10-2002

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Dennis C. Smolarski
Santa Clara University, dsmolarski@scu.edu

Recommended Citation

Smolarski, D. C. (2002). Teaching Mathematics in the Seventeenth and Twenty-First Centuries. Mathematics Magazine, 75(4), 256–262. https://doi.org/10.2307/3219160

Teaching Mathematics in the Seventeenth and Twenty-first Centuries

DENNIS C. SMOLARSKI, S.J.
Santa Clara University
Santa Clara, CA 95053
dsmolarski@math.scu.edu

In the late 1960s, many people saw a fictional vision of the beginning of the twenty-first century via the movie, *2001: A Space Odyssey*. Early in the movie, a lunar expedition uncovers a large, black monolith in the crater Clavius. Although the movie was fictional, and computers have not yet reached HAL's ability to speak and read lips, the lunar crater Clavius does exist and is named after a sixteenth century scholar who was instrumental in introducing mathematics into the university curriculum.

Christopher Clavius (1538–1612) is often associated with the astronomical and mathematical justification for shifting from the Julian to the Gregorian calendar. He was also a university professor who was convinced that mathematics should be a standard part of a university curriculum and who saw the need to train instructors of mathematics. He exerted his influence on the *Ratio Studiorum (The Plan of Studies)*, a 1599 document that included administrative norms and curricular guidelines for Jesuit schools as well as offering pedagogical suggestions in the form of "Rules" for teachers of various subjects (the Latin text **[15]** and English translations **[6, 7]** are available).

This 400 year old document provided guidelines for teaching mathematics in Jesuit schools in the seventeenth century and contains what may seem to be novel and even modern suggestions for student involvement in learning mathematics. In particular, the *Ratio Studiorum* called for what today might be called *student colloquia* and *interactive review sessions*. For this reason, Clavius might be considered the father of certain contemporary pedagogical techniques in mathematics.

The last forty years have seen numerous attempts to revitalize the mathematical curriculum and mathematical pedagogy, especially in the United States. Particularly in the last twenty years, much time and energy has been focused on university-level calculus courses, with such well-known results as the Harvard Reform Calculus approach (see also two volumes in the *MAA Notes* series **[5, 23]** and a book by Krantz **[13]** with its useful appendices). The Mathematical Association of America conducted special sessions on calculus reform at its annual meetings in 1996 and 1997. The October 1997 issue of the *American Mathematical Monthly* contained several articles on the topic and *The College Mathematics Journal* **[17, 10]** frequently addresses these concerns. Many websites, for instance at Cornell, Swarthmore, and Harvard, are devoted to this discussion.

The catch-phrase *cooperative learning* is often used to describe techniques such as student presentations and interactive classroom sessions. These interactive components arise from the experience, common among teachers, that one comes to a deeper understanding of a subject by explaining it to others. This experience has prompted instructors to assign essays on mathematical topics **[20]** or oral presentations in classes. Although such ideas may seem revolutionary, there are, in fact, references to these practices in the *Ratio Studiorum*. In particular, a guideline in the Rules for the Professor of Mathematics in the *Ratio* requires that students present a solution to "some famous mathematical problem" before an assembly of other students [**6**, p. 46; **7**, p. 175], a precursor of contemporary student presentations.

Christopher Clavius, Jesuit mathematician

Christopher Clavius was born in Bamberg, Germany in 1538. In 1555, he traveled to Rome to join the Society of Jesus (commonly known as the Jesuits), a religious order in the Catholic Church. Nine years later, in 1564, he was ordained a priest while finishing his theological studies at the Jesuit-run Roman College, now known as the Gregorian University. He began teaching mathematical subjects on a regular basis at the Roman College around 1564. Clavius taught for more than 45 years in Rome, until his death in 1612 (except for two years in 1596 and 1597 when he was in Naples and Spain). Documents from the Roman College [**24**, p. 7] indicate that Clavius was the sole teacher of mathematics for at least 22 years between 1564 and 1595.

Clavius lived at the beginning of what is now called the "Scientific Revolution." This was the time of Copernicus (who published *De revolutionibus* in 1543) and the time of Galileo (who was condemned by the Inquisition in 1633) with whom Clavius discussed astronomical phenomena. Newton would be born 30 years after the death of Clavius. These were times of great scientific speculation as well as intellectual inertia and ecclesiastical opposition, particularly to Galileo.

