# APPLICATION OF SHORT-TIME FOURIER TRANSFORM IN MACHINE FAULT DETECTION

By

**M.S. SAFIZADEH, A.A. LAKIS and M. THOMAS**

Department of Mechanical Engineering
École Polytechnique de Montréal

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# NOMENCLATURE

| | |
|---|---|
| $s(t)$ | The magnitude of the vibration signal with zero mean |
| $P(x)$ | The distribution of the signal $s(t)$ |
| $\mu_n$ | The $n^{th}$ moment of the signal $s(t)$ |
| $C(\tau)$ | The Cepstrum of a signal |
| $FFT$ | Fast Fourier transform |
| $G_{xx}(f)$ | The complex spectrum of the signal $s(t)$ |
| $P(t,\omega)$ | The joint distribution function of time and frequency |
| $S(\omega)$ | The spectrum of the signal $s(t)$ |
| $s_t(\tau)$ | The short-time Fourier transform of the signal $s(t)$ |
| $h(\tau)$ | Window function |
| $S_t(\omega)$ | The spectrum of the short-time Fourier transform of the signal $s(t)$ |
| $STFT$ | Short-time Fourier transform |

# LEGENDS OF FIGURES

# Abstract

The detection of faults in machinery is based on the verification of classical vibration parameters, including both time domain and frequency domain parameters. There are several methods by which one can estimate these parameters and each of the methods has advantages and disadvantages. In certain cases, such as transient events in machinery or varying speed rotating machinery, traditional methods of vibration analysis either in time or in frequency are incapable of reflecting changes in the operating conditions. The use of time-frequency methods is one step towards a solution of some of the problems and the Short-Time Fourier Transform (STFT) is the simplest method of time-frequency analysis.

This paper proposes the application of the STFT as a time-frequency method which can provide more information about a signal both in time and in frequency, and give a better representation of the signal than the conventional methods used in machinery diagnosis.

In this paper, we review the traditional vibration analysis techniques which are widely used in practice. Secondly, we discuss the necessity of time-frequency analysis in the field of machinery diagnostics. Thirdly, the theory of the Short-Time Fourier Transform is briefly explained. Some practical examples of defective bearings and defective gearboxes are analyzed by the STFT method and, in conclusion, the efficacy and advantages of the STFT are demonstrated.

# 1. Introduction

With increased competition in the production and greater pressure on the price of industrial rotating machinery, the necessity for efficient methods of the condition monitoring and detecting faults in machinery has become apparent. It is necessary to find, on the one hand, ways to protect the productivity of critical equipment and, on the other hand, ways to reduce operating and maintenance costs. The most efficient method will be one which recognises that a problem exists before damage has occurred in machine, so that ample time is available to schedule repairs with minimum disruption to operations and production [1].

The wave forms of vibration signals from rotating machinery are often recorded and analyzed by processing the data using analysis techniques. Each different technique gives some information about the condition of the machinery but the need exists for a technique which gives all the necessary information.

In practice, after deciding on the type of sensor, its location and the parameter to be monitored [2], the processing technique to be chosen will depend upon the precise condition we wish to monitor. If *fault detection* is our objective, then the speed and reliability of the processing technique are important but, if *fault diagnosis* is our objective, the accuracy of the method is critical.

There are several conventional methods for the detection and identification of faults. Some of these methods provide a representation of signals in the time domain and others in the frequency domain. In all of the methods, it is assumed that signals are stationary. This assumption, however, is not always accurate. In certain machines, in the early stages of defects, vibrational signals become non-stationary; in this case, conventional methods are not applicable.

In recent years, a number of new analytical methods have been developed in the field of signal processing: these are called "joint time-frequency analysis methods". However, they are not generally used in the field of machinery diagnostics. There has been considerable progress in research into the development of the theory of joint time-frequency methods and other non-stationary signal processing methods, but more work must be carried out to prove to industry that these new methods are effective in the condition-monitoring of mechanical systems.

The objective of this work is, firstly, to outline the limitations of conventional methods and, secondly, to demonstrate the speed and accuracy which can be obtained by using joint time-frequency analysis methods in the field of machinery diagnostics. In this paper we first present a review of traditional methods with their advantages and disadvantages. Secondly, we discuss the necessity for using time-frequency methods, and present a brief theory of the Short-Time Fourier Transform as the fastest and the easiest method among other time-frequency methods. Thirdly, a technique of adaptively adjusting the window length used in the Short-Time Fourier Transform is presented. Finally, some examples of fault detection and the identification of real problems are given, using the Short-Time Fourier Transform.

## 2. Time-Based and Frequency-Based Vibration Analysis Techniques

There are a large number of vibration analysis techniques which may be applied to the processing of a vibrational signal. These techniques can highlight different characteristics of the signal which may be used in the detection and diagnosis of faults in machinery. Many studies have been carried out to find the most effective technique for the analysis, monitoring and diagnostics of machines. Unfortunately, none of these techniques has been proven to be efficient. In the following section, the advantages and disadvantages of conventional methods are described in order to understand why time-frequency methods are needed.

Conventional vibration analysis methods fall into two categories:

a) Time-domain vibration analysis techniques

b) Frequency-domain vibration analysis techniques

## 2.1 Time-domain vibration analysis techniques:

### 2.1.1) <u>Time Wave form</u>:

Using an instrument as simple as an oscilloscope or FFT analyzer, it is possible to view the wave form of the vibration. It may be possible to identify the period of events existing in a machine and any amplitude modulation in the vibration signal [3]. However, although the time domain often shows the nature of the mechanical problem better than the frequency domain, there are several reasons which lead us to avoid the use of the time domain display in machine monitoring. For instance, in the case of a complex machine, the vibration signature may combine several signals with different frequencies, amplitudes and phases, and it would be virtually impossible to decompose the signature into its separate components. However, used in conjunction with other methods, it could prove helpful.

### 2.1.2) <u>Overall level (R.M.S.) Measurements</u>:

Overall level measurements [4-6] are the most common vibration measurement in use. It is a simple and inexpensive type of measurement, which is calculated by estimating the root mean square (RMS) level of the time record. It has been found that, in rotating machinery, velocity is the best indicator of general condition. Charts are available which indicate acceptable levels, for example VDI 2056 (table 1). The greatest limitation is the lack of precise information to be extracted from the data. These charts are extremely generalized in conception, and have little regard to mobility. The mobility relationship is defined as:

$$Vibration = Force \times Mobility$$

where the mobility is the ability of a structure to move under force. Since the mobility changes from machine to machine, vibration level changes accordingly. For example, the measurement of a damaged bearing must be made on the outside of the bearing housing support. The signal detecting procedure is affected by the transmission path to the sensor. Unless a problem is severe, the overall level measurements may not change significantly. Unfortunately, people have relied too heavily on these measurements alone, and have been surprised to see machines fail, apparently without warning.

### 2.1.3) <u>Peak level detection</u>:

As an alternative to RMS, the peak level of the signal can be used [7, 8]. A baseline "peak" level is defined for a new machine, and any variations from this norm would be indicative of a change in machine condition. Operational standards have been developed which recommend vibration boundary levels for satisfactory or unsatisfactory running conditions. For example standard API 610 [API standard 610, 7$^{th}$ edition, Centrifugal Pumps for General Refinery Services] defines vibration limits for centrifugal pumps. This is particularly useful for monitoring the change in the amount of impulsion, possibly due to the occurrence of impacts. However, this method is not reliable, since resonant behavior often dominates the vibration signal, and therefore only a very severe transient impact will bring about any change in the peak level.

### 2.1.4) Crest factor:

The crest factor [4, 6] (sometimes called the impact index) is the ratio of the peak level to the RMS level of the vibrational signal. The time waveform of a machine in good health is mostly random. When a localized fault appears, a periodic peak is seen to occur in the signal. As the fault increases, the waveform becomes far more impulsive, with higher peak levels, but the RMS value is not affected significantly. In work carried out by the General Electric Company [9], it was shown that the crest factor could be used as an indicator of bearing condition. The crest factor limits are as follows: 2 to 3 indicates a normal bearing, 3 to 8 indicates fault initiation and 8 to 10 indicates fault growth. However, this method has certain limitations. The RMS level is significantly increased in bearings with multiple or spreading defects, resulting in the reduction of the crest factor. Background noise is also a problem because it increases the RMS level and consequently decreases the crest factor.

### 2.1.5) Shock Pulse:

The shock pulse method [7, 10] detects the development of a mechanical shock wave caused by increasing damage. For example, impacts produced by small defects in a bearing may excite resonances in the bearing and the machine. The periodic signals with characteristic frequencies from a bearing may indicate deformations or defects in the bearing, but they are not always visible and frequently follow the rhythm in higher frequencies (higher of 2kHz). To observe the shock pulses, first the vibration is measured by a transducer mounted on the casing of the machine. Then, the signal from the transducer is passed through a bandpass filter to isolate one of the resonances. Finally, the filtered signal is transformed into a train of impulses by passing it through a pulse converter (Figure 1).

By observing the increase of the level and the rhythm, it is possible to determine which of the bearing elements is damaged (Figure 2). It is also useful to calculate the amplitude spectrum of the train of impulses; the complete procedure is also known as the high frequency resonance technique or high frequency shock pulse [11]. This approach is efficient but has certain disadvantages. When there is more than one fault in a machine, the pulse repetition will not correspond to one single fault. For example, if there are simultaneous cavities on the inner and outer races of a bearing, the frequency of the shock pulses will correspond to neither BPFI (*ball-pass frequency on the inner race*) nor BPFO (*ball-pass frequency on the outer race*) but with their sum: BPFI+BPFO.

#### 2.1.6) Spike Energy:

This method was developed in 1970 to measure the condition of rolling bearings [10]. It is based on the high frequency peak value of the acceleration. Spike Energy shows the intensity of impact energy caused by mechanical faults. To measure Spike Energy, as with the Shock Pulse method, the output signal of the accelerometer is filtered through a bandpass filter, and the time variation of the signal is measured by a peak-to-peak detector as an indicator of the severity of the impact measurements. Spike Energy is expressed in “gSE” units. Spike Energy Spectrum may also be obtained by using FFT analysis. This technique is often used in high frequency vibration such as metal-to-metal contact and cavitation. Details of an application of this technique to vibration monitoring of seal-less pumps are given in [12].

This method has proven satisfactory in fault detection, but it has problems similar to those of the Shock Pulse method and may be misleading in the case of simultaneous faults.

**2.1.7) <u>Demodulation</u>:**

An alternative way to monitor rotating machines is demodulation or the enveloping method. This method is based on the properties of amplitude-modulated signals which are often encountered in machine monitoring. The type of fault is indicated by the impact rates. The envelope of a wave modulated in amplitude reveals the repetition frequency of the impacts (Figure 3). The potential applications of these properties hinge on the availability of mathematical tools that enable assessment of the envelope function characteristics. Demodulation analysis of a bandpass filtered signal is based on the Hilbert Transform which generates the envelope of the time signal. The Hilbert Transform $(H)$ of the signal in time $s(t)$ is defined as

$$H[s(t)] = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(t)}{t-\tau} d\tau = \bar{s}(t) \tag{1}$$

This constitutes the imaginary part of the analytical signal defined as

$$z(t) = s(t) + j\bar{s}(t) = |s(t)| e^{i\theta(t)} \tag{2}$$

where $\theta(t) = arctg\left[\frac{\bar{s}(t)}{s(t)}\right]$ and the module of the analytical signal, $|z(t)| = \sqrt{s^2(t) + \bar{s}^2(t)}$.

represents the envelope of the time signal $s(t)$. The time envelope calculated by the Hilbert Transform can be useful in bearing defect detection [6, 13, 14] or gearbox defect detection [15]. The problem with the demodulation method [16] is that the Hilbert Transform cannot be used to demodulate the whole vibration signal. The signal first has to be filtered by passing it through a bandpass filter in order to separate one of the dominant harmonics and all of its sidebands. If a narrow bandpass filter is chosen, it is likely to miss some of the higher order sidebands of the chosen harmonic. And if a broad bandpass filter is used, it is likely to pass some of the sidebands from adjacent harmonics. In both cases, the modulation calculated by the Hilbert Transform will not represent the amplitude modulation of the signal. As a solution to this problem, the rectification process in the demodulation method is replaced by peak value waveform [17].

