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S duality and dyonic p-brane solutions in type II string theory

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We show how a solitonic ''magnetically'' charged p-brane solution of a given supergravity theory, with the magnetic charge carried by an antisymmetric tensor gauge field, can be generalized to a dyonic solution. We discuss the cases of ten-dimensional and eleven-dimensional supergravity in more detail and a new dyonic five-brane solution in ten dimensions is given. Unlike the purely electrically or magnetically charged five-brane solution the dyonic five-brane contains nonzero Ramond-Ramond fields and is, therefore, an intrinsically type II solution. The solution preserves half of the type II spacetime supersymmetries. It is obtained by applying a solution-generating $\mathrm{SL}(2,R) \times \mathrm{SL}(2,R)$ S duality transformation to the purely magnetically charged five-brane solution. One of the $\mathrm{SL}(2,R)$ duality transformations is basically an extension to the type II case of the six-dimensional Z_2 string-string duality. We also present an action underlying the type IIB supergravity theory.

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I. INTRODUCTION

Recently, an active field of research has been the search for p-brane solutions (i.e., solutions with p translational spacelike isometries) in supergravity theories (for some recent reviews, see, e.g., [1–3]). One of the motivations is that, in case the supergravity theory is an effective superstring theory, it might give us information about the strong-coupling behavior of the superstring. Two particularly interesting examples corresponding to the ten-dimensional heterotic string effective action are the fundamental string solution found by Dabholkar *et al.* [4] and the five-brane soliton found by Strominger [5]. Many more p-brane solutions have been found, both in $D=10$ [1–3,6,7] as well as in $D=11$ [8].

Most of the p-brane solutions found so far are either elementary solutions requiring a singular source term (that we will call ''electrically charged'') or solitonic solutions (''magnetically charged''). It is natural to investigate the possibility of constructing *dyonic* p-brane solutions. So far, most of the research has concentrated on the study of four-dimensional dyonic black holes (0-branes), see, e.g., Refs. [9,10]. In this work we will consider this problem for higher p and higher dimensions.[1] To explain the basic idea we start by succinctly describing the ten-dimensional five-brane

electric-magnetic (EM) Z_2 map. Any ten-dimensional heterotic five-brane solution $5_{(10)}$ can be reinterpreted via dimensional reduction as a six-dimensional string (1-brane) solution $1_{(6)}$. If the original $5_{(10)}$ solution does not give rise to six-dimensional vector fields, then one can use the string-string duality symmetry [11,12] to generate another six-dimensional solution $1'_{(6)}$, which in turn can be reinterpreted via ''inverse dimensional reduction'' as another ten-dimensional five-brane solution $5'_{(10)}$. Since the six-dimensional transformation is an EM duality transformation on the axion two-form, we get a *ten-dimensional EM duality for heterotic five-branes*.

As an example, let us consider, the magnetic $5_{(10)m}$ solution[2] [5] in the string frame[3]:

$$\mathbf{5}_{(10)m}\begin{cases} ds^2 = (dx^0)^2 - (dx^1)^2 - (dx^m)^2 - e^{2\hat{\phi}}dx^a\,dx^a, \\ \hat{H}_{abc} = \frac{2}{3}\epsilon_{abcd}\partial^d\hat{\phi}, \end{cases} \tag{1}$$

where $x^{\hat{\mu}} = (x^0, x^1, x^m; x^a), m \in \{6,7,8,9\}, a \in \{2,3,4,5\}$. The five-brane world-volume is parametrized by $\{x^0, x^1, x^m\}$. The dilaton only depends on the x^a's and satisfies $\Box e^{2\hat{\phi}} = 0$ where $\Box = \delta^{ab}\partial_a\partial_b$. This solution can be reinterpreted as a string in six dimensions by eliminating the x^m's:

$$\mathbf{1}_{(6)m}\begin{cases} ds^2 = (dx^0)^2 - (dx^1)^2 - e^{2\phi}(dx^a)^2, \\ H_{abc} = \frac{2}{3}\,\epsilon_{abcd}\partial^d\phi, \end{cases} \tag{2}$$

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[1]We will construct solutions with nonzero Ramond-Ramond fields. Since the Ramond-Ramond fields do not couple via a standard sigma model action to the type II string it is nontrivial to deal with source terms. In this paper we postpone a proper treatment of this issue and only solve the source-free equations of motion.

[2]Our conventions are those of Ref. [13]. We only discuss the so-called ''neutral five-brane'' [14,15]. This example has previously been studied in Refs. [16–18].

[3]The Einstein frame metric $\hat{g}^E_{\hat{\mu}\hat{\nu}}$ and the string frame metric $\hat{g}^S_{\hat{\mu}\hat{\nu}}$ in ten dimensions are related by $\hat{g}^E_{\hat{\mu}\hat{\nu}} = e^{-1/2\hat{\phi}}\hat{g}^S_{\hat{\mu}\hat{\nu}}$.

