Surrogate-based optimum design of 3D reinforced concrete building frames to Eurocodes

Panagiotis E. Mergos [a,*]

[a] *Department of Civil Engineering, City, University of London, London EC1V 0HB, UK*

Abstract.

The optimum structural design of real-world 3D concrete building frames to modern design standards is a complex and computationally expensive task. Hence, the use of surrogate-based optimization (SBO) methodologies must be investigated to reduce computational cost. The present study applies, for first time, a fully-fledged SBO algorithm to the optimum design of 3D concrete building frames. More particularly, the algorithm is applied to the minimum material cost design of a 4-storey and a 12-storey 3D RC building according to Eurocodes. It is found that the SBO algorithm can converge earlier than other well-established metaheuristic optimization algorithms reducing considerably the required computational effort. Nevertheless, it is likely to get trapped in local optima for large-scale RC frames. To overcome this drawback, a novel hybrid approach is also proposed herein that offers improved computational performance for large-scale concrete building frames.

Keywords: Structural optimization; Surrogates; Metamodels; Reinforced concrete; 3D; Building frames

1 Introduction

Reinforced concrete (RC) building frames represent a large part of the built environment and they are associated with significant economic costs and environmental impacts (Olivier *et al*. 2015). Therefore, design of these structural systems for minimum economic cost and/or environmental impact represents an urgent need for modern societies (Mergos 2018a, 2018b). At the same time, the optimum structural design of RC building frames to modern design standards, such as the Eurocodes, may be so highly complex that cannot be addressed adequately by manual trial and error procedures. In these cases, the use of automated optimization algorithms is recommended. Optimization algorithms can be divided into gradient-based and metaheuristic. The latter category includes algorithms such as the Genetic Algorithm (GA) (Holland 1975), Simulated Annealing (SA) (Kirkpatrick *et al* 1983), Particle Swarm Optimization (PSO) (Kennedy 2001), the Flower Pollination Algorithm (FPA) (Yang 2012), and many others. Metaheuristic optimization algorithms may require more

[*] Corresponding author. Panagiotis E. Mergos, Senior Lecturer in Structural Engineering, Research Centre for Civil Engineering Structures, City, University of London, London, EC1V 0HB, UK.
E-mail address: panagiotis.mergos.1@city.ac.uk, Tel. 0044 (0) 207040 8417

computational cost to converge but they are less likely to get trapped in local optima than gradient-based algorithms (Yang 2008).

A significant amount of research has been dedicated in the previous years on optimizing the structural design of concrete frames (Sarma and Adeli 1997). Nevertheless, the vast majority of these studies concentrate on either single concrete members (i.e. beams, columns and others) (e.g. Mergos 2018a, Yeo and Gabbai 2011, Medeiros and Kripka 2014, Kayabekir *et al*. 2021 and Kayabekir *et al*. 2022) or 2D concrete frames (e.g. Paya *et al*. 2008, Akin and Saka 2015, Mergos 2018c, Rakici *et al*. 2020). To the best of the author's knowledge, the research studies addressing optimum structural design of realistic 3D RC buildings to modern design codes are only limited to: Fadaee and Grierson (1996), BaIling and Yao (1997), Sahab *et al*. (2005), Govindaraj and Ramasany (2007), Sharafi *et al*. (2012), Kaveh and Behnam (2013), Lagaros (2014), Esfandiari *et al*. (2018), Dehnavipour *et al*. (2019), Martins *et al*. (2020) and Mergos (2021). The limited number of these research studies can be attributed to the high level of complexity and significant computational effort involved in the structural design of 3D RC buildings frames (Sarma and Adeli 1997).

The previous observations reveal the need to investigate the applicability and efficiency of surrogate-assisted methodologies in the optimum design of real-world concrete building frames. Surrogates or else metamodels are prediction models that provide fast approximations of computationally expensive objective and/or constraint functions at new design points based on a limited number of previous design points of these functions. In this manner, the computational burden drastically decreases making parametric, sensitivity and optimization studies more feasible. There exists a wide range of available surrogate models in literature, characterized by different levels of accuracy and complexity, such as the classic polynomial Response Surface Models (RSM), Radial Basis Functions (RBFs), Kriging model, Support Vector Regression and Artificial Neural Networks (ANN) (Forrester *et al*. 2008).

Surrogate-based optimization (SBO) is the process of employing surrogates to drastically reduce the computational effort of optimization problems involving computationally intensive objective and/or constraint functions. SBO is a far more elaborated procedure than developing a surrogate model. Furthermore, it is not limited to the identification of the optimum solution of a surrogate model. The latter is not the case because surrogates are only approximations of real functions.

SBO has been widely used in aerospace and mechanical engineering designs mainly due to the intensive finite element analyses involved (Queipo et al. 2005, Forrester and Keane 2009). In civil engineering, there exists a significant number of studies developing surrogate models to predict structural response and performance (e.g. Gudipati et al. 2018, Du and Padgett 2020, Shekhar and Gosh 2020). Nevertheless, application of SBO to the optimum design of civil engineering structures, such as buildings and bridges, is still quite limited. In the latter applications, SBO has been mainly used within probabilistic optimization frameworks such as the robust (e.g. Battacharjya and Chakraborty 2011, Battacharjya *et al*. 2018, Penadés-Plà *et al*. 2020), reliability-based (e.g. Papadrakakis and Lagaros 2002, Khatibinia *et al*. 2013, Jia *et al*. 2014), risk-based (e.g. Ruiz *et al*. 2018) and life-cycle cost based (e.g. Gidaris and Taflanidis 2015) optimum structural design. These studies employ surrogates to address the high computational cost arising from the numerous numerical simulations required to obtain reliable statistical results of the objective functions and/or design constraints involved. Furthermore, SBO has been used in the context of the optimum performance-based design of structures (e.g. Gholizadeh and Salajegheh 2009, Mokarram and Banan 2018), where structural performance has to be evaluated by computationally expensive nonlinear structural analyses.

Recently, SBO has been used in the optimum, code-based design of complex bridge structures. García-Segura *et al*. (2018) developed a multi-objective, surrogate-assisted optimization framework for the sustainable design of post-tensioned concrete box-girder bridges. The application of surrogates in this study is justified by the large computational cost from the existence of numerous design variables and objective functions in addition to the need for time-consuming finite element analyses. Therefore, a surrogate model is used to predict the structural behaviour of the bridge designs. Furthermore, Penadés-Plà *et al*. (2019) examined kriging-based heuristic optimization to obtain the optimal solution of a continuous box-girder pedestrian bridge of three spans. The authors conclude that kriging-based optimization offers similar results to metaheuristic optimization algorithms using less computational effort. More particularly, the SBO reduces the computational effort by approximately 100% while it offers only 3% more expensive optimal solutions with respect to metaheuristic algorithms.

From the previous literature review, it can be concluded that SBO methodologies have not yet been applied to the structural design of 3D RC building frames. This is despite the fact that the design of these systems is accompanied by high computational costs prohibiting efficient optimization efforts in reasonable computational times. The latter may hinder the widespread use of optimization solutions in the design of real-world concrete buildings. Furthermore, most of the existing SBO studies in civil engineering adopt simplified frameworks, where surrogate models are only built once, and the optimum solutions of the surrogates are treated as the optimum solutions of the real functions. As discussed, this approach can be misleading since surrogates are only approximations of real functions.