In the academic community of sixteenth century Italy, there were two factions in contention over the role of mathematics in science and education. One faction consisted of a number of prominent Italian philosophers who denied that pure mathematics should be regarded as *scientia*, that is, scientific knowledge in the Aristotelean sense [**3**, pp. 36–37; **12**, p. 5; **14**, p. 33; **24**, p. 136]. The other faction was represented by Clavius. At that time in Italy, the only accepted professional disciplines for university study were theology, medicine, and law [**8**, p. 126; **12**, p. 81]. Mathematical topics were presented in the study of logic in the *trivium* (alongside grammar and rhetoric) and also in the *quadrivium* of arithmetic, geometry, and astronomy (taught as applied geometry), and music (taught as applied arithmetic). But these were preparatory subjects to be learned by students before entering a university to study philosophy as a basis for professional studies. Mathematics itself was not usually taught in a university [**3**, p. 35]. Clavius attempted to change that, at least within Jesuit schools.

The Jesuit Order was approved in 1540 and almost immediately set about establishing schools. In 1556, the year of the death of Ignatius of Loyola, founder of the Jesuit Order, the number of Jesuit schools was 35. Fifty-nine years later, in 1615, three years after Clavius died, the number had grown to 372 [**11**, p. 201]. In the middle of this period, the creation of the *Ratio Studiorum* was authorized by the head of the Jesuit Order, Father General Claude Aquaviva, in order to provide common guidelines for the curriculum, pedagogy, and organization of these schools.

In 1584, six Jesuit teachers, elected from different European Jesuit provinces, gathered in Rome and reviewed the various local educational documents in use at Jesuit schools. In August 1585, this commission submitted a report to Aquaviva who, in turn, submitted the draft to the faculty of the Roman College, where Clavius had been teaching for some 20 years [**7**, pp. 28–30]. After revision, the first draft of *Ratio* appeared in 1586 and was sent to Jesuit schools throughout Europe with a request for comments. A second draft ("1586B") also was written in 1586 but was not published or circulated. The collected comments on the original 1586 *Ratio* were again studied by the faculty of the Roman College and a revised *Ratio* was published in 1591 to be circulated for review. After more study in Rome, both by Father General Aquaviva and the faculty of the Roman College, a definitive version was promulgated in 1599 [**7**, pp. 31–33] (also see the introductory letter promulgating the 1599 *Ratio*, [**6**, pp. xii–xiii; **7**, pp. 119–20]).

The *Ratio* provided a common list of subjects to be taught, including Latin, Greek, Hebrew, scripture, theology, philosophy (which included logic, physics, and meteorology), moral philosophy, and mathematics, forming an early type of core curriculum. In addition, it included various suggestions for pedagogical techniques and rules for teachers. By providing such guidelines, the *Ratio* sought to offer a uniform approach to secondary and higher education in schools associated with the Jesuit Order throughout Europe and elsewhere. The definitive version of the *Ratio* appeared in 1599 at a crucial time in the history of Jesuit education when there was a rapid increase in the number of Jesuit schools.

During the two decades before the 1599 *Ratio* was published, Clavius wrote several treatises on mathematics which were reviewed by Aquaviva and, most probably, by others involved in the drafting and revision of the *Ratio* [**12**, pp. 61, 64] (the Latin text of these documents is also available [**16**, pp. 109–22]). The two primary documents were *Modus quo disciplinæ mathematicæ in scholis Societatis possent promoveri (A Method of Promoting the Mathematical Disciplines in the Schools of the Society)* (written around 1582) and *De re mathematica instructio (On Teaching Mathematics)* (written prior to 1593) [**22**].