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where the dilaton satisfies[4] $\square e^{2\phi}=0$. Now we make use of the six-dimensional Z_2 EM duality. This is most easily described in the Einstein frame[5] by

$$\phi'=-\phi, \quad H'=e^{-2\phi}{*}H, \tag{3}$$

with $*$ the six-dimensional Hodge star. In the string frame this transformation acts on the metric. The result is the *six-dimensional fundamental string solution*

$$\mathbf{1}_{(6)e} \begin{cases} ds^2=e^{2\phi}[(dx^0)^2-(dx^1)^2]-(dx^a)^2, \\ B_{01}=e^{2\phi}. \end{cases} \tag{4}$$

Observe that the dilaton now satisfies $\square e^{-2\phi}=0$. Finally, we can reinterpret this solution as a five-brane solution in ten dimensions:[6]

$$\mathbf{5}_{(10)e} \begin{cases} ds^2=e^{2\hat\phi}[(dx^0)^2-(dx^1)^2]-(dx^m)^2-(dx^a)^2, \\ \hat B_{01}=e^{2\hat\phi}. \end{cases} \tag{5}$$

The dilaton depends only on the x^a's and satisfies $\square e^{-2\hat\phi}=0$. This solution can be thought of as an electric ten-dimensional heterotic five-brane solution which is the EM dual of the solitonic five-brane $5_{(10)m}$.

It is natural to try to generalize the EM duality Z_2 to SL(2,R) so that we can use it to build dyonic solutions. In doing so we encounter the following two problems.

(1) We have restricted ourselves to the case in which no six-dimensional vector fields arise. Otherwise, in the framework of the heterotic string compactified on T^4, the dual theory does not coincide with the original theory. A way of seeing this is the following. In the presence of vector fields there are Chern-Simons terms in the field-strength for the axion. In dualizing the theory, these Chern-Simons terms get interchanged with topological terms in the action and, vice versa, every topological term in the action gives rise to a Chern-Simons term[7]. Therefore, the duality just described can only be a symmetry of the equations of motion if for every Chern-Simons term there is a corresponding topological term. However, in the heterotic case there are no topological terms whatsoever. Therefore, the duality just described would not be a symmetry of the equations of motion of a single theory, but would relate two different (dual string) theories.

(2) To extend the Z_2 symmetry to SL(2,R) one needs more fields coupled to the axion in a specific way. This is similar to the four-dimensional case where, in the absence of the pseudoscalar axion a, one only has a Z_2 EM duality. The introduction of a suitably coupled to the vectors enhances the symmetry to SL(2,R).

The solution to both of these problems lies in the type II theories: In the type IIB theory there are topological terms present from the beginning, and they are such that they get interchanged with the Chern-Simons terms that appear in the compactification, leaving the theory invariant. At the same time, in these topological terms the axion is coupled to the four-form $\hat D$ [which is a Ramond-Ramond (RR) field] in the "right way," so we can extend the above five-brane Z_2 to an SL(2,R) and we can build dyonic five-branes. The price to pay is that, although we can always start with a type I five-brane, in general we will obtain type II five-branes with extra nonzero RR fields which seem to be necessary for the symmetry enhancement (see, however, the conclusions).

In the following section we will briefly review type IIB supergravity. As a new result, which is useful for our present purposes, we will present an action underlying the type IIB theory. In the next section we will dimensionally reduce this action using an ansatz simple but rich enough to show the symmetry enhancement mechanism. In Sec. IV we will obtain a new ten-dimensional type II dyonic five-brane solution which continually interpolates between the $5_{(10)m}$ and $5_{(10)e}$ solutions given in (1) and (5), respectively. Finally, in the conclusions we will discuss more general applications of our techniques.

Part of the results of this article have been presented in Ref. [19].

II. *D*=10 TYPE IIB SUPERGRAVITY

It is known [20] that the field equations of $D=10$ type IIB supergravity [21] cannot be derived from a covariant action. Nevertheless, it is useful to think about an "action" in the restricted sense explained below. The only equation of motion that cannot be obtained from an action is that of the four-form gauge field $\hat D$. This equation of motion states that the field strength $\hat F$ of $\hat D$ is self-dual: $\hat F={}^\star\hat F$. It follows that if one sets $\hat F=0$ everywhere in the equations of motion, one should be able to obtain the resulting reduced set of equations from an action, by varying with respect to all fields but $\hat D$. This was done in Ref. [13].

