EPM/RT-94/11

"PATH-TRACKING FOR A MOBILE WHEELED ROBOT WITH A DIFFERENTIAL DRIVE"

par

Romano M. DeSantis

gratuit

Département de génie électrique et génie informatique

Section automatique

École Polytechnique de Montréal

mars 1994

Dépôt légal, mars 1994
Bibliothèque nationale du Québec
Bibliothèque nationale du Canada

Pour se procurer une copie de ce document, s'adresser au:

Éditions de l'École Polytechnique de Montréal
École Polytechnique de Montréal
Case Postale 6079, Succursale centre-ville
Montréal (Québec) H3C 3A7
(514) 340-4000

Compter 0,10$ par page (arrondir au dollar le plus près) et ajouter 3,00$ (Canada) pour la couverture, les frais de poste et la manutention. Régler en dollars canadiens par chèque ou mandat-poste au nom de l'École Polytechnique de Montréal. Nous n'honorerons que les commandes accompagnées d'un paiement, sauf s'il y a eu entente préalable dans le cas d'établissements d'enseignement, de sociétés ou d'organismes canadiens.

PATH-TRACKING FOR A MOBILE WHEELED ROBOT WITH A DIFFERENTIAL DRIVE

DeSantis, R.M.,

Ecole Polytechnique de Montreal, DEGEGI
2900 Edouard Montpetit
Montreal, Canada H3C 3A7
Fax: **514-3404174,** Phone: **514-3404552**

0. Summary

Questions relevant to the design of a path-tracking controller for a mobile wheeled robot with a differential drive are considered. The development assumes a motion that is planar and free from lateral and longitudinal slippages, uses a model that takes into account both dynamic and kinematic properties of the vehicle, and is based on the concept of geometric path-tracking. The results lead to a path-tracking action that is a memoryless function of the lateral, heading, and velocity path-tracking offsets. If these offsets are kept small and the assigned tracking velocity is constant, then the ensuing controller turns out to have a linear, time-invariant and decoupled PID structure.

Keywords: mobile-robots, motion-control, path-tracking, slippage, heading, autonomous, non-holonomic constraints

1. Introduction.

One of the problems encountered in the application of mobile wheeled robots to automate office, hospital, and factory floors operations [An.1, Co.1, Gi.1, He.1, Ka.1, Sh.1] is the design of adequate path-tracking controllers. Currently, these controllers are designed by tacitly assuming that the forward speed of the vehicle is constant and that the path is a straight line. The typical approach is to develop an ad hoc dynamic model of the vehicle and to then consider a heuristic path-tracking controller of which the action is usually proportional to the lateral and orientation path-tracking offsets. A number of test-bench and simulation experiments are then implemented to analyze the properties of the proposed controller [Bo.1, De.1, Fe.1, He.1, Le.1, Ka.1].

While this ad hoc approach has led to a number of concrete applications, a more systematic and more comprehensive approach to the design of path-tracking controllers is desirable. Ideally, this approach should remain applicable when the path is not necessarily a straight line, or when the required velocity is not constant, or when these eventualities both occur at the same time. It should enable one to clearly relate the ensuing control to such physically meaningful tracking measurements as lateral, heading and velocity offsets. It should clarify how the various characterizations of path tracking assignments (such as required velocity and radius of curvature of the path) influence the controller's structure and gains. Finally, whenever possible, one would like this approach to lead to a control law characterized by such practical features as a possibly time-invariant, decentralized and memoryless structure.

The objective of the present paper is to develop such an approach in the special case of a mobile wheeled robot equipped with a differential drive. The locomotion platform of this robot is equipped with a pair of front castors and a pair of rear co-axial drive wheels (figure 1); each of these latter wheels is independently driven by a DC motor which is in turn energized by a control voltage. Robots with this geometry are found in a number of applications and their study has already been considered by several authors [Bo.1, De.1, He.1, Sa.1].

The steps characterizing our approach incorporate many of the ideas that have been recently established in the context of the control of robotic manipulators [Cr.1, Spo.1]: the development of a dynamic model relating the state of the vehicle to the control action; nonlinear feedback linearization and decoupling by means of a partition of the control action; the establishment of a feedback controller on the basis of the linearized model. A difficulty in carrying out these steps, is in selecting a mathematical formulation that is both sufficiently simple for an analytical development to be feasible, and meaningful enough as to retain the physical features of the problem. In most of the robotic literature [Co.1, Da.1, Gi.1, Ka.1, Ke.1, Mu.1], path-tracking is meant to imply the convergence of the state of the vehicle to a desired state which is itself a prescribed function of time. In what follows, we prefer to adopt the interpretation that is traditional in automotive applications [Fe.1] and that has more recently also been adopted (albeit rather informally) in robotic applications [De.1, De.4, De.5, He.1, Sh.2]. According to this interpretation, path-tracking still entails the convergence of the vehicle's state to a desired state. This latter state, however, rather than a prescribed function of time, is now a function of the position and of the orientation of the vehicle with respect to the path to be followed.

2. Vehicle's Basic Dynamic Model

The mobile wheeled robot will be modeled under the standard assumptions that the motion of the vehicle is planar, that the vehicle's properties are symmetrical with respect to its longitudinal axis, that the contact between tires and surface of motion is point-wise, and that no force is exerted from the castors on the vehicle. In describing this model we adopt the following notations: (x, y) is the position of the center of mass of the vehicle (c.o.m.) with respect to a fixed frame; (v_u, v_w) denotes the velocity of the c.o.m. expressed in vehicle's frame coordinates; θ gives the orientation (heading) of the vehicle; Ω is the angular velocity (yaw rate); F_{u1} and F_{u2} denote the longitudinal forces exerted on the vehicle by the right (1) and left tires (2); F_{w1} and F_{w2} are the lateral forces exerted by the right (1) and left tires (2); a is the distance between the c.o.m and the drive wheels' axle; ℓ is the distance between a drive wheel and the longitudinal axis; m is the mass of the vehicle; j: is the yaw moment of inertia with respect to the c.o.m..

