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Published in:
Neurocomputing

DOI:
10.1016/j.neucom.2022.02.081

Neurocomputing 488 (2022) 130–143

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Bipartite consensus for a class of nonlinear multi-agent systems under switching topologies: A disturbance observer-based approach

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ARTICLE INFO

Article history:
Received 18 October 2021
Revised 3 January 2022
Accepted 26 February 2022
Available online 1 March 2022
Communicated by Zidong Wang

Keywords:
Nonlinear MASs
Bipartite consensus
Directed fixed and switching topologies
Disturbance observer
Output feedback control

ABSTRACT

This paper considers the leader-following bipartite consensus for a class of nonlinear multi-agent systems (MASs) subject to exogenous disturbances under directed fixed and switching topologies, respectively. Firstly, two new output feedback control protocols involving signs of link weights are introduced based on relative output measurements of neighboring agents. In order to estimate the disturbances produced by an exogenous system, a disturbance observer-based approach is developed. Then, sufficient conditions for leader-following bipartite consensus with directed fixed topologies are derived. Furthermore, by assuming that each switching topology contains a directed spanning tree, it is proved that the leader-following bipartite consensus can be realized with the designed output feedback control protocol if the dwell time is larger than a non-negative threshold. Finally, numerical simulations inspired by a real-world DC motors are provided to illustrate the effectiveness of the proposed controllers.

1. Introduction

In recent years, a tremendous amount of attention has been devoted to cooperative control of MASs [1,2]. This can be ascribed to its wide applications in varieties of fields such as sensor networks [3], unmanned air vehicles [4], power systems [5], formation control [6], and so on.Among the dynamic behavior of MASs, consensus is a fundamental issue, which is committed to driving each participating agent to the same state by only collecting the local information from its neighbors [7]. Leader-following consensus reduces the tracking consensus when there is a leader in MASs, where the followers' states will gradually become consistent with that of the leader over time [8–10].

Most literature consider that agents interact cooperatively to finish a common task. However, competitive interaction may also exist in practical situations. For instance, it was introduced in [11] that companies not only collaborate but also compete in the industrial market. If there was competition for market resources, the challenges would appear because it would generate competition among company relatives and decrease chances of cooperation. Also, in view of the leader–follower framework, the authors investigated the cooperation between employer and employees, and cooperation or competition among employees in the management control system [12]. Then, the signed graph has been a universal tool to describe the networks with both cooperative and competitive relationships. The study on dynamic behavior of MASs over signed graphs can be traced back to [13], where the agents exhibit dynamic phenomena of bipartite consensus and consensus for signed graphs with balanced and unbalanced structures, respectively. In particular, all agents eventually evolve into two subgroups with different consensus states if the signed graph is structurally balanced, and they gradually converge to a common state 0 if the signed graph is structurally unbalanced. Inspired by this pioneering work [13], there have been made much efforts toward addressing the dynamic behaviors of consensus and bipartite consensus for MASs on signed digraphs from different aspects [14,15]. In particular, concerning leader-following dynamics, we mention leader-following bipartite consensus under fixed-time [16], fractional-order nonlinear dynamics [17], event-triggered control [18], and finite-time control [19].

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https://doi.org/10.1016/j.neucom.2022.02.081

It is worthwhile noticing that most of current works on competitive interactions depend on the assumption that communication topologies are undirected and fixed [20]. However, for many practical situations, communication channels among agents may be interrupted or be reconstructed. The interactions among agents may be time-varying over a period of time, and it is essential to consider MASs over directed switching topologies. Therefore, a large number of works with regard to consensus subject to switching topologies have been reported. In [21], based on Lyapunov theory and the property of M-matrix, the leader-following consensus control problem of nonlinear MASs was solved under switching topologies. In [22], the authors investigated leader-following consensus for MASs over directed fixed and switching topologies, respectively. As interactions among agents include competitive and cooperative links, constructing an effective control protocol to realize bipartite consensus under switching topologies is in great demand and challenging. In [23], based on a distributed observer and the certainty equivalence principle, the leader-following bipartite consensus problem of multiple uncertain Euler–Lagrange systems over signed switching networks was solved. Later, the signed consensus for MASs over fixed and switching topologies was concerned in [24]. And the pinning control problem for MASs subjected to switching topologies has been resolved in [25]. As we have seen, although many studies have been performed on leader-following bipartite consensus control of MASs, it is usually supposed that the dynamic of each agent is linear. While in practical situations, nonlinear MASs are common [26]. Then in [27], the authors presented a distributed impulsive controller for nonlinear MASs under a pinning control strategy. Therefore, it is essential to investigate the bipartite consensus of nonlinear MASs with switching communication topologies, including competitive interactions.

As an interesting issue continued from single systems, how to deal with exogenous disturbances, cyber-attacks and noises has received more and more attention recently [28]. In [29], the team-triggered fixed-time consensus control for a class of double-integrator agents under the uncertain disturbance was investigated. In [30], a distributed filtering algorithm was developed to realize the distributed implementation subject to the cyber-attacks and non-Gaussian noises. In [31], the discrete time-varying linear system with superposed system noises was investigated. Meanwhile, how to reject disturbances is also an expressly important topic under switching topologies. In [32], under switching topologies, the semi-global output consensus of MASs with exogenous disturbances was addressed. Furthermore, disturbance observer-based control strategies were provided to deal with disturbances from network-connected dynamic systems [33,34]. Motivated by the forgoing related works, a disturbance observer-based control law is utilized to investigate the bipartite consensus for MASs subject to exogenous disturbances under switching topologies in this paper.

Many existing works on bipartite consensus mainly employed full relative states of neighboring agents to construct control laws. However, the relative state information of agents can not always be obtained in practical engineering. Therefore, as one of the central topics in cooperative control, output feedback control played an important role in achieving asymptotic tracking by designing a distributed controller [35,36]. And it should be pointed out that only linear MASs with an output feedback control approach has been taken into account during a relatively long period [37,38]. Indeed, nonlinear dynamics are more general in practical environments. By employing the output feedback control approach, consensus control strategies of nonlinear MASs have been presented to achieve robust adaptive output consensus in [39]. The output feedback consensus problem of high-order linear systems subject to switching topologies was also solved by designing output control strategies in [40]. However, the above mentioned literatures only study bipartite consensus problem under switching topologies without exogenous disturbances, and the limited information for unknown disturbances makes it difficult to consider the bipartite consensus, involving how to construct the output feedback control strategy without using any state information, how to combine the nonlinear control condition, and how to deal with the effects of competitive relationship between agents. These are challenging problems for leader-following bipartite consensus for nonlinear MASs subject to exogenous disturbances.

In this paper, we fill in this gap by considering a leader-following bipartite consensus for a class of nonlinear MASs under directed fixed and switching topologies. After having formally defined the two controllers and illustrated the algorithms to set the gain matrices, we performed a theoretical analysis of the proposed approaches. Through a Lyapunov-based argument, we prove that the two controllers are able to guarantee convergence of the system to a leader-following bipartite consensus. Our algorithms and theoretical findings are then illustrated via numerical simulations on some case studies based on a real-world MAS inspired by [22,25] and formed by single-link manipulators with revolute joints actuated by DC motors, which interact over different (static or switching) topologies. The numerical findings show the good performances of the proposed controllers in different scenarios, corroborating our theoretical results. The main novelties of our approach with respect to the literature discussed in the above are summarized as follows:

- Compared with some results using the full relative states of neighboring agents [9,28], two new distributed bipartite consensus control strategies based on output measurements are proposed, in which only the relative output information of neighboring agents is utilized. Also different from [1,16] that the dynamic of system state is linear, we consider a class of Lipschitz nonlinear condition, which is more significant from practical point of view.
- Since the upper bounds of exogenous disturbances are not always obtainable, it is comparatively challenging for estimating the states of agents due to the unknown exogenous disturbances. In order to deal with this problem, a disturbance observer-based control protocol is formulated to estimate the exogenous disturbances and system states, which provides an effective solution to the MASs subject to exogenous disturbances.
- As interactions among agents might be time-varying, cooperative or antagonistic, and exogenous disturbances are inevitably generated from exogenous systems, both switching topologies and exogenous disturbances are investigated in the context of bipartite consensus under signed communication topologies in this paper. The derived results are more general.

The rest of the paper is organized as follows. In Section 2 we report some definitions and preliminaries used in this paper. In Section 3, we formulate the problem. In Section 4, we present our main results, with proofs reported in Appendices A and B. In Section 5, we discuss three numerical examples. Section 6 concludes the paper and outlines future research.

2. Notation and Preliminaries

2.1. Notation

$\mathbb{R}^n$ stands for n -dimensional Euclidean space. I_N denotes the $N \times N$ identity matrix. $\mathbf{1}_N$ and $\mathbf{0}_N$ represent the $N \times 1$ column vector of all ones and all zeros, respectively. $|\cdot|$ represents an absolute symbol, $\otimes$ denotes the Kronecker product. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ represent the minimum and maximum eigenvalues of a matrix. $\text{sgn}(\cdot)$ is a sign function, which satisfies

$$\text{sgn}(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$

2.2. Preliminaries

Let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be the sighed topology of the followers, with a set of nodes $\mathscr{V} = \{v_1, v_2, v_3, \cdots, v_N\}$ and a set of edges $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$. Define $N_i = \{j \in \mathscr{V} : (j, i) \in \mathscr{E}\}$ as neighbors of agent i. $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the signed weighted adjacency matrix, in which $a_{ij} > 0$ if $(j, i) \in \mathscr{E}, i \neq j$, and $a_{ij} = 0$,$(j, i) \notin \mathscr{E}$. A path from node v_i to v_j is a sequence of edges in $\mathscr{G}, (v_i, v_{i1}), (v_{i1}, v_{i2}), \cdots, (v_{im}, v_j)$ with distinct nodes $v_{ik}, k = 1, 2, \cdots, m$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ is defined as

$$l_{ii} = \sum_{j=1, j\neq i}^{N} |a_{ij}|; l_{ij} = -a_{ij}, i \neq j.$$

Definition 1. A signed graph $\mathscr{G}$ is structurally balanced if the graph possess a bipartition $\mathscr{V}_1, \mathscr{V}_2$ satisfying $\mathscr{V}_1 \cup \mathscr{V}_2 = \mathscr{V}$ and $\mathscr{V}_1 \cap \mathscr{V}_2 = \varnothing$ and $a_{ij} \geqslant 0, v_i, v_j \in \mathscr{V}_k (k \in \{1, 2\})$, and $a_{ij} \leqslant 0, v_i \in \mathscr{V}_k, v_j \in \mathscr{V}_{3-k} (k \in \{1, 2\})$. Otherwise, it is called structurally unbalanced.

Hence, define $\mathscr{S} = \{S = \text{diag}(s_1, s_2, \cdots, s_N)., s_i \in \{-1, 1\}\}$. The lemma about structural balance can be obtained.

Lemma 1 [13]. *For a structurally balanced graph $\mathscr{G}$, there exists a diagonal matrix $S \in \mathscr{S}$ such that all diagonal elements of $S\mathscr{A}S$ are nonnegative. On the other hand, one could conclude that the diagonal entries of SLS are nonnegative and all off-diagonal entries of SLS are non-positive. Besides, $S \in \mathscr{S}$ provides a partition, i.e., $\mathscr{V}_1 = \{i | s_i > 0\}$ and $\mathscr{V}_2 = \{i | s_i < 0\}$.*

Assumption 1. Suppose that the digraph $\mathscr{G}$ consisting of a leader and the N followers contain a directed spanning tree with the leader located at the root.

Define $R = \text{diag}(a_{10}, a_{20}, \cdots, a_{N0})$, where $a_{i0} > 0$ if the information of the leader is available to follower i and $a_{i0} = 0$ otherwise. Let $L_R = L + R$. Based on Assumption 1, L_R is positive definite.

