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Essays on Vertical Mergers, Advertising, and Competitive Entry

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Essays on Vertical Mergers, Advertising, and Competitive Entry

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TO MY FAMILY: Huseyin Ayar, Nuriye Ayar, Elif Ozer Ayar, Huseyin Ozer, Ayse Ozer, Ayse Ayar, Emine Ayar, Ismail Ayar, Fatma Ayar Oztekin, Ibrahim Ayar, Ihsan Ayar, Huriye Ayar Intepe, Sengul Ayar Imamoglu, Sevde Ozer, and Talha Ozer.

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Essays on Vertical Mergers, Advertising, and Competitive Entry

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This dissertation consists of three independent essays. We briefly introduce these essays in chapter 1 and leave a comprehensive introduction to each essay. Chapter 2 considers a vertically separated industry where production takes time and vertical mergers shorten production time. We investigate the impact of vertical mergers on the downstream firms' ability to collude and show that vertical mergers facilitate downstream collusion. Chapter 3 provides a theoretical foundation for a puzzling empirical observation that advertising follows an inverted U shape for some new products. Chapter 4 analyzes an incumbent's response to a competitive entry. We show that if the quality of the entrant is uncertain, the incumbent can "jam" the quality signalling of the entrant. Finally, chapter 5 summarizes main conclusions of three essays.

Contents

Chapter 1

Introduction

Chapter 2 analyzes the effect of a vertical merger on downstream collusion when vertically integrated firms have shorter production lags. Vertical integration is pro-competitive and not profitable if firms behave competitively. However, we show that vertical integration facilitates collusion, in which case it is profitable.

In chapter 3, a monopolist introduces a new product of either low or high quality. It advertises to make consumers aware of the product and signals product quality using both price and advertising. When consumption does not reveal product quality, price is higher and advertising is lower than they would be if product quality is observable. Price rises and advertising falls as the fraction of aware consumers increases. When consumption reveals product quality, price is higher and advertising is lower than they would be if product quality is observable. Price declines as the fraction of aware consumers increases and advertising follows an inverted U shape. We find support for these empirical predictions from a data set on Direct-to-Consumer advertising on pharmaceutical drugs.

Chapter 4 considers an industry in which a monopolist incumbent with known quality faces a competitive entry. When the quality of the entrant is certain, we show that the income distribution of consumers and the quality difference between the entrant and the incumbent determine whether the entrant can secure a positive market share or not. When the quality of the entrant is uncertain, we show that the incumbent and the entrant have opposing strategic incentives. While a high quality entrant wants to signal its quality to consumers, the incumbent wants to prevent this signalling attempt. Under certain parametric values, we show that by increasing its own price, the incumbent not only prevents the entrant from quality signaling, but also increases its own profit by doing so.

Chapter 2

Do Vertical Mergers Facilitate Downstream Collusion?

2.1 Introduction

The growing number of cases of collusion involves downstream industries in which intermediate goods are used as an input in production.[1] In most of these industries, since production of intermediate goods is time-consuming, advanced order is often necessary. Vertical integrations are observed in a significant portion of those collusive industries.[2] The purpose of this paper is to analyze the impact of a vertical merger in a vertically separated industry in which input production takes time, and hence has to be ordered in advance. More specifically, we focus on the effect of a vertical merger on collusion among

[1] See Naughton (2004) describes some alleged abuse of monopsony power in intermediate good markets such as cattle (*Pickett v. Tyson Fresh Meats, inc.*), tobacco (*Deloach v.Philip Morris*), and timber (*Washington Alder LLC v. Weyerhaeuser Co.*). Similarly, Tosdal (1917) describes German steel cartel and Hendricks, Porter, and Tan (2000) analyze the joint bidding for oil and gas tracks.

[2] As an example, Tyson, ConAgra Beef Companies, Cargill, Smithfield,and Farmland National Beef Corporation, control 70 percent of beef packing industry and they also participate in livestock production through vertical integrations. The cartel in German Steel Industry before WWI and the bromine cartel in the US Levinstein (1997) also feature vertical mergers.

downstream firms. We raise the relevant question of whether or not a vertical merger facilitates downstream collusion.

Vertical mergers in particular and vertical restraints in general were considered as most likely to be pro-competitive or neutral by Chicago School and antitrust authorities during the 1970's and 1980's, Robert H. Bork (1978) due to their potential efficiency enhancing effects such as eliminating the double-markup problem and improving supply chain management. Recently, however, academics and anti-trust authorities have challenged this conclusion on the ground that vertical mergers could be anti-competitive by raising rival's costs and facilitating collusion.

Many intermediate goods industries experience an order-to-delivery lag, because order management, production, and delivery take time. For example, in the apparel industry the order-to-deliver lag averages 9 months while it averages only 42 days in the poultry meat industry. Vertical mergers can shorten this lag dramatically by improving coordination and management between upstream and downstream firms. For example, Zara, a famous Spanish apparel maker, is vertically integrated and its production and distribution needs only two weeks to get a product to its stores, rather than the nine-month industry average.[3]

We consider an industry in which, in each period, upstream firms produce a perfectly homogeneous intermediate good at a constant uniform marginal cost and announce a unit price. Then downstream firms place their quantity orders, which are observable and take time to produce and deliver. At the time of delivery, downstream firms make their payments and transform it into a perfectly homogenous final good and then compete in quantities to supply to the downstream market. Both upstream and downstream firms repeat this interaction forever. We focus on the stationary collusive equilibrium in which the

[3]See Richardson's (1996) description of vertical mergers and rapid response in apparel industry.

downstream firms form an all-inclusive cartel, maximizing joint profits, and using reversion to the static Nash equilibrium forever, Friedman (1971) to punish any deviation from the monopoly outcome. The cartel's ability to collude is measured by the minimum discount factor above which this monopoly outcome is sustainable for each cartel member. We say vertical merger facilitates collusion if it reduces this critical discount factor.

We first consider a vertically separated industry where upstream and downstream firms are separated. First notice that advanced quantity orders of intermediate goods need to be done before the downstream market opens, and there is no way downstream firms can reverse their orders and input cost is sunk. Hence quantity orders carry a commitment value for downstream firms. Moreover, the amount that a downstream firm orders in a period determines the maximum amount that it can supply to the market in that period. As a result, in the case of a downstream collusion, the only way to deviate for a disloyal cartel member is to order more than its assigned monopoly share at the order stage. This deviation attempt is observable, but cannot be responded to in that period.

We now consider the impact of a vertical merger between one of the downstream firms and an upstream firm on collusion. We assume that a vertically integrated firm can process its own orders more rapidly. In other words, the order-to-delivery lag for within-firm orders is shorter than the order-to-delivery lag between firms. Also, realize that intra-firm orders have no commitment value because the integrated entity can reverse the cost of input orders through better coordination with its upstream affiliate: hence, input cost is not sunk. Resulting from the lack of pre-commitment, the unintegrated firm makes its quantity commitment before the integrated firm; hence, in the punishment phase the unintegrated firm, the '(Stackelberg) leader', can exercise a preemptive advantage over the integrated firm, the '(Stackelberg) follower'. In other words, in vertically related industries with order-to-delivery lags, a vertical merger is not profitable if firms behave competitively.

The asymmetry between intra-firm and inter-firm orders has two effects on the ability of downstream firms to collude. A vertically merged firm can punish any deviation by a downstream firm within the same period, thereby eliminating any incentive for that firm to deviate. We call this effect of a vertical merger the *quick response effect*. The possibility for collusion then depends upon the vertically integrated firm's incentive to deviate. Its one period gain from deviating is the same as it was when the firm is not vertically integrated. However, the downstream firm's ability to punish a deviation by the vertically integrated firm is greater than it is when the firm is not vertically integrated. The vertically integrated firm's payoff during the punishment phase is lower than it is when the firm is not vertically integrated. We call this effect of a vertical merger the *lack of quantity precommitment effect*. Both effects work towards making collusion more likely; hence we conclude that in vertically related industries with order-to-delivery lags, a vertical merger facilitates collusion, which is profitable.

To our knowledge, the closest related article in the literature is Nocke and White (2005) in which they focus on the impact of vertical merger on upstream collusion in vertically related industries. In our model, downstream firms first order their input, then transform it and compete to sell to consumers. Hence, the input orders are sunk and carry a commitment power. In other words, downstream firms can use input orders as a strategic decision variable to improve their payoff in competition. In their setup, downstream firms first compete to sell the final product then start ordering inputs to produce it. Hence, downstream firms' orders have no commitment power. In fact, Nocke and White's and our papers focus on different vertically related industries.

The rest of the article is organized as follows. In section 2, we outline the baseline model. In section 3, we analyze the equilibrium and the impact of vertical merger on downstream collusion with only two retailers. Section 4 provides an extension of the model where we investigate the impact of a vertical merger on collusion with a more competitive downstream industry. Section 5 concludes the article.

2.2 Model

There are $n \geq 2$ upstream firms producing a perfectly homogeneous intermediate good at a constant uniform marginal cost, c. There are $m \geq 2$ buyers in the intermediate good market who compete against each other in a downstream market. Each of the downstream firms uses the same technology for transforming one unit of intermediate good into one unit of homogenous final output and does so at a constant per unit cost. For simplicity, production cost of downstream firms is normalized to zero. Time is discrete and there is an infinite number of periods. The downstream inverse demand in each period is given by

$$P = a - \sum_{j=1}^{m} q_j$$

where P is the final good market price and q_j $(j = 1, ..., m)$ is the amount that downstream j supplies to the market.

The timing of the game in each period is as follows:

1. Contract offer stage:

Upstream firms simultaneously post wholesale prices.

2. Order stage:

Downstream firms simultaneously place their orders with upstream firms.

3. Competition stage:

Downstream firms receive their orders at the end of the period, make their payments and decide how much quantity to supply to the downstream market.

The orders are assumed to be publicly observable and take time to process. Therefore, the amount that a firm orders in a period determines the maximum amount that it can supply to the market in that period. The future payoffs are discounted by the same discount factor δ for all the firms. Each firm maximizes the present value of the sum of the infinite sequence of one-period stage game payoffs.

2.3 Equilibrium Analysis: Two Downstream Firms

In this section, we analyze the impact of a vertical merger on downstream collusion when there are only two downstream firms and upstream firms behave competitively. We extend the analysis to m firms in a later section. The solution concept is subgame perfect. We focus on the stationary collusive equilibrium in which the downstream firms form an all-inclusive cartel, maximize joint profits, and use reversion to the static Nash equilibrium forever, Friedman (1971) to punish any deviation from the monopoly outcome.[4] Although deviation may be profitable for a period, the continuation profits will be lower. The tradeoff depends on the discount factor. There is a critical discount factor below which each cartel member prefers the short-run gain against the long-term loss, Tirole (1989). The highest critical discount factor among the members of the cartel is defined as the critical discount factor for the cartel.

In order to analyze the impact of a vertical merger on the collusion, we first calculate the critical discount factor in a vertically separated market. This is a market with no vertically integrated firm. Then, we compare this critical discount factor with the critical discount factor when one of the downstream firms merges with an upstream firm. If the latter is smaller than the former, we conclude that the vertical merger facilitates collusion.

[4]Renegotiation and side payments are not possible.

2.3.1 Vertically Separated Industries

Non-Collusive Equilibrium

Since upstream firms are assumed to compete in price and downstream firms only buy from the cheapest supplier, in any subgame perfect equilibrium, upstream firms post wholesale prices equal to marginal cost c. This is the standard undercutting logic of Bertrand equilibrium.

Lemma 1. *The static Nash equilibrium is the Cournot equilibrium.*

In the equilibrium each downstream firm purchases $(a-c)/3$ in the order stage and sells this quantity in the downstream market. Each downstream firm makes a per period profit of

$$\Pi^C = \frac{(a-c)^2}{9},$$

where the superscript C stands for Cournot. Repeated play of the static Nash equilibrium is a subgame perfect equilibrium in the repeated game.

Collusive Equilibrium

In the collusive equilibrium, downstream firms set quantities so that total supply in the downstream market is at the monopoly level. To implement the monopoly outcome, downstream firms have to share the monopolistic quantity. Let α_j denote the market share of downstream firm j. Its profits are given by

$$\Pi_j^M = \alpha_j \frac{(a-c)^2}{4},$$

where the superscript M denotes monopoly.

As Scherer (1990) notes, "the very act of fixing the price at a monopolistic level creates incentives for sellers to expand output beyond the quantity that will sustain the agreed-upon price" (p. 244). Profits from a deviation depend on the sharing rule of the collusive agreement. The most profitable deviation for downstream firm j is to purchase more than its assigned monopoly share in intermediate good market. The firm solves the following problem:

$$\max_{q_j^d} \quad q_j^d \left[a - c - [q_j^d + (1-\alpha_j)\frac{(a-c)}{2}] \right].$$

Solving for the optimal order and substituting this quantity into the firm's profits yields

$$\Pi_j^D = \frac{(1+\alpha_j)^2(a-c)^2}{16},$$

where the superscript D refers to deviation. The deviation will trigger the infinite reversion to the Cournot Nash equilibrium. Thus, downstream firm j prefers to collude as long as

$$\frac{\Pi_j^M}{1-\delta} \geq \Pi_j^D + \frac{\delta \Pi_j^C}{1-\delta}.$$

Each downstream firm is willing to collude if its discounted present value of collusive profit is bigger than its deviation profit plus discounted present value of its profit in the punishment phase. The range of discount factors in which downstream firm j has no incentive to deviate can be written as follows

$$\delta \geq \underline{\delta}_j \equiv \frac{\Pi_j^D - \Pi_j^M}{\Pi_j^D - \Pi_j^C} = \frac{9(1+\alpha_j)^2 - 36\alpha_j}{9(1+\alpha_j)^2 - 16}.$$

Here $\underline{\delta}_j$ denotes downstream firm j's critical discount factor.

The cartel is sustainable if and only if neither downstream firm has an incentive to deviate, that is if, and only if,

$$\delta \geq \underline{\delta} \equiv \max\{\underline{\delta}_1, \underline{\delta}_2\}.$$

We are interested in determining the lowest value for the critical discount factor $\underline{\delta}$. Since $\underline{\delta}_j$ is decreasing in α_j, symmetry implies that equal sharing minimizes the critical discount factor for the cartel. Plugging this solution into the above equation yields the following lemma.

Lemma 2. *In a vertically separated market, $\underline{\delta} = \frac{9}{17}$ is the minimum critical discount factor that sustains downstream collusion.*

2.3.2 Vertically Related Industries

We now consider the case in which one of the downstream firms, say firm 1, has merged with an upstream firm. We assume that a vertically integrated firm can process its own orders more rapidly. In other words, the order lag for within-firm orders is shorter than the order lag between firms.

Non-collusive Equilibrium

The vertically integrated firm has two kinds of order strategies. It can order input from another independent upstream firm or it can order from itself. If the vertically integrated firm places an order with an independent upstream firm, then it is committed to that order. Thus, one subgame perfect equilibrium to the stage game consists of both firms ordering the Cournot quantities from upstream firms. Given these orders, the vertically integrated firm has no incentive to order additional quantity from itself. However, there is another subgame perfect equilibrium in which the vertically integrated firm supplies itself. In this equilibrium, the downstream firm (firm 2) gets to commit to a quantity before the vertically integrated firm. The equilibrium in this case is the Stackelberg equilibrium. In what follows, we will focus on the Stackelberg equilibrium. The justification for doing so is that if costs of filling within-firm orders are slightly less than c, the cost of buying from another upstream firm, then the vertically integrated firm's best reply to the other firm's choice of the Cournot quantity is always to order the Cournot quantity from itself rather than from an upstream competitor.

The following lemma characterizes the Stackelberg equilibrium.

Lemma 3. *The downstream firm 2's equilibrium profit is* $\Pi_2^S = (a-c)^2/8$ *and the vertically integrated firm's equilibrium profit is* $\Pi_1^S = (a-c)^2/16$.

The existence of production lags is the main characteristics of this model, which derives this equilibrium. Since production takes time, the unintegrated firm gains the first mover advantage through quantity orders. However, as a second mover, the integrated firm produces its inputs before the production period starts due to production lag. [5]

As before, repeated play of the static Nash equilibrium is a subgame perfect equilibrium to the repeated game.

In the Stackelberg equilibrium, downstream firm 2's profits exceed the profits of the vertically integrated firm. The vertically integrated firms's inability to commit to an order at the same time as its downstream rival means that the latter can afford to order more than the Cournot quantity knowing that the vertically integrated firm's best response is to order less than the Cournot quantity.

Collusive Equilibrium

When can the vertically integrated firm and the downstream firm collude on the monopoly outcome in each period? Suppose the vertically integrated firm's share of the

[5]It can be also shown that if the input production is instantaneous, it is optimal for the integrated firm to produce its input before competition takes place. Indeed, in the equilibrium the integrated firm produces instantaneously at the order stage. That takes away the first mover advantage of unintegrated firm, resulting in the Cournot equilibrium.

monopoly output is α_1 and the downstream firm share is $\alpha_2 = 1 - \alpha_1$. Then vertically integrated firm's profits in each period are

$$\Pi_1^M = \alpha_1(a-c)^2/4$$

and the downstream firm's profits in each period are

$$\Pi_2^M = \alpha_2(a-c)^2/4.$$

The downstream firm cannot respond to any deviation by the vertically integrated firm. Therefore, the optimal deviation for the vertically integrated firm is the same as it was when it was not vertically integrated. Recall that profits from that deviation are

$$\Pi_1^D = (1+\alpha_j)^2(a-c)^2/16.$$

On the other hand, when the downstream firm deviates from its share of the monopoly output, the vertically integrated firm can respond within the same period. Hence, the downstream firm's best deviation is to choose the Stackelberg quantity, which implies that $\Pi_2^D = \Pi_2^S$.

The downstream firm has no incentive to deviate as long as

$$\frac{\Pi_2^M}{1-\delta} > \Pi_2^D + \frac{\delta\Pi_2^S}{1-\delta} \Leftrightarrow \Pi_2^M > \Pi_2^S.$$

This result follows from the fact that $\Pi_2^D = \Pi_2^S$. Thus, the downstream firm has no incentive to deviate if its share of the monopoly profits exceeds its Stackelberg payoff. This result is independent of the discount factor, which implies that $\underline{\delta}_2 = 0$.

The vertically integrated firm has no incentive to deviate as long as

$$\frac{\Pi_1^M}{1-\delta} > \Pi_1^D + \frac{\delta \Pi_1^S}{1-\delta},$$

which does depend upon the discount factor. Thus, the critical discount factor for the cartel is determined by the critical discount factor of the vertically integrated firm. The critical discount factor of the vertically integrated firm reduces to

$$\delta \geq \underline{\delta}_1 \equiv \frac{(1+\alpha_1)^2 - 4\alpha_1}{(1+\alpha_1)^2 - 1}$$

The vertically integrated firm's critical discount factor is decreasing in α_1. Hence, the critical discount factor for the cartel is achieved by setting

$$\alpha_2 = \frac{\Pi_2^S}{\Pi^M}.$$

Substituting share values in the corresponding equation for the vertically integrated firm's critical discount factor leads to the following lemma.

Lemma 4. *The critical discount factor with single vertical merger is* $\underline{\delta}^I = \frac{1}{5}$.

Since the critical discount factor for the cartel in a vertically separated market is $\frac{9}{17}$, we obtain the following result.

Proposition 1. *In a vertically separated market with two downstream firms, a vertical merger facilitates downstream collusion.*

The intuition for the result is as follows. A vertically merged firm can punish any deviation by a downstream firm within the same period, thereby eliminating any incentive for that firm to deviate. We call this effect of a vertical merger the *quick response effect.*

The possibility for collusion then depends upon the vertically integrated firm's incentive to deviate. Its one period gain from deviating is the same as it was when the firm is not vertically integrated. However, the downstream firm's ability to punish a deviation by the vertically integrated firm is greater than it was when the firm is not vertically integrated. The vertically integrated firm's payoff during the punishment phase is $\Pi_1^S < \Pi^C$. We call this effect of a vertical merger the *lack of quantity precommitment effect.* Both effects work towards making collusion more likely.

It is easy to observe that these effects are only relevant in vertically related industries with order-to-delivery lags. If the lag for intra-firm orders is the same as for inter-firm orders, then vertical integration has no impact on the ability of firms to collude in the downstream markets. Moreover, the observability of orders or alternatively credible announcement of orders is crucial for these effects to be relevant. Therefore, the main prediction of the model is that when orders are observable, we would expect to observe more vertical mergers in the industries with order-to-delivery lags compared to the ones without order-to-delivery lags.

An important result of this section is that if the market were competitive, we would not see a vertical merger, because it is not profitable. However, If downstream firms are colluding, we expect to see a vertical merger, because it facilitates collusion, in which case it is profitable. Another interesting point to note is that vertical merger does not lead to counter merger by the unintegrated downstream rival. If downstream firms behave competitively, the unintegrated firm receives '(Stackelberg) leader' profit by staying unintegrated while it receives only Cournot profit by integrating due to the "lack of quantity pre-commitment effect". Moreover, If both downstream firms are integrated, they both produce internally and end up being symmetric Cournot competitors with unobservable orders, which hinders collusion.

We can conclude that a vertical merger could be an evidence that collusion is actually taking place. Therefore, in vertically related industries with observable orders and order-to-delivery lags, whether there is a vertical merger or not could be a test of collusion for antitrust authorities.

2.4 Extension

2.4.1 Equilibrium Analysis: Many Downstream Firms

In this section, we analyze a model with more than two retailers to check whether facilitating collusion property of vertical merger still holds in a more competitive retail industry.

No-Vertical Merger

Most of the analysis is similar to the case with two retailers; hence, we give the results for similar parts without elaborating on details. Let's say there are m downstream firms and $n \geq 2$ input producers . If firms behave competitively, the equilibrium profit for a downstream firm j can be written as follows

$$\Pi_j^C = \frac{(a-c)^2}{(m+1)^2}.$$

To implement the monopoly outcome, downstream firms have to share monopolistic quantity. In the vertically separated industry, all downstream firms are symmetric. We know that the minimum critical discount factor can be achieved by sharing monopolistic quantity equally. Its profits are given by

$$\Pi_j^M = \frac{(a-c)^2}{4m}.$$

The only way of deviation for a downstream firm is to purchase more than its assigned quantity at the order stage. A downstream firm's deviation profit is given by

$$\Pi_j^D = \frac{(m+1)^2(a-c)^2}{16m^2}.$$

Since downstream firms are symmetric, the cartel's critical discount factor can be calculated from a downstream firm's incentive compatibility constraint.

$$\frac{\Pi_j^M}{(1-\delta)} \geq \Pi_j^D + \frac{\delta \Pi_j^C}{(1-\delta)}.$$

The minimum discount factor above which downstream firms can successfully collude can be written as follows

$$\underline{\delta} = \frac{(m+1)^2}{(m+1)^2 + 4m}.$$

Lemma 5. *A decrease in the number of downstream firms facilitates collusion.*

On the contrary, perfectly competitive downstream market makes it impossible to collude, i.e., as $m \to \infty$, $\underline{\delta} \to 1$.

Single Vertical Merger

*Non-collusive Equilibrium:*The following lemma summarizes the profits of integrated and unintegrated retailers

Lemma 6. *The vertically integrated firm's equilibrium profit is firm is $\Pi_1^S = \frac{(a-c)^2}{4m^2}$ and an unintegrated firm's equilibrium profit is $\Pi_2^S = \frac{(a-c)^2}{2m^2}$.*

In fact, this lemma presents an extended version of lemma 3 in which $m = 2$.

We now introduce the following notation. The integrated firm receives α_1 while each downstream firms receives α_2 where $\alpha_1 + (m-1)\alpha_2 = 1$. Now we can specify the integrated firm's profit as $\Pi_1^M = \alpha_1(a-c)^2/4$ and the unintegrated firm's as $\Pi_2^M = \alpha_2(a-c)^2/4$.

