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Plasma-neutral coupling allows electrostatic ion cyclotron waves to propagate below ion cyclotron frequency

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The effect of ion-neutral collisions on the propagation characteristics of electrostatic ion cyclotron (EIC) waves in a partially ionized plasma is investigated. The dispersion relation of EIC waves is derived using a fluid model taking neutral dynamics into account. The propagation properties of EIC modes, including the damping factor, are examined for various ionization degrees and collision frequencies, which determine the momentum transferred from ions to neutral particles. It is found that the motion of neutral particles driven by plasma-neutral coupling leads to an increase in the effective ion mass, and consequently, EIC waves can propagate even below the ion cyclotron frequency. In a hot neutral gas, the gas-thermal mode can also propagate as well as the EIC mode. The possibility of observing in the laboratory and the Earth's ionosphere is discussed.

I. INTRODUCTION

Partially ionized plasmas exhibit unique behaviors that are never observed in fully ionized plasmas, and such specificity is sometimes observed as a great beauty, such as auroras. Many phenomena involving neutral particles have also been observed in laboratory plasmas, such as neutral depletion,[1] vortex formation,[2,3] axial neutral gas flow reversal,[4] and oscillation/intermittent phenomena.[5–8] When the momentum transferred from the ions drives the neutral flow, the partially ionized plasma should be treated as a multi-component fluid composed of an electrically conducting fluid and a neutral fluid.

It is commonly assumed that neutral particles in plasmas are stationary because the flow velocity of the neutral gas is usually much smaller than that of the ions. This treatment is convenient for simplifying the equations and reducing the number of quantities required to be measured experimentally. However, it should be noted that in weakly ionized plasmas, the momentum and energy of neutral particles per unit volume can be comparable to those of plasma. Hence, it is essential to appropriately handle neutral particles to understand the structure formation and transport phenomena.

In partially ionized plasmas, collisions with neutral particles are an important factor in studying the damping of waves and the growth rate of instabilities. When collisions are sufficiently frequent, it is generally considered is that the damping rate increases with neutral density. The dynamical behavior of neutral particles in wavefields, however, yields opposite results. The importance of the neutral gas flow on the electrostatic modes was studied in the early days of plasma research by Sessler, and the effect of collisional damping on ion and electron waves in an unmagnetized plasma was discussed by taking into account the neutral-particle motion in a wavefield.[9] Vranjes and Poedts reported the remarkable features of ion acoustic (IA) waves in a partially ionized plasma, i.e., the dynamical behavior of neutral particles reduces the damping rate compared with the case of stationary neutrals.[10] A recent experiment by Sharma *et al.* has verified this by showing that the IA waves excited with a grid exciter can propagate farther at higher neutral gas pressure.[11] These previous studies demonstrate that the behavior of neutral particles needs to be treated correctly in order to understand wave phenomena in partially ionized plasmas. On the other hand, the effect of neutral particles on wave phenomena in magnetized plasmas has not been fully discussed.

Electrostatic ion cyclotron (EIC) waves identified by D'Angelo and Motley[12] have been studied extensively.[13–23] A general background and the history of research on EIC waves can be found in the appropriate literature[24] and recent papers.[25,26] The effect of collisions with neutrals on EIC waves was investigated by Suszcynsky *et al.*[27,28] They showed experimentally that EIC waves could be excited even when the collision frequency and the ion cyclotron frequency are comparable, demonstrating the existence of EIC waves in weakly ionized plasmas in the bottom of the E region in the Earth's ionosphere. However, previous studies have not addressed the role of neutral-particle dynamics for the propagation of EIC waves in weakly ionized plasmas.

In this paper, we focus on EIC waves in weakly ionized plasmas. Keeping the physical situation as simple as possible without losing the physics of interest, we investigate the propagation properties of EIC waves using a multi-component fluid model. In Sec. II, a dispersion relation of EIC waves, which propagate nearly perpendicular to the background magnetic field, is derived for a cold neutral gas. We show that the dynamical behavior of neutral particles in a wavefield reduces the frequency of propagation modes as well as the damping factor. In Sec. III, we investigate the effect of finite neutral gas temperature. The dispersion relation indicates that the EIC and gas thermal modes can be simultaneously excited. Finally, we summarize the present study in Sec. IV.

II. DISPERSION RELATION OF EIC WAVES IN A PARTIALLY IONIZED PLASMA

The dispersion relation of EIC waves in a uniform magnetic field (**B**) is obtained from the momentum balance equations for ions and neutral particles given by

$$Mn_{\mathrm{i}}\left(\frac{\partial \mathbf{u}_{\mathrm{i}}}{\partial t}+\mathbf{u}_{\mathrm{i}}\cdot\nabla\mathbf{u}_{\mathrm{i}}\right)=-k_{\mathrm{B}}T_{\mathrm{i}}\nabla n_{\mathrm{i}}+en_{\mathrm{i}}(-\nabla\phi+\mathbf{u}_{\mathrm{i}}\times\mathbf{B})-Mn_{\mathrm{i}}\nu_{\mathrm{in}}(\mathbf{u}_{\mathrm{i}}-\mathbf{u}_{\mathrm{n}}), \tag{1}$$

$$Mn_{\mathrm{n}}\left(\frac{\partial \mathbf{u}_{\mathrm{n}}}{\partial t}+\mathbf{u}_{\mathrm{n}}\cdot\mathbf{u}_{\mathrm{n}}\right)=-k_{\mathrm{B}}T_{\mathrm{n}}\nabla n_{\mathrm{n}}-Mn_{\mathrm{n}}\nu_{\mathrm{ni}}(\mathbf{u}_{\mathrm{n}}-\mathbf{u}_{\mathrm{i}}), \tag{2}$$

and the continuity equations by

$$\frac{\partial n_{\mathrm{i}}}{\partial t}+\nabla\cdot(n_{\mathrm{i}}\mathbf{u}_{\mathrm{i}})=0,\ \ \frac{\partial n_{\mathrm{n}}}{\partial t}+\nabla\cdot(n_{\mathrm{n}}\mathbf{u}_{\mathrm{n}})=0. \tag{3}$$