Lemma 2 [21]. *If Assumption 1 holds, then there exists a matrix $\Theta = \text{diag}\{\varphi_1, \cdots, \varphi_N\}$, with $\varphi_i > 0$ such that $\Theta L_R + L_R^{\mathrm{T}}\Theta > 0$, where $L_R^{\mathrm{T}}\varphi = \mathbf{1}_N$ and $\varphi = [\varphi_1, \cdots, \varphi_N]^{\mathrm{T}} \in \mathbb{R}^N$.*

3. Problem Formulation

We consider a MAS consisting of N follower agents and one leader agent. Define a n-dimensional *state vector* $x_i(t) \in \mathbb{R}^n$ and a r-dimensional *output measurement* $z_i(t) \in \mathbb{R}^r$, which evolve in continuous time $t \in \mathbb{R}$. We assume that all the followers have the same internal dynamics, which may then be subject to external inputs and exogenous disturbances. Specifically, the dynamic of the ith follower agent subject to exogenous disturbances is described by

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t), t) + Bu_i(t) + Dw_i(t), z_i(t) = Cx_i(t), \tag{1}$$

in which $u_i(t) \in \mathbb{R}^m$ is the m-dimensional input vector; $f(\cdot) : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is a nonlinear function, common to all the agents, which is continuous and differentiable in $t; w_i(t) \in \mathbb{R}^p$ denotes the exogenous disturbance, which is generated by

$$\dot{w}_i(t) = Mw_i(t), \tag{2}$$

in which $M \in \mathbb{R}^{p\times p}$ is an external matrix; $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{r\times n}$, and $D \in \mathbb{R}^{n\times p}$ are constant matrices. We assume that the leader's state $x_0(t) \in \mathbb{R}^n$ is not influenced by external inputs and disturbances. Hence, the dynamic of the leader, labeled by 0, is defined as

$$\dot{x}_0(t) = Ax_0(t) + f(x_0(t), t). \tag{3}$$

In the rest of this paper, we will make the following assumptions on the agents' dynamics.

Assumption 2. The pair (A, B, C) is stabilizable and detectable.

Assumption 3. There exists a matrix $F \in \mathbb{R}^{m\times p}$ such that $D = BF$.

Assumption 4. The matrix M has k distinct eigenvalues and the real part of each eigenvalue is zero. Moreover, (M, D) is observable.

Assumption 5. There exists a positive constant $\rho > 0$ such that

$$\|f(a_1, t) - s_i f(a_2, t)\| \leqslant \rho\|a_1 - s_i a_2\|, \forall a_1, a_2 \in \mathbb{R}^n. \tag{4}$$

In our analysis, we focus on the study of the convergence to a leader-following bipartite consensus for the MAS of N followers and one leader defined in (1) and (3), which is defined as follows.

Definition 2 (*Leader-following bipartite consensus*). The leader-following bipartite consensus for MAS (1) and (3) can be achieved for some $k \in \{1, 2\}$ if

$$\lim_{t\to\infty}\|x_i(t) - x_0(t)\| = 0, \forall i \in \mathscr{V}_k,$$
$$\lim_{t\to\infty}\|x_i(t) + x_0(t)\| = 0, \forall i \in \mathscr{V}_{3-k},$$

which can be further written by

$$\lim_{t\to\infty}\|x_i(t) - s_i x_0(t)\| = 0, i = 1, 2, \cdots, N.$$

Note that, differently from other notions of consensus [21], in Definition 2, we are not requiring that the leader converges to a fixed point, but we say that a leader-following bipartite consensus is achieved if the entire system synchronizes toward a trajectory in which a set of followers has the same state of the leader, and the states of remaining followers will asymptotically track the opposite values of those of the leader.

Remark 1. As indicated in [22], Assumption 3 presents a matching condition under which the disturbance effects can be compensated through the control action. A sufficient criterion for the existence of the matrix F is that $\text{rank}(B, D) = \text{rank}(B)$. Since F may not be equal to I_m, the disturbances can be imposed on some channels other than the control input channels.

Remark 2. Eq. (2) is mainly motivated by the disturbance observer-based approach for MASs in the work of [33]. It is to reflect the deterministic disturbances such as constants and sinusoidal disturbances, and it covers a wide range of periodic disturbances such as the sinusoidal functions upon which many other functions can be approximated with a bias. Moreover, Assumption 4 on the eigenvalues of M is commonly used for disturbance rejection and output regulation. If the eigenvalues of the matrix M are strictly located in the left-half plane, the disturbance is stable.

Remark 3. Different from conventional consensus, the interactions among agents are usually described by a signed graph with positive/negative weights. This means that the row sum of Laplacian is generally not zero, and Laplacian of a signed graph may be positive definite, which are the main differences with standard graph theory for conventional consensus [8–10]. Based on Lamma 1 and the premise of structural balance, we can finally define $\bar{L}_R = SLS + R$. Then, one obtains $\bar{L}_R$ is positive definite, i.e., $\bar{L}_R > 0$. That is, a leader-following bipartite consensus problem can be converted into a conventional leader-following consensus problem on a standard graph. In addition, the main contribution of the sighed graph is that it can depict the relationships between agents not only cooperative but also competitive. And the leader in the MASs (3) can be a real or a virtual agent that provides a reference state being tracked by the followers. Based on the relative output information among neighboring agents, the nonlinear MASs can acheve bipartite consensus under switching topologies, whilst the disturbances are fully rejected by the designed disturbance observer. Examples of practical physical systems are the formation control of unmanned air vehicles [4], the cooperative control of mobile robots [6], the design of distributed moving sensor networks [7], and so forth. Therefore, the bipartite consensus is much more general and the consensus can be recognized as a special case of it.

4. Main Results

In this section, we have solved the leader-following bipartite consensus problem for nonlinear MASs with deterministic exogenous disturbances over directed topologies by utilizing an observer-based approach. Specifically, we will start considering fixed topologies. Then, we will generalize our results to switching topologies.

4.1. Bipartite consensus under directed fixed topologies

In this subsection, we consider the bipartite consensus for the MASs in (1)-(3) over a directed fixed topology. An observer-based controller based on the output measurements is developed by defining the following input functions for the followers:

$$u_i(t) = \beta K_1\left[\sum_{j=1}^{N}|a_{ij}|\left(\hat{x}_i(t) - \mathrm{sgn}(a_{ij})\hat{x}_j(t)\right) + a_{i0}(\hat{x}_i(t) - s_i x_0(t))\right] - F\hat{w}_i(t), \quad i = 1, \cdots, N, \tag{5}$$

where $\beta > 0$ is a coupling strength, K_1 is the feedback gain matrix, $\hat{x}_i(t)$ is the state observer, and $\hat{w}_i(t)$ is the disturbance estimation vector. The evolution of these two functions are defined as follows:

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + f(\hat{x}_i(t), t) + Bu_i(t) + D\hat{w}_i(t) - \alpha\tilde{F}\left[\sum_{j=1}^{N}|a_{ij}|\cdot\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right) + a_{i0}\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{i0})\tilde{\xi}_0(t)\right)\right], \tag{6}$$

and

$$\dot{\hat{w}}_i(t) = M\hat{w}_i(t) - G_1\left[\sum_{j=1}^{N}|a_{ij}|\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right) + a_{i0}\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{i0})\tilde{\xi}_0(t)\right)\right], \tag{7}$$

where $\alpha > 0$ is a coupling strength, $\widetilde{F}$ is the state observer gain matrix, G_1 is the disturbance observer gain matrix, and $\tilde{\xi}_i(t) =: z_i(t) - C\hat{x}_i(t)$ is the error between the measurement output and the corresponding quantity computed from the state observer. Since the leader acts as a reference signal generator, it is supposed that $\hat{x}_0(t) = x_0(t)$, i.e., the leader does not need to observe its own state. Then, one has $\tilde{\xi}_0(t) = z_0(t) - C\hat{x}_0(t) = z_0(t) - Cx_0(t) = 0$. Define the following three errors:

$$\bar{\xi}_i(t) = x_i(t) - \hat{x}_i(t), \quad \hat{\xi}_i(t) = \hat{x}_i(t) - s_i x_0(t), \quad e_i(t) := w_i(t) - \hat{w}_i(t), \tag{8}$$

in which $\bar{\xi}_i(t)$ denotes the observer error between the state of agent i and its observer, $\hat{\xi}_i(t)$ denotes consensus tracking error between the state observer of agent i and the state of the leader or its opposite side, and $e_i(t)$ denotes the error between the disturbance and its observer, respectively. Note that, if the three quantities in (8) converge to 0 as $t \to \infty$, then the observer-based controller is well-defined and a bipartite leader-following consensus is achieved according to Definition 2. Hence, we will utilize these three quantities to study the system. Specifically, utilizing the definitions in (8), we can write the dynamics for the state and its observer as

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t), t) + \beta BK_1\left[\sum_{j=1}^{N}|a_{ij}|\left(\hat{x}_i(t) - \mathrm{sgn}(a_{ij})\hat{x}_j(t)\right) + a_{i0}(\hat{x}_i(t) - s_i x_0(t))\right] + De_i(t), \tag{9}$$

and

$$\dot{\hat{x}}_i(t) = A\hat{x}_i(t) + \beta BK_1\left[\sum_{j=1}^{N}|a_{ij}|\left(\hat{x}_i(t) - \mathrm{sgn}(a_{ij})\hat{x}_j(t)\right) + a_{i0}(\hat{x}_i(t) - s_i x_0(t))\right] + f(\hat{x}_i(t), t) - \alpha\tilde{F}\left[\sum_{j=1}^{N}|a_{ij}|\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right) + a_{i0}\tilde{\xi}_i(t)\right], \tag{10}$$

respectively. Since $s_i s_j a_{ij} \geqslant 0, i, j = 1, \cdots N$, one obtains $a_{ij}s_i = |a_{ij}|s_j$ and $|a_{ij}|s_i = a_{ij}\mathrm{sgn}(a_{ij})s_i = |a_{ij}|s_i\mathrm{sgn}(a_{ij})$. Then the following dynamics for the three errors in (8) can be obtained:

$$\dot{\bar{\xi}}_i(t) = A\bar{\xi}_i(t) + f(x_i(t), t) - f(\hat{x}_i(t), t) + De_i(t) + \alpha\tilde{F}\left[\sum_{j=1}^{N}|a_{ij}|\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right) + a_{i0}\tilde{\xi}_i(t)\right], \tag{11}$$

$$\dot{\hat{\xi}}_i(t) = A\hat{\xi}_i(t) + f(\hat{x}_i(t), t) - s_i f(x_0(t), t) + Bu_i(t) - \alpha\tilde{F}\left[\sum_{j=1}^{N}|a_{ij}|\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right) + a_{i0}\tilde{\xi}_i(t)\right], \tag{12}$$

$$\dot{e}_i(t) = Me_i(t) + G_1\left[\sum_{j=1}^{N}|a_{ij}|\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right) + a_{i0}\tilde{\xi}_i(t)\right] \tag{13}$$

In the following, we will show that, if Assumptions 1–5 hold, then one can establish an algorithmic procedure to design the gain matrices for the observer-based control law in (17)-(19). Specifically, we propose the following algorithm.

Algorithm 1. Assume Assumptions 1–5 hold. Then, we define the following steps:

1. Solve the following two matrix inequalities:

$$\begin{bmatrix} A^TP + PA - \mu_0 C^T C + \gamma_1 I + c_1 P & P & D^T P \\ P & -I & 0 \\ * & 0 & -I \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} A^TP + PA - \mu_1 PB^T BP + \gamma_2 I + c_2 P & P \\ P & -I \end{bmatrix} < 0, \tag{15}$$

to get a matrix P, with $\mu_0 > 0, \mu_1 > 0, c_1 > 0, c_2 > 0$, $\gamma_1 > (\alpha+1)\lambda_{\max}\left(\Theta^{-1}L_R^{\mathrm{T}}\Theta L_R\right) + \frac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2$, $\gamma_2 > \frac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 + \alpha\lambda_{\max}\left(C^{\mathrm{T}}CC^{\mathrm{T}}C\right)$, and ρ is defined in Assumption 5.

2. Solve the LMI as follows:

$$QM + M^{\mathrm{T}}Q + \gamma_3 I + c_3 Q < 0, \tag{16}$$

to get a matrix Q, with $c_3 > 0$ and $\gamma_3 > \lambda_{\max}\left(C^{\mathrm{T}}CC^{\mathrm{T}}C\right) + 1$.

3. Choose coupling strengths $\alpha > \mu_0/\lambda_0, \beta > \mu_1/\lambda_0$, where $\lambda_0 c = \lambda_{\min}\left(L_R + \Theta^{-1}L_R^{\mathrm{T}}\Theta\right)$ and $\Theta = \mathrm{diag}\{\varphi_1, \cdots, \varphi_N\}$.