The first document (*A Method* ...) seems to have been written in response to a request by Aquaviva in 1582 that the faculty of the Roman College convey its feelings about the teaching of various subjects [**16**, p. 109]. In the suggestions Clavius makes, one sees a hint of the intellectual climate of the time. To change the common opinion regarding mathematics, Clavius recommends that "*invitandus erit magister [artium mathematicarum] ad actus solemniores, quibus doctores creantur et disputationes publicæ instituuntur* (the mathematics teacher should be invited to the more solemn events at which doctorates are conferred and public disputations held ...)" [**16**, p. 115]. (The implication is that in many places they were not so invited!) He then takes to task, in particular, instructors of philosophy since, as he writes, "*docent, scientias mathematicas non esse scientias* (they teach that mathematical sciences are not sciences)" [**16**, p. 116]. He says that teachers should encourage students to learn mathematics, impressing on them how important the discipline is [**16**, p. 117]. Clavius also argues that one cannot understand various natural phenomena without mathematics. It is evident that some of Clavius's advice was incorporated by the drafting commission into the various drafts of the *Ratio*. It is also possible to see some similarities between the praise given to mathematics in the first, 1586 draft of the *Ratio* and the praise given to mathematics in Clavius's first document.

The second document (*On Teaching Mathematics*) is cited in a 1593 decree on the topic of educating teachers of mathematics [**16**, p. 622], written by the rector of the Roman College, Robert Bellarmine, at the wish of Aquaviva. The document emphasizes how vital it is to train future mathematics teachers well. It also offers specific suggestions on what mathematical topics should be taught, namely Euclid, the spherical elements of Theodosius, the *Conics* of Apollonius, applied topics pertaining to astronomy and physics (by studying Archimedes), and algebra [**16**, p. 118]. These topics briefly summarize a more detailed list of mathematical topics Clavius prepared prior to 1581 (*Ordo servandus in addiscendis disciplinis mathematicis [The Order to be Observed in Teaching Mathematical Disciplines]*, [**16**, pp. 110–15]). This document also proposes how this training should be undertaken, by suggesting private study of mathematics for a full year after young Jesuit seminarians had completed their study of philosophy and before studying theology [**16**, p. 118]. Near its end, the document includes a cry of exasperation, "*cum egeamus magistris mathematices* (We need mathematics teachers)" [**16**, p. 118].

In both documents, we notice the tension between Clavius's promoting the importance of mathematics in a university curriculum and the position held by Italian philosophers, some of whom were also Jesuits. This tension is also evident in the various drafts of the *Ratio*. For example, the 1591 draft of the *Ratio* contains a curious section that mentions Clavius and then instructs administrators of schools to guard that the philosophy instructors not disparage the dignity of mathematics (a topic alluded to in *A Method* ...). It then ends with the statement, "*fit enim sæpe, ut qui minus ista novit, his magis detrahat* (often, the less one knows about such things, the more he detracts)" (1591 *Ratio Studiorum*, Rules for the Provincial Superior: On Mathematics, n. 44 [**15**, p. 236]).

Readers of the various drafts of the *Ratio* and the documents written by Clavius can see the influence of someone who values mathematics highly and who is concerned that the young Jesuit Order train students well in the mathematical sciences, an unpopular position in late sixteenth century Italy. The personal opinion of Clavius about mathematics seems best summarized in his words: "Since ... the mathematical disciplines in fact require, delight in, and honor truth ... there can be no doubt that they must be conceded the first place among all the other sciences" [**3**, p. 38]. This praise is similar to the well-known statement of Carl F. Gauss (1777–1855), that "mathematics is the queen of the sciences," but Clavius lived two centuries earlier!

Mathematics pedagogy in the *Ratio Studiorum*

It seems reasonable to conclude that Clavius had some effect on the *Ratio Studiorum*. Aquaviva and those who drafted and revised the *Ratio* not only would have read the treatises described above, but they would have solicited personal comments from faculty of the Roman College, such as Clavius. The fact that Clavius is mentioned in both the 1586 and the 1591 drafts is proof enough of his influence. As noted earlier, Clavius was for many years the sole professor of mathematics at the Roman College whose faculty reviewed drafts of the *Ratio*. What is common to the three drafts and the final text is an emphasis on mathematics that was unusual for that time in Italy. These texts prescribe that all students be taught mathematics for one year, specifically, Euclid and topics related to geography (that is, applied geometry), and the sphere. (Since Clavius had published several mathematical texts, further detail in the final version of the *Ratio* was not needed.) In fact, the earlier drafts of the *Ratio* also recommended a second year of mathematics for those interested in further study.