We will show in this section that one can even write down an action involving $\hat F$. A useful property of such an action is that, when properly used, it leads to the correct action for the *dimensionally reduced* type IIB supergravity theory. We thus may avoid the dimensional reduction of the ten-dimensional type IIB field equations which is more complicated. This property will be exploited in the next section.

The idea is the following: we keep $\hat F$ different from zero but eliminate the self-duality constraint. Of course, we would like to have an equation of motion for $\hat D$ replacing $\hat F={}^\star\hat F$. As a matter of fact, there is a perfect "spare equation of motion" at our disposal. One of the consequences of the self-duality constraint is that the equation of motion of $\hat D$ is equal to its Bianchi identity. Therefore, it is natural to take,

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[4]We express all solutions in terms of their own dilatons.

[5]The Einstein frame metric $g^E_{\mu\nu}$ and the string frame metric $g^S_{\mu\nu}$ in six dimensions are related by $g^E_{\mu\nu}=e^{-\phi}g^S_{\mu\nu}$.

[6]The $5_{(10)e}$ solution can be viewed as a special case of the Dabholkar string $1_{(10)e}$ where the eight-dimensional Laplace equation for the dilaton has been solved in the presence of four extra isometries. As a consequence, the Dabholkar string has an eight-dimensional spherical symmetry while the $5_{(10)e}$ solution has a four-dimensional spherical symmetry.

[7]For instance, in four dimensions, the axion field strength has Chern-Simons terms so $H=\partial B+\frac{1}{2}AF$ and there are no topological terms present in the action. In the dual theory the axion is substituted by the pseudoscalar a and the topological term $aF^\star F$ appears in the action.

as the equation of motion[8] for $\hat{D}$,

$$\nabla_{\hat{\mu}}\hat{F}^{\hat{\mu}\hat{\mu}_1\hat{\mu}_2\hat{\mu}_3\hat{\mu}_4} = \frac{3}{5!4}\epsilon^{ij}\epsilon^{\hat{\mu}_1\cdots\hat{\mu}_{10}}\hat{\mathcal{H}}^{(i)}_{\hat{\mu}_5\hat{\mu}_6\hat{\mu}_7}\hat{\mathcal{H}}^{(j)}_{\hat{\mu}_8\hat{\mu}_9\hat{\mu}_{10}}. \quad (6)$$

This equation is compatible with the self-duality constraint (which we have temporarily abolished), but it does not imply it. So, in fact, we have just eliminated the self-duality constraint in a consistent way.

Now, is there an action for the new set of equations of motion obtained by eliminating the self-duality constraint and substituting it with Eq. (6)? Note that in the original equations, the self-duality of $\hat{F}$ was already taken into account and therefore only $\hat{F}$ occurs. In this nonself-dual (NSD) theory we expect both $\hat{F}$ and $^\star\hat{F}$ to occur. The NSD theory is defined by the property that it has the same field content as the original theory but $\hat{F}$ is not self-dual and, if one imposes self-duality in the field equations, one recovers it.

It turns out that the easiest way to find the NSD theory and its action is to make the most obvious ansatz for its action: add to the action for the $\hat{F}=0$ case an $\hat{F}^2$ (kinetic) term and a topological term with numerical factors to be adjusted. One easily finds that the action we are looking for, in the string frame and with the notation and conventions of Ref. [13], is given by[9]

$$\hat{S}^{\text{string}}_{\text{NSD-IIB}} = \frac{1}{2}\int d^{10}x\sqrt{-\hat{j}}\left\{e^{-2\hat{\varphi}}[-\hat{R}(\hat{j})+4(\partial\hat{\varphi})^2\right.$$
$$\left.-\tfrac{3}{4}(\hat{\mathcal{H}}^{(1)})^2]-\tfrac{1}{2}(\partial\hat{\ell})^2-\tfrac{3}{4}(\hat{\mathcal{H}}^{(2)}-\hat{\ell}\,\hat{\mathcal{H}}^{(1)})^2-\tfrac{5}{6}\hat{F}^2\right.$$
$$\left.-\frac{1}{96\sqrt{-\hat{j}}}\epsilon^{ij}\epsilon\hat{D}\hat{\mathcal{H}}^{(i)}\hat{\mathcal{H}}^{(j)}\right\}. \quad (7)$$

For the sake of completeness we list the definitions of the field strengths and gauge transformations for the type IIB fields $\{\hat{D}_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}},\hat{j}_{\hat{\mu}\hat{\nu}},\hat{\mathcal{B}}^{(i)}_{\hat{\mu}\hat{\nu}},\hat{\ell},\hat{\varphi}\}$, $(i=1,2)$:

$$\hat{\mathcal{H}}^{(i)}=\partial.\hat{\mathcal{B}}^{(i)}, \quad \delta\hat{\mathcal{B}}^{(i)}=\partial\hat{\Sigma}^{(i)},$$

$$\hat{F}=\partial\hat{D}+\tfrac{3}{4}\epsilon^{ij}\hat{\mathcal{B}}^{(i)}\partial\hat{\mathcal{B}}^{(j)}, \quad \delta\hat{D}=\partial\hat{\rho}-\tfrac{3}{4}\epsilon^{ij}\partial\hat{\Sigma}^{(i)}\hat{\mathcal{B}}^{(j)}. \quad (8)$$

Varying with respect to all the fields one gets the equations of motion of the NSD theory and, imposing the self-duality constraint, these become the equations of the type IIB theory.

