluctant to offer insurance products against catastrophe risks at regular prices. This is the first in-depth study on the pricing of flood insurance for three different layers in the Netherlands that uses of the full probability density of flood damage in all 53 dyke-areas of the country. Therefore, it provides several useful practical insights for insurers, reinsurers, and the government that is now considering introducing flood insurance in the Netherlands, which is currently not broadly available.

Our premium estimates for the different layers show that risk-based premiums charged by RA insurers for flood insurance can be considerably above the RN premiums, which may make insurance unaffordable for many homeowners. In practice, commercial insurance and reinsurance companies are generally RA, which may result in high flood insurance premiums, as they normally require a premium surcharge for covering extreme risks, such as floods (Froot, 2001). In practice, premiums may be even higher than those reported in this study because of administrative expenses that are not included in our premium estimates. With the involvement of a RN government as the insurer or reinsurer of last resort, flood insurance can be provided at significantly lower premiums, as our results of RN premiums indicate (Ericson and Doyle, 2004). In such a system, the government covers part of the extreme damage, for which it receives, from the insurance sector, compensation that reflects the risk of this coverage. As an RN agent, the government will not charge the RA surcharge for the coverage of extreme damage as a commercial reinsurer would do, which results in a lower premium and more affordable coverage of flood insurance (Botzen and van den Bergh, 2008; Paudel *et al.*, 2012). As an illustration, our results show that, on average, an RA insurer charges €85 per year, while an insurance system with RN participants that can be achieved by establishing a PP partnership charges on average €51 per year. Botzen and van den Bergh (2012) estimate demand curves for flood insurance in the Netherlands using a choice experiment of flood insurance demand, which shows that a market penetration of about 50% under homeowners could be achieved in a voluntary market for the RN premiums estimated in this paper. Higher (RA) premiums would result in lower market penetration (Paudel *et al.*, 2012). It should be noted that, in practice, the difference between private flood insurance premiums and the premiums charged by a PP partnership may be even larger than the difference between our RA and RN premium estimates because private reinsurers will included a profit margin in setting premiums, which is not included in our RA estimates. An additional advantage of having the government as a reinsurer is, therefore, that the government would view the provision of flood insurance as a public good that does not require making a profit. This is an additional reason for why government involvement in a PP insurance or compensation system makes flood insurance more feasible and affordable, which results in greater market penetration. A potential disadvantage of a PP flood insurance system is that it creates a financial liability for the government. For example, it has been argued that in the United States, the National Flood Insurance Program (NFIP) in which the federal government carries the flood risk has created a large liability. The NFIP needed to borrow large amount from the US congress to cover the severe losses that occurred in the wake of hurricane Katrina (Michel-Kerjan, 2010). This liability is somewhat limited in our proposed PP flood insurance system for the Netherlands in which it is assumed that private insurance companies carry a middle-sized layer of losses, and the government acts as a reinsurer of only very large and infrequent losses (see Figure 1).

Our premium estimates show that flood insurance premiums differ significantly per dyke-ring area. If flood insurance premiums were fully differentiated with respect to actual flood risk, then large differences in premiums would also arise between homeowners within one dyke-ring area because of a heterogeneous distribution of risk within a dyke-ring: for example, because of variations in potential water levels. It may be that certain households will decide not to purchase flood insurance if their (risk-based) premiums are too high, which raises the questions whether flood insurance should be mandatory, or whether a cross-subsidisation of policies should be introduced, or whether alternative policies can be designed to cope with the affordability issues of flood insurance. It may be advisable to make flood insurance compulsory in order spread the costs of insurance over a larger number of policyholders, which makes the insurance system more feasible and reduces the need for subsidising insurance. However, it may be questionable whether such a compulsory insurance scheme is fair to homeowners who live in areas with a low flood risk and whether a compulsory flood insurance system may undermine any incentives to policyholders for risk reduction and the implementation of measures that mitigate flood damage. A certain degree of differentiation of flood insurance premiums with respect to actual flood risk is desirable in order to provide adequate incentives for risk reduction: for example, by rewarding homeowners who protect their house against flooding with a discount on their flood insurance premium (Botzen *et al.*, 2009). This can be done by setting collective premiums on the basis of the dyke-ring area and a risk-zone or risk profile, and not by setting flat premiums at a national level. If risk-based flood insurance premiums have undesirable equity consequences, Kunreuther and Michel-Kerjan (2010) propose that a programme to provide subsidies or insurance vouchers could be designed for low-income people who live in flood-prone areas. This programme would deal with the affordability issue of flood insurance and does not distort incentives to mitigate flood risk (Kunreuther and Michel-Kerjan, 2011).