Profits from a deviation depends on the sharing rule of the collusive agreement

$$\Pi_1^D = \frac{(1+\alpha_1)^2(a-c)^2}{16} \text{ and } \Pi_2^D = \frac{(2-(m-2)\alpha_2)(a-c)^2}{32}.$$

Proposition 2. *In a vertically separated market with many downstream firms, a vertical merger facilitates downstream collusion.*

The result that a vertical merger in industries with production lag facilities collusion also arises with an oligopolistic downstream market. This gives an additional motive for anti-trust authorities to scrutinize this type of industry.

Do further mergers also facilitate collusion? Since the market share of the first integrated firm is higher than the other unintegrated firms, initially there is a market share motive among other firms for further integration. However, as the number of integrations increases the symmetry among downstream firms, it starts hindering downstream collusion. For example, if all firms but one are integrated, then the last possible integration will make the market completely symmetric and the critical discount which sustains collusion will be equal to the one in complete vertical separation. In summary, we can conclude that there is a certain amount of integration above which further integration hinders collusion and thus we expect to see an intermediate number of vertical mergers in vertically related industries with order-to-delivery lags.

Chapter 3

The Dynamics of Price and Advertising as Signals of Quality

3.1 Introduction

When a firm introduces a new product, it advertises to make consumers aware of the product and signals product quality using both price and advertising. Over time, as information about the product diffuses, more and more consumers become aware of the product. This paper examines the impact of increasing product awareness on advertising and on price. We study this issue in a static model under two kinds of information environments. For products like fire alarms and hair loss drugs, product quality is not easily verified since consumption is a highly imperfect signal of product quality. In these cases, consumers who are aware of the product remain uninformed about product quality. Hence, the value of signaling increases as more consumers become aware of the product. For other products like anti-histamine drugs and CD players, consumption reveals product quality. In these cases, there are likely to be three kinds of consumers: informed consumers who are aware of the product and know its quality, uninformed consumers who are aware of the product but do not know its quality, and unaware consumers who do not know about the product. We model this situation by assuming that consumers who are aware of the product at the beginning of the period are informed, but consumers who learn about the product from advertising during the period are uninformed. In this case, the value of signaling declines as more consumers become aware of the product and are informed.

In characterizing the predictions of the signaling model, we focus on the unique separating equilibrium that survives the Intuitive Criterion of Cho and Kreps (1987). In most cases, this is also the unique equilibrium. Our main findings are as follows. When product awareness does not lead to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable and, as the fraction of aware consumers increases, price rises and advertising decreases. Thus, the distortion on price gets larger and the distortion on advertising gets smaller. When awareness leads to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable. As the fraction of aware consumers increases, price declines and advertising follows an inverted U shape. Thus, the distortion on both price and advertising decreases as more consumers become aware and informed. We find support for these empirical implications from a data set on Direct-to-Consumer advertising on pharmaceutical drugs. After being approved by the FDA in December 1997, annual advertising for the hair loss prescription drug, Propecia, declined over the period 1998 to 2002. On the other hand, annual advertising for Singulair, an allergy prescription drug, over the same period follows an inverted U shape.

There is a voluminous theoretical literature on price and/or advertising as signals of product quality. The seminal paper is by Nelson (1974) and an excellent review of the literature can be found in Bagwell (2005). Cooper and Ross (1984), Bagwell and Riordan

(1991), and Linnemer (2002) study signaling models in which all consumers are aware of the product but not all are informed about the quality of the product. Consequently, advertising can signal but not inform. Bagwell and Riordan show that the high quality firm will distort price upward and that the price will decline with the fraction of informed consumers. Linnemer allows the firm to use advertising as well as price to signal quality and characterizes conditions under which the firm will engage in dissipative advertising. He argues that advertising is zero during introductory and mature phases of the product cycle, but positive during the expansion phase. Our main contribution to this strand of the literature is to give advertising a positive role in making consumers aware of the product and to examine how price and advertising will change as more consumer become aware of the product.

Overgaard (1991), Zhao (2000), Orzach et al. (2002), and Bagwell and Overgaard (2005) study signaling models in which advertising enhances demand but product quality is not observable. In the language of this paper, advertising makes consumers aware of the product but they remain uninformed. These papers show that the high quality firm will distort price upward and advertising downward relative to the case in which product quality is observable. Our contribution to this literature is the comparative static result that price increases and advertising decreases with the fraction of aware consumers.

The empirical literature on advertising and product quality has failed to find a consistent relationship. The correlation between quality and advertising varies not only across different markets and products but also across time. For example, Caves and Green (1996) find that the quality-advertising correlation is generally weak in the early ages of the product, but it becomes stronger as the product matures. Horstmann and MacDonald (2003) study data on advertising and price from the compact disc player market. They find that price falls at an accelerating pace and that advertising exhibits an inverted U shape.

They were not able to reconcile these results with existing signaling models. However, the results are consistent with the model developed in this paper under the assumption that some of the consumers who are aware of the product are also informed, and that the fraction of informed consumers grows over time.

This paper is organized as follows. In Section 2, we describe the basic model. In Section 3, we study product markets in which consumers may and may not be aware of the product but are never informed about product quality. We characterize the separating equilibrium and obtain closed form solutions for advertising and price as functions of the fraction of aware consumers. In Section 4, we study product markets in which consumers who are aware of the product may also be informed about product quality. We characterize the separating equilibrium and solve for the solution numerically. In section 5, we document several advertising patterns for prescription drugs. Section 6 provides some concluding remarks, while the proofs are collected in the Appendix.

3.2 The Model

A monopolist manufactures a new product of uncertain quality. For simplicity, we will assume that product quality is either high or low: $q \in \{H, L\},\ H > L$. Let ρ_0 denote the ex ante probability of high quality. The monopolist knows product quality. Production costs of the high (low) quality product are constant and equal to c_H (c_L). We impose the following assumptions on product costs and quality: (i) $c_H > c_L$ and (ii) $c_H/H < c_L/L$. Condition (i) states that high quality product is more costly to produce and condition (ii) implies that cost per unit of quality is lower for the high quality product.

There is a continuum of consumers for the new product, each with a potential demand for one unit. A consumer's utility for a product of quality q is given by

$$u(q, p) = \theta q - p$$

where p is the price of the product. Consumers are differentiated in their willingness to pay which is modeled by assuming θ is uniformly distributed on $[0, R]$. All consumers are willing to pay more for the higher quality good.

Some consumers are aware of the product while others are not. Let λ denote the fraction of consumers who are not aware of the product at the beginning of a period. The monopolist can increase the fraction of consumers who are aware of the product by advertising during a period. The probability of an unaware consumer learning about the product from advertising is given by $a/(1 + a)$ where a denotes advertising expenditures. Thus, the fraction of potential consumers at the end of a period is

$$1 - \lambda + \frac{a\lambda}{1 + a}.$$

In what follows, we distinguish two kinds of product markets. In the first case, we assume consumers who are aware of the product do not know its quality. This situation would apply to a product whose quality is not observable and cannot be learned, at least not until some time elapses. Examples would include hair-loss products or fire alarms. In the second case, we assume that the fraction of consumers who are aware of the product at the beginning of the period (i.e., $1 - \lambda$) also know its quality but that the fraction of consumers who learn about the product during the period from advertising (i.e., $\lambda\frac{a}{1+a}$) do not know the quality of the product. This situation would apply to a product like an anti-histamines drug whose quality is not observable but is quickly learned from experience. We will refer to consumers who are aware of the product and know its quality as *informed* consumers and consumers who are aware of the product and do not know its quality as *uninformed*. The key difference between these two cases is in regard to how the monopolist responds to changes in the value of λ, which will be decreasing over time. In the first case, decreases in λ reduces the monopolist's incentive to advertise but increases its incentive to signal high quality; in the second case, decreases in λ reduces the monopolist's incentive to advertise and to signal quality.

Before analyzing these two cases, it will be useful to characterize the solution to the model when product quality is observable so all consumers who are aware of the product are informed. This benchmark case does not imply that all consumers are aware of the existence of the product, but whoever is aware of the product knows its quality. In this case, the monopolist who supplies product quality q chooses price and advertising to maximize

$$\Pi_q^o(p,a) = \left[1 - \lambda + \lambda \frac{a}{1+a}\right](R - \frac{p}{q})(p - c_q) - a,$$

where superscript o stands for the fact that product quality is *observable*. The high quality monopolist's (unique) profit-maximizing price is

$$p_H^o = \frac{RH + c_H}{2}$$

and advertising expenditure is

$$a_H^o = \begin{cases} \frac{\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}}{2\sqrt{H}} & \text{if} \quad \lambda \in (\overline{\lambda}_H, 1] \\ 0 & \text{if} \quad \lambda \in [0, \overline{\lambda}_H] \end{cases},$$

where

$$\overline{\lambda}_H = \left(\frac{2\sqrt{H}}{RH - c_H}\right)^2.$$

Similarly, the solution for the low quality monopolist is

$$p_L^o = \frac{RL + c_L}{2}$$

and

$$a_L^o = \begin{cases} \frac{\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}}{2\sqrt{L}} & \text{if} \quad \lambda \in (\overline{\lambda}_L, 1] \\ \\ 0 & \text{if} \quad \lambda \in [0, \overline{\lambda}_L] \end{cases},$$

where

$$\overline{\lambda}_L = \left(\frac{2\sqrt{L}}{RL - c_L} \right)^2.$$

For each type of monopolist, optimal prices are independent of λ and advertising levels are nondecreasing in λ.

The following lemma compares the solutions of the high and low quality monopolists.

Lemma 7. *(i) If $\lambda \in [\overline{\lambda}_H, 1]$, then $a_H^o > a_L^o$ (ii) $p_H^o > p_L^o$.*

The Lemma states that the high quality monopolist advertises more and charges a higher price.

3.3 Case I: No Informed Consumers

In this section, we study the case where only a fraction of the consumers are aware of the product and they are not informed. The monopolist uses advertising to increase the fraction of potential consumers. The high quality monopolist wants to distinguish itself from the low quality monopolist and can use both advertising and price to do so.

Let the consumer assessment of the probability that the quality is H after observing some price and advertising pair, (p, a), be denoted by $\rho(p, a) \in [0, 1]$. How consumers make the inference requires an explanation at this point. First, as it is widely assumed in signaling literature, an unaware consumer who receives an advertisement observes all advertising spending[1]. Second, all aware consumers can observe advertising spending and price. Similarly, in Milgrom and Robert (1986), all consumers are aware of the product and they all observe advertising spending and price. With these assumptions, consumers who become aware of the product observe the firm's total advertising spending and price; thus, they hold the same inferences about the firm's quality, $\rho(p, a)$. The payoff of the monopolist who supplies quality q and chooses (p, a) is

$$\Pi_q(p, a; \rho) = D(p, a; \rho)(p - c_q) - a,$$

where

$$D(p, a; \rho) = \left[1 - \lambda + \lambda \frac{a}{1 + a}\right] (R - \frac{p}{\rho H + (1 - \rho)L}).$$

Two observations are in line at this point. First, the higher the consumer assessment of the probability that quality is high, the bigger is the payoff of the monopolist. In other words, for given (p, a), an increase in $\rho(p, a)$ increases the payoff of each type of monopolist. Hence, the low quality firm has an incentive to mimic the price and advertising selection of the high quality firm, if this fools potential costumers. Second, when quality is observable, consumers correctly form the belief of $\rho(p, a) = 1$ ($\rho(p, a) = 0$) for any pair of (p, a) for the high quality firm (the low quality firm). In case of an information environment where quality is not observable, we first define our equilibrium concept and present some basic characteristics of separating equilibria.

[1]This assumption enables consumers to make the same inference for the product's quality. However, consumers do not have to observe all advertising spending. Only having a positive correlation between the firm's total advertising and consumer's observed advertising would be qualitatively sufficient.

A *Perfect Bayesian Equilibrium* is a set of strategies $\{(p_L, a_L), (p_H, a_H)\}$ and beliefs $\rho(p, a)$, such that: (i) each strategy is optimal given the beliefs (i.e. (p_q, a_q) maximizes $\Pi_q(p, a; \rho(p, a))$, and (ii) the beliefs, derived from the equilibrium strategies, are consistent with Bayes' rule whenever possible. In a *separating equilibrium*, each type plays a different strategy (i.e., $(p_L, a_L) \neq (p_H, a_H)$); hence, uninformed consumers can infer quality from the strategy of the monopolist (i.e. $\rho(p_H, a_H) = 1 > 0 = \rho(p_L, a_L)$). In a *pooling equilibrium*, both types play the same strategy (i.e, $(p_L, a_L) = (p_H, a_H)$); hence, uninformed consumers can infer nothing from the strategy of the monopolist (i.e., $\rho(p_H, a_H) = \rho(p_L, a_L) = \rho_0$)

In a separating equilibrium, the low quality firm is revealed and acts as in observable quality benchmark case, (p_L^o, a_L^o), and earn the corresponding profit $\Pi_L(p_L^o, a_L^o; 0) = \Pi_L^o$. Therefore, to separate itself, the high quality firm must choose a pair (p_H, a_H) which the low quality firm has no incentive to mimic. Hence, (p_H, a_H) is incentive compatible for the low quality firm if

$$\Pi_L(p_H, a_H; 1) \leq \Pi_L(p_L^o, a_L^o; 0) = \Pi_L^o.$$

The following lemma shows that the low quality firm has an incentive to mimic the high quality firm's observable quality price and advertising pair, (p_H^o, a_H^o), if this fools potential costumers.

Lemma 8. $\Pi_L(p_H^o, a_H^o; 1) > \Pi_L(p_L^o, a_L^o; 0)$.

Thus, if the high quality firm is to separate, it must distort its selection (p_H, a_H), away from the observable quality maximizer, $(p_H^o, a_H^o\ )$.

Least-cost separating equilibrium

The only equilibrium outcome that survives Intuitive Criterion of Cho and Kreps (1987) is so-called *least-cost* separating outcome. In this equilibrium, the high quality firm chooses (p_H, a_H) to solve the following problem:

$$\begin{aligned} \max_{p,a} \ \Pi_H(p,a;1) &= \left[1-\lambda+\lambda\frac{a}{1+a}\right](R-\frac{p}{H})(p-c_H)-a \\ \textit{subject to} & \\ \Pi_L(p,a;1) &= \left[1-\lambda+\lambda\frac{a}{1+a}\right](R-\frac{p}{H})(p-c_L)-a \leq \Pi_L^o \end{aligned}$$

where

$$\Pi_L^o = \begin{cases} \frac{(RL-2\sqrt{L}-c_L)^2}{4L} + \frac{(1-\sqrt{\lambda})(RL-c_L)}{\sqrt{L}} & \text{if} \quad \lambda \in (\overline{\lambda}_L, 1] \\ \frac{(RL-c_L)^2}{4L} & \text{if} \quad \lambda \in [0, \overline{\lambda}_L] \end{cases}.$$

The following propositions and corollaries characterize the solution to the high quality firm's maximization problem.

Proposition 3. *In the unique separating equilibrium that satisfies the Intuitive Criteria,* $(p_L^s, a_L^s) = (p_L^o, a_L^o)$ *and*

$$(p_H^s, a_H^s) = \begin{cases} \left(\frac{(RH+c_L)\sqrt{\lambda}+\sqrt{\Delta}}{2\sqrt{\lambda}}, \frac{[\sqrt{\lambda}(RH-c_L)-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}}\right) & if \quad \lambda \in (\overline{\lambda}_K, 1] \\ \left(\frac{RH+c_L+\sqrt{\Delta_1}}{2}, 0\right) & if \quad \lambda \in (\overline{\lambda}_L, \overline{\lambda}_K] \\ \left(\frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-c_L{}^2)(H-L)}{4L}}, 0\right) & if \quad \lambda \in [0, \overline{\lambda}_L] \end{cases}$$

where $\overline{\lambda}_K$=max$\{\lambda : a_H^s(\lambda) = 0\}$, and $\overline{\lambda}_K > \overline{\lambda}_L$.

In the separating equilibrium, denoted by superscript s, the high quality monopolist employs advertising and/or price to separate itself depending on the fraction of unaware consumers. In the first region, the fraction of unaware consumers is high enough, (i.e., $\lambda \in (\overline{\lambda}_K, 1]$), that the high quality monopolist efficiently uses both advertising and price to separate itself. In the second region, the fraction of unaware consumers is in an intermediate range (i.e., $\lambda \in (\overline{\lambda}_L, \overline{\lambda}_K]$) so that the monopolist uses only price to separate. In the third region, the fraction of unaware consumers is so low that the monopolist charges a fixed price and does not advertise at all. In what follows, we explain the characteristics and the underlying intuition of the separating equilibrium as the fraction of aware consumers changes.

Corollary 1. p_H^s *is strictly decreasing in* λ *and greater than* p_H^o*.*

The high quality monopolist distorts price above monopoly price and distortion decreases with λ. Why does the high quality firm distort price upward? The answer can be seen by considering the mimicry incentive for the low quality firm. The low quality firm has a lower marginal cost and would like to set a price lower than p_H^o when consumers believe that it is of the high quality. Hence, in order to decrease mimicry incentive of the low quality, the distortion in price should be an increase from p_H^o. When consumers who are aware of the product remain uninformed about product quality, the value of signaling decreases with λ, which in turn decreases price distortion.

Corollary 2. ***(i)*** *If $\lambda > \overline{\lambda}_K$, then a_H^s is strictly increasing* ***(ii)*** *If $\lambda > \overline{\lambda}_L$, then $a_H^s < a_L^o$.*

The advertising is lower than it would be if product quality is observable and it falls as the fraction of aware consumers increases. More interestingly, the high quality firm advertises less than the low quality firm in the least-cost separating equilibrium. From Lemma 1, remember that when quality is observable, the high quality firm advertises more than the low quality firm. Why does low advertising expenditure signal product quality? When believed as the the high quality firm, the low quality firm enjoys an increase in profit margin since it can charge a higher price. Then the mimicry incentive of the low quality firm is to expand the market by increasing advertising. By doing this, the low quality firm takes advantage of high profit margin. As a result, the distortion should be a decrease in advertising not an increase. Moreover, advertising falls as the fraction of aware consumers increases for two reasons. First, the need for informative advertising decreases as the fraction of aware consumers increases. Second, the marginal cost of advertising is the same while the marginal benefit of advertising decreases as the fraction of aware consumers increases. Therefore, price becomes a more efficient signal compared to advertising, which in turn results in further decrease in advertising.

Finally, in the Proposition 1, consider the region where $\lambda \in [0, \overline{\lambda}_L)$. Why does the high quality firm set a constant high price and advertise at zero level? In this region, the marginal benefit of advertising is less than its marginal cost since the fraction of unaware consumers is so small. Hence, advertising expenditure is dissipative and can only be used as money burning. It turns out that price is a more efficient signal for the high quality firm compared to dissipative advertising. The cost of money burning is the same for both types of the monopolist while decreasing demand through price hurts the low quality monopolist more due to its higher price margin. That is why, higher quality product does not advertise and charges high and constant price.

Existence of the separating equilibrium

The least-cost separating equilibrium exists, when the high quality firm prefer the equilibrium pair of advertising and price (a_H^s, p_H^s) to any other choice of advertising and price where it is mistakenly considered as the low quality firm, that is,

$$\Pi_H(p_H^s, a_H^s; 1) \geq \max_{p,a} \Pi_H(p, a; 0).$$

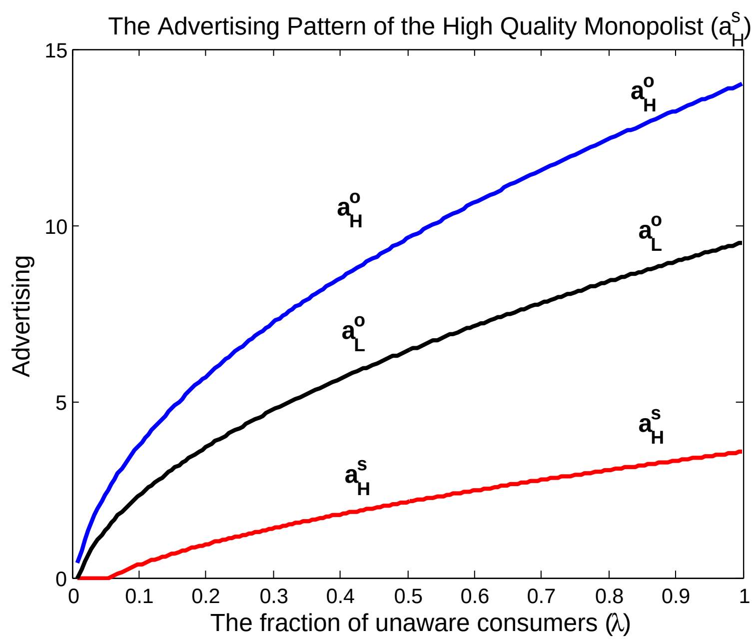
The following proposition characterizes conditions under which the separating equilibrium exists.

Proposition 4. *A separating equilibrium satisfying the Intuitive Criterion exists if (i)(H-L) is not too small and (ii)R is not too small.*

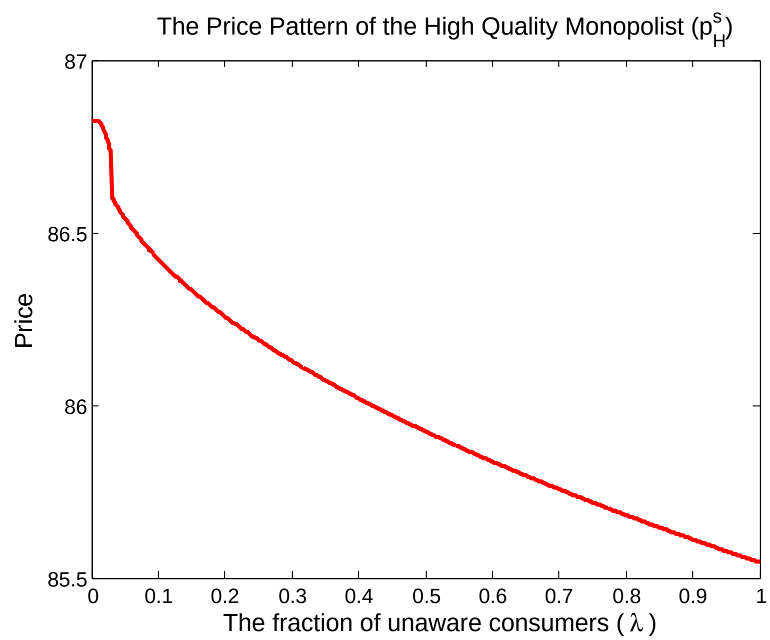
3.3.1 Numerical Example:

In this section, we propose a fully specified numerical example that give rise to above mentioned least-cost separating equilibrium. Assume that $R = 10, H = 10, c_H = 5, L = 5,$ and $c_L = 3$ with which it is easy to check that both the efficiency condition, ($c_H/H > c_L/L$), and the conditions required for the existence are satisfied.

The following figure presents advertising expenditure pattern when (i) quality is observable (ii) quality is not observable and aware consumers are uninformed about product quality. When product quality is not observable, the high quality firm advertises less than the low quality firm; which in turn implies that there is a negative relationship between product quality and advertising. Furthermore, advertising decreases as the fraction of unaware consumers decreases.



The following figure illustrates the unique least-cost separating equilibrium price pattern for the high quality firm.



When consumption does not reveal product quality, price rises as the fraction of unaware consumers decreases.

3.4 Case II: Informed Consumers

Consumption reveals product quality for some products like anti-histamine drugs and CD players. For such products, there are likely to be three kinds of consumers: informed consumers who are aware of the product and know its quality, uninformed consumers who are aware of the product but do not know its quality, and unaware consumers who do not know about the product. We assume that the fraction of consumers who are aware of the product at the beginning of the period (i.e., $1-\lambda$) also know its quality but that the fraction of consumers who learn about the product during the period from advertising (i.e., $\frac{a\lambda}{1+a}$) do not know the product quality.

When product quality is observable so all consumers who are aware of the product are informed, the profit maximizing price and advertising solutions of the monopolist are characterized in Section 2. To sum up, when quality is observable, the high quality monopolist advertises more and charges a higher price than the low quality firm (i.e., $a_H^o > a_L^o$ and $p_H^o > p_L^o$).