Theorem 1. Under Assumptions 1–5, the leader-following bipartite consensus for nonlinear MAS (1) and (3) with the deterministic disturbances (2) is achieved by observer-based control law (5) if $K_1 = -B^TP, G_1 = Q^{-1}C^T$ and $\tilde{F} = -P^{-1}C^{\mathrm{T}}$.

Proof: At this stage, we can analytically prove that the observer-based control law in (17)-(19) with gain matrices defined in Algorithm 1 can solve the leader-following bipartite consensus problem for nonlinear MAS in (1)-(3). The following result formally guarantees our claim. The proof, which is based on the Lyapunov stability theory to show the convergence of the three quantities in (8), is quite cumbersome and is thus reported in Appendix A.

Remark 4. For general nonlinear MASs, it may be challenging to design distributed protocols only based on relative states of neighboring agents over directed networks, and the state feedback control approach is no longer applicable. To amend the drawback of this fact, the disturbance observer approach and output feedback control approach have been proved to be significant in dealing with the bipartite consensus problem of nonlinear MASs with exogenous disturbances. Furthermore, as far as we know, the measurement outputs can be transmitted to a remote data-processing center performing a signal estimation task in network systems. However, it is still an open topic that how to combine the observer-based approach with networked system, and the designed approach presented in [31] might be useful for investigating this topic.

Remark 5. Noticeably, based on the Finsler's lemma in [44] and bounded real lemma in [45], one can conclude that the Assumption 2 is a necessary condition for the feasibility of LMIs (14) and (15). Also the Assumption 5 is the so-called Lipschitz condition and all linear and piecewise-linear time-invariant continuous functions satisfy this condition. However, there are some related works that considered the continuous-time nonlinear dynamics which do not satisfy the Lipschitz condition [2,9,20]. For instance, in [26], the nonlinear functions $f_i(\cdot)(i = 1, 2, \cdots, n)$ are unknown, and it involves the periodically time-variant disturbances $d_i(t)(i = 1, 2, \cdots, n)$, which can be estimated by fourier series expansion. Therefore, how to relax the constraints one the system's dynamic and combine nonlinear conditions with the time-varying periodic disturbances will be considered in our future studies.

4.2. Bipartite consensus under directed switching topologies

In this subsection, the leader-following bipartite consensus is studied for MASs over switching topologies. The set of all directed switching communication topologies can be denoted as $\tilde{\mathscr{G}} = \{\mathscr{G}^1(\mathscr{V}, \mathscr{E}^1, \mathscr{A}^1), \cdots, \mathscr{G}^\tau(\mathscr{V}, \mathscr{E}^\tau, \mathscr{A}^\tau)\}$, where $\mathscr{E}^\tau$ is the set of directed links, $\mathscr{A}^\tau = \left[a_{ij}^\tau\right] \in \mathbb{R}^{N\times N}, \tau = 1, \cdots, N$ denotes the graph adjacency matrix, $a_{ij}^\tau > 0$, if $j \in N_i^\tau$, otherwise $a_{ij}^\tau = 0$. Define $N_i^\tau = \{j : (j, i) \in \mathscr{E}^\tau\}$ as the set of the neighbors of agent i, and if $\{j, i\} \subseteq \mathscr{E}^\tau$, it is said that i and j are adjacent. Let $\varpi(t) : \mathbb{R}^+ \to \{1, 2, \cdots, N\}$ represents the switch signal of communication topologies. Assume that there is an infinite sequence of nonoverlapping time intervals $[t_k, t_{k+1}), k \in N$, with $t_1 = 0, \hat{\tau}_0 \leqslant t_{k+1} - t_k \leqslant \hat{\tau}_1$, in which the positive constant $\hat{\tau}_0$ denotes the dwell time.

Assumption 6. Assume that the switching digraph $\mathscr{G}^\Gamma$contains a directed spanning tree for each $\Gamma \in \{1, 2, \cdots, \tau\}$, and the leader (agent 0) is located at the root node.

Lemma 3. If Assumption 6 holds, then there exists a matrix $\Theta^\Gamma = \mathrm{diag}\{\varphi_1^\Gamma, \cdots, \varphi_N^\Gamma\}$, with $\varphi_i^\Gamma > 0$ such that $\Theta^\Gamma L_R^{(\Gamma)} + \left(L_R^{(\Gamma)}\right)^{\mathrm{T}}\Theta^\Gamma > 0$, where $\left(L_R^{(\Gamma)}\right)^{\mathrm{T}}\varphi^\Gamma = \mathbf{1}_N$ and $\varphi^\Gamma = [\varphi_1^\Gamma, \cdots, \varphi_N^\Gamma]^{\mathrm{T}} \in \mathbb{R}^N$.

Based on the directed switching topologies and the disturbance observer, we propose a leader–follower bipartite consensus controller for the ith agent as follows:

$$u_i(t) = \tilde{\beta}K_2\left[\sum_{j=1}^N |a_{ij}^{\varpi(t)}|\left(\hat{x}_i(t) - \mathrm{sgn}\left(a_{ij}^{\varpi(t)}\right)\hat{x}_j(t)\right) + a_{i0}^{\varpi(t)}(\hat{x}_i(t) - s_i x_0(t))\right] - F\hat{w}_i(t), \tag{17}$$

where $\tilde{\beta} > 0$ is a coupling strength, K_2 is the feedback gain matrix, $\hat{x}_i(t)$ is the state observer, and $\hat{w}_i(t)$ is the disturbance estimation vector. The evolution of these two functions are defined as follows:

$$\begin{aligned}\dot{\hat{x}}_i(t) &= A\hat{x}_i(t) + f(\hat{x}_i(t), t) + Bu_i(t) + D\hat{w}_i(t) \\ &\quad - \tilde{\alpha}\hat{F}\left[\sum_{j=1}^N \left|a_{ij}^{\varpi(t)}\right| \cdot \left(\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right)\right.\right. \\ &\quad \left.\left. + a_{i0}^{\varpi(t)}\left(\tilde{\xi}_i(t) - \mathrm{sgn}\left(a_{i0}^{\varpi(t)}\right)\tilde{\xi}_0(t)\right)\right)\right],\end{aligned} \tag{18}$$

and

$$\begin{aligned}\dot{\hat{w}}_i(t) &= M\hat{w}_i(t) - G_2\left[\sum_{j=1}^N |a_{ij}^{\varpi(t)} t|\left(\left(\tilde{\xi}_i(t) - \mathrm{sgn}(a_{ij})\tilde{\xi}_j(t)\right)\right.\right. \\ &\quad \left.\left. + a_{i0}^{\varpi(t)}\left(\tilde{\xi}_i(t) - \mathrm{sgn}\left(a_{i0}^{\varpi(t)}\right)\tilde{\xi}_0(t)\right)\right)\right],\end{aligned} \tag{19}$$

respectively, where $\tilde{\alpha} > 0$ is a coupling strength, $\widehat{F}$ is the state observer gain matrix, G_2 is the disturbance observer gain matrix, and $\tilde{\xi}_i(t) =: z_i(t) - C\hat{x}_i(t)$ is the error between the measurement output, $z_i(t)$, and the corresponding quantity computed from the state observer, $C\hat{x}_i(t)$. Similar to the scenario with fixed topology, since the leader acts as a reference signal generator, it is supposed that $\hat{x}_0(t) = x_0(t)$, i.e., the leader does not need to observe its own state, consequently, $\tilde{\xi}_0(t) = 0$. Similarly, we will investigate the convergence of the system to a leader-following bipartite consensus under switching topologies.

In the following, we will show that, if Assumptions 2–4 hold, then a procedure to design the gain matrices for the observer-based control law in (17)-(19) can be designed, according to the following algorithm.

Algorithm 2. Assume Assumptions 2–4 hold. Then, we define the following steps:

1. Solve the following two matrix inequalities:
$$\begin{bmatrix} A^{\mathrm{T}}\bar{P}+\bar{P}A-\tilde{\mu}_0C^{\mathrm{T}}C+\gamma_4I+\tilde{c}_1\bar{P} & \bar{P} & D^{\mathrm{T}}\bar{P} \\ \bar{P} & -I & 0 \\ * & 0 & -I \end{bmatrix} < 0, \tag{20}$$
$$\begin{bmatrix} A^{\mathrm{T}}\bar{P}+\bar{P}A-\tilde{\mu}_1\bar{P}B^{\mathrm{T}}B\bar{P}+\gamma_5I+\tilde{c}_2\bar{P} & \bar{P} \\ \bar{P} & -I \end{bmatrix} < 0, \tag{21}$$
to get a matrix $\bar{P}$, with scalars $\tilde{\mu}_0>0, \tilde{\mu}_1>0, \tilde{c}_1>0,\ \tilde{c}_2>0$, $\gamma_5>\frac{\lambda_{\max}(\Theta^{\varpi(t)})}{\lambda_{\min}(\Theta^{\varpi(t)})}\rho^2+\tilde{\alpha}\lambda_{\max}\left(C^{\mathrm{T}}CC^{\mathrm{T}}C\right), \rho$ is defined in Assumption 5, and $\gamma_4>\frac{\lambda_{\max}(\Theta^{\varpi(t)})}{\lambda_{\min}(\Theta^{\varpi(t)})}\rho^2+ (\tilde{\alpha}+1)\lambda_{\max}\left(\Theta^{-\varpi(t)}\left(L_R^{\varpi(t)}\right)^{\mathrm{T}}\Theta^{\varpi(t)}L_R^{\varpi(t)}\right)$. Then, take $K_2=-B^T\bar{P}$ and $\widehat{F}=-\bar{P}^{-1}C^T$.
2. Solve the LMI as follows:
$$\bar{Q}M+M^{\mathrm{T}}\bar{Q}+\gamma_6I+\tilde{c}_3\bar{Q}<0, \tag{22}$$
to get a matrix $\bar{Q}$, with scalars $\tilde{c}_3>0, \gamma_6>\lambda_{\max}\left(C^{\mathrm{T}}CC^{\mathrm{T}}C\right)+1$. Then, take $G_2=\bar{Q}^{-1}C^{\mathrm{T}}$.
3. Choose coupling strengths $\tilde{\alpha}>\tilde{\mu}_0/\tilde{\lambda}_0, \tilde{\beta}>\tilde{\mu}_1/\tilde{\lambda}_0$, where $\tilde{\lambda}_0 \triangleq \min_{\Gamma=1,\cdots,n}\left(\lambda_{\min}\left(L_R^{\Gamma}+\left(\Theta^{\Gamma}\right)^{-1}\left(L_R^{\Gamma}\right)^{\mathrm{T}}\Theta^{\Gamma}\right)\right)$ and $\Theta^{\Gamma}=\mathrm{diag}\{\varphi_1^{\Gamma},\cdots,\varphi_N^{\Gamma}\}$.

Theorem 2. Under Assumptions 2,3,4 and 6, the leader-following bipartite consensus for nonlinear MASs (1), (3) subject to the deterministic disturbances (2) is realized by utilizing observer-based controller (17) if the dwell time $\hat{\tau}_0>\ln\ell_0/\hat{c}_0$, where $\hat{c}_0=\min_{i\in\{1,2,3\}}\{\hat{c}_i\}, \ell_0= \varphi_{\max}/\varphi_{\min}, \varphi_{\min}= \min_{\Gamma,i}\left\{\varphi_i^{(\Gamma)}\right\}, \varphi_{\max}=\max_{\Gamma,i}\left\{\varphi_i^{(\Gamma)}\right\}, \Gamma\in\{1,2,\cdots,\tau\}, i\in\{1,2,\cdots,N\}$.

Proof: Similar to the scenario with fixed topology, we can analytically prove that the observer-based control law in (17)-(19) with gain matrices defined in Algorithm 2 can resolve the leader-following bipartite consensus problem for nonlinear MAS in (1)-(3) with switching topologies. The following result, whose proof is reported in Appendix B, formally guarantees our claim.