The main objective of the present study is to investigate the efficiency and applicability of SBO frameworks to the optimum design of real-world RC buildings. To serve this goal, a computational platform for optimizing the structural design of real-scale 3D RC frame buildings is used that employs, for first time, a fully-fledged SBO algorithm to this challenging optimization problem. The performance of the SBO algorithm is compared with several established metaheuristic optimization algorithms and useful conclusions are drawn with respect to its computational efficiency and limitations. Furthermore, recommendations are made regarding the most efficient use of the SBO algorithm in the context of the optimum design of real-world RC building frames according to modern design guidelines. Finally, a novel hybrid approach is also proposed in this study that offers high computational performance and efficiency in optimizing complex and/or large-scale RC frames.

2 Framework for optimum structural design of RC building frames

The optimum structural design of concrete frames is treated herein as a single-objective optimization task with discrete design variables. The vector $\boldsymbol{x}$ of these design variables consists of the cross-sections assigned to different groups of structural members in the frame. These cross-sections are taken from discrete lists of cross-sections specified by the designers following standard construction practices. Any shape of cross-sections can be used in this optimization framework. However, for reasons of simplicity, in the present study square sections are considered for columns and rectangular sections for beams with the corresponding steel reinforcement configurations shown in Fig. 1. Furthermore, sizing optimization is only considered herein by assuming that the geometry, material properties, concrete cover, boundary conditions and loadings of the concrete frames are fixed.

The objective function $f(\boldsymbol{x})$ of the optimization problem is the total construction cost of concrete and reinforcing steel materials. These costs are calculated by summing the individual costs of all structural members in frames. The steel reinforcement of concrete members is calculated for the ULS based on standard structural design procedures in accordance with Eurocode 2 (EC2) (CEN 2000) and Eurocode 8 (EC8) (CEN 2004) for low ductility class (DCL) design rules. More particularly, concrete beams are designed for major direction bending, shear and torsion and concrete columns are designed for biaxial bending moments accounting for axial load effects and biaxial shear forces using the procedures described in CSI (2016).

Following this approach, $f(\boldsymbol{x})$ is determined by Eq. (1), where V_c (m^3) stands for the total concrete volume and m_s (kg) the total mass of reinforcing steel accounting for both the longitudinal and transversal steel reinforcement of concrete members. In Eq. (1), f_{co} and f_{so}

represent the prices of concrete per unit volume and reinforcing steel per unit mass, respectively.

$$f(\boldsymbol{x}) = V_c(\boldsymbol{x}) \cdot f_{co} + m_s(\boldsymbol{x}) \cdot f_{so} \tag{1}$$

It is worth noting at this point that other design objectives such as the life-cycle economic cost and environmental impact of concrete buildings can be considered in the optimum design of concrete buildings to offer more holistic design solutions (Mergos 2018a). Furthermore, the design of concrete buildings to maximize structural robustness (i.e. the capacity of sustaining local failures of elements e.g. via alternate load path strategy or redundancy) should be further investigated to prevent catastrophic progressive collapses in extreme events such as earthquakes and blasts (Biagi and Chiaia 2013, Kiakojouri *et al*. 2020). Nevertheless, the objective function used herein is deemed as adequate for the purposes of the present study that is focussing on the numerical efficiency of the surrogate-based optimization framework.

The design constraints in the optimization problem herein reflect the rules for the design of concrete frames in EC2 – Part 1 (CEN 2000) and EC8 – Part 1 (CEN 2004) for DCL. They include structural detailing prescriptions and safety verifications for the ultimate (ULS) and serviceability (SLS) limit states in terms of both displacements and forces. More particularly, for the ULS, a design constraint is assumed not to be satisfied when the corresponding safety check (i.e. for bending, shear and torsion) cannot be fulfilled by any permissible amount of steel reinforcement in the concrete sections. This is the case because only concrete sections are treated as independent variables herein. Furthermore, a design constraint for a column or a beam member is assumed not to be satisfied when the design shear forces and torsional moments exceed the maximum capacity of compressive concrete struts. For the SLS, a beam member is assumed not to fulfil the design constraints when the corresponding check for deflections is not satisfied. Beam members are checked for deflections using the limiting span-to-depth ratio approach (Moss and Brooker 2006).

The design constraints are treated indirectly in the formulation of the optimization problem by following the penalty function approach. A more detailed description of the optimization framework used in this study for concrete buildings can be found in Mergos (2021).

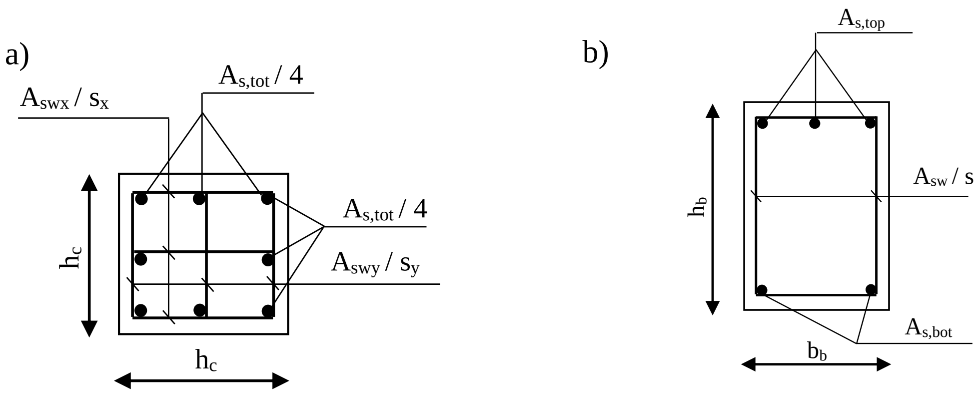

Fig. 1: Concrete cross-sections and steel reinforcement configurations assumed for; a) columns; b) beams

For the implementation of the optimization framework described above, a MATLAB (MathWorks 2020a) application, namely STROLAB (STRuctural Optimization LABoratory), has been developed (Mergos 2021). STROLAB is interacting, for the purposes of structural analysis and design, with the well-established integrated structural analysis and design software SAP2000 (CSI 2020) via its Application Programming Interface. A detailed presentation of the computational procedures followed by STROLAB can be found in Mergos (2021).

Closing this section, it is important to clarify that a code-based approach is followed in the present study that is consistent with standard engineering practice. However, code-based design is not guaranteed to offer maximum structural performance of concrete frames. Additional considerations are required for the optimum performance-based design of concrete frames as explained in Mergos (2018c).

3 Surrogate-based optimization (SBO)

From the description of the optimization framework of the previous section, it is clear that the evaluation of the objective function $f(\boldsymbol{x})$, which is the materials cost of the 3D concrete building frames, entails significant computational effort. This is the case because of the several computationally costly 3D finite element analyses required to calculate design action effects and the numerous structural design checks needed to examine compliance with the ULS and SLS constraints of the Eurocodes. Therefore, the applicability and potential benefits of SBO approaches to the computational cost of the optimum design of real-world concrete building frames have to be further investigated.

Most fully-fledged SBO methodologies follow a similar generic procedure. First, a set of initial sampling designs is decided where the computationally expensive objective function $f(\boldsymbol{x})$ is evaluated. This procedure is also called the Design of Experiments (DoE) or the initial sampling plan (Queipo *et al*. 2005, Forrester and Keane 2009). Next, the examined initial designs are used to construct the surrogate model $s(\boldsymbol{x})$. The surrogate model should offer reliable predictions of the real objective function $f(\boldsymbol{x})$ landscape especially in the vicinity of the optimum design. Then, a search of the surrogate model takes place to identify new promising design solutions. These new designs are called adaptive sampling designs or infill points. The adaptive sampling designs have been determined by using the surrogate models. Therefore, they must be re-evaluated by calling the true functions. Finally, the adaptive sampling designs are added to the previous designs and the procedure returns to the construction of the surrogate model phase until convergence is reached.