The quantity M is the mass, n the density, $\mathbf{u}$ the flow velocity, T the temperature, e the elementary charge, ϕ the electrostatic potential, and k_{B} the Boltzmann constant. The subscripts i and n refer to ions and neutrals, respectively. Ignoring the mass difference between ions and neutral particles, we have left out the subscript for M. For simplicity, each temperature is spatiotemporally constant throughout the paper. The interaction between ions and neutrals is given by the friction forces $-Mn_{\mathrm{i}}\nu_{\mathrm{in}}(\mathbf{u}_{\mathrm{i}}-\mathbf{u}_{\mathrm{n}})$ and $-Mn_{\mathrm{n}}\nu_{\mathrm{ni}}(\mathbf{u}_{\mathrm{n}}-\mathbf{u}_{\mathrm{i}})$, where ν_{in} is the collision frequency for ions with the neutrals and $\nu_{\rm ni}$ is the reverse. In the present study, we assume momentum conservation for collisions and adopt the relationship $n_{\rm i}\nu_{\rm in} = n_{\rm n}\nu_{\rm ni}$.

We use the Boltzmann relation for electrons, $n_e = n_0 \exp[e\phi/(k_{\rm B}T_{\rm e})]$, and a charge neutrality limit condition to close the set of equations instead of the momentum balance equation for the electrons and the Poisson equation. Here, the subscript e refers to electrons. The assumption is reasonable when the following conditions are satisfied: 1) the frequency of interest is sufficiently lower than the ion plasma frequency, 2) the electrons move along the magnetic field line indicating the finite parallel wavenumber. In addition, the parallel wavenumber ($k_\parallel$) should be much smaller than the perpendicular wavenumber ($k_\perp$), and the small-$k_\parallel$ limit approximation is valid in the range $\tan^{-1}(k_\parallel/k_\perp) \gg \sqrt{m/M}$,[29] where m is the electron mass. The derivation of the dispersion relation taking the parallel wavenumber into account is shown in the Appendix.

For simplicity, we choose a rectangular-coordinate system and assume that the uniform magnetic field is in the z direction ($\mathbf{B} = B\mathbf{e}_z$). It is assumed that each plasma parameter is uniform in space and that there are no flows everywhere ($u_{\rm i0} = u_{\rm n0} = 0$) in steady state. We also assume a perturbation of $\exp[{\rm i}(kx-\omega t)]$ corresponding to a limit condition of $k_\parallel \to 0$. By linearizing Eqs. (1)-(3) and using the linearized Boltzmann relation $n_1/n_0 = e\phi_1/(k_{\rm B}T_{\rm e})$, we obtain the following equation

$$\begin{aligned}\omega^4 &+ {\rm i}2\nu_{\rm in}\left(1+\frac{n_0}{n_{\rm n0}}\right)\omega^3 - \left[\omega_{\rm ci}^2 + c_{\rm s}^2k^2 + \nu_{\rm in}^2\left(1+\frac{n_0}{n_{\rm n0}}\right)^2\right]\omega^2 \\ &- {\rm i}\nu_{\rm in}\left[c_{\rm s}^2k^2 + 2(\omega_{\rm ci}^2 + c_{\rm s}^2k^2)\frac{n_0}{n_{\rm n0}}\right]\omega + \nu_{\rm in}^2\left[c_{\rm s}^2k^2 + (\omega_{\rm ci}^2 + c_{\rm s}^2k^2)\frac{n_0}{n_{\rm n0}}\right] = 0, \end{aligned} \tag{4}$$

where $\omega_{\rm ci} = eB/M$ is the ion cyclotron frequency and $c_{\rm s} = \sqrt{k_{\rm B}(T_{\rm e}+T_{\rm i})/M}$ the ion sound velocity. Equation (4), a fourth-degree algebraic equation in ω, gives the dispersion relation of an EIC wave in a partially ionized plasma under the small parallel wavenumber limit condition. In a collisionless plasma, Eq. (4) results in the well-known dispersion relation, $\omega^2 = \omega_{\rm ci}^2 + c_{\rm s}^2k^2$.[30] Two non-dimensional parameters, i.e., $\hat{\nu} = \nu_{\rm in}/\omega_{\rm ci}$ and $\eta = n_0/n_{\rm n0}$, can characterize the plasma-neutral coupling on the wave propagation: the parameters reflect the strength of coupling between ion and neutral fluids (plasma-neutral coupling). If the neutral particles are unperturbed, i.e., $\mathbf{u}_{\rm n1} = 0$, the dispersion relation is expressed by taking the limit $\eta \to 0$. It is noted that there is no solution that permits wave propagation in the region of $\omega < \omega_{\rm ci}$ in both the collisionless and stationary neutral cases.

Figure 1 shows a dispersion curve of a collisional EIC wave at $\hat{\nu} = 4$ and $\eta = 10^{-1}$, where

$\rho_s = c_s/\omega_{ci}$. To study the mode damping, we denote the complex frequency by $\omega = \omega_R + i\omega_I$ and evaluate the imaginary part in ω associated with the magnitude of mode damping ($\omega_I < 0$) as well as the frequency ω_R. In the high-frequency region ($\omega/\omega_{ci} \gg \hat{\nu}$), the dispersion curve lies on the collisionless curve (dotted line). The collision does not affect the propagation properties because the momentum of the ions is barely transferred to the neutral particles within the wave period.