The earlier drafts of the *Ratio*, which specifically mention Clavius, also suggested certain practices to help students understand the subject better. As noted above, these practices may be best understood in terms of contemporary counterparts: the *colloquium* and the *review session*.

In the 1586B draft, the text pertaining to mathematics includes a recommendation for colloquia or presentations by students: "*Semel aut iterum in mense auditorum aliquis in magno philosophorum theologorumque conventu illustre aliquod problema mathematicum enarret, prius a magistro, sicut oportet, edoctus*. (Once or twice a month, let some one of the students explain in detail some famous mathematical problem before a large gathering of students of philosophy and theology, after first having been taught it thoroughly by the teacher, as necessary)" [**15**, p. 177]. In the final 1599 *Ratio*, this recommendation is rephrased: "*Singulis aut alternis saltem mensibus ab aliquo auditorum magno philosophorum theologorumque conventu illustre problema mathematicum enodandum curet; posteaque, si videbitur, argumentandum*. (Let [the professor of mathematics] arrange that every month or every other month some one of the students before a large gathering of students of philosophy and theology has some famous mathematical problem to work out and afterwards, if it seems well, to defend his solution)" [**15**, p. 402] (also [**6**, p. 46; **7**, p. 175]). One is tempted to see the shift from "once or twice a month" to "every month or every other month" as a concession to the voices of opposition raised by other faculty.

In the 1586B draft, there is also mention of an interactive review session in the following recommendation: "*In cuiusque etiam mensis sabbato uno, prælectionis loco, præcipua, quæ per eum mensem explicata fuerint, publice repetantur, non perpetua oratione, sed se mutuo percunctantibus auditoribus hoc fere modo: Repete illam propositionem. Quomodo demonstratur? Potestne aliter demonstrari? Quem usum habet in artibus et in reliqua vitæ communis praxi?* (On one Saturday of each month, in place of the lecture, let the principal points that during that month had been explained be publicly repeated, not in an uninterrupted speech, but with the students mutually asking questions of themselves, generally in this manner: 'Repeat that proposition.' 'How is it proven?' 'Can it be proven otherwise?' 'What use does it have in the arts or in the other practices of common life?')" [**15**, p. 177].

As a result of recommendations by a general meeting of Jesuit delegates in 1593–94 that the definitive *Ratio* be briefer [**7**, p. 32], the details about student interaction and the types of questions students should be asking were no longer included. But the definitive 1599 *Ratio* kept the "Repetition" (review session) as a standard practice in its recommendation: "Once a month and generally on Saturday in place of the lecture, let the principal points that have been explained during that month be publicly repeated." [**15**, p. 402] (also [**6**, p. 46; **7**, p. 175]).

Similar pedagogical practices such as a presentation by students (or "disputation") and reviews are also prescribed by the 1599 *Ratio* for other subjects (for instance, Philosophy and Theology [**6**, p. 20–21; **7**, p. 144–45], and Scripture [**6**, p. 32; **7**, p. 159]). What is especially notable is the inclusion of versions of these practices, modified for teaching mathematics, in the 1599 *Ratio*. Admittedly there is no direct evidence, either from the documents of Clavius on mathematics or from other documents, that Clavius originated these pedagogical practices. Nevertheless, the fact that his name appears in earlier drafts of the *Ratio* in conjunction with the listing of these practices (and of the more detailed questions proposed) and that, as a member of the faculty at the Roman College, he reviewed the drafts of the *Ratio*, suggests that he did exert his influence on the text in the definitive 1599 *Ratio*.

The guidelines found in the 1599 *Ratio*, as well as the sample student questions found in the earlier versions, encouraged regular student participation and recommended that as part of a review of the subject matter, students reflect on alternative proofs of mathematical theorems and the application of mathematics to other areas of life. Similar recommendations have reappeared in the last decade, but, it is interesting to note, at the core of some of these modern pedagogical suggestions is a tradition over 400 years old.