### 2.1.8) Kurtosis:

The technique of Kurtosis analysis is another method used to indicate the "peakedness" of the signal. Kurtosis ( $Ku$ ) is a statistical parameter, derived from the statistical moments of the probability density function of the vibrational signal. " $Ku$ " is defined as

$$Ku = \frac{\int_{-\infty}^{\infty} s^4 p(s)\, ds}{\left[\int_{-\infty}^{\infty} s^2 p(s)\, ds\right]^2} \qquad (3)$$

where $s$ is the magnitude of the vibration signal with zero mean and $p(s)$ is the distribution of $s$.

To give a simple explanation of this parameter, knowing that the first moment about zero gives the mean value of distribution:

$$\mu = \int_{-\infty}^{\infty} s p(s) ds \tag{4}$$

The second moment called variance gives the standard deviation and is defined as:

$$\mu_2 = \sigma^2 = \int_{-\infty}^{\infty} s^2 p(s) ds \tag{5}$$

Higher order moments are defined by the general integral.

$$\mu_n = \int_{-\infty}^{\infty} s^n p(s) ds \tag{6}$$

Then, Kurtosis is just the fourth moment , $\mu_4$, normalized with respect to the square of the variance.

$$Kurtosis = \frac{\mu_4}{\sigma^4} \tag{7}$$

A bearing in good condition has a Gaussian distribution function and the Kurtosis value of its signal is equal to three, but a damaged bearing has a Kurtosis value which will be greater than three. Advantages of this methods are: a) Kurtosis value is independent of load and speed conditions, b) it has been found that the amplitude distributions, and therefore the *Ku* value, are relatively unaffected by variations in the transmission paths of vibrational signals. But for modulated signals this technique may lead to inaccurate predictions [15].

### 2.1.9) Orbits:

Orbits display or Lissajous curves [10, 18] are obtained by displaying time base waveforms from two transducers whose outputs are phase shifted by 90 degrees (Figure 4). Orbits are particularly useful in the analysis of the vibration of a shaft during rotation. The shaft orbit can provide basic amplitude, frequency and phase lag angle information. It is able to indicate wear in a journal bearing, shaft misalignment, shaft imbalance and shaft rub. Since the orbits are directly constructed in the time domain, they are deformed by noise, surface quality and self-excited low frequency vibration of the rotating shaft [19]. Consequently, the detection of faults in rotating machinery by this method is often unsuccessful. Nevertheless, Orbits display is used to complement other methods [20].

### 2.1.10) Shaft centerline:

The shaft centerline position is used to estimate the shaft centerline relative to the geometric centerline and clearance of the bearing. From these data, the shaft attitude or position, its angle and eccentricity ratio can be calculated and may be used as an indication of bearing wear and misalignment generated by heavy loads [20]. The Shaft centerline position method has the same limitations as the Orbits method.

## 2.2 Frequency domain vibration analysis techniques:

### 2.2.1) Spectrum Analysis:

A spectrum is derived from the vibration waveform by performing a "Fast Fourier Transform"

[3]. The benefit of the spectrum is that each rotating element in a machine generates identifiable frequencies; the peaks in frequencies define the type of fault and the amplitude of the peaks indicates the severity of the fault [21, 22]. Spectrum analysis information may be used in different ways to recognize defects in machines. The spectral indices such as R.M.S. levels [23] can show the difference between the current spectrum and the baseline or the previous spectrum. These indices are good indicators of the overall performance of machinery. An alternative way is to define an allowable tolerance limit on the baseline spectrum such that, if there is a fault in the machinery, the spectrum will exceed the limit. The narrow bandwidth spectrum may be replaced by a constant percentage bandwidth spectrum in order to simplify its application.

Although spectrum analysis is one of the best vibration indicators of machine condition, it must be pointed out that the defect frequency may be close to frequencies excited by other components in machines; therefore, by a small change in speed, the position of the peaks may change and give incorrect results. To prevent this problem, a new spectrum called the Synthesized Spectrum may be used [24]. However, these fault frequencies and fault conditions are not always easily identifiable. A discussion on how spectrum analysis may provide erroneous results and therefore false warnings is presented in [25] and a number of conditions which are necessary in order to obtain correct results with different types of spectrum are given.

### 2.2.2) Waterfall plot:

A Waterfall plot (also known as a cascade plot) is composed of FFT magnitude spectrums displayed at different machine speeds (Figure 5). This method is used to examine sub-synchronous and super-synchronous components during run-up or run-down stages of a machine [26]. The advantage of this format over single or overlaid spectrum displays is that changes in the spectrum versus changes of speed may be identified visually. This method is specially applied to certain types of fault such as oil whirl/whip, cracked shaft [27] and rubs. Although Waterfall display is useful, it has some limitations. When the characteristics of the signal are changing rapidly in time, the spectral representation of the signal at each speed is degraded. If the speed varies relatively slowly (1000-4000 RPM over 60 sec) the Waterfall technique may give data of an acceptable order of magnitude [28]. If the speed varies more rapidly, this technique may provide inaccurate results.

### 2.2.3) <u>Holospectrum</u>:

This method [19] provides information not only about peak frequency and amplitude, but also about phase relationships. In general, vibrations of a rotor are measured by two accelerometers, as in the Orbits method. Holospectrum is formed by a simple vector in each frequency (Figure 6). It is composed of a circle, line and ellipse placed on the frequency axis. A circle is obtained if the amplitudes of two components are equal and their phases are 90 or 270. A circle is obtained in the rotating frequency of a shaft if there is imbalance in the shaft. A line is obtained if the phase lag between two elements of a machine is $0^\circ$ or $180^\circ$ and the slope of the line depends on their amplitude proportion. This method has the same limitations as the Orbits method.

### 2.2.4) <u>Cascade Holospectrum</u>:

Following the Cascade spectrum diagram principle, the 2-dimensional Holospectrum can be used to construct the Cascade Holospectrum diagram (Figure 7). The Cascade Holospectrum diagrams [19] may provide us with more information about the transient events of a machine during run-up and run-down, and may intuitively demonstrate the change in the Holospectrum components of different orders. In contrast to the Cascade diagram, the Cascade Holospectrum gives the phase relations between the two accelerometers. This method has the disadvantages of Waterfall diagram and Holospectrum technique.

### 2.2.5) Bode and Polar plot:

The Bode plot is a log-log diagram of the amplitude of a complex signal (often transfer function) versus the frequency accompanied by a semi-log diagram of the phase of the signal. The Polar plot, also known as the Nyquist plot, is another representation of the signal in polar coordinates with the radius of the curve corresponding to the amplitude of the signal, and the phase signal corresponding to the value of the phase to the horizontal axis. Frequency or running speed is changed along different points (Figure 8). Although these plots are useful in the field of balancing problems, mode shape of the rotor, cracked shaft detection and rubs [20, 26, 29, 30], they are of little use in the case of machine monitoring. In this case, spectrum representation is preferred to the Bode plot. The Polar plot is also of little help in finding the natural frequencies of a system because it needs curve-fitting algorithms. These plots are a different way of displaying results, but do not offer a new method of analysis.

### 2.2.6) Cepstrum Analysis:

If the inverse Fourier Transform of the logarithm of the correlation function is taken, we obtain what is termed as the Cepstrum which is a function of the independent variable "Quefrency" in milliseconds [24].

$$C(\tau) = FFT^{-1}[\log G_{xx}(f)] \tag{8}$$

It is used to highlight periodicities in the spectrum, in the same way that the spectrum is used to highlight periodicities in the time waveform. Thus, the harmonics in the spectrum are summed into one peak in the cepstrum, making it easier to identify, and observe trends in, specific fault frequencies. Quefrency shows frequency spacing in the spectrum but it shows nothing about absolute frequency. On the other hand, it is possible to edit out the effect of the transmission path because both this and the excitation, which are multiplicative in the spectrum, become additive in the Cepstrum. It has been found to be useful in bearing and gear-box analysis [4, 31]. However, this method has some disadvantages. Firstly, the spectrum sometimes has several harmonics and sidebands in the low and middle frequencies; these appear in the Cepstrum and distort the harmonics and sidebands in the higher frequencies. In such a case, it may be hard to identify the type of defect by the Cepstral method. Secondly, there is no relationship between the magnitude of the Cepstrum and the severity of the defect.

## 3. Time-Frequency Analysis

### 3.1 Is Time-Frequency analysis really necessary?

As mentioned in the last section, each of the conventional vibrational methods used in fault detection and identification for steady speed machines has several limitations. The assumption of constant speed in the above methods results in stationary and pseudo-stationary vibration signals. However, even if we take this assumption into account, the limitations of the above methods reduce their performance. On the other hand, there are presently several types of varying and variable speed rotating machinery for which the stationary or pseudo-stationary vibration signals cannot be assumed to be accurate. These types of rotating machinery include gear drives, rolling element bearings, internal combustion engines, cam-driven mechanisms and reciprocating machinery. Rapidly varying speed in this group of rotating machinery generates non-stationary vibrational signals. The application of conventional techniques to the analysis of non-stationary vibrational signals may yield incorrect results.

The most common traditional techniques of vibration analysis are based on frequency domain analysis and, among these, spectral analysis plays a major role. The use of spectral analysis techniques with machines of rapidly varying speed often results in a smeared spectrum because the frequency components are changed over time, and averaging over several blocks of analysis may result in an obscured spectral representation. Although the presentation of the signal in the time domain may indicate some modulations, it will be very difficult to identify the sources of these modulations. It appears, therefore, that it is necessary to employ a new technique which would combine frequency information with amplitude changes in time. Furthermore, the initial appearance of a defect in a machine can produce transient phenomena in the vibrational signal. Passage of a ball over a localized defect in a bearing, contact of a damaged tooth with other teeth in the gearbox, and piston slap in the engine are examples of well-known industrial problems generating transient events. Frequency domain vibration analysis methods, such as the power spectrum, average the transient events so that they do not appear clearly in the spectral lines. Time domain methods, which are also used to analyse transitory signals, can lose the frequency information of different machine components. Finally, if both methods are used, it will be difficult to relate the frequency information to the forces causing the amplitude variation of the signal in the time domain. Therefore, rather than separate observation of the time from observation of the frequency characteristics of a signal, it is necessary to use a joint time-frequency technique.

In 1946, time-frequency (TF) analysis was applied to speech communication [32] for the first time, but application of this method to the field of mechanical signature analysis started only in the early 1990's. The earliest time-frequency method is known as the Spectrogram or Short-Time Fourier Transform (STFT). In recent years, various TF techniques, such as the Wigner-Ville Distribution and Wavelet transforms, have been developed in the signal processing field.

Only the STFT method will be discussed in the following sections, as being a technique which is very fast and very easy to interpret. This technique will be applied to fault detection and identification in two well-known industrial elements, the ball-bearing and gearbox. The suitability of this method in the field of machinery diagnostics will be demonstrated.

### 3.2 The Short-Time Fourier Transform

The STFT may be considered a method that breaks down the non-stationary signal into many small segments which can be assumed to be locally stationary, and applies the conventional FFT to these segments.