We now consider the case where quality is not observable. In a separating equilibrium, the low quality firm is revealed and acts as if quality is observable, (p_L^o, a_L^o) , and earn the corresponding profit $\Pi_L^o = \Pi_L(p_L^o, a_L^o; 0)$. Therefore, to separate itself, the high quality firm must choose a pair (p_H, a_H) which the low quality firm has no incentive to mimic. Hence, the price and advertising pair (p_H, a_H) is incentive compatible for the low quality firm if

$$\Pi_L(p_H, a_H; 1) = \lambda \frac{a}{1+a}(R - \frac{p}{H})(p - c_L) + (1-\lambda)(R - \frac{p}{L})(p - c_L) - a \leq \Pi_L^o$$

When some consumers have knowledge of the product's quality, the LHS of the inequality represents the low quality firm's mimicry profit $(\Pi_L(p_H, a_H; 1))$. By masquerading as high quality, the low quality monopolist could only deceive the uninformed consumers, represented by $\frac{\lambda a}{1+a}$, but not the informed consumers, represented by $(1-\lambda)$. Moreover, since the LHS of the inequality is increasing in λ, an increase in the fraction of informed consumers decreases the mimicry profit of the low quality monopolist.

In the separating equilibrium, the high quality firm chooses (p_H, a_H) to solve the following problem:

$$\max_{p,a} \; \Pi_H(p,a;1) = \left[1-\lambda+\lambda\frac{a}{1+a}\right](R-\frac{p}{H})(p-c_H)-a$$

subject to

$$\Pi_L(p,a;1) = (1-\lambda)(R-\frac{p}{L})(p-c_L)+\lambda\frac{a}{1+a}(R-\frac{p}{H})(p-c_L)-a \leq \Pi_L^o$$

It is not possible to get a closed form solution easily, because the first order condition with respect to price (advertising) is a nonlinear function of advertising (price). Instead, in the following proposition, we characterize the properties of the solution to the high quality firm's maximization problem.

Proposition 5. *In the unique separating equilibrium that satisfies the Intuitive Criteria,* $(p_L^{si}, a_L^{si}) = (p_L^o, a_L^o)$ *and*

$$\begin{aligned} &(i)\ \text{if}\ \lambda \in (\overline{\lambda}_I, 1], \text{then} \quad && a_H^{si} < a_H^o \ \text{with}\ \tfrac{d(a_H^o - a_H^{si})}{d\lambda} > 0 \ \text{and} \\ & && p_H^{si} > p_H^o \ \text{with}\ \tfrac{dp_H^{si}}{d\lambda} > 0. \\ &(ii)\ \text{if}\ \lambda \in [0, \overline{\lambda}_I], \text{then} \quad && (p_H^{si}, a_H^{si}) = (p_H^o, a_H^o). \end{aligned}$$

The intuition goes as follows. As the fraction of informed consumers increases, it becomes more costly for the low quality firm to masquerade as the high quality firm. Thus, it is optimal for the high quality firm to decrease the distortion in both price and advertising. When the fraction of informed consumers reaches a certain threshold, the high quality firm is able to charge its observable quality price and advertising pair while the low quality firm does not mimic and acts as if quality is observable.

Existence of the separating equilibrium

The least-cost separating equilibrium exists, when the high quality firm prefer the equilibrium pair of advertising and price (a_H^{si}, p_H^{si}) to any other choice of advertising and price where it is mistakenly considered as the low quality firm, that is,

$$\Pi_H(p_H^{si}, a_H^{si}; 1) \geq \max_{p,a} \Pi_H(p, a; 0).$$

The following proposition characterizes conditions under which the separating equilibrium exists.

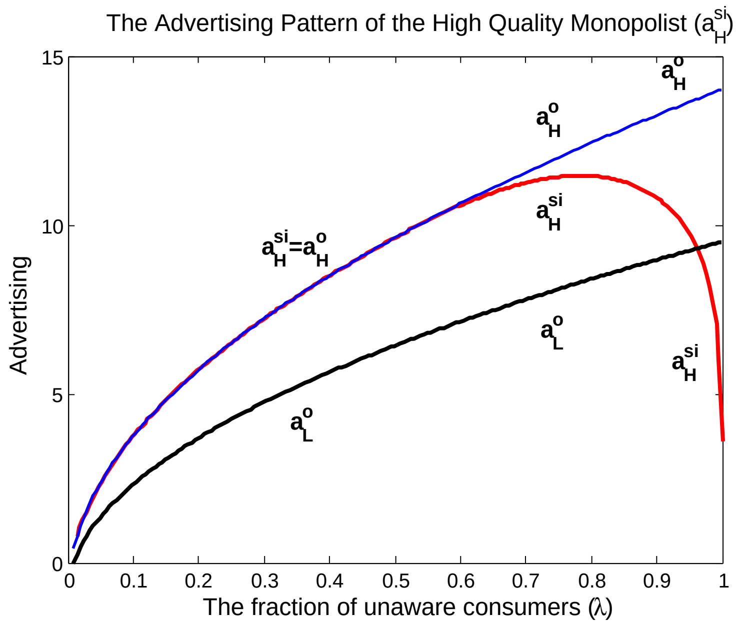
Proposition 6. *A separating equilibrium satisfying the Intuitive Criterion exists if (i)(H-L) is not too small and (ii)R is not too small.*

The following section numerically solves the high quality firm's maximization problem and characterizes the separating equilibrium price and advertising levels, (p_H^{si}, a_H^{si}) and $(p_L^{si}, a_L^{si}) = (p_L^o, a_L^o)$, where superscript *si* stands for *separating* when some consumers are *informed.*

3.4.1 Numerical Example:

We assume the same parametrization as in the numerical example of previous section i.e., that $R = 10, H = 10, c_H = 5, L = 5$, and $c_L = 3$. When awareness leads to knowledge of product quality, the following graph illustrates the advertising pattern of the high quality, (p_H^{si}, a_H^{si}), and the low quality firm, (p_L^o, a_L^o) in the separating equilibrium.

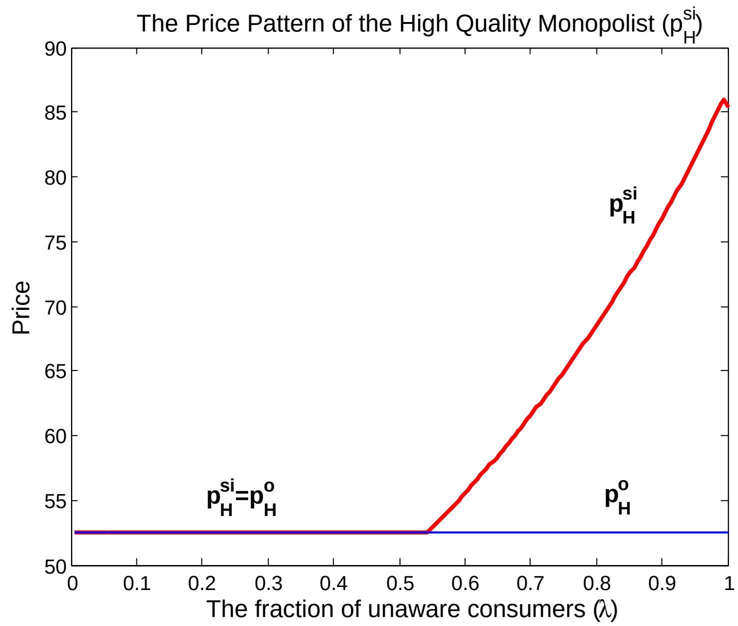
Advertising is lower than it would be if product quality is observable. As the fraction of informed consumers increases, advertising follows an inverted U shape. The



advertising of the high quality monopolist (i.e., a_H^{si}) first increases, but at a decreasing rate and then turns downward and converges to its observable quality advertising level (i.e., a_H^o). More importantly, as more consumers become aware and informed, the distortion in advertising decreases. The reason is simple. As the fraction of informed consumers increases, it becomes more costly for the low quality monopolist to signal a high quality falsely to uninformed consumers. As a result, the high quality monopolist can signal quality with a smaller advertising distortion.

When awareness leads to knowledge of product quality, the following graph presents the price pattern in the separating equilibrium.

Price is higher than it would be if product quality is observable. The intuition is as follows. A low quality monopolist would lose more sales from informed consumers by charging a high price; hence, uninformed consumers rationally infer higher quality from



the higher price. As the fraction of aware consumers increases, price falls and converges to observable quality price. In other words, as more consumers become aware and informed, distortion in price decreases because it becomes more costly for the low quality monopolist to mimic a high quality and fool uninformed consumers.

3.5 Empirical Predictions

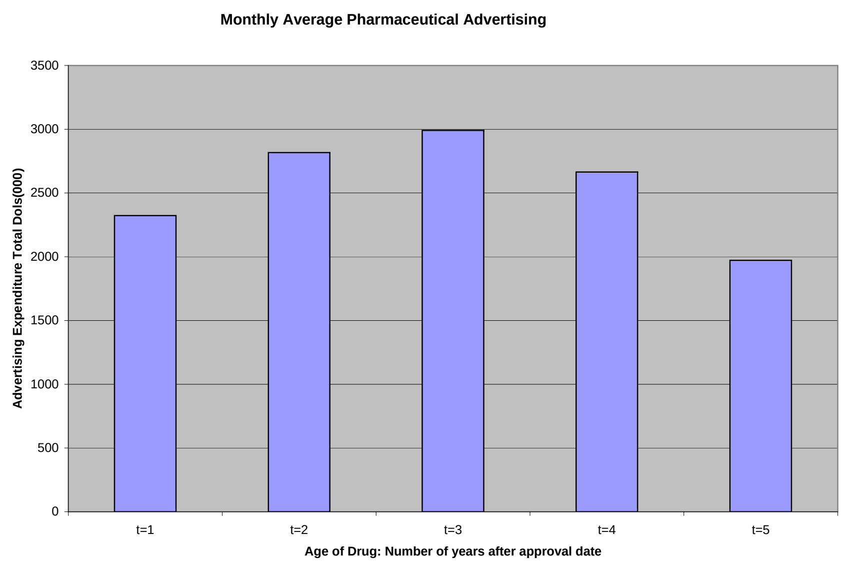
In both the marketing and economics literature, the theoretical and empirical relationship between price, advertising and product quality has been studied extensively. Starting with Nelson (1974), this relationship has mostly been explained through the signaling approach where price and advertising could function as a signal of unobservable product quality. In what follows, we present our contribution to this literature and find support for our empirical predictions from a data set on Direct-to-Consumer advertising on pharmaceutical drugs.

When all consumers are aware of the product but not all are informed about the quality of the product, Bagwell and Riordon (1991) show that the high quality firm will distort price upward and that the price will decline with the fraction of informed consumers. Hence, they predict a positive correlation between price and quality. Linnemer (2002) uses the same model with Bagwell and Riordan except he allows the firm to use (dissipative) advertising as well as price to signal quality. He shows that as the fraction of informed consumers increases, prices are high and decreasing while advertising is zero during introductory and mature phases of the product cycle, but positive during the expansion phase. A positive relationship between advertising and quality follows for only expansion phase of the product cycle.

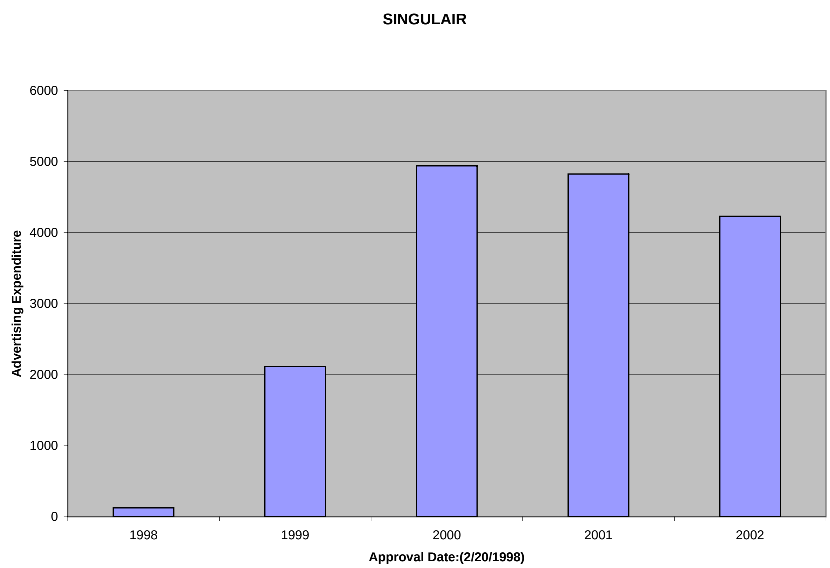
By giving advertising a positive role in making consumers aware of the product, we show that as the fraction of aware consumers increases, advertising takes on an inverted U-shape. In early phase of the product cycle, the correlation between advertising and quality is negative while it becomes positive during the expansion and the mature phases. Horstmann and MacDonald (2003) study data on advertising and price from the compact disc player market. They find that price falls at an accelerating pace and that advertising exhibits an inverted U shape. They were not able to reconcile these results with existing signaling models. However, the results are consistent with the model developed in this paper under the assumption that some of the consumers who are aware of the product are also informed, and that the fraction of informed consumers grows over time.

We next propose some evidence to inverted U shaped advertising pattern from a data set on Direct-to-Consumer advertising on pharmaceutical drugs. Direct to Consumer Advertising (DTCA) expenditure, obtained from TNS Media Intelligence, consists of individual brand-name drugs. TNS Media Intelligence monitors advertising expenditures for various media such as radios, newspapers, magazines, and TVs. Their database include all advertising expenditure for prescriptions drug that appears in these media. We have total monthly advertising expenditure from 1996 to 2002. The FDA's Orange Book is used for the approval dates. In most of the cases, the approval date and the launch dates of the products coincide while sometimes there is only small difference. To calculate the age of the drug, we consider the approval date as a launch date of the drug.

We first consider the drugs which have approval dates between 1996 and 1998 and have stayed in the market for at least five years. There are 25 brand-name drugs in this category. The following first graph summarizes the average monthly advertising level of these drugs as a function of age of the drug.



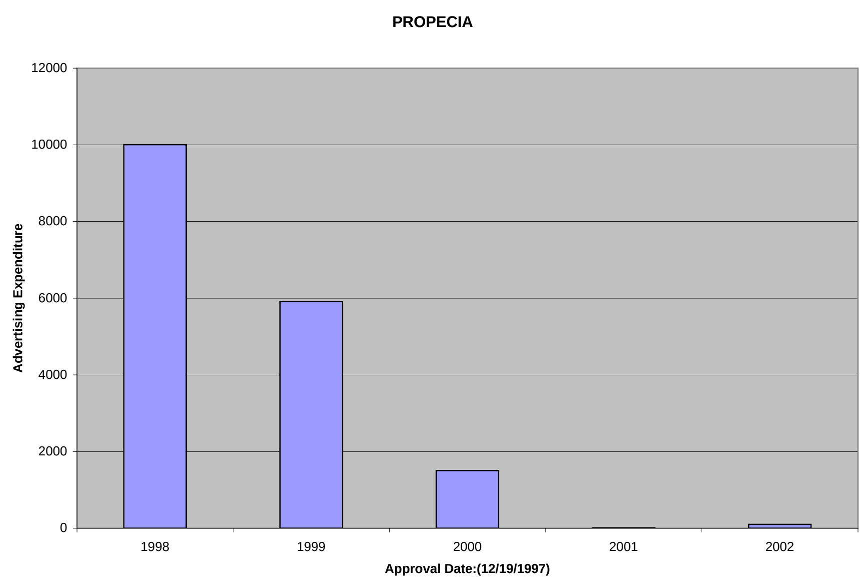
However, the advertising expenditure pattern differs for individual prescription drugs in our sample. For instance, Singulair, an allergy relief prescription drug, is approved by the FDA in February 1998. Average annual advertising pattern of Singulair is illustrated in second one of the following figures



Consumption of Singulair is likely to reveal its quality. If the fraction of informed consumers grows over time, our model predicts that the advertising takes an inverted U shape, which is consistent with the advertising pattern of Singulair.

In related literature, Overgaard (1991), Zhao (2000), Orzach et al. (2002), and Bagwell and Overgaard (2005) study signaling models in which advertising enhances demand but product quality is not observable. These papers show that the high quality firm will distort price upward and advertising downward relative to the case in which product quality is observable. In other words, they predict a positive correlation between price and quality and a negative correlation between adverting and quality. When consumption does not reveal product quality (i.e., consumers who are aware of the product remain uninformed about the product quality), our model predicts that advertising decreases over entire life cycle of the product and the correlation between price and quality is positive and strengthens as the fraction of aware consumers increases.

For example, the FDA approved Propecia, hair loss prescription drug [2], in December 1997 and Propecia is an example of the goods for which consumption does not reveal product quality easily. Our model predicts that advertising decreases over entire life cycle of the product. The following graph presents annual average advertising expenditures for Propecia, which is consistent with the empirical prediction of our model.



[2]From Merck's webpage "Propecia was developed to treat mild to moderate male pattern hair loss ...Remembering to take your pill each day is important...Most men see results 3 to 12 months after starting Propecia...If Propecia has not worked within 12 months, further treatment is not likely to help."

Chapter 4

Response to Competitive Entry: Signal Jamming

4.1 Introduction

We show that if the quality of the entrant is certain, the income distribution of consumers and the quality difference between the entrant and the incumbent determine whether entry of a new firm to an industry with an incumbent is possible or not. If the income distribution is small or qualities are "far apart", it is likely that the incumbent captures all of the market in which each and every consumer prefers the product of the incumbent to the product of the entrant. As the income distribution increases or if qualities are "close", it is more likely to observe a profitable entry. In other words, as the income distribution increases, the industry evolves from having a monopoly to having duopoly with a fully covered market and from there the industry evolves to a partially covered market with a duopoly. This part of the paper is mostly similar to the model of Shaked and Sutton (1982). We also show that if the quality of the entrant is uncertain, the high quality entrant may have an incentive to separate itself from the low quality entrant while the incumbent has an incentive to jam the quality signalling attempt. This part of the paper is related to Kalra et al. (1998) in the sense that they propose the similar idea of signal jamming and a candidate for a signal-jamming equilibrium in their paper. We also propose a signal-jamming (pooling) equilibrium and numerically show that, for a large set of parametric values, this signal-jamming equilibrium indeed exists.

This part of the paper is also related to Fudenberg and Tirole (1986) in which they propose a new theory of predation based on "signal-jamming". In their model, the entrant, uncertain about its future profitability, uses its initial period profit to estimate its future profit and decide whether to exit or not while the incumbent jams the inference of the entrant to induce it to exit. Moreover, the quality signalling attempt of the entrant is also related to the signalling literature. As an example of the signalling under monopoly setting, Hertzendorf and Overgaard (2001a) show that pure price separation is not possible if the vertical differentiation is small. Moreover, Fluet and Garella (2002) and Hertzendorf and Overgaard (2001a, 2001b, and 2002) study price signalling along with the possibility of advertising in a multi-sender context with two competing firms. These papers are different from our paper for three reasons. First, some of them have only a single sender while in our setup there are two competing firms. Second, the papers with a multi-sender context have the common informational assumption that consumers do not know the quality of either firm so that both firms try to convey their quality to consumers. For example, if the firms adopt the same strategy (pooling), consumers will not be able to distinguish the two. Finally, none of these models considers the possibility of signal jamming.

The rest of the article is organized as follows. In section 2, we outline the basic model and characterize the best response functions and possible market structures when the quality of the entrant is observable. In section 3, we present signal-jamming equilibrium when the quality of the entrant is unobservable. We also show that in this equilibrium the incumbent jams the entrant's separation attempt and increases its profit by doing so. Section 4 concludes the article. Proofs are in the Appendix.

4.2 The Model

The incumbent produces a product with the quality, q_I, at the marginal cost, c_I and faces a new entry to the market. The entrant manufactures an inferior product and for simplicity we assume that it has two possible quality levels either low, q_L, or high, q_H. The corresponding marginal costs are c_L and c_H respectively. We impose the following assumptions on product quality and cost: (i)$q_L < q_H < q_I$ and (ii) $c_L < c_H < c_I$. The condition (i) states that the entrant has an inferior product compared to the incumbent and the high quality entrant has a better quality product compared to the low quality entrant. The condition (ii) states that the higher quality is more costly to produce. Moreover, we postulate that the incumbent can observe the entrant's quality through reverse engineering. Finally, the incumbent and the entrant set their prices simultaneously.

There are total mass of N consumers in the market. They all know the incumbent's product quality. This is a reflection of the fact that consumers are already familiar with the product or have consumed it before. Furthermore, consumers can not observe the entrant's product quality while they know that the entrant's quality could be either low or high. This reflects the fact that firms generally have better information than the consumers have and reverse engineering is prohibitively expensive to undertake for an individual consumer. Moreover, consumers are identical in tastes, but differentiated in income. Incomes, represented by θ, are uniformly distributed on some support $0 < a \leq \theta \leq b$ with unitary density. Each consumer can purchase either one unit of the product or make no purchase. In case of no purchase, they consume a Hicksian "composite commodity" with the quality of q_o and the price of $p_o = 0$. The net surplus of a consumer with income θ is given by

$$u(p_i, q_i|\theta) = q_i(\theta - p_i) \quad i \in \{o, L, H, I\}$$

where p_i represents the market price for the product i and similarly q_i denotes the quality for the product i. Without loss of generality, we normalize the total mass of consumers to be unity, i.e., N=1.

4.3 The Entrant's Quality is Observable

In what follows, we first characterize the solution to the model when the entrant's quality is observable by consumers. We need to consider two cases: (i) the entrant is of high quality and (ii) the entrant is of low quality. Instead of analyzing case (i) and (ii) separately, we denote the entrant as E $\in \{L, H\}$ so that $q_E \in \{q_H, q_L\}$ denotes the E-type entrant's quality while $c_E \in \{c_H, c_L\}$ denotes its marginal cost.

We first derive the incumbent's and the entrant's demand and profit functions. A pair set of prices (p_I, p_E) represents prices of the incumbent and the entrant respectively. A consumer is indifferent in purchasing from the incumbent and the entrant, if $q_I(\theta_I - p_I) = q_E(\theta_I - p_E)$; that is the indifferent consumer at θ_I gets the same net surplus from consuming either product. A consumer is indifferent in purchasing the product of the entrant and not making a purchase at all, if $q_E(\theta_E - p_E) = q_o(\theta_E - p_o)$.

The critical thresholds θ_I and θ_E can be calculated as follows

$$q_I(\theta_I - p_I) = q_E(\theta_I - p_E) \;\; \Leftrightarrow \;\; \theta_I = \frac{q_I p_I - q_E p_E}{q_I - q_E}$$

$$q_E(\theta_E - p_E) = q_o(\theta_E - p_o) \;\; \Leftrightarrow \;\; \theta_E = \frac{q_E p_E}{q_E - q_o}$$

In what follows, we focus on the pair of prices (p_I, p_E) in which $\theta_I > \theta_E$.[1] Then it is easy to see that consumers with income $\theta > \theta_I$ strictly prefer the product of the incumbent at price p_I to the product of the entrant at p_E. Similarly, consumers with income $\theta_I > \theta > \theta_E$ strictly prefer the product of the entrant at price p_E to the composite good at $p_o = 0$. The rest of the consumers with income $\theta_E > \theta$, does not make a purchase and consumes the composite good.