Proposition 1. (Vehicle's basic dynamic model). The dynamic model of the mobile wheeled robot with a differential drive is given by

$$\dot{v}_u = v_w\Omega + \frac{F_{u1} + F_{u2}}{m} \tag{1}$$

$$\dot{v}_w = - v_u\Omega + \frac{F_{w1} + F_{w2}}{m} \tag{2}$$

$$\dot{\Omega} = - \frac{(F_{w1} + F_{w2})a}{j} + \frac{(F_{u1} - F_{u2})\ell}{j} \tag{3}$$

$$\dot{x} = \cos\theta v_u - \sin\theta v_w \tag{4}$$

$$\dot{y} = \sin\theta v_u + \cos\theta v_w \tag{5}$$

$$\dot{\theta} = \Omega. \tag{6}$$

Proof: The Newton equation in vehicle's frame coordinates gives

$$ma_u = F_{u1} + F_{u2}$$

$$ma_w = F_{w1} + F_{w2}$$

where a_u and a_w denote the acceleration of the center of mass of the vehicle (c.o.m) expressed in vehicle's frame coordinates. By observing that

$$a_u = \dot{v}_u - v_w\Omega$$

$$a_w = \dot{v}_w + v_u\Omega$$

we obtain (1,2). Eqn (3) is the expression of the Euler equation; kinematic eqns (4,5,6) give the velocity of the vehicle in fixed frame coordinates starting from the expression of this velocity in vehicle's frame coordinates.

In what follows, the vectors $q:=[x, y, \theta]$, $t:= [v_u\ v_w\ \Omega]$, and $a_c := [\dot{v}_u\ \dot{v}_w\ \dot{\Omega}]$ will be, respectively, referred to as the vehicle's configuration, twist and acceleration; the vector $X:= [x\ y\ \theta\ v_u\ v_w\ \Omega]$ is referred to as the state of the vehicle.

2. Absence of Slippage and Servomotors' Dynamics.

The absence of slippage in the motion of the wheels creates an interdependence among the forward, lateral and angular velocities that considerably simplifies the model given by (1-6).

Proposition 2 (Dynamic model in the absence of slippage).
In the absence of a lateral slippage, (1-3) are equivalent to

$$\dot{v}_u = a\Omega^2 + \frac{F_{u1} + F_{u2}}{m} \tag{7}$$

$$\dot{\Omega} = \frac{(F_{u1} - F_{u2})\ell}{(j + ma^2)} - \frac{mv_u\Omega a}{(j + ma^2)} \tag{8}$$

$$v_w = a\Omega. \tag{9}$$

In the absence of a longitudinal slippage, we also have

$$v_u = \frac{(\dot{\theta}_1 + \dot{\theta}_2)R_a}{2} \tag{10}$$

$$\Omega = \frac{(\dot{\theta}_1 - \dot{\theta}_2)R_a}{2\ell} \tag{11}$$

where: θ_i is the angular displacement of the i-th drive wheel, and

R_a is the radius of the drive wheel (assumed to be identical for the two wheels).

Proof: Because of the absence of a lateral slippage, the lateral velocity of the center-point of the drive wheels' axle is null; it follows

$$v_w = a\Omega \tag{12}$$

from which we obtain

$$\dot{v}_w = a\dot{\Omega}. \tag{13}$$

Combining (2) with (13), it follows

$$F_{w1} + F_{w2} = m(a\dot{\Omega} + v_u\Omega) \tag{14}$$

Inserting (12) into (2) and (14) into (3), yields (7-9). To complete the proof, note that the longitudinal velocity of the vehicle's c.o.m. is given by the mean value of the velocities of the points of contact of the tires with the surface on which the vehicle moves. Since in the absence of a longitudinal slippage these velocities are given by $\theta_i R_a$, one has (10). A similar reasoning gives (11).

Remark 1. A general method to model mechanical systems subject to holonomic and nonholonomic constraints (in particular, mobile wheeled robots amy be found in [De.6] and [Sa.1].

Proposition 3 (Influence of the DC motors' dynamics). In the absence of lateral and longitudinal slippage, and with the propulsion forces (F_{u1}, F_{u2}) produced by DC motors applied on the drive wheels' axes, the vehicle's dynamic model is described by

$$\dot{v}_u = a\Omega^2 - \frac{K_\Phi K_b v_u}{R_e R_a{}^2 m} + \frac{2K_\Phi U_1}{R_e R_a m} \tag{15}$$

$$\dot{\Omega} = - \frac{m v_u \Omega a}{(j + ma^2)} - \frac{2K_\Phi K_b \ell^2 \Omega}{R_e R_a{}^2 (j + ma^2)} + \frac{2K_\Phi \ell U_2}{R_e R_a (j + ma^2)} \tag{16}$$

$$v_w = a\Omega, \tag{17}$$

where

$$U_1 := (V_1 + V_2)/2 \tag{18}$$

$$U_2 := (V_1 - V_2)/2 \tag{19}$$

and where V_i is the control voltage applied to the i-th motor.

Proof: Using the classical equation of a DC motor [An.2], the torques applied by the motors to the drive wheels' axes are given by

$$\Gamma_i = K_{\Phi i} I_i \tag{20}$$

where

$$V_i = R_{ei} I_i + K_{bi} \theta_i \tag{21}$$

with I_i the current flowing in the inductor of the i-th motor, and K_Φ, K_b and R_{e1} the DC motor characteristic parameters (assumed to be identical for the two motors). In the absence of a longitudinal slippage,

$$F_{ui} = \frac{\Gamma_i}{R_a} \tag{22}$$

and therefore, from (20,21),

$$F_{ui} = \{V_i - K_{bi} \dot{\theta}_i\} \frac{K_{\Phi i}}{R_{ei} R_a} . \tag{23}$$

The desired result follows by inserting (23) in (7,8), by using (18,19) and by conveniently re-arranging the terms of the ensuing equations.

Remark 2: In the special case where a=0, the model described by proposition 3 coincides with that given in [De.1]. If in addition to a=0, we also impose $m=4j/\ell$ then $\tau\eta\iota\sigma$ model also coincides with that developed in [Bo.1]. The relations among these models is further illustrated by the following proposition.