The NSD theory defined by Eq. (7) has all the symmetries of the type IIB theory, including the global SL(2,R)$_{\text{IIB}}$, and an additional global Z_2 EM duality of the $\hat{D}$ field that inter-

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[8]The term at the right-hand side (RHS) follows from the Chern-Simons term in $\hat{F}$, see below.

[9]In this paper we will use the convention that $\hat{\varphi}$ is the dilaton and $\hat{\mathcal{B}}^{(1)}$ the NS-NS axion. Note that other definitions of the dilaton and Neveu-Schwarz–Neveu-Schwarz (NS-NS) axion are possible which differ from $\hat{\varphi}$ and $\hat{\mathcal{B}}^{(1)}$ by an SL(2,R) rotation.

changes $\hat{F}$ and $^\star\hat{F}$. To exhibit the SL(2,R)$_{\text{IIB}}$ symmetries, it is useful to go to the Einstein frame because the Einstein metric is inert under them:

$$S^{\text{Einstein}}_{\text{NSD-IIB}} = \frac{1}{2}\int d^{10}x\sqrt{-\hat{g}}\left[-\hat{R}+\tfrac{1}{4}\text{Tr}(\partial_\mu\hat{\mathcal{M}}\partial^\mu\hat{\mathcal{M}}^{-1})\right.$$
$$\left.-\tfrac{3}{4}\tilde{\hat{\mathcal{H}}}_{(i)}\hat{\mathcal{H}}^{(i)}-\tfrac{5}{6}\hat{F}^2-\frac{1}{96\sqrt{-\hat{g}}}\epsilon^{ij}\epsilon\hat{D}\hat{\mathcal{H}}^{(i)}\hat{\mathcal{H}}^{(j)}\right], \quad (9)$$

where $\hat{g}_{\hat{\mu}\hat{\nu}}=e^{-1/2\hat{\varphi}}\hat{j}_{\hat{\mu}\hat{\nu}}$ is the Einstein-frame metric, $\hat{\mathcal{M}}$ is the 2×2 matrix

$$\hat{\mathcal{M}}=(\hat{\mathcal{M}}_{ij})=\frac{1}{\text{Im}\lambda}\begin{pmatrix}|\hat{\lambda}|^2 & -\text{Re}\hat{\lambda} \\ -\text{Re}\hat{\lambda} & 1\end{pmatrix}, \quad (10)$$

where $\hat{\lambda}=\hat{\ell}+ie^{-\hat{\varphi}}$ is a complex scalar that parametrizes SL(2,R)$_{\text{IIB}}$ and

$$\tilde{\hat{\mathcal{H}}}_{(i)}=\hat{\mathcal{H}}^{(j)}\hat{\mathcal{M}}_{ji}. \quad (11)$$

The action (9) is invariant under the SL(2,R)$_{\text{IIB}}$ transformations

$$\hat{\mathcal{H}}'=\Lambda\hat{\mathcal{H}},$$
$$\hat{\mathcal{M}}'=(\Lambda^{-1})^T\hat{\mathcal{M}}\Lambda^{-1}. \quad (12)$$

If Λ is the SL(2,R)$_{\text{IIB}}$ matrix

$$\Lambda=\begin{pmatrix}d & c \\ b & a\end{pmatrix}, \quad (13)$$

the transformation Eq. (12) of the matrix $\hat{\mathcal{M}}$ implies the usual transformation of the complex scalar $\hat{\lambda}$:

$$\hat{\lambda}'=\frac{a\hat{\lambda}+b}{c\hat{\lambda}+d}. \quad (14)$$

Although this symmetry does not involve any EM duality rotation $\hat{\mathcal{H}}\to{}^\star\hat{\mathcal{H}}$ in the ten-dimensional space-time, from the string theory point of view it is a genuine S duality symmetry [22] since some of its transformations ($\hat{\lambda}'=-1/\hat{\lambda}$) interchange the strong- and weak-coupling regimes of string theory and world-sheet elementary excitations (NS-NS states) with solitons (RR states) [13]. In fact, as we will see in Sec. IV, it is the SL(2,R)$_{\text{IIB}}$ which transforms electric solutions into magnetic solutions and which can be used by itself to construct dyonic solutions. However, these dyonic solutions are restricted in the sense that if the electric (magnetic) charge is carried by the NS-NS axion then the magnetic (electric) charge is carried by the RR axion. In the next section we will introduce another SL(2,R) transformation, which we will call SL(2,R)$_{\text{EM}}$, and which will enable us to construct dyonic solutions where both axions carry an electric as well as a magnetic charge.