Now we can write down the demands of the entrant and the incumbent as follows

$$\begin{aligned} D_E &= \begin{cases} \theta_I - a = \dfrac{q_I p_I - q_E p_E}{q_I - q_E} - a & \text{if } \theta_E \leq a \\ \theta_I - \theta_E = \dfrac{q_I p_I - q_E p_E}{q_I - q_E} - \dfrac{q_E p_E}{q_E - q_o} & \text{if } \theta_E \geq a \end{cases} \\ D_I &= b - \theta_I = b - \frac{q_I p_I - q_E p_E}{q_I - q_E}. \end{aligned}$$

The profit of the entrant and the incumbent can be written as follows

$$\begin{aligned} \Pi_E &= \begin{cases} (p_E - c_E)(\dfrac{q_I p_I - q_E p_E}{q_I - q_E} - a) & \text{if } \theta_E \leq a \\ (p_E - c_E)(\dfrac{q_I p_I - q_E p_E}{q_I - q_E} - \dfrac{q_E p_E}{q_E - q_o}) & \text{if } \theta_E \geq a \end{cases} \\ \Pi_I &= (p_I - c_I)(b - \frac{q_I p_I - q_E p_E}{q_I - q_E}). \end{aligned}$$

[1] In other words, the price pair of (p_E, p_I) has to satisfy the following inequality: $\frac{p_I}{p_E} > \frac{q_E(q_I - q_o)}{q_I(q_E - q_o)}$. By restricting ourself to this price region, we exclude the possibility that the entrant can drive the incumbent out of the market.

Lemma 9. *The best response functions take the form of*

$$p_E = \begin{cases} \dfrac{q_I p_I - a(q_I - q_E) + c_E q_E}{2q_E} & if \quad \theta_E \leq a \\ \dfrac{q_I(q_E - q_o)p_I + q_E c_E(q_I - q_o)}{2q_E(q_I - q_o)} & if \quad \theta_E \geq a \end{cases},$$

$$p_I = \frac{q_E p_E + b(q_I - q_E) + c_I q_I}{2q_I}.$$

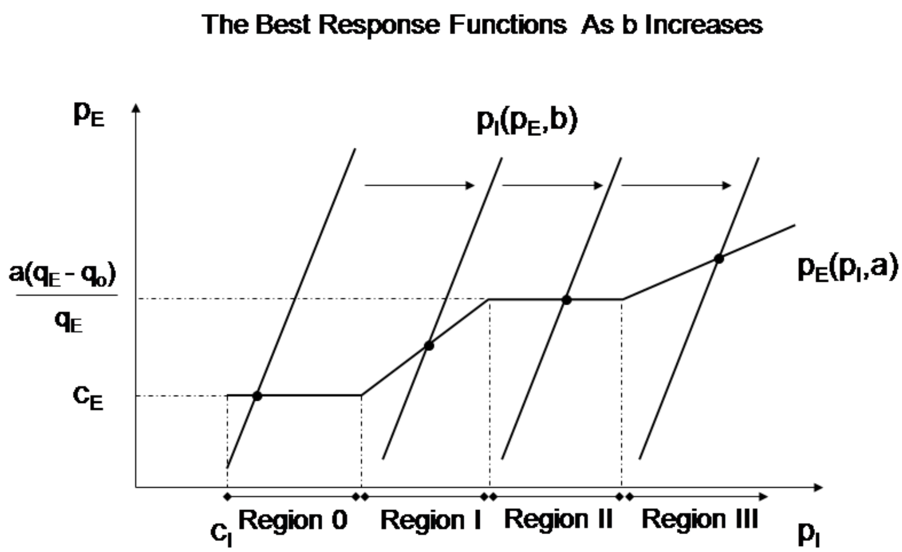
Note that the entrant's reaction function, $p_E(p_I)$, is increasing in the incumbent's price, p_I while the incumbent's reaction function, $p_I(p_E)$, is increasing in the entrant's price, p_E. This indicates the fact that the entrant's and the incumbent's price strategies are strategically complementary to each other.

We identify four separate market structures and corresponding regions depending on income distributions. The following lemma presents the conditions on the range of income distribution which in turn determines whether the solution to lie in region 0,I, II, or III.

Lemma 10. *The solution lies in*

$$\begin{array}{lll} Region\, 0 & if & \frac{c_I q_I - q_E c_E}{q_I - q_E} \leq b \leq 2a - \frac{c_I q_I - q_E c_E}{q_I - q_E}, \\ Region\, I & if & 2a - \frac{c_I q_I - q_E c_E}{q_I - q_E} \leq b \leq (\frac{2q_I + q_E - 3q_o}{q_I - q_E})a - \frac{c_I q_I - 2q_E c_E}{q_I - q_E}, \\ Region\, II & if & (\frac{2q_I + q_E - 3q_o}{q_I - q_E})a - \frac{c_I q_I - 2q_E c_E}{q_I - q_E} \leq b \leq (\frac{4q_I - q_E - 3q_o}{q_I - q_E})a - \frac{c_I q_I(q_E - q_o) + 2q_E c_E(q_I - q_o)}{q_I - q_E}, \\ Region\, III & if & (\frac{4q_I - q_E - 3q_o}{q_I - q_E})a - \frac{c_I q_I(q_E - q_o) + 2q_E c_E(q_I - q_o)}{q_I - q_E} \leq b. \end{array}$$

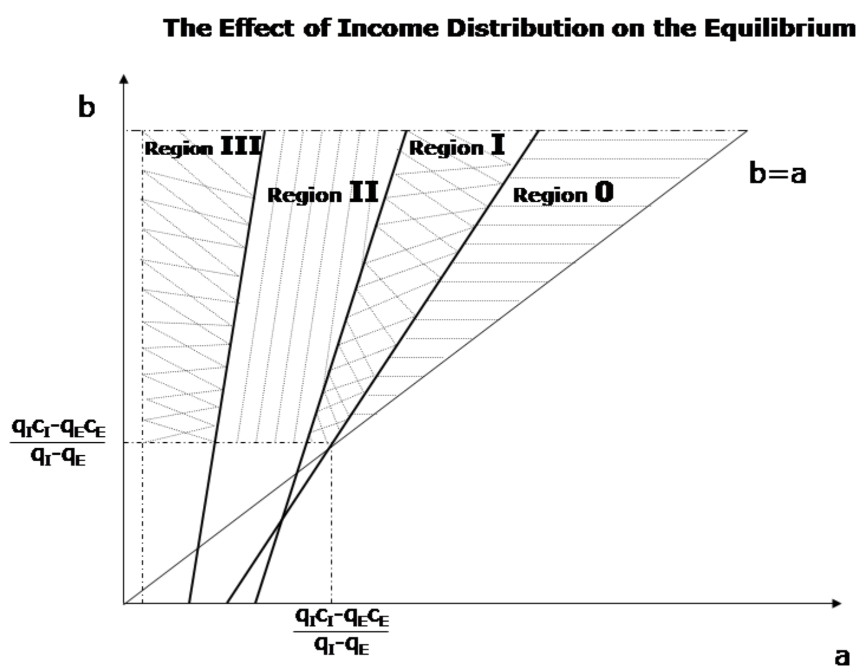
The intersection of the best response functions of the entrant and the incumbent will determine the solution and, in turn, market structure. For various values of income distributions, i.e., only b changes, the following figure presents the best response functions of the entrant and the incumbent and the corresponding solutions



Whether solution lies in region 0, I, II or III depends on where the increasing best response function of the incumbent intersects with the best response function of the entrant and that in turn depends on the range of income distribution.

If the range of income distribution is small, i.e., b is close to a, the solution lies in Region 0 where the incumbent stays as the sole provider of the product. In this region, the entrant can not make a sale even though it sells its product at its marginal cost of c_E. Technically speaking, θ_I is less than the lowest income, a, so that all consumers prefer the product of the incumbent to both the product of the entrant and the composite product. If the range of income distribution is intermediate, the solution lies in Region I where the entrant can also make a sale along with the incumbent. Technically speaking, θ_I (θ_E) is bigger (less) than the lowest income level, a. In this region, the market is fully covered; that is, each consumer purchases from either the entrant or the incumbent. Moreover, Region II corresponds to a certain range of b and a such that $\theta_E = a$. Over this range of parameter values, the entrant leaves its price constant while the price of incumbent varies. In other words, the entrant faces a kinked demand schedule at this price. Finally, if the range of income distribution is high, i.e., b is very high compared to a, both firms coexist and make a sale in the market. Technically speaking, both θ_E and θ_I are bigger than the lowest income level. In this region, some consumers purchase neither product and hence, the market is not covered.

The regions 0, I, II, and III are illustrated for various values of a and b in the following figure.



It is clear in this figure that the range of income distribution plays a key role in determining market structure. For example, on the vertical axis, if we fix any value of b with a small value of a, the range of income distribution is high at this point. As a result the solution lies in Region III. With the same value of b, if we keep increasing the value of a, the range of income distribution gets smaller and finally the solution lies in Region 0. Alternatively, as the range of income distribution increases, the market evolves from being a monopoly in Region 0 to a duopoly with coexistence of the entrant and the incumbent in Region III where the market is not fully covered.

4.4 The Entrant's Quality is not Observable

In what follows, we consider the case where the entrant manufactures the product of uncertain quality, either low or high. Let ρ_o denote the ex ante probability of the entrant being high quality. Consumers perfectly observe the incumbent's quality, q_I. The timing of the game is as follows. In the first stage, nature chooses a quality level of the entrant: q_L or q_H, which is observable by both the incumbent and the entrant, but not by consumers. In the second stage, the incumbent and the entrant simultaneously choose their prices p_I and p_E, which are observable by the consumers. Then consumers form their belief about the quality of the entrant. The consumer belief, $\rho(p_I, p_E)$, denotes the probability of the entrant being high type. Finally consumers make the purchase decision. The solution concept, we use in this paper, is sequential equilibrium by Kreps and Wilson (1981).

A *Sequential Equilibrium* is a set of pricing strategies $\{(p_L, p_I), (p_H, p_I)\}$ and beliefs $\rho(p_I, p_E)$, such that: (i) each pricing strategy of the entrants is optimal given the optimal action on the part of the incumbent firm and optimal purchasing strategy and beliefs of consumers, (ii) the incumbent's pricing strategy is optimal given the optimal type-contingent strategies of the entrant and the optimal purchasing strategy and beliefs of consumers, (iii) consumers make the optimal purchase decision given the optimal type-contingent pricing strategies of the entrant and the pricing strategy of the incumbent, and (iv) the beliefs, derived from the equilibrium strategies, are consistent with Bayes' rule whenever possible. In our analysis, we focus on the separating and pooling sequential equilibria. In a *separating equilibrium*, each type plays a different strategy, i.e., $(p_L, p_I) \neq (p_H, p_I)$. Hence, consumers can infer the quality from the pricing strategy of the entrant, i.e., $\rho(p_H, p_I) = 1 > 0 = \rho(p_L, p_I)$. In a *pooling equilibrium*, both types of the entrants play the same pricing strategy and the incumbent plays the same strategy irrespective of the entrant's type, i.e., $(p_L, p_I) = (p_H, p_I)$. Hence, consumers cannot infer the quality of the entrant from the pricing strategies and consumers' ex post belief of entrant being high quality is equal to the consumers ex ante belief of entrant being high quality, i.e., $\rho(p_H, p_I) = \rho(p_L, p_I) = \rho_0$.

In what follows, we focus on the region or market structure in which both the incumbent and the entrant coexist while the market is not fully covered, i.e., Region III. This market structure corresponds to the specific market structure analyzed in Kalra et al. (1998) [2]

4.4.1 Least-Cost Separation Attempt by the Entrant

This part presents and models the the fact that the H-type entrant has incentive to signal its quality. When consumers are uncertain about the quality of entrant's product, they base their purchase decision on the expected quality of the entrant's product, $\tilde{q}_E = \rho q_H + (1-\rho)q_L$, and q_I. Since $q_H > \tilde{q}_E$, the H-type entrant's profit decreases compared to the case where the entrant's quality is observable by consumers. Therefore, the H-type entrant stands to gain from revealing its true type to consumers. In fact, it is easy to show that

[2] By following Kalra et al. (1998), in this paper we ignore the possibility that the incumbent can drive out the entrant completely from the market by charging very low price i.e., predatory pricing. In a future research project, it would be interesting to show that an entrant who can enter under complete information may not able to do so under incomplete information because of signal jamming by the incumbent.

$\partial \Pi_H^H / \partial \tilde{q}_E > 0$. The following lemma summarizes this result.

Lemma 11. *Given the price of the incumbent, the H-type entrant's profit increases in its perceived quality.*

This provides the necessary incentive for the H-type entrant to engage in quality signalling. However, the H-quality entrant has to adopt a pricing strategy which makes mimicry unprofitable for the L-type entrant. In the next proposition, we show that the H-type entrant can signal its quality by increasing its price above its complete information level.

Proposition 7. *For any prices of the incumbent, the H-type entrant can signal its quality by increasing its price above its complete information level, i.e.,*

$$\frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H} \Big/ \frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H} > 1 \;\; for\; any\; p_H \geq p_H^*,$$

where p_H^* stands for the complete information price of the H-type entrant.

In other words, this proposition indicates that the single-crossing condition is satisfied in this environment. Hence, the H-type entrant can discourage the mimicry of the L-type entrant by increasing its own price above its optimal complete information price.

The following constraint optimization problem gives us the least cost separating price of the H-type entrant, Quality-Signalling (QS), as a function of the incumbent's price.

$$
\begin{aligned}
p_H^{QS} \in arg \max_{p_H} \Pi_H^H(p_I, p_H) &= (p_H - c_H)\left[\frac{q_I p_I - q_H p_H}{q_I - q_H} - \frac{q_H p_H}{q_H - q_o}\right] \\
subject \quad to & \\
\Pi_L^H &= (p_H - c_L)\left[\frac{q_I p_I - q_H p_H}{q_I - q_H} - \frac{q_H p_H}{q_H - q_o}\right] \\
\leq \Pi_L^L &= (p_L^* - c_L)\left[\frac{q_I p_I - q_L p_L^*}{q_I - q_L} - \frac{q_L p_L^*}{q_L - q_o}\right]
\end{aligned}
$$

where the subscript in Π_j^i stands for the entrant's true quality while the superscript stands for the entrant's perceived quality by consumers and p_L^* denotes the complete information price of the L-type entrant and is derived in Lemma 9.

In its separation attempt, the high quality firm needs to distort its full information price upwards to avoid the mimicry by the L-type entrant. This distortion successfully and profitably signals to consumers that p_H^{QS} is indeed set by the H-type entrant.

Lemma 12. *For a given price of the incumbent, p_I, the least cost separating price strategy of the H-type entrant is*

$$
p_H^{QS} = \frac{q_I p_I (q_H - q_o)(1 + \lambda_1) + q_L c_L (q_I - q_o)(c_H + c_L \lambda_1)}{2q_H (q_I - q_o)(1 + \lambda_1)}
$$

where

$$
\lambda_1 = -1 + \frac{q_H (q_I - q_o)(c_H - c_L)}{\sqrt{\left[q_I p_I (q_H - q_o) + q_H c_L (q_I - q_o)\right]^2 - 4q_H (q_I - q_o)(q_I - q_H)(q_H - q_o)\Pi_L^L}}
$$

For any given price of the incumbent, $p_H^{QS}(p_I)$ is the H-type entrant's the least cost separating price strategy.

4.4.2 Signal-Jamming Pricing by the Incumbent

In this section, we show that for certain parameter values, even though the H-type attempts to separate, the attempt will not be successful due to signal jamming by the incumbent.

It is easy to show that the incumbent's profit decreases in perceived quality of the entrant, i.e., $\partial \Pi_I / \partial \tilde{q}_E < 0$. The following lemma summarizes this result.

Lemma 13. *Given the price of the entrant, p_E, the incumbent's profit decreases in the perceived quality of the entrant.*

The following lemma states that the incumbent can increase the signalling cost of the entrant by distorting its own price upwards.

Lemma 14. *As the incumbent's price increases, the signalling cost of the high quality entrant increases.*

In the following lemma, we show that an increase in price of the incumbent increases the mimicry incentive of the low type entrant.

Lemma 15. *As the incumbent's price increases, the mimicry incentive of the low quality incentive increases.*

These three lemmas provide the necessary incentive for the incumbent to engage in signal-jamming, in which the entrant's attempt to signal quality is no longer be optimal and it prefers to be pooled with the L-quality entrant.

The signal-jamming (SJ) pricing strategy of the incumbent can be derived from the following constraint optimization problem

$$
\begin{aligned}
p_I^{SJ} \in arg\max_{p_I} \Pi_I(p_I, p_H) &= (p_I - c_I)\left[b - \frac{q_I p_I - \tilde{q}_E p_E}{q_I - \tilde{q}_E}\right] \\
subject \;\; to & \\
\Pi_H^H(p_I) &= (p_H^{QS} - c_H)\left[\frac{q_I p_I - q_H p_H^{QS}}{q_I - q_H} - \frac{q_H p_H^{QS}}{q_H - q_o}\right] \\
\leq \Pi_H^{\rho_o} &= (p_H^{\rho_o} - c_H)\left[\frac{q_I p_I - \tilde{q}_E p_H^{\rho_o}}{q_I - \tilde{q}_E} - \frac{\tilde{q}_E p_H^{\rho_o}}{\tilde{q}_E - q_o}\right]
\end{aligned}
$$

where $p_H^{\rho_o}$ is the price set by the H-type entrant when consumers believe that it has an average quality of $\tilde{q}_E = \rho_o q_H + (1 - \rho_o) q_L$.

The constraint is that the H-type entrant weakly prefers being pooled with the L-type entrant to engaging in quality signalling. This constraint optimization problem of the incumbent gives us the incumbent's signal-jamming best response function for any given price of the entrant.

We can now propose our candidate for the signal-jamming equilibrium. We define (p_I^{SJ}, p_E^{SJ}) as the equilibrium to the complete information game in which the entrant's type is of $\tilde{q}_E$, i.e., the expected value of q_H and q_L. We propose that the pair of prices (p_I^{SJ}, p_E^{SJ}), the intersection of the incumbent's and the entrant's best replies in this complete information game, is a Signal-Jamming equilibrium. The off-the-equilibrium beliefs are as follows. Consumers believe that the quality of the entrant is low if price is not equal to p_E^{SJ} but less than the least-cost separating price response to p_E^{SJ} and high otherwise. Also, consumers believe that the quality of the entrant is high if the incumbent price is not equal to p_I^{SJ}.

The first necessary condition to be satisfied is that the profit of the low quality entrant if it reveals itself and sets its price equal to best reply to p_I^{SJ} is not greater than its pooling profit at (p_I^{SJ}, p_E^{SJ}). In other words, as it is represented in the following inequality, the low quality firm should not have any incentive to deviate from the signal-jamming (pooling) equilibrium.

$$\Pi_L^{\rho_o}(p_I^{SJ}, p_E^{SJ}) \geq \Pi_L^L(p_I^{SJ}, p_L^*(p_I^{SJ})) = \max_{p_E} \Pi_L^L(p_I^{SJ}, p_E)$$

where

$$\Pi_L^L(p_I^{SJ}, p_L^*(p_I^{SJ})) = \frac{\left[q_I p_I^{SJ}(q_L - q_o) - q_L c_L(q_I - q_o)\right]^2}{4q_L(q_I - q_o)(q_I - q_L)(q_L - q_o)}$$

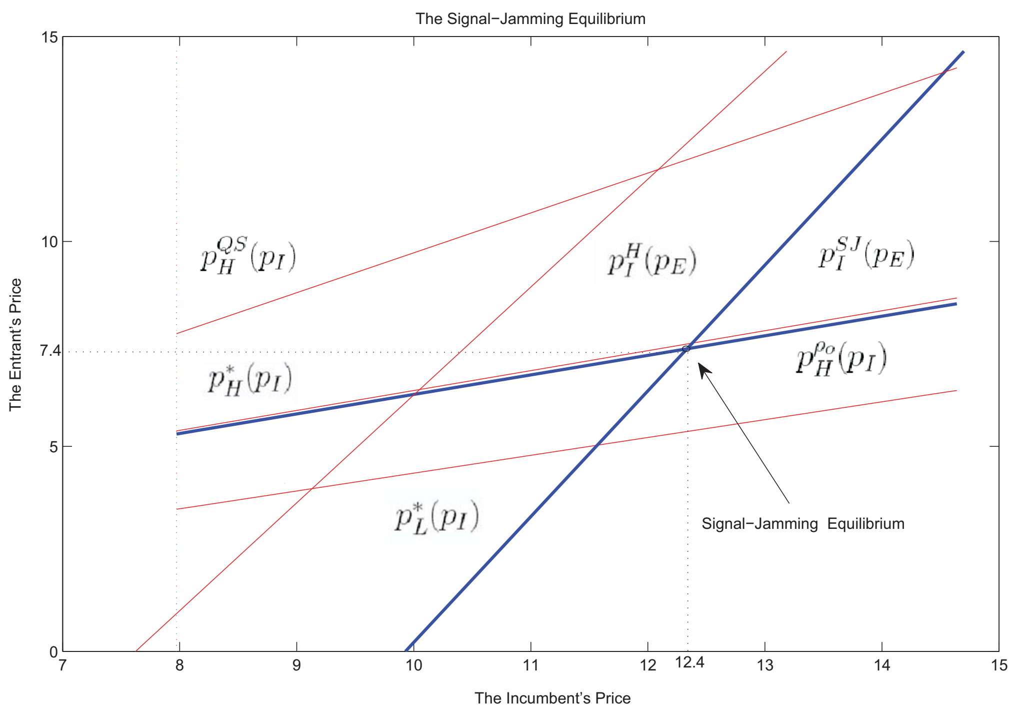
and

$$\Pi_L^{\rho_o}(p_I^{SJ}, p_E^{SJ}) = \frac{\left[q_I p_I^{SJ}(\tilde{q}_E - q_o) + \tilde{q}_E(c_H - 2c_L)(q_I - q_o)\right]\left[q_I p_I^{SJ}(\tilde{q}_E - q_o) - \tilde{q}_E c_H(q_I - q_o)\right]}{4\tilde{q}_E(q_I - q_o)(q_I - \tilde{q}_E)(\tilde{q}_E - q_o)}$$

The second necessary condition to be satisfied is that the profit of the high quality entrant if it reveals itself and sets the price equal to the least-cost separating response to p_I^{SJ} is not greater than its pooling profit at (p_I^{SJ}, p_E^{SJ}). This is equivalent to showing that the constraint in the signal jamming optimization problem above is satisfied. However, if this condition is satisfied, i.e., the constraint of H-type is not binding, then, by definition, the p_I^{SJ} is a best response to the p_E^{SJ}. This means that the incumbent has no incentive to deviate as well.

It is possible to simplify these necessary conditions, especially the first one, since it is is a quadratic function of either p_E^{SJ} or p_I^{SJ}. However, overall even the simplified versions will be complex expressions of either p_E^{SJ} or p_I^{SJ}. Instead, we solve these complex expression numerically to show that this signal-jamming equilibrium exists. It is also easy to show that the equilibrium exists for large set of parameter values.

In what follows, a numerical example illustrates the signal-jamming equilibrium, we just proposed. Let the marginal cost of the incumbent, c_I, be 5, the marginal costs of the H- (c_H) and the L-type (c_L) entrants be 3 and 0, respectively. Moreover, the quality of the incumbent product, q_I, is 25, while those of the H- (q_H), the L-type (q_L) entrants and the composite product are 19, 10, and 2, respectively. The ex-ante probability of the entrant's product quality being high, ρ_o, is 0.7. Under this set of parametric values, the complete information, separating, and signal-jamming equilibrium are presented in the following figure.



In the figure, $p_I^*(p_E)$ and $p_I^{SJ}(p_E)$ denote the complete information and signal-jamming best response functions of the incumbent respectively while $p_H^*(p_I)$, $p_H^{\rho_o}(p_I)$, and $p_H^{QS}(p_I)$ denote the high quality under complete information, the average quality under complete information, and the quality signalling least cost price strategy of the high quality entrant respectively. Moreover, the signal-jamming equilibrium in this numerical example satisfies all the necessary conditions, identified above. At this point, it is in line to make three important observations about the signal-jamming equilibrium.

The first observation is that given price of the incumbent, p_I, the equilibrium price of the entrant with observable quality, $p_H^*(p_I)$, is lower than the equilibrium price with unobservable quality, $p_H^{QS}(p_I)$, in the relevant region. In other words, the H-type entrant increases its price above the complete information level in its separation attempt. Also, the equilibrium price of the entrant when its quality is observable, p_H^*, is 6.38 and it increases to 11.75 under quality signalling without signal jamming. This increase is to deter the mimicry of the L-type entrant.