The SBO computational framework adopted herein is part of the Global Optimization Toolbox of MATLAB version R2020b (MathWorks 2020b). It follows the same generic methodology steps as the general SBO procedure described above. In the following, the numerical techniques used by the adopted SBO framework to implement these generic methodology steps are discussed in more detail.

To construct the surrogate, the SBO framework generates first a number of quasi-random initial designs within bounds as part of the DoE phase. For these designs, the real objective function $f(\boldsymbol{x})$ is evaluated. It is clarified that $f(\boldsymbol{x})$ is evaluated in this study by STROLAB calling SAP2000 to conduct structural analysis and design and by calculating the materials cost from Eq. (1) and adding potential penalties due to constraints violation. Then, the SBO framework uses the random points to construct a surrogate $s(\boldsymbol{x})$ as an approximation to the real function by using a Radial Basis Function (RBF) interpolator. RBF interpolators are beneficial because they use the same basic formula for any number of problem dimensions and points. Furthermore, they can take prescribed $f(\boldsymbol{x})$ values at the points where the function has been evaluated. Moreover, constructing an RBF interpolator is computationally efficient since it only requires a system of *N*-by-*N* linear equations to be solved, where *N* represents the number of evaluation points. In the adopted SBO framework, a cubic RBF with a linear trail is assumed (Gutmann 2001) as shown in Eq. (2), where λ_i are coefficients (weights) to be determined by the construction of the surrogate, the norm $\|\cdot\|$ is the Euclidean norm, $\boldsymbol{x}$ is the prediction point location, $\boldsymbol{x_i}$ are the locations of the previously evaluated points, $\varphi\ (r) = r^3$ for a cubic RBF and $p(x)$ is a linear polynomial.

$$s(\boldsymbol{x}) = \sum_{i=1}^{N} \lambda_i \varphi(\|\boldsymbol{x} - \boldsymbol{x_i}\|) + p(\boldsymbol{x}) \qquad (2)$$

In the next stage, the algorithm searches the surrogate for new promising design solutions. The search procedure followed is mainly based on the recommendations by Regis and Shoemaker (2007). The search begins from the incumbent, which is the best evaluated point since the last surrogate reset. The algorithm generates randomly a great number of sample points within a scaled area around the incumbent and within specified bounds of the design variables. Special sampling and rounding provisions are also taken so that the sample points consist of integer variables as required in the present study (MathWorks 2020b). Next, the sample points are evaluated based on the merit function. The merit function $f_{mer}(\boldsymbol{x})$ is the weighted sum of two terms as shown in Eq. (3), where *w* is a weight value between 0 and 1. $\bar{s}(\boldsymbol{x})$ is the scaled surrogate value given by Eq. (4), where s_{max} and s_{min} are the maximum and minimum respectively surrogate values of the sample points. Furthermore, $\bar{d}(\boldsymbol{x})$ represents the scaled distance value given by Eq. (5), where $d(\boldsymbol{x})$ is the minimum distance of the sample point $\boldsymbol{x}$ from any evaluated point, d_{max} is the maximum of all distances between the sample points and the evaluation points and d_{min} is the minimum of all distances between the sample points and the evaluation points. Clearly, as *w* increases the search method focusses on the surrogate values leading the search to minimize the surrogate. On the other hand, as *w* decreases the search places more emphasis to points that are distant from the evaluated points driving the search to new regions.

$$f_{mer}(\boldsymbol{x}) = w \cdot \bar{s}(\boldsymbol{x}) + (1-w) \cdot \bar{d}(\boldsymbol{x}) \quad (3)$$

$$\bar{s}(\boldsymbol{x}) = \frac{s(\boldsymbol{x}) - s_{min}}{s_{max} - s_{min}} \quad (4)$$

$$\bar{d}(\boldsymbol{x}) = \frac{d_{max} - d(\boldsymbol{x})}{d_{max} - d_{min}} \quad (5)$$

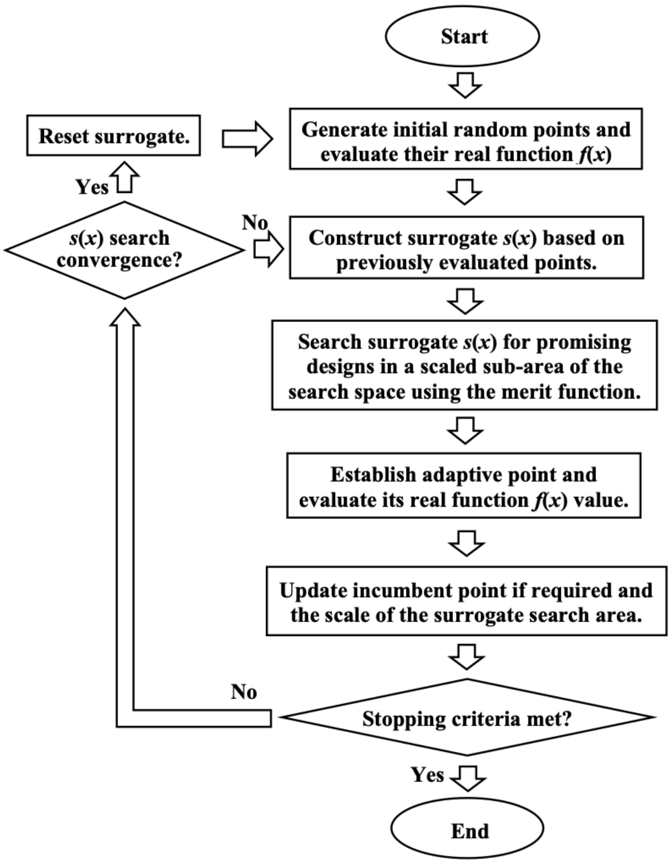

Fig. 2: SBO flowchart

The algorithm evaluates the merit function for all sample points and selects the point with the lowest value that is the adaptive point. Then, it evaluates the real objective function at the adaptive point. If the $f(\boldsymbol{x})$ value at the adaptive point is smaller than the incumbent, then the search is deemed as successful and the incumbent point is updated. If the latter is not the case then the search is deemed as unsuccessful. If a number of consecutive successful searches take place then the algorithm increases the scale of the search area to accelerate the exploration of the search space. On the other hand, if a number of unsuccessful searches occur then the algorithm decreases the scale of the search area to exploit better the examined location. Following this approach, the algorithm eventually converges to an incumbent with near optimal $f(\boldsymbol{x})$ value. When the search area becomes sufficiently small and all sample points are tightly clustered around the incumbent then convergence is assumed and the algorithm resets the surrogate which means that it returns to the stages of generating new random initial points and reconstructing the surrogate (MathWorks 2020b).

The analysis terminates when one of the stopping criteria set by the user is met such as the maximum number of the real objective function evaluations. The final solution is the best incumbent point of all surrogate resets. The afore-described procedure of the adopted SBO framework is illustrated in the flowchart of Fig. 2.