Finally, we expect that a NSD type IIB theory including fermions can also be found. Of course, the full NSD type IIB action cannot be supersymmetric. However, the supersymmetry should be recovered in the field equations when the (super-) self-duality constraint is imposed.[10]

III. TYPE II EM DUALITY IN SIX DIMENSIONS

In this section we will reduce the NSD type IIB action (7) to six dimensions using the following simplified ansatz for the fields ($\mu, \nu, \ldots$ are six-dimensional spacetime indices and $m, n, \ldots$ are the four internal directions)[11]:

$$\hat{J}_{\mu\nu} = J_{\mu\nu}, \quad \hat{j}_{mn} = -e^{\bar{G}}\delta_{mn},$$

$$\hat{\mathscr{B}}^{(i)}_{\mu\nu} = \mathscr{B}^{(i)}_{\mu\nu},$$

$$\hat{D}_{\mu\nu\rho\sigma} = D_{\mu\nu\rho\sigma}, \quad \hat{D}_{mnpq} = D_{mnpq}, \tag{15}$$

$$\hat{\ell} = \ell, \quad \hat{\varphi} = \bar{\varphi} + \bar{G}.$$

All other components are zero. This ansatz contains extra scalars (as compared to the ansatz used in the example of the introduction) which are the RR fields necessary to extend Z_2 to SL(2,R) as we will show below. Note that for the D field we only consider the scalars that arise in six dimensions consistently with self-duality. This gives precisely one scalar: $D = \epsilon^{mnpq}D_{mnpq}$. In the dimensional reduction we dualize the field strength $F(D)_{\mu_1 \ldots \mu_5}$ to the field strength of a scalar $\tilde{D}$ and a suitable normalization turns the self-duality constraint into $\tilde{D} = D$, which can now be substituted into the action. The kinetic and topological terms for D and $\tilde{D}$ then give equal contributions to the reduced action. Therefore it suffices to collect the D_{mnpq} terms only and multiply these terms by a factor of 2.

The resulting reduced action is

$$S = \frac{1}{2}\int d^6x \sqrt{-J}\{e^{-2\varphi}[-R(J) + 4(\partial\varphi)^2 - (\partial\bar{G})^2$$

$$-\frac{3}{4}(\mathscr{H}^{(1)})^2] - \frac{1}{2}e^{2\bar{G}}(\partial\ell)^2 - \frac{3}{4}e^{2\bar{G}}(\mathscr{H}^{(2)} - \ell\mathscr{H}^{(1)})^2$$

$$-\frac{1}{72}72\,e^{-2\bar{G}}(\partial D)^2 + \frac{1}{8}D\mathscr{H}^T\mathscr{L}^*\mathscr{H}\}, \tag{16}$$

where $(^*\mathscr{H})_{\mu\nu\rho} = (1/6\sqrt{-j})\,\epsilon_{\mu\nu\rho\alpha\beta\gamma}\mathscr{H}^{\alpha\beta\gamma}$ and where we have introduced the 2×2 matrix

$$\mathscr{L} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{17}$$

As we did in the Introduction we go to the Einstein metric $g = e^{-\varphi}J$ and obtain

$$S = \frac{1}{2}\int d^6x \sqrt{-g}\left(-R(g) + \frac{2\partial\lambda\partial\bar{\lambda}}{(\lambda - \bar{\lambda})^2} + \frac{2\partial\kappa\partial\bar{\kappa}}{(\kappa - \bar{\kappa})^2}\right.$$

$$\left. -\kappa_2\mathscr{H}^T\hat{\mathscr{M}}\mathscr{H} + \kappa_1\mathscr{H}^T\mathscr{L}^*\mathscr{H}\right). \tag{18}$$

The complex scalars λ, κ are

$$\kappa = \kappa_1 + i\kappa_2 = \frac{1}{8}D + \frac{3}{4}ie^{2G},$$

$$\lambda = \lambda_1 + i\lambda_2 = \ell + ie^{-\varphi}, \tag{19}$$

with

$$\varphi = \bar{G} + \bar{\varphi}, \quad 2G = \bar{G} - \bar{\varphi}. \tag{20}$$