When the wave frequency approaches the collision frequency ($\omega/\omega_{ci} \sim \hat{\nu}$), the collision effect becomes important. As the wavenumber decreases, the damping factor defined by $\gamma \equiv |\omega_I|/\omega_R$ ($\omega_I < 0$) increases, and the mode finally disappears at $k\rho_s = 3.2$ where the damping factor becomes unity. It is worth noting that the propagation mode reappears for $k\rho_s < 1.9$, in which the frequency is lower than the ion cyclotron frequency. If the neutral particles are stationary in the wavefield, there is no propagation mode in this region. In other words, the propagation modes in the frequency of $\omega < \omega_{ci}$ exist only by including the effect of neutral particle dynamics.

As seen in Fig. 1, in the small wavenumber region, the dispersion curve approaches $\omega_{ci}^* = \omega_{ci}/(1+\eta^{-1})$ asymptotically instead of ω_{ci}. The following estimation provides an intuitive understanding of this trend. First, we consider the limit $k = 0$ and neglect the pressure term in Eqs. (1) and (2). By substituting Eq. (2) into Eq. (1), a momentum balance equation of the multi-component fluid is obtained

$$\frac{\partial \mathbf{u}_1^*}{\partial t} = \frac{eB}{M}\left(1+\frac{1}{\eta}\right)^{-1} \mathbf{u}_{i1} \times \mathbf{e}_z = \omega_{ci}^* \mathbf{u}_{i1} \times \mathbf{e}_z, \tag{5}$$

where $\mathbf{u}_1^* = (n_0 \mathbf{u}_{i1} n_{n0} \mathbf{u}_{n1})/(n_0 + n_{n0}) = (\mathbf{u}_{i1} + \mathbf{u}_{n1}/\eta)/(1+\eta)$. If the force acting on the ions per unit time and per unit volume is the same as in the collisionless case, Eq. (5) indicates that the mass of the fluid will be effectively heavier by $M^* = M(1+\eta^{-1})$.

In order to get a better understanding, we seek the phase relation between ions and neutrals. From Eqs. (2) and (3), the relationship between the ion and neutral flow velocities is expressed as

$$\mathbf{u}_{n1} = \frac{i\eta\hat{\nu}}{(\omega/\omega_{ci}) + i\eta\hat{\nu}} \mathbf{u}_{i1}. \tag{6}$$

The ratio of the ion flow velocity with the neutral flow velocity is rewritten as $u_{i1}/u_{n1} = A\exp(i\chi)$. When the damping factor is small enough, the phase and magnitude can be expressed, respectively, as

$$\chi = \tan^{-1}\left(-\frac{\omega_R}{\eta\nu_{in}}\right) \quad , \quad A \simeq \frac{\omega_R}{\eta\nu_{in}}\left[1+\left(\frac{\eta\nu_{in}}{\omega_R}\right)^2\right]^{\frac{1}{2}}. \tag{7}$$

Equation (7) indicates that ions and neutrals move in-phase ($\chi \sim 0$) when the momentum of ions is sufficiently transferred to the neutral particles within the wave period ($\omega_R/(\eta\nu_{in} \ll 1)$. For

$\nu_{\rm in} \sim \omega_{\rm R}$ and $\eta = 10^{-1}$, the estimated velocity ratio is $A \sim 10$. In weakly ionized plasmas, the velocity of neutral particles perturbed by ions is not large, but the neutral particle dynamics has a significant effect on the propagation of EIC waves.

The dispersion curves for the different sets of collision frequencies and density ratios are presented in Fig. (2). When the normalized collision frequency is less than unity, i.e., $\hat{\nu} < 1$, the neutral particles do not play an essential role in the dispersion properties because not enough momentum is transferred. The modification of the dispersion curve from the collisionless EIC waves becomes apparent when the normalized collision frequency exceeds unity ($\hat{\nu} > 1$). Under this condition, the frequency approaches $\omega \simeq \omega_{\rm ci}^*$ asymptotically at low wavenumbers, and the intuitive explanation regarding the multi-fluid behavior mentioned above gives a reasonable approximation for the mode frequency. Moreover, the forbidden region of propagation modes vanishes for a higher value of η.

Although the characteristics of the propagation mode can be determined by $\hat{\nu}$ and η, these are not independent of each other, as both depend on the neutral density. Therefore, when demonstrating the propagation of the EIC wave in actual circumstances, one should be careful to ensure that these quantities are a possible combination.

The interest here is whether the neutral flow effect on the EIC waves can be confirmed experimentally in laboratory plasmas. Considering a typical laboratory plasma, we calculate Eq. (4) for a low-temperature argon plasma with $n_0 = 10^{18}$ m^{-3}, $T_{\rm e} = 4$ eV, and $T_{\rm i} = 0.1$ eV in a magnetic field of $B = 0.01$ T and take the cross-section of ion-neutral collisions in Ref.[31]. Figure 3 shows the frequency for $k\rho_{\rm s} = 1$ as a function of ionization degree defined by $R_{\rm iz} = n_0/(n_{\rm n0} + n_0) = \eta/(1+\eta)$; the damping factor, $\gamma = |\omega_{\rm I}|/\omega_{\rm R}$, is also depicted. In the fully-ionized and collisionless case ($R_{\rm iz} = 1$), the corresponding mode frequency is $\omega/\omega_{\rm ci} = 1.4$.