The second observation is that the incumbent increases its own price to decrease the separation incentive of the entrant. The incumbent's equilibrium price when the entrant's quality is observable, p_I^*, is 10.05. However, in case of unobservable quality, the incumbent's equilibrium price under separation attempt of the entrant is 12.08 while its price under signal-jamming, p_I^{SJ}, is 12.4.

The third observation is that the incumbent increases its profit more than 10% by signal jamming: hence, it has an incentive to jam the separation attempt of the entrant. The incumbent's profit when the entrant's quality is known is only 106.26 and it increases to 209.41 when the entrant's quality is not observable and the incumbent does not engage in signal-jamming. However, the incumbent's profit with signal jamming is 232.35.

We have repeated the numerical analysis for large set of parameter values and find that the signal-jamming equilibrium does generally exist.

Chapter 5

Conclusion

In chapter 2, our analysis has revealed that in vertically related industries with production lag, vertical integration is pro-competitive and not profitable if firms behave competitively. However, we show that vertical integration facilitates collusion, in which case it is profitable. Two effects are important for this result: the quick response effect and the lack of pre-commitment effect.

The quick response effect arises because the integration creates efficiency by shortening the order-to-delivery lag thereby enabling the integrated downstream firm to respond to deviations of cartel members faster. This effect eliminates all deviation incentive of the unintegrated downstream firm. As a result, whether the vertical merger facilitates collusion or not solely depends on the integrated firm. Within period deviation profit of the integrated firm stays the same as it were in vertically separated case. However, ordering quantity in advance does not carry a commitment value for the integrated firm, which results in the lack of pre-commitment effect. This effect reduces the profit of the integrated firm in the punishment phase and in turn facilitates collusion.

Chapter 3 gives advertising a positive role in making consumers aware of the product and examines the impact of increasing product awareness on advertising and on price. We study this issue in a static model under two kinds of information environments. When awareness leads to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable. As the fraction of aware consumers increases, price declines and advertising follows an inverted U shape. Thus, the distortion on both price and advertising decreases as more consumers become aware and informed. When product awareness does not lead to knowledge of product quality, price is higher and advertising is lower than they would be if product quality is observable and, as the fraction of aware consumers increases, price rises and advertising decreases. Thus, the distortion on price gets larger and the distortion on advertising gets smaller. We also find support for these empirical observations from a data set on Direct-to-Consumer advertising on pharmaceutical drugs.

In chapter 4, we show that if the quality of the entrant is certain, whether the entrant can receive a positive market share or not depends on the income distribution of consumers and the quality difference between the entrant and the incumbent. If the income distribution is small or if qualities are "far apart", it is more likely that the incumbent captures all of the market in which each and every consumer prefers the product of the incumbent to the product of the entrant. As the income distribution increases, the entrant is more likely to have a positive market share along with the incumbent. In other words, as the income distribution increases, the industry evolves from having a monopoly to having duopoly with a fully covered market and from there it evolves to a partially covered market with a duopoly.

By following Kalra et al.(1998), we also observe that if the quality of the entrant is uncertain, the H-type entrant has incentive to signal its quality while the incumbent can profitably prevent this signalling attempt. We propose a pooling equilibrium as a candidate signal-jamming equilibrium. We solve the model numerically and show that the proposed equilibrium indeed exists.

Appendix

Proof of Lemma 1:

Two downstream firm, say firm 1 and firm 2, make quantity orders order of k_1 and k_2 respectively at the order stage. The price of input is c. Each firm takes into account the fact that the amount it orders in a period determines the maximum amount that it can supply to the market in that period. In other words, supplied quantity, q_i can not be more than k_i. Then firms compete in Cournot fashion in the competition stage by setting quantities, $q_i \leq k_i$. We are looking for the subgame perfect equilibrium of this static game. We start solving the game from the last stage.

Competition Stage: At this stage, let k_1 and k_2 denote quantity order of Firm 1 and Firm 2 respectively. Each firm maximizes its profit by setting q_i

$$\max_{q_i} \; q_i(a - q_i - q_j) - ck_i \; \; s.t. \; q_i \leq k_i$$

There are four separate regions to consider depending on the values of k_1 and k_2, which determines whether constraints are binding or not.

Region 1: $k_1 \leq \frac{a-k_2}{2}$ and $k_2 \leq \frac{a-k_1}{2}$

In this region, the constraints of both firms bind. This means that it is profitable for each firm to supply to the market up to their full capacity, i.e., $q_i = k_i$. The profit functions are

$$\Pi_i = k_i(a - k_i - k_j) - ck_i \quad for \; i, j = 1, 2.$$

Region 2: $k_1 \leq \frac{a-k_2}{2}$ and $k_2 > \frac{a-k_1}{2}$

In this region, it is easy to see that only firm 1's constraint is binding, i.e. $q_1^* = k_1$. The profit function of firm 2 is

$$\Pi_2 = q_2(a - k_1 - q_2) - ck_2$$

The firm 2's best response is $q_2^* = \frac{a-k_1}{2}$.

Region 3: $k_1 > \frac{a-k_2}{2}$ and $k_2 \leq \frac{a-k_1}{2}$

In this region, it is easy to see that only firm 2's constraint is binding,i.e. $q_2^* = k_2$. The profit function of firm 1 is

$$\Pi_1 = q_1(a - q_1 - k_2) - ck_1$$

The firm1's best response is $q_1^* = \frac{a-k_2}{2}$

Region 4: $k_1 > \frac{a-k_2}{2}$ and $k_2 > \frac{a-k_1}{2}$

In this region, the constrains of both firms are non-binding: hence, each firm maximize the following profit functions.

$$\Pi_i = q_i(a - q_i - q_j) - ck_i \ \ for \ \ i, j = 1, 2.$$

The equilibrium is characterized by Cournot quantities under zero marginal cost, i.e. $q_i^* = \frac{a}{3}$. The equilibrium profits are

$$\Pi_i = \frac{a^2}{9} - ck_i \ \ for \ \ i = 1, 2.$$

Order Stage: We can move up to the order stage to derive what the subgame perfect equilibrium is. Let's start from Region 4. In this region, both firms are ordering more than what they need in competition stage, and it is optimal for both to reduce their orders to $k_i = q_i^* = \frac{a}{3}$ to minimize their costs. However, $k_i = \frac{a}{3}$ is not in the Region 4: hence, subgame perfect equilibrium can not be in this region. Now let's analyze the Region 3. In this region, the firm 1's best response is to reduce it order to $q_1^* = \frac{a-k_2}{2}$ to minimize its cost. However, $k_1 = \frac{a-k_2}{2}$ is not in the Region 3: hence, the subgame perfect equilibrium can not be in Region 3. Similarly, the equilibrium can not be in Region 2. The only region where the equilibrium reside is Region 1. In this region, both firms produces up to their capacities k_1 and k_2. The profit functions are

$$\Pi_i = k_i(a - k_i - k_j) - ck_i \quad for \ i, j = 1, 2.$$

At the order stage both firms choose their capacities to maximize the profit function above. Hence, the unique subgame perfect equilibrium of the stage game is the Cournot equilibrium, i.e. $k_1 = k_2 = \frac{a-c}{3}$. ■

Proof of lemma 2:

We first show that downstream firm j's critical discount factor, $\underline{\delta}_j$, decreases as its own share, α_j, increases. The downstream j's critical discount factor is

$$\underline{\delta}_j = \frac{9(1+\alpha_j)^2 - 36\alpha_j}{9(1+\alpha_j)^2 - 16} = \frac{(1+\alpha_j)^2 - 4\alpha_j}{(1+\alpha_j)^2 - \frac{16}{9}}$$

We first introduce the following lemma.

Lemma 16. *(i)If $z < 2(1+\alpha)$, $f(\alpha, z) = \frac{(1+\alpha)^2-4\alpha}{(1+\alpha)^2-z}$ decreases in α.*

(ii) If $z < (1+\alpha)^2$, $f(\alpha, z)$ increases in z.

Proof:

$$\begin{aligned}\frac{df}{d\alpha} &= \frac{[2(1+\alpha)-4]\left[(1+\alpha)^2-z\right]-2(1+\alpha)\left[(1+\alpha)^2-4\alpha\right]}{\left[(1+\alpha)^2-z\right]^2}\\ &= \frac{-2z(1+\alpha)-4(1-\alpha)^2+4z+8(1+\alpha)\alpha}{\left[(1+\alpha)^2-z\right]^2}\\ &= \frac{2\left[z-2(1+\alpha)\right](1-\alpha)}{\left[(1+\alpha)^2-z\right]^2}\end{aligned}$$

From the last expression, it is easy to see that if $z < 2(1+\alpha)$, $f(\alpha, z)$ decreases in α. ■

Since the value of z in the downstream firm j's critical discount factor is $\frac{16}{9}$ and it is less than $2(1+\alpha_j)$, the downstream firm j's critical discount factor decreases in α_j.

The cartel's critical discount factor is $\underline{\delta} = max\{\underline{\delta}_1, \underline{\delta}_2\}$. This discount factor is minimized at $\alpha_1 = \alpha_2 = \frac{1}{2}$ since both $\underline{\delta}_1$ and $\underline{\delta}_2$ decreases in its share and $\alpha_1 + \alpha_2 = 1$. Now plugging this solution $\alpha_1 = \alpha_2 = \frac{1}{2}$ into the cartel's discount factor yields that $\underline{\delta} = \frac{9}{17}$. ■

Proof of Lemma 3:

Let Firm 1 be the integrated firm and and Firm 2 the unintegrated firm. The price of input is c. Here is the timing of the stage game. The Firm 2 moves first and chooses its capacity, k_2 at the *order stage*. Then the Firm 1 moves second and chooses its capacity, k_1. Since input production takes time, the Firm 1 must produce its capacity before the competition stage. Let's call this stage where the Firm 1 moves as *interim stage*. Finally, at the *competition stage*, both firms set their quantities,q_1 and q_2, in Cournot fashion taking into account that $q_1 \leq k_1$ and $q_2 \leq k_2$.

Let't solve the game starting from the last stage.

Competition Stage: At this stage, Firm 1 and Firm 2 have already installed their capacities,

k_1 and k_2. They simply play a Cournot game with capacity constraints: $q_1 \leq k_1$ and $q_1 \leq k_1$.

We have already analyzed this subgame in the proof of lemma 1. The result is that firms produce up to their capacities (*Region 1:* $k_1 \leq \frac{a-k_2}{2}$ and $k_2 \leq \frac{a-k_1}{2}$). The payoff functions of Firm 1 and Firm 2 are as follows

$$\Pi_1 = k_1(a - k_1 - k_2) - ck_1$$

$$\Pi_2 = k_2(a - k_1 - k_2) - ck_2$$

Interim Stage: This is the stage between the order and competition stages. At this stage, Firm 1 moves and installs its capacity, k_1 by taking the Firm 2's capacity, k_2, as given. Firm 1 maximizes the following profit function

$$\Pi_1 = k_1(a - k_1 - k_2) - ck_1$$

The solution gives us the Firm 1's best response function

$$k_1 = \frac{a - c - k_2}{2}$$

Order Stage: This is the stage Firm 2 installs its capacity,k_2 by taking the Firm 1's best response function as given. Firm 2 maximizes the following profit function

$$\Pi_2 = k_2(a - k_1 - k_2) - ck_2 = k_2(a - \frac{a - c - k_2}{2} - k_2) - ck_2$$

Firm 2's equilibrium capacity can be written as

$$k_2^* = \frac{a - c}{2}$$

And its equilibrium payoff is

$$\Pi_2^S = \frac{(a-c)^2}{8}$$

Finally Firm 1's equilibrium payoff is

$$\Pi_1^S = \frac{(a-c)^2}{16}$$

■

Proof of Lemma 4:

We need to first show that the integrated firm's critical discount, $\underline{\delta}_1$, decreases in its own share, α_1. The integrated firm's critical discount factor is

$$\delta \geq \underline{\delta}_1 \equiv \frac{(1+\alpha_1)^2 - 4\alpha_1}{(1+\alpha_1)^2 - 1}$$

Notice that the condition, $z < 2(1+\alpha)$, of the lemma 16 is satisfied in this function in which z=1. Hence, we can directly conclude that this critical discount factor decreases in α_1.

The downstream firm 2's share is equal to

$$\alpha_2 = \frac{\Pi_2^S}{\Pi^M} = \frac{\frac{(a-c)^2}{16}}{\frac{(a-c)^2}{8}} = \frac{1}{2}$$

Since the integrated firm 1's critical discount factor, δ_1, decreases in α_1, the downstream firm 1's the minimum discount factor is minimized at the maximum value of α_1, which is $\frac{1}{2}$. Plugging this solution, $\alpha_1 = \frac{1}{2}$, into δ_1 yields the minimum discount factor for the cartel, which is $\underline{\delta^I} = \frac{1}{5}$. ■

Proof of Proposition 1:

From lemma 2, we know that in vertically separated industry, the minimum discount factor $\underline{\delta}^{\neg I}$ is $\frac{9}{17}$. However, in lemma 4, we show that the minimum discount factors with vertical merger is $\frac{1}{5}$, which is less than $\frac{9}{17}$. Hence, the vertical merger facilitates collusion. ■

Proof of Lemma 5:

$$\begin{aligned}\frac{d\underline{\delta}}{dm} &= \frac{[2(1+\alpha)-4]\left[2(m+1)[(m+1)^2+4m\right]-[2(m+1)+4](m+1)^2}{\left[(m+1)^2+4m\right]^2} \\ &= \frac{8(m+1)m-4}{\left[(m+1)^2+4m\right]^2} > 0\end{aligned}$$

■

Proof of Lemma 6:

Let's call the integrated firm as the Firm 1 and the other unintegrated downstream as the Firm j, where $j \in \{2, 3, ..., m\}$. The Firm 1 chooses its production in Stage 3 by taking the production of the unintegrated firms as given

$$\max_{q_1} q_1[a - c - q_1 - (m-1)q_j]$$

We can write the first order conditions as follows

$$a - c - q_1 - (m-1)q_j - q_1 = 0 \iff q_1 = \frac{a - c - (m-1)q_j}{2}.$$

In Stage 2, the Firm j chooses its production q_j while it takes into account the Firm 1's production best response, q_1.

$$\max_{q_j} q_j[a - c - q_j - (m-2)\tilde{q}_j + q_1] = q_j[a - c - q_j - (m-2)\tilde{q}_j + \frac{a - c - q_j - (m-2)\tilde{q}_j}{2}]$$

The first order conditions can be written as follows

$$a - c - (m-1)q_j + \frac{a - c - (m-1)q_j}{2} + q_j(-1 - \frac{1}{2}) = 0.$$

$$q_j = \frac{a-c}{m} \ \ and \ \ q_1 = \frac{a-c}{2m}$$

Plugging q_1 and q_j into the profit functions results in the the following profits of the integrated firm (Firm 1) and an unintegrated downstream firm (Firm 2)

$$\Pi_1^S = \frac{(a-c)^2}{4m^2} \ \ and \ \ \Pi_2^S = \frac{(a-c)^2}{2m^2}.$$

■

Proof of Proposition 2:

The Firm 1's discount factor can be calculated as follows.

$$\frac{\Pi_1^M}{(1-\delta)} > \Pi_1^D + \frac{\delta \Pi_1^C}{1-\delta}$$

The discount factor is equal to

$$\underline{\delta}_1 = \frac{\frac{(1+\alpha_1)^2(a-c)^2}{16} - \frac{\alpha_1(a-c)^2}{4}}{\frac{(1+\alpha_1)^2(a-c)^2}{16} - \frac{(a-c)^2}{4m^2}} = \frac{m^2(1+\alpha_1)^2 - 4m^2\alpha_1}{m^2(1+\alpha_1)^2 - 4}$$

Second, we derive the critical discount factor of an unintegrated downstream firm,

$\underline{\delta}_2$). The unintegrated firm has no incentive to deviate as long as the following inequality is satisfied

$$\frac{\Pi_2^M}{1-\delta} > \Pi_2^D + \frac{\delta \Pi_2^C}{1-\delta}$$

The critical discount factor for the unintegrated downstream firm is equal to

$$\underline{\delta}_2 = \frac{\frac{(2-(m-2)\alpha_2)^2(a-c)^2}{8} - \frac{\alpha_2(a-c)^2}{4}}{\frac{(2-(m-2)\alpha_2)^2(a-c)^2}{8} - \frac{(a-c)^2}{2m^2}} = \frac{m^2(2-(m-2)\alpha_2)^2 - 8m^2\alpha_2}{m^2(2-(m-2)\alpha_2)^2 - 16}$$

Next, we show that maximum of the critical discount factor of the integrated downstream firm, $\underline{\delta}_1$, and the unintegrated downstream firm, $\underline{\delta}_2$, is less than the one under no vertical merger, $\underline{\delta} = \frac{(m+1)^2}{(m+1)^2+4m}$.

We first show that $\underline{\delta} > \underline{\delta}_1$.

$$\underline{\delta} = \tfrac{(m+1)^2}{(m+1)^2+4m} > \underline{\delta}_1 = \tfrac{m^2(1+\alpha_1)^2-4m^2\alpha_1}{m^2(1+\alpha_1)^2-4}$$

Plugging $\alpha_1 = \frac{1}{m}$ into the above equation yields the following

$$\underline{\delta}_1 = \tfrac{m-1}{m+3} < \underline{\delta} = \tfrac{(m+1)^2}{(m+1)^2+4m}$$

for $m > 2$

We next show that $\underline{\delta}_2$ at $\alpha_2 = \frac{1}{m}$ is also less than $\underline{\delta}$.

$$\underline{\delta}_2 = \tfrac{m-2}{m+6} < \underline{\delta} = \tfrac{(m+1)^2}{(m+1)^2+4m}$$

When downstream firms share the monopolistic outcome equally, the critical discount factor with vertical merger is less than the one without vertical merger. This finding is enough for us to conclude that when there are m downstream firm, vertical merger facilitates collusion.

$\blacksquare$

Proof of Lemma 7:

(i) We first show that $\overline{\lambda}_L > \overline{\lambda}_H$

$$\overline{\lambda}_L = (\frac{2\sqrt{L}}{RL - c_L})^2 > \overline{\lambda}_H = (\frac{2\sqrt{H}}{RH - c_H})^2$$

$$\sqrt{L}(RH - c_H) > \sqrt{H}(RL - c_L) \Longleftrightarrow H\sqrt{L}(R - \frac{c_H}{H}) > L\sqrt{H}(R - \frac{c_L}{L})$$

In the last inequality, $H\sqrt{L} > L\sqrt{H}$ is always the case because of the assumption $H > L$. Also, observe that

$$(R - \frac{c_H}{H}) > (R - \frac{c_L}{L}) \Longleftrightarrow \frac{c_L}{L} > \frac{c_H}{H}$$

From the last inequality, we conclude that $\overline{\lambda}_L > \overline{\lambda}_H$ because $\frac{c_L}{L} > \frac{c_H}{H}$ is the efficiency assumption in this paper.

Now, consider the second part of (i) of Lemma 1. If $\lambda \in (\overline{\lambda}_H, 1]$, then

$$a_H^o > a_L^o \Longleftrightarrow a_H^o = \frac{\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}}{2\sqrt{H}} > a_L^o = \frac{\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}}{2\sqrt{L}}$$

$$2\sqrt{L}\sqrt{\lambda}(RH - c_H) > 2\sqrt{H}\sqrt{\lambda}(RL - c_L) \Longleftrightarrow \sqrt{L}(RH - c_H) > \sqrt{H}(RL - c_L)$$

$$H\sqrt{L}(R - \frac{c_H}{H}) > L\sqrt{H}(R - \frac{c_L}{L}) \tag{5.1}$$

We know that $H\sqrt{L} > L\sqrt{H}$ since $H > L$. Then, the inequality (5.1) is satisfied if

$$(R - \frac{c_H}{H}) > (R - \frac{c_L}{L}) \Longleftrightarrow \frac{H}{c_H} > \frac{L}{c_L}$$

The efficiency assumption of $\frac{c_L}{L} > \frac{c_H}{H}$ leads to $a_H^o > a_L^o$ for all $\lambda \in (\overline{\lambda}_H, 1]$

(ii)

$$p_H^c > p_H^c \Longleftrightarrow p_H^c = \frac{RH + c_H}{2} > p_H^c = \frac{RL + c_L}{2}$$

$$R(H - L) + c_H - c_L > 0$$

where $H > L$ and $c_H > c_L$. Therefore, the high quality monopolist charges higher prices compared to the low quality monopolist. ■

Proof of Lemma 8:

Observe first that

$$\Pi_L(p_H^o, a; 1) > \Pi_L(p_L^o, a; 0) \text{ for all } \lambda \in [0, 1]$$

$$\Pi_L(p_H^o, a; 1) = [\frac{\lambda a}{1 + a} + (1 - \lambda)](\frac{RH - c_H}{2H})(\frac{RH + c_H - 2c_L}{2}) - a \tag{5.2}$$

$$\Pi_L(p_L^o, a; 0) = [\frac{\lambda a}{1 + a} + (1 - \lambda)](\frac{RL - c_L}{2L})(\frac{RL - c_L}{2}) - a \tag{5.3}$$

The payoff in the equation (5.2) is bigger than the payoff in the equation (5.3) if following inequality is satisfied

$$(\frac{RH - c_H}{2H})(\frac{RH + c_H - 2c_L}{2}) > (\frac{RL - c_L}{2L})(\frac{RL - c_L}{2}) \tag{5.4}$$

It is always the case that

$$(\frac{RH + c_H - 2c_L}{2}) > (\frac{RL - c_L}{2}) \Longleftrightarrow R(H - L) + c_H - c_L > 0$$

Moreover, it is also the case that

$$(\frac{RH - c_H}{2H}) > (\frac{RL - c_L}{2L}) \Longleftrightarrow \frac{c_L}{L} > \frac{c_H}{H}$$

As a result, in the inequality (5.4) both elements of the right hand side are bigger than the elements of the left hand side. Consequently, we can conclude that $\Pi_L(p_H^o, a; 1) > \Pi_L(p_L^o, a; 0)$ for all $\lambda \in [0, 1]$

Now recall from Lemma 1 that if $\lambda \in (\overline{\lambda}_H, 1]$, then $a_H^o > a_L^o$ while if $\lambda \in [0, \overline{\lambda}_H]$, then $a_H^o = a_L^o = 0$

Start with the region where $\lambda \in [0, \overline{\lambda}_H]$. Since $a_H^o = a_L^o = 0$ (i.e. they are equal), the following inequality is the result of first step in this lemma.

$$\Pi_L(p_H^o, a_H^o; 1) > \Pi_L(p_L^o, a_L^o; 0)$$

Now, consider the region,$\lambda \in (\overline{\lambda}_H, 1]$. One can show that $\Pi_L(p_H^o, a_H^o; 1) > \Pi_L(p_L^o, a_L^o; 0)$ also holds with $a_H^o > a_L^o$ by the following

$$\frac{d\Pi_L(p_H^o, a; 1)}{da} |_{a=a_H^o} > 0$$

This last condition means that if the low quality firm can mimic the high quality firm, it prefers higher level of advertising to a_L^o (i.e. its profit is higher at a_H^o)

$$\frac{d\Pi_L(p_H^o, a; 1)}{da}\Big|_{a=a_H^o} = \frac{\lambda}{(1+\frac{\sqrt{\lambda}(RH-c_H)-2\sqrt{H}}{2\sqrt{H}})^2}(\frac{RH-c_H}{2H})(\frac{RH+c_H-2c_L}{2}) - 1 > 0$$

After some straightforward calculations, the inequality reduces to the following

$$(RH - 2c_L + c_H) > RH - c_H \Longleftrightarrow c_H > c_L.$$

Thus, the observable quality price and advertising spending (p_H^o, a_H^o) cannot be a separating equilibrium. Hence, if the high quality firm is to separate, it has to distort price and/or advertising from (p_H^o, a_H^o). In other words, signaling issue is relevant. ■

Proof of Proposition 3:

The Lagrangian for the maximization problem is the following;

$$\Lambda = [\lambda\frac{a}{1+a} + (1-\lambda)](R - \frac{p}{H})(p - c_H) - a]$$
$$+\mu[\Pi_L^o - [\lambda\frac{a}{1+a} + (1-\lambda)](R - \frac{p}{H})(p - c_L) + a]$$

The first order conditions are;

$$\frac{\partial\Lambda}{\partial p} = [R - \frac{2p}{H} + \frac{c_H}{H}] + \mu[R - \frac{2p}{H} + \frac{c_L}{H}] = 0 \tag{5.5}$$

$$\frac{\partial \Lambda}{\partial a} = [\frac{\lambda}{(1+a)^2}(R-\frac{p}{H})(p-c_H)-1] + \mu[\frac{\lambda}{(1+a)^2}(R-\frac{p}{H})(p-c_L)-1] = 0 \tag{5.6}$$

$$\frac{\partial \Lambda}{\partial \mu} = \Pi_L^o - [\lambda\frac{a}{1+a} + (1-\lambda)](R-\frac{p}{H})(p-c_L) + a = 0 \tag{5.7}$$

By solving (5.5) and (5.6)the optimal level of advertising and price in the equilibrium is

$$a = \frac{\sqrt{\lambda}(RH-p) - \sqrt{H}}{\sqrt{H}} \tag{5.8}$$

In order to find the equilibrium pair of p_H and a_H , we solve equation (5.7) and equation (5.8). The optimal advertising level, a_H, is the solution of the following equation

$$\sqrt{H}{a_H}^2 - [\sqrt{\lambda}(RH-c_L) - 2\sqrt{H}]a_H - [(1-\lambda)[\sqrt{\lambda}(RH-c_L) - 2\sqrt{H}] - \sqrt{H}(1-\lambda+a_L^o)^2 - (1-\lambda)\lambda\sqrt{H}] = 0$$

The next step is to find the available roots of this function. The roots are a_H^1 and a_H^2 where $a_H^1 < a_H^2$. Also, there is a corresponding price p_H^1 and p_H^2 for each level of advertising a_H^1 and a_H^2 respectively.