There are two SL(2,R)/U(1) scalar cosets in the action (18) and correspondingly there are two SL(2,R) symmetries of the equations of motion. One of them is the original SL(2,R)$_\text{IIB}$ symmetry of the NSD type IIB action (7). Note that G (not $\bar{G}$) is SL(2,R)$_\text{IIB}$ invariant. The second is the EM SL(2,R)$_\text{EM}$ duality of the two-form potentials we were looking for, and it is only a symmetry of the equations of motion. It acts on κ and $\mathscr{H}$ as

$$\kappa' = \frac{p\kappa + q}{r\kappa + s},$$

$$\mathscr{H}'_{\mu\nu\rho} = (r\kappa_1 + s)\mathscr{H}_{\mu\nu\rho} + r\kappa_2\mathscr{L}\hat{\mathscr{M}}^*\mathscr{H}_{\mu\nu\rho}, \tag{21}$$

with $ps - qr = 1$. Note that these SL(2,R)$_\text{EM}$ transformations are similar in form to the *S* duality of the heterotic string compactified to four dimensions,[12] with vector fields replaced by two-form fields and with the axion/dilaton field replaced by[13] κ.

We finally recall that the complete noncompact symmetry group of $D = 6$ type IIA supergravity [24] is SO(5,5). In this section we have dimensionally reduced a truncated version of the type IIB theory, using the special ansatz (15). In this way we recovered an

$$\text{SO}(2,2) \equiv \text{SL}(2,R)_\text{EM} \times \text{SL}(2,R)_\text{IIB} \tag{22}$$

noncompact symmetry of the equations of motion. In the next section we will apply this symmetry to construct a class of dyonic five-brane solutions.

IV. DYONIC FIVE-BRANES

Using the solution-generating transformations constructed in the previous section, it is now straightforward to construct ten-dimensional dyonic five-brane solutions $5_{(10)d}$. To this end we apply the most general SL(2,R)$_\text{IIB} \times$ SL(2,R)$_\text{EM}$ transformation, with parameters $a, b, c, d(ad - bc = 1)$ and $p, q, r, s(ps - qr = 1)$, respectively, to the $5_{(10)m}$ solution given in (1). The result is given by[14]

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[10]We thank M. Green for a discussion on this point.

[11]It turns out that not $\bar{\varphi}, \bar{G}$ but combinations of them for which we reserve the names φ, G, naturally fit into a SL(2,R) coset.

[12]See, e.g., the review of Ref. [23].

[13]This is the reason that we use the name SL(2,R)$_\text{EM}$.

[14]All results in this section are ten-dimensional and in string frame. We omit all carets.

$$ds^2 = A[(dx^0)^2 - (dx^1)^2] - B(dx^m)^2 - Ae^{2C}(dx^a)^2,$$

$$\begin{pmatrix} \mathscr{H}^{(1)} \\ \mathscr{H}^{(2)} \end{pmatrix} = \begin{pmatrix} asH - \frac{3}{4}bre^{-2C}*H \\ csH - \frac{3}{4}dre^{-2C}*H \end{pmatrix}, \qquad (23)$$

$$e^{-\varphi} = \frac{e^{-C}}{a^2 + b^2 e^{-2C}},$$

$$\ell = \frac{ac + bde^{-2C}}{a^2 + b^2 e^{-2C}},$$

$$D_{mnpq} = \frac{1}{3}\epsilon_{mnpq}\frac{qs + \frac{9}{16}pre^{-2C}}{s^2 + \frac{9}{16}r^2 e^{-2C}},$$

where the functions A, B and H_{abc} are functions of C,

$$A = \sqrt{a^2 + b^2 e^{-2C}}\sqrt{s^2 + \frac{9}{16}r^2 e^{-2C}},$$

$$B = \frac{\sqrt{a^2 + b^2 e^{-2C}}}{\sqrt{s^2 + \frac{9}{16}r^2 e^{-2C}}}, \qquad (24)$$

$$H_{abc} = \frac{2}{3}\epsilon_{abcd}\partial^d C, \qquad (25)$$

and C depends only on the x^a's and satisfies $\Box e^{2C} = 0$.

A characteristic feature of the above dyonic five-brane solutions is that nonzero RR fields are needed in order for the solution to carry electric as well as magnetic charge. Setting the RR axion and the other RR fields ℓ and D_{mnpq} equal to zero, leads to a purely electric or purely magnetic solution.

The above family of dyonic five-brane solutions contains the known four purely electrically or magnetically charged fivebrane solutions (see Introduction) as special cases. First of all, for $a = d = p = s = 1$, $b = c = q = r = 0$ (unit transformation) we recover $5_{(10)m}$ which we call $5^{(1)}_{(10)m}$ here.[15]

Secondly, the $(Z_2)_{IIB}$ transformation $b = p = w = 1$, $c = -1, a = d = q = r = 0$, when acting on the $5^{(1)}_{(10)m}$ solution, leads to the electrically charged solution[16]

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[15]The superscript $(1),(2)$ indicates that the charge of the solution is carried by the NS-NS or RR axion, respectively.

