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Accounting for Variability

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ACCOUNTING FOR VARIABILITY: IDENTIFYING CRITICAL ACTIVITIES AS A SUPPLEMENT TO THE CRITICAL PATH

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ABSTRACT

Successful realization of construction activities requires simultaneous integration of various resource input flows, giving rise to considerable sources of flow variability. Such variability might manifest as schedule variations which jeopardize the project performance, especially when using deterministic scheduling. Current scheduling techniques fail to efficiently tackle variability and rely on deterministic approaches. Therefore, this study fills the gap by developing a Discrete Event Simulation model, where activity durations are modeled using beta distributions and Program Evaluation and Review Technique assumptions. By applying the Spearman correlation coefficient, activities with higher influence on the schedule are identified, highlighting where to reduce variability. An application example was conducted involving a Critical Path Method (CPM) network containing eleven activities. Two types of waste emerging due to variability are identified as waiting time and variation gaps. Out of the eleven activities in the example network, two sets of critical activities are identified. Results reveal that an 80% reduction in variability in these critical activities leads to a 51.9% increase in likelihood of completing the project on schedule, 30% decrease in waiting time, and 28.6% decrease in variation gap. An important implication of this research is that near critical paths could become critical based on the amount of variability contained in the activities lying on each path. Acquiring such information early on during planning provides proactive, eye-opening insights into potential problematic scheduling areas. The study's contribution includes investigating the variability effect on two types of waste in production and providing project planners with a stochastic approach to manage the hidden waste in production systems;

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the approach examines the effect of reducing variability on the overall project performance characterized by meeting deadlines, avoiding trade idling (reducing waiting time), and exploring potential opportunities for enhancing performance (reducing variation gaps).

INTRODUCTION

Variability is a common phenomenon across many industries, including construction. Variability is characterized by unevenness or non-uniformity, and it manifests in many forms (Hamzeh, 2009; Schonberger, 2008). It has a considerable downgrading impact on production performance metrics, leading to productivity loss through causing excessive delays, inflated completion times, and longer queues of incomplete jobs (Arashpour and Arashpour, 2015). For instance, one of the effects of variability is rework that causes process completion time to be delayed significantly (Arashpour and Arashpour, 2015). Therefore, analyzing the workflow variability is an essential area of the production management approach (Palaniappan et. al, 2007). In particular, production flows carry two types of variability which are process-time variability and flow variability (Koskela, 2000). The former is concerned with the time needed to handle a task at one workstation, or in other words, it refers to the completion time of a task. As for the latter, it refers to the variability in the advent of a task or a job to a workstation, or securing all the prerequisites of the task to ensure that it can start on time (Koskela, 2000; Spearman and Hopp, 1996).

The preconditions or resource input flows necessary for carrying out a construction task are classified into construction design, components and materials, workers, equipment, space, connecting works, and external conditions (Koskela, 2000). Successful realization of a task requires synchronization of these resource flows. Thus, a problem encountered in any of these flows has a considerable effect on successfully achieving the task (Lindhard et. al, 2020). In fact, many sources of flow variability exist within the construction industry, and this is why studying variability is crucial, especially schedule variations. Actually, the high level of variability in construction activities, which can amount to around 60% of the activity average duration, is a major factor in causing project-level delays (Ballesteros-Perez et. al, 2020). Variation in the duration or dates of one activity can affect other activities along the way and possibly result in schedule interruptions and reduced productivity (Wambeke et. al, 2012). Moreover, given that a construction schedule might contain numerous activities/relationships and can become complex, it is frequently hard to predict the effect of such variations.