The STFT of a signal $s_t(\tau)$ is achieved by multiplying the signal by a window function, $h(\tau)$, centered at "$t$", to produce a modified signal. Since the modified signal emphasises the signal around time "$t$", Fourier Transforms will reflect the distribution of frequency around that time.

$$S_t(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega\tau} s(\tau)\, h(\tau - t)\, d\tau \tag{9}$$

We may consider $S_t(\omega)$ as the sum of the Fourier base functions but the base functions are a modulated version of the window function (Figure 9).

The energy density spectrum at time "$t$" may be written as follows:

$$P(t,\omega) = |S_t(\omega)|^2 = \left| \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-j\omega\tau} s(\tau)\, h(\tau - t)\, d\tau \right|^2 \tag{10}$$

For each different time we get a different spectrum and the ensemble of these spectra provide the time-frequency distribution $P(t,\omega)$.

Resolutions in time and frequency will be determined by the width of window h(τ). A large window width is chosen when we need greater accuracy in frequency and a small window width when we want to have greater accuracy in time. However, the STFT depends greatly on the width of the window and by varying the window used, one can exchange resolution in time for resolution in frequency. Figure 10 shows a signal composed of a constant frequency and an impulse. A great difference in the STFT representation of the signal is apparent if the width of the window is changed.

### 3.3 Adapting the Short-Time Fourier Transform

To solve the problem of the "trade-off" in resolution between the time and frequency domains in the Short-Time Fourier Transform, we must consider the origin of the problem. Use of a single fixed window during the analysis of the signal is the origin of the STFT problem.

We compute the adaptive STFT using a window of variable length. The criterion for adjusting the window length is the Kurtosis parameter. As mentioned in 2.1.8 of this article, this parameter is an indicator of the signal's "peakedness" as a consequence of the presence of defects in the machine.

The window length is determined by considering an initial T length for the window, thus computing the Kurtosis parameter for this slice of the signal. If it is greater than 3, the window length is divided into two. This work is repeated until the Kurtosis parameter for the signal segment in the window is less than or equal to 3. During this time, the spectrum of the signal segment is calculated. Then we move the window and repeat the same steps for the whole of the signal.

By this technique, the window length is adjusted depending on the characteristics of the signal. The benefits of this technique are:

- obtaining a performance surpassing that of the fixed window length STFT.
- providing a technique that is better suited to the diagnosis of mechanical signals than other methods.

# 4. Applications of time-frequency analysis to machinery diagnosis

## 4.1 Some Industrial Applications Of Time-frequency Analysis In Mechanical Systems

In the last few decades, many methods of time-frequency analysis have been applied to various areas of physics and engineering, such as speech processing and image processing. In the field of machinery diagnostics, Forrester [33] has used time-frequency methods in the detection of damaged gears in helicopter gearboxes. He has shown that, with the signal enhancement techniques (conventional methods) offered by Stewart and McFadden, it is difficult to distinguish one type of fault, e.g. tooth-cracking or pitting, from another; but the Wigner-Ville Distribution (one of the time-frequency methods) can more accurately reveal the type of defect. Wang and McFadden [34-37] have also studied the application of time-frequency analysis to the detection of gear damage. They have demonstrated that direct use of the Wigner-Ville Distribution can produce a complicated time-frequency representation of the signal and, furthermore, it has been suggested that the application of an appropriate window function in the time domain can improve the results of Wigner-Ville [38]. On the other hand, they have shown that the complexity of the Wigner-Ville representation, although reduced, still remains and still makes it difficult to detect mechanical failure in gear systems. They have proven that application of the Spectrogram (STFT) for the early detection of damage in gears has some advantages over the application of Wigner-Ville Distribution [39].

In another work, Rohrbaugh [40] has applied time-frequency analysis to several sets of marine machinery. He compared the Spectrogram (STFT) with cone-kernel time-frequency representation from Cohen class distributions. He showed that, while the Spectrogram can reveal the general time-varying characteristics of a vibrational signal, for more information about the signal we must consider other time-frequency methods.

Rohrbaugh and Cohen [41] outlined another new time-frequency method developed by Loughlin, Pitton and Atlas [42] for the detection of faults in pumps, and found it to have several advantages over the Spectrogram when dealing with reciprocating machinery. This method is known as "positive time-frequency distribution" and is based on a minimum cross-entropy scheme (MCE). Some applications of time-frequency analysis to the monitoring of machining processes, such as drilling and grinding operations, have been presented by Loughlin, Atlas, Bernard and Pitton [43]. They showed that, although the Spectrogram (STFT) is an efficient method for the demonstration of the time-varying characteristics of a process, sometimes newer time-frequency methods can provide more detail on the signal. They concluded that the newer methods of time-frequency analysis may assist in the early detection of problems. Atlas, Bernard and Narayanan [44] summarized some applications of time-frequency analysis in different domains of machinery diagnostics. They emphasised the importance of using time-frequency analysis in manufacturing and monitoring applications. In a recent work, Loughlin and Bernard [45] presented some applications of MCE time-frequency methods to different machine vibrational signals.

The papers cited above give some examples of the application of different time-frequency methods, including the STFT, to condition monitoring of mechanical systems. Each of these papers shows the way in which a new time-frequency method can reveal certain information about the signal that can not be obtained by traditional methods.

Today, one of the most important factors limiting the progress of machine diagnostic techniques is the lack of familiarity of mechanical engineers with new signal processing methods. The complicated theory of time-frequency analysis and the absence of an operational software for time-frequency analysis restrict engineers from using these methods in machine diagnosis.

Among the various time-frequency methods, the Short-Term Fourier Transform is the easiest and the fastest method.

This work is an attempt to present the limitations of conventional methods of vibration analysis in machine diagnosis and to emphasise the application of the STFT to fault detection and identification.

A user-friendly software has also been developed to facilitate the use of time-frequency methods by engineers whether or not they are familiar with time-frequency analysis.

### 4.2 Software For Time-frequency Analysis Of Signals

The software is designed to be run interactively; it produces and represents results in the energy-time-frequency plane for sampled time signals. Firstly, we choose the required signal from the list of available signals in the principal window, by mouse. Secondly, we select the method of analysing the signal: the FFT, the STFT, or the adaptive-window STFT, and provide information about the signal and options relating to the chosen method, such as the length and type of window for the STFT. Finally, the program represents the results in the form of the time-frequency plane projection of the signal and a three dimensional representation of the signal in the energy-time-frequency space. The energy intensity is conveyed by different colors. The three dimensional representation can be rotated in order to obtain the best point of view.

The working and accuracy of the program can be verified by a theoretical signal. The first example is a sum of sines (Figure 11). The signal is composed of the sum of three sines: 100 Hz, 300 Hz and 1000 Hz. As predicted, the time-frequency plane of the STFT shows three lines at 100 Hz, 300 Hz and 1000 Hz parallel with the time axis and the time-frequency-energy space shows three peaks constant in time (Figure12).

The next examples are an amplitude-modulated sine at 200 Hz (Figures 13 to 14) and a frequency-modulated sine at 200 Hz (Figures 15 to 16). The modulation may easily be seen in the time domain and it appears to be unnecessary to use the STFT; however, in real cases we never have a signal without noise and it is often impossible to find a modulation in time. A frequency modulation is particularly difficult to identify by its spectrum. In the time-frequency domain, the modulations are very clearly displayed. The importance of the STFT is more apparent when applied to industrial signals. It is noted that signals with different modulations in time and frequency are very usual in machinery diagnosis and the time-frequency representation gives a good interpretation of these signals.

### 4.3 Experimental application of Short-Time Fourier Transform

After verifying the program by computer-simulated signals, we can investigate the data obtained from an experimental case: the application of the STFT method to pin-point a defect, the characteristics of which are known, located on a rolling bearing. The test was conducted on a bearing having a simple defect on the inner raceway. This test was performed in a laboratory using the test setup shown in Figure 17. An interchangeable rotating shaft was supported by two journal bearings (SKF 1210 EKTN9 self-aligning double row) labelled A and B. An electric motor provided 12.2Hz rotation for the bearing shaft. Load was imposed by the break which was installed on the gearbox output. The defective bearing was mounted on support A.

The defect was created by scratching the bearing raceway with an electric pen. Figure 18 shows the signal measured on bearing A and its spectrum. The results for the defective bearings were also verified by calculating the frequency at which the rolling elements passed over the defects [1]. The geometric characteristics of the system are as follows:

pitch diameter D=69 mm

Diameter of the rolling body d=10.32 mm

Contact angle $\alpha$ =7.87 deg

Number of rolling elements $N$ =17

Bearing frequency of rotation $F_r$ =12.2 Hz

On the inner raceway, the frequency of rolling body defect impact is:

$$F_i = \frac{F_r N}{2}\left[1 + \frac{d}{D}\cos(\alpha)\right] \tag{11}$$

The pass frequency on a point of the inner raceway computed by using equation (11) is at approximately 238 Hz. We can find the default frequency among other frequencies by the spectrum in Figure 18 but we cannot be certain that this is indeed the default frequency because the default frequency must have a special characteristic. In this case, the default frequency must be an amplitude-modulated wave at approximately 238 Hz with the frequency of modulation being equal to the rotating frequency.

The time-frequency representation of the signal provided by the STFT shows the amplitude-modulated signal at the default frequency and its harmonics (Figure 19). We can easily calculate the frequency of modulation and verify that it is correct and equals the rotating frequency.

## 4.4 Industrial application of Short-Time Fourier Transform

The second case of data obtained from a real case concerns the defective gear train of a hoist drum in a large shovel operating at an open pit iron mine. The data are measured by International Measurement Solutions company in order to find the problem in the machine.

Gears generate a mesh frequency equal to the number of teeth on the gear multiplied by the rotational speed of the shaft driving it. A high vibration level at the mesh frequency is often caused by tooth error, wear of the meshing surfaces, or any other problem that would cause the profiles of meshing teeth to deviate from their ideal geometry. Sidebands at the mesh frequency, on the other hand, are typically due to a failure of mating teeth. Imagine a cracked tooth which is not yet broken, and will consequently not be noticed by the operating personnel. However, it will, due to its weakened mechanical condition, deflect more under load than the other (healthy) teeth when it goes into mesh. This results in a signal with amplitude modulation. Thus, an increasing level in the sidebands spaced with rotation speed in the frequency spectrum results from the cracked tooth.

A minimum length of time is required to perform an FFT analysis of each process. The time resolution required will depend on the period of each tooth mesh and the desired level of accuracy. Sometimes, it is not possible to measure the signal for long enough to provide the periodicity of shock in the FFT spectrum.

In this particular case, the process did not even last one revolution of the driven gear. The case was investigated by time-frequency distribution precisely because it is known that time-frequency methods do not need as much time signal as the FFT spectrum.

Figure 20 shows respectively the signal and its spectrum. The spectrum of the signal indicates some large peaks around 200 Hz and some other smaller peaks in the vicinity of 400 Hz, 800 Hz and 1200 Hz. However, it is very difficult to assume or confirm any defects at this point. On the other hand, the amplitude-modulated characteristic of the signal is clearly displayed in the representation of the signal in the time-frequency domain, as shown in figure 21. It is very simple to read the gear-meshing frequency at approximately 200Hz and three large impacts due to three partially broken teeth at a frequency of approximately 400 Hz and obtain the frequency of the modulation. In addition, the time and frequency of each peak are easily identified.

## 5. Conclusion

It has been shown that, although the majority of conventional methods may give good results when detecting a single fault in various simple elements of machines, no single technique can provide all the answers for all cases. It is difficult to decide which method gives the best result, in particular when the precise type of fault is not known.

The Short-Term Fourier Transform is an effective method of time-frequency analysis and a powerful tool in machine condition monitoring. The short-time spectrum gives a clear representation of the time-frequency plane and a simple interpretation of the energy variation due to damage. There is, unfortunately, a fundamental problem with this approach: high resolution cannot be obtained simultaneously in the time domain and the frequency domain. Although this method gives the time-frequency information with limited precision, in order to achieve greater precision we must turn to advanced time-frequency methods such as the adaptive Short-Term

Fourier Transform. This method produces reasonable and useful window lengths for the Short-Term Fourier Transform.