Remark 6. Note that when selecting the coupling strength $\alpha, \beta, \tilde{\alpha}$ and β, the parameters $\mu_0, \mu_1, \tilde{\mu}_0, \tilde{\mu}_1$ should be chosen relatively small so that $\alpha>\mu_0/\lambda_0, \beta>\mu_1/\lambda_0, \tilde{\alpha}>\tilde{\mu}_0/\tilde{\lambda}_0$, and $\tilde{\beta}>\tilde{\mu}_1/\tilde{\lambda}_0$ hold. However, both the convergence rates for bipartite consensus tracking and states observing can be improved by enlarging the coupling strength. This indicates that though the bipartite consensus tracking problem can be solved by setting the coupling strength $\alpha>\mu_0\lambda_0, \beta>\mu_1/\lambda_0, \tilde{\alpha}>\tilde{\mu}_0/\tilde{\lambda}_0$, and $\tilde{\beta}>\tilde{\mu}_1/\tilde{\lambda}_0$, the convergence rates may be quite small when the coupling strengths $\alpha, \beta, \tilde{\alpha}$and β are, respectively, only slightly larger than their corresponding threshold values. Therefore, the utilize of larger coupling strength could speed up the convergence process of the controllers. These also reflect that our observer-based control strategies are flexible from the practical point of view.

Remark 7. The control protocols (5) and (17) are partly motivated by the observer-type protocols for MASs proposed in [21,25,33]. However, the observer designed here is indeed different from that in related works. First, our assumptions are less restrictive, as we only require that a follower can access the information on its state observer, and not the actual state. Second, when considering the observer error between follower i and the leader, the requirement for state information of follower agent is relaxed, i.e., $\hat{\xi}_i(t)=\hat{x}_i(t)-s_ix_0(t)$, as we only require the state of the observer embedded in follower i. However, the state information of follower agent is necessary in existing related works [21,28,33]. Furthermore, our controllers could realize not only the common consensus but also the bipartite consensus involved with the competitive relationships between agents. And the nonlinear dynamics and switching signals are also considered into the controllers, which reflects that the research of our paper is more extensive.

Remark 8. According to Algorithms 1 and 2, the existence of the gain matrices to be used in the controller depends on the possibility to solve a set of matrix inequalities. It is easy to observe that the matrix inequalities in Algorithm 2 are feasible if and only if the matrix inequalities in Algorithm 1 are feasible. Therefore, in the process of solving matrix inequalities in Algorithms 1 and 2, they need to be scaled by the Yang's inequality correspondingly. If the matrix inequality remains in its original form, the coefficients of the matrix will appear in the non-diagonal position of the matrix. Since the diagonal element is 0, it will be impossible to solve the matrix inequality. Then, after the introduction of some parameters, they will increase the complexity, but these parameters also increase the flexibility of the conditions, making it easier to find the solution of matrix inequality. To overcome this drawback, we use the property of the Kronecker product to reduce the computational complexity. Also, the introduction of Lipschitz nonlinear condition could combine two terms of the inequality into one, thereby reducing the computational burden. In general, the application of the Young's inequality could result in some new decision variables, this is an inevitable problem when using this approach. However, we believe that the increase in time complexity caused by the calculation of Yang's inequality is controllable.

5. Numerical examples

In this section, we propose and discuss three examples to illustrate our theoretical findings and demonstrate the performance of the algorithms we developed. In the first example, we consider a scenario with a fixed topology and we show that the observer-based controller proposed in Algorithm 1 is able to steer the system to leader-following bipartite consensus. We consider the scenario of a linear dynamics and we discuss the characteristics of the consensus state reached, depending on the topology structure. In the second example, we consider a nonlinear dynamics on a static signed topology, and we show that our algorithm is also able to deal with nonlinear scenarios, where other methods proposed in the literature fail [16,20]. In the third example, we consider a scenario of nonlinear dynamics on switching topologies, and we illustrate how Algorithm 2 can be used to design an observer-based controller for the system, while in [21], the linear controllers and observers proposed cannot be used. In these examples, we consider a network of six agents interacting according to four different topologies, labeled as $\mathscr{G}_1, \mathscr{G}_2, \mathscr{G}_3$, and $\mathscr{G}_4$, and illustrated in Fig. 2. Observe that all the six topologies are structurally balanced, with the same partition equal to $\mathscr{V}_1=\{1,2\}, \mathscr{V}_2=\{3,4,5\}$. Similar to [21,22], we consider a MAS consisting of six single-link manipulators with revolute joints actuated by DC motors: $N=5$ follower agents and one leader agent, labeled as agent 0. A schematic illustration of the physical system is reported in Fig. 1. The state of each agent is characterized by a 4-

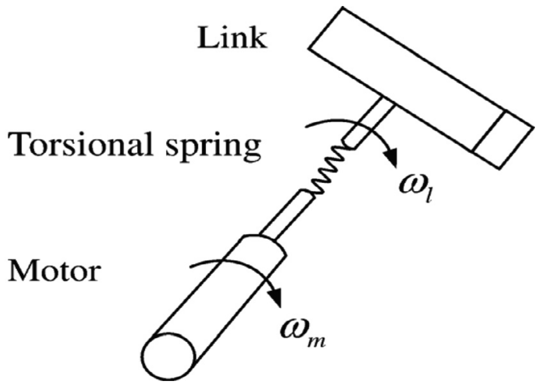


Fig. 1. Schematic of the single-link manipulator with a flexible joint.

dimensional vector $x_i(t) = (x_{i1}(t), x_{i2}(t), x_{i3}(t), x_{i4}(t))^\top$, where $x_{i1}(t)$ is the angular rotation of the motor, $x_{i2}(t)$ is the angular velocity of the motor, $x_{i3}(t)$ is the angular rotation of the link of the ith manipulator, and $x_{i4}(t)$ is the angular velocity of the link of the ith manipulator. Similar to [21,22], the dynamic of the ith manipulator can be written in the form of (1), withFig. 3

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.26 & 48.6 & 0 \\ 0 & 0 & 0 & 10 \\ 1.95 & 0 & -1.95 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}^\top, \quad D = B \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix}^\top, \tag{23}$$

and nonlinear function $f(x_i(t), t) = (0, 0, 0, 0.333 \sin(x_{i3}(t)))^\top$. The disturbances are generated by (2) with $M = \begin{bmatrix} 0 & 1.5 \\ -1.5 & 0 \end{bmatrix}$. It is easy to obtain that $D = BF$. Also, it is easy to verify that Assumptions 2–4 hold, and that Assumptions 5 holds with $\rho = 0.333$. In the simulations, the initial conditions are assigned randomly.

Example 1. We consider the MAS made of six single-link manipulators with dynamics defined in (1) and (23), interacting on the fixed topologies $\mathcal{G}_1$ and $\mathcal{G}_2$, illustrated in Figs. 2(a) and (b), respectively, where the first topology does not contain antagonistic edges, whereas the second does. We construct the observer-based controller utilizing the proposed Algorithm 1, for which convergence is guaranteed by Theorem 1. Specifically, by simple calculations, we get $\Theta = \mathrm{diag}\{2.0422, 1.0060, 0.9012, 2.3980\}$. We take $c_1 = 1, c_2 = 1, c_3 = 3, \gamma_1 = 5, \gamma_2 = 7, \gamma_3 = 6$, and the following step 1) of Algorithm 1, we set $\alpha = 34, \beta = 21, \mu_0 = 10$, and $\mu_1 = 6$, it yields that $\lambda_0 = 0.3173$. Note that the Algorithm 1 holds if

$$\begin{bmatrix} A^\top P + PA + \gamma_2 I + c_2 P & P \\ P & -I \end{bmatrix} < 0. \tag{24}$$

Then, based on (24) and Algorithm 1, we compute the following matrices P and Q

$$P = \begin{bmatrix} 0.7373 & 0.0147 & -0.6210 & -0.2588 \\ 0.0147 & 0.0162 & -0.0063 & 0.0175 \\ -0.6210 & -0.0063 & 1.0708 & -1.0625 \\ -0.2588 & 0.0175 & -1.0625 & 3.7443 \end{bmatrix},$$

$$Q = \begin{bmatrix} 3.9492 & 0.1854 & -1.2871 & -0.6210 \\ 0.1854 & 4.5754 & -1.2934 & -0.1762 \\ -1.2871 & -1.2934 & 3.6787 & -1.2641 \\ -0.6210 & -0.1762 & -1.2641 & 3.3694 \end{bmatrix},$$

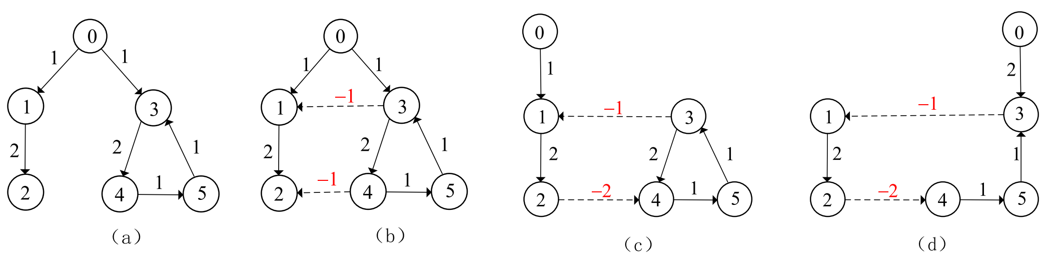


Fig. 2. The four different topologies considered in the three examples: (a) $\mathcal{G}_1$, (b), $\mathcal{G}_2$, (c) $\mathcal{G}_3$, and (d) $\mathcal{G}_4$.

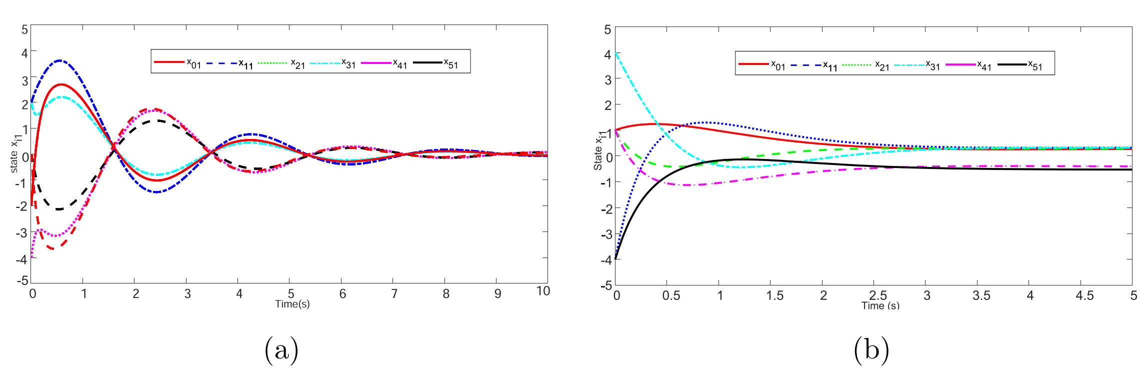


Fig. 3. Temporal evolution of $x_{i1}(t)$ under the linear dynamics in Example 1.