In the region $R_{\rm iz} > 3 \times 10^{-2}$, the mode frequency is almost the same as in the collisionless case, and the damping factor increases with a decrease of the ionization degree. This feature of collisional damping is identical to the conventional understanding, and hence, the dynamics of neutral particles are not crucial in this region. The mode disappears in the region $4 \times 10^{-3} < R_{\rm iz} < 3 \times 10^{-2}$ because the damping factor is larger than unity. The propagation mode reappears in $R_{\rm iz} < 4 \times 10^{-3}$. It is worth pointing out that in this region, the damping rate decreases with a decrease of $R_{\rm iz}$. Moreover, the mode frequency decreases with the ionization degree. These features are attributed to the neutral particles moving together with the ions, resulting in a smaller frictional force.

Qualitatively, since the wavelength becomes longer as the magnetic field becomes weak, the required device size perpendicular to the magnetic field increases. If the magnetic field is several hundred gauss, the typical wavelength is on the order of 10^{-1} m or shorter. Such an environment would be feasible in a typical low-temperature plasma device. In addition, early experiments on EIC waves were carried out using plasmas with low ionization degrees ($R_{\rm iz} > 3\times10^{-4}$) but relatively low collisionality ($\nu_{\rm in}/\omega_{\rm ci} < 1$).[27] Therefore, to confirm the reduction of the mode frequency and damping factor, it is necessary to use a more collisional plasma where sufficient momentum transfer to the neutrals is ensured.

EIC waves have also been observed in the E and F regions of the Earth's ionosphere. In the lower altitude of the E region (altitude down to 120 km), the ion-neutral collision frequency is comparable or several times higher than the ion cyclotron frequency, typically $\nu_{\rm in}/\omega_{\rm ci} \sim 3$ at ~ 110 km,[32,33] and the typical ionization degree is $R_{\rm iz} \sim 10^{-6}$-10^{-5}. Hence, we may be able to observe EIC waves with a frequency below the ion cyclotron frequency. In the F region, since the normalized collision frequency decreases and is less than unity ($\nu_{\rm in}/\omega_{\rm ci} < 1$), it could be difficult to observe the effect of plasma-neutral coupling on EIC waves.

III. EFFECT OF FINITE NEUTRAL GAS TEMPERATURE ON WAVE PROPAGATION

Let us now take a look at the effect of finite neutral gas temperature on the propagation of EIC waves. By keeping the pressure term of neutral fluid in Eq. (2) and using Eq. (3), the relationship between the ion and neutral flow velocities can be expressed as

$$u_{\rm n\mathit{x}} = \frac{{\rm i}u_{{\rm i}x}}{f_T}\left(\frac{\omega}{\omega_{\rm ci}} + {\rm i}\eta\hat{\nu}\right)^{-1}, \quad u_{\rm ny} = {\rm i}u_{\rm ny}\left(\frac{\omega}{\omega_{\rm ci}} + {\rm i}\eta\hat{\nu}\right)^{-1}, \tag{8}$$

where f_T is defined by

$$f_T = 1 - \frac{\tau k^2\rho_{\rm s}^2}{\omega(\omega + {\rm i}\eta\hat{\nu})}, \quad \tau = \frac{T_{\rm n}}{T_{\rm e}+T_{\rm i}}. \tag{9}$$

Substituting Eq. (8) into the linearized Eq. (1), we obtain a dispersion relation given by the fifth-order algebraic equation

$$\sum_{j=0,1,\cdots,5} a_j \left(\frac{\omega}{\omega_{\rm ci}}\right)^j = 0, \tag{10}$$

and the coefficients a_j can be written as

$$\begin{aligned}
a_5 &= 1, \\
a_4 &= 2\mathrm{i}\hat{v}(1+\eta), \\
a_3 &= -\left[1+(1+\tau)K^2\hat{v}^2(1+\eta)^2\right], \\
a_2 &= -\mathrm{i}\hat{v}\left\{K^2[1+\tau(2+\eta)]+2(1+K^2)\eta\right\}, \\
a_1 &= \eta\hat{v}^2\left[(1+\tau)K^2+(1+K^2)\eta\right]+\tau K^2(1+\hat{v}^2+K^2), \\
a_0 &= \mathrm{i}\tau\hat{v}\left[K^2+(1+K^2)\eta\right],
\end{aligned} \tag{11}$$

where $K = k\rho_{\rm s}$. The dispersion relation contains two eigenmodes: one is the EIC mode, and the other is the gas thermal (GT) mode[10] given by $f_T = 0$, in which the GT mode is excited by coupling with acoustic waves in a neutral fluid.

The dispersion relations for two different values of τ are shown in Fig. 4, and the parameters except τ are the same as those described in Fig. 1. It can be found that the GT mode, which obeys a similar dispersion relation of the neutral sound wave ($\omega = kV_{\rm tn}$; $V_{\rm tn} = \sqrt{k_{\rm B}T_{\rm n}/M}$), propagates in the higher wavenumber region. When the normalized neutral gas temperature is $\tau = 10^{-2}$ [Fig. 4(a)], it is easy to distinguish the EIC mode from the GT mode, in which the GT mode is non-propagating for $k\rho_{\rm s} < 1$. The dispersion curve of EIC waves is identical to that in the cold neutral case. In a typical laboratory plasma, the GT mode will essentially not affect a proof-of-principle experiment of collisional EIC waves because the typical value of τ is on the order of 10^{-2} or less.