The development of a user-friendly software package facilitates the use of time-frequency techniques in machine diagnosis. The time-frequency methods, including the short-time spectrum, have been implemented on a computer and used, along with conventional methods, in the analysis of vibrational signals. The advantages of the short-time spectrum have been demonstrated by using this method, not only on measured signals from bearings installed in an experimental set-up, but also on vibrational signals from an industrial gearbox.

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**Table 1: Vibration standard VDI 2056**

<table>
<tr><td>18</td><td>145</td><td rowspan="3">Not Permissible</td><td rowspan="2">Not Permissible</td><td>Not Permissible</td><td rowspan="2">Just Tolerable</td></tr>
<tr><td>11.2</td><td>141</td><td rowspan="2">Just Tolerable</td></tr>
<tr><td>7.1</td><td>137</td><td rowspan="2">Just Tolerable</td><td rowspan="2">Allowable</td></tr>
<tr><td>4.5</td><td>133</td><td rowspan="2">Just Tolerable</td><td rowspan="2">Allowable</td></tr>
<tr><td>2.8</td><td>129</td><td rowspan="2">Allowable</td><td rowspan="6">Good Turbo-machines</td></tr>
<tr><td>1.8</td><td>125</td><td rowspan="2">Allowable</td><td rowspan="5">Good Large machines > 75 KW</td></tr>
<tr><td>1.12</td><td>121</td><td rowspan="4">Good Medium machines 15-75 KW</td></tr>
<tr><td>0.71</td><td>117</td><td rowspan="3">Good Small machines < 15 KW</td></tr>
<tr><td>0.45</td><td>114</td></tr>
<tr><td>0.28</td><td>109</td></tr>
<tr><td>RMS Velocity (mm/s)</td><td>V dB (Ref 10E-6 mm/s)</td><td>Group L</td><td>Group M</td><td>Group G</td><td>Group T</td></tr>
</table>

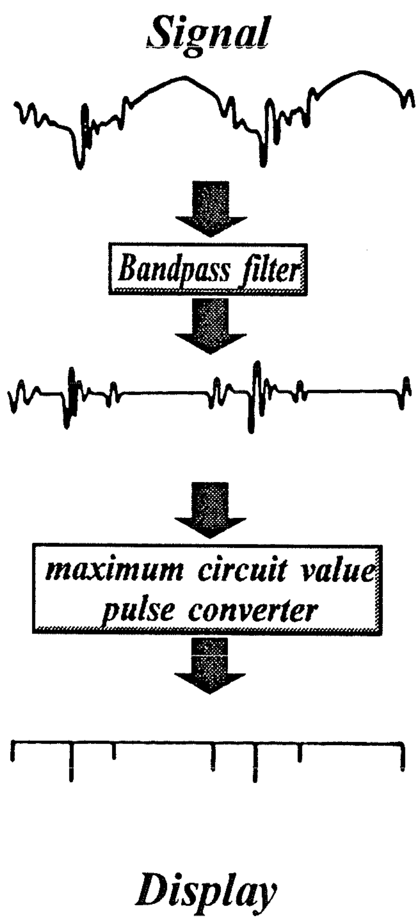



Figure 1: The stages in the conversion of shock pulse waves to high frequency pulses.

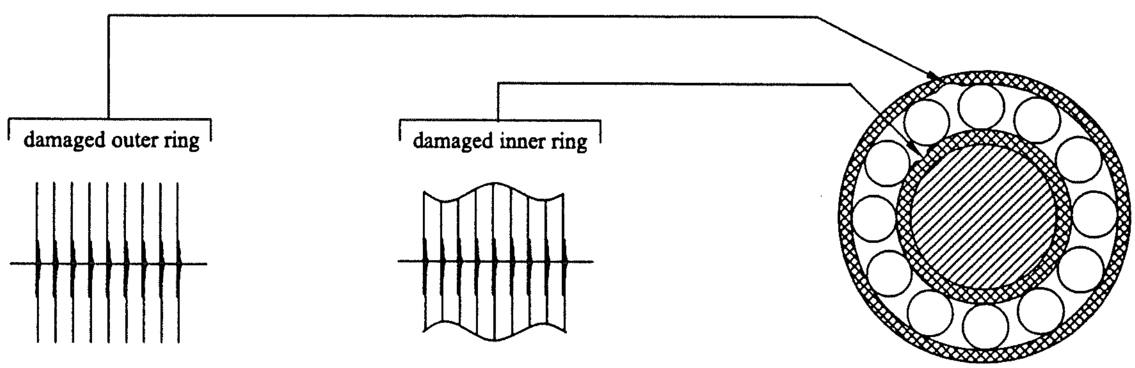



Figure 2: pulse shocks from damaged inner race and damaged outer race of a bearing.

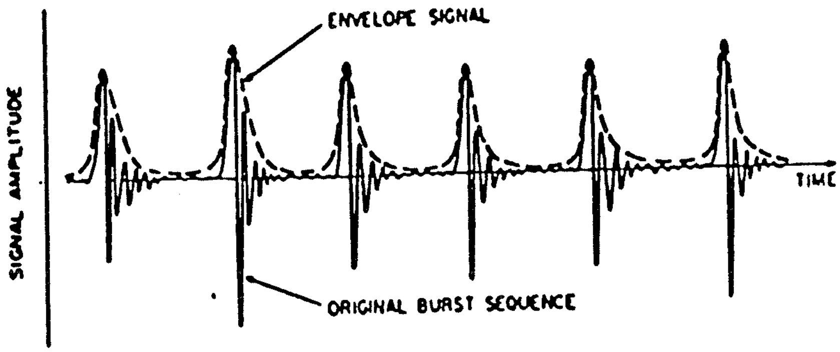



Figure 3: The envelope method shows the behavior of a signal.

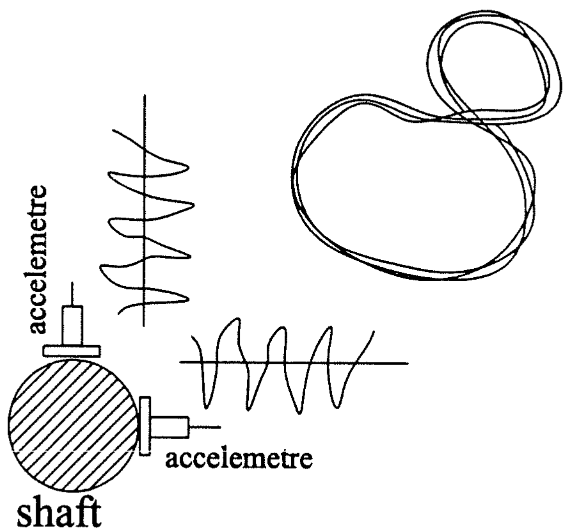



Figure 4: Typical Shaft orbital motion (Lissajous' figure).

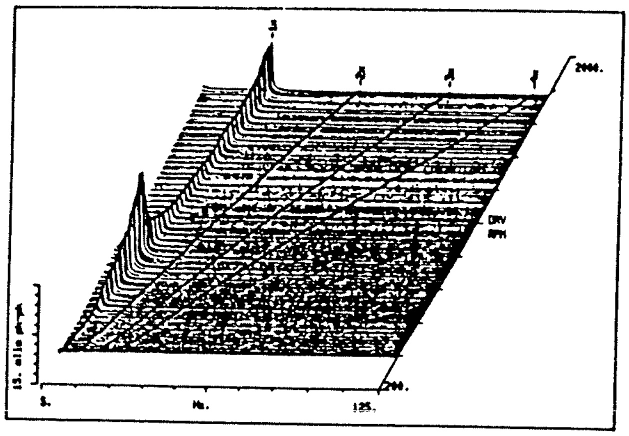
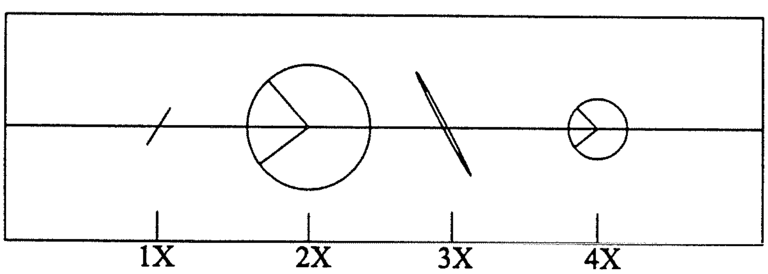
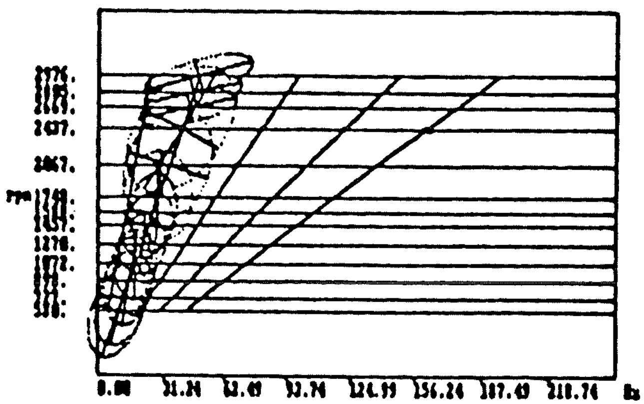


Figure 5: Waterfall plot of a turbine coast down [26].

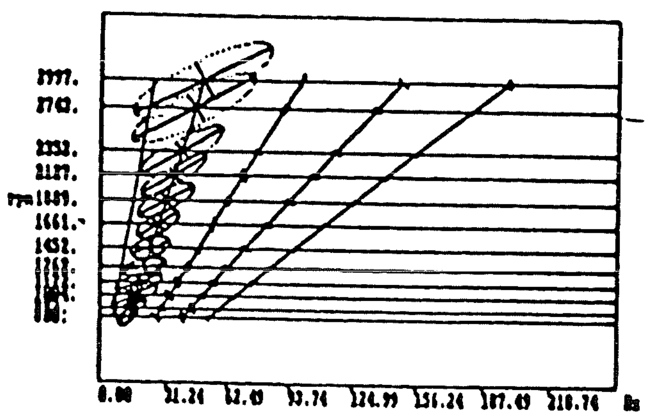



| R.P.M. | MAX | DEG. |
|---|---|---|
| 3816.02 | 4.14 | 49.5 |
| 5381.20 | 22.61 | 145.0 |
| 18759.51 | 10.9 | 43.5 |

Figure 6: The Holospectrum showing the radial vibration of a rotor.



a)



b)

Figure 7: The cascade Holospectrum Diagrams of a turbine [19].
a) run-up stages.
b) run-down stages.

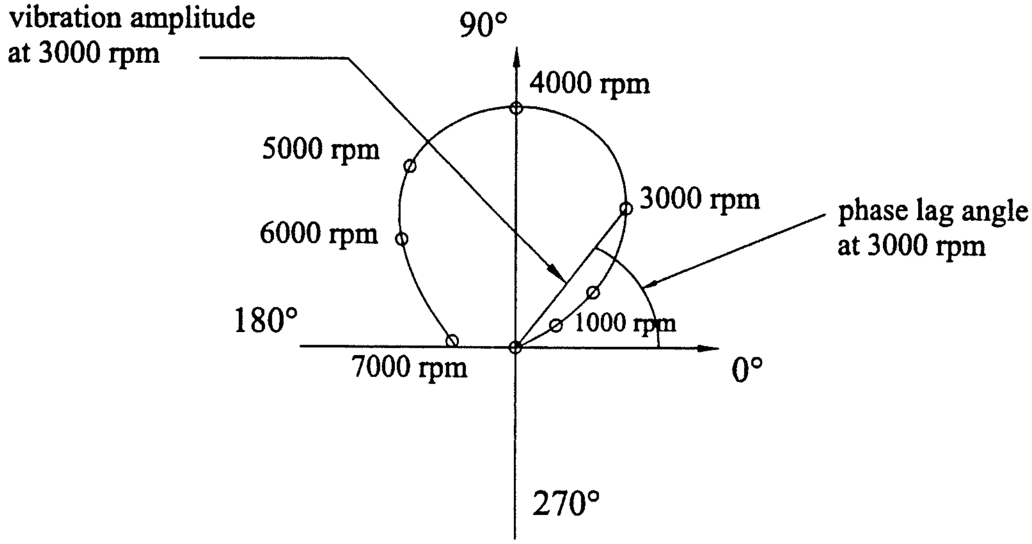



Figure 8: Nyquist plot.