In brief, variations in flow cause project completion to be delayed and waste to be increased (Tommelein et al., 1999). To mitigate this issue of variability, probabilistic scheduling techniques such as Program Evaluation and Review Technique (PERT) have been adopted. Although PERT has become a standard scheduling tool for projects having activities with uncertain durations, it underestimates the duration average and neglects the merge event bias (Ballesteros-Perez, 2017). Also, the focus in PERT is only on the activities lying on the critical path. Failure to carefully watch the activities that are near critical, and that can become critical in certain situations, would also impact the schedule negatively. In fact, CPM/PERT could become ineffective since it necessitates time-consuming and recurrent revising of calculations (Ragel, 2021); and it is unwieldy to model the dependence of ongoing tasks between trades using CPM, as it fails to clearly represent variability (Tommelein et al., 1999). Previous research has focused on modelling variability through assuming a longer mean process time or considering a greater

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variance in process times (Arashpour et al., 2013), and the existing scheduling techniques have failed to efficiently address variability and relied on deterministic approaches. However, variability is more complex and needs to be modeled more accurately. Therefore, this study fills the gap through adopting a stochastic approach for assessing the impact of variability on the project schedule. Here comes into play the simulation as a powerful tool to model potential variations and assess their effects on the critical paths and overall schedule. This study employs Discrete Event Simulation (DES) in identifying the activities that have the highest potential for improving schedule performance by reducing variability and corresponding waste in the corresponding activities. The Spearman coefficient, combined with the characteristics of the schedule network, are used to assess the criticality of an activity. The simulated variability reflects both process-time and flow variability, and the study considers two types of waste which are waiting time and variation gaps that are explained in more detail in the "Waste in Construction" subsection. A mathematical model, that integrates a beta distribution with PERT assumptions, is built in MATLAB to generate activities' durations. Using this model, DES runs for a CPM network are done and the results are analyzed. In this approach, the activities' variability is combined with the network characteristics allowing identification of how much impact the variability has on the overall project duration. Results from the approach provides the project manager with an insight regarding where to focus the effort of reducing variability, whether on the highly variable activities laying on the critical and near critical paths, or on the activities occurring on multiple paths. The study contribution lies in examining through simulation the effects of variability on two overlooked types of waste in production systems, and providing schedulers with a proactive approach towards managing the hidden waste in a production system through examining the effect of mitigating variability on project performance characterized by meeting deadline, avoiding trade idling (reducing waiting time), and unveiling potential opportunities for speeding the project (reducing variation gaps). It aims at improving scheduling and project performance by reducing productivity loss. A discussion on reducing variability using Lean Construction concepts follows, as Lean Construction complements the aim of this study by promoting reducing variability and increasing predictability of various workloads. Finally, the conclusion and future recommendations are presented.

LITERATURE REVIEW

PERT and Simulation in Construction

There are few studies in the literature that integrate PERT with simulation. For instance, Lu and AbouRizk (2000) proposed a simulation model that aims at simplifying the classic CPM/PERT analysis. This is done through integrating the approach of discrete event modeling with a simplified method for determining critical activities. The model showed remarkable improvement in analyzing project risks portrayed as schedule overruns and in determining criticality of activities. Lee and Arditi (2006) developed a scheduling system that combines CPM, PERT, and optimized discrete event simulation (DES). The model includes statistical testing to help eliminate outliers and increase its accuracy. Lee (2005) proposed a stochastic simulation model to measure the probability of completing a project within a specified duration through amending traditional PERT concepts to include more statistical modeling flexibility. The model can be used by a contractor to assess the possibility of meeting the contractual requirements prior to bidding.

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Trietsch and Baker (2012), stating that PERT underestimates project duration, developed PERT 21 which is an amendment of PERT through modeling processing times as lognormal distributions with linear association, and using historical data to calibrate the estimates. Using such distributions allows balancing the criticalities of project activities and controlling projects through monitoring that stochastic balance. Lu (2002) presented an approach that embeds artificial neural network (ANN) into simulation to find proper distributions for activity durations. The beta distribution is more accurately and efficiently fitted using the developed model. Kirytopoulos et. al (2008) showed the importance of using historical data and selecting the right distributions for estimating project duration using both PERT and Monte Carlo Simulation (MCS). Results show that MCS is superior to PERT as it reveals the difference in results when using accurate historical information to select distributions versus when such information is not available.