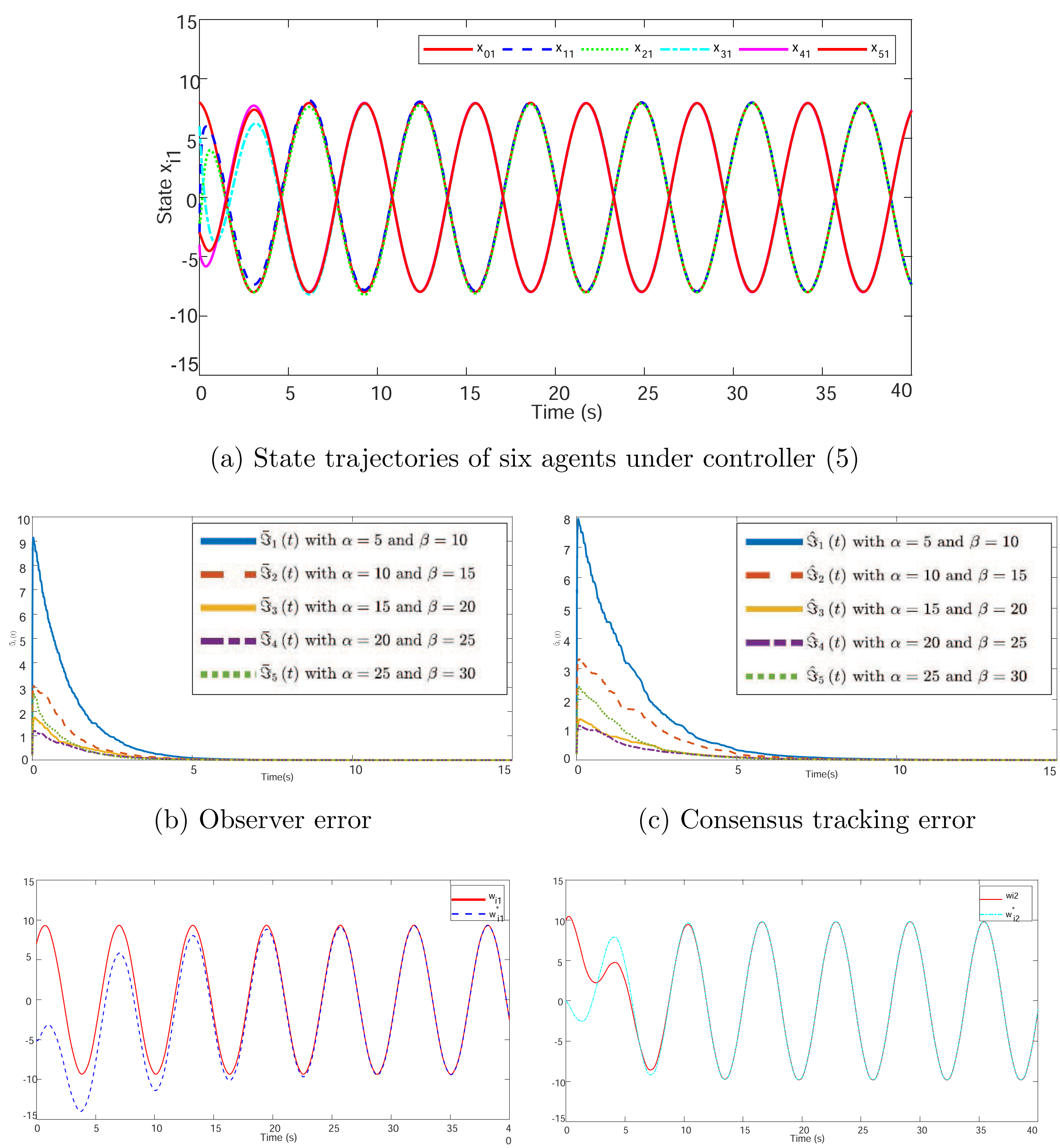
respectively, which are used in Algorithm 1 to compute the gain matrices:

$$G_1 = \begin{bmatrix} 0.3679 & 0.2960 & 0.3001 & 0.1959 \\ 0.2997 & 0.1963 & 0.6546 & 0.6079 \end{bmatrix}^\top,$$

$$\tilde{F} = \begin{bmatrix} -1.1711 & -60.8337 & -1.1634 & -0.1267 \\ -15.5325 & 14.2425 & -15.6654 & -5.8523 \end{bmatrix}^\top,$$

and $K_1 = [-0.3167\ -0.3508\ 0.1363\ -0.3781]$. Under the controller (5), the states of five followers and the leader without and with antagonistic edges are shown in Figure. 3, which reflects the control law also can solve the bipartite consensus problem for linear MASs under directed fixed topology. In contrast, the linear controller and observer design in [22] does not hold in this model simulation. Interestingly, when considering the trajectories with antagonistic edges, it can be observed that the states of agents 1 and 2 can asymptotically approach those of the leader, while the states of agents 3, 4 and 5 asymptotically track the opposite values of those of the leader, which is in accordance with the cluster partition $\mathcal{V}_1$ and $\mathcal{V}_2$.

Example 2. In this case, consider a MAS of six single-link manipulators with fixed topology $\mathcal{G}_2$. Fig. 4a depicts that the followers approach the leader's state with antagonistic edges under nonlinear control protocol (5). Note that many existing controllers proposed in the literature cannot handle this scenario [21,25]. Figs. 4b and 4c depict that the time evaluation of bipartite consensus error between the leader and each follower agent as $t \to \infty$. The evolution of disturbances and disturbance observers are plotted in



(a) State trajectories of six agents under controller (5)

(b) Observer error

(c) Consensus tracking error

(d) Disturbances and observers

(e) Disturbances and observers

Fig. 4. Temporal evolution of state $x_{i1}(t)$, observer error $\bar{\xi}_i(t)$, consensus error $\hat{\xi}_i(t)$, disturbances $\omega_i(t)$, and disturbance observers $\tilde{\omega}_i(t)$ under the nonlinear dynamics in Example 2..

Figs. 4c and 4d, which show that the disturbance observer exhibits an excellent estimation performance by utilizing the observer based control method.

Example 3. Finally, we consider the nonlinear MAS in the scenario of switching topologies, and we provide some simulation results to illustrate the effectiveness of the control protocol proposed in Algorithm 2, whose effectiveness is discussed in Theorem 2. We consider the MAS of six single-link manipulators under switching topologies with antagonistic edges shown in panels (c) and (d) of Fig. 2. Note that it is not difficult to conclude the pair (A, B, C) is stabilizable and detectable. Assume that the network topology switches periodically between $\mathscr{G}_3$ and $\mathscr{G}_4$ every 0.4s. Assume Assumption 6 holds, then, following Algorithm 2, let $\tilde{\alpha} = 29$, $\tilde{\beta} = 32$, $\tilde{\mu}_0 = 20, \tilde{\mu}_1 = 8, \gamma_4 = 9$, $\gamma_5 = 6, \gamma_6 = 5, \tilde{c}_1 = 1, \tilde{c}_2 = 2$, and $\tilde{c}_3 = 4$, it yields that $\tilde{\lambda}_0 = 0.7218$. Similar to Example 2, following steps 2) and 3) of Algorithm 2, we compute

$$\bar{P} = \begin{bmatrix} 0.3109 & 0.0083 & -0.2887 & -0.0387 \\ 0.0083 & 0.0526 & -0.0283 & 0.0652 \\ -0.2887 & -0.0283 & 0.8722 & -1.0888 \\ -0.0387 & 0.0652 & -1.0888 & 2.7165 \end{bmatrix},$$

$$\bar{Q} = \begin{bmatrix} 4.1127 & 0.1678 & -1.3310 & -0.6656 \\ 0.1678 & 4.7540 & -1.3341 & -0.2042 \\ -1.3310 & -1.3341 & 3.8466 & -1.3126 \\ -0.6656 & -0.2042 & -1.3126 & 3.5213 \end{bmatrix},$$

respectively. Then, following Algorithm 2, we compute the three gain matrices, obtaining

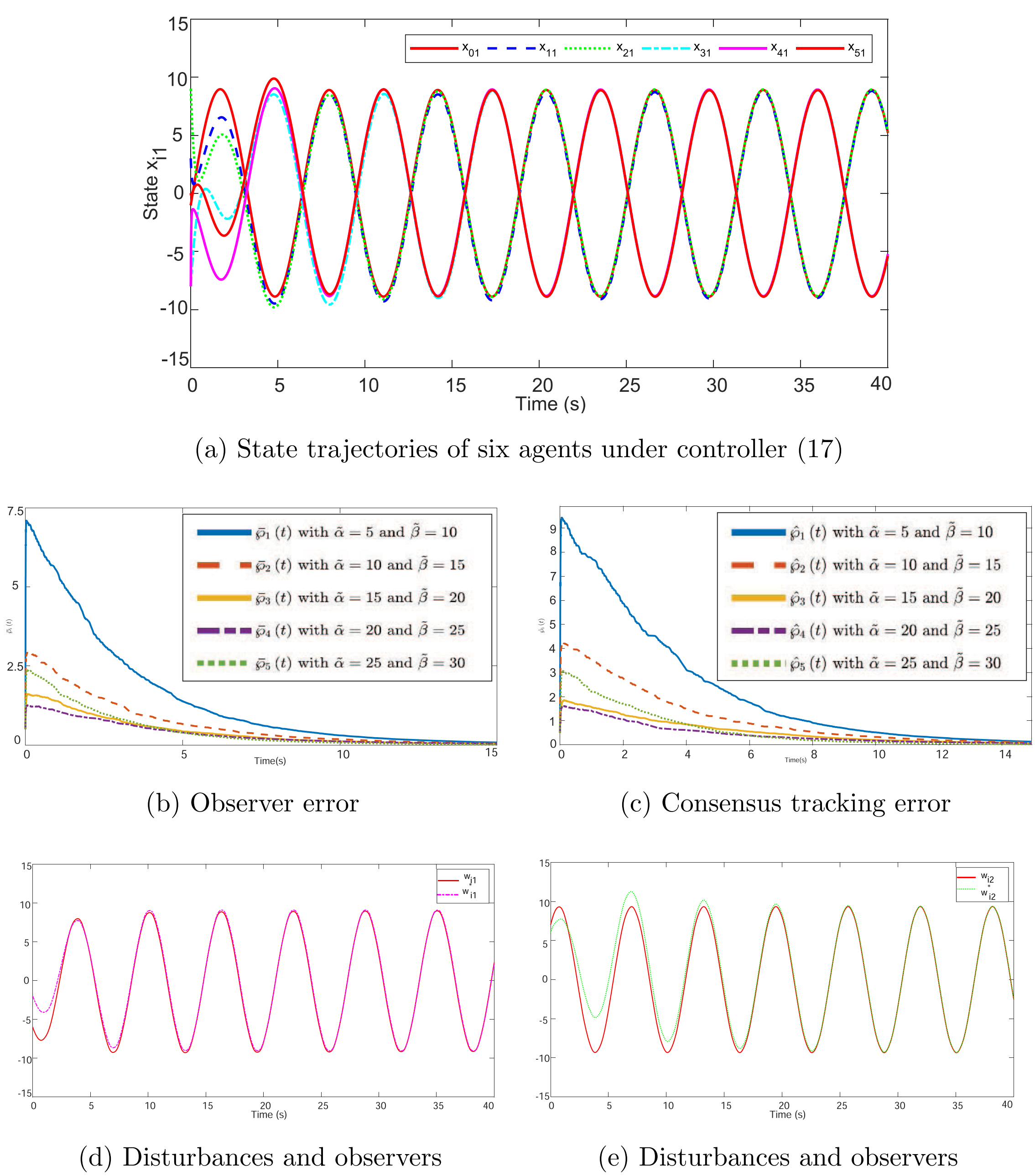


Fig. 5. Temporal evolution of state $x_{i1}(t)$, observer error $\bar{\xi}_i(t)$, consensus error $\hat{\xi}_i(t)$, disturbances $\omega_i(t)$, and disturbance observers $\tilde{\omega}_i(t)$ under the nonlinear dynamics in Example 3.

$$G_2 = \begin{bmatrix} 0.3554 & 0.2867 & 0.2876 & 0.1910 \\ 0.2886 & 0.1901 & 0.6244 & 0.5823 \end{bmatrix}^\top,$$
$$\hat{F} = \begin{bmatrix} -8.7655 & -18.2778 & -6.2112 & -2.1757 \\ -10.2680 & 1.8811 & -10.3730 & -4.7172 \end{bmatrix}^\top,$$

and $K_2 = [0.3303\ -0.7992\ 0.2282\ -0.1146]$. Based on Theorem 2, bipartite consensus control subject to the deterministic disturbances can be realized if $\hat{\tau}_0 = 0.4 > 0.3715$. In particular, under the controller (17), the states of five followers and the leader with antagonistic edges are shown in Fig. 5a, which reflects the control law could solve the bipartite consensus problem for nonlinear MASs under directed switching topologies. Differently from most linear controller and observer designed in the literature [26], which cannot deal with nonlinear scenario. Furthermore, profiles of the observer error $\bar{\xi}_i(t)$ and the bipartite consensus tracking error $\hat{\xi}_i(t)$ are shown in Figs. 5b and 5c. Our numerical simulations illustrate that both the convergence for leader-following bipartite consensus and state's observer are typically fast and that their rates can be improved by enlarging the coupling strength $\tilde{\alpha}$ and $\tilde{\beta}$. This indicates that, even though the theoretical guarantees in Theorem 2 ensures that the bipartite consensus can be solved by setting any coupling strengths $\tilde{\alpha} > \tilde{\mu}_0/\tilde{\lambda}_0$ and $\tilde{\beta} > \tilde{\mu}_1/\tilde{\lambda}_0$, the convergence rates may be quite small if the coupling strengths are only slightly larger than the requirements, suggesting the use of larger couplings to speed up the convergence process. The evolution of disturbances and disturbance observers are plotted in Figs. 5d and 5e, which show that the exogenous disturbances are well estimated.