Proposition 4 (Vehicle's model when its c.o.m. is located on the wheels' axle). If a=0 the vehicle's model becomes

$$\frac{v_u(s)}{U_1(s)} = \frac{K_{SPEED}}{(1 + s\tau_{SPEED})} \qquad (24)$$

$$\frac{\Omega(s)}{U_2(s)} = \frac{K_{ORIENT}}{(1 + s\tau_{ORIENT})} \qquad (25)$$

where

$$K_{SPEED} := \frac{R_a}{K_b} \qquad \tau_{SPEED} := \frac{R_e R_a^2 m}{K_\Phi K_b} \qquad (26)$$

$$K_{ORIENT} := \frac{R_a}{K_b \ell} \qquad \tau_{ORIENT} := \frac{R_e R_a^2 j}{2K_\Phi K_b \ell^2} \qquad (27)$$

Proof: Eqns (24,25) follow by simply setting a=0 in (15-17), by applying Laplace transforms and by introducing notations (26,27) (see figure 4).

4. The Path-Tracking Problem

A path-tracking assignment is the combination of a path and of a profile of linear and angular velocities and accelerations with which this path has to be followed. A path [La.1, ch.9] is described by a set of continuous functions,

$$q_p(s) := [x_p(s) \;\; y_p(s) \;\; \theta_p(s)],$$

where $s \in [0, \infty)$ is a parameter defining a point of the path, and $q_p(s)$ is the value of the configuration that the vehicle is required to have at the point of the path defined by s. Similarly, a velocity and acceleration profile along a path is described by a set of continuous functions

$$t_p(s) := [v_{up}(s) \;\; v_{wp}(s) \;\; \Omega_p(s)],$$

$$a_p(s) := [a_{up}(s) \;\; a_{wp}(s) \;\; a_{\theta p}(s)], \qquad s \in [0, \infty),$$

where $t_p(s)$ and $a_p(s)$ represent the desired twist and acceleration of the vehicle at point s.

A path tracking assignment is admissable if there exists a smooth function $s(t)$, $t \in [0, \infty)$, such that, using the notations

$$x_p(t) := x_p(s(t)); \qquad y_p(t) := y_p(s(t)); \qquad \theta_p(t) := \theta_p(s(t));$$

$$v_{up}(t) := v_{up}(s(t)); \qquad v_{wp}(t) := v_{wp}(s(t)); \qquad \Omega_p(t) := \Omega_p(s(t));$$

$$a_{up}(t) := a_{up}(s(t)); \qquad a_{wp}(t) := a_{wp}(s(t)); \qquad a_{\theta p}(t) := a_{\theta p}(s(t)),$$

one has

$$v_{up}(t) = \dot{x}_p(t)\cos\theta_p(t) + \dot{y}_p(t)\sin\theta_p(t)$$

$$v_{wp}(t) = -\dot{x}_p(t)\sin\theta_p(t) + \dot{y}_p(t)\cos\theta_p(t)$$

$$\Omega_p(t) := \dot{\theta}_p(t)$$

and

$$a_{up}(t) = \dot{v}_{up}(t); \;\; a_{wp}(t) = \dot{v}_{wp}(t); \;\; a_{\theta p}(t) = \dot{\Omega}_p(t).$$

We require the function $X_p(s) := [q_p(s)\ t_p(s)]$ to be continuous with respect to its projection into the work-space. More specifically, $[q_p(s)\ t_p(s)]$ must be such that, given any $\epsilon>0$, there exists a $\mu(\epsilon)>0$ with the property that, for any $s_1\ s_2 \in [0, \infty)$ one has

$$|[q_p(s_1)\ t_p(s_1)] - [q_p(s_2)\ t_p(s_2)]| < \epsilon,$$

provided that

$$|(x_p(s_1), y_p(s_1)) - (x_p(s_2), y_p(s_2))| < \mu(\epsilon),$$

where

$[q_p(s_i)\ t_p(s_i)] := [x_p(s_i)\ y_p(s_i)\ \theta_p(s_i)\ v_{up}(s_i)\ v_{wp}(s_i)\ \Omega_p(s_i)]$, $i = 1,2$.

Given an admissable path-tracking assignment, path-tracking is the problem of generating the control action required for the vehicle to follow the assigned path with the specified velocity. More formally, the control action must be selected so that

$$\lim_{t \to \infty} X(t) = X_p(s), \tag{28}$$

for some $s \in [0, \infty)$.

For technical purposes (soon to become evident), it is convenient to re-state this problem by using the concept of a desired state. Given a state of the vehicle,

$$X := [x\ y\ \theta\ v_u\ v_w\ \Omega], \tag{29}$$

the vehicle's desired state in correspondence to a path-tracking assignment is defined by

$$X_d := [q_d\ t_d] \tag{30}$$

$$q_d := [x_d\ y_d\ \theta_d] \tag{31}$$

$$t_d := [v_{ud}\ v_{wd}\ \Omega_d], \tag{32}$$

where

$$[x_d\ y_d\ \theta_d] := [x_p(s')\ y_p(s')\ \theta_p(s')], \tag{33}$$

and

$$[v_{ud}\ v_{wd}\ \Omega_d] := [v_{up}(s')\ v_{wp}(s')\ \Omega_p(s')]. \tag{34}$$

The value of $s' \epsilon [0, \infty)$, in (33, 34), is selected so that $(x_p(s'), y_p(s'))$ is the point of the path (in work space) closest to (x, y). More specifically, s' is such that

$$(x-x_p(s'))^2+(y-y_p(s'))^2 < (x-x_p(s))^2+(y-y_p(s))^2 \tag{35}$$

for each $s \ne s'$, $s \epsilon [0, \infty)$.

Proposition 5. Path-tracking is equivalent to generating the control action required to have

$$\lim_{t \to \infty} X(t) = X_d(t) \tag{36}$$

where $X(t)$ is the state of the vehicle at time t, and $X_d(t)$ the desired state associated with $X(t)$.

