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"PATH-TRACKING FOR CAR-LIKE AND TRACTOR-TRAILER-LIKE ROBOTS"

par

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PATH-TRACKING FOR CAR-LIKE AND TRACTOR-TRAILER-LIKE ROBOTS

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0. Summary

Path-tracking controllers for car-like and tractor-trailer-like robots are designed by generalizing the geometric path-tracking approach currently adopted for car-like robots. This is done by formalizing the concepts of speed, lateral and heading offsets and by assuming a slippage-free motion. The main result is that for straight-line or circular-arc paths to be tracked with a constant velocity, path-tracking may be ensured by means of a linear, time-invariant and decoupled controller of which the gains may be determined using the very familiar PID and state-feedback techniques.

1. Introduction.

While appropriate for many practical applications, most of the results concerning path-tracking controllers for mobile wheeled robots are of an ad hoc nature and are limited to car-like robots, straight-line paths and a constant tracking velocity (see, for example, Anderson 1985, Cox 1990, Giralt 1988, Borenstein and Koren 1987, DeSantis and Hurteau 1990, Lee 1992). Recently, the search for greater flexibility of motion, a greater payload and a greater variety of vehicle formations has created an interest in extending these results into a context where more general path assignments may be considered and where the car-like robot may be replaced with an articulated robot (in particular, a tractor-trailer-like robot).

Typical of efforts addressing this problem, is the paper by Sampei et al. 1990, which gives a systematic procedure for the design of a path-tracking controller for a tractor-trailer-like robot. Basic elements of this procedure are a slippage-free motion, a purely kinematic model of the vehicle, straight-line paths to be followed with a constant velocity, exact and Lyapunov linearizations, and time-scale transformation techniques. Similar contributions based on the same techniques propose more general and more rigorous procedures applicable to a larger class both of path-assignments and articulated vehicles (see, for example, Kanayama et al. 1990, Walsh et al. 1992, D'Andrea-Novel et al. 1992).

A common feature of these developments is a state-trajectory-following approach whereby the path-tracking controller is required to ensure the convergence of the vehicle's state to a desired state which is itself a prescribed function of time. While mathematically elegant and producing many interesting results, state-trajectory-following may not necessarily be the best approach for designing path-tracking controllers. Indeed, the approach that is routinely adopted in industrial car-like robots and that is based on the notion of geometric path-tracking, appears to lead to simpler controllers that are more intuitive and easier to implement. Traditional in automotive applications, this latter approach still entails the convergence of the vehicle's state to a prescribed state. However, rather than being a pre-assigned function of time, this prescribed state is now a function of the configuration of the vehicle with respect to the path to be tracked (see, for example, Fenton et al. 1976, Hemami et al. 1992, Shin et al. 1992).

Based on these observations, it is of interest to explore the design of path-tracking controllers for car-like and tractor-trailer-like robots by adopting the geometric path-tracking approach. We will carry out such an extension here by using a configuration-space setting (Latombe 1991, Fernandez, Gurvitz and Li 1991), by formalizing the notions of speed, lateral and heading offsets, and by assuming a slippage-free motion (Alexander and Maddocks 1988). We consider first the case of a tractor-trailer-like robot. The case of a car-like robot is then treated by specializing the tractor-trailer results.

2. Vehicle's Dynamic Model

Consider a tractor (equipped with two rear-drive wheels and two front-steering wheels) linked to a trailer (with two rear wheels) by means of a revolute vertical joint (Figure 1). Assume the vehicle's motion to be planar, the geometric kinematic and dynamic properties of both tractor and trailer to be symmetrical with respect to their longitudinal axes, and the contact between the tires and the surface to be point-wise. Assume further that the difference between the orientation of the tractor and that of its front wheels (**steering angle**) is sufficiently small that these wheels can be represented in terms of a "median" wheel located at the center of the front axle. The center of the tractor's rear axle will be referred to here as the vehicle's **guide-point.**

To discuss the vehicle's dynamics we introduce the following notations (see Figure 1):

(x, y): work-space coordinates of the tractor's guide-point;

Θ_1, Θ_2: tractor's, trailer's heading;

Φ: trailer's orientation with respect to the tractor;

δ: steering angle;

v_u, v_w: velocity of the tractor's center of mass (**c.o.m.**) in tractor-frame coordinates;

Ω_1, Ω_2 : angular velocities of the tractor and of the trailer;

F_u, : vector, the entries of which describe the longitudinal forces exerted by the tires on the vehicle (see Figure 2);

F_p: propulsion control;

F_s: steering control;

a: distance from tractor's c.o.m. to front axle;

b: distance from tractor's rear axle to vertical joint;

L_1: distance from tractor's rear axle to front axle;

L_2: distance from trailer's axle to vertical joint.

With these notations, by writing the Newton-Euler equations (see Kane Levinson 1985) and imposing well-known holonomic and nonholonomic constraints (see Latombe 1991), we obtain the following dynamic model of the vehicle. A general methodology for the development of models of this nature may be found in Saha and Angeles 1989.

Proposition 1. Under a slippage-free motion, the dynamics of the tractor-trailer robot is described by

$$\dot{v}_u = g_0 + g_uF_u + g_pF_p + g_sF_s \tag{2.1}$$

$$\dot{x} = \cos\theta_1 v_u \tag{2.2}$$

$$\dot{y} = \sin\theta_1 v_u \tag{2.3}$$

$$v_w = \frac{av_u\tan\delta}{L_1} \tag{2.4}$$

$$\dot{\theta}_1 = \frac{v_u\tan\delta}{L_1} \tag{2.5}$$

$$\dot{\Phi} = -\frac{v_u}{L_1L_2}\{L_1\sin\Phi + (L_2 + b\cos\Phi)\tan\delta\} \tag{2.6}$$

$$\dot{\delta} = F_s, \tag{2.7}$$

where g_0, g_u, g_p and g_s are well-defined functions of x, y, θ_1 and Φ, and of the vehicle's mass and geometric parameters.

The vectors

$$\mathbf{q} := [x \; y \; \theta_1 \; \Phi]', \tag{2.8}$$

$$\mathbf{v} := [v_u \; v_w \; \Omega_1 \; \Omega_2]', \tag{2.9}$$

$$\mathbf{X} := [x \; y \; \theta_1 \; \Phi \; v_u \; v_w \; \Omega_1 \; \Omega_2]' := [q \; v]' \tag{2.10}$$

and

$$\mathbf{a}_c := [\dot{v}_{u1} \; \dot{v}_{w1} \; \dot{\Omega}_1 \; \dot{\Omega}_2]', \tag{2.11}$$

are referred to as the **configuration (q)**, the **velocity (v)**, the **state (X)** and the **acceleration ($\mathbf{a}_c$)** of the vehicle.