4 Case studies

4.1 Four-storey RC frame

In this section, a 3D regular 4-storey concrete building frame is examined with 3 equal spans of 5m in each direction and uniform storey height of 3m (Fig. 3). Concrete class C25/30 and reinforcing steel class B500C are used following the specifications of EC2. Concrete cover to the centroid of the longitudinal steel bars is taken as 50mm. Due to symmetry, one cross-section is used for all interior columns, one section for all corner columns and one section for the rest of perimeter columns. Furthermore, one section is used for all exterior beams and one section for all interior beams of the first 3 storeys. Two more sections are used for the exterior and interior beams respectively of the top storey due to the high dead loads applied at this level, as explained in the following. In total, 7 different cross-sections are used for this frame setting the number of dimensions d in this optimization problem (i.e. d = 7).

The beam and column cross-sections are assumed to have the general form of Fig. 1. For beams, a list of 8 different rectangular cross-sections is considered having a width of 0.30m and heights that increase from 0.30m to 0.65m with a constant step of 0.05m. For columns, a list of 8 possible square cross-sections is considered with heights ranging from 0.30m to 0.65m again with a constant step of 0.05m. Following these considerations, the size of the search space for this optimization problem is 8^7 possible design configurations.

The concrete building is designed to withstand static and wind loads. Slab dead loads are taken as 6kN/m^2 (inclusive of self-weight) for all storeys apart from the top storey where they become 16kN/m^2 because of the existence of a roof garden. Slab live loads are 5kN/m^2 for all storeys except for the top storey, where they are set as 2kN/m^2. The slab loads are transferred to the beams following standard procedures. In addition, a wind uniform lateral pressure, of 1.5kN/m^2 magnitude, is assumed to be acting to the external surface of the building. Concrete and reinforcing steel unit prices are considered to be $f_{co} = 100\ €/m^3$ and $f_{so} = 1\ €/kg$ respectively. The building is designed according to the specifications of EC2.

Figure 4 shows an indicative optimization history exhibited by the adopted SBO framework for the 3D RC frame under examination in terms of material cost versus the number of real function evaluations (i.e. number of structural designs of the RC frame). For this analysis, 50 initial random points were used for the first construction of the surrogate function $\boldsymbol{s}(\boldsymbol{x})$. This is clear in Fig. 4, where the first 50 function evaluations correspond to initial random points indicated by inverted triangles. Next, function evaluations related to adaptive sample points take place that are indicated by small black asterisks in the figure. At the same time, the progression of the incumbent (blue x markers) and the best of all evaluated points (green circle markers) with the number of function evaluations is demonstrated. The best points always coincide with the incumbent points since there is no surrogate reset taking place within the function evaluations shown in the figure. As anticipated, the cost of the best points gradually decreases until the SBO framework reaches the optimum solution to this problem, with a minimum cost of approximately 12,477 Euros, after 285 function evaluations.

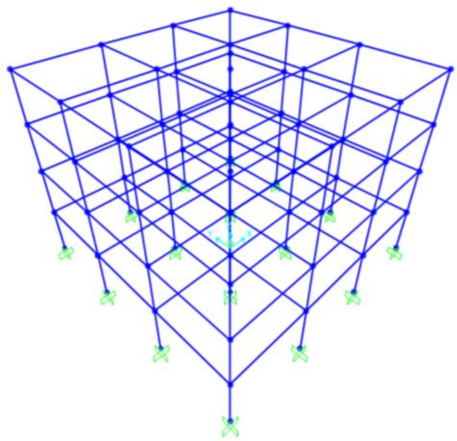

Fig. 3: 3D view of the 4-storey concrete frame

Table 1 presents the cross-sections and costs of the design solutions obtained at the start, at 100 function evaluations and at the end of the optimization history of Fig. 4. It can be seen that the 1st feasible solution uses larger beam sections and smaller interior and perimeter column sections than the final optimal solution. The best solution after 100 evaluations uses smaller sections for the interior beams of the first three storeys, the exterior beams of the top storey and the interior columns than the final solution. On the other side, it employs larger sections for the interior beams of the top storey. All other sections are the same as the final solution. It can be concluded from the previous comparisons that the identification of the optimal design solution for this concrete frame is not a straightforward task as it affected by the complex interaction of concrete members in structural analysis and the subsequent calculation of the steel reinforcement that contributes to the frame cost.

Furthermore, Fig. 5 demonstrates the exterior and interior frames of the final optimum solution of the concrete building with the corresponding cross-sections drawn to scale. It can be concluded that the interior frames require larger sections than the exterior and that the beam sections of the top storey are larger than the lower storeys due to the existence of the additional dead load at the roof of the building. The same figure presents the calculated flexural and shear steel reinforcement areas of the exterior and interior frames of the optimum solution of the concrete building as calculated by SAP2000. It is emphasised herein that it is not implied that this optimal solution is the most efficient structural solution for the concrete building under investigation. It is simply the best solution following the specifications of the optimization problem described at the beginning of this section.

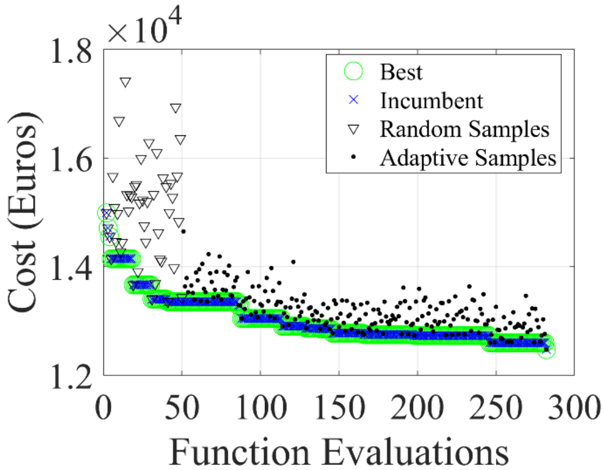

Fig. 4: Optimization history of the SBO framework with 50 initial random points

Table 1: Design solutions cross-sections and costs

Members group	*Cross sections (m)*		
	1st feasible solution	Best solution after 100 iterations	Final optimal solution
Exterior beams – storeys 1 - 3	0.40 X 0.30	0.30 X 0.30	0.30 X 0.30
Interior beams – storeys 1 - 3	0.45 X 0.30	0.35 X 0.30	0.40 X 0.30
Exterior beams – storey 4	0.40 X 0.30	0.30 X 0.30	0.35 X 0.30
Interior beams – storey 4	0.60 X 0.30	0.60 X 0.30	0.45 X 0.30
Interior columns	0.40 X 0.40	0.35 X 0.35	0.45 X 0.45
Perimeter columns	0.30 X 0.30	0.35 X 0.35	0.35 X 0.35
Corner columns	0.35 X 0.35	0.30 X 0.30	0.30 X 0.30
Frame Cost (Euros)	**14,888**	**13,139**	**12,477**

Figure 6 compares the computational performance of the SBO framework with well-established metaheuristic optimization algorithms such as the GA, PSO, SA and FPA optimization algorithms. For each algorithm, 5 independent runs are conducted to account for their stochastic formulation. For the SBO algorithm, 50 initial random points are employed based on the findings of a preliminary analysis. For the GA, PSO and SA algorithms, the parameters values used are the ones recommended in MATLAB R2020b – Global Optimization Toolbox (MathWorks 2020b) that maximize their overall computational performance. For the FPA algorithm, which is not included in the Global Optimization Toolbox, a population size of n = 25 and a switch probability value of p = 0.5 is assumed as these values provide in general good performance for this algorithm (Yang 2008, Mergos

2021). It is noted herein that the parameter values of all previous metaheuristic algorithms have not been specifically tuned for the optimization problems of this study. Parameter tuning tailored to these optimization problems could further improve the computational performance of the optimization algorithms. For all algorithms, 3500 (= 500·d) maximum objective function evaluations are set as a stopping criterion for each run. This limit is deemed as a reasonable computational cost for practical applications of the present optimization framework.