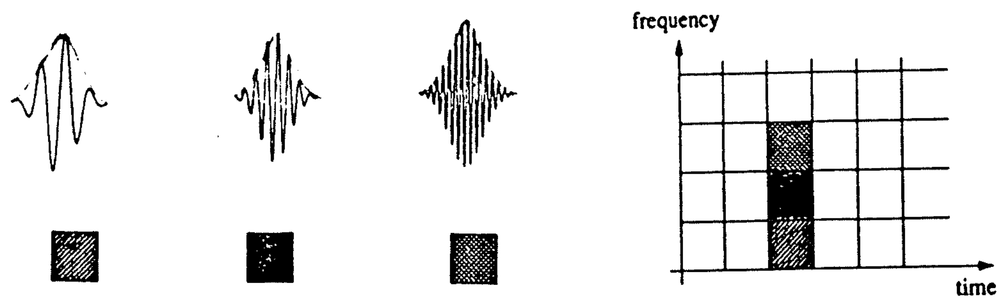



Figure 9: Basis functions and time frequency resolution of the short-time Fourier transform (STFT).

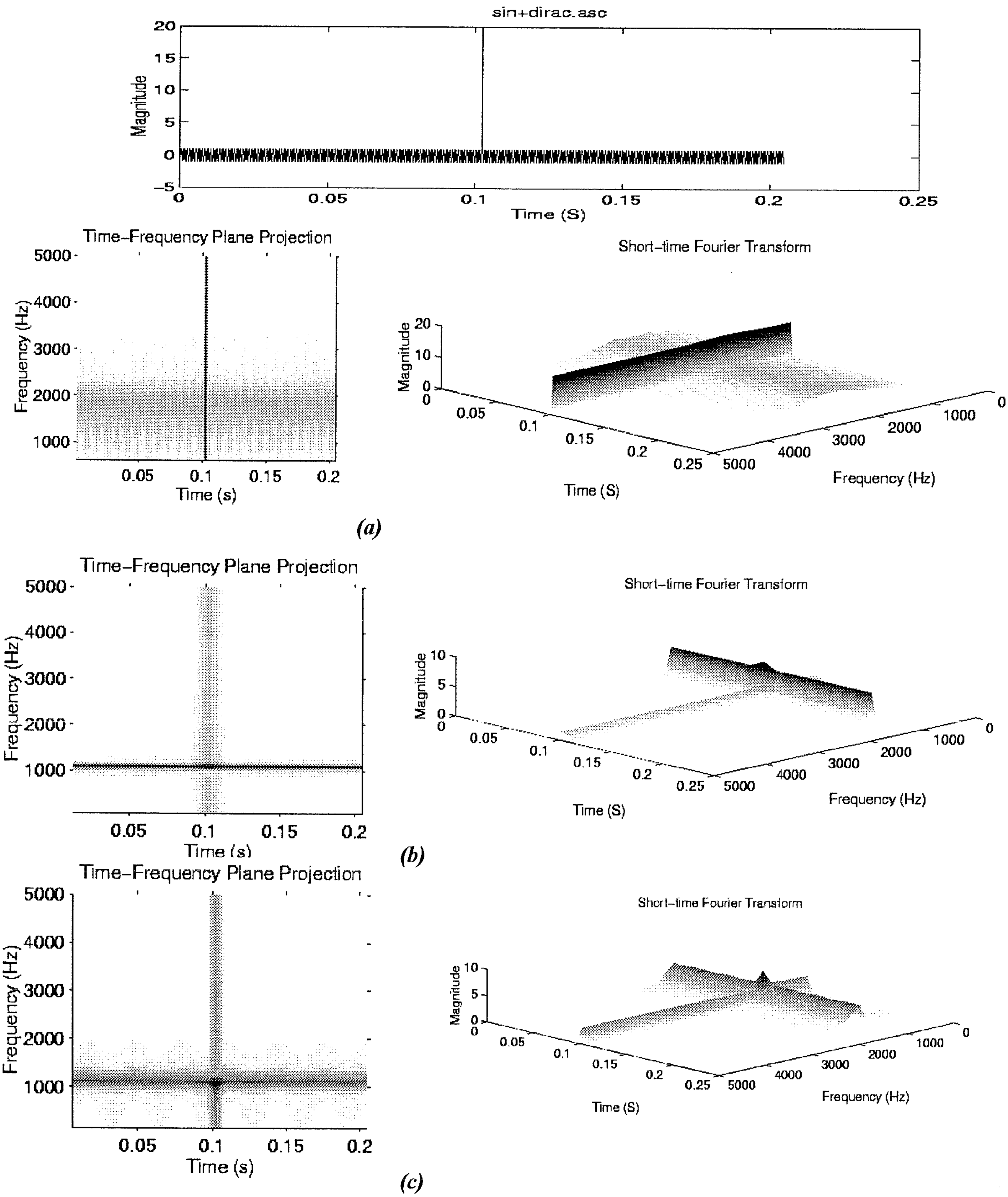



Figure 10: The STFT of a signal composed of a constant frequency plus an impulse.
(a) we use a short window, which gives a good indication of when the impulse occurred but gives abroad localization for the frequency.
(b) we use a long duration window, which gives the opposite effect.
(c) a compromise window is used.

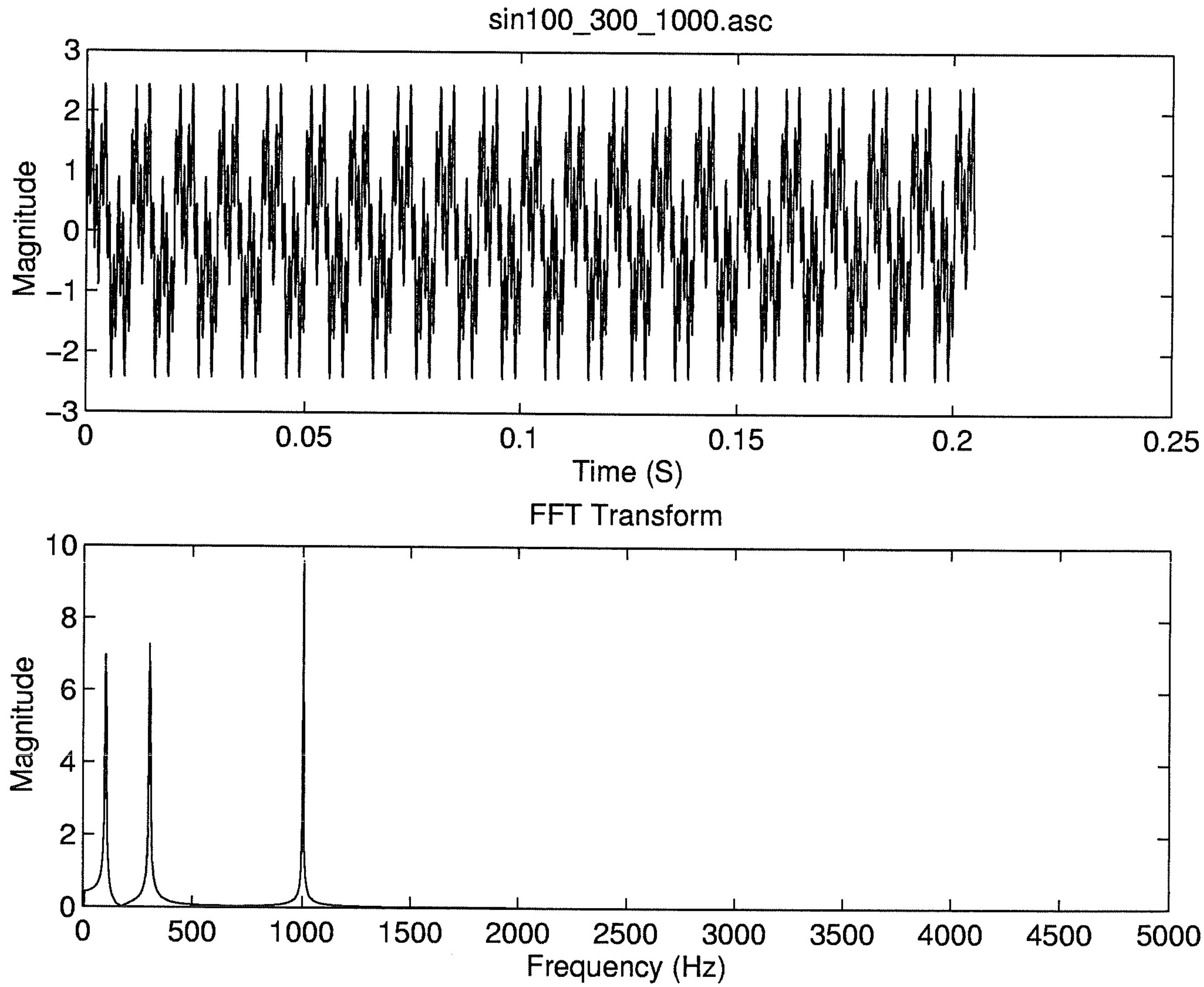



Figure 11: Sum of sines signal and its spectrum.