3. Path-Tracking

A path-tracking controller receives as input actual and desired values of the vehicle's speed, heading and position relative to the path. It provides as output the propulsion and steering required to bring to zero the difference between actual and desired values (see Figure 3). In spite of its simplicity, a formal statement of this task requires a careful definition of the concepts of admissible path-tracking assignment, the vehicle's desired state and path-tracking offsets. A **path-tracking assignment** is the combination of a path in the configuration-space and a profile of the linear and angular velocities and accelerations with which this path is to be followed. A **path** (see Latombe 1991) is described by a smooth vector function,

$$q_p(s) := [x_p(s) \; y_p(s) \; \theta_{p1}(s) \; \Phi_p(s)], \tag{3.1}$$

where $s \in [0, \infty)$ is a parameter defining a point of the path and $q_p(s)$ represents the vehicle's required configuration at point s. A **velocity** and **acceleration profile** along a path is described by a set of smooth functions

$$v_p(s) := [v_{up}(s) \; v_{wp}(s) \; \Omega_{p1}(s) \; \Omega_{p2}(s)], \tag{3.2}$$

$$a_p(s) := [a_{up}(s) \; a_{wp}(s) \; a_{\theta p1}(s) \; a_{\theta p2}(s)], \; s \in [0, \infty), \tag{3.3}$$

where $v_p(s)$ and $a_p(s)$ are the velocity and the acceleration that the vehicle should have when its work-space position corresponds to $(x_p(s)$ $y_p(s))$. A path-tracking assignment is **admissible** if eqns (3.2, 3.3) are compatible with eqns (2.2-2.7).

Given a state of the vehicle $X:=[x\ y\ \Theta_1\ \Phi\ v_u\ v_w\ \Omega_1\ \Omega_2]'$, the vehicle's **desired state** corresponding to an admissible path-tracking assignment is defined by

$$X_d:=[\mathbf{q}_d\ \mathbf{v}_d] \tag{3.4}$$

$$q_d:=[x_d\ y_d\ \Theta_{d1}\ \Phi_d] \tag{3.5}$$

$$v_d:=[v_{ud}\ v_{wd}\ \Omega_{d1}\ \Omega_{d2}], \tag{3.6}$$

with

$$[x_d\ y_d\ \Theta_{d1}\ \Phi_d]:=[x_p(\sigma)\ y_p(\sigma)\ \Theta_{p1}(\sigma)\ \Phi_p(\sigma)], \tag{3.7}$$

and

$$[v_{ud}\ v_{wd}\ \Omega_{d2}\ \Omega_{d2}]:=[v_{up}(\sigma)\ v_{wp}(\sigma)\ \Omega_{p1}(\sigma)\ \Omega_{p2}(\sigma)], \tag{3.8}$$

where $\sigma\epsilon[0,\infty)$ has the property that $(x_p(\sigma), y_p(\sigma))$ is the point of the work-space path closest (in Euclidean norm) to (x, y).

The **path-tracking offsets (velocity (v_{os}), heading (Θ_{1os}, Φ_{os}), lateral (L_{os}) and steering offsets)** are defined by

$$v_{os}(t):=v_u(t)-v_{ud}(t) \tag{3.9}$$

$$\Theta_{os}(t):=\Theta_1(t)-\Theta_{d1}(t) \tag{3.10}$$

$$\Phi_{os}(t):=\Phi(t)-\Phi_d(t) \tag{3.11}$$

$$L_{os}(t):=-\{x(t)-x_d(t)\}\sin\Theta_{d1}(t)+\{y(t))-y_d(t)\}\cos\Theta_{d1}(t) \tag{3.12}$$

$$\delta_{os}(t):=\delta(t)-\delta_d(t). \tag{3.13}$$

where,

$$\delta_d:=\tan^{-1}\frac{\Omega_{d1}L_1}{v_{ud}}. \tag{3.14}$$

It should be noted that while the meanings of v_{os}, θ_{os}, Φ_{os} and δ_{os} are rather obvious the lateral offset, L_{os}, represents the (signed) distance between the guide-point of the vehicle and the assigned path in the work-space.

The task of a **path-tracking** controller may be now formally stated as that of generating the propulsion and steering controls required to have

$$\lim_{t \to \infty} [v_{os}(t) \;\; \theta_{os}(t) \;\; \Phi_{os}(t) \;\; L_{os}(t)] = 0. \tag{3.15}$$

Remark 1. The above statement gives a precise mathematical formulation of the intuitive notion of geometric path-tracking that is currently used in actual implementations of car-like robots. By simply replacing Φ_{os} with Φ_{osi}, i= 1,2,..., where Φ_{osi} would denote the heading offset of the i-th trailer, this statement becomes applicable to tractors with multiple trailers.

4. Controller Design

With the above background, the design of a path-tracking controller may be viewed as equivalent to the following stabilization problem.

Proposition 2. Under a slippage-free motion, path-tracking is equivalent to stabilizing the dynamic system

$$\dot{v}_{os} = u_1(t) \tag{4.1}$$

$$\dot{\Theta}_{os} = \frac{(v_{ud}+v_{os})}{L_1} (\tan(\delta_d+\delta_{os}) - \tan\delta_d) \tag{4.2}$$

$$\dot{\Phi}_{os} = \frac{(v_{ud}+v_{os})}{L_1 L_2} \{L_1(\sin\Phi_d - \sin(\Phi_d+\Phi_{os})) + (L_2 + c\cos\Phi_d)\tan\delta_d - (L_2 + b\cos(\Phi_d+\Phi_{os}))\tan(\delta_d+\delta_{os})\} \tag{4.3}$$

$$\dot{L}_{os} = (v_{ud}+v_{os})\{\sin\Theta_{os} + \frac{a}{L_1}\tan(\delta_d+\delta_{os})\cos\Theta_{os} - \frac{a}{L_1}\tan\delta_d\} \tag{4.4}$$

$$\dot{\delta}_{os} = u_2(t), \tag{4.5}$$

where

$$u_1 := -\dot{v}_{ud} + g_0 + g_u F_u + g_p F_p + g_s F_s, \tag{4.6}$$

$$u_2 := -\dot{\delta}_d + F_s. \tag{4.7}$$

With this result, a path-tracking controller may now be designed by bringing to bear well-known control theory techniques.