[16]Note that according to our point of view this solution is of electric character because the string coupling constant e^φ is small ($e^{-2\varphi}$ is singular). This property is due to the fact that the axion involved is a RR axion which from the string theory point of view is a (world-sheet) soliton. *All* solutions have the property that the dilaton corresponding to the electrically (magnetically) charged solutions satisfies $\Box e^{-2\varphi} = 0$ ($\Box e^{2\varphi} = 0$). In this way the electric and magnetic solutions are always connected via a strong or weak coupling duality.

$$5^{(2)}_{(10)e} \begin{cases} ds^2 = e^\varphi[(dx^0)^2 - (dx^1)^2 - (dx^m)^2] - e^{-\varphi}(dx^a)^2, \\ \mathscr{H}^{(2)}_{abc} = \frac{2}{3}\epsilon_{abcd}\partial^d\varphi, \\ \varphi = \varphi(x^a), \quad \Box e^{-2\varphi} = 0. \end{cases}$$
$$(26)$$

Thirdly, the $5^{(1)}_{(10)e}$ solution in Eqs. (5) is obtained by applying the $(Z_2)_{IIB} \times (Z_2)_{EM}$ transformation $b = 1$, $c = -1$, $q = -3/4$, $r = 4/3$, $a = d = p = s = 0$ on the $5^{(1)}_{(10)m}$ solution.

Finally, by applying the $(Z_2)_{EM}$ transformation $a = d = 1$, $q = -3/4$, $r = 4/3$, $b = c = p = s = 0$ on the $5^{(1)}_{(10)m}$ solution we obtain the second magnetically charged solution $5^{(2)}_{(10)m}$

$$5^{(2)}_{(10)m} \begin{cases} ds^2 = e^{-\varphi}[(dx^0)^2 - (dx^1)^2] - e^\varphi[(dx^m)^2 + (dx^a)^2], \\ B^{(2)}_{01} = -e^{-2\varphi}, \\ \varphi = \varphi(x^a), \quad \Box e^{2\varphi} = 0. \end{cases}$$
$$(27)$$

Observe that the original EM Z_2-transformation discussed in the Introduction that transforms the $5^{(1)}_{(10)m}$ solution into the $5^{(1)}_{(10)e}$ solution, is the product of a $(Z_2)_{IIB}$ and a $(Z_2)_{EM}$.

Finally, the $5^{(1)}_{(10)m}$ and $5^{(1)}_{(10)e}$ solutions are also solutions of the heterotic superstring whereas the $5^{(2)}_{(10)m}$ and $5^{(2)}_{(10)e}$ solutions are solutions of the type I superstring [25]. These particular solutions have been used in [25] to confirm the ten-dimensional duality between the heterotic and type I superstring.

V. CONCLUSIONS AND OUTLOOK

In this article we have constructed dyonic five-brane solutions in ten dimensions by applying a solution-generating $SL(2,R)_{IIB} \times SL(2,R)_{EM}$ S duality transformation. The ten-dimensional dyonic five-brane solutions can also be interpreted as six-dimensional dyonic string solutions. It is interesting to compare our results with the six-dimensional dyonic string solution that was recently constructed in Ref. [26]. Our solution differs from that of Ref. [26] in the following two respects. First of all, the dyonic string of Ref. [26] contains no RR fields whereas our solution does. For instance, our solution contains two axions while the one of Ref. [26] contains one. Secondly, the solution of Ref. [26] is a heterotic solution that breaks 3/4 of the spacetime supersymmetries. Our solution is a type II solution that breaks 1/2 of the type II spacetime supersymmetries. This is necessarily so because the purely electrically or magnetically charged five-brane has this property and we know that the SL $(2,R)_{IIB} \times SL(2,R)_{EM}$-transformation, viewed as a noncompact symmetry of six-dimensional supergravity is consistent with the full set of type II supersymmetries. One could also compare the dyonic solutions of this paper with those of Ref. [26] in ten dimensions. The solutions constructed in Ref. [26] contain 3 arbitrary functions. One difference is that our solution contains 1 dilatonic scalar (ϕ) and two RR scalars (ℓ and D), whereas those of Ref. [26] contain a single dilatonic scalar. In order that the two types of solutions coincide

one should therefore take $\ell = D = 0$ in our case. This immediately leads to one of the 4 special cases that are purely electrically or magnetically charged. There seems to be no further overlap between the dyonic solutions of Ref. [26] and those constructed in this work.