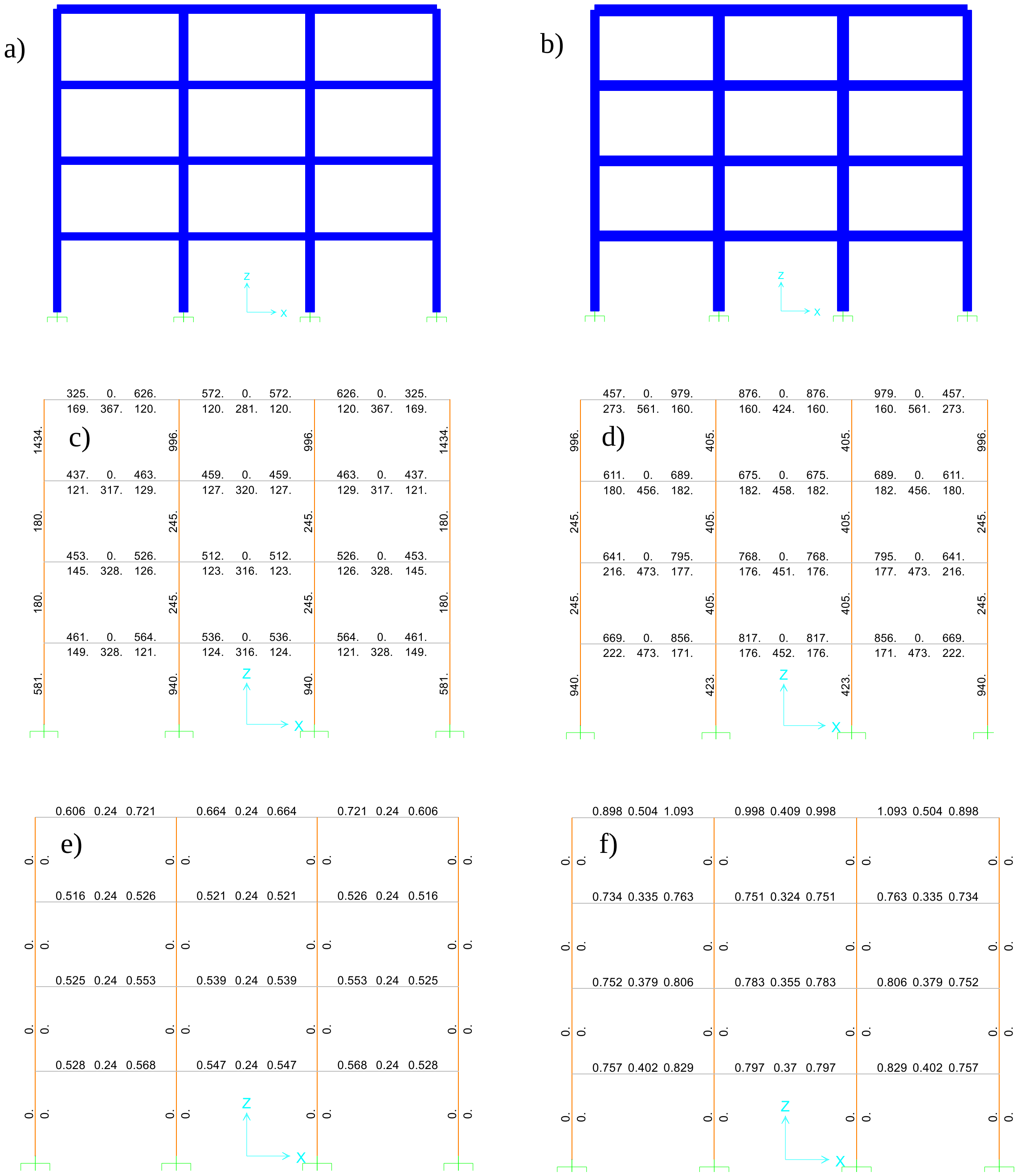

Fig. 5: Optimal design solution a) exterior frame with cross-sections drawn to scale; b) interior frames with cross-sections drawn to scale; c) flexural reinforcement (mm^2) - exterior frames; b) flexural reinforcement – interior frames (mm^2); c) shear reinforcement – exterior frames (mm^2/mm); d) shear reinforcement – interior frames (mm^2/mm)

Figure 6a compares the mean optimization histories of the 5 independent runs of the different algorithms. It is interesting to note that all mean optimization histories converge to almost the same minimum cost after approximately 600-800 function evaluations. It is seen in this figure that the SBO and SA algorithms converge on average significantly faster than then other algorithms in the first 100 function evaluations. At approximately 200 evaluations, all algorithms seem to offer similar performance apart from the FPA algorithm, which converges more slowly. After 200 function evaluations, the SBO algorithm exhibits best average performance converging after approximately 400 function evaluations, which is considerably faster than all other algorithms converging after more than 600 evaluations. It is also worth noting that all algorithms' independent runs, within the 3500 function evaluations limit, converge to the same optimum design solution, shown in Table 1, apart from one run of the PSO algorithm. Figure 6b presents, in the form of box plots, the numbers of function evaluations at which convergence to the optimum solution was achieved by all independent runs of the different algorithms. The box plots show the minimum, maximum and median (red line) function evaluations. Inside the boxes, the 25th to 75th percentiles are contained. It is clear that that the SBO algorithm convergences faster than all algorithms and it does so more robustly with smaller variations in the numbers of function evaluations at convergence.

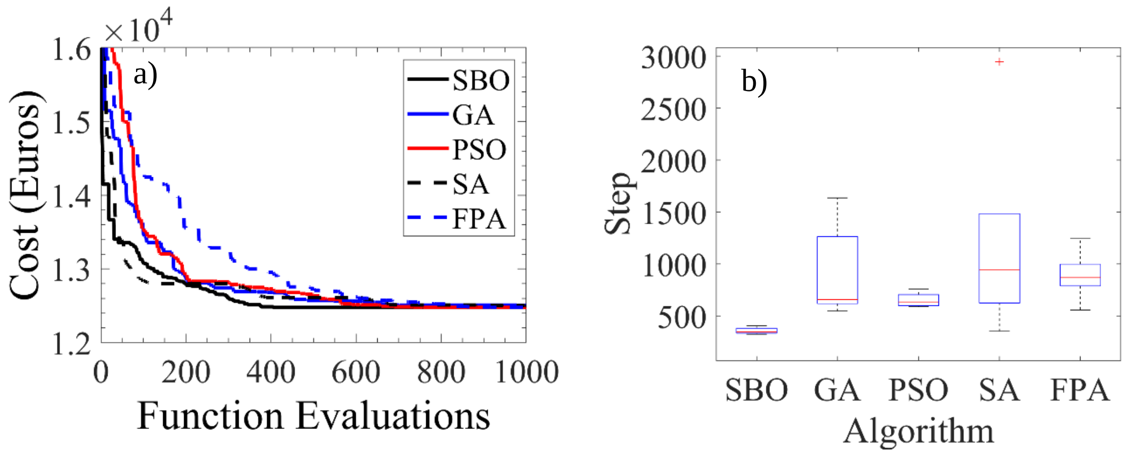

Fig. 6: Comparison of the SBO framework with other optimization algorithms: a) mean optimization histories; b) number of function evaluations at convergence.