Proposition 3. Under a slippage-free motion and with small offsets, path-tracking is equivalent to stabilizing the linear system

$$\dot{x} = Ax + Bu, \tag{4.8}$$

where

$$x := [v_{os}\ \Theta_{os}(t)\ \Phi_{os}(t)\ L_{os}(t)\ \delta_{os}(t)]' \tag{4.9}$$

$$B := \begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix}\ ' \tag{4.10}$$

and

$$A := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{v_{ud}}{L_1\cos^2\delta_d} \\ 0 & 0 & a_{33} & 0 & a_{35} \\ 0 & v_{ud} & 0 & 0 & \dfrac{av_{ud}}{L_1\cos^2\delta_d} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{4.11}$$

with

$$a_{33} := -v_{ud}\left\{\frac{\cos\Phi_d}{L_2} - \frac{b\tan\delta_d\sin\Phi_d}{L_1L_2}\right\} \tag{4.12}$$

and

$$a_{35} := -v_{ud}\,\frac{L_2 + c\cos\Phi_d}{L_2L_1\cos^2\delta_d}. \tag{4.13}$$

Proposition 4. If the path in the work-space is a straight-line or a circular-arc, the tracking velocity is constant and the path-tracking offsets are kept sufficiently small, then path-tracking may be ensured by combining two linear and time-invariant controllers; first, a steering controller providing the steering action

$$F_s(t) = - K_s[\theta_{os}(t) \;\; \Phi_{os}(t) \;\; L_{os}(t) \;\; \delta_{os}(t)], \tag{4.14}$$

where K_s is a row-vector of constant gains; second, a speed controller providing the propulsion

$$F_p(t) = F_{p1}(t) + F_{p2}(t), \tag{4.15}$$

where

$$F_{p1} = g_p^{-1}\{\dot{v}_{ud} - g_0 - g_u F_u - g_s F_s\} \tag{4.16}$$

with F_s given by eqn (4.14), and

$$F_{p2} = -\,g_p^{-1}\{K_{p1} v_{os} + K_{p2} \int v_{os}\, dt\} \tag{4.17}$$

with K_{p1} and K_{p2} constant gains.

Proposition 5. The controller described by Proposition 4 has the following properties:

a) the dynamics of v_{os} is described by

$$\ddot{v}_{os} - (p_{11} + p_{12})\dot{v}_{os} + p_{11}p_{12}v_{os} = 0, \tag{4.18}$$

where

$$K_{p1} = -\,(p_{11} + p_{12}), \qquad K_{p2} = p_{11}p_{12}, \tag{4.19}$$

and p_{11}, p_{12} are the eigenvalues of eqn (4.18);

b) gains $K_s := [K_{s1}\ K_{s2}\ K_{s3}\ K_{s4}]$ may be chosen so as to stabilize

$$\dot{x} = Ax + Bu$$

$$u = -\,K_s x, \tag{4.20}$$

where

$$B := [0\ 0\ 0\ 1]' \tag{4.21}$$

and

$$A := \begin{matrix} 0 & 0 & 0 & \dfrac{v_{ud}}{L_1 \cos^2\delta_d} \\ 0 & a_{22} & 0 & a_{24} \\ v_{ud} & 0 & 0 & \dfrac{a v_{ud}}{L_1 \cos^2\delta_d} \\ 0 & 0 & 0 & 0 \end{matrix} \tag{4.22}$$

with

$$a_{22} := - \quad v_{ud}\left\{\frac{\cos\Phi_d}{L_2} - \frac{b\tan\delta_d\sin\Phi_d}{L_1 L_2}\right\} \tag{4.23}$$

and

$$a_{24} := -v_{ud}\,\frac{(L_2 + c\cos\Phi_d)}{L_1 L_2\,\cos^2\delta_d}\,. \tag{4.24}$$

Proposition 6. The steering controller described by eqn (4.14) may be replaced with

$$F_s(t) = - K_s[L_{os}(t) \;\; \dot{L}_{os}(t) \;\; \Phi_{os}(t) \;\; \delta_{os}(t)]', \tag{4.25}$$

where K_s is a row-vector of constant gains. In the case of a forward motion, ($v_{ud} > 0$), a suitable steering controller is given by

$$F_s(t) = - K_s[L_{os}(t) \;\; \dot{L}_{os}(t) \;\; \delta_{os}(t)]' \tag{4.26}$$

where, once again, K_s is a row-vector of constant gains.

Remark 2. The structure of eqns (4.1-4.7) and (4.8-4.13) reveals that, under the hypothesis of a slippage-free motion, the influence of external perturbations and mass-parameters variations over the vehicle's dynamics satisfies the so-called matching conditions (see Corless 1993). It follows that the stabilization problem considered in Proposition 2 may be given a solution that is robust with respect this influence; the performance of the controllers in Propositions 3-6 could be made robust by equipping these controllers with the addition of an appropriate feedback loop (see DeSantis 1994b).

5. An Application Example

Consider the design of a path-tracking controller for a tractor-trailer-like robot characterized by the following geometric parameters:

$$a = 1 \text{ m}, \quad b = 1 \text{ m}, \quad L_1 = 2 \text{ m and } L_2 = 4 \text{ m}. \tag{5.1}$$

Let the path-tracking assignment require this vehicle to follow a circular path of radius R = 20 m, with a velocity equal to 2.5 m/s. By following the procedure suggested by Propositions 4 and 5, path-tracking may be ensured by means of a controller made up of a steering component and a speed component.

The steering controller is described by

$$F_s(t) = - K_s[\Theta_{os}(t) \;\; \Phi_{os}(t) \;\; L_{os}(t) \;\; \delta_{os}(t)] \tag{5.2}$$

where gain row-vector K_s is chosen so as to stabilize A - BK_s, with matrices A and B as in eqns (4.10, 4.11). The values of δ_d and Φ_d, on which matrix A depends, are computed using eqns (2.5, 2.6). These equations imply

$$\delta_d = \tan^{-1} \left(\frac{\Omega_{1d} L_1}{v_{ud}} \right) = \tan^{-1} \left(\frac{L_1}{R} \right) \tag{5.3}$$

and

$$R\sin\Phi_d + b\cos\Phi_d) = - L_2. \tag{5.4}$$

With the assigned values of a, b, L_1, L_2 and R, it follows that δ_d = .1 and Φ_d = -.251. Corresponding to a forward motion (v_{ud} = 2.5: the tractor pulls the trailer), we then have

$$A := \begin{matrix} 0 & 0 & 0 & 1.2 \\ 0 & -.73 & 0 & -1.57 \\ 2.5 & 0 & 0 & 1.26 \\ 0 & 0 & 0 & 0 \end{matrix} \quad , \tag{5.5}$$

and corresponding to a backward motion ($v_{ud} = -2.5$: the tractor pushes the trailer):

$$A := \begin{matrix} 0 & 0 & 0 & -1.2 \\ 0 & .73 & 0 & 1.57 \\ -2.5 & 0 & 0 & -1.26 \\ 0 & 0 & 0 & 0 \end{matrix} \quad . \tag{5.6}$$