6. Conclusion

In this article, we have investigated the leader-following bipartite consensus control of nonlinear MASs subject to exogenous disturbances under directed fixed and switching topologies, respectively. Firstly, a disturbance observer-based approach is proposed to estimate the disturbances generated from the exogenous system. Secondly, an observer-based control law based on relative output measurements of neighboring agents is introduced. Then, by assuming that each switching topology contains a directed spanning tree, it is proved that the leader-following bipartite consensus can be achieved with the designed output feedback control law if the dwell time is larger than a non-negative threshold. Finally, based on a real-world physical system, the effectiveness of the developed algorithms in different scenarios is verified via three numerical examples.

Our work advances the literature along several directions. Compared to previous works [21,32], our approach is more general, as it considered nonlinear MASs, antagonistic interactions, output feedback control, disturbance observer, and switching topologies simultaneously, allowing to deal with more general and realistic scenarios. In particular, different from [16,18], the disturbance observer is incorporated into the controller to actively compensate for the disturbance effects on leader-following bipartite consensus. This enables our controller to be robust to disturbances produced by an exogenous system, making our algorithms suitable for direct application in many control areas, such as game control, formation control, containment control, and flocking control for MASs [6,14,22].

Our promising results, supported by the examples illustrated in Section 5, suggest the possible extension of our methodology to different scenarios. In particular, following [1,8], it would be interesting to investigate event-based bipartite consensus of MASs under directed switching topologies, and further extending our algorithms to deal with the disturbances that cannot be estimated or may be unbounded. Furthermore, following [29], a promising idea can be that of implementing a team-triggered control (TTC) strategy, which incorporates the event-triggered control (ETC) and self-triggered control (STC), to realize the bipartite consensus of MASs under the uncertain disturbances. This idea will be investigated in our future study.

CRediT authorship contribution statement

Qiang Wang: Conceptualization, Methodology, Writing - original draft. **Wangli He:** Visualization, Investigation. **Lorenzo Zino:** Validation. **Dayu Tan:** Supervision. **Weimin Zhong:** Supervision, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

This work is supported by National Natural Science Fund for Distinguished Young Scholars (61925305), National Natural Science Foundation of China (61890930-3, 61922030), Shanghai International Science & Technology Cooperation Program (21550712400) and Fundamental Research Funds for the Central Universities(222202217006).

Appendix A. Proof of Theorem 1

In the proof, we will show that $\bar{\xi}(t) \to 0, \hat{\xi}(t) \to 0$, and $e(t) \to 0$ as $t \to \infty$, guaranteeing convergence to the leader-following bipartite consensus.

First, utilizing the Kronecker product notation, (5)-(7) can be written into a compact matrix form as

$$\begin{aligned}\dot{\bar{\xi}}(t) &= \left[I_N \otimes A + \alpha\left(L_R \otimes \tilde{F}C\right)\right]\bar{\xi}(t) + (I_N \otimes D)e(t) \\ &\quad + I_N \otimes (f(x(t),t) - f(\hat{x}(t),t)), \qquad (A.1)\\ \dot{\hat{\xi}}(t) &= [I_N \otimes A + \beta(L_R \otimes BK_1)]\hat{\xi}(t) - \alpha\left(L_R \otimes \tilde{F}C\right)\bar{\xi}(t) \\ &\quad + (f(\hat{x}(t),t) - (SI_N \otimes f(x_0(t),t))), \qquad (A.2)\\ \dot{e}(t) &= (I_N \otimes M)e(t) + (L_R \otimes G_1 C)\bar{\xi}(t), \qquad (A.3)\end{aligned}$$

where we use the notation $f(x(t),t) := \left[f^\top(x_1(t),t),\cdots,f^\top(x_N(t),t)\right]^\top$, $f(\hat{x}(t),t) := \left[f^\top(\hat{x}_1(t),t),\cdots,f^\top(\hat{x}_N(t),t)\right]^\top$, and $e(t) := [e_1^\top(t), e_2^\top(t), \cdots, e_N^\top(t)]^\top$. Then, we choose a Lyapunov function candidate $V_1(t)$ as follows

$$V_1(t) = V_{11}(t) + V_{12}(t) + V_{13}(t), \qquad (A.4)$$

where $V_{11}(t) = \bar{\xi}^T(t)(\Theta \otimes P)\bar{\xi}(t), V_{12}(t) = \hat{\xi}^T(t)(\Theta \otimes P)\hat{\xi}(t)$, and $V_{13}(t) = e^T(t)(\Theta \otimes Q)e(t)$. Taking the derivatives of $V_{11}(t), V_{12}(t)$ and $V_{13}(t)$, we obtain

$$\begin{aligned}\dot{V}_{11}(t) &= \bar{\xi}^T(t)\left[\Theta \otimes (A^\top P + PA) + 2\alpha\left(\Theta L_R \otimes P\tilde{F}C\right)\right]\bar{\xi}(t) \\ &\quad + 2\bar{\xi}^T(t)(\Theta \otimes P) \times (f(x(t),t) - f(\hat{x}(t),t)) + 2\bar{\xi}^T(t)(\Theta \otimes PD)e(t),\\ \dot{V}_{12}(t) &= \hat{\xi}^T(t)\left[\Theta \otimes (A^\top P + PA) + 2\beta(\Theta L_R \otimes (PBK_1))\right]\hat{\xi}(t) \\ &\quad + 2\hat{\xi}^T(t)(\Theta \otimes P) \times (f(\hat{x}(t),t) - (SI_N \otimes f(x_0(t),t))) \\ &\quad - 2\alpha\hat{\xi}^T(t)\left(\Theta L_R \otimes P\tilde{F}C\right)\bar{\xi}(t),\\ \dot{V}_{13}(t) &= e^\top(t)\left[\Theta \otimes (QM + M^\top Q)\right]e(t) + 2e^\top(t)(\Theta L_R \otimes QG_1C)\bar{\xi}(t).\end{aligned} \qquad (A.5)$$

According to Assumption 5 and Young's inequality, we have

$$\begin{aligned}2\quad & \bar{\xi}^\top(t)(\Theta\otimes P)(f(x(t),t)-f(\hat{x}(t),t))\\ \leqslant\quad & \bar{\xi}^\top(t)\left(\Theta\otimes PP^\top\right)\bar{\xi}(t)\\ & +\left[(f(x(t),t)-f(\hat{x}(t),t))^\top(\Theta\otimes I)(f(x(t),t)-f(\hat{x}(t),t))\right]\\ \leqslant\quad & \bar{\xi}^\top(t)\left(\Theta\otimes PP^\top\right)\bar{\xi}(t)\\ & +\left[\lambda_{\max}(\Theta)(f(x(t),t)-f(\hat{x}(t),t))^\top(f(x(t),t)-f(\hat{x}(t),t))\right]\\ \leqslant\quad & \bar{\xi}^\top(t)\left(\Theta\otimes PP^\top\right)\bar{\xi}(t)+\lambda_{\max}(\Theta)\rho^2\bar{\xi}^\top(t)\bar{\xi}(t)\\ \leqslant\quad & \bar{\xi}^\top(t)\left[\Theta\otimes\left(PP^\top+\tfrac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)\right]\bar{\xi}(t),\end{aligned}\tag{A.6}$$

where

$$\lambda_{\max}(\Theta)\rho^2\bar{\xi}^\top(t)\bar{\xi}(t)\leqslant\frac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2\bar{\xi}^\top(t)(\Theta\otimes I)\bar{\xi}(t).$$

Similar to (A.6), we obtain

$$\begin{aligned}2\quad & \hat{\xi}^\top(t)(\Theta\otimes P)(f(\hat{x}(t),t)-SI_N\otimes f(x_0(t),t))\\ & \leqslant\hat{\xi}^\top(t)\left(\Theta\otimes PP^\top\right)\hat{\xi}(t)\\ & +\left[(f(\hat{x}(t),t)-SI_N\otimes f(x_0(t),t))^\top\cdot\right.\\ & \left.\cdot(\Theta\otimes I)(f(\hat{x}(t),t)-SI_N\otimes f(x_0(t),t))\right]\\ \leqslant\quad & \hat{\xi}^\top(t)\left(\Theta\otimes PP^\top\right)\hat{\xi}(t)\\ & +\left[\lambda_{\max}(\Theta)(f(\hat{x}(t),t)-SI_N\otimes f(x_0(t),t))^\top\cdot\right.\\ & \left.\cdot(f(\hat{x}(t),t)-SI_N\otimes f(x_0(t),t))\right]\\ & \leqslant\hat{\xi}^\top(t)\left(\Theta\otimes PP^\top\right)\hat{\xi}(t)+\lambda_{\max}(\Theta)\rho^2\hat{\xi}^\top(t)\hat{\xi}(t)\\ & \leqslant\hat{\xi}^\top(t)\left[\Theta\otimes\left(PP^\top+\tfrac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)\right]\hat{\xi}(t).\end{aligned}\tag{A.7}$$

Substituting $K_1=-B^TP, G_1=Q^{-1}C^T$, and $\tilde{F}=-P^{-1}C^\top$ into (A.5), and by using (A.6) and (A.7), we obtain the following inequalities:

$$\begin{aligned}\dot{V}_{11}(t)\leqslant&\ \bar{\xi}^\top(t)[\Theta\otimes(PA+A^\top P)-\alpha((\Theta L_R+L_R^\top\Theta)\otimes C^\top C)\\ &+\Theta\otimes\left(PP^\top+\frac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)]\bar{\xi}(t)+2\bar{\xi}^T(t)(\Theta\otimes PD)e(t),\\ \dot{V}_{12}(t)\leqslant&\ \hat{\xi}^T(t)[\Theta\otimes(A^\top P+PA)-\beta((\Theta L_R+L_R^\top\Theta)\otimes PBB^\top P)\\ &+\Theta\otimes\left(PP^\top+\frac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)]\hat{\xi}(t)+2\alpha\hat{\xi}^T(t)(\Theta L_R\otimes C^\top C)\bar{\xi}(t),\\ \dot{V}_{13}(t)=&\ e^\top(t)[\Theta\otimes(QM+M^\top Q)]e(t)+2e^\top(t)\left(\Theta L_R\otimes C^TC\right)\bar{\xi}(t).\end{aligned}$$

Let $\hat{A}=L_R+\Theta^{-1}L_R^\top\Theta$ and $H=-\alpha((\Theta L_R+L_R^\top\Theta)\otimes C^\top C)$, by utilizing Lemma 2, we obtain $H\leqslant-\alpha\Theta\hat{A}\otimes C^\top C\leqslant-\alpha\lambda_0\Theta\otimes C^\top C$, where $\lambda_0 c\triangleq=\lambda_{\min}\left(L_R+\Theta^{-1}L_R^\top\Theta\right)$ and $\Theta=\mathrm{diag}\{\varphi_1,\cdots,\varphi_N\}$. According to Lemma 1 in [23], we obtain

$$\begin{aligned}\dot{V}_{11}(t)\leqslant&\ \bar{\xi}^T(t)\\ &\times\left[\Theta\otimes\left(PA+A^\top P-\alpha\lambda_0C^\top C+PP^\top+\frac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)\right]\bar{\xi}(t)\\ &+2\bar{\xi}^T(t)(\Theta\otimes PD)e(t).\end{aligned}\tag{A.8}$$

Similarly, calculating the derivative of $V_{12}(t)$, we obtain

$$\begin{aligned}\dot{V}_{12}(t)\leqslant&\ \hat{\xi}^T(t)\\ &\times\left[\Theta\otimes\left(A^\top P+PA-\beta\lambda_0PBB^\top P+PP^\top+\frac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)\right]\hat{\xi}(t)\\ &+2\alpha\hat{\xi}^T(t)(\Theta L_R\otimes C^\top C)\bar{\xi}(t).\end{aligned}\tag{A.9}$$

Based on Lemma 2 and the facts $\alpha>\mu_0/\lambda_0, \beta>\mu_1/\lambda_0$, we obtain

$$\begin{aligned}\dot{V}_{11}(t)\leqslant&\ \bar{\xi}^T(t)\left[\Theta\otimes\left(PA+A^\top P-\mu_0C^\top C+PP^\top+\tfrac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)\right]\bar{\xi}(t)\\ &+2\bar{\xi}^T(t)(\Theta\otimes PD)e(t),\\ \dot{V}_{12}(t)\leqslant&\ \hat{\xi}^T(t)\left[\Theta\otimes\left(A^\top P+PA-\mu_1PBB^\top P+PP^\top+\tfrac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2 I\right)\right]\hat{\xi}(t)\\ &+2\alpha\hat{\xi}^T(t)(\Theta L_R\otimes C^\top C)\bar{\xi}(t).\end{aligned}$$