Proof. If (36) holds, then

$$\lim_{t \to \infty} X(t) = X_p(s), \tag{37}$$

where s is such that $X_p(s) := X_d(t)$. Conversely, if (37) holds, then

$$\lim_{t \to \infty} |(x(t),y(t)) - (x_d(t),y_d(t))| < \lim_{t \to \infty} |(x(t),y(t)) - (x_p(s),y_p(s))| = 0. \tag{38}$$

hence

$$\lim_{t \to \infty} |(x_d(t),y_d(t)) - (x_p(s),y_p(s))| = 0. \tag{39}$$

By invoking the continuity of $X_p(.)$ with respect to its workspace projection, it follows

$$\lim_{t \to \infty} X_d(t) = X_p(s). \tag{40}$$

which combined with (37) gives (36).

Remark 3. The above statement of the path-tracking problem can be easily modified so as to be applicable to mobile wheeled robots the geometry of which is not necessarily of the power-wheel-chair type. The modifications required in the case of car-like and tractor-trailer-like robots are illustrated in [De.4, De.5].

5. The Path Tracking Controller

The accuracy with which the vehicle motion complies with the path-tracking assignment may be described in terms of velocity (v_{os}), heading (θ_{os}), and lateral (ℓ_{os}) offsets. These offsets are defined as follows

$$v_{os}(t) := v_u(t) - v_{ud}(t) \tag{41}$$

$$\theta_{os}(t) := \theta(t) - \theta_d(t) \tag{42}$$

$$\ell_{os}(t) := -\{x(t)-x_d(t)\}\sin\theta_d(t) + \{y(t))-y_d(t)\}\cos\theta_d(t) \tag{43}$$

where ℓ_{os} represents the (signed) distance between the position of the center of mass of the vehicle and the projection of the assigned path in work-space.

Proposition 6. Path-tracking is equivalent to generating the control required to have

$$\lim_{t \to \infty.} [v_{os}(t)\ \theta_{os}(t)\ \ell_{os}(t)] = 0 \tag{44}$$

Proof. Clearly, if

$$\lim_{t\to\infty} X(t) = X_d(t) \tag{45}$$

then (44) holds. Conversely, if (45) holds then

$$\lim_{t\to\infty} [x(t)\ y(t)\ \theta(t)] = \lim_{t\to\infty} [x_d(t)\ y_d(t)\ \theta_d(t)]. \tag{46}$$

By the smoothness of these functions, it follows

$$\lim_{t\to\infty} [\dot{x}(t)\ \dot{y}(t)\ \Omega(t)] = \lim_{t\to\infty} [\dot{x}_d(t)\ \dot{y}_d(t)\ \Omega_d(t)], \tag{47}$$

and therefore (44).

Proposition 7 (Path-tracking offsets dynamics). The dynamics of the path-tracking offsets is described by

$$\dot{v}_{os} = -\dot{v}_{ud} + a\Omega^2 - \frac{K_\phi K_b v_u}{R_e R_a^2 m} + \frac{K_\phi U_1}{R_e R_a m} \tag{48}$$

$$\dot{\theta}_{os} := -\dot{\Omega}_d - \frac{m v_u \Omega a}{(j + ma^2)} - \frac{2K_\phi K_b \ell^2 \Omega}{R_e R_a^2 (j + ma^2)} + \frac{2K_\phi \ell U_2}{R_e R_a (j + ma^2)} \tag{49}$$

$$\dot{\ell}_{os} = (v_{ud} + v_{os})\sin\theta_{os} + (\Omega\cos\theta_{os} - \Omega_d)a. \tag{50}$$

Proof: Eqns (48,49) follow from (17,18) by simply subtracting, respectively, v_{ud} and Ω_d from the left and right members of these equations and by using the definition of v_{os} and θ_{os}. To prove (50), compute the time derivative of (43) so as to obtain

$$\begin{aligned}\dot{\ell}_{os}(t) := &-\{\dot{x}(t)-\dot{x}_d(t)\}\sin\theta_d(t) + \{\dot{y}(t))-\dot{y}_d(t)\}\cos\theta_d(t) \\ &-\{x(t)-x_d(t)\}\Omega_d\cos\theta_d(t) - \{y(t))-y_d(t)\}\Omega_d\sin\theta_d(t)\end{aligned} \tag{51}$$

Noting that, by the definition of (x_d, y_d), the vector $(x-x_d, y-y_d)$ is perpendicular to the tangent to the path, $(\cos\theta_d, \sin\theta_d)$, it follows

$$\dot{\ell}_{os}(t):=-\dot{x}(t)\cos\theta_d(t)+\dot{y}(t)\sin\theta_d(t) \; +\dot{x}_d(t)\sin\theta_d(t)-\dot{y}_d(t)\cos\theta_d(t) \quad (52)$$

and therefore

$$\dot{\ell}_{os}(t):=-\dot{x}(t)\sin\theta(t)+\dot{y}(t)\cos\theta(t) \; + \dot{x}_d(t)\sin\theta_d\,(t)-\dot{y}_d(t)\cos\theta_d(t)$$

$$-\dot{x}(t)\{\sin\theta_d - \sin\theta\} + \dot{y}(t)\{\cos\theta_d - \cos\theta\}. \quad (53)$$

Using (4,5), simple algebraic manipulations lead to

$$\dot{\ell}_{os} = v_w\cos\theta_{os} - v_{wd} + v_u\sin\theta_{os}. \quad (54)$$

which combined with (17) gives (50).

Proposition 8 (Decoupling and Nonlinear Feedback Linearization). By partitioning the control according to the relation

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \begin{pmatrix} u_p \\ u_s \end{pmatrix} + \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} \quad (55)$$

where $(u_p,\ u_s)$ is viewed as the new control, and a_1, a_2, β_1 and β_2 are given by

$$a_1:= \frac{R_eR_am}{K\Phi} \qquad a_2:= \frac{R_eR_a(j + ma^2)}{2K\Phi\ell} \quad (56)$$

$$\beta_1:= a_1\{\dot{v}_{ud} - a\Omega^2 + \frac{K_\Phi K_b v_u}{R_eR_a{}^2m}\}$$

$$\beta_2:= a_2\{\dot{\Omega}_d + \frac{mv_u\Omega a}{(j + ma^2)} + \frac{2K_\Phi K_b\ell^2\Omega}{R_eR_a{}^2(j + ma^2)}\}, \quad (57)$$

the dynamics of the path-tracking offsets is given by

$$\dot{v}_{os} = u_p \tag{58}$$

$$\dot{\theta}_{os} = u_s \tag{59}$$

$$\dot{\ell}_{os} = (v_{ud} + v_{os})\sin\theta_{os} + (\Omega\cos\theta_{os} - \Omega_d)a. \tag{60}$$

Proposition 8 suggests that path tracking may be now studied by bringing to bear well-known χοντρολ theory results [Ch.1, Ka.2].