By computing K_s via the LQR approach, we have $K_s = H^{-1} B'P$ where P the solution of the Riccati Equation (see Chen 1987)

$$A'P - PBH^{-1}B'P + PA + Q = 0 \tag{5.7}$$

with Q and H are conveniently selected positive definite matrices. Note that an appropriate selection depends on whether a forward or a backward motion is considered and is usually more laborious in the case of a backward motion. For a forward motion, with the selection $H = .1$ and $Q = I_4$, we obtain

$$K_{s1} = 6.9 \qquad K_{s2} = -.25 \qquad K_{s3} = 3.16 \qquad K_{s4} = 6. \tag{5.8}$$

For a backward motion, with $H = 1$ and $Q = \mathrm{diag}[q_i]$, and with $q_1 = q_2 = .1$, $q_3=100$ and $q_4=10$, we obtain

$$K_{s1} = 98.8 \qquad K_{s2} = 95.7 \qquad K_{s3} = -10.0 \qquad K_{s4} = 9.2. \tag{5.9}$$

The speed controller generates a propulsion given by

$$F_p = g_p^{-1} \{- g_0 - g_u\hat{F}_u - g_sF_s\}$$
$$- g_p^{-1}\{K_{p1}v_{os} + K_{p2} \int v_{os}dt\}. \tag{5.10}$$

where $\hat{F}_u$ denotes the estimated effect of longitudinal perturbation forces acting on the tires, F_s is as in eqn (5.2), and g_0, g_u, g_s and

g_p are as in Proposition 1. By requiring the dynamics of the velocity offset to be characterized by the poles $p_{11} = -6$ and $p_{12} = -.1$ and applying eqn (4.19), we obtain $K_{p1} = 6.1$, $K_{p2} = .6$.

Figures 5-6 show typical (simulated) vehicle responses corresponding to a forward motion under nominal operating conditions (that is, actual values of the vehicle's kinematic and mass parameters equal to expected values and absence of external perturbations). Figures 7-8 show typical responses obtained under the same conditions for a backward motion. These results suggest that under quite realistic path-tracking offsets the performance of the controller is satisfactory in both types of motion. Note that in the backward motion, completely in accordance with everyday experience, the region of attraction in the offset space is smaller than in forward motion and the offsets have a more pronounced transient dynamics.

6. The Case of a Car-like Robot

By considering the mass parameters of the trailer to be equal to zero, and by focusing attention on the dynamics of the tractor, eqns (2.1, 2.7) coincide with the model of a car-like robot. It follows that Propositions 1-6 enable a rediscovery and an extension of results relevant to path-tracking controllers for car-like robots. This extension means that the procedures currently available for designing these controllers, now become applicable to more general paths, as well as to the case where the tracking velocity is no longer constrained to be constant. More formally, we have the following results.

Proposition 7. The dynamics of a car-like robot is given by

$$\dot{x} = v_u \cos\theta - v_w \sin\theta \tag{6.1}$$

$$\dot{y} = v_u \sin\theta + v_w \cos\theta \tag{6.2}$$

$$\dot{\theta} = \Omega \tag{6.3}$$

$$\dot{v}_u = v_w \Omega + \frac{F_{u1}\cos\delta_1}{M} - \frac{F_{w1}\sin\delta_1}{M} + \frac{F_{u2}}{M} + \frac{F_p}{M} \tag{6.4}$$

$$\dot{v}_w = - v_u \Omega + \frac{F_{u1}\sin\delta_1}{M} + \frac{F_{w1}\cos\delta_1}{M} + \frac{F_{w2}}{M} \tag{6.5}$$

$$\dot{\Omega} = \frac{F_{u1} a \sin\delta_1}{J} + \frac{F_{w1} a \cos\delta_1}{J} - \frac{F_{w2} b}{J} \tag{6.6}$$

Proposition 8. Under a slippage-free motion, the vehicle's velocity is submitted to the constraint

$$\mathbf{Hv} = 0, \tag{6.7}$$

where

$$H := \begin{matrix} -\dfrac{b\tan\delta_1}{L} & 1 & 0 \\ -\dfrac{\tan\delta_1}{L} & 0 & 1 \end{matrix} \; . \tag{6.8}$$

Moreover,

$$\delta_1 := \tan^{-1}\{\frac{v_w + a\Omega}{v_u}\} \quad . \tag{6.9}$$

Proposition 9. Under a slippage-free motion, the lateral forces exerted by the tires on the vehicle are given by

$$\mathbf{F}_w = G_1 + G_2\mathbf{F}_u + G_3F_p + G_4\mathbf{F}_s, \tag{6.10}$$

where

$$\mathbf{F}_w := [F_{w1} \ \ F_{w2}]' \tag{6.11}$$

$$\mathbf{F}_u := [F_{u1} \ \ F_{u2}]' \tag{6.12}$$

$$G_1 := -[HG_w]^{-1}AG_0 \tag{6.13}$$

$$G_2 := -[HG_w]^{-1}AG_u \tag{6.14}$$

$$G_3 := -[HG_w]^{-1}AG_p \tag{6.15}$$

$$G_4 := -[HG_w]^{-1}G_s \tag{6.16}$$

$$G_0 := [v_w\Omega \ \ -v_u\Omega \quad 0]' \tag{6.17}$$

and

$$G_w := \begin{matrix} -\frac{\sin\delta_1}{M} & 0 \\ \frac{\cos\delta_1}{M} & \frac{1}{M} \\ \frac{a\cos\delta_1}{J} & -\frac{b}{J} \end{matrix} \qquad G_u := \begin{matrix} \frac{\cos\delta_1}{M} & \frac{1}{M} \\ \frac{\sin\delta_1}{M} & 0 \\ \frac{a\sin\delta_1}{J} & 0 \end{matrix} \tag{6.18, 6.19}$$

$$G_p := [\ \frac{1}{M} \quad 0 \quad 0]' \tag{6.20}$$

$$G_s := \begin{bmatrix} -\dfrac{bv_u}{L\cos^2\delta_1} \\ \\ -\dfrac{v_u}{L\cos^2\delta_1} \end{bmatrix} \tag{6.21}$$

Proposition 10. Under a slippage-free motion, the dynamics of the vehicle is described by

$$\dot{v}_u = g_0 + g_u\mathbf{F}_u + g_pF_p + g_s\mathbf{F}_s \tag{6.22}$$

$$v_w = \frac{v_u(b\tan\delta_1)}{L} \tag{6.23}$$

$$\Omega = \frac{v_u(\tan\delta_1)}{L} \tag{6.24}$$

$$\dot{\delta}_1 = F_{s1} \tag{6.25}$$

$$\dot{x} = \cos\theta v_u - \sin\theta v_w \tag{6.26}$$

$$\dot{y} = \sin\theta v_u + \cos\theta v_w \tag{6.27}$$

$$\dot{\theta} = \Omega, \tag{6.28}$$

where

$$g_0 := [1 \;\; 0 \;\; 0]\{G_0 + G_wG_1\} \tag{6.29}$$

$$g_u := [1 \;\; 0 \;\; 0]\{G_u + G_wG_2\} \tag{6.30}$$

$$g_p := [1 \;\; 0 \;\; 0]\{G_p + G_wG_3\} \tag{6.31}$$

$$g_s := [1 \;\; 0 \;\; 0]G_wG_4 \tag{6.32}$$

and G_0, G_1, G_2, G_3, G_4, G_u, G_p, G_s are as in eqs. 9-16.