When the neutral gas temperature is relatively high, the frequencies of the EIC and GT modes are comparable at higher wavenumbers, as shown in Fig. 4(b). In this situation, it is difficult to identify each mode around $k\rho_{\rm s} \sim 1$, and the modes show continuous dispersion characteristics approaching EIC modes for the low wavenumber side and GT modes for the high wavenumber side. Since the GT mode originates from an eigenmode of the neutral fluid, this result indicates the possibility of EIC wave excitation by neutral sound waves as an energy source.

IV. CONCLUSIONS

We have derived the linear dispersion relation of EIC waves with a multi-component fluid mode that takes the neutral dynamics into account. The EIC modes can propagate below the ion cyclotron frequency by effectively increasing the ion mass. Furthermore, the damping factor of EIC modes decreases compared with the stationary neutral case. The modification of EIC modes due to the dynamical behavior of neutral particles is observable in weakly ionized laboratory plasmas and the E region of the Earth's ionosphere. In the hot neutral gas case, the GT modes can propagate as well as the EIC modes, which indicates that sound waves in neutral fluids can excite waves in plasmas and vice versa.

The neutral particle dynamics plays an essential role in electrostatic wave propagation. It has been shown that the dispersion relation of EIC waves is modified by plasma-neutral coupling. In order to deal with instability and heating issues, it is necessary to consider the neutral particle dynamics as well as the inhomogeneity of plasma parameters and the kinetic effects. The present work provides a fundamental understanding in terms of the propagation properties of collisional EIC waves, which is essential for addressing the above issues. In partially ionized plasmas, it is crucial to handle neutral particles appropriately.

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Appendix A: Dispersion relation of EIC waves for a finite parallel wavenumber

It is assumed that the perturbation is given by $\exp[\mathrm{i}(k_\perp x + k_\parallel z - \omega t)]$, where $k_\perp$ and $k_\parallel$ are the wavenumbers perpendicular and parallel to the magnetic field, respectively. The linearized equations of momentum conservation can be written for ions as

$$(\omega - \mathrm{i}\nu_{\mathrm{in}})\mathbf{u}_{\mathrm{i}} = \mathbf{k}c_{\mathrm{s}}^2 \frac{n_{\mathrm{i1}}}{n_0} + \mathrm{i}\omega\mathbf{u}_{\mathrm{i}} \times \mathbf{e}_{\mathrm{z}} + \mathrm{i}\nu_{\mathrm{in}}\mathbf{u}_{\mathrm{n}}, \tag{A1}$$

and for neutrals as

$$\left(\omega + \mathrm{i}\nu_{\mathrm{in}}\frac{n_0}{n_{\mathrm{n0}}}\right)\mathbf{u}_{\mathrm{i}} = \mathbf{k}V_{\mathrm{tn}}^2 \frac{n_{\mathrm{n1}}}{n_{\mathrm{n0}}} + \mathrm{i}\nu_{\mathrm{in}}\frac{n_0}{n_{\mathrm{n0}}}\mathbf{u}_{\mathrm{i}}, \tag{A2}$$

where $V_{\mathrm{tn}} = \sqrt{k_{\mathrm{B}}T_{\mathrm{n}}/M}$ is the neutral thermal velocity. Also, the linearized continuity equations can be expressed as

$$\frac{n_{\mathrm{j1}}}{n_{\mathrm{j0}}} = \frac{k_\perp u_{\mathrm{j}x} + k_\parallel u_{\mathrm{j}z}}{\omega}, \quad \mathrm{j} = \mathrm{i},\ \mathrm{n}. \tag{A3}$$

To avoid complicating the equation, we have used the normalized variables defined as follows:

$$\Omega = \frac{\omega}{\omega_{\mathrm{ic}}}, \quad \hat{\nu} = \frac{\nu_{\mathrm{in}}}{\omega_{\mathrm{ci}}}, \quad \mathbf{U}_{\mathrm{j}} = \frac{\mathbf{u}_{\mathrm{j}}}{c_{\mathrm{s}}}, \quad \mathbf{K} = \mathbf{k}\rho_{\mathrm{s}}, \quad \eta = \frac{n_0}{n_{\mathrm{n0}}}, \quad \tau = \frac{T_{\mathrm{n}}}{T_{\mathrm{e}} + T_{\mathrm{i}}}, \tag{A4}$$

where $n_{0,1} = n_{i0,1}$ and the quasi-neutrality limit assumption is used. After a few steps, we obtain the dispersion relation as

$$\Omega\left[(\alpha+\gamma)\left(\alpha+\gamma\frac{f_\|}{f_\mathrm{T}}-1\right)\right]\left(\alpha+\gamma\frac{f_\perp}{f_\mathrm{T}}\right)-(\alpha+\gamma)\left(\alpha+\gamma\frac{f_\perp}{f_\mathrm{T}}\right)K_\perp^2$$
$$-\left[(\alpha+\gamma)\left(\alpha+\gamma\frac{f_\|}{f_\mathrm{T}}-1\right)\right]K_\|^2-(\alpha+\gamma)\gamma\frac{f_\times}{f_\mathrm{T}}\left(\frac{\gamma}{\beta f_\mathrm{T}}-2\right)K_\times^2=0, \tag{A5}$$

where

$$\alpha=\Omega+\mathrm{i}\hat{v},\;\; \beta=\Omega+\mathrm{i}\eta\hat{v},\;\; \gamma=\frac{\eta\hat{v}^2}{\beta},$$

and

$$f_\mathrm{T}=1-\frac{\tau K^2}{\Omega\beta},\;\; f_{\perp,\|}=1-\frac{\tau K_{\perp,\|}^2}{\Omega\beta},\;\; f_\times=\frac{\tau K_\times^2}{\Omega\beta},\;\; K_\times\sqrt{K_\perp K_\|}.$$