Furthermore, under Assumptions 2 and 3, by utilizing Lemma 1, we have

$$\begin{aligned}&2\bar{\xi}^T(t)(\Theta\otimes PD)e(t)\\ &\leqslant\bar{\xi}^T(t)(\Theta\otimes D^\top PPD)\bar{\xi}(t)+e^\top(t)(\Theta\otimes I)e(t),\\ &2\alpha\hat{\xi}^T(t)(\Theta L_R\otimes C^\top C)\bar{\xi}(t)\leqslant\alpha\hat{\xi}^T(t)(\Theta\otimes C^\top CC^\top C)\hat{\xi}(t)\\ &+\alpha\bar{\xi}^\top(t)(\Theta\otimes I)\left(\Theta^{-1}L_R^\top\Theta L_R\otimes I\right)\bar{\xi}(t)\\ &\leqslant\alpha\lambda_{\max}(C^\top CC^\top C)\hat{\xi}^\top(t)(\Theta\otimes I)\hat{\xi}(t)\\ &+\alpha\lambda_{\max}\left(\Theta^{-1}L_R^\top\Theta L_R\right)\bar{\xi}^\top(t)(\Theta\otimes I)\bar{\xi}(t),\\ &2e^\top(t)(\Theta L_R\otimes C^\top C)\bar{\xi}(t)\leqslant e^\top(t)(\Theta\otimes C^\top CC^\top C)e(t)\\ &+\bar{\xi}^\top(t)(\Theta\otimes I)\left(\Theta^{-1}L_R^\top\Theta L_R\otimes I\right)\bar{\xi}(t)\\ &\leqslant\lambda_{\max}(C^\top CC^\top C)e^\top(t)(\Theta\otimes I)e(t)\\ &+\lambda_{\max}\left(\Theta^{-1}L_R^\top\Theta L_R\right)\bar{\xi}^\top(t)(\Theta\otimes I)\bar{\xi}(t).\end{aligned}\tag{A.10}$$

Then, we have $\dot{V}_1(t)\leqslant\dot{\bar{V}}_{11}(t)+\dot{\bar{V}}_{12}(t)+\dot{\bar{V}}_{13}(t)$, where

$$\begin{aligned}\dot{\bar{V}}_{11}(t)=&\ \bar{\xi}^T(t)[\Theta\otimes(PA+A^\top P-\mu_0C^\top C+D^\top PPD+PP^\top\\ &+\left(\tfrac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2+(\alpha+1)\lambda_{\max}\left(\Theta^{-1}L_R^\top\Theta L_R\right)\right)I)]\bar{\xi}(t),\\ \dot{\bar{V}}_{12}(t)=&\ \hat{\xi}^T(t)[\Theta\otimes(A^\top P+PA-\mu_1PBB^\top P+PP^\top\\ &+\left(\tfrac{\lambda_{\max}(\Theta)}{\lambda_{\min}(\Theta)}\rho^2+\alpha\lambda_{\max}(C^\top CC^\top C)\right)I)]\hat{\xi}(t),\\ \dot{\bar{V}}_{13}(t)=&\ e^\top(t)[\Theta\otimes(QM+M^\top Q+(\lambda_{\max}(C^\top CC^\top C)+1)I]e(t).\end{aligned}$$

It can be yielded from (14), (15) and (16) that

$$\begin{aligned}&\dot{\bar{V}}_{11}(t)\leqslant\bar{\xi}^T(t)[\Theta\otimes(PA+A^\top P-\mu_0C^\top C+D^\top PPD+\gamma_1 I+PP^\top)]\bar{\xi}(t),\\ &\dot{\bar{V}}_{12}(t)\leqslant\hat{\xi}^T(t)[\Theta\otimes(A^\top P+PA-\mu_1PBB^\top P+PP^\top+\gamma_2 I)]\hat{\xi}(t),\\ &\dot{\bar{V}}_{13}(t)\leqslant e^\top(t)[\Theta\otimes(QM+M^\top Q+\gamma_3 I)]e(t).\end{aligned}$$

According to the Schur complement lemma, with $\rho>0$, we can conclude there exist three nonnegative parameters c_1, c_2, and c_3 such that $\dot{\bar{V}}_{11}(t)<-c_1\bar{\xi}^\top(t)(\Theta\otimes P)\bar{\xi}(t), \dot{\bar{V}}_{12}(t)<-c_2\hat{\xi}^\top(t)(\Theta\otimes P)\hat{\xi}(t)$, and $\dot{\bar{V}}_{13}(t)<-c_3e^\top(t)(\Theta\otimes Q)e(t)$. Then, we obtain $\dot{V}_1(t)<-c_0V_1(t)$, where $c_0=\min_{i\in\{1,2,3\}}\{c_i\}$. Thus, one has $V_1(t)<e^{-c_0t}V_1(0)$. By summing the three terms in (A.4), we conclude that when $t\to 0, V_1(t)\to 0$, which implies $\bar{\xi}(t)\to 0, \hat{\xi}(t)\to 0$, and $e(t)\to 0$. This completes the proof. ■

Appendix B. Proof of Theorem 2

Under Assumption 2, and utilizing the definitions in (8), we can write the following dynamics for the state and its estimator:

$$\begin{aligned}\dot{x}_i(t)=&Ax_i(t)\\ &+\tilde{\beta}BK_2\left[\sum_{j=1}^N\left|a_{ij}^{\varpi(t)}\right|\left(\hat{x}_i(t)-\mathrm{sgn}\left(a_{ij}^{\varpi(t)}\right)\hat{x}_j(t)\right)+a_{i0}^{\varpi(t)}(\hat{x}_i(t)-s_ix_0(t))\right)\right]\\ &+f(x_i(t),t)+De_i(t),\end{aligned}\tag{B.1}$$

and

$$\dot{\hat{x}}_i(t)=A\hat{x}_i(t)+\tilde{\beta}BK_2\left[\sum_{j=1}^{N}|a_{ij}^{\varpi(t)}|\left(\hat{x}_i(t)-\mathrm{sgn}\left(a_{ij}^{\varpi(t)}\right)\hat{x}_j(t)\right)+a_{i0}^{\varpi(t)}(\hat{x}_i(t)-s_ix_0(t))\right)]+f(\hat{x}_i(t),t)-\tilde{\alpha}\hat{F}\left[\sum_{j=1}^{N}|a_{ij}^{\varpi(t)}|\left(\tilde{\xi}_i(t)-\mathrm{sgn}\left(a_{ij}^{\varpi(t)}\right)\tilde{\xi}_j(t)\right)+a_{i0}^{\varpi(t)}\tilde{\xi}_i(t)\right], \tag{B.2}$$

respectively, from which we derive the following dynamics for the three errors in (8):

$$\begin{aligned}\dot{\tilde{\xi}}(t)&=\left[I_N\otimes A+\tilde{\alpha}\left(L_R^{\varpi(t)}\otimes\hat{F}C\right)\right]\tilde{\xi}(t)+(I_N\otimes D)e(t)\\&\quad+I_N\otimes(f(x(t),t)-f(\hat{x}(t),t)),\\\dot{\hat{\xi}}(t)&=\left[I_N\otimes A+\tilde{\beta}\left(L_R^{\varpi(t)}\otimes BK_2\right)\right]\hat{\xi}(t)-\tilde{\alpha}\left(L_R^{\varpi(t)}\otimes\hat{F}C\right)\tilde{\xi}(t)\\&\quad+(f(\hat{x}(t),t)-(SI_N\otimes f(x_0(t),t))),\\\dot{e}(t)&=(I_N\otimes M)e(t)+\left(L_R^{\varpi(t)}\otimes G_2C\right)\tilde{\xi}(t).\end{aligned}$$

For $t\in[t_k,t_{k+1}),k\in\mathbb{N}$, the multiple Lyapunov functions $V_2(t)$ can be constructed as follows:

$$V_2(t)=V_{21}(t)+V_{22}(t)+V_{23}(t), \tag{B.3}$$

where $V_{21}(t)=\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)\tilde{\xi}(t),V_{22}(t)=\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)\hat{\xi}(t)$, and $V_{23}(t)=e^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{Q}\right)e(t)$. Taking the derivatives of $V_{21}(t),V_{22}(t)$ and $V_{23}(t)$, which yield

$$\begin{aligned}\dot{V}_{21}(t)&=\tilde{\xi}^\top(t)\left[\Theta^{\varpi(t)}\otimes(A^\top\bar{P}+\bar{P}A)+2\tilde{\alpha}\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes\bar{P}\hat{F}C\right)\right]\\&\quad\times\tilde{\xi}(t)2\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)(f(x(t),t)\\&\quad-f(\hat{x}(t),t))+2\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}D\right)e(t),\\\dot{V}_{22}(t)&=\hat{\xi}^\top(t)\left[\Theta^{\varpi(t)}\otimes(A^\top\bar{P}+\bar{P}A)+2\tilde{\beta}\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes\left(\bar{P}BK_2\right)\right)\right]\hat{\xi}(t)\\&\quad+2\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)(f(\hat{x}_i(t),t)-(SI_N\otimes f(x_0(t),t))),\\&\quad-2\tilde{\alpha}\hat{\xi}^\top(t)\left[\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes\left(\bar{P}\hat{F}C\right)\right]\tilde{\xi}(t),\\\dot{V}_{23}(t)&=e^\top(t)\left[\Theta^{\varpi(t)}\otimes(\bar{Q}M+M^\top\bar{Q})\right]e^\top(t)\\&\quad+2e^\top(t)\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes\overline{Q}G_2C\right)\tilde{\xi}(t).\end{aligned} \tag{B.4}$$

To simplify the analysis, based on Assumption 5 and Young's inequality, we obtain

$$\begin{aligned}&2\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)(f(x(t),t)-f(\hat{x}(t),t))\\&\leqslant\tilde{\xi}^\top(t)\left[\Theta^{\varpi(t)}\otimes\left(\bar{P}\bar{P}^\top+\frac{\lambda_{\max}(\Theta^{\varpi(t)})}{\lambda_{\min}(\Theta^{\varpi(t)})}\rho^2I\right)\right]\tilde{\xi}(t),\\&2\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)(f(\hat{x}(t),t)-SI_N\otimes f(x_0(t),t))\\&\leqslant\hat{\xi}^\top(t)\left[\Theta^{\varpi(t)}\otimes\left(\bar{P}\bar{P}^\top+\frac{\lambda_{\max}(\Theta^{\varpi(t)})}{\lambda_{\min}(\Theta^{\varpi(t)})}\rho^2I\right)\right]\hat{\xi}(t).\end{aligned} \tag{B.5}$$

Substituting $K_2=-B^T\bar{P},G_2=\bar{Q}^{-1}C^T$, and $\hat{F}=-\bar{P}^{-1}C^\top$ into (B.4), and in view of (B.5), we obtain

$$\begin{aligned}\dot{V}_{21}(t)&\leqslant\tilde{\xi}^\top(t)\Big[\Theta^{\varpi(t)}\otimes(\bar{P}A+A^\top\bar{P})\\&\quad-\tilde{\alpha}\left(\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}+\left(L_R^{\varpi(t)}\right)^\top\Theta^{\varpi(t)}\right)\otimes C^\top C\right)\\&\quad+\Theta^{\varpi(t)}\otimes\left(\bar{P}\bar{P}^\top+\frac{\lambda_{\max}(\Theta^{\varpi(t)})}{\lambda_{\min}(\Theta^{\varpi(t)})}\rho^2I\right)\Big]\tilde{\xi}(t)\\&\quad+2\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}D\right)e(t),\\\dot{V}_{22}(t)&\leqslant\hat{\xi}^\top(t)\Big[\Theta^{\varpi(t)}\otimes(A^\top\bar{P}+\bar{P}A)\\&\quad-\tilde{\beta}\left(\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}+\left(L_R^{\varpi(t)}\right)^\top\Theta^{\varpi(t)}\right)\otimes\bar{P}BB^\top\bar{P}\right)\\&\quad+\Theta^{\varpi(t)}\otimes\left(\bar{P}\bar{P}^\top+\frac{\lambda_{\max}\left(\Theta^{\varpi(t)}\right)}{\lambda_{\min}\left(\Theta^{\varpi(t)}\right)}\rho^2I\right)\Big]\hat{\xi}(t)\\&\quad+2\tilde{\alpha}\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes C^\top C\right)\tilde{\xi}(t),\\\dot{V}_{23}(t)&=e^\top(t)\left[\Theta^{\varpi(t)}\otimes(\bar{Q}M+M^\top\bar{Q})\right]e(t)\\&\quad+2e^\top(t)\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes C^\top C\right)\tilde{\xi}(t).\end{aligned}$$