Lee et. al (2010) presented a stochastic simulation model for scheduling construction operations through conducting sensitivity analysis of various resource combinations and feeding the optimal combination into the project schedule to execute simulation. Ingalls and Morrice (2004) addressed PERT with resource problems using the qualitative simulation graph methodology (QSGM). The approach carries the potential of generating solid scheduling decisions that eliminate the need to rerun the schedule due to the differences between the actual and estimated activity durations. Poshdar et. al (2014) carried out a study to explore a suitable probability distribution function to be used in case a beta distribution cannot be used. Results show that a Burr distribution is a more accurate distribution to model processes that have variability levels between 100% and 150%. Karabulut (2017) compared the results from applying traditional CPM-PERT with those from performing risk analysis using Monte Carlo Simulation (MCS). It has been shown that studying the risk through MCS gives more realistic outcomes. In a similar study, Hendradewa (2019) assessed the schedule risks of a project using MCS and CPM-PERT. It was concluded that the simulation results can be projected to evaluate the potential of completing the project in a timely manner as per the given schedule. Forcael et al. (2018) presented a DES model to schedule activities on a construction project and compared the obtained durations to the ones generated by PERT. The results show that the model could be adopted as a valid management tool to address the effect of variability in construction processes.

Variability in Construction

As for variability in construction, several studies investigated the reasons and remedies of variability. Wambeke et. al (2011) classified the causes of variations into eight main categories which are: prerequisite work, labor force, detailed design/working method, material and components, tools and equipment, management/supervision/information flow, work/job site conditions, and weather or external conditions. Wambeke et. al (2012) presented a study where a risk assessment matrix is integrated with LPS, with the aim of reducing or eliminating variations in the task durations. The results show that the productivity performance is 35% higher on projects adopting LPS method than on traditional projects. Thomas et. al (2002) investigated the effect of reducing output variability on improving labor performance. The results show that the project performance is correlated with variability in labor productivity. Palaniappan et. al (2007) presented a special purpose simulation (SPS) template for workflow analysis with the aim of reducing variability. The template helps the modeler spend less time on assessing different outputs involved in workflow analysis. Garcia-Lopez and Fischer (2016) concluded

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that the current methods for managing workflow are insufficient for understanding variability in activity flows. They proposed a theoretical workflow model to help managers analyze the workflow variability and its effect on downstream activities. Arashpour and Arashpour (2015) presented a study that helps managers on construction projects stabilize the workflow in order to improve productivity and performance. Brodetskaia et al. (2013) applied DES in implementing pull production control that aims at improving the stability of project workflow through mitigating flow variability. The method is presented at the operational level where pending work packages undergo real-time prioritization, and crew assignments and production capacities of trades undergo daily regulation. The results shed the light on the importance of performing dynamic control when allocating the available production resources. Other simulation applications in mitigating variability in construction include employing MCS in predicting dimensional variability when producing assemblies in offsite construction (Rausch et al., 2019). Results from this study reveal that MCS method produces more accurate values as compared to traditional methods that are either overly conservative or overly ambitious.

Waste in Construction

Due to variability, construction processes accommodate different types of waste, some of which are hidden. Two different types of waste are tackled in this research. Type I is what Lindhard (2014) refers to as “waiting time”; it is the waste that emerges when an activity is completed late, and it represents the time that the next work crew must wait until they are able to start working on their activity. According to Tommelein et al. (1999), such wasted time during which a crew fails to achieve its production capacity due to constraints, results in lost productivity and leads to schedule delays and cost overruns. Type II is what Lindhard (2014) refers to as “variation gaps”. It is the waste that emerges when an activity has the potential to be completed ahead of schedule, but due to Parkinson’s law stating that “work expands to fill the time available” (Parkinson, 1957), the activity is completed on schedule, or in any case where the activity actually ends up being completed ahead of schedule. In both cases the variation gap represents the time gap from when the activity is completed or theoretically could have been completed until the following activity is started. Parkinson’s Law does not affect the simulation results since a part of the variation gap is attributed to it, but the other part will always be there; thus, the total size of the variation gap is preserved. Parkinson’s Law affects the variation gap in the sense that the contractor will choose their next tasks based on the available time, thus, prioritizing another task thinking it can be completed faster, thereby introducing a variation gap before the task and eliminating the potential of finishing an activity ahead of time. Type II waste is a hidden waste in the production system, and it implies missing an opportunity for enhancing schedule performance. Furthermore, the relationship between teams matters. For example, let’s assume there is a foundation team working in area 1 on a project that is divided into sub-areas. If this team anticipates that they will finish early, they might not communicate it, or might communicate it very late, because they prefer to keep their buffer internal to be used for their advantage by increasing their earned value. Trades seek to optimize their own work; they don’t care about the continuity of the project workflow (Lindhard and Wandahl, 2014). By then, the following team (structural team for instance) will not have enough time to mobilize immediately to area 1. In short, if a trade is late in one area but has a buffer in another area, this buffer might be used by the trade for their flexibility. Therefore, optimizing one trade’s work at the expense of optimizing the project flow denotes optimizing