When $K_\|$ is sufficiently smaller than $K_\perp$, it is easy to check that neglecting the last two terms in LHS of Eq. (A5) yields a good approximation. By taking a limit of $K_\| \to 0$ and adopting a cold neutral assumption, i.e., $\tau = 0$, the dispersion relation can be rewritten as

$$\Omega-\frac{\alpha+\gamma}{(\alpha+\gamma)^2-1}K_\perp^2=0, \tag{A6}$$

and Eq. (A6) is identical to Eq. (4). If $\tau \neq 0$, Eq. (A5) is attributed to Eq. (10). Considering the opposite limit, i.e., $K_\perp \to 0$, for $\tau = 0$, Eq. (A5) becomes

$$\Omega^3+\mathrm{i}\hat{v}(1+\eta)\Omega^2-K_\|^2\Omega+\mathrm{i}\eta\hat{v}K_\|^2=0, \tag{A7}$$

and this corresponds with Eq. (8) in Ref.[10].

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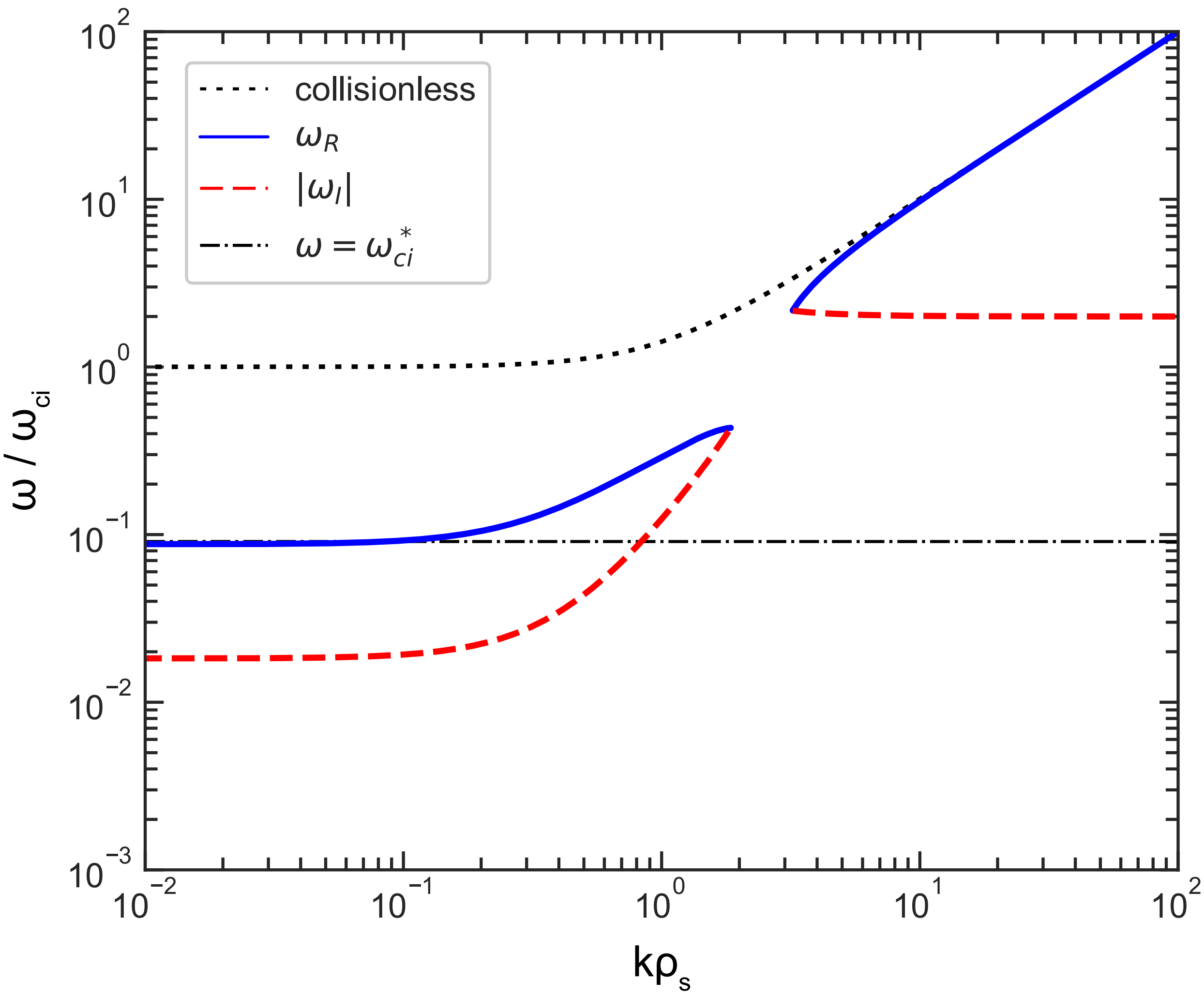

FIG. 1. (color online) Typical dispersion relation of an EIC wave with the neutral flow effect ($\hat{v} = 4$ and $\eta = 10^{-1}$). The real part (ω_{R}) and imaginary part associated with mode damping (ω_{I}) in ω are shown by solid (blue) and dashed (red) lines, respectively. The dispersion relation of collisionless EIC waves is depicted by the dotted line, and the dotted-dashed line indicates $\omega = \omega_{\mathrm{ci}}^*$.