The legacy of Clavius

From a contemporary viewpoint, Clavius created relatively little original mathematics, but the time was not congenial for this to happen. Today we know that mathematicians need the right climate to nurture new developments. However, Clavius did prove geometrically that, given a regular polygon with $2n + 1$ sides, the base angle is n times the summit angle in an isosceles triangle formed by a vertex of the polygon and its opposite side. (Thus, for a pentagon, n equals 2, and a base angle would be twice the summit angle.) Two centuries later Gauss would repeat the proof algebraically and use the result to construct a regular 17-sided polygon by rule and compass [**18**, p. 25; **19**, p. 336].

Perhaps, the greatest legacy of Clavius was his influence on the definitive version of the *Ratio Studiorum* which led to the inclusion of mathematics as a standard subject taught in Jesuit schools [**3**, pp. 34–36]. This inclusion is especially significant, given the common opinion in Italian academic circles about mathematics at the time. This was particularly crucial because the founding of Jesuit schools essentially coincided with the beginning of the scientific revolution.

In *A Method ...*, Clavius even proposed establishing an "academy" for more advanced study in mathematics. Such a specialized academy was explicitly mentioned in the first two public draft versions of the *Ratio*, but in the definitive 1599 version of the *Ratio* this mandate was reduced to the suggestion that "private lessons" be given to those more inclined toward mathematics (Rules for Provincials, n. 20, [**6**, p. 8; **7**, p. 130]). Although a special advanced mathematics curriculum was not mentioned in the definitive *Ratio* for all schools, Clavius's vision of establishing a special school to train Jesuit mathematicians was nevertheless realized in Antwerp near the end of Clavius's life. In 1611 François d'Aguilon, S.J. founded a school for mathematics and was joined by Gregory St. Vincent, S.J. around 1616 who had studied under Clavius in Rome. During his life, St. Vincent researched mathematical concepts we would now see as precursors to the infinitesimal calculus. Clavius's vision of a special "academy" may be viewed as a precursor of honors, seminar, or directed study classes in contemporary schools where students with an exceptional talent and love for mathematics may have their mathematical curiosity nurtured and challenged.

One additional contribution of Clavius to the mathematical community of later generations was his set of mathematical texts. For example, in 1574 Clavius published *The Elements of Euclid.* This was not simply a translation, but a text containing Euclid's work as well as comments on it, many taken from previous commentators and editors, but also including Clavius's own criticisms and elucidations of Euclid's axioms. This text has been held up as a model and led to Clavius being called the *Euclid of the 16th century* [**4**, p. 311]. It was this text that the Jesuit missionary Matteo Ricci brought to China and translated into Chinese, giving the Orient its first exposure to Euclid. Ricci was a student of Clavius in the mid-1570s and, because of his scientific knowledge which, to a large extent, was imparted to him by Clavius, is one of the few Westerners still honored in China. Ricci's grave near Beijing is still a place of pilgrimage. Other major works of Clavius include *In Sphæram Ioannis de Sacro Bosco commentarius* (1581), *Epitome arithmeticæ practicæ* (1583), *Astrolabium* (1593), *Geometria practica* (1604), *Algebra* (1608), and *Triangula sphærica* (1611). These well-written texts were reprinted numerous times and widely used in Jesuit schools. They were also used by such mathematicians as Leibniz and Descartes (who studied at the Jesuit College of La Flèche) [**18**, pp. 7, 48; **3**, p. 34].

In his texts, Clavius used notation that has become standard in mathematics today, such as the square root sign, parentheses, the x-like symbol for an unknown [**2**, pp. 369, 151, 154,], and the decimal point [**9**; **1**, p. 303], although Cajori seems less certain about Clavius's understanding of the period as a decimal point [**2**, p. 322]. Although some of these symbols had been used by others in German lands earlier, Clavius helped to bring these symbols to Italy and to standardize this mathematical notation due to their appearance in his widely-disseminated books.

Summary

The historian of science George Sarton calls Clavius, "the most influential teacher of the Renaissance" [**21**, p. 70]. Clavius was convinced of the importance of the study of mathematics in universities and helped to influence the Jesuit *Ratio Studioum* to include mathematics as a required part of the curriculum. This 400-year old document also advanced the insight that students would understand mathematical concepts better by explaining them to others and by questioning mathematical derivations during a review. Such practices are still effective tools for helping students of the twenty-first century appreciate the beauty and wonders of mathematics.

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