4.2 Twelve-storey RC frame

In the present section, a 3D regular 12-storey concrete building frame is examined with 3 equal spans of 5m in each direction and uniform storey height of 3m (Fig. 7). Concrete class C25/30 and reinforcing steel class B500C are used following the specifications of EC2. Concrete cover to the centroid of the longitudinal steel bars is taken as 50mm. Due to symmetry, one cross-section is used for all interior columns, one section for all corner columns and one section for the rest of perimeter columns. Furthermore, for simplicity, one cross-section is assumed for all exterior beams and one cross-section for all interior beams of every two consecutive storeys. Totally, 15 different cross-sections are used (i.e. d = 15) for this frame constituting the design variables of this optimization problem.

For beams, a list of 10 different rectangular cross-sections is considered having a width of 0.30m and heights that increase from 0.30m to 1.2m with a constant step of 0.10m. For columns, a list of 10 possible square cross-sections is considered with heights ranging from 0.30m to 1.20m again with a constant step of 0.10m. It is clarified herein that it is it not implied that square columns are more structurally efficient than rectangular columns for the present case study. Square columns are used as they are assumed to serve better architectural considerations of the building. Following these considerations, the size of the search space for this optimization problem is 10^{15} potential design solutions.

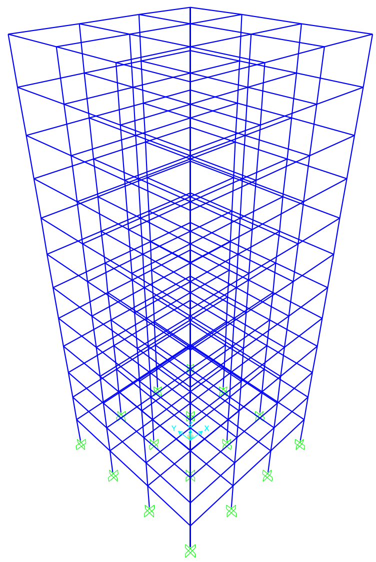

Fig. 7: 3D view of the 12-storey concrete frame

The concrete building is designed to withstand static and seismic loads. Slab dead loads are taken as 6kN/m^2 (inclusive of self-weight) for all storeys apart from the top storey where they become 16kN/m^2 because of the existence of a roof garden. Slab live loads are 2kN/m^2 for all storeys. Fos static loads, the building is designed in accordance with EC2. Moreover, the concrete building is designed against earthquake loads following EC8 for the low ductility class (DCL). The seismic action is applied via the Type 1 response spectrum of EC8 assuming type D soil conditions. The building is assumed to be of importance class II. The design peak ground acceleration (PGA) is 0.36g as recommended for seismic zone III in Greece. The behaviour factor is taken as 1.5 in accordance with DCL requirements in EC8. To satisfy the damage limitation (DL) prescriptions of EC8, it is specified that inter-storey drifts should remain below 0.75% for the frequent earthquake, assuming ductile non-structural elements attached to the frame. Moreover, to prevent large lateral displacements under the design seismic action, the roof displacement of the frame is constrained to 1% of the total height.

Figure 8 presents a sample optimization history exhibited by the SBO algorithm for the RC frame in terms of material cost versus number of function evaluations. In this history, 20 initial random points were used for the construction of the surrogate. Again, initial points are represented by inverted triangles, adaptive points by asterisks, incumbents by blue x markers and best points by green circles. As in the previous example, the analysis starts with initial points followed by adaptive points and the best points match with incumbent points. However, in this design example, the search of surrogate, for promising new designs, phase of the algorithm converges multiple times before the stopping criterion is met leading to an equal number of surrogate resets as highlighted by the vertical blue lines in the figure. For each surrogate reset, new initial points are calculated followed by corresponding adaptive points. Interestingly, in the subsequent surrogate resets, the incumbent points do not always coincide with the best achieved points of the algorithm. This is because incumbent points represent the best points achieved from the last surrogate reset and not from the start of the analysis. In this optimization history, surrogate resets seem to fail updating the best solution after approximately 3200 function evaluations.

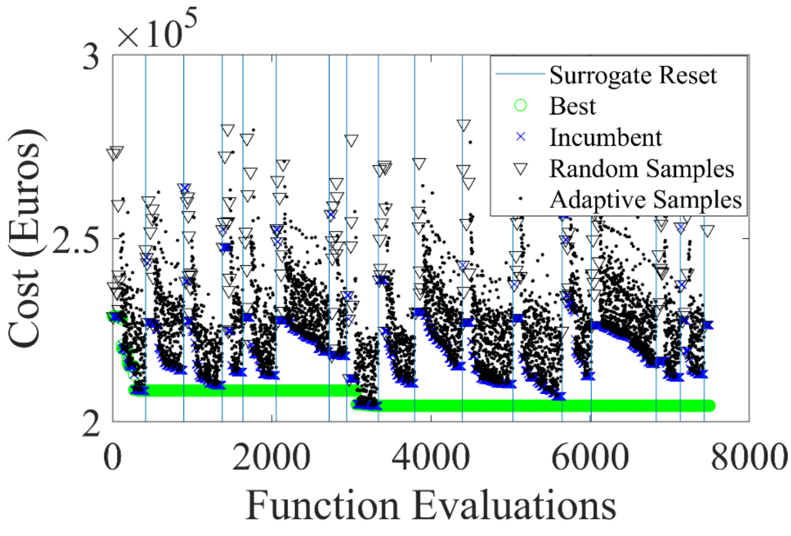

Fig. 8: Optimization history of the SBO framework with 20 initial random points

Figure 9 compares the performance of the SBO framework with the GA, PSO, SA and FPA optimization algorithms. For all algorithms, 5 independent runs with 7500 (= 500·*d*) maximum function evaluations are conducted to account for the random procedures of these algorithms. This maximum number of function evaluations is used herein as a termination criterion to account for computational budget limitations when the optimization framework is applied in standard engineering practice.

For the SBO framework, 20 random points are used for the initial surrogate construction based on the results of a preliminary analysis. For the FPA, a population size of n = 25 flowers and a switch probability of p = 0.5 are assumed in this comparison (Mergos 2021). For all other algorithms, the recommended parameter values in MATLAB R2020b – Global Optimization Toolbox (MathWorks 2020b) are used that maximize their performance. Figures 9a and 9b compare the mean optimization histories of these algorithms after the 5 independent runs. The latter figure is only a zoom of the former figure in the first 1000 function evaluations. It can be seen that the SBO outperforms significantly all other algorithms in the first approximately 300 function evaluations showing initially a high exploitation capability. This can be considered as a clear advantage of this algorithm when the computational budget for the optimization analysis is limited to a very low number of function evaluations. After the first 300 evaluations, however, the SBO framework gets stuck in local optima exhibiting rather poor performance and gradually it is outperformed by all other optimization algorithms. Figure 9c presents, in the form of box plots, the final costs obtained by the various optimization runs after 7500 function evaluations. It is verified that the SBO demonstrates the worst median performance of all algorithms. The best performance is obtained by the FPA algorithm with a final cost of 192,694.3 Euros. This is due to the high degree of diversification and exploration capacity of the FPA algorithm, which is able to track global optimum solutions in complex and large-scale problems (Mergos 2021, Mergos and Yang 2021, Mergos and Yang 2022).