Proposition 11. Under a slippage-free motion, path-tracking is equivalent to stabilizing the dynamic system

$$\dot{v}_{os} = u_1(t) \tag{6.33}$$

$$\dot{\theta}_{os} = \frac{(v_{ud}+v_{os})\tan(\delta_{d1}+\delta_{os1})}{L} - \frac{(v_{ud}+v_{os})\tan\delta_{d1}}{L} \tag{6.34}$$

$$\dot{L}_{os} = (v_{ud}+v_{os})\sin\theta_{os} + \frac{(v_{ud}+v_{os})b\tan(\delta_{d1}+\delta_{os1})\cos\theta_{os}}{L} - \frac{(v_{ud}+v_{os})b\tan\delta_{d1}}{L} \tag{6.35}$$

$$\dot{\delta}_{os1} = u_2(t) \tag{6.36}$$

$$\dot{\delta}_{os2} = u_3(t). \tag{6.37}$$

where

$$u_1 := -\dot{v}_{ud} + g_0 + g_u\mathbf{F}_u + g_pF_p + g_s\mathbf{F}_s, \tag{6.38}$$

$$u_2 := -\dot{\delta}_{d1} + F_{s1}, \tag{6.39}$$

and g_0, g_u, g_p, g_s are as in Proposition 11.

Proposition 12. Under a slippage-free motion and small offsets, path-tracking is equivalent to stabilizing

$$\dot{x} = Ax + Bu, \tag{6.40}$$

$$y = Cx$$

where

$$B := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}' \tag{6.41}$$

$$C := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{6.42}$$

and

$$A := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{v_{ud}}{L\cos^2\delta_{d1}} \\ 0 & v_{ud} & 0 & \dfrac{bv_{ud}}{L\cos^2\delta_{d1}} \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.43}$$

Proposition 13. If the path in work-space is a straight line or a circular arc, the desired tracking velocities are constant and the path-tracking offsets are kept sufficiently small, then path-tracking may be ensured by the combination of two time-invariant controllers. First, a position/ orientation controller providing the steering action

$$F_s(t) = - K_s[\theta_{os}(t) \; L_{os}(t) \; \delta_{os1}(t)]', \tag{6.44}$$

where K_s is a matrix of constant gains. Second, a speed controller providing the propulsion

$$F_p(t) = F_{p1}(t) + F_{p2}(t), \qquad (6.45)$$

where

$$F_{p1} = g_p^{-1} \{\dot{v}_{ud} - g_0 - g_u F_u - g_s F_s\} \qquad (6.46)$$

with F_s given by eq. 6.13, and

$$F_{p2} = - g_p^{-1} \{K_{p1} v_{os} + K_{p2} \int v_{os} dt\} \qquad (6.47)$$

with K_{p1} and K_{p2} two constant gains.

Proposition 14. The controller described by eqs. 6.13-.16 has the following properties:

a) the dynamics of v_{os} is given by

$$\ddot{v}_{os} - (p_{11} + p_{12})\dot{v}_{os} + p_{11}p_{12}v_{os} = 0, \qquad (6.48)$$

where poles p_{11} and p_{12} are such that

$$-(p_{11} + p_{12}) = K_{p1} \quad \text{and} \quad p_{11}p_{12} = K_{p2}; \qquad (6.49)$$

b) gains K_s may be chosen so as to stabilize the system

$$\dot{x} = (A - BK_s)x \qquad (6.50)$$

where

$$B := \quad [0 \ 0 \ 1]' \qquad (6.51)$$

and

$$A := \begin{matrix} 0 & 0 & \dfrac{v_{ud}}{L\cos^2\delta_{d1}} \\ v_{ud} & 0 & \dfrac{bv_{ud}}{L\cos^2\delta_{d1}} \\ 0 & 0 & 0 \end{matrix} \qquad (6.52)$$

Conclusions

By assuming a slippage-free motion, it is possible to simplify considerably the model of the kinematic and dynamic behavior of a tractor-trailer-like robot (Proposition 1). Combined with a geometric notion of path-tracking, this simplification leads, in turn, to a simplified description of the dynamics of lateral, heading and velocity path-tracking offsets (Proposition 2). It follows that, for small offsets, path-tracking may be ensured by means of an affine controller, linear with respect to these offsets (Proposition 3). Corresponding to straight-line or circular paths to be followed with a constant velocity, this controller may be implemented by means of a steering component generating steering as a function of lateral and heading offsets, and of a speed component providing propulsion as a function of steering action and speed offset (Proposition 4). The design of these components may be carried out by using PID and state-feedback time-invariant techniques (Proposition 5). Simplified versions of the steering controller, in which the tractor's heading offset may be replaced by a measurement of the rate of lateral offset, are also available (Proposition 6). Finally, by removing from our equations the influence of the trailer, we rediscover and generalize results available for car-like robots (Propositions 7-14).

Although not the focus of the current study, the potential for a practical implementation of this controller is promising. Preliminary simulations suggest the controller's performance to be reasonably robust not only with respect to path-tracking offsets, external perturbations (longitudinal and lateral drag) and parameter variations (mass and kinematic properties of the vehicle), but also with respect to a relaxation of the hypothesis of a slippage-free motion on which the controller design is based. As for technological requirements, these should be adequately met by means of standard components such as optical encoders, acceleromoters, gyros, CCD cameras, laser range finders, sonars, and other similar components.

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Dedicated to Giovanna on the double occasion of our 30-th marriage anniversary and her 50-th birth anniversary.