Although we give in this article explicit results only for the five-brane in ten dimensions, we expect that the techniques we use can be applied to more general cases. To explain the basic idea,[17] consider a supergravity theory containing a metric $g_{\mu\nu}$, dilaton ϕ and a $(p+1)$-form gauge field[18] $B_{\mu_1 \cdots \mu_{p+1}}$. The Lagrangian for these fields takes a standard form, as in Ref. [3]. From the general analysis of Ref. [3] it follows that this theory has an elementary p-brane solution p_e and a solitonic $(D-p-4)$-brane solution $(D-p-4)_m$. We next observe that the dual of a $(p+1)$-form field is again a $(p+1)$-form field in $2(p+2)$ spacetime dimensions. In order to allow for a Z_2 S duality transformation we therefore reinterpret the $(D-p-4)_m$ solution in D dimensions as a p_m solution in $2(p+2)$ dimensions via a dimensional reduction over $D-2p-4$ spacelike *world volume* directions. Similarly, a dimensional reduction of the p_e solution in D dimension over $D-2p-4$ spacelike *transverse* directions leads to a p_e solution in $2(p+2)$ dimensions. Given the standard form of the Lagrangean in D dimensions, and assuming a simple ansatz for the D-dimensional fields that includes the D-dimensional p_e and $(D-p-4)_m$ solutions [as in (15)], one can show that the field equations corresponding to the dimensionally reduced theory in $2(p+2)$ dimensions are invariant under a Z_2 S duality transformation that maps the p_e and p_m solutions into each other. In summary, the special case of the p_e solution in D dimensions where there are $(D-2p-4)$ extra Abelian isometries in the transverse directions can be viewed, via a Z_2 S duality transformation in $2(p+2)$ dimensions, as the purely ''electrically'' charged partner $(D-p-4)_e$ of the ''magnetically'' charged $(D-p-4)_m$ soliton solution in D dimensions.

It is instructive to consider a few examples of the above general analysis. Consider for instance ten-dimensional type IIA supergravity. The theory contains a 1,2- and 3-form gauge fields and therefore has the following solutions (see, e.g., Ref. [16] or the table in Ref. [27]):

$$(0_e, 6_m), \quad (1_e, 5_m), \quad (2_e, 4_m). \tag{28}$$

Applying the above analysis for $D=10$ and $p=0,1,2$, respectively, we see that all the elementary solutions can be reinterpreted as purely ''electrically'' charged parners of the ''magnetically'' charged soliton solutions by a Z_2 S duality transformation in 4, 6, and 8 dimensions, respectively:

$$0_e \rightarrow 6_e, \quad 1_e \rightarrow 5_e, \quad 2_e \rightarrow 4_e. \tag{29}$$

Note that only the string/five-brane solution can also be considered as a solution of the heterotic superstring.

Next, we consider type IIB supergravity. It contains a complex 2-form and a self-dual 4-form gauge field. The complex 2-form gauge field leads to the following solutions:

$$(1_e + 1_e, 5_m + 5_m). \tag{30}$$

In addition, the self-dual 4-form gauge field leads to the self-dual three-brane solution 3_{EM} of Ref. [6,7]. By reducing to six dimensions we find that

$$1_e + 1_e \rightarrow 5_e + 5_e. \tag{31}$$

The self-dual 3_{EM} solution is special in the sense that our general formulas given above lead to a Z_2-duality transformation in ten dimensions itself. However, since type IIB supergravity is already self-dual there is no such Z_2 transformation.[19]

Finally, we consider the case of eleven-dimensional supergravity. There is only one three-form in eleven dimensions which leads to an elementary membrane 2_e and a solitonic five-brane 5_m. By performing a Z_2 S duality transformation in 8 dimensions we find that

$$2_e \rightarrow 5_e. \tag{32}$$

The results of this work suggest that in all examples given above, the Z_2 S duality transformation can be extended to an $SL(2,R)$ transformation in a relatively simple way. The $SL(2,R)$-transformations so obtained can then be applied to construct dyonic 4, 5- and 6-brane solutions in $D=10$ and a dyonic 5-brane solution in $D=11$. It would be interesting to construct these dyonic p-brane solutions and to investigate their properties, like, e.g., their singularity structure.

Note added: After this work was completed, an interesting paper appeared [28] where, for quite different purposes, also the reduction of $D=10$ type II supergravity on T^4 is considered. The authors of [28] reduce the type IIA theory while in this letter the reduction of the (truncated) type IIB theory is considered.

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[17]The discussion below is similar to that of Ref. [16].

[18]We only consider p-brane solutions where the charge is carried by $B_{\mu_1 \cdots \mu_{p+1}}$. In particular, we do not consider here p-brane solutions where the charge is carried by a vector component of g and/or B.

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[19]See, however, Sec. II where a related Z_2-transformation, which is present in the NSD theory, is mentioned.

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