The cross-sectional dimensions of the best design solution are presented in Table 2. For comparison purposes, the cross-sectional dimensions of the 1st feasible solution and the best design solution after 3750 (= 50% of total) function evaluations and corresponding frame costs are also presented in this table. As anticipated, the beam section sizes of the final optimal solution are larger at the lower stories than the upper stories. This is justified by the higher seismic actions at the lower stories. Moreover, the interior beams at the upper floors are larger than the exterior beams at the same floors as they attract higher static loads. It is also noted that the interior columns are larger than the perimeter columns and that the perimeters columns are larger than then corner columns. The previous anticipated trends are not consistently met in the earlier design solutions. For example, the exterior beams of the 5th and 6th storey in the first feasible design solution are larger than the interior beams of the same floors and all the beams of the lowest four storeys. Similarly, the beams of the 9th and 10th storey of the best solution after 3750 evaluations are larger than all other beams in the frame. Again, it is emphasised that it is not implied herein that this optimal solution is the most efficient structural solution for the concrete building under investigation. It is simply the best solution following the specifications of the optimization problem described at the beginning of this section.

Table 2: Design solutions cross-sectional dimensions (in m) and costs

	Beam Groups					
	1st feasible solution		**Best solution after 3750 iterations**		**Final Optimal Solution**	
Storeys	**Exterior Beams**	**Interior Beams**	**Exterior Beams**	**Interior Beams**	**Exterior Beams**	**Interior Beams**
1 - 2	0.4 X 0.3	1.1 X 0.3	0.7 X 0.3	0.8 X 0.3	1.2 X 0.3	1.2 X 0.3
3 – 4	0.9 X 0.3	0.9 X 0.3	0.8 X 0.3	0.3 X 0.3	1.2 X 0.3	1.2 X 0.3
5 - 6	1.1 X 0.3	0.4 X 0.3	0.3 X 0.3	0.8 X 0.3	0.3 X 0.3	1.0 X 0.3
7 - 8	0.4 X 0.3	0.4 X 0.3	0.7 X 0.3	0.3 X 0.3	0.3 X 0.3	0.7 X 0.3
9 - 10	0.6 X 0.3	0.4 X 0.3	0.8 X 0.3	0.9 X 0.3	0.3 X 0.3	0.7 X 0.3
11 - 12	0.9 X 0.3	0.4 X 0.3	0.4 X 0.3	0.6 X 0.3	0.3 X 0.3	0.4 X 0.3
	Column Groups					
	1st feasible solution		**Best solution after 3750 iterations**		**Final Optimal Solution**	
Interior columns	0.9 X 0.9		1.1 X 1.1		0.9 X 0.9	
Perimeter columns	1.1 X 1.1		0.7 X 0.7		0.7 X 0.7	
Corner columns	0.9 X 0.9		0.8 X 0.8		0.5 X 0.5	
Frame Costs (Euros)	**228,700**		**205,910**		**192,694**	

Furthermore, Fig. 10 shows the lateral deflections of the exterior and interior frames of the obtained optimum solution of the RC building when subjected to the design earthquake where the corresponding cross-sections are drawn to scale. It is interesting to note in Fig. 10 that the top lateral displacement under the design earthquake is slightly lower than the 1% limit of the building height satisfying marginally the respective constraint of the optimization problem. Based on the previous discussion, it is further investigated in this study a novel hybrid approach combining the SBO and FPA algorithms. The goal is to examine whether the proposed hybrid approach combines the benefits of these algorithms (i.e. exploitation capacity of the SBO algorithm and exploration capability of the FPA algorithm). Two potential combinations of these algorithms are considered. In the first combination, termed SBO-FPA, the analysis starts with the SBO algorithm followed by FPA algorithm and in the second combination, termed as FPA-SBO, the analysis starts with the FPA algorithm followed by the SBO algorithm. In both cases, 1000 function evaluations are allocated to the SBO algorithm and 6500 evaluations to the FPA algorithm leading to a total of 7500 total evaluations as the case with the single algorithms. In the hybrid approach, the best solution obtained by the first algorithm serves as a starting point of the second algorithm ensuring the continuity of the solution procedure.

Figure 9d compares the mean optimization histories obtained by the single and hybrid algorithms after 5 independent runs with 7500 function evaluations in total. As expected, the SBO and SBO-FPA algorithms exhibit similar performance in the first 1000 evaluations, which outperforms the other two algorithms. After 1000 evaluations, the SBO-FPA algorithms performs better than the SBO algorithm taking advantage of the exploration capabilities of the FPA algorithm. Nevertheless, the final solutions obtained after 7500 evaluations are on average significantly worse than the original FPA algorithm. This practically means that there is no benefit of using the SBO-FPA hybrid algorithm since it seems to be outperformed by the single algorithms (i.e. either SBO or FPA) in the full range of the response.

The FPA-SBO algorithm exhibits initially a very similar performance to the FPA algorithm but after the first 2000 evaluations it seems to outperform the FPA algorithm. This is just due to the random procedures of the FPA algorithm since both solutions use the FPA in the first 6500 evaluations. It is therefore more meaningful to compare the mean performance of these algorithms after the 6500 function evaluations. This comparison is given in Fig. 9e. It can be seen in this figure that both the FPA and the SBO algorithms improve considerably the previously obtained solutions after the 6500 evaluations. However, the SBO does so significantly earlier than the single FPA (i.e. approximately 250 function evaluations in this example as opposed to 1000 evaluations of the FPA algorithm after the first 6500 function evaluations). This is an interesting conclusion since it means that the SBO has the potential to improve FPA’s performance even in the later stages of the response and with a very limited amount of additional function evaluations. Furthermore, Fig. 9f shows the final costs obtained at the end of the analysis for the single and hybrid algorithms. It can be seen in this figure that the best solution obtained by the FPA-SBO algorithm out of the 5 independent runs is the same as the single FPA algorithm presented in Table 2. Even more, the FPA-SBO algorithm performs on average better and more robustly than the single FPA algorithm. This means that the recommended hybrid algorithm is a more reliable alternative to the single FPA algorithm.