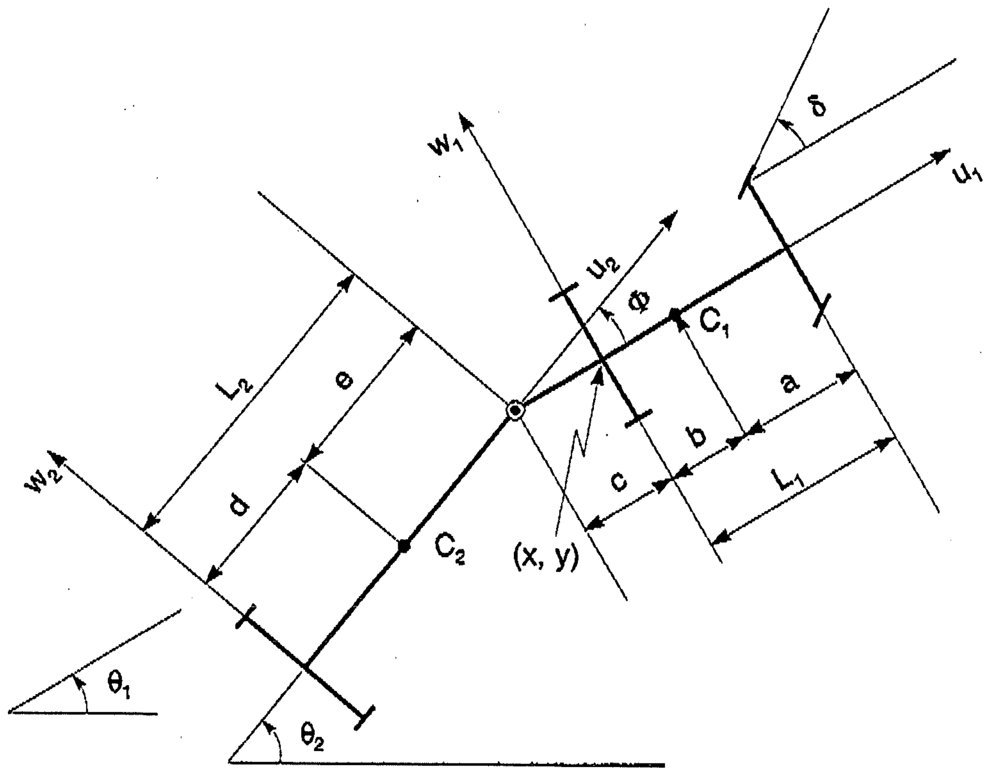

Figure 1: Vehicle's geometry

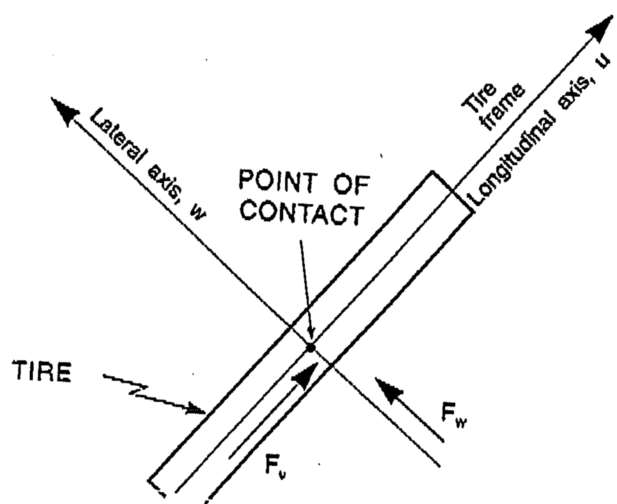

Figure 2: Longitudinal and lateral forces exerted by the tires

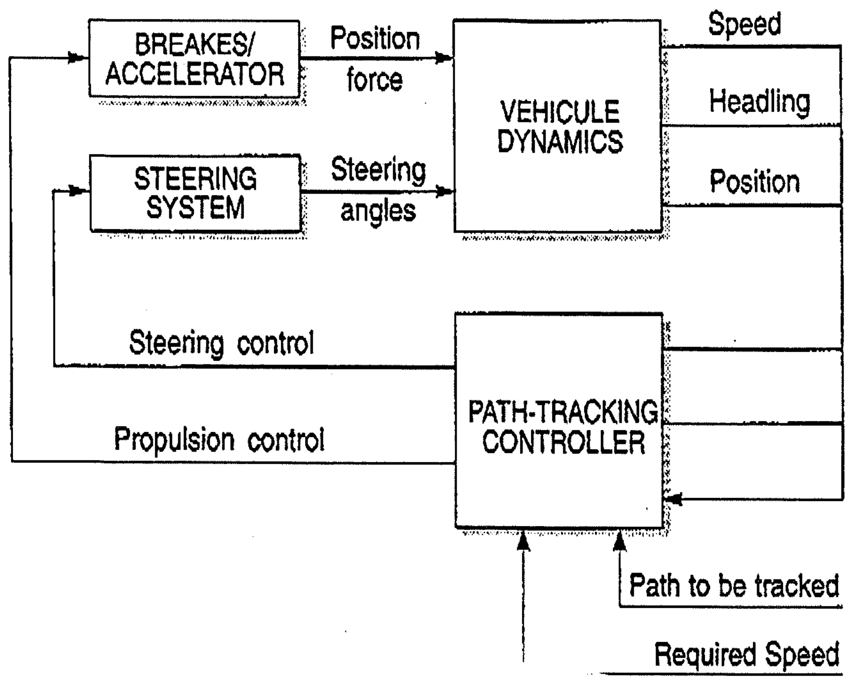

Figure 3: Principle of operation of a path-tracking controller