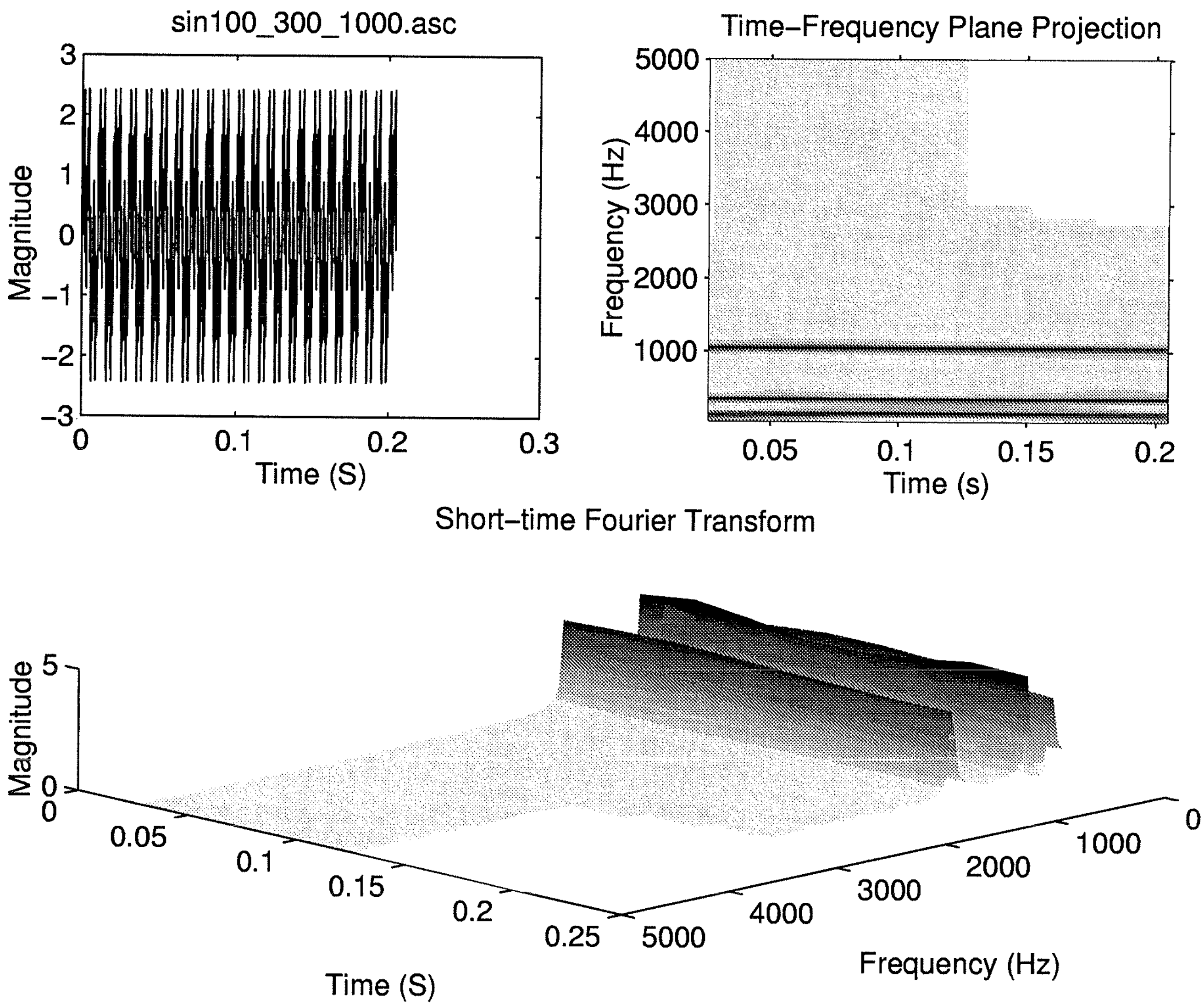



Figure 12: Short-time Fourier transform of the sum of sines signal

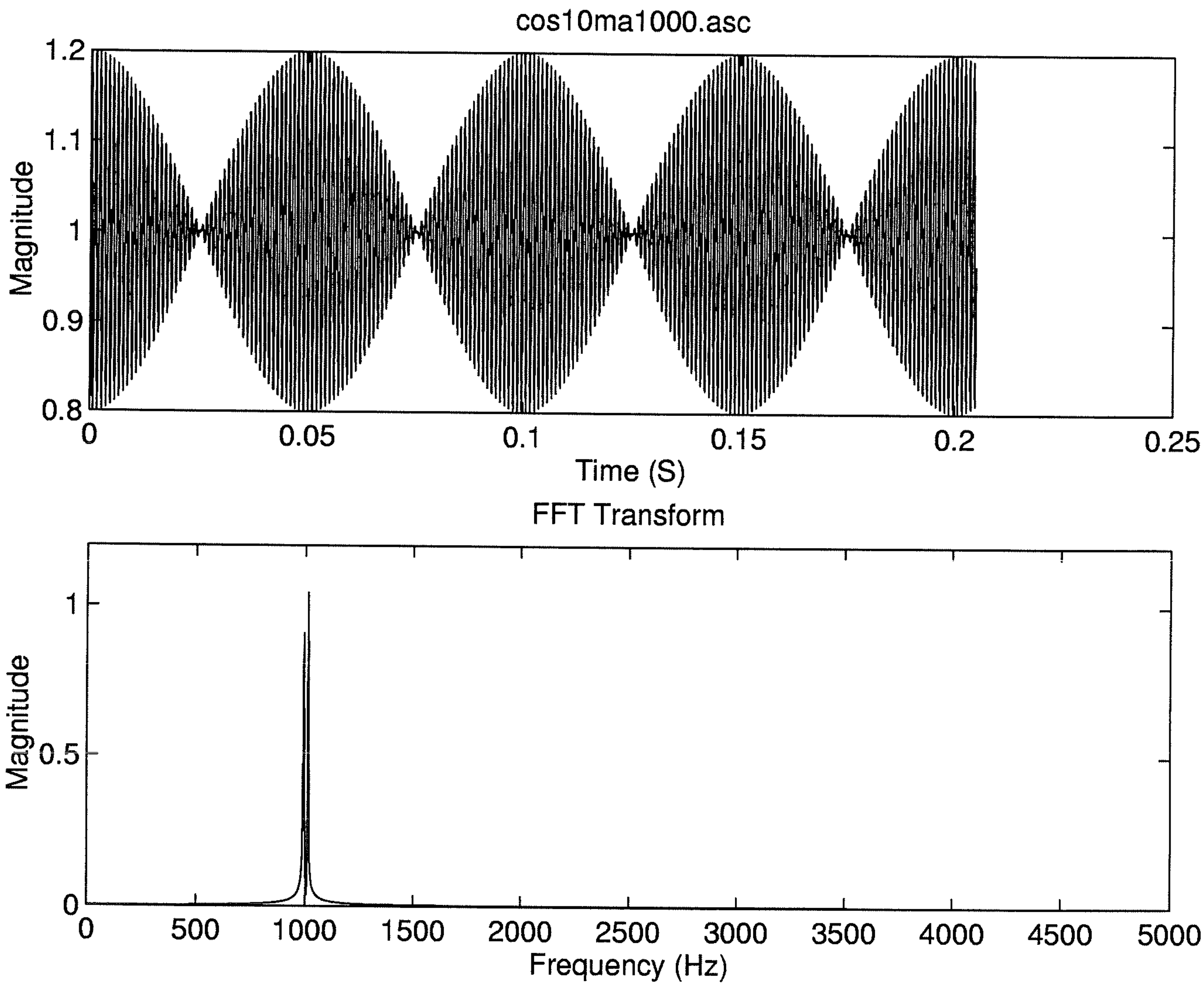



Figure 13: Amplitude-modulated signal and its spectrum.

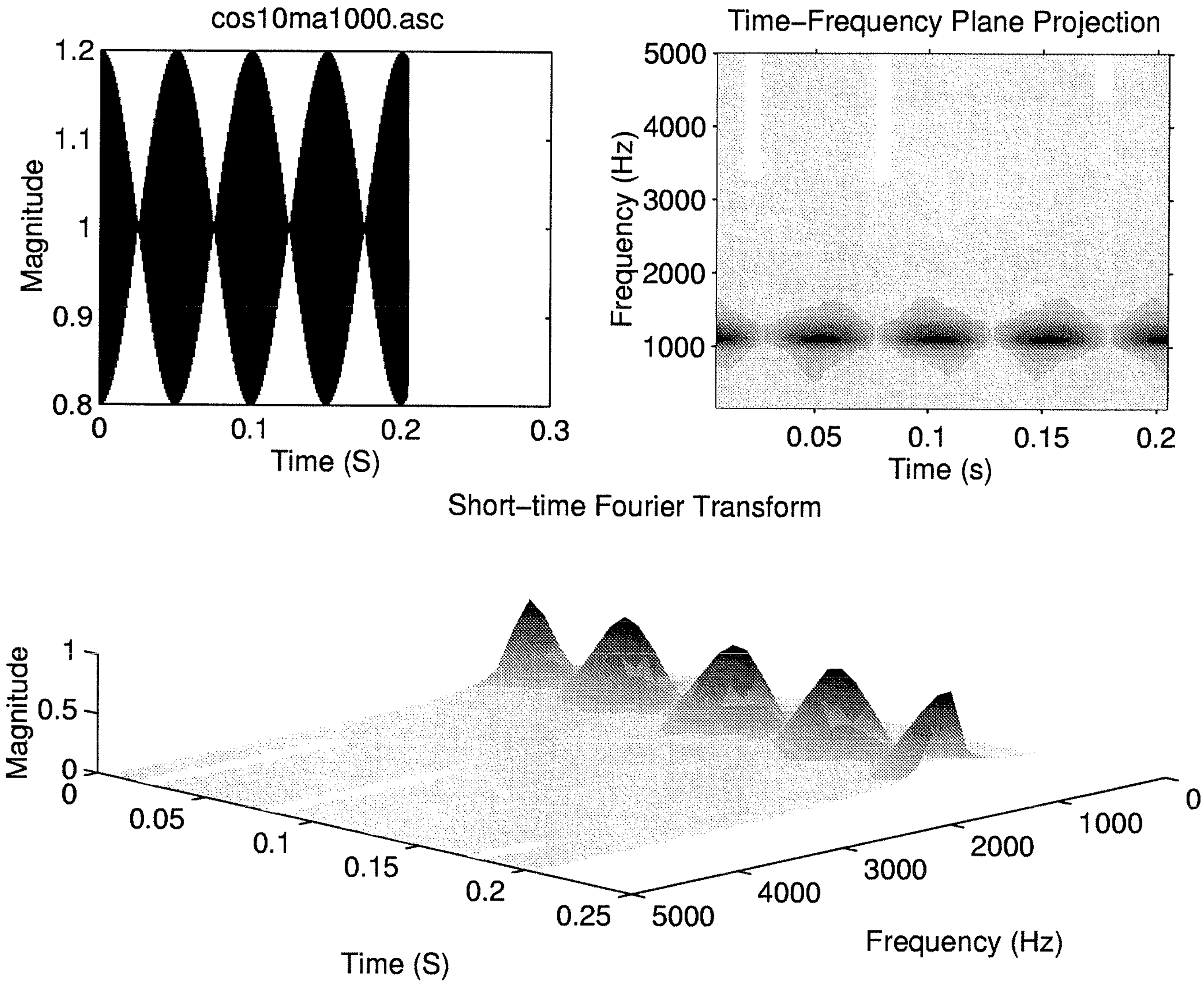



Figure 14: Short-time Fourier transform of the amplitude modulated signal.