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the parts instead of the whole, which increases cycle time, results in growing buffers and increased waiting time (Hopp and Spearman, 2000), and might destroy the production system's performance (Tommelein et al., 1999).

Problem Statement & Contributions

The existing scheduling techniques have failed to efficiently tackle the inconstant nature of processes in the construction industry, particularly the uncertainties pertinent to the duration needed to accomplish a task. This kind of random variation, referred to as natural variability that stems from inconstancies between machines, materials and operators, labor availability, rework, random detentions, etc., has been little incorporated in traditional construction scheduling processes (Forcael et al., 2018). The established practice has seen assigning activity durations using a deterministic approach, which is far from the reality of construction (Forcael et al., 2018). For instance, previous research applied PERT as a deterministic method by sampling a fixed mean for durations without generating a probability distribution that is sampled stochastically. Moreover, running construction projects the traditional way encourages aggregating pending works between consecutive crews into wasteful buffers, as a response to mitigating uncertainties and variabilities in flow (Brodetskaia et al., 2013). Simulation has been applied to analyze schedule overruns, modify the activity durations, model productivity, achieve dynamic planning, etc. Nonetheless, no research has addressed the issue of identifying the activities with high variability that have the most influence on the schedule, coupled with the network characteristics, or addressed the impact of such activities on waiting time and variation gaps. Failure to address variability reflects negatively on project performance as the variability has a detrimental impact on productivity, and this is clearly shown in the simulation of Parade of Trades (Tommelein et al., 1999). All that being said, this study presents a DES model that incorporates variability into scheduling through a stochastic approach to help practitioners assess the effect of mitigating such variability on the project performance pertaining to meeting the specified deadline, avoiding trade idling, and enhancing schedule performance. In each simulation run, a random duration, which is based on a stochastic distribution, is generated for each activity, reflecting thereby the reality of construction. Depending on the distribution parameters, the variability of an activity can be assessed as high or low, and thus, the activity's impact on the overall schedule can be determined. Because there are precedence relationships between activities, variability in one activity influences the start dates of dependent ones, which in turn impacts waiting time and variation gap wastes. Through DES, which respects the network characteristic represented by precedence relationships, calculation of waiting time and variation gap wastes is conducted; that is not addressed in other studies. This way, not only delays are addressed but diagnostics into the production system are performed. The model helps determine the criticality of paths, identifying thereby the activities that should be deliberated on to avoid productivity loss. The model is further elaborated in the following section.

RESEARCH METHODOLOGY

A stochastic simulation approach is adopted as a methodology to achieve the goals of this study. Generally, in the construction field, simulation is used for developing and experimenting with computer-based exemplifications of real situations and construction systems with the aim of understanding the

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underlying behavior. The application of simulation in construction research has seen significant growth over the last two decades (AbouRizk, 2010). Using simulation, one can experiment with various scenarios and different conditions to make informative decisions without having to build the model and waste time, money, and resources. Specifically, the DES modeling approach attempts at modeling the behavior of a system while its state advances over time by tracking the system events, which are perceived as the initiators of change in the system state (Alvanchi et al., 2011). DES is most widely used to model systems which are viewed as queuing networks, it has been applied to model various construction operations (Moradi et al., 2015), and it has proven over many years the ability to handle variability which is understood as a quality of non-uniformity and has a close tie with the randomness of a phenomenon (Forcael et al., 2018). Entities which are the individual objects go through a series of activities where they wait in queues (Moradi et al., 2015). Generally, DES modules produce two types of data which are observational and time-persistent data. Observational data, which are collected through the occurrence of relevant events, can be counted and averaged to generate probability distributions. The time-persistent value is one that remains valid from the event time that generated it up until the following event that changes it (Moradi et al., 2015). All that being said, and since the most convenient way to model a process or a queuing network is using DES, and CPM is a series of processes that is similar to a queuing network, the durations assigned for individual activities in this study are modeled as independent stochastic discrete events. Each duration, referred to as work effort, represents the time taken to complete the task, expressed in man-hours. It is assumed for this study that the variation in the required man-hours of a specific task comes from variation in labor productivity (Arashpour and Arashpour, 2015). Also, for simplicity reasons, the number of laborers or the manning for each task is set to one in the simulation. If the manning is changed, the new duration can be calculated through dividing the required man-hours by the number of laborers working on the task, and it can be adjusted for any expected productivity loss caused by overmanning (Singh, 2003).