Conclusions and recommendations

This study has applied a practical Pareto efficient allocation model for estimating the required maximum insurance and reinsurance coverage (respectively, RMIC and RMRC) within a practical range of deductibles and stop-losses, and for providing insights into flood risk allocations between the stakeholders of a multilayer insurance system. Flood risk insurance in the 53 dyke-ring areas in the Netherlands is used as a case study. The RMIC and RMRC amounts are estimated under the 99.9% TVaR damage constraint, in such a way that the best trade-offs between the deductible (stop-loss) and the (re)insurance premiums can be found. Premiums have been estimated for an insurance system that consists of RA private insurers and reinsurers who require a surcharge on the premium for covering events with a highly uncertain risk, such as floods. Moreover, premiums are estimated for a PP partnership in which the RN government participates as a reinsurer in order to prevent the high flood insurance premiums that can arise in a private insurance system. This study could be of practical relevance for commercial insurance and reinsurance companies, the government, and policy makers who are aiming to establish catastrophe risk insurance systems, in general, and flood risk insurance in the Netherlands, in particular, which is currently not generally available.

Large differences are observed between the RA and the RN (re)insurance premiums, which provide important insights for the affordability and feasibility of an insurance system for catastrophe risk. In general, the flood losses located on the right tail of the loss density function are relatively expensive to insure compared with those located on the left or middle part of the loss distribution. This is clearly evident from our premium estimates that were derived for the case where the insurance and the reinsurance are provided by an agency that is strongly RA to catastrophe risk. The assumption of insurer and reinsurer risk aversion or risk neutrality also has an impact on the choice for the stop-loss and deductible levels. In general, if insurance is offered by an RN agency, then there will be no extra surcharge on the premium, which will result in choices for a lower deductible and stop-loss because (re)insurance premiums are lower compared with the insurance system with RA insurance agents.

The participation of governments or other RN agencies, either as a full insurer or a reinsurer, in a catastrophe risk insurance system could make a system much cheaper for the consumer and more feasible to implement in practice. On the basis of this study, we can draw four important lessons for how a feasible and affordable flood risk insurance system could be established in the Netherlands. First, a PP insurance system in which the government acts as a reinsurer of last resort could be a good solution for providing coverage for extreme risks, which are difficult to insure for affordable premiums, as is the case for flood risk in the Netherlands. This could reduce the reinsurance premiums significantly as our estimates made under the RN assumption show. Private insurance companies could participate in such a system by providing insurance coverage for regular, less extreme, flood losses. Second, government participation in a multilayer insurance system could lead to the choice of lower deductibles and stop-losses by, respectively, the insured and insurer because the insurance and reinsurance could be purchased for relatively low premiums. Therefore, government participation in a catastrophe risk insurance system could help such an insurance to function more like a regular insurance system. In particular, the government role could result in a financially solvent and affordable insurance system with fewer exclusions or limitations on coverage. Second, the differences in insurance and reinsurance premiums between the dyke-ring areas are significant, which makes flood insurance for those living in relatively flood-prone dyke-ring areas expensive. In some cases, this may lead to a low take-up of flood insurance, which would impair the spreading of risk and increase the costs of insurance per policyholder and result in high premiums. A solution could be to make flood insurance compulsory and introduce some degree of cross-subsidisation of premiums or implement other policies such as insurance vouchers to deal with the equity and affordability issues of risk-based premiums. Nevertheless, some degree of differentiation of insurance premiums with respect to risk is desirable to provide policyholders with incentives for risk reduction. Fourth, because of their low-frequency/high-impact characteristics, catastrophe risks generally require a long-term strategy and planning for the adequate management of risk by building strong financial capacity and reserves for compensating damage. Such a long-term strategy may not always be in the interests of commercial insurance companies, which have a more short-term view, because they have the possibility to stop offering insurance or augment policy conditions unilaterally, which can be undesirable for policyholders. This is an additional reason for the government to have a role in an insurance system for natural disaster risks because the government is able to take a long-term perspective on risk management and consumer interests into consideration. Further research could focus on deriving empirical estimates of insurer's behaviour towards risk and explore the suitability of different utility function (as suggested by Gerber and Pafumi, 1997, to provide coverage for flood risk in the Netherlands. Moreover, future research could examine in more detail the sensitivity of results to model uncertainty, for example, by performing stress test and scenarios analysis for relevant parameter values and loss data for inputs. Another important topic for future studies could be to examine how robust the multilayer insurance system proposed in this paper is to long-term trends that influence flood risk, such as climate change.

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Appendices

Appendix A

Derivation of the deductible and stop-loss levels

In this optimisation problem, there are two main conditions in determining the range for the optimal level of the deductible, which can be justified as follows:

The deductible is higher than the maximum damage $D > \nu$, which leads to the situation where the insured does not pay any insurance premium and receives no coverage; i.e. $I(E[x]) = \bar{I} = \pi = 0$. Hence,

$$P(x \le \nu) \ge 1 - \alpha \tag{A1}$$

Eqn (A1) implies that the insured does not need to purchase insurance if the damage amount is lower than the α TVaR amount.