Proposition 9. A path tracking controller exists and its action may be computed in terms of the path tracking assignment and of the heading, lateral and velocity offsets.

Proof: By virtue of proposition 8, to prove existence it is sufficient to show the stabilizability of (58-60). To do this, we first prove local stabilizability. Applying Lyapunov's first method (lemma 1 below), this may be done by proving the stabilizability of the linear approximant

$$\dot{x} = Ax + Bu, \tag{61}$$

where

$x := (x_1 \; x_2 \; x_3 \; x_4)$ $\qquad$ $u := (u_1 \; u_2)$

with

$x_1 := v_{os}$ $\qquad$ $x_2 := \theta_{os}$ $\qquad$ $x_3 := \dot{\theta}_{os}$ $\qquad$ $x_4 := \ell_{os}$

$u_1 := u_p$ $\qquad$ $u_2 := u_s$.

and

$$A := \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & v_u & a & 0 \end{matrix} \qquad B := \begin{matrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{matrix} \tag{62}$$

The stabilizability of this system may, in turn, be obtained by proving its controllability. Controllability is a consequence of the fact that dim{B AB}=4, and of the forthcoming lemma 2. To complete the proof, it suffices to observe that, in the specific case under consideration, local stabilizability implies stabilizability in the large. Indeed, by appropriately re-defining a first segment of the path tracking assignment, the assumption of small initial values of $v_{os}(t)$, $\Theta_{os}(t)$ and $\ell_{os}(t)$ may always be satisfied.

Lemma 1 [Ka.2, p. 184]. Let x=0 be an equilibrium point for the nonlinear system

$$\dot{x} = f(t, x) \tag{63}$$

where f: $[0, \infty) \times D \rightarrow R^n$ is continuously differentiable, D:= $\{x \in R^n,\ x^2 < r\}$, and the Jacobian matrix $[\delta f/\delta x]$ is bounded and uniformly Lipschitz continuous on D. Then, the origin is an exponentially stable equilibrium point for the nonlinear system, if and only if it is an exponentially stable equilibrium point for the linear approximant

$$\dot{x} = A(t)x. \tag{64}$$

where

$$A(t) := \left.\frac{\delta f(t, x)}{\delta x}\right|_{x=0}. \tag{65}$$

Lemma 2 [Ch.1, p.179]. The system

$$\dot{x} = A(t)x + B(t)u, \tag{66}$$

with matrices A(t), B(t) continuously differentiable n-1 times, is controllable if

$$\dim\{[M_0(t)\,|\,M_1(t)\,|\ldots|\,M_{n-1}(t)]\} = n \tag{67}$$

where

$$M_{k+1}(t) = -A(t)M_k(t) + \frac{d}{dt}M_k(t), \qquad k = 0, 1, \ldots, n-1$$

with

$$M_0(t) = B(t). \tag{68}$$

Among the options opened up by Proposition 9, a particularly attractive design is represented by a decentralized controller consisting of a steering component generating u_s as a function of $\theta_{os}(t)$, and $\ell_{os}(t)$, and of a propulsion component providing u_p as a function of $v_{os}(t)$.

Proposition 10. (Path-tracking by means of a decentralized controller). If the path-tracking offsets are kept sufficiently small, then path-tracking may be obtained by means of a speed controller

$$u_p = -K_{p1}v_{os} + K_{p2}\int v_{os}\,dt \tag{69}$$

and of a steering controller

$$u_s(t) = -K_{s1}\dot{\theta}_{os}(t) + K_{s2}\theta_{os}(t) + K_{s3}\ell_{os}(t). \tag{70}$$

Proposition 11. (Path-tracking by means of decentralized PID controllers). If the path-tracking offsets are kept sufficiently small, and the desired tracking velocity is constant, then the gains in (69, 70) are constant.

Remark 4. The control voltages to be applied to the DC motors are obtained from (69,70) by taking into account (18,19) and (55).

Remark 5. While it is clear that (69) represents a speed controller with a PI (proportional + integral) feedback, it is noted that, when the desired tracking velocity is constant, the steering controller (70) may also be interpreted as a PID (proportional + integral + derivative) feedback. To justify this interpretation, it suffices to note that from (50) one obtains (for small offsets)

$$L_{os} = v_{ud} \int \theta_{os} + \theta_{os}. \tag{71}$$

The interest of this interpretation is in that it enables to design a path-tracking controller by applying both classical and more recent PID technology [Ku.1, De.2, De.3].

Remark 6. As clarified by the following proposition, the gains of the PID controller described by (69,70) may be selected so as to obtain a dynamics for the speed offset and for the lateral and heading offsets that is completely decoupled. Moreover, the poles associated with this dynamics are arbitrarily selectable.

Proposition 12. (Design of a path-tracking PID controller). The path-tracking PID controller described by (69,70) has the following properties:

a) the dynamics of v_{os} is described by

$$\ddot{v}_{os} - (p_{11} + p_{12})\dot{v}_{os} + p_{11}p_{12}v_{os} = 0, \tag{72}$$

where

$$K_{p1} = -(p_{11} + p_{12}), \qquad K_{p2} = p_{11}p_{12}; \tag{73}$$

b) if the desired path tracking velocity is constant, then the dynamics of θ_{os} is described by

$$\dddot{\theta}_{os} - (p_{21} + p_{22} + p_{23})\ddot{\theta}_{os} + (p_{21}p_{22} + p_{21}p_{23} + p_{22}p_{23})\dot{\theta}_{os} - p_{21}p_{22}p_{23}\theta_{os} = 0 \tag{74}$$

where

$$K_{s1} = p_{21}p_{22} + p_{21}p_{23} + p_{22}p_{23} - K_{s3}a$$
$$K_{s2} = p_{21} + p_{22} + p_{23}$$
$$K_{s3} = -p_{21}p_{22}p_{23}/v_{ud}. \tag{75}$$

6. An Application Example

Consider a mobile wheeled robot equipped with the locomotion unit of the power wheel-chair Model 6755, by Fortress Eng. Co., Montreal. The parameters characterizing this unit have the values indicated in table 1. Consider the task of tracking with a constant velocity v_{ud}=5m/s a circle of radius equal to 10 m. The design of PID controllers capable of implementing this task may be carried out by applying propositions 8-12.