It should be shown that $(a_H^1, p_H^1) = (a_H^s, p_H^s)$ gives higher profit compared to (a_H^2, p_H^2) where $a_H^1 < a_H^2$ and $p_H^1 > p_H^2$ and both (a_H^1, p_H^1) and (a_H^2, p_H^2) yield the same for a mimicking low quality firm. Basically, it is required to show that $\Pi^H(a_H^1, p_H^1; 1) > \Pi_H(a_H^2, p_H^2; 1)$ when $\Pi_L(a_H^1, p_H^1; 1) = \Pi_L(a_H^2, p_H^2; 1)$. Then, we show that (a_H^1, p_H^1) is the least-cost separating equilibrium or the one survives by standard refinement (i.e., Cho and Kreps (1987)). Observe that

$\Pi_H(a_H^1, p_H^1; 1) - \Pi_H(a_H^2, p_H^2; 1)$
$= [\Pi_H(a_H^1, p_H^1; 1) - \Pi_H(a_H^2, p_H^2; 1)] - [\Pi_L(a_H^1, p_H^1; 1) - \Pi_L(a_H^2, p_H^2; 1)]$
$= [c_H - c_L][D(a_H^2, p_H^2; 1) - D(a_H^1, p_H^1; 1)]$

Since $c_H > c_L$, the high-quality firm gains more at (a_H^1, p_H^1) if demand is lower. It is easy to show that, however, demand at (a_H^1, p_H^1) is always lower than demand at (a_H^2, p_H^2) since $a_H^1 < a_H^2$ and $p_H^1 > p_H^2$. This equilibrium can also be called least-cost separating equilibrium. The only plausible root is;

$$a_H^s = a_H^1 = \frac{[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{\Delta}}{2\sqrt{H}}$$

where $\Delta = [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1-\lambda+a_L^o)^2 - (1-\lambda)\lambda\sqrt{H}]$

Then by using the equality $p_H^s = RH - \frac{(1+a_H^s)\sqrt{H}}{\sqrt{\lambda}}$, the separating equilibrium price is;

$$p_H^s = p_H^1 = \frac{(RH + c_L)\sqrt{\lambda} + \sqrt{\Delta}}{2\sqrt{\lambda}}$$

In case of $\lambda \in [\overline{\lambda}_L, \overline{\lambda}_K)$, a_L^o is positive in the separating equilibrium with the profit $\Pi_L^o = \frac{(1+a_L^o-\lambda)^2}{\lambda} - (1-\lambda) = \frac{[(RL-c_L)^2+4L] - 4\sqrt{L}\sqrt{\lambda}(RL-c_L)}{4L}$. Then, the incentive compatibility condition for low-quality firm is satisfied if

$$(1-\lambda)(R - \frac{p}{H})(p - c_L) = \Pi_L^o = \frac{[(RL - c_L)^2 + 4L] - 4\sqrt{L}\sqrt{\lambda}(RL - c_L)}{4L} \qquad (5.9)$$

The problem reduces to find price levels p_H^s that satisfies the equation (5.9). The only plausible root of this function is

$$p_s^H = \frac{RH + c_L + \sqrt{\Delta_1}}{2}$$

where $\Delta_1 = (RH + c_L)^2 - 4(RHc_L + \frac{H\Pi_L^o}{1-\lambda})$ Now, let's check the boundary values of p_H^s at $\overline{\lambda}_L$ and $\overline{\lambda}_K$. At $\lambda = \overline{\lambda}_L = (\frac{2\sqrt{L}}{RL-c_L})^2$, the incentive compatibility condition (5.9) reduces to

$$(R - \frac{p_H}{H})(p_H - c_L) = \frac{(RL-c_L)^2}{4L}$$

It reduces to

$$p_H^s(\overline{\lambda}_L) = \overline{p} = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-{c_L}^2)(H-L)}{4L}}$$

Next thing to solve is that what would be the value of p_H^s at $\overline{\lambda}_K$. First, remember that at $\overline{\lambda}_K = max\{\lambda : a_H^s(\lambda) = 0\}$

$$a_H^s = \frac{[\sqrt{\lambda}(RH-c_L)-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}} = 0$$

where $\Delta = [\sqrt{\lambda}(RH-c_L) - 2\sqrt{H}]^2 - \frac{H}{L}[\sqrt{\lambda}(RL-c_L) - 2\sqrt{L}] + 4(1-\lambda)\sqrt{\lambda}\sqrt{H}\sqrt{L}[\sqrt{L}(RH-c_L) - \sqrt{H}(RL-c_L)]$

By plugging Δ into $a_H^s = 0$, we get the following equation:

$$H(RL-c_L)^2\overline{\lambda}_K + 4LH = 4\sqrt{\overline{\lambda}_K}\sqrt{H}\sqrt{L}[(1-\overline{\lambda}_K)\sqrt{L}(RH-c_L) + \overline{\lambda}_K\sqrt{H}(RL-c_L)] \quad (5.10)$$

Now, find the optimal price from the incentive compatibility (5.9) condition of the low quality firm

$(R - \frac{p}{H})(p - c_L) = \frac{[(RL-c_L)^2+4L]-4\sqrt{L}\sqrt{\overline{\lambda}_K}(RL-c_L)}{4L(1-\overline{\lambda}_K)}$ Now, lets use the equation (5.10) in incentive compatibility condition. $(R - \frac{p}{H})(p - c_L) = \sqrt{\overline{\lambda}_K}[\frac{RH-c_L}{\sqrt{H}} - \frac{RL-c_L}{\sqrt{L}}]$

Then, $p_H^s(\overline{\lambda}_K) = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-{c_L}^2)(H-L)}{4L} - H\sqrt{\overline{\lambda}_K}[\frac{RH-c_L}{\sqrt{H}} - \frac{RL-c_L}{\sqrt{L}}]}$

$p_H^s = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-{c_L}^2)(H-L)-4\sqrt{\overline{\lambda}_K}\sqrt{H}\sqrt{L}[\sqrt{L}(RH-c_L)-\sqrt{H}(RL-c_L)]}{4L}}$

This last equation is equal to the $p_H^s(\overline{\lambda}_K)$.

In the region of $\lambda \in [0, \overline{\lambda}_L]$, we start with analyzing the case where all consumers are aware of the product ($\lambda = 0$). Consumers consist of only aware-type so that advertising spending only has the role of dissipative signaling (*money burning*) and does not directly enhance demand.

The payoff function of q-quality is

$$\Pi_q(p, a; \rho) = D(p, a; \rho)(p - c_q) - a$$

The profit maximizing equilibrium price and advertising are $p_L^o = \frac{RL+c_L}{2}$ and $a_L^o = 0$ for the low-quality firm and $p_H^o = \frac{RH+c_H}{2}$ and $a_H^o = 0$ for the high-quality firm and the profits are $\Pi_L^o = \frac{(RL-c_L)^2}{4L}$ and $\Pi_H^o = \frac{(RH-c_H)^2}{4L}$.

In the separating equilibrium, the low-quality firm would play the strategy $(p_L^o, a_L^o) = (\frac{RL+c_L}{2}, 0)$. The incentive compatibility condition for the low quality firm is as follows

$$\Pi_L(p, a; 1) = (R - \frac{p}{H})(p - c_L) - a \leq \Pi_L^o = \frac{(RL-c_L)^2}{4L} \qquad (IC_L)$$
$$(R - \frac{p}{H})(p - c_L) - \frac{(RL-c_L)^2}{4L} \leq a \qquad (IC_L)$$

Then, by using IC_L, define a function $a(p)$ as the level of advertising required to deter imitation by a low quality firm for a given price p. Another way to think of the advertising decision is asking the question, how much advertising should the high-quality firm employ just to have the incentive compatibility condition of the low-quality firm satisfied?

$$a(p) = max\{0, (R - \frac{p}{H})(p - c_L) - \frac{(RL-c_L)^2}{4L}\}$$

It is easy to show that $\Pi_L^o = \frac{(RL-c_L)^2}{4L} = \Pi_L(\underline{p}; 1) = \Pi_L(\overline{p}; 1) < \Pi_L(p_L^o; 1) < \Pi_L(p_H^o; 1) < \Pi_L(p_L^H; 1)$ where $\underline{p} < p_L^o < p_L^H < p_H^o < \overline{p}$ along with the values

$$\underline{p} = \frac{RH+c_L}{2} - \sqrt{\frac{(R^2HL-{c_L}^2)(H-L)}{4L}}$$
$$p_L^o = \frac{RL+c_L}{2}$$

$$p_L^H = \frac{RH+c_L}{2}$$

$$p_H^o = \frac{RH+c_H}{2}$$

$$\overline{p} = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-c_L{}^2)(H-L)}{4L}}$$

Even under the most favorable beliefs, the low quality firm does not mimic any price below $\underline{p}$ and above $\overline{p}$ since corresponding profit is less than its observable quality profit. Hence, $p \notin (\underline{p}, \overline{p})$, advertising spending is not required to ensure separation. However, given that price is in the region of $(\underline{p}, \overline{p})$, at least an amount $a(p)$ of advertising has to be spent to deter the mimicry of lower quality. Therefore, the maximization problem for the high quality firm could be written in the following form;

$$\max_{p,a} \quad \Pi_H(p,a;1) = \frac{(RH-p)(p-c_H)}{H} - a$$

$$\textit{subject to}$$

$$(i) \qquad a \geq a(p)$$

$$(ii) \qquad p \; \epsilon \; [\underline{p}, \overline{p}]$$

The firm will choose the lowest possible advertising, $a = a(p)$, to minimize the cost; then, its profit and the maximization problem reduce to

$$\Pi_H(p,a;1) = \frac{(RH-p)(p-c_H)}{H} - a(p) = \frac{(RH-p)(p-c_H)}{H} - [\frac{(RH-p)(p-c_L)}{H} - \frac{(RL-c_L)^2}{4L}]$$

$$\max_{p,a} \Pi_H(p,a;1) = \frac{(c_H-c_L)p}{H} - \frac{(c_H-c_L)RH}{H} + \frac{(Rl-c_L)^2}{4L}$$

$$\textit{subject to}$$

$$p \; \epsilon \; [\underline{p}, \overline{p}]$$

Since the payoff function of the high-quality firm increases in price, it is optimal to increase the price to $\overline{p}$. Also, for the region $p\epsilon(0,\underline{p})\bigcup(\overline{p},\infty)$, the price itself is enough to ensure separation; therefore, it is again optimal to choose $\overline{p}$. To sum up, higher quality would price at $\overline{p}$ and does not advertise in the separating equilibrium.

For $\lambda \in (0, \overline{\lambda}_L]$, the idea of the proof is similar. The low quality firm does not mimic the high quality.

$$(1-\lambda)(R-\tfrac{p_H}{H})(p_H-c_L) = \Pi_L^o = (1-\lambda)\tfrac{(RL-c_L)^2}{4L} \qquad IC_L$$

Hence, the equilibrium price is $p_H^s = \overline{p} = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-{c_L}^2)(H-L)}{4L}}$ ■

Proof of Corollary 1:

If $\lambda \in (\overline{\lambda}_K, 1]$, the separating equilibrium price is

$$p_H^s = \frac{(RH+c_L)\sqrt{\lambda}+\sqrt{\Delta}}{2\sqrt{\lambda}} = \frac{RH+c_L}{2} + \frac{\sqrt{\Delta}}{2\sqrt{\lambda}}$$

where $\Delta = \frac{\lambda[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}{L}$

$$p_H^s = \frac{RH+c_L}{2} + \frac{\sqrt{\frac{\lambda[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}{L}}}{2\sqrt{\lambda}}$$

$$p_H^s = \frac{RH+c_L}{2} + \frac{\sqrt{(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}}\sqrt{(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}}}{2\sqrt{L}}$$

Let's take the derivative of p_H^s with respect to λ

$$\frac{dp_H^s}{d\lambda} = \frac{\sqrt{(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}}}{2\sqrt{L}} \frac{1}{\sqrt{(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}}} \frac{-2\sqrt{H}\sqrt{L}}{\sqrt{\lambda}} < 0$$

As λ increases, the high quality firm's price p_H^s decreases.

Now, let's turn to the second part of the Corollary, $p_H^s > p_H^o$. The proof of $p_H^s > p_H^o$ follows immediately from the proof of $a_H^s < a_H^o$.

$$a_H^s - a_H^o = \frac{[(RH-c_L)\sqrt{\lambda}-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}} - \frac{(RH-c_H)\sqrt{\lambda}-2\sqrt{H}}{2\sqrt{H}}$$

$$a_H^s - a_H^o = \frac{(c_H-c_L)\sqrt{\lambda}-\sqrt{\Delta}}{2\sqrt{H}}$$

All we need is the sign of $(c_H - c_L)\sqrt{\lambda} - \sqrt{\Delta}$; hence, multiplying it with some other positive expression is not going to affect the sign.

$$[\sqrt{\Delta} - (c_H - c_L)\sqrt{\lambda}][\sqrt{\Delta} + (c_H - c_L)\sqrt{\lambda}] = \Delta - \lambda(c_H - c_L)^2$$

$$sign\{\Delta - \lambda(c_H - c_L)^2\} = -sign\{(c_H - c_L)\sqrt{\lambda} - \sqrt{\Delta}\}$$

We are interested in the sign of $\Delta - \lambda(c_H - c_L)^2$

$$\begin{aligned}\Delta - \lambda(c_H - c_L)^2 = \quad & [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1-\lambda)\sqrt{\lambda}(RH - c_L) - \sqrt{H}] \\ & -4H(1-\lambda + a_L^o)^2 - 4H(1-\lambda)\lambda - \lambda(c_H - c_L)^2\end{aligned}$$

$$\begin{aligned}= \quad & [\sqrt{\lambda}(RH - c_H) - 2\sqrt{H} + (c_H - c_L)\sqrt{\lambda}]^2 + 4\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] \\ & -\sqrt{H}(1-\lambda + a_L^o)^2 - (1-\lambda)\lambda\sqrt{H}] - \lambda(c_H - c_L)^2\end{aligned}$$

$$\begin{aligned}= \quad & [\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}]^2 + 2[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}](c_H - c_L)\sqrt{\lambda} \\ & 4\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1-\lambda + a_L^o)^2 - (1-\lambda)\lambda\sqrt{H}]\end{aligned}$$

Let's plug a_L^o into the equation and multiply it by L.

$$\begin{aligned}\Delta - \lambda(c_H - c_L)^2 = & L[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}]^2 + 2L[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}](c_H - c_L)\sqrt{\lambda} + \\ & 4L\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1-\lambda + \tfrac{\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}}{2\sqrt{L}})^2 - (1-\lambda)\lambda\sqrt{H}]\end{aligned}$$

$$\begin{aligned}> \quad & 4L\sqrt{H}(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - 4H\sqrt{L}[(1-\lambda)[\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}] \\ & -4HL(1-\lambda)^2 - 4HL(1-\lambda)\lambda\end{aligned}$$

$$= \quad 4\sqrt{L}\sqrt{H}[(1-\lambda)\sqrt{\lambda}[\sqrt{L}(RH - c_H) - \sqrt{H}(RL - c_L) > 0$$

First inequality follows form the fact that

$$L[\sqrt{\lambda}(RH - c_H) - 2\sqrt{H}]^2 > H[\sqrt{\lambda}(RL - c_L) - 2\sqrt{L}]^2.$$

The second inequality follows from the fact that

$$\frac{H}{c_H} > \frac{L}{c_L}.$$

Hence $\Delta - \lambda(c_H - c_L)^2 > 0$ and $sign\{(c_H - c_L)\sqrt{\lambda} - \sqrt{\Delta}\} < 0$ so that $a_H^s < a_H^o$

$$p_H^s - p_H^o = \frac{(RH+c_L)\sqrt{\lambda}+\sqrt{\Delta}}{2\sqrt{\lambda}} - \frac{(RH+c_H)}{2}$$

$$p_H^s - p_H^o = \frac{\sqrt{\Delta}-(c_H-c_L)\sqrt{\lambda}}{2\sqrt{\lambda}}$$

In fact, we have just shown that

$$sign\{\sqrt{\Delta} - (c_H - c_L)\sqrt{\lambda}\} > 0.$$

Hence, $p_H^s > p_H^o$. ■

Proof of Corollary 2:

(i) If $\lambda \in (\overline{\lambda}_K, 1]$, the separating equilibrium advertising level is

$$a_H^s = \frac{[\sqrt{\lambda}(RH-c_L)-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}}$$

$$a_H^s = \sqrt{\lambda}[\frac{(RH-c_L)}{2\sqrt{H}} - \frac{\sqrt{[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}}{2\sqrt{H}\sqrt{L}}] - 1$$

$$\frac{da_H^s}{d\lambda} = \frac{\lambda^{-0.5}}{2}[\frac{(RH-c_L)}{2\sqrt{H}} - \frac{\sqrt{[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}}{2\sqrt{H}\sqrt{L}}]+$$

$$\sqrt{\lambda}[-\frac{\left[((RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H})((RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L})\right]^{-0.5}}{2\sqrt{H}\sqrt{L}}(\frac{-2\lambda^{-0.5}\sqrt{H}\sqrt{L}}{2\sqrt{H}\sqrt{L}})] > 0$$

$$\frac{da_H^s}{d\lambda} > 0$$

(ii) If $\lambda \in (\overline{\lambda}_K, 1]$, $a_H^s - a_L^s$ can be written as follows

$$a_H^s - a_L^o = \frac{[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{\Delta} - 2\sqrt{H}a_L^o}{2\sqrt{H}} = \frac{A - B - \sqrt{\Delta}}{2\sqrt{H}}$$

where $A = \sqrt{\lambda}(RH - c_L) - 2\sqrt{H}$ and $B = 2\sqrt{H}a_L^o$.

Now, the task is to write down Δ in terms of A and B.

$$\Delta = [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1-\lambda)[\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}] - \sqrt{H}(1-\lambda+a_L^o)^2 - (1-\lambda)\lambda\sqrt{H}]$$

$$\Delta = [\sqrt{\lambda}(RH - c_L) - 2\sqrt{H}]^2 + 4\sqrt{H}[(1-\lambda)\sqrt{\lambda}(RH - c_L) - 4H[(a_L^o)^2 + 2(1-\lambda)a_L^o + 2(1-\lambda)]]$$

We also know that $a_L^o = \frac{(RL-c_L)\sqrt{\lambda}-2\sqrt{L}}{2\sqrt{L}} \Leftrightarrow A = (RL - c_L) - 2\sqrt{L}$

$$\begin{aligned}
\Delta &= A^2 + 4\sqrt{H}(1-\lambda)[A + 2\sqrt{H}] - 4H[(a_L^o)^2 + 2(1-\lambda)a_L] + 8H(1-\lambda)] \\
&= A^2 + 4\sqrt{H}(1-\lambda)A - 4H[(a_L^o)^2 + 2(1-\lambda)a_L^o] \\
&= [A + 2\sqrt{H}(1-\lambda)]^2 - 4H(1-\lambda)^2 - 4H[a_L + (1-\lambda)]^2 + 4H(1-\lambda)^2 \\
&= [A + 2\sqrt{H}(1-\lambda)]^2 - [2\sqrt{H}a_L + 2\sqrt{H}(1-\lambda)]^2 \\
&= [A + 2\sqrt{H}(1-\lambda)]^2 - [B + 2\sqrt{H}(1-\lambda)]^2 = (A-B)[A + B + 4\sqrt{H}(1-\lambda)]
\end{aligned}$$

$$\Delta = (A-B)[A + B + 4\sqrt{H}(1-\lambda)]$$

Now, let's go back to our original problem and substitute for Δ

$$a_H^s - a_L^o = \frac{A-B-\sqrt{\Delta}}{2\sqrt{H}}$$

$$a_H^s - a_L^o = \frac{A-B-\sqrt{(A-B)[A+B+4\sqrt{H}(1-\lambda)]}}{2\sqrt{H}} = \frac{\sqrt{A-B}\left[\sqrt{A-B}-\sqrt{A+B+4\sqrt{H}(1-\lambda)}\right]}{2\sqrt{H}} < 0$$

■

Proof of Proposition 4:

Let's first calculate the high quality firm's profit in the separating equilibrium

If $\lambda \in (\bar{\lambda}_L, 1]$, then $(p_H^s, a_H^s) = (\frac{(RH+c_L)\sqrt{\lambda}+\sqrt{\Delta}}{2\sqrt{\lambda}}, \frac{[(RH-c_L)\sqrt{\lambda}-2\sqrt{H}]-\sqrt{\Delta}}{2\sqrt{H}})$ and

$$\Pi_H(p_H^s, a_H^s; 1) = \frac{[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}\lambda][(RH+c_L-2c_H)\sqrt{\lambda}+\sqrt{\Delta}]-2\sqrt{H}\lambda[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}]}{4H\lambda}$$

where $\Delta = \frac{\lambda[(RH-c_L)\sqrt{L}-(RL-c_L)\sqrt{H}][(RH-c_L)\sqrt{L}+(RL-c_L)\sqrt{H}-4\sqrt{\lambda}\sqrt{H}\sqrt{L}]}{L}$

And the H-quality firm's profit is in case of deviation from the separating equilibrium

$$\max_{p,a} \Pi_H(p,a;0) = \frac{(RL-c_H)^2 - 4\sqrt{L}\sqrt{\lambda}(RL-c_H)+4L}{4L}$$

The separating equilibrium exists if the H-quality prefers the separating equilibrium pair to any other pair where consumer mistakenly believes that it is of a low quality firm.

$$\Pi_H(p_H^s, a_H^s; 1) > \max_{p,a} \Pi_H(p,a;0)$$

$$\Pi_H(p_H^s, a_H^s; 1) - \max_{p,a} \Pi_H(p,a;0) > 0$$

$$\frac{[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}\lambda][(RH+c_L-2c_H)\sqrt{\lambda}+\sqrt{\Delta}]-2\sqrt{H}\lambda[(RH-c_L)\sqrt{\lambda}-\sqrt{\Delta}-2\sqrt{H}]}{4H\lambda}$$
$$-\frac{(RL-c_H)^2-4\sqrt{L}\sqrt{\lambda}(RL-c_H)+4L}{4L} > 0$$

After some algebra, the inequality reduces to the following;

$$\Pi_H(p_H^s, a_H^s; 1) - \max \Pi_H(p,a;0) = \frac{(c_H-c_L)\lambda[2L(\frac{\sqrt{\Delta}}{\sqrt{\lambda}}+\lambda c_L+2\sqrt{H}\sqrt{\lambda})-H(c_H+c_L+4\sqrt{L}\sqrt{\lambda})]}{16LH\lambda} > 0$$

The increasing marginal cost assumption (i.e., $c_H > c_L$) is necessary for the existence. The following part of the last inequality determines the conditions under which the separating equilibrium exists.

$$[2L(\frac{\sqrt{\Delta}}{\sqrt{\lambda}} + \lambda c_L + 2\sqrt{H}\sqrt{\lambda}) - H(c_H + c_L + 4\sqrt{L}\sqrt{\lambda})] > 0$$

$$\Longleftrightarrow$$

$$2L\frac{\sqrt{\Delta}}{\sqrt{\lambda}} > H(c_H + c_L + 4\sqrt{L}\sqrt{\lambda}) - 2L(\lambda c_L + 2\sqrt{H}\sqrt{\lambda})$$

Let's take the square of both sides

$$2L^2\frac{\Delta}{\lambda} > [H(c_H + c_L + 4\sqrt{L}\sqrt{\lambda}) - 2L(\lambda c_L + 2\sqrt{H}\sqrt{\lambda})]^2.$$

Now substitute $\Delta = \frac{\lambda[(R^2LH - {c_L}^2)(H-L) - 4\sqrt{\lambda LH}(R\sqrt{LH} - c_L)(\sqrt{H} - \sqrt{L})]}{L}$ into the equation;

$$4L(R^2LH - {c_L}^2)(H - L) > [H(c_H + c_L + 4\sqrt{L}\sqrt{\lambda}) - 2L(\lambda c_L + 2\sqrt{H}\sqrt{\lambda})]^2 + 16L\sqrt{\lambda LH}(R\sqrt{LH} - c_L)(\sqrt{H} - \sqrt{L}).$$

This inequality is satisfied if (i) (H-L) is not too small and (2) R is not too small.