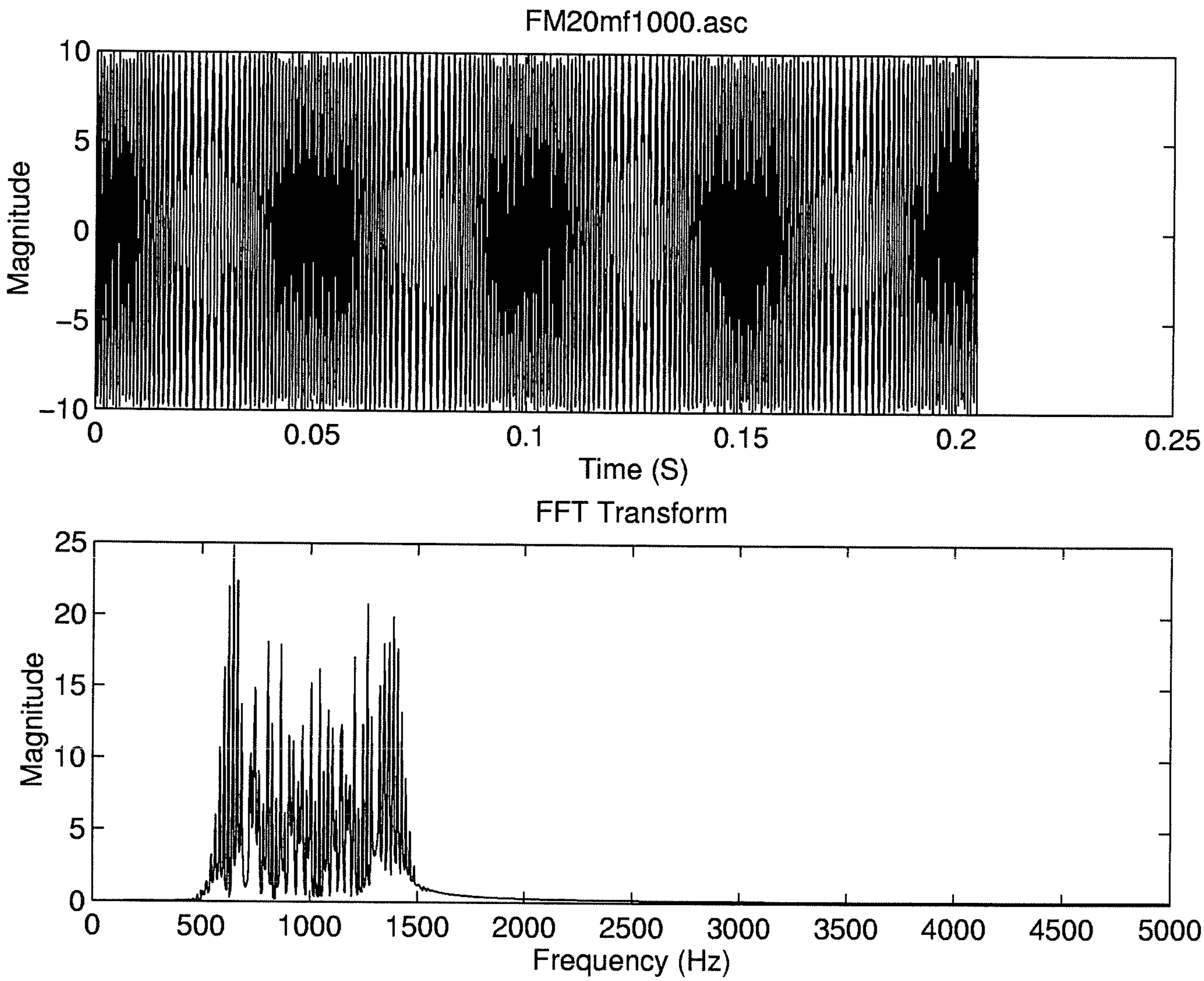



Figure 15: Frequency-modulated signal and its spectrum.

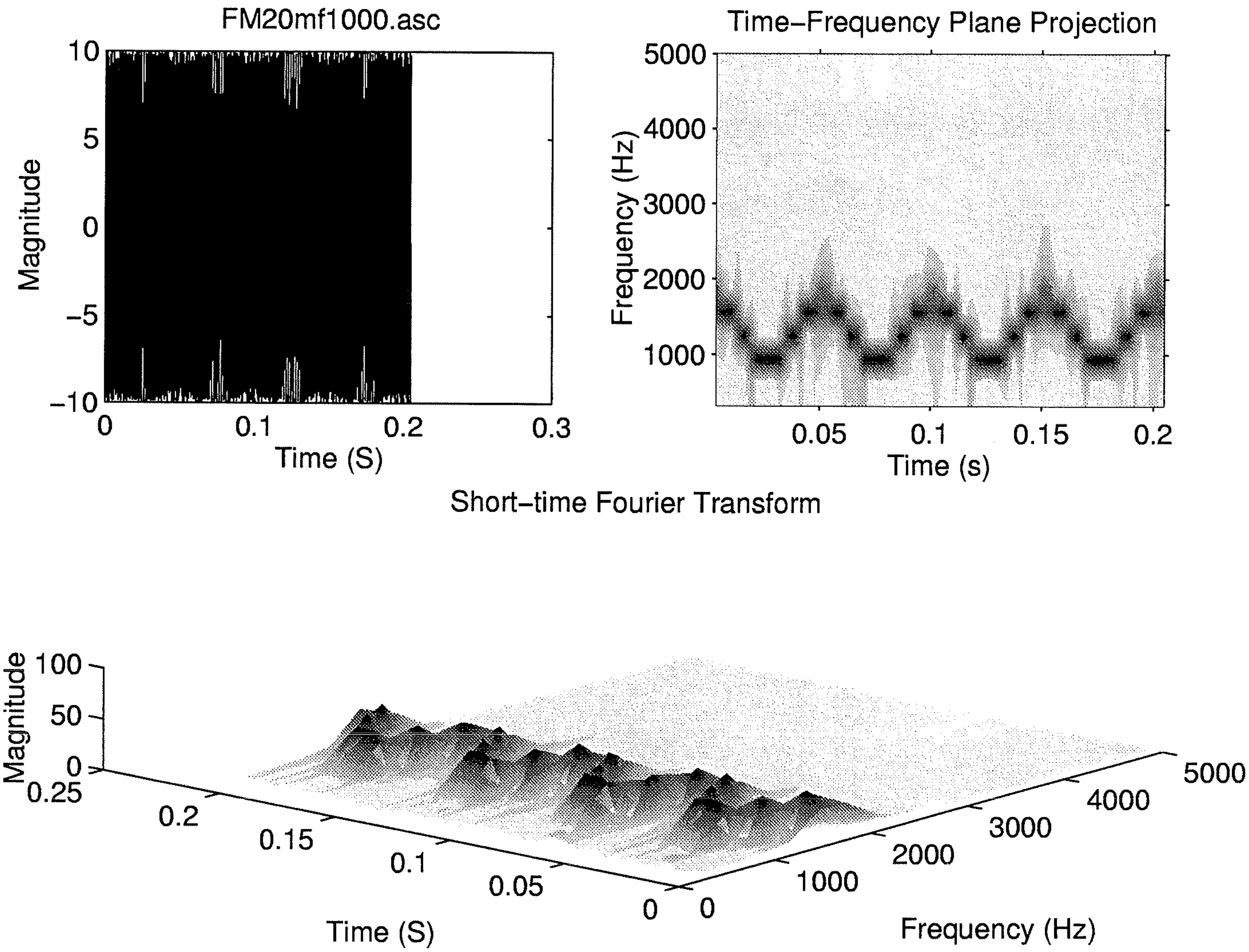



Figure 16: Short-time Fourier transform of the frequency-modulated signal

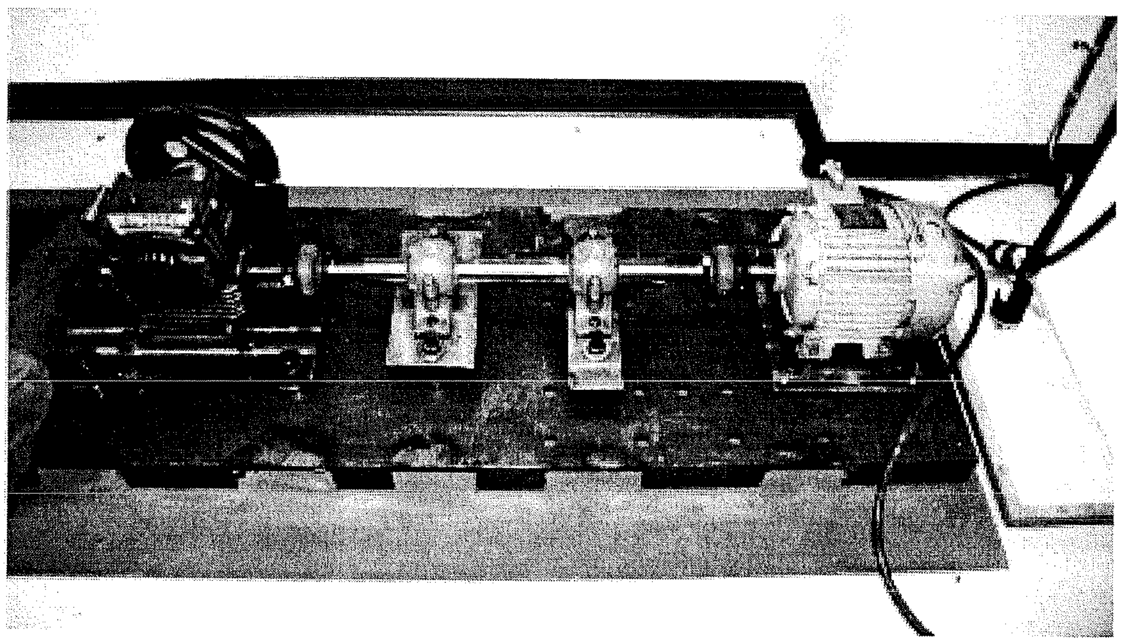


Figure 17: Test setup.

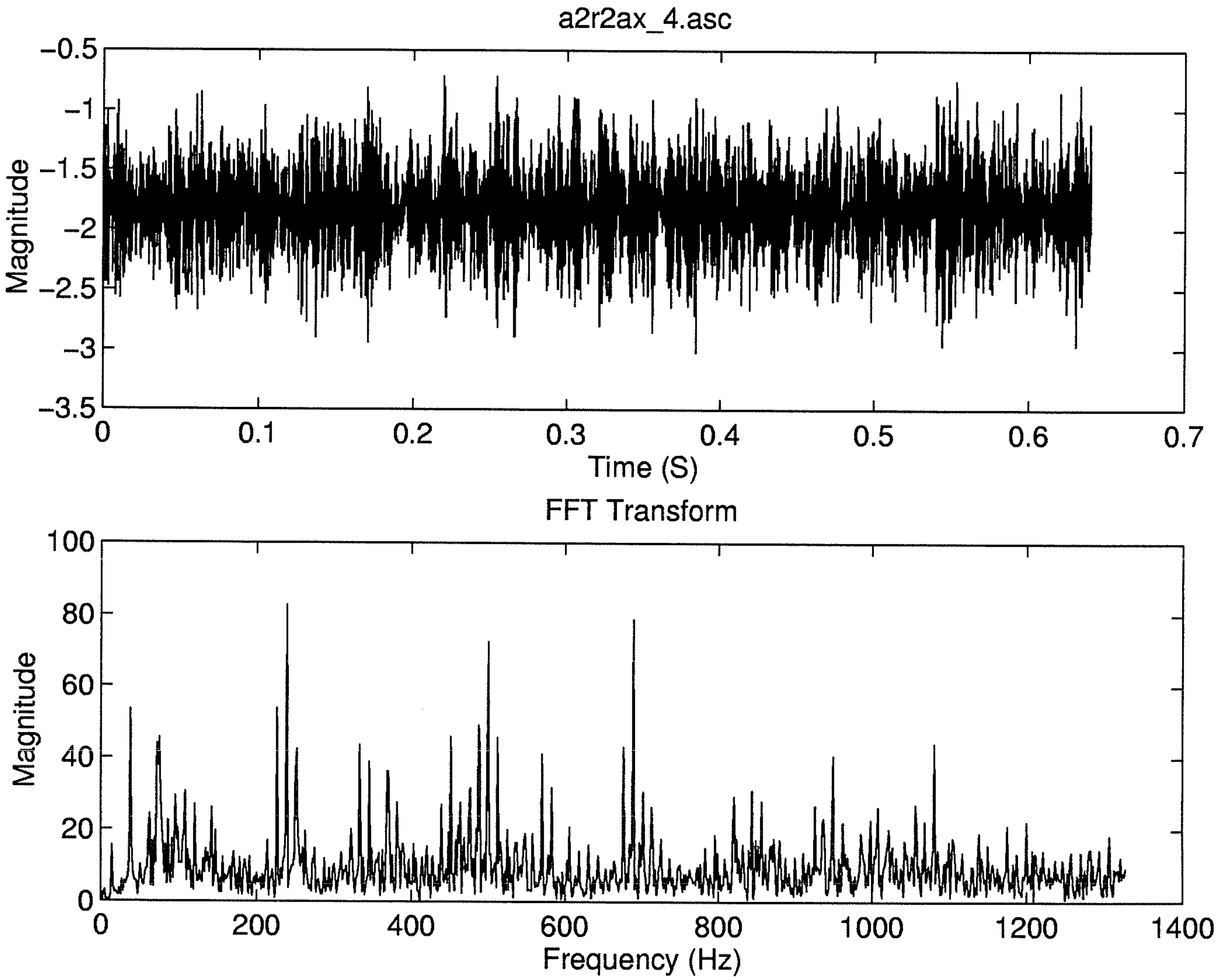



Figure 18: The signal measured on a defective bearing and its spectrum.