Following the procedure suggested by these propositions, one first selects the poles associated with the desired dynamics of the speed offset (p_{11}, p_{12}) and of the lateral and orientation offset

(p_{21}, p_{22}, p_{23}). Let

$p_{11} = -3 \quad p_{12} = -5$

$p_{21} = -1.5 \qquad p_{22} = -1.5 \quad p_{23} = -3$

The gains K_{p1} and K_{p2}, and K_{s1}, K_{s2}, and K_{s3} may then be computed by using (73) and (75). This gives

$K_{p1} = 8 \qquad K_{p2} = 15$

$K_{s1} = 11 \qquad K_{s2} = 6 \qquad K_{s3} = 6.75/25 = .025.$

At this point, one computes (u_p, u_s) using (69,70), and (U_1,U_2) using (55-57). Finally, V_1 and V_2, the voltages to apply to the DC motors, are computed from U_1 and U_2 by applying (18,19).

Table I: Vehicle's Parameters
==============================

a = .5 m (distance between the c.o.m and the drive wheels' axle);

ℓ = .5 m (distance between a drive wheel and the longitudinal axis);

m = 200 Kg (mass of the vehicle);

j = 12.5 Kg m^2 (yaw moment of inertia with respect to the c.o.m);

R_a = .12 m (drive wheel's radius);

R_e = 8 Ω (resistance of the DC motor inductor);

K_ϕ, K_b: .35 volt/rad/s (characteristic parameters of the DC motor);

Conclusion

The assumption of a slippage-free motion and the concept of geometric path-tracking adopted in the present paper, allow an analytical discussion of the questions of existence, structure and design of path-tracking controllers for mobile wheeled robots with a differential drive. In particular, they lead to useful descriptions of the vehicle's kinematic and dynamic properties (Propositions 1-4), and they allow to establish the equivalence of various formulations of the path-tracking problem (Propositions 5,6). This equivalence leads to the equivalence between path-tracking and the stabilization of an appropriate dynamic system (Proposition 7,8). This in turn, makes it possible to design a path-tracking controller by bringing to bear decoupling and nonlinear feedback linearization techniques (Propositions 9, 10). From the application of these techniques, it follows that such controller may consist of two simple, linear and decoupled controllers (Proposition 11), the gains of which may be selected by means of well-familiar PID techniques (Proposition 12). All these results can be easily extended to other types of mobile wheeled robots as car-like or tractor-trailer-like robots (see, for example, [De.4] and [De.5]).

References

[Al.1] Alexander, J.C., Maddocks, J.H., On the Maneuvering of Vehicles, **Siam J. Applied Math**, Vol 48, n.1, Feb. 1988.

[An.1] Anderson, S.E., (Editor), **Proceedings of the 3rd International Conference on Automated Vehicle Systems,** Stockholm, Sweden, October 1985.

[An.2] Anonimous, **Dc Motors Speed Controls and Servo Systems,** An Engineering Handbook by Electro-Craft Corporation 1977

[Bo.1] Borenstein, J., Koren, Y., Motion Control Analysis of a Mobile Robot, **Journal of Dynamic Systems Measurement and Control,** Trans ASME, Vol. 109, June 1987.

[Ch.1] Chen, C.-T., **Linear System Theory and Design**, HRW 1987.

[Co.1] Cox, I., J., Wilfong, G. T., (Editors), **Autonomous Robot Vehicles**, Springer-Verlag 1990.

[Da.1] D'Andrea-Novel, B., Bastin, G., Campion, G., Dynamic Feedback Linearization of Nonholonomic Wheeled Mobile Robots, **Proceedings of the 1992 International Conference on Robotics and Automation,** Nice, France, May 1992.

[De.1] DeSantis, R.M., Hurteau, R., On the Motion Control of Electric AGVs, **Automazione e Strumentazione,** Special issue on AGV, Vol.3, March 1990, pp. 137-150.

[De.2] DeSantis, R.M., An Adaptive PI/Sliding-Mode Controller for a Speed Drive, **Journal of Dynamic Systems Measurement and Control,** Trans ASME, Vol. 111, September 1989, pp. 409-415.

[De.3] DeSantis, R.M., On PI and PID Sliding Mode Controllers, **Advances in Computing and Control**, Springer-Verlag, 1989.

[De.4] DeSantis, R.M., 1993a, Path-Tracking for Car-Like and Tractor-Trailer-Like Robots, **EPM/RT-93/25**, Ecole Polytechnique de Montréal, 1993 (to appear in the **Int. Journal of Robotic Research,** Vol 13, N.3, Dec 1994).

[De.5] DeSantis, R.M., 1994, Path-Tracking for a Car-Like Robot with Double Steering, (to appear in the **IEEE Trans on Vehicular Technology**).

[De.6] DeSantis, R.M., 1994, Dynamic Modeling of Mechanical Systems subject to Holonomic and Nonholonomic constraints, **EPM/RT-94/04,** Ecole Polytechnique de Montréal, 1994.

[Fe.1] Fenton, R.E., On the Steering of Automated Vehicles: Theory and experiments, **IEEE Trans on Automatic Control**, Vol AC-21, No.3, June 1976

[Gi.1] Giralt, G., **Les Robots Mobiles Autonomes**, LAAS, Toulouse, Publication No. 87308, 1988.

[He.1] Hemami, A., Mehrabi, M.G., Cheng, R.M.H., A Synthesis of an Optimal Control Law for Path Tracking in Mobile Robots, **Automatica,** Vol.8, N.2, pp. 383 - 387, 1992.