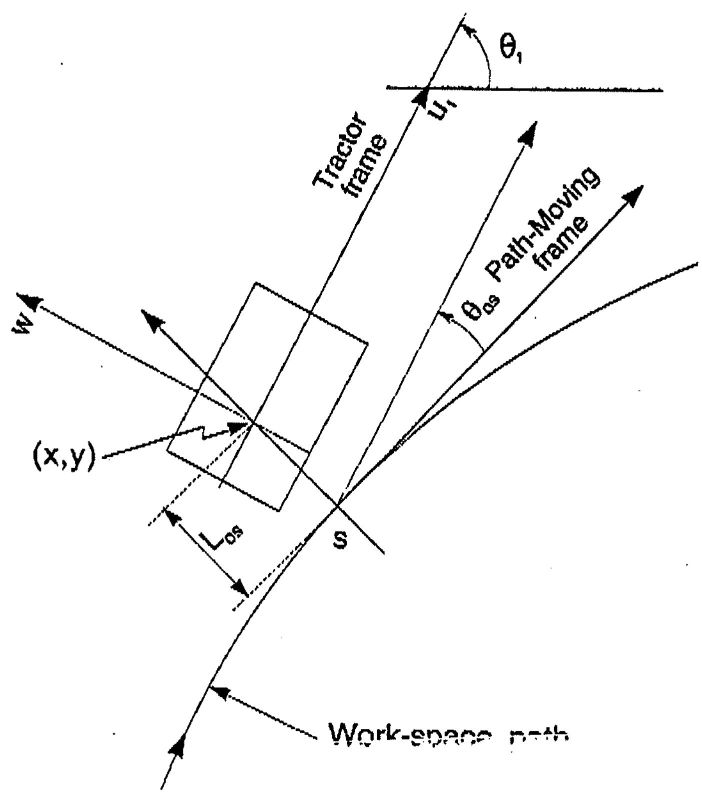

Figure 4: Lateral and tractor's heading offsets

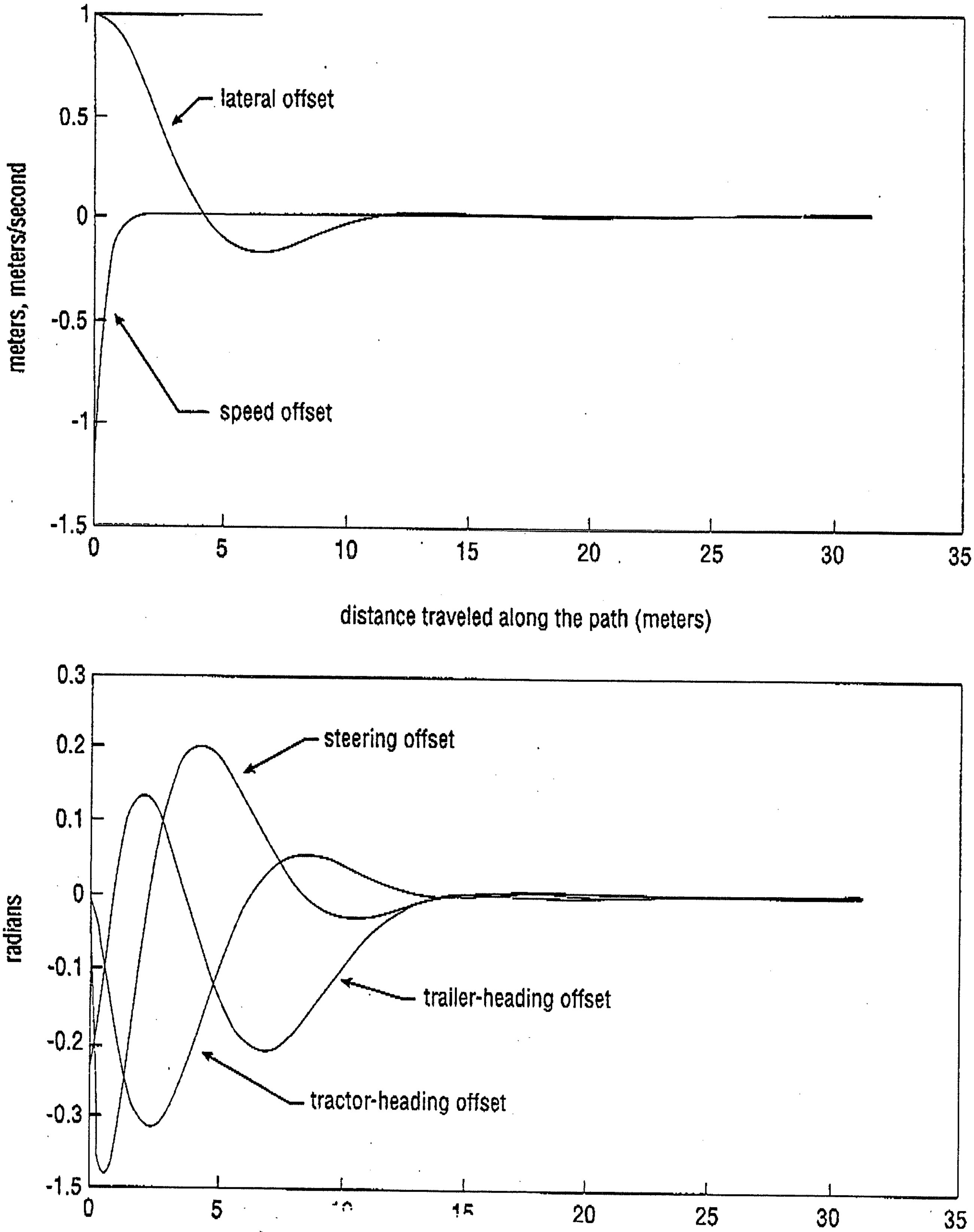

Figure 5: Forward circular maneuver: path-tracking offsets

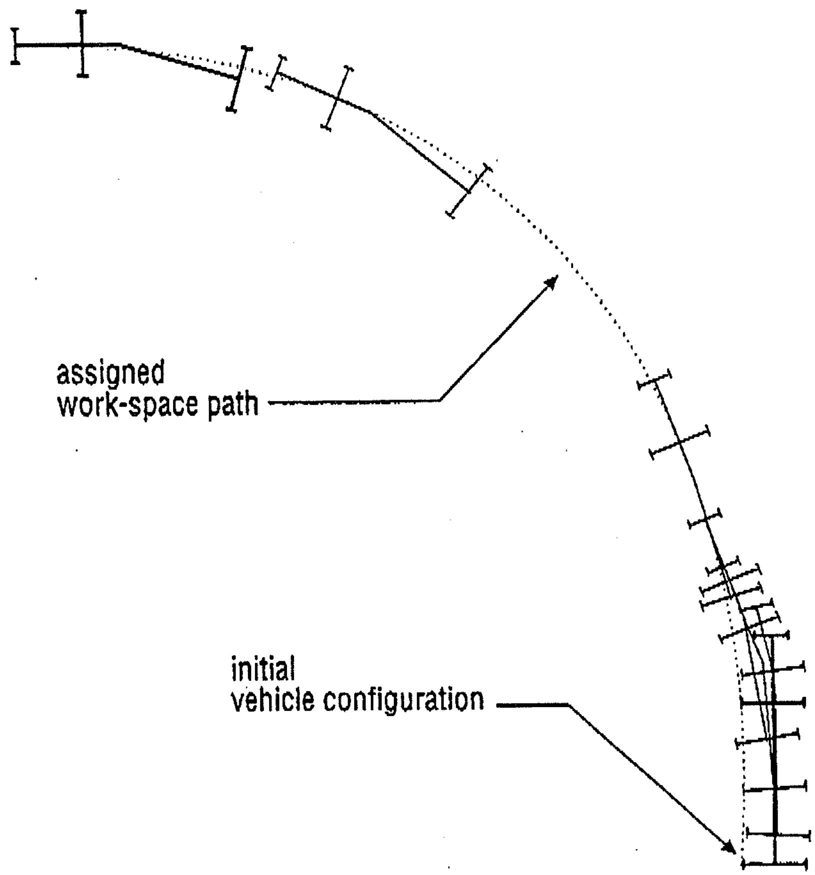