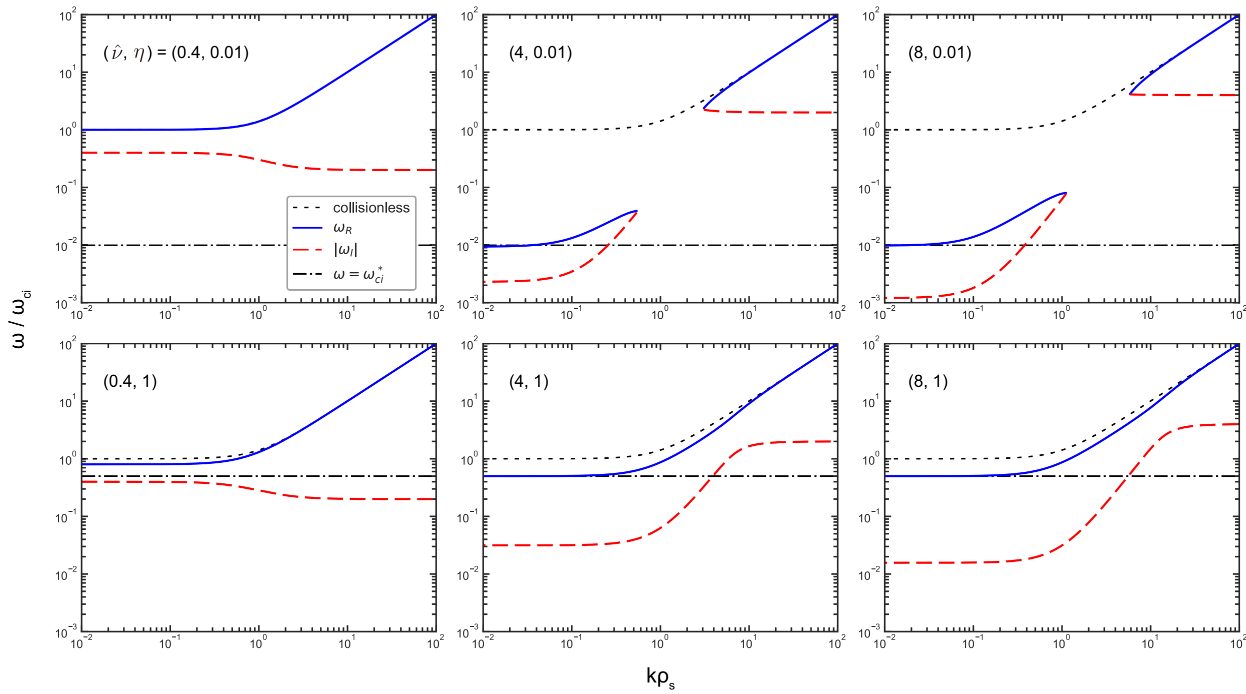

FIG. 2. (color online) Dispersion relations of an EIC wave in several sets of $\hat{\nu}$ and η. $\omega_{\rm R}$ and $|\omega_{\rm I}|$ ($\omega_{\rm I} < 0$) are shown by the solid (blue) and dashed (red) lines, respectively. In each figure, the upper dotted line indicates the dispersion relation of a collisionless EIC wave and the lower dotted-dashed line shows the modified ion cyclotron frequency $\omega_{\rm ci}^{*} = \omega_{\rm ci}(1+\eta^{-1})$.