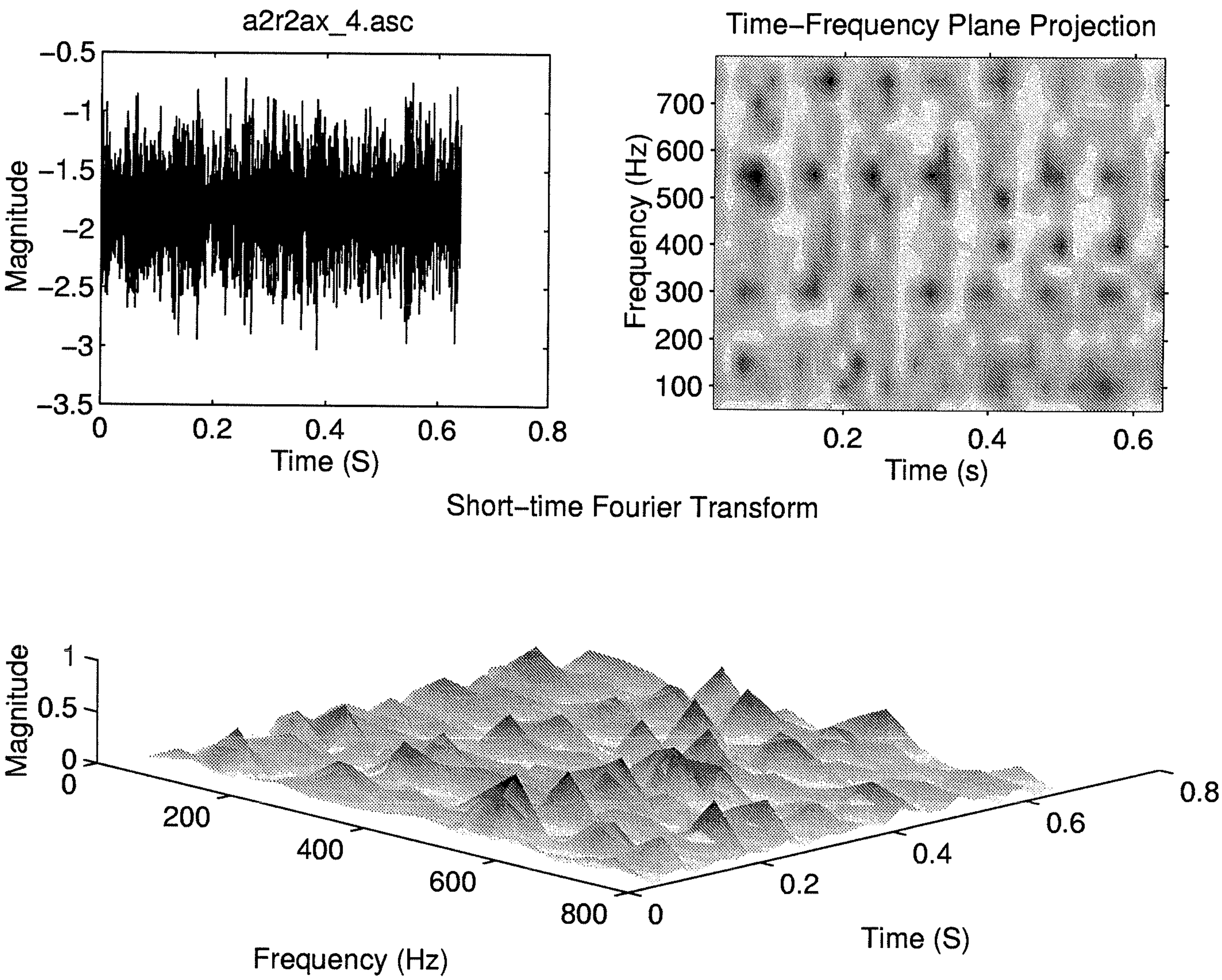



Figure 19: Short-time Fourier transform of the defective bearing signal.

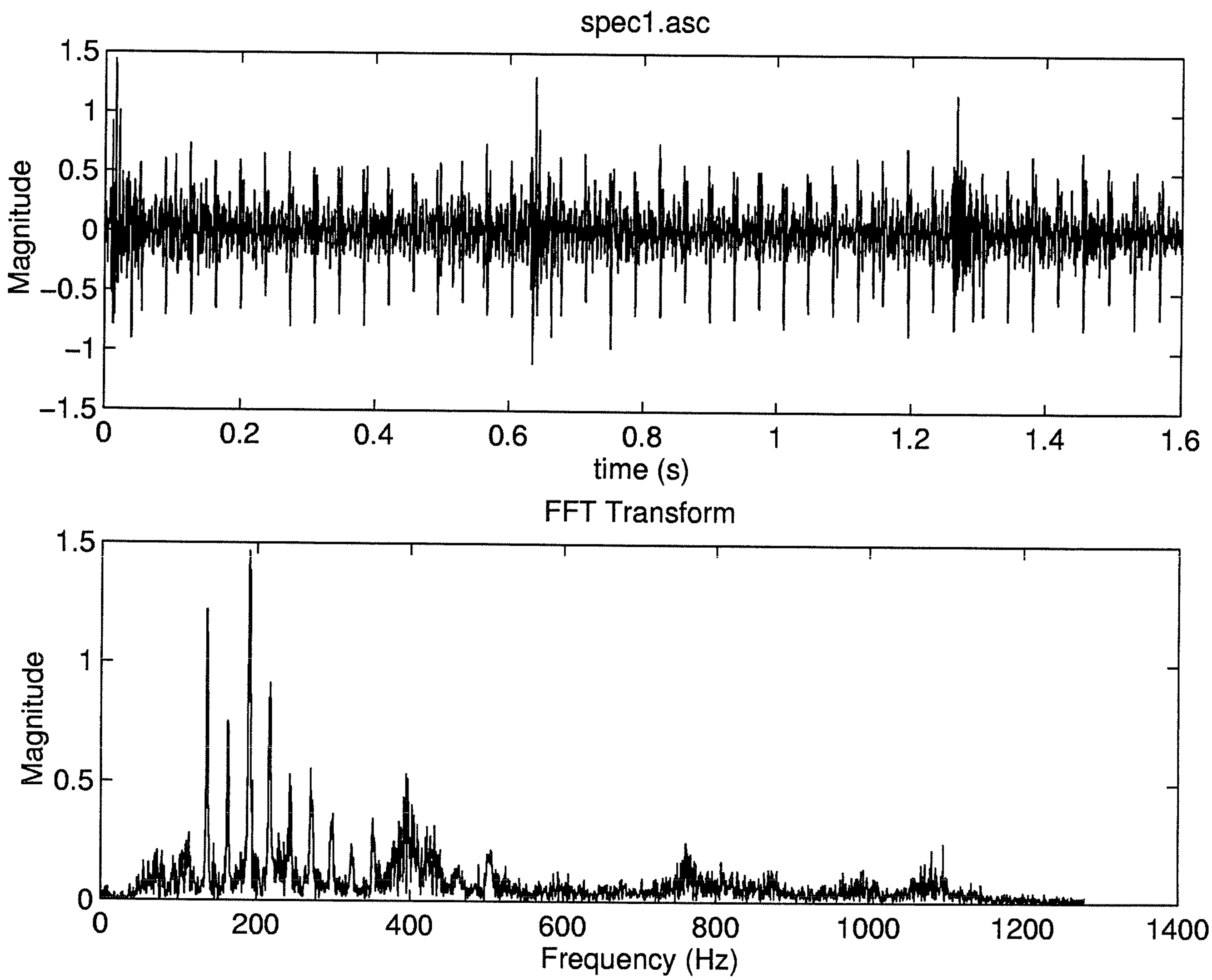



Figure 20: The signal measured on a defective gearbox and its spectrum.

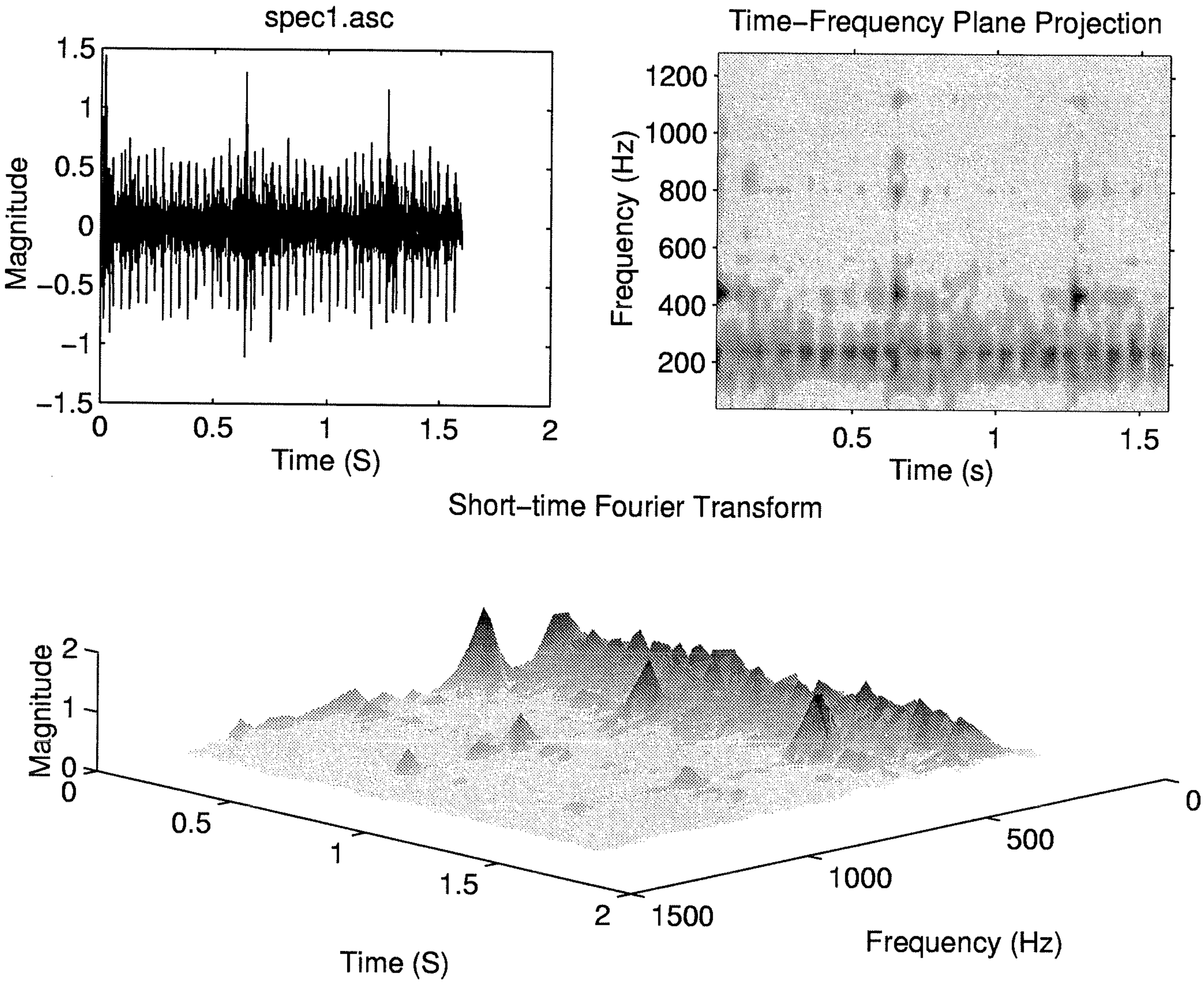



Figure 21: Short-time Fourier transform of the defective gearbox signal.

École Polytechnique de Montréal
C.P. 6079, Succ. Centre-ville
Montréal (Québec)
H3C 3A7