Since $\tilde{\alpha}>\tilde{\mu}_0/\tilde{\lambda}_0$ and $\tilde{\beta}>\tilde{\mu}_1/\tilde{\lambda}_0$, and according to Lemma 4 in [23], we obtain

$$\begin{aligned}&\dot{V}_{21}(t)\leqslant\tilde{\xi}^T(t)\left[\Theta^{\varpi(t)}\otimes\left(\bar{P}A+A^T\bar{P}-\tilde{\mu}_0C^TC+\bar{P}\bar{P}^T+\frac{\lambda_{\max}(\Theta^{\varpi(t)})}{\lambda_{\min}(\Theta^{\varpi(t)})}\rho^2I\right)\right]\tilde{\xi}(t),\\&\dot{V}_{22}(t)\leqslant\hat{\xi}^\top(t)\left[\Theta^{\varpi(t)}\otimes(\bar{P}A+A^\top\bar{P}-\tilde{\mu}_1\bar{P}BB^\top\bar{P}+\bar{P}\bar{P}^\top+\frac{\lambda_{\max}(\Theta^{\varpi(t)})}{\lambda_{\min}(\Theta^{\varpi(t)})}\rho^2I)\right]\\&\hat{\xi}(t)+2\tilde{\alpha}\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes C^\top C\right)\tilde{\xi}(t),\\&\dot{V}_{23}(t)=e^\top(t)\left[\Theta^{\varpi(t)}\otimes(\bar{Q}M+M^\top\bar{Q})\right]e(t)+2e^\top(t)\left(\Theta^{\varpi(t)}L_R^{\varpi(t)}\otimes C^\top C\right)\tilde{\xi}(t).\end{aligned}$$

Furthermore, under Assumptions 3 and 5, by utilizing Lemma 1, one has

$$\begin{aligned}&2\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}D\right)e(t)\leqslant\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes D^\top\bar{P}\bar{P}D\right)\tilde{\xi}(t)\\&\quad+e^\top(t)\left(\Theta^{\varpi(t)}\otimes I\right)e(t),2\tilde{\alpha}\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}L_R\otimes C^\top C\right)\tilde{\xi}(t)\\&\leqslant\tilde{\alpha}\lambda_{\max}(C^\top CC^\top C)\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes I\right)\hat{\xi}(t)\\&\quad+\tilde{\alpha}\lambda_{\max}\left(\left(\Theta^{\varpi(t)}\right)^{-1}\left(L_R^{\varpi(t)}\right)^\top\Theta^{\varpi(t)}L_R^{\varpi(t)}\right)\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes I\right)\tilde{\xi}(t),\end{aligned}$$

and

$$\begin{aligned}&2e^\top(t)\left(\left(\Theta^{\varpi(t)}\right)^{-1}L_R^{\varpi(t)}\otimes C^\top C\right)\tilde{\xi}(t)\\&\leqslant\lambda_{\max}(C^\top CC^\top C)e^\top(t)\left(\Theta^{\varpi(t)}\otimes I\right)e(t)\\&\quad+\lambda_{\max}\left(\left(\Theta^{\varpi(t)}\right)^{-1}\left(L_R^{\varpi(t)}\right)^\top\Theta^{\varpi(t)}L_R^{\varpi(t)}\right)\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes I\right)\tilde{\xi}^\top(t).\end{aligned}$$

Similarly, one can conclude $\dot{V}_2(t)\leqslant\dot{\tilde{V}}_{21}(t)+\dot{\tilde{V}}_{22}(t)+\dot{\tilde{V}}_{23}(t)$, where

$$\begin{aligned}&\dot{\tilde{V}}_{21}(t)\leqslant\tilde{\xi}^\top(t)\left[\Theta^{\varpi(t)}\otimes\left(A\bar{P}+\bar{P}A^\top-\tilde{\mu}_0C^\top C+D^\top\bar{P}\bar{P}D+\gamma_4I+barP^\top\bar{P}\right)\right]\tilde{\xi}(t),\\&\dot{\tilde{V}}_{22}(t)\leqslant\hat{\xi}^\top(t)\left[\Theta^{\varpi(t)}\otimes(A^\top\bar{P}+\bar{P}A-\tilde{\mu}_1\bar{P}BB^\top\bar{P}+\gamma_5I+\bar{P}\bar{P}^\top)\right]\hat{\xi}(t),\\&\dot{\tilde{V}}_{23}(t)\leqslant e^\top(t)\left[\Theta^{\varpi(t)}\otimes(\bar{Q}M+M^\top\bar{Q}+\gamma_6I)\right]e(t).\end{aligned}$$

Based on the Schur's complement lemma, one can conclude there exist three non-negative parameters $\tilde{c}_1,\tilde{c}_2$ and $\tilde{c}_3$ such that $\dot{\tilde{V}}_{21}(t)<-\tilde{c}_1\tilde{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)\tilde{\xi}(t),\dot{\tilde{V}}_{22}(t)<-\tilde{c}_2\hat{\xi}^\top(t)\left(\Theta^{\varpi(t)}\otimes\bar{P}\right)\hat{\xi}(t),$

and $\dot{\tilde{V}}_{23}(t) < -\tilde{c}_3 e^{\top}(t)\left(\Theta^{\varpi(t)} \otimes \bar{Q}\right)e(t)$. Hence, one has $\dot{V}_2(t) < -\tilde{c}_0 V_2(t), t \in [t_k, t_{k+1})$, for $k \in \mathbb{N}$, where $\tilde{c}_0 = \min_{i \in \{1,2,3\}}\{\tilde{c}_i\}$.

It is noted that the MASs with control law (17) switches when $t = t_k, k \in \mathbb{N}$. Then one can obtain $V_2(t_2^-) < V_2(t_1)e^{-\tilde{c}_0(t_2 - t_1)} < e^{-\tilde{c}_0\hat{\tau}_0}V_2(t_1), t \in [t_1, t_2)$, where $V_2(t_2^-) = \lim_{t \to t_2} V_2(t)$. Let $\Gamma = \varpi(t_k)$, $\bar{\Gamma} = \varpi(t_{k^-}), \widehat{A}_{\min} = \lambda_{\min}\left(\Theta^{\Gamma}\right) \otimes \bar{P}$, $\widehat{A}_{\max} = \lambda_{\max}\left(\Theta^{\Gamma}\right) \otimes \bar{P}$, $\widetilde{A}_{\max} = \lambda_{\max}\left(\Theta^{\bar{\Gamma}}\right) \otimes \bar{P}, \widetilde{A}_{\min} = \lambda_{\min}\left(\Theta^{\bar{\Gamma}}\right) \otimes \bar{P}$, $\widehat{B}_{\min} = \lambda_{\min}\left(\Theta^{\Gamma}\right) \otimes \bar{Q}, \widehat{B}_{\max} = \lambda_{\max}\left(\Theta^{\Gamma}\right) \otimes \bar{Q}, \widetilde{B}_{\max} = \lambda_{\max}\left(\Theta^{\bar{\Gamma}}\right) \otimes \bar{Q}$, and $\widetilde{B}_{\min} = \lambda_{\min}\left(\Theta^{\bar{\Gamma}}\right) \otimes \bar{Q}$, according to (B.3), one gets $\bar{\xi}^{\mathrm{T}}(t_k)\widehat{A}_{\min}\bar{\xi}(t_k) \leqslant V_{21}(t_k) \leqslant \bar{\xi}^{\mathrm{T}}(t_k)\widehat{A}_{\max}\bar{\xi}(t_k)$, $\hat{\xi}^{\mathrm{T}}(t_k)\widehat{A}_{\min}\hat{\xi}(t_k) \leqslant V_{22}(t_k) \leqslant \hat{\xi}^{\mathrm{T}}(t_k)\widehat{A}_{\max}\hat{\xi}(t_k)$, $e^{\mathrm{T}}(t_k)\widehat{B}_{\max}e(t_k) \leqslant V_{23}(t_k) \leqslant e^{\mathrm{T}}(t_k)\widehat{B}_{\min}e(t_k)$, $\bar{\xi}^{\mathrm{T}}(t_{k^-})\ \widetilde{A}_{\min}\bar{\xi}(t_{k^-}) \leqslant V_{21}(t_{k^-}) \leqslant \bar{\xi}^{\mathrm{T}}(t_{k^-})\widetilde{A}_{\max}\bar{\xi}(t_{k^-})$, $\hat{\xi}^{\mathrm{T}}(t_{k^-})\widetilde{A}_{\min}\hat{\xi}(t_{k^-}) \leqslant V_{22}(t_{k^-}) \leqslant \hat{\xi}^{\mathrm{T}}(t_{k^-})\ \widetilde{A}_{\max}\hat{\xi}(t_{k^-})$, and $e^{\mathrm{T}}(t_{k^-})\widetilde{B}_{\max}e(t_{k^-}) \leqslant V_{23}(t_{k^-}) \leqslant e^{\mathrm{T}}(t_{k^-})\widetilde{B}_{\min}e(t_{k^-})$. In summary, we obtain $V_{21}(t_k) \leqslant \ell_0 V_{21}(t_{k^-})$, $V_{22}(t_k) \leqslant \ell_0 V_{22}(t_{k^-})$, and $V_{23}(t_k) \leqslant \ell_0 V_{23}(t_{k^-})$, where

$$\ell_0 = \frac{\max_{\Gamma=1,\cdots,L}\left(\lambda_{\max}\left(\Theta^{\Gamma}\right)\right)}{\min_{\bar{\Gamma}=1,\cdots,L}\left(\lambda_{\min}\left(\Theta^{\bar{\Gamma}}\right)\right)} = \frac{\varphi_{\max}}{\varphi_{\min}}.$$

Thus, we obtain $V_2(t_2) < \ell_0 e^{-\tilde{c}_0\hat{\tau}_0}V_2(t_1)$, i.e., $V_2(t_2) < e^{(-\tilde{c}_0\hat{\tau}_0 + \ln \ell_0)}V_2(0)$. Hence, if the dwell time satisfies $\hat{\tau}_0 > \ln(\ell_0/\tilde{c}_0)$, then the following holds $V_2(t_2) < e^{-\kappa\hat{\tau}_0}V_2(0)$, where $\kappa = \tilde{c}_0 - (\ln \ell_0)/\hat{\tau}_0 > 0$. For $t > t_2$, there exists a non-negative integer $s \geqslant 2$ such that $t_s < t \leqslant t_{s+1}$. In addition, for an arbitrary non-negative integer $w \in N$, one has $V_2(t_{w+1}) < e^{-\kappa\hat{\tau}_0}V_2(t_w) < e^{-\kappa w\hat{\tau}_0}V_2(0)$. Similarly, when $t \in (t_s, t_{s+1})$, one has

$$V_2(t) < e^{-\tilde{c}_0(t-t_s)}V_2(t_s) < e^{-[\tilde{c}_0(t-t_s)+(s-1)\kappa\hat{\tau}_0]}V_2(0) < e^{-\frac{(s-1)\hat{\tau}_0}{s\hat{\tau}_1}\kappa t}V_2(0) < e^{-\frac{\hat{\tau}_0\kappa}{2\hat{\tau}_1}t}V_2(0),$$

where $s \geqslant 2$ and $\hat{\tau}_0 \leqslant t_{k+1} - t_k \leqslant \hat{\tau}_1$. When $t = t_{s+1}$, we obtain $V_2(t) < e^{-\frac{\hat{\tau}_0\kappa}{\hat{\tau}_1}t}V_2(0)$, which implies $\bar{\xi}(t) \to 0, \hat{\xi}(t) \to 0$, and $e(t) \to 0$ as $t \to \infty$. ■

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