However, because the insured wants to optimise the deductible level under the condition $\{x \le \nu\} \ge 1 - \alpha$, the insured should purchase insurance, which leads to $D \le \nu$ with conditions $I(X) \ge 0 \Rightarrow \bar{I} \ge 0 \Rightarrow \pi \ge 0$, and the insured amount necessary to cover the corresponding damage is $I(X) = \lambda\bar{I} \le \nu$. If the deductible is lower than the TVaR amount, and larger than the random damage, then Eqn (6) can be modified as follows:

$$P\{D \le \nu - I(X)\} \ge 1 - \alpha \tag{A2}$$

where $I(X) = \lambda\bar{I}$. Eqn (A2) implies that when an insured has to purchase insurance, the following condition must hold:

$$D \le \nu - I(X) \tag{A3}$$

Because $I(X)$ is an decreasing function with increasing D, D can be approximated with $D \approx \nu - I(X)$.

Because the insurer's stop-loss level shifts with D, the optimality problem for insurers can be proved in the same way as for the insured, which gives us: $M \approx \nu - I(X) - D$, with $I(X) = \lambda\gamma\bar{I}$. From this, it follows that $M \approx \nu - \lambda\gamma\bar{I}$.

Appendix B

Total premium and coverage estimates per dyke-ring area

Table B1 depicts the allocation of total premiums – under the RA and RN assumptions – and the corresponding total annual expected value of maximum coverage based on the 99% TVaR level, including both insurance and reinsurance coverage. The calculations in this section are based, unless otherwise stated, on the 15% deductible level for the insured and the 84% stop-loss for the insurer. The table shows the total annual amounts per dyke-ring area in thousand euros, which are shown in ascending order of the premium amount. Column 2 depicts the mean deductible amounts estimated at the 15% damage level; columns 3 and 4 respectively show the RA insurance and reinsurance premiums; and columns 5 and 6 respectively represent the premiums for the RN insurer and reinsurer. Column 7 shows the corresponding total annual expected value of required maximum coverage amount – to be provided collectively by the insurer and the reinsurer. The last row presents the average over the 53 dyke-ring areas for each column.

Table B1 The expected value of the deductible ($E[D]$), the total annual expected value of required maximum insurance coverage (RMIC), and the corresponding RA and RN premiums charged by the insurer and reinsurer, shown per dyke-ring area (in 1000 euros per year)

Dyke-ring	$E[D]$	RA premium		RN premium		RMIC
Nr.	Homeowner	Insurer	Reinsurer	Insurer	Reinsurer	At 99% TVaR
37	1	1	1	1	1	5
33	1	2	2	1	1	8
4	2	4	3	2	2	15
39	8	14	11	7	6	50
23	10	18	13	9	7	62
1	10	19	14	10	7	66
18	16	30	22	16	12	104
3	22	42	31	22	16	146
40	26	48	36	25	19	166
46	26	48	36	25	19	167
2	34	63	47	33	24	219
6	43	81	61	42	31	281
19	45	84	63	43	33	291
28	80	149	112	77	58	518
49	118	221	166	115	86	771
51	126	235	176	122	92	819
5	129	241	181	125	94	838
47	133	248	186	129	97	863
42	147	274	206	142	107	954
7	151	283	212	147	110	985
12	174	324	243	168	126	1129
32	180	337	253	175	131	1173
11	191	358	268	186	139	1245
10	240	449	337	233	175	1563
27	260	486	365	253	189	1692
21	282	527	395	274	205	1835
35	392	732	549	380	285	2547
25	421	786	590	408	306	2735
31	480	897	673	466	349	3121
26	500	934	701	485	364	3251
8	504	941	706	489	367	3275
24	575	1075	807	559	419	3743
34	628	1173	880	609	457	4081
13	633	1183	888	614	461	4118
50	673	1257	943	653	490	4375
17	726	1357	1018	705	529	4723
9	764	1428	1072	742	557	4971
38	807	1508	1131	783	588	5248
52	881	1647	1236	855	642	5732
45	926	1731	1299	899	675	6025
44	977	1825	1369	948	711	6352
30	1059	1979	1484	1028	771	6886
36	1091	2040	1530	1059	795	7099
48	1116	2086	1565	1083	813	7260
15	1225	2290	1718	1189	892	7969
22	1440	2691	2019	1398	1049	9366
29	1600	2990	2243	1553	1165	10 407
41	2508	4687	3516	2434	1826	16 310
20	3232	6041	4532	3137	2354	21 023
14	3273	6117	4589	3177	2383	21 287
53	5000	9345	7011	4853	3641	32 523
43	7297	13 636	10 230	7082	5313	47 456
16	8504	15 892	11 922	8254	6192	55 308
Average	937	1752	1314	910	683	6097

RA, risk averse; RN, risk neutral; TVaR, tail value at risk.