Figure 6: Forward circular maneuver: vehicle's motion in work-space

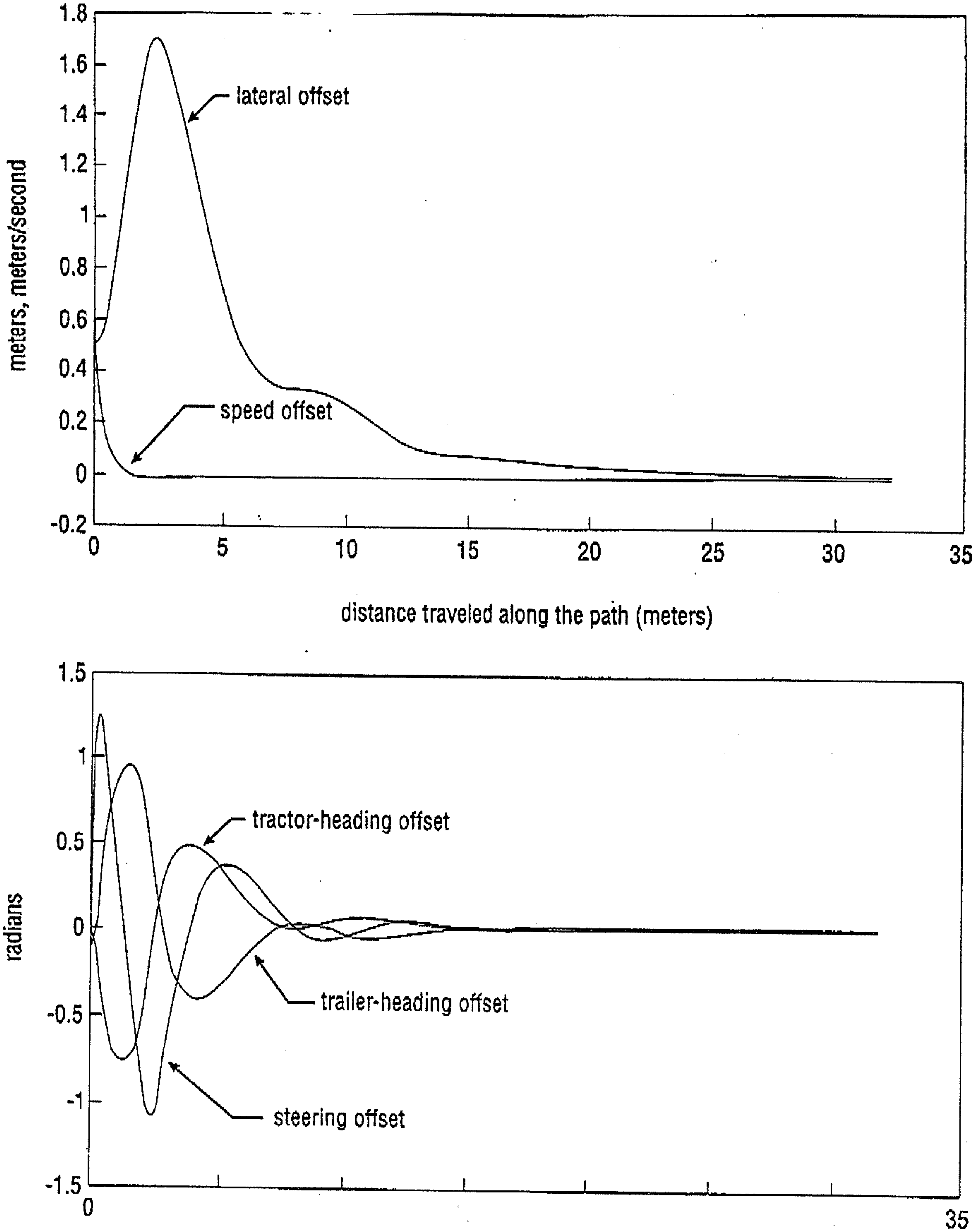

Figure 7: Backward circular maneuver: path-tracking offsets

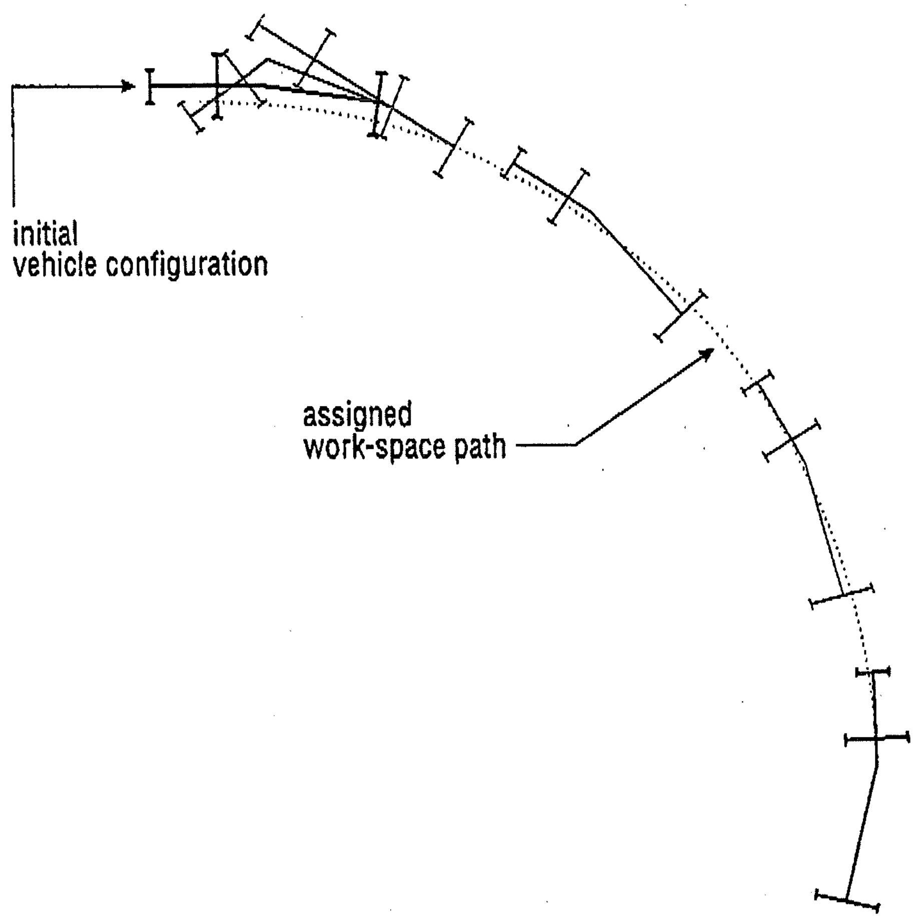

Figure 8: Backward circular maneuver: vehicle's motion in work-space