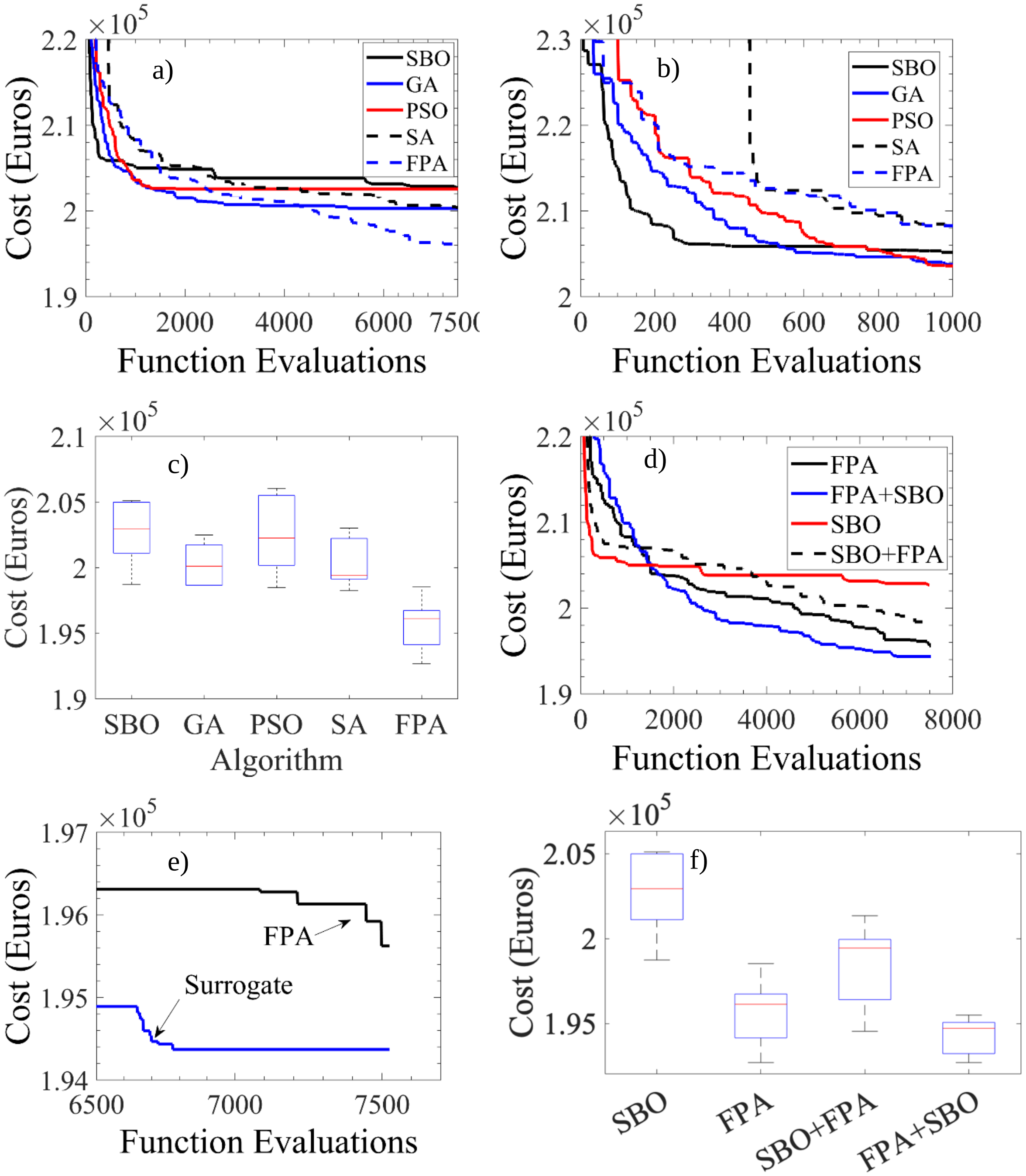

Fig. 9: Comparisons of algorithm performances: a) mean histories; b) mean histories in the first 1000 evaluations; c) final costs after 7500 evaluations; d) mean histories of hybrid algorithms; e) FPA and surrogate mean response after 6500 FPA evaluations; f) final costs of hybrid algorithms after 7500 evaluations

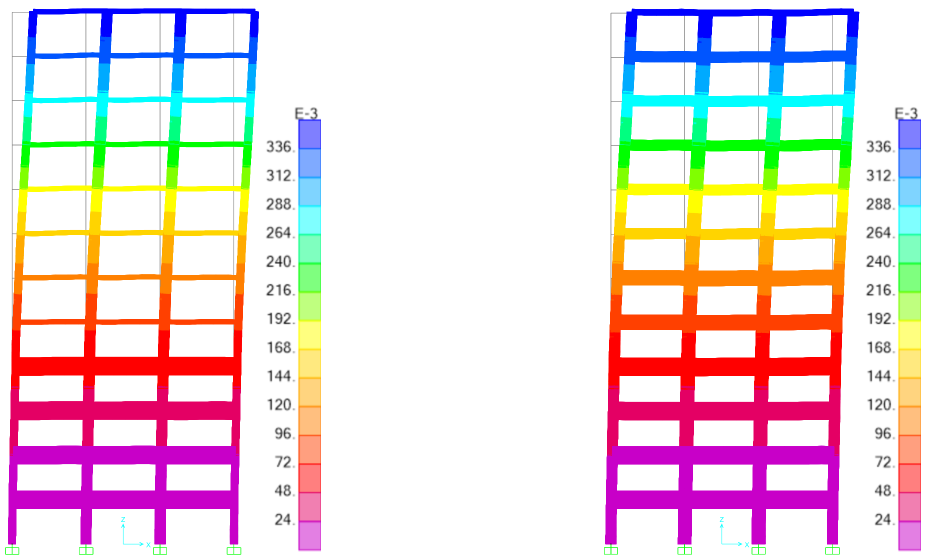

Fig. 10: Lateral deflections response of the optimal design solution with cross-sections drawn to scale: a) exterior frames; b) interior frames (note: displacements in m)

5 Conclusions

Reinforced concrete frame buildings are associated with high economic and environmental costs on a global scale. Therefore, the optimum design of these structural systems is an imperative need. Nevertheless, the structural design of real-world concrete building frames to modern design guidelines is highly complex and accompanied by significant computational costs undermining the application of optimization methodologies in everyday practice. Therefore, the applicability and efficiency of surrogate-based optimization (SBO) approaches in this optimization problem is investigated.

To support the purposes of the current research, a versatile computational platform, namely STROLAB (i.e. Structural Optimization Laboratory), is applied. The platform applies, for first time, a fully-fledged SBO algorithm to the optimum design of 3D concrete building frames. In particular, the SBO algorithm is applied in the design of a 4-storey and 12-storey 3D building RC frames for minimum cost and according to Eurocodes. The performance of the SBO algorithm is then compared with several metaheuristic algorithms including SA, GA, PSO and FPA. Useful conclusions are made with respect to the solution efficiency of the SBO framework in the optimum structural design of concrete frames.

It is found that for the smaller-scale concrete building the SBO algorithm drives to the same optimum design solution as the other algorithms and in a smaller number of function evaluations leading to significant savings in the required computational effort. For the larger-scale concrete building, it is observed that the SBO outperforms the other algorithms for very small numbers of function evaluations showing high early exploitation capability. Therefore, it represents the best choice for this building when the computational budget is very limited. Nevertheless, as the number of function evaluations increases, the SBO seems to get trapped in local optima. As a result, it is outperformed by other optimization algorithms, with larger exploration capacity, such as the FPA algorithm.

To combine the high exploration ability of the FPA algorithm and the exploitation capacity of the SBO algorithm, a novel hybrid approach, termed FPA-SBO, is also proposed in this study where the efficient global search of the FPA algorithm is followed by an intensive local search of the SBO algorithm. It is found that the SBO algorithm, when applied after the FPA algorithm, considerably improves the outcomes of the FPA search within a very limited number of additional function evaluations and that the FPA-SBO hybrid approach offers improved quality and more robust computational performance.

At this point, it is important to clarify that a code-based approach is followed in the present study that is consistent with standard engineering practice. A performance-based design approach is more appropriate to control structural performance of concrete frames. Therefore, the use of SBO algorithms in the optimum performance-based design of concrete frames needs to be examined. Furthermore, additional design objectives such as the life-cycle cost and/or environmental impact as well as structural robustness should be further investigated.

Closing this study, the need for adopting efficient SBO procedures in the optimum design of real-world reinforced concrete structures is highlighted that can reduce drastically the computational cost and promote optimization efforts in standard engineering practice. Hence, further research is required to explore the applicability and efficiency of different existing SBO methodologies in the optimum design of concrete structures as well as to develop new SBO techniques that are specifically tailored to this optimization problem.

Conflict of interest

On behalf of all authors, the corresponding author states that there is no conflict of interest.

Replication of results

The SBO, GA, SA, PSO algorithms used in this study are parts of the MATLAB Global Optimization Toolbox (MathWorks 2020b). The Flower Pollination Algorithm (FPA) code is readily available in the MATLAB file exchange system. It is also noted that all the algorithms used in this study are based on stochastic processes. Hence, exact replication of the results presented herein is not possible.

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