[La.1] Latombe, J.C., **Robot Motion Planning,** Kluwer 1991.

[Le.1] Lee, A.Y., A Preview Steering Autopilot Control Algorithm for Four-Wheel Steering Passenger Vehicles, **Journal of Dynamic Systems Measurement and Control**, Trans ASME, Vol. 114, Sept 1992, pp.401-409.

[Ka.1] Kanayama, Y., Kimura, Y., Miyazaki, F., Noguchi, T., A Stable Tracking Control Method for an Autonomous Mobile Robot, **Proc. International Conference on Robotics and Automation**, Cincinnati, Ohio, 1990, p.384-389.

[Ka.2] Khalil, H. K., **Nonlinear Systems,** MacMillan 1992.

[Ke.1] Kehtarnavaz, N., Griswold, N.C., Lee, J.S., Visual Control of an Autonomous Vehicle (BART): The Vehicle-Following Problem, **IEEE Trans on Vehicular Technology,** Vol. 40, No.3, Aug 1991.

[Ku.1] Kuo, B. C., **Automatic Control Systems,** Prentice Hall, 1987.

[Sa.1] Saha, S.K., Angeles, Kinematics and Dynamics of a Three-wheeled 2-DOF AGV, **1989 IEEE, International Conference on Robotics and Automation** , Scottsdale, Arizona 1989.

[Sp.1] Spong, M.W., Vidyasagar, M., **Robot Dynamics and Control**, John Wiley & Sons 1989.

[Sh.1] Shladover, S.E., @ Co. (13 co-authors), Automatic Vehicle Control Developments in the Path Program, **IEEE Trans on Vehicular Technology,** Vol. 40, No.1, Feb 1991.

[Sh.2] Shin,D.H., Singh, S., Lee, J.J., Explicit Path-Tracking by Autonomous Vehicles, **Robotica,** Vol. 16, pp. 537-554, 1992.