Let's now calculate the high quality firm's profit at the proposed separating equilibrium $(p_H^s, a_H^s) = (\overline{p}, 0)$ when $\lambda \in [0, \overline{\lambda}_L]$

$$\begin{aligned}\Pi_H(p_H^s, a_H^s; 1) = \Pi_H(\overline{p}, 0; 1) &= (1-\lambda)(R - \frac{\overline{p}}{H})(\overline{p} - c_H) \\ &= (1-\lambda)(\frac{(RL - c_L)^2}{4L} - \frac{(RH - \overline{p})(c_H - c_L)}{H})\end{aligned}$$

The following is the high quality firm's profit when it deviates from the separating equilibrium. Realize that the high quality firm would not advertise in case of deviation because $\lambda \in [0, \overline{\lambda}_L]$ i.e., even the low quality firm with lower marginal cost does not advertise.

$$\max_p \Pi_H(p, 0; 0) = \max_p (1-\lambda)(R - \tfrac{p}{L})(p - c_H) = (1-\lambda)\tfrac{(RL - c_H)^2}{4L}$$

The separating equilibrium exists if the the following condition is satisfied;

$$\Pi_H(p_H^s, a_H^s; 1) = (1-\lambda)(\tfrac{(RL - c_L)^2}{4L} - \tfrac{(RH - \overline{p})(c_H - c_L)}{H}) > \max_p \Pi_H(p, 0; 0) = (1-\lambda)\tfrac{(RL - c_H)^2}{4L}$$

$$\tfrac{(c_H - c_L)[H(2RL - c_L - c_H) - 4L(RH - \overline{p})]}{4LH} > 0.$$

In what follows, we show that the following equation is satisfied if (i) (H-L) is not too small and (2) R is not too small.

$$sign\{H(2RL - c_L - c_H) - 4L(RH - \overline{p})\} > 0$$

Let's substitute $\overline{p} = \frac{RH+c_L}{2} + \sqrt{\frac{(R^2HL-c_L{}^2)(H-L)}{4L}}$ into the last equation.

$$sign\{-Hc_H - Hc_L + 2Lc_L + 2\sqrt{L}\sqrt{(R^2HL - c_L^2)(H-L)}\}$$

$$sign\{2\sqrt{L}\sqrt{(R^2HL - c_L^2)(H-L)} - [H(c_H + c_L) - 2Lc_L]\}.$$

Let's multiply the last equation with the following positive equation

$$2\sqrt{L}\sqrt{(R^2HL - c_L^2)(H-L)} + [H(c_H + c_L) - 2Lc_L].$$

Then the inequality turns out to be;

$$sign\{4L(R^2HL - c_L^2)(H-L) - [H(c_H + c_L) - 2Lc_L]^2\}.$$

After some algebra, this equality reduces to the following;

$$sign\{[2R\sqrt{H}L\sqrt{H-L}]^2 - [H(c_H + c_L)]^2\}$$

$$sign\{[2R\sqrt{H}L\sqrt{H-L} - H(c_H + c_L)][2R\sqrt{H}L\sqrt{H-L} + H(c_H + c_L)]\}.$$

Finally, the separating equilibrium$(p_H^s, a_H^s) = (\overline{p}, 0)$ exists if

$$sign\{2R\sqrt{H}L\sqrt{H-L} - H(c_H + c_L)\} > 0.$$

This inequality holds if (i)(H-L) is not too small (ii) R is not too small.

Finally, if $\lambda \in [\overline{\lambda}_L, \overline{\lambda}_K]$, then $(p_H^s, a_H^s) = (\frac{RH+c_L+\sqrt{\Delta_1}}{2}, 0)$ and $\Pi_H(p_H^s, a_H^s) = (1-\lambda)(R - \frac{p_H^s}{H})(p_H^s - c_H)$. In case of deviation, we have specified the off-the-equilibrium path beliefs such that consumers believe it is of low quality. In what follows, we describe the deviation profit of high quality

$$\Pi_H(p, a; 0) = (1-\lambda) + \lambda\frac{a}{1+a}(R - \frac{p}{L})(p - c_H) - a$$

$$(p_H^L, a_H^L) = (\tfrac{RL+c_H}{2}, \tfrac{\sqrt{\lambda}(RL-c_H)-2\sqrt{L}}{2\sqrt{L}}) \quad .$$

With $\lambda \in [\overline{\lambda}_L, \overline{\lambda}_K]$, there are two separate cases: (i)$a_H^s = 0 < a_H^L < a_L^o$, and (ii)$a_H^L = a_H^s = 0 < a_L^o$.

Case(i): The H-quality firm's deviation advertising is positive, i.e. $a_H^s = 0 < a_H^L = \frac{\sqrt{\lambda}(RL-c_H)-2\sqrt{L}}{2\sqrt{L}} < a_L^o$. Hence, the following inequality $\Pi_H(p_H^s, a_H^s; 1) > \Pi_H(p_H^L, a_H^L; 0)$ where

$$\begin{aligned}\Pi_H(p_H^s, a_H^s; 1) &= (1-\lambda)(R - \frac{p_H^s}{H})(p_H^s - c_H) \\ &= (1-\lambda)(R - \frac{RH + c_L + \sqrt{\Delta_1}}{2H})(\frac{RH + c_L + \sqrt{\Delta_1}}{2} - c_H)\end{aligned}$$

$$\Pi_H(p_H^L, a_H^L; 0) = \tfrac{(RL-c_H)^2 - 4\sqrt{L}\sqrt{\lambda}(RL-c_H)+4L}{4L}$$

After some algebra, the inequality reduces to

$$(c_H - c_L)[2RL - c_L - c_H - 4\sqrt{L}\sqrt{\lambda} - (1-\lambda)(RH - c_L - \sqrt{\Delta_1})] \geq 0$$

Case(ii): By using the previous case where $\lambda \in [0, \overline{\lambda}_L]$, it is easy to show that $\overline{p} > p_H^s > p_H^L$. The following is the incentive compatibility condition for H-quality firm

$$\begin{aligned}\Pi_H(p_H^s, a_H^s) &= (1-\lambda)(R - \frac{p_H^s}{H})(p_H^s - c_H) \\ &> (1-\lambda)(R - \frac{\overline{p}}{H})(\overline{p} - c_H) \\ &> \Pi(p_H^L, a_H^L; 0) = (1-\lambda)\frac{(RL - c_H)^2}{4L}.\end{aligned}$$

However, this is the same inequality with the case where $\lambda \in [0, \overline{\lambda}_L]$; hence, the separating equilibrium exists if (i)(H-L) is not too small (ii) R is not too small. ■

No pooling equilibrium:

The proof is similar to Bagwell (2005). Before destabilization of pooling equilibria, we first introduce the method by Bagwell and Ramey (1988). Let's define the demand of any firm when the initial prior of being high-quality is ρ_0

$$D(p, a; \rho_0) = \lambda \frac{a}{1+a}(R - \frac{p}{\rho_0 H + (1-\rho_0)L}) + (1-\lambda)(R - \frac{p}{\rho_0 H + (1-\rho_0)L})$$

Let's define the following heuristic payoff function

$$\tilde{\Pi}(p, a; c, \rho) = (p - c)D(p, a; \rho) - a$$

where ρ represents the probability that the firm is of high quality. In fact, there are only two marginal cost levels: c_L and c_H while $\tilde{\Pi}(p, a; c)$ is heuristic payoff function with marginal costs c and demand $D(p, a; 1)$. Let's assume that for any given c, there exists a unique $p(c)$ and $a(c)$ that maximizes the payoff function which is concave in both p and a.

$$\gamma(c) = (p(c), a(c)) = \underset{p,a}{\text{argmax}}\, \tilde{\Pi}(p, a; c, 1)$$

As a special case, $\gamma(c_H) = (p^o_H(c_H), a^o_H(c_H))$ where p^o_H and a^o_H are observable quality price and advertising spending of a high-quality firm respectively. Furthermore, let's assume that there exists $\overline{c} > c_L$ and $c_L > \underline{c}$ with the boundary condition as follows

$$\max\{\Pi_L(\gamma(\overline{c}); 1), \Pi_L(\gamma(\underline{c}); 1)\} < \Pi^o_L = \Pi(\tilde{\gamma(c_L)}).$$

In a candidate pure strategy pooling equilibrium such as $(\tilde{p}, \tilde{a})$, let's assume that both type of firms play this strategy with probability one and all exposed consumers believe that the firm is indeed a high-quality with probability ρ_0. In our case, under the condition that $c_L < c_H$ and the low-quality firm is indifferent, demand reducing changes shall make the high-quality better off as we have shown before. With $c_L < c_H$ and the boundary conditions , there exists $\dot{c} > c_L$ that gives the following *Indifference* equality

$$(\tilde{p} - c_L)D(\tilde{p}, \tilde{a}; \rho_0) - \tilde{a} - (p(\dot{c}) - c_L)D(p(\dot{c}), a(\dot{c}); 1) + a(\dot{c}) = 0$$

Here, there might be a $\ddot{c} < c_L$ that satisfies the last equality but we prefer $\dot{c}$ since it induces a profitable deviation by decreasing the demand for the high-quality firm while $\ddot{c}$ does the opposite. In order to destabilize the candidate pooling equilibrium all we need is another pair of price and advertising in which high-quality firm becomes better off while low-quality firm is indifferent(Cho and Kreps (1987) refinement).

We also have the following inequality by construction

$$(p(\dot{c}) - \dot{c})D(p(\dot{c}), a(\dot{c}); 1) - a(\dot{c}) - (\tilde{p} - \dot{c})D(\tilde{p}, \tilde{a}; \rho_0) + \tilde{a} > 0$$

By adding up last two equation, We drive the following inequality

$$(\dot{c} - c_L)[D(\tilde{p}, \tilde{a}; \rho_0) - D(p(\dot{c}), a(\dot{c}); 1)] > 0$$

Hence, it is a fact that $D(\tilde{p}, \tilde{a}; \rho_0) > D(p(\dot{c}), a(\dot{c}); 1)$ since $\dot{c} > c_L$

The next step is to show that this pair of strategies $(p(\dot{c}), a(\dot{c}))$ makes the high-quality firm better off compared to pooling strategy $(\tilde{p}, \tilde{a})$. The sign of the following equation determines whether deviation would be profitable for the high-quality firm;

$$(\tilde{p} - c_H)D(\tilde{p}, \tilde{a}; \rho_0) - \tilde{a} - (p(\dot{c}) - c_H)D(p(\dot{c}), a(\dot{c}); 1) + a(\dot{c})$$

Now, subtract the *indifference* equation to get

$$(c_L - c_H)[D(\tilde{p}, \tilde{a}; \rho_0) - D(p(\dot{c}), a(\dot{c}); 1)] < 0$$

Therefore, the high-quality firm has incentive to deviate from the candidate pooling equilibrium pair $(\tilde{p}, \tilde{a})$ to the pair $(p(\dot{c}), a(\dot{c}))$ and also consumers correctly believes that this deviation is an act of high quality firm with Cho and Kreps (1987). So, no pooling equilibria can survive under Cho and Kreps refinement. ■

Proof of Proposition 5:

The incentive compatibility condition of the low quality firm $(IC_L)$is

$$\Pi_L(p,a;1) = \lambda\frac{a}{1+a}(R-\frac{p}{H})(p-c_L) + (1-\lambda)(R-\frac{p}{L})(p-c_L) - a \le \Pi_L^o.$$

If the IC_L is satisfied for the pair (p_H^o, a_H^o), the high quality firm prefers setting (p_H^o, a_H^o) to maximize its payoff for any value $\lambda \in [0,1]$. We characterize the properties of the (p_H^{si}, a_H^{si}) in three steps.

In the first step, we argue that at $\lambda = 1$, aforementioned maximization problem of the high quality firm perfectly coincides with the maximization problem in Section 3. When all consumers are unaware of the product (i.e., $\lambda = 1$), the fraction of aware consumers is zero so that whether aware consumers have the knowledge of product quality does not matter. Therefore, we conclude that the solutions at $\lambda = 1$ have following properties: $a_H^{si} < a_L^o < a_H^o$ and $p_H^{si} > p_H^o > p_L^o$.

In the second step, we show that Π_L^o (i.e., the RHS of IC_L) is decreasing in λ and $\Pi_L(p_H^o, a_H^o; 1)$ (i.e., the LHS of IC_L) is increasing in λ. The optimal observable quality profit for the low quality firm is

$$\Pi_L^o = \begin{cases} \frac{(RL-2\sqrt{L}-c_L)^2}{4L} + \frac{(1-\sqrt{\lambda})(RL-c_L)}{\sqrt{L}} & \text{if} \quad \lambda \in (\overline{\lambda}_L, 1] \\ \frac{(RL-c_L)^2}{4L} & \text{if} \quad \lambda \in [0, \overline{\lambda}_L] \end{cases}.$$

It is easy to show that if $\lambda \in (\overline{\lambda}_L, 1]$, then

$$\frac{d\Pi_L^o}{d\lambda} = -\frac{1}{2}\lambda^{-\frac{1}{2}}\frac{RL-c_L}{\sqrt{L}} < 0$$

and

$$\frac{d^2\Pi_L^o}{d\lambda^2} = \frac{1}{4}\lambda^{-\frac{3}{2}}\frac{RL-c_L}{\sqrt{L}} > 0.$$

Thus, Π_L^o is decreasing and convex in λ. Similarly,

$$\Pi_L(p_H^o, a_H^o; 1) = (\lambda(RH - c_H) - 2\sqrt{\lambda}\sqrt{H})(\frac{RH + c_H - 2c_L}{4H})$$

$$+(1-\lambda)(R(2L - H) - c_H)(\tfrac{RH+c_H-2c_L}{4L})$$

$$\frac{d\Pi_L(p_H^o, a_H^o; 1)}{d\lambda} = ((RH - c_H) - \lambda^{-\frac{1}{2}}\sqrt{H})(\frac{RH + c_H - 2c_L}{4H}) - (R(2L - H) - c_H)(\frac{RH + c_H - 2c_L}{4L})$$

After some simplification, this equality reduces to the following

$$\frac{d\Pi_L(p_H^o, a_H^o; 1)}{d\lambda} = (\frac{H - L}{L})(\frac{HR + c_H}{\sqrt{H}}) - \lambda^{-\frac{1}{2}}$$

At $\lambda = 1$, $\frac{d\Pi_L(p_H^o, a_H^o; 1)}{d\lambda} > 0$. Moreover, the $\frac{d\Pi_L(p_H^o, a_H^o; 1)}{d\lambda}$ takes the value of zero at

$$\lambda^* = \left[\frac{1}{(\frac{H-L}{L})(\frac{HR+c_H}{\sqrt{H}})}\right]^2$$

It is also easy to show that

$$\frac{d^2\Pi_L(p_H^o, a_H^o; 1)}{d\lambda^2} > 0$$

Hence, if $\lambda \in (\lambda^*, 1]$, $\Pi_L(p_H^o, a_H^o; 1)$ is increasing and convex in λ. To be able to argue that $\Pi_L(p_H^o, a_H^o; 1)$ decreasing in whole region of $\lambda \in [\overline{\lambda}_H, 1]$, we need to show that the value λ^* is less than $\overline{\lambda}_H$.

$$\overline{\lambda}_H = (\frac{2\sqrt{H}}{RH - c_H})^2 > \lambda^* = \left[\frac{1}{(\frac{H-L}{L})(\frac{RH+c_H}{\sqrt{H}})}\right]^2$$

$$\Longleftrightarrow$$

$$\left[\frac{2\sqrt{H}}{RH - c_H}\right] > \left[\frac{1}{(\frac{H-L}{L})(\frac{RH+c_H}{\sqrt{H}})}\right]$$

$$\Longleftrightarrow$$

$$2(\frac{H-L}{L})(HR + c_H) > (RH - c_H)$$

Thus, λ^* is always smaller than $\overline{\lambda}_H$.

In the third step, we argue that (i) at $\lambda = 1$, Π_L^o (i.e., the RHS of IC_L) is less than $\Pi_L(p_H^o, a_H^o; 1)$ (i.e., the LHS of IC_L) (ii) at $\lambda = \overline{\lambda}_H$, Π_L^o is bigger than $\Pi_L(p_H^o, a_H^o; 1)$. At $\lambda = 1$, the maximization problem in section 3 coincides with the one we are analyzing here. From Lemma 2, it is the case that $\Pi_L(p_H^o, a_H^o; 1) > \Pi_L^o$ at $\lambda = 1$. To show part (ii), remember that if $\lambda \leq \overline{\lambda}_H$, then $a_H^o = a_L^o = 0$. By definition of p_L^o, the following inequality is always satisfied at $\lambda = \overline{\lambda}_H$

$$\Pi_L(p_H^o, 0; 1) = (1-\lambda)(R - \frac{p_H^o}{L})(p_H^o - c_L) < \Pi_L^o = (1-\lambda)(R - \frac{p_L^o}{L})(p_L^o - c_L)$$

Now, by combining step two and three, we can conclude that there exists a unique $\overline{\lambda}_I$ such that

$$\Pi_L(p_H^o, a_H^o; 1) < \Pi_L^o \quad \text{if} \quad \lambda \in [0, \overline{\lambda}_I)$$
$$\Pi_L(p_H^o, a_H^o; 1) = \Pi_L^o \quad \text{if} \quad \lambda = \overline{\lambda}_I$$
$$\Pi_L(p_H^o, a_H^o; 1) > \Pi_L^o \quad \text{if} \quad \lambda \in (\overline{\lambda}_I, 1]$$

Thus, the high quality firm has to distort price and advertising from (p_H^o, a_H^o) if $\lambda \in (\overline{\lambda}_I, 1]$. For all other values of λ, it sets its optimal observable price and advertising (p_H^o, a_H^o) and there is no distortion.

Now, from second step, we know that $\Pi_L^o - \Pi_L(p_H^o, a_H^o; 1)$ is maximized at $\lambda = 1$ and decreases as λ decreases. Basically, the distortion in both price and advertising is highest when there is no informed consumer. Over time, as the fraction of informed consumers increases, the distortion in both price and advertising decreases. ■

Proof of Proposition 6:

The nice property of the solution pair (p_H^{si}, a_H^{si}) is that the distortion decreases as λ decreases. In other words, if one can find the conditions under which

$$\Pi_H(p_H^{si}, a_H^{si}; 1) > \max_{p,a} \Pi_H(p, a; 0)$$

at $\lambda = 1$. Then, as the fraction of informed consumers increases, distortion decreases and $\Pi_H(p_H^{si}, a_H^{si}; 1)$ increases. As a result, the existence is satisfied for all other values of λ under the same conditions. However, the conditions under which this inequality is satisfied at $\lambda = 1$ is already characterized in Section 3. The separating equilibrium (p_H^{si}, a_H^{si}) exists if (i) (H-L) is not too small (ii) R is not too small. ■

Proof of Lemma 9: When $\theta_E \leq a$, i.e., $p_E \leq \frac{(q_E - q_o)a}{q_E}$, the entrant's pricing problem is

$$arg \max_{p_E} \Pi_E = (p_E - c_E)(\frac{q_I p_I - q_E p_E}{q_I - q_E} - a)$$

Taking derivative of Π_E w.r.t. p_E gives the entrant's reaction function in the following form

$$\begin{aligned}
0 &= \frac{q_I p_I - q_E p_E}{q_I - q_E} - a - (p_E - c_E)\frac{q_E}{q_I - q_E} \\
p_E &= \frac{q_I p_I - a(q_I - q_E) + c_E q_E}{2q_E} \\
BR_E(p_I) &= \frac{q_I p_I - a(q_I - q_E) + c_E q_E}{2q_E}
\end{aligned}$$

When $\theta_E \geq a$, i.e., $p_E \geq \frac{(q_E - q_o)}{q_E}$, the entrant's pricing problem is

$$arg \max_{p_E} \Pi_E = (p_E - c_E)(\frac{q_I p_I - q_E p_E}{q_I - q_E} - \frac{q_E p_E}{q_E - q_o})$$

Taking derivative of Π_E w.r.t. p_E gives the entrant's reaction function in the following form

$$\begin{aligned}
0 &= \frac{q_I p_I - q_E p_E}{q_I - q_E} - \frac{q_E p_E}{q_E - q_o} - (p_E - c_E)(\frac{q_E}{q_I - q_E} + \frac{q_E}{q_E - q_o}) \\
p_E &= \frac{q_I(q_E - q_o)p_I + q_E c_E(q_I - q_o)}{2q_E(q_I - q_o)} \\
BR_E(p_I) &= \frac{q_I(q_E - q_o)p_I + q_E c_E(q_I - q_o)}{2q_E(q_I - q_o)}
\end{aligned}$$

The incumbent's pricing problem is

$$arg \max_{p_I} = \Pi_I = (p_I - c_I)(b - \frac{q_I p_I - q_E p_E}{q_I - q_E})$$

Taking derivative of Π_I w.r.t. p_I gives the incumbent's reaction function in the following form

$$
\begin{aligned}
0 &= b - \frac{q_I p_I - q_E p_E}{q_I - q_E} - (p_I - c_I)\frac{q_I}{q_I - q_E} \\
p_I &= \frac{q_E p_E + b(q_I - q_E) + c_I q_I}{2q_I} \\
BR_I(p_E) &= \frac{q_E p_E + b(q_I - q_E) + c_I q_I}{2q_I}
\end{aligned}
$$

■

Proof of Lemma 10: The intersection of best response functions of the incumbent and the entrant determines the equilibrium that could be in region 0,I, II, or III.