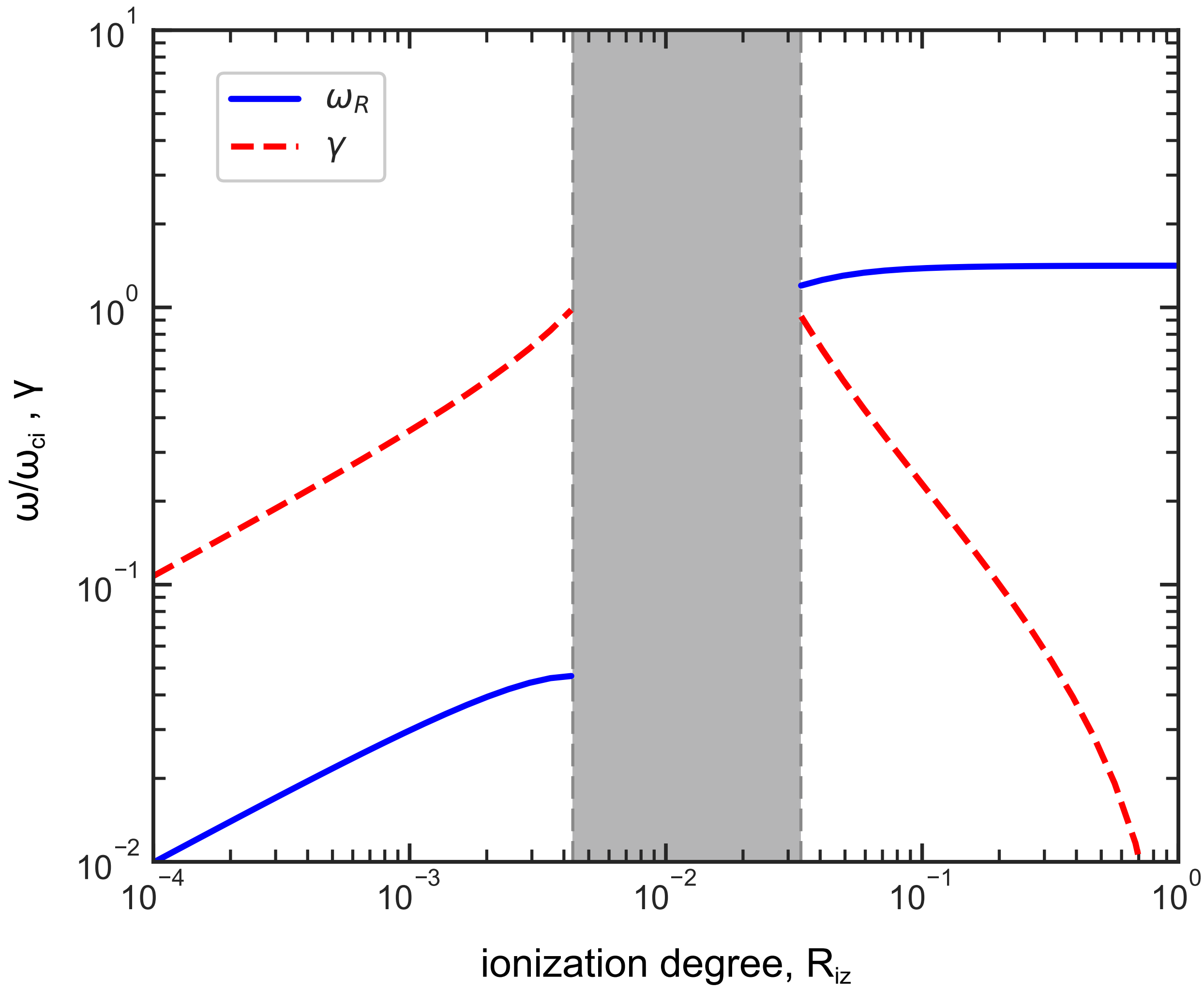

FIG. 3. (color online) Frequency as a function of ionization degree in a partially ionized argon plasma with $n_0 = 10^{18}$ m^{-3}, $T_e = 4$ eV, and $T_i = 0.1$ eV in a magnetic field of $B = 0.01$ T(solid line). The damping factor is also shown (dashed line). There are no modes satisfying $\gamma < 1$ in the hatched (grey) region.

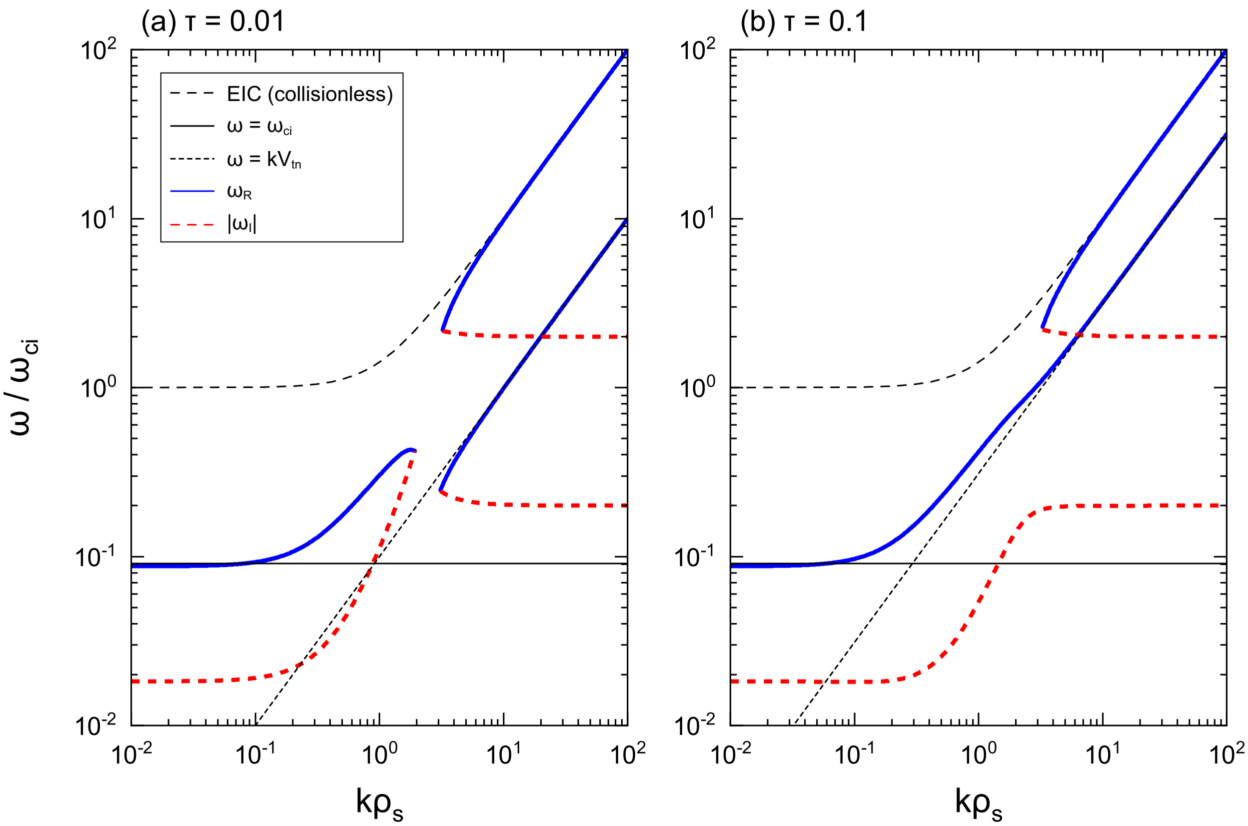

FIG. 4. (color online) Dispersion relations of collisional EIC wave with a finite neutral gas temperature of (a) $\tau = 10^{-2}$ and (b) $\tau = 10^{-1}$. The collision frequency is $\hat{\nu} = 4$ and the density ratio is $\eta = 10^{-1}$. Dispersion relations of the collisionless EIC mode (dashed) and neutral-acoustic mode (dotted, $\omega = kV_{\rm tn}$) are also depicted, and the solid horizontal line indicates $\omega = \omega_{\rm ci}^{*}$.