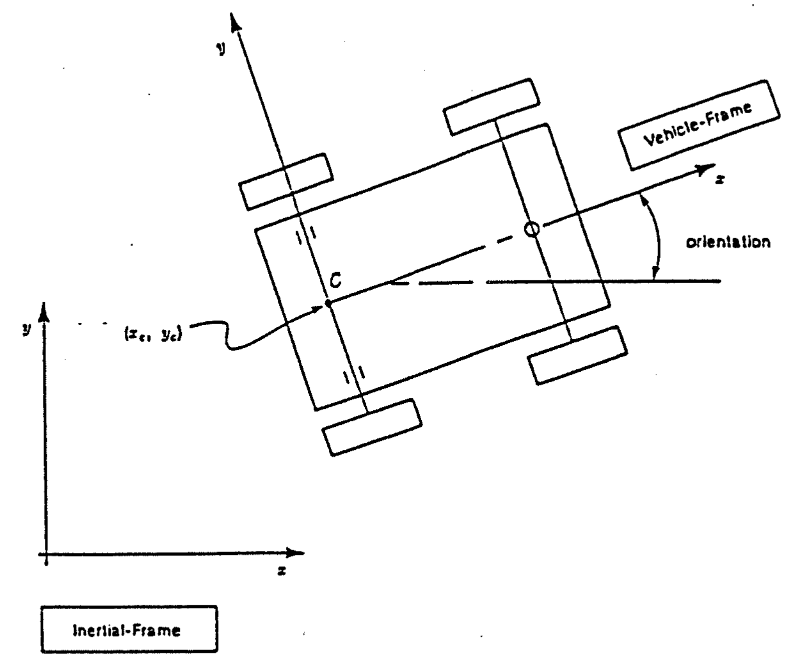

Figure 1: Vehicle Configuration

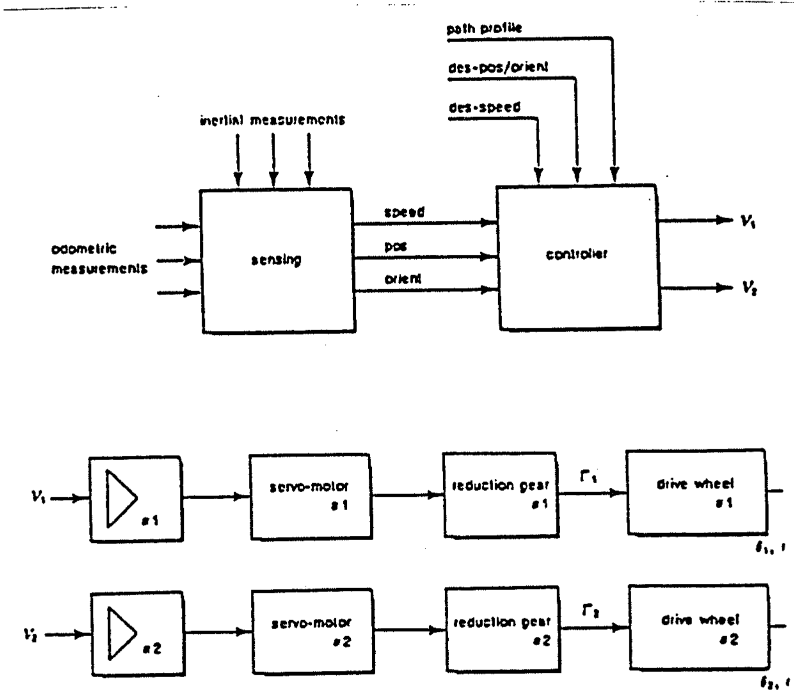

Figure 2: Automatic Path-Tracking

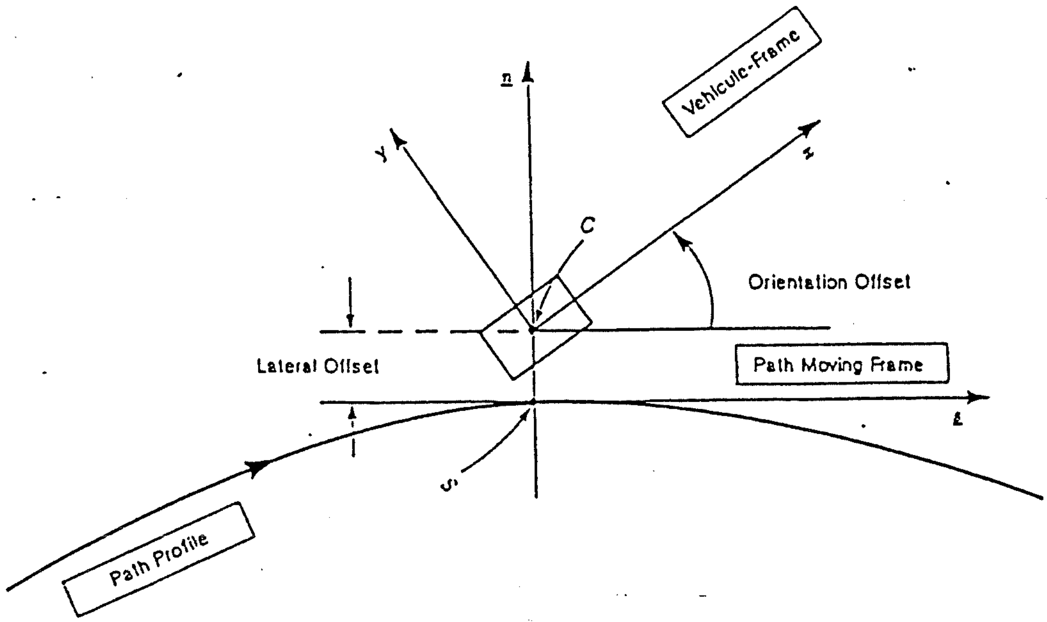

Figure 3: Lateral and Orientation Offsets

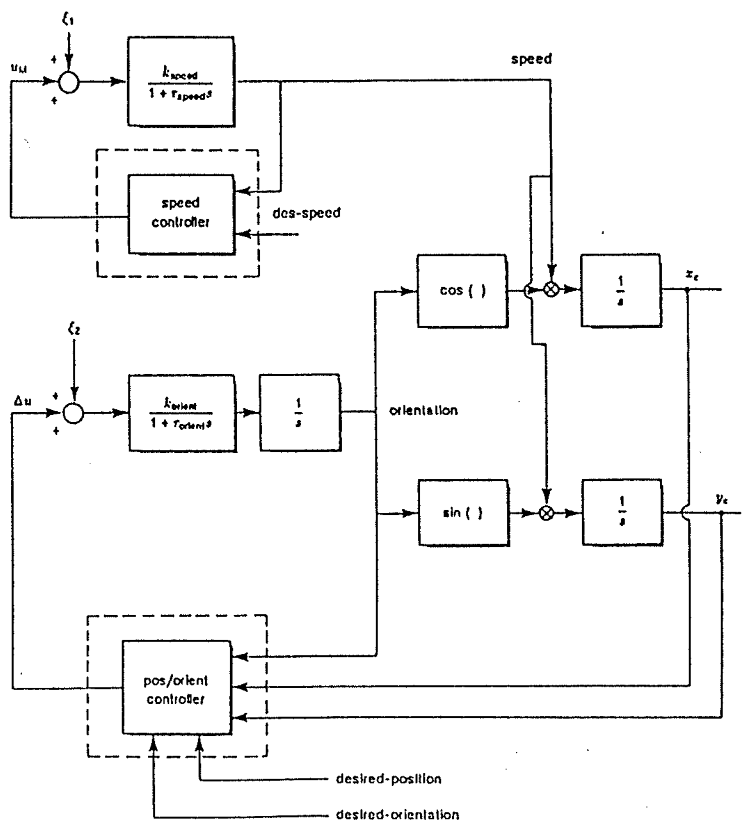

Figure 4: Structure of the vehicle's model when a=0.

List of Symbols

(x, y): position of the center of mass of the vehicle (c.o.m.) with respect to a fixed frame;

(v_u, v_w): velocity of the c.o.m. expressed in vehicle's frame coordinates;

θ: orientation (heading) of the vehicle;

Ω: angular velocity (yaw rate);

F_{u1}, F_{u2}: longitudinal forces exerted on the vehicle by the right (1) and left tires (2);

F_{w1}, F_{w2}: lateral forces exerted by the right (1) and left tires (2);

a : distance between the c.o.m and the drive wheels' axle;

ℓ : distance between a drive wheel and the longitudinal axis;

m: mass of the vehicle;

j: yaw moment of inertia with respect to the c.o.m.

(a_u, a_w): acceleration of the c.o.m expressed in vehicle's frame coordinates;

(u_p, u_s): propulsion, steering control;

K_ϕ, K_b, R_e: DC motor characteristic parameters;

V_1, V_2: control voltages applied to the DC motors;

R_a: drive wheel's radius (identical for the two);

R_e: resistance of the DC motor inductor;

K_ϕ, K_b: characteristic parameters of the DC motor;

$q(t):=[x(t), y(t), \theta(t)]'$: vehicle configuration vector;

$[x_p(s)\ y_p(s)\ \theta_p(s)]$, $s\epsilon[0, \infty)$: assigned path in configuration space;

$v_{up}(s)$ $v_{wp}(s)$ $w_p(s)$ $a_{up}(s)$ $a_{wp}(s)$ $a_{\theta p}(s)$: linear and angular velocities and accelerations with which the assigned path has to be followed;

$X_d := [x_d\ y_d\ \theta_d\ v_{ud}\ v_{wd}\ \Omega_d]'$: desired state;

$\theta_{os} := \theta - \theta_d$: heading offset;

ℓ_{os} := lateral offset, i.e.: distance between the c.o.m. of the vehicle and the path;

$v_{os} := v_u - v_{ud}$: velocity offset;

p_{11} p_{12}: poles associated with the speed offset dynamics;

p_{21} p_{22} p_{23}: poles associated with the orientation offset dynamics;

K_{p1} K_{p2}: gains of the speed controller feed-back component;

K_{s1} K_{s2} K_{s3}: gains of the steering controller feed-back component;