The best response function of the entrant can be written as follows

$$
\begin{aligned}
BR_E(p_I) &= \begin{cases} c_E & if \quad c_I \leq p_I \leq \frac{c_E q_E + a(q_I - q_E)}{q_I} \\ \frac{q_I p_I - a(q_I - q_E) + c_E q_E}{2q_E} & if \quad \frac{c_E q_E + a(q_I - q_E)}{q_I} \leq p_I \leq \frac{a(q_I + q_E - 2q_o) - c_E q_E}{q_I} \\ \frac{a(q_E - q_o)}{q_E} & if \quad \frac{a(q_I + q_E - 2q_o) - c_E q_E}{q_I} \leq p_I \leq \frac{(q_I - q_o)[2a(q_E - q_o) - c_E q_E]}{q_I(q_E - q_o)} \\ \frac{q_I(q_E - q_o)p_I + q_E c_E(q_I - q_o)}{2q_E(q_I - q_o)} & if \quad \frac{(q_I - q_o)[2a(q_E - q_o) - c_E q_E]}{q_I(q_E - q_o)} \leq p_I \end{cases}, \\
BR_I(p_E) &= \frac{q_E p_E + b(q_I - q_E) + c_I q_I}{2q_I}.
\end{aligned}
$$

Region 0:($\theta_I < a$ & $\theta_E < a$) In this parametric region, the incumbent optimally charges a price that even the poorest consumer prefers to buy from the incumbent and the entrant can not capture market share even by charging its marginal cost, c_E. In other words, the indifference cutoff between the incumbent and the entrant, θ_I, is lower than a, even if the entrant's price is c_E. The best response function of the entrant is $BR_E(p_I) = c_E$ where

$$c_I \leq \quad p_I \quad \leq \frac{c_E q_E + a(q_I - q_E)}{q_I}$$

The best response functions of the entrant and the incumbent intersect in this region if the following condition is satisfied

$$\begin{aligned}
c_I \leq & \quad p_I & \leq \frac{a(q_I - q_E) + q_E c_E}{q_I} \\
c_I \leq & \quad BR_I(c_E) = \frac{q_E c_E + b(q_I - q_E) + c_I q_I}{2q_I} & \leq \frac{a(q_I - q_E) + q_E c_E}{q_I} \\
\frac{c_I q_I - q_E c_E}{q_I - q_E} \leq & \quad b & \leq 2a - \frac{c_I q_I - c_E q_E}{q_I - q_E}
\end{aligned}$$

where $BR_I(c_E)$ is the incumbent's best response at the price of c_E by the entrant.

In this region, the incumbent is the only active seller in the market.

Region I:($a < \theta_I$ & $\theta_E < a$)

In this region, some consumers potentially may buy from the entrant and all these consumers strictly prefer consuming the product of the entrant instead of consuming the outside good. The best response function of the entrant is $BR_E(p_I) = \frac{q_I p_I - a(q_I - q_E) + c_E q_E}{2q_E}$ where

$$\frac{a(q_I - q_E) + q_E c_E}{q_I} \quad < \quad p_I$$

Plugging best response function of the incumbent into the inequality induces first condition of Region I

$$\begin{aligned}
\frac{a(q_I - q_E) + q_E c_E}{q_I} &< p_I \\
\frac{a(q_I - q_E) + q_E c_E}{q_I} &< BR_I(c_E) = \frac{q_E c_E + b(q_I - q_E) + c_I q_I}{2q_I} \\
2a - \frac{c_I q_I - c_E q_E}{q_I - q_E} &< b
\end{aligned}$$

The best response function of the entrant is $BR_E(p_I) = \frac{q_I p_I - a(q_I - q_E) + c_E q_E}{2q_E}$ where

$$p_I < \frac{a(q_E + q_I - 2q_o) - c_E q_E}{q_I}$$

Since $\theta_E = \frac{p_E q_E}{(q_E - q_o)} < a$, the price charged by the entrant in this region is can not be more than $\frac{a(q_E - q_o)}{q_E}$. Plugging best response function of the incumbent into the inequality induces first condition of Region I

$$\begin{aligned}
p_I &< \frac{a(q_E + q_I - 2q_o) - c_E q_E}{q_I} \\
BR_I(\frac{a(q_E - q_o)}{q_E}) &< \frac{a(q_E + q_I - 2q_o) - c_E q_E}{q_I} \\
b &< \frac{2q_I + q_E - 3q_o}{q_I - q_E} a - \frac{c_I q_I - 2q_E c_E}{q_I - q_E}
\end{aligned}$$

Now by combing these two conditions, we derive the condition to have the equilibrium in the Region I

$$2a - \frac{c_I q_I - c_E q_E}{q_I - q_E} \quad < b < \quad (\frac{2q_I + q_E - 3q_o}{q_I - q_E})a - \frac{c_I q_I - 2q_E c_E}{q_I - q_E}$$

Region II: ($\theta_I > a$ & $\theta_E = a$)

In this region, some consumers potentially may buy from the entrant while the poorest consumer is indifferent between consuming the product of the entrant and consuming the outside good. Since $\theta_E = \frac{p_E q_E}{(q_E - q_o)} = a$, the price charged by the entrant in this region is $\frac{a(q_E - q_o)}{q_E}$. In other words, the best response function of the entrant is $BR_E(p_I) = \frac{a(q_E - q_o)}{q_E}$ where

$$\frac{a(q_I + q_E - 2q_o) - c_E q_E}{q_I} \leq p_I \leq \frac{(q_I - q_o)[2a(q_E - q_o) - c_E q_E]}{q_I(q_E - q_o)}$$

Plugging best response function of the incumbent into the inequality will give us the condition of Region II

$$\frac{a(q_I + q_E - 2q_o) - c_E q_E}{q_I} \leq BR_I(\frac{a(q_E - q_o)}{q_E}) \leq \frac{(q_I - q_o)[2a(q_E - q_o) - c_E q_E]}{q_I(q_E - q_o)}$$
$$(\frac{2q_I + q_E - 3q_o}{q_I - q_E})a - \frac{c_I q_I - 2q_E c_E}{q_I - q_E} \leq b \leq (\frac{4q_I - q_E - 3q_o}{q_I - q_E})a - \frac{c_I q_I(q_E - q_o) + 2q_E c_E(q_I - q_o)}{q_I - q_E}$$

Region III:($\theta_I > a$ & $\theta_E > a$)

In this region, some consumers potentially may buy from the entrant while some of these consumers strictly prefer buying the product of the entrant and the others prefer consuming the outside good. Since $\theta_E = \frac{p_E q_E}{(q_E - q_o)} > a$, the price charged by the entrant in this region can not be less than $\frac{a(q_E-q_o)}{q_E}$. The best response function of the entrant in this region is $BR_E(p_I) = \dfrac{q_I(q_E-q_o)p_I + q_E c_E(q_I-q_o)}{2q_E(q_I-q_o)}$ where

$$\frac{(q_I-q_o)[2a(q_E-q_o)-c_E q_E]}{q_I(q_E-q_o)} \leq p_I$$

Plugging best response function of the incumbent into the inequality induces condition of Region III

$$\begin{aligned} \frac{(q_I-q_o)[2a(q_E-q_o)-q_E c_E]}{q_I(q_E-q_o)} &< BR_I(\frac{a(q_E-q_o)}{q_E}) \\ (\frac{4q_I-q_E-3q_o}{q_I-q_E})a - \frac{c_I q_I(q_E-q_o)+2q_E c_E(q_I-q_o)}{q_I-q_E} &< b \end{aligned}$$

■

Proof of Lemma 11: Given the price of the incumbent, the H-type entrant's profit can be written as

$$\Pi_H(\tilde{q}_E) = (p_H(\tilde{q}_E)-c_H)D_H(\tilde{q}_E) = (p_H(\tilde{q}_E)-c_H)\left[\frac{q_I p_I(\tilde{q}_E)-\tilde{q}_E p_E(\tilde{q}_E)}{q_I-\tilde{q}_E} - \frac{\tilde{q}_E p_E(\tilde{q}_E)}{\tilde{q}_E-q_o}\right].$$

where $p_H(\tilde{q}_E)$ and $p_I(\tilde{q}_E)$ are the complete information optimal prices of the H-type entrant and the incumbent, respectively.

We first show that the H-type entrant's demand, $D_H(\tilde{q}_E)$, increases in perceived quality, $\tilde{q}_E$.

$$\begin{aligned}\frac{dD_H(\tilde{q}_E)}{d\tilde{q}_E} &= \frac{-p_H(q_I-\tilde{q}_E)+q_Ip_I-\tilde{q}_Ep_H}{(q_I-\tilde{q}_E)^2} - \frac{p_H(\tilde{q}_E-q_o)-\tilde{q}_Ep_H}{(\tilde{q}_E-q_o)^2} \\ &= \frac{q_I(p_I-p_H)}{(q_I-\tilde{q}_E)^2} + \frac{q_op_H}{(q_I-\tilde{q}_E)^2} > 0\end{aligned}$$

Now by using Envelope Theorem, we can write down the following

$$\frac{d\Pi_H(\tilde{q}_E, p_H(\tilde{q}_E), p_I(\tilde{q}_E))}{(d\tilde{q}_E)} = sign\left[\frac{\partial \Pi_H(\tilde{q}_E)}{\partial \tilde{q}_E}\right] = sign\left[\frac{\partial D_H(\tilde{q}_E)}{\partial \tilde{q}_E}\right] > 0$$

Hence, we can conclude that the H-type entrant's profit increases in its perceived quality, $\tilde{q}_E$. ■

Proof of Proposition 7: We write down the profit of the E-type entrant as

$$\begin{aligned}\Pi_E(\tilde{q}_E, p_I, c_E) &= (p_E-c_E)\left[\frac{q_Ip_I-\tilde{q}_Ep_E}{q_I-\tilde{q}_E} - \frac{\tilde{q}_Ep_E}{\tilde{q}_E-q_o}\right] \\ &= (p_E-c_E)\left[\frac{q_Ip_I(\tilde{q}_E-q_o)-\tilde{q}_Ep_E(\tilde{q}_I-q_o)}{(q_I-\tilde{q}_E)(\tilde{q}_E-q_o)}\right]\end{aligned}$$

where $E \in \{L, H\}$ and $\tilde{q}_E = \rho_o H + (1-\rho_o)L$.

From Lemma 9, we know that when the perceived quality of the entrant is $\tilde{q}_E$, the incumbent optimal price is

$$p_I(\tilde{q}_E) = \frac{\tilde{q}_Ep_E + b(q_I-\tilde{q}_E+c_Iq_I)}{2q_I}$$

Plugging $p_I(\tilde{q}_E)$ into the profit,$\Pi_E(\tilde{q}_E, p_I, c_H)$, of the H-type entrant results in

$$\Pi_E(\tilde{q}_E, p_I, c_E) = (p_E-c_E)\left[\frac{b(q_I-\tilde{q}_E)(\tilde{q}_E-q_o)+c_Iq_I(\tilde{q}_E-q_o)-\tilde{q}_Ep_E(2q_I-\tilde{q}_E-q_o)}{2(q_I-\tilde{q}_E)(\tilde{q}_E-q_o)}\right].$$

Differentiating this profit function w.r.t. p_E, it can be shown that

$$\frac{\partial \Pi_E(\tilde{q}_E, p_I, c_E)}{\partial p_E} = \left[\frac{b(q_I - \tilde{q}_E)(\tilde{q}_E - q_o) + c_I q_I(\tilde{q}_E - q_o) + \tilde{q}_E c_E(2q_I - \tilde{q}_E - q_o) - 2\tilde{q}_E p_E(2q_I - \tilde{q}_E - q_o)}{2(q_I - \tilde{q}_E)(\tilde{q}_E - q_o)}\right].$$

and

$$\frac{\partial^2 \Pi_E(\tilde{q}_E, p_I, c_E)}{\partial p_E \partial c_E} = \frac{\tilde{q}_E(2q_I - \tilde{q}_E - q_o)}{2(q_I - \tilde{q}_E)(\tilde{q}_E - q_o)} > 0$$

which implies that

$$\frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H} - \frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H} > 0$$

However, it is easy to see that for any values of $p_H \geq p_H^*$, the profit of both the H- and L-type entrant decreases. Hence, the last inequality implies that

$$\left|\frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H}\right| < \left|\frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H}\right|$$

When $p_H \geq p_H^*$, both $\frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H}$ and $\frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H}$ have the negative sign, we can conclude that

$$\frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H} \Big/ \frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H} > 1 \ \ for\ any\ p_H \geq p_H^*,$$

■

Proof of Lemma 12:

We first derive the profit, Π_L^L of the L-type entrant. From Lemma 9, we know that

$$p_L^* = \frac{q_I p_I(q_L - q_o) + q_L c_L(q_I - q_o)}{2q_L(q_I - q_o)}$$

By plugging the complete information price, p_L^*, into the profit function, Π_L^L, we get the following

$$\Pi_L^L = \frac{\left[q_I p_I(q_L - q_o) - q_L c_L(q_I - q_o)\right]^2}{4q_L(q_I - q_o)(q_I - q_L)(q_L - q_o)}.$$

Furthermore, we rewrite Π_L^H as follows

$$\Pi_L^H = (p_H - c_L)\left[\frac{q_I p_I(q_H - q_o) - q_H p_H(q_I - q_o)}{(q_I - q_H)(q_H - q_o)}\right].$$

The constraint optimization problem can be written in the following Lagrangian form

$$\begin{aligned}
\Lambda &= \Pi_H^H + \lambda_1(\Pi_L^L - \Pi_L^H) \\
&= (p_H - c_H)\frac{\left[q_I p_I(q_H - q_o) - q_H p_H(q_I - q_o)\right]}{(q_I - q_H)(q_H - q_o)} \\
&\quad + \lambda_1\left[\frac{\left[q_I p_I(q_L - q_o) - q_L c_L(q_I - q_o)\right]^2}{4q_L(q_I - q_o)(q_I - q_L)(q_L - q_o)} - (p_H - c_L)\frac{\left[q_I p_I(q_H - q_o) - q_H p_H(q_I - q_o)\right]}{(q_I - q_H)(q_H - q_o)}\right]
\end{aligned}$$

The first-order conditions can be written as follows

$$\begin{aligned}
\frac{\partial \Lambda}{\partial p_H} &= q_I p_I(q_H - q_o)(1 + \lambda_1) - 2q_H p_H(q_I - q_o)(1 + \lambda_1) + q_H(q_I - q_o)(c_H + c_L\lambda_1) = 0 \\
\frac{\partial \Lambda}{\partial \lambda_1} &= (p_H - c_L)\left[q_I p_I(q_H - q_o) - q_H p_H(q_I - q_o)\right] - (q_I - q_H)(q_H - q_o)\Pi_L^L = 0 \\
& p_H\frac{\partial \Lambda}{\partial p_H} = 0, \lambda_1\frac{\partial \Lambda}{\partial \lambda_1}, p_H \geq 0, \lambda_1 \leq 0
\end{aligned}$$

The constraint of this optimization problem is binding due to focus on the least-cost separating equilibrium, i.e., $\lambda_1 < 0$. Hence, solving p_H from the $\frac{\partial \Lambda}{\partial p_H}$ equation leads to

$$p_H^{QS} = \frac{q_I p_I (q_H - q_o)(1+\lambda_1) + q_L c_L (q_I - q_o)(c_H + c_L \lambda_1)}{2 q_H (q_I - q_o)(1+\lambda_1)}$$

In order to guarantee that this is indeed the separating price which maximizes the H-type entrant profit, the following second order condition has to be satisfied

$$\frac{\partial^2 \Lambda}{\partial p_H^2} = -2 q_H (q_I - q_o)(1+\lambda_1) < 0$$
$$\lambda_1 > -1$$

Now, we substitute p_H^{QS} into $\frac{\partial \Lambda}{\partial \lambda_1}$ equation above to derive the value of λ_1

$$\left[\frac{q_I p_I (q_H - q_o)(1+\lambda_1) + q_H (q_I - q_o)(c_H - c_L)(2+\lambda_1)}{2 q_H (q_I - q_o)(1+\lambda_1)}\right]\left[\frac{q_I p_I (q_H - q_o)(1+\lambda_1) - q_H (q_I - q_o)(c_H + c_L \lambda_1)}{2(1+\lambda_1)}\right]$$
$$-(q_I - q_H)(q_H - q_o)\Pi_L^L = 0$$

Now, by substituting $x = q_I p_I (q_H - q_o)$, $y = q_H (q_I - q_o)$, and $z = 4(q_I - q_o)(q_I - q_H)(q_H - q_o)\Pi_L^L$, the equation reduces to following one

$$\begin{aligned}
0 &= (x^2 - z)(1+\lambda_1)^2 - 2xy(1+\lambda_1)^2 c_L - y^2 \left[c_H - c_L(2+\lambda_1)\right](c_H + c_L \lambda_1) \\
0 &= (x^2 - 2xyc_L + y^2 c_L^2 - z)\lambda_1^2 + 2(x^2 - 2xyc_L + y^2 c_L^2 - z)\lambda_1 \\
&+ (x^2 - 2xyc_L - z) - y^2(c_H - 2c_L)c_H \\
0 &= \left[(x - yc_L)^2 - z\right]\lambda_1^2 + 2\left[(x - yc_L)^2 - z\right]\lambda_1 + \left[(x - yc_L)^2 - z\right] - y^2(c_H - c_L)^2
\end{aligned}$$

We can rewrite the last equation in the form of $a\lambda_1^2 + b\lambda_1 + c = 0$ where $a = [(x - yc_L)^2 - z]$, $b = 2a$, and $c = a - y^2(c_H - c_L)^2$. The roots can be calculated as follows

$$\begin{aligned}\lambda_1 &= \frac{-2a \mp \sqrt{b^2 - 4ac}}{2a} = \frac{-2a \mp \sqrt{(2a)^2 - 4ac}}{2a} \\ &= \frac{-2a \mp \sqrt{b^2 - 4ac}}{2a} = -1 \mp \sqrt{\frac{a-c}{a}} = -1 \mp \frac{y(c_H - c_L)}{\sqrt{(x - yc_L)^2 - z}}\end{aligned}$$

Since the second order condition requires that $\lambda_1 > -1$, the only plausible root is the following one

$$\begin{aligned}\lambda_1 &= -1 + \frac{y(c_H - c_L)}{\sqrt{(x - yc_L)^2 - z}} \\ \lambda_1 &= -1 + \frac{q_H(q_I - q_o)(c_H - c_L)}{\sqrt{[q_I p_I(q_H - q_o) - q_H(q_I - q_o)c_L]^2 - 4q_H(q_I - q_o)(q_I - q_H)(q_H - q_o)\Pi_L^L}}\end{aligned}$$

where we first plug a, c and then x, y, and z into the equation(s). ■

Proof of Lemma 13: Given the price of the H-type entrant, the incumbent's profit can be written as

$$\Pi_I(\tilde{q}_E) = (p_I(\tilde{q}_E) - c_I)D_I(\tilde{q}_E) = (p_I(\tilde{q}_E) - c_I)\left[b - \frac{q_I p_I(\tilde{q}_E) - \tilde{q}_E p_E(\tilde{q}_E)}{q_I - \tilde{q}_E}\right].$$

where $p_H(\tilde{q}_E)$ and $p_I(\tilde{q}_E)$ are the complete information optimal prices of the H-type entrant and the incumbent, respectively.

We first show that the incumbent's demand, $D_I(\tilde{q}_E)$, increases in perceived quality,

$\tilde{q}_E$.

$$\begin{aligned} \frac{dD_I(\tilde{q}_E)}{d\tilde{q}_E} &= -\frac{-p_H(q_I - \tilde{q}_E) + q_I p_I - \tilde{q}_E p_H}{(q_I - \tilde{q}_E)^2} \\ &= -\frac{q_I(p_I - p_H)}{(q_I - \tilde{q}_E)^2} < 0 \end{aligned}$$

Now by using Envelope Theorem, we can write down the following

$$\frac{d\Pi_I(\tilde{q}_E, p_H(\tilde{q}_E), p_I(\tilde{q}_E))}{(d\tilde{q}_E)} = sign\left[\frac{\partial \Pi_I(\tilde{q}_E)}{\partial \tilde{q}_E}\right] = sign\left[\frac{\partial D_I(\tilde{q}_E)}{\partial \tilde{q}_E}\right] < 0$$

Hence, we can conclude that the incumbent's profit decreases in the entrant's perceived quality, $\tilde{q}_E$. ■

Proof of Lemma 14:

Due to the complementarity between the entrant's and the incumbent's pricing strategies, the price of entrant increases as the price of the incumbent increases. We need to show that as the price of the high quality entrant increases, the ratio of the marginal profit of the low type entrant over the marginal profit of the high cost entrant decreases. In other words, the following has to be true

$$\frac{\partial\left[\frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H} \Big/ \frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H}\right]}{\partial p_H} < 0 \;\; for\ any\ p_H \geq p_H^*$$

From Proposition 7,

$$\frac{\frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H}}{\frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H}} = \frac{\frac{b(q_I - \tilde{q}_E)(\tilde{q}_E - q_o) + c_I q_I(\tilde{q}_E - q_o) + (\tilde{q}_E c_L - 2\tilde{q}_E p_H)(2q_I - \tilde{q}_E - q_o)}{2(q_I - \tilde{q}_E)(\tilde{q}_E - q_o)}}{\frac{b(q_I - \tilde{q}_E)(\tilde{q}_E - q_o) + c_I q_I(\tilde{q}_E - q_o) + (\tilde{q}_E c_H - 2\tilde{q}_E p_H)(2q_I - \tilde{q}_E - q_o)}{2(q_I - \tilde{q}_E)(\tilde{q}_E - q_o)}}$$

$$\frac{\partial \left[\frac{\frac{\partial \Pi_L(\tilde{q}_E, p_I, c_L)}{\partial p_H}}{\frac{\partial \Pi_H(\tilde{q}_E, p_I, c_H)}{\partial p_H}} \right]}{\partial p_H} = -\frac{2\tilde{q}_E^2(c_H - c_L)(2q_I - \tilde{q}_E - q_o)}{[b(q_I - \tilde{q}_E)(\tilde{q}_E - q_o) + c_I q_I(\tilde{q}_E - q_o) + (\tilde{q}_E c_H - 2\tilde{q}_E p_H)(2q_I - \tilde{q}_E - q_o)]^2} < 0$$

Therefore, the high type entrant's ability to separate decreases as the price of the incumbent increases. ■

Proof of Lemma 15:

The following profit function denotes the profit of the low quality entrant when it is perceived as an average (ρ_o) quality.

$$\Pi_L^{\rho_o}(p_I) = (p_H^* - c_L) \left[\frac{q_I p_I(\tilde{q}_E - q_o) - \tilde{q}_E p_H^*(q_I - q_o)}{(q_I - \tilde{q}_E)(\tilde{q}_E - q_o)} \right]$$

The following profit function denotes the profit of the low quality entrant when it is perceived as a low quality.

$$\Pi_L^L(p_I) = (p_L^* - c_L) \left[\frac{q_I p_I(\tilde{q}_E - q_o) - \tilde{q}_E p_L^*(q_I - q_o)}{(q_I - q_L)(q_L - q_o)} \right]$$

In the next equation, we analyze whether the increasing price of incumbent, p_I, increases the mimicing incentive of the low quality entrant or not.

$$\begin{aligned} \frac{\partial \left[\Pi_L^{\rho_o}(p_I) - \Pi_L^L(p_I)\right]}{\partial p_I} &= \frac{q_I(p_H^* - c_L)}{q_I - \tilde{q}_E} - \frac{q_I(p_L^* - c_L)}{q_I - q_L} \\ &= q_I \left[\frac{p_H^*(q_I - q_L) - p_L^*(q_I - \tilde{q}_E) - c_L(q_I - q_L - q_I + \tilde{q}_E)}{(q_I - \tilde{q}_E)(q_I - q_L)} \right] \end{aligned}$$

Plugging $\tilde{q}_E = q_L + \Delta$ for $\Delta > 0$ into the equation above reduces it to the following one

$$\frac{\partial\left[\Pi_L^{\rho_o}(p_I) - \Pi_L^L(p_I)\right]}{\partial p_I} = q_I\left[\tfrac{(p_H^* - p_L^*)(q_I - q_L) + \Delta(p_L^* - c_L)}{(q_I - \tilde{q}_E)(q_I - q_L)}\right] > 0$$

As the price of the incumbent increases, the mimicry incentive of the low quality entrant increases as well. ■

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Vita

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