For each activity, the best-case duration, most-likely duration, and worst-case duration have been evaluated. Based on this three-point estimate, the variance and mean duration is calculated using classic PERT formulas. The mean duration is considered the expected duration of the activity and the value to be used in a deterministic schedule. The calculation of the needed work effort is based on a beta distribution function. A beta distribution is selected since previous studies have found this distribution as the most suitable for this problem (Abourizk et. al, 1994; Farid and Koning, 1994; Nguyen et. al, 2013; Lindhard et. al, 2019). Ideally, scheduling should be based on the entire distribution functions, but in practice, such distributions often are difficult and costly to determine (Che-Hao et al., 1995). A point estimate is a simple way to map the probability despite the lack of knowledge regarding the exact distribution function (Tsai, Franceschini, 2005). A beta distribution can be approximated based on its maximum, mode, and minimum values (Moitra 1990). Therefore, the simulation uses a three-point estimate [best-case (a_i), most-likely (m_i), worst-case (b_i)] to determine the beta distribution. This three-point estimate approach is similar to the one used in PERT. However, unlike PERT which is a deterministic approach where the three values are used to create a fixed mean (Lee and Arditi, 2006; Khamooshi and Cioffi, 2013), the three values are used to generate a probability distribution which is sampled stochastically. The three-point estimates (a_i , m_i , b_i) define the range and mode of the considered beta distribution, meaning that outside the interval [a_i , b_i], the distributions are identically zero and the modal value is located at m_i . The

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shape parameters of the PERT-beta distributions are obtained from (a_i, m_i, b_i) by following the procedure presented in (Davis, 2008) as stated in expressions 1 and 2.

$$\alpha = \frac{2(b + 4m - 5a)}{3(b - a)} \times \left(1 + 4 \frac{(m - a)(b - m)}{(b - a)^2} \right) \quad (1)$$

$$\beta = \frac{2(5b - 4m - a)}{3(b - a)} \times \left(1 + 4 \frac{(m - a)(b - m)}{(b - a)^2} \right) \quad (2)$$

Having obtained the PERT-beta shape parameters, the associated first two moments of the obtained distributions can be calculated as stated in expressions 3 and 4.

$$\mu \stackrel{\text{def}}{=} \frac{a + 4m + b}{6} \equiv a + \frac{\alpha}{\alpha + \beta} \times (b - a) \quad (3)$$

$$\sigma^2 \stackrel{\text{def}}{=} \frac{b - a}{6} \equiv \frac{\alpha}{\alpha + \beta} \times \frac{\beta}{\alpha + \beta} \times \frac{(b - a)^2}{\alpha + \beta + 1} \quad (4)$$

Utilizing expressions 1 and 2 ensures that the formulas for the moments of the general beta distribution are equivalent to the special case of the PERT-beta distribution. Having obtained the shape parameters, the corresponding beta distributions modelling each activity's duration can be sampled stochastically in various software, and the network structure can simply be encoded algebraically in that setting. Here, MATLAB R2020a has been used.

In the DES, each stochastically sampled duration of an activity is compared to the expected duration (mean duration), and the larger of the two represents the time span from starting the task until the successor task is commenced. Even if an activity is completed ahead of schedule, it is expected that the work crew of the next activity follows the given schedule and will not start working after the task is completed ahead of schedule (Lindhard, 2014). Based on the defined work crew, the duration of each activity is calculated, as is the duration of the entire project. It is assumed that the schedule is updated when a delay is encountered in the execution of the preceding activities.

Based on these assumptions, the duration of the entire project is calculated, and the paths exceeding the deadline are identified. Afterwards, the activities contributing to overruns are identified through calculating the correlation between each activity's duration and the projects duration. Two common correlation coefficients that are widely adopted in statistical analysis are Spearman and Pearson correlation coefficients. Spearman coefficient measures the relationship quality between two factors. It is used mainly to describe monotonic relationships, or in other words, relationships in which variables tend to move in the same or opposite directions with possibly inconstant rates, whereas for relationships that are linear with constant rates, Pearson coefficient is used (Thirumalai et al., 2017). Thus, being more inclusive, the Spearman correlation coefficient between individual activity workload and the total project duration is used to identify the activities contributing to the overruns. The methodology is summarized in Fig. 1.

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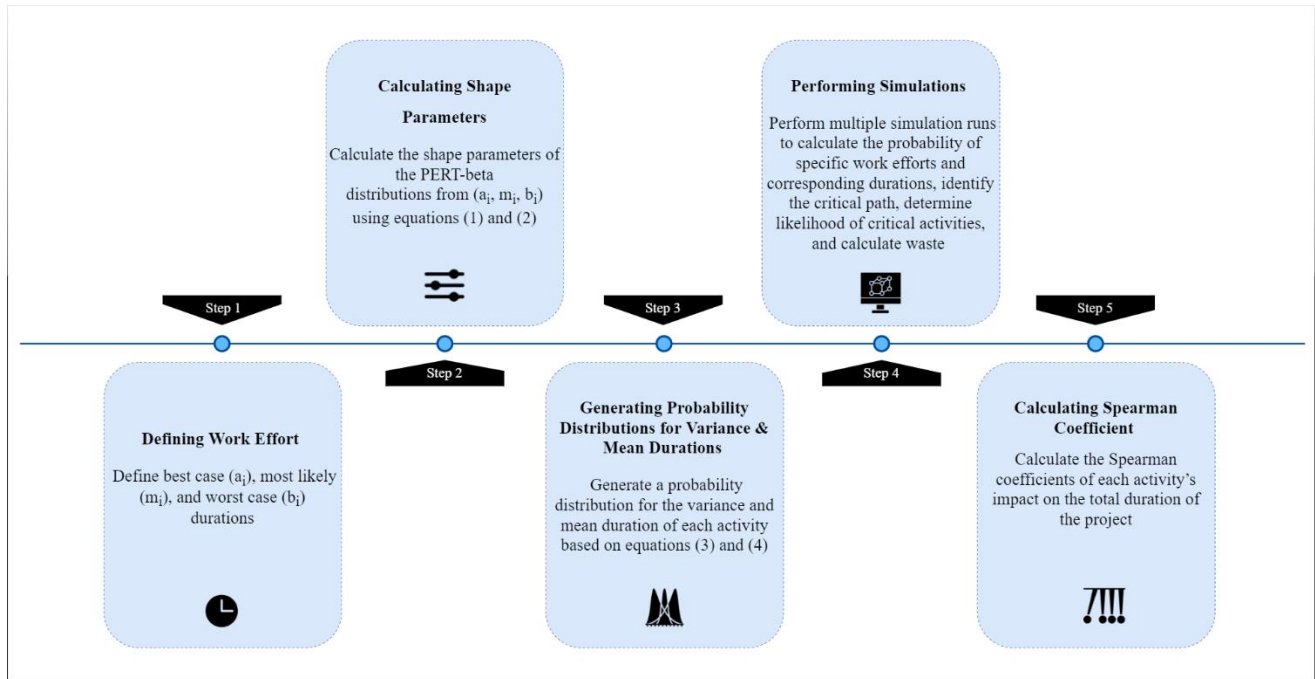


Fig. 1. Research methodology.

APPLICATION EXAMPLE

The stochastic discrete event network is encoded in a script in MATLAB. The script contains information regarding the activities, the network diagram, and the defined deadline. The number of activities is defined together with the work effort (e.g. best-case, most-likely and worst-case). The network diagram in Fig. 2 shows the precedence relationships of activities. The network encoded in the simulation and the corresponding activity durations are adopted from Nicholas and Steyn (2017). The network diagram has five paths, and the number of activities in each path range from two to five. The paths are shown in Table 1. The gray path in Fig. 2 is P3. According to PERT, P3 is the critical path, while P1 and P5 are near-critical paths. Fard et al. (2017) explain that “the concept of near-criticality is based on the CPM calculations that evaluate how close any activity in a logic network is to becoming a critical-path activity.” The latter is known to be the longest continuous activity sequence that forms the minimum project duration. A sequence of activities that is less critical is considered a near-critical path (Fard et al., 2017). Table 2 contains the applied three-point estimates, along with the activities’ calculated mean durations and variances. Finally, the deadline is the number of days allowed from start to finish in the network diagram. The deadline is important, because it affects the number of overruns. As the deadline becomes tighter, the more important it becomes to identify the critical paths and activities. In the simulation, the deadline has been defined as 33 days.

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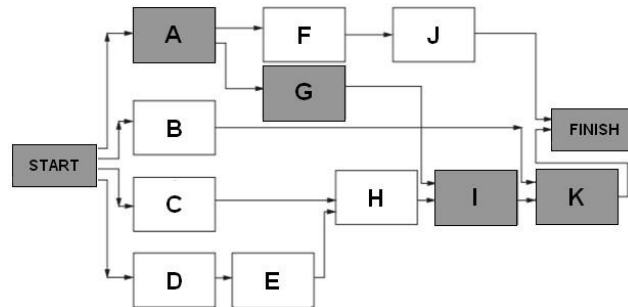


Fig. 2. The network diagram, based on Nicholas and Steyn (2017).

Table 1. The five paths in the network diagram

Path	Activity	Summed mean duration	Summed variance
P1	A-F-J	29	8.22
P2	B-K	20	17.00
P3	A-G-I-K	31	9.66
P4	C-H-I-K	19	4.89
P5	D-E-H-I-K	30	13.89

Table 2. The basic characteristics to the activities (A-K) in the network diagram

Activity	Best Case (a)	Most Likely (m)	Worst Case (b)	Variance (V)	Mean (t_e)
A	7	14	21	5.44	14
B	6	15	30	16.00	16
C	2	5	8	1.00	5
D	1	4	7	1.00	4
E	3	12	21	9.00	12
F	7	10	13	1.00	10
G	5	7	9	.44	7
H	2	4	6	.11	4
I	1	6	11	2.78	6
J	1	5	9	1.78	5
K	1	4	7	1.00	4

The subsequent analysis revealed activities A, E, I, and K as being critical. Because the distribution skewness affects delays, it was decided to use symmetrical distributions instead of right-skewed ones in order to show variability in the best-case scenarios. Right-skewed distributions amplify the effect and show more extreme results. Using symmetrical distributions reveals how, even in best-case scenarios, variability is detrimental in the production system. Therefore, a symmetrical distribution was used for the four critical activities.

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RESULTS

In the simulation, the required work effort of each activity is generated using the identified beta distribution. The strength of simulation is that various scenarios can be built to reflect a close to real-life situation. By conducting multiple simulation runs, the probabilities of a specific work effort and the corresponding duration can be calculated each time. Thus, each simulation run results in identifying the project duration, the critical path, the waste, and the likelihood of critical activities.

First, the network shown in Fig. 2 is analyzed by simulating the completion time of the project. For the initial simulation, the activities' estimated min, mode, and max durations are applied. Based on the simulation, the overall durations and waste are calculated.

Based on the initial durations, the likelihood of exceeding the deadline is estimated to be 64.7%, while the mean duration is estimated to be 34.03 days. The high likelihood of exceeding the deadline is an indication of the importance of incorporating the network complexity, especially when having near critical paths. In accordance with PERT calculations, where only the critical path is considered, the mean duration is 31 days. The simulation also allows for calculating waiting time and variation gaps. The mean waiting time is 4.65 days and the mean variation gap is 4.86 days. The cumulative distribution function of the work effort, the probability distributions of waiting time, and the variation gaps are shown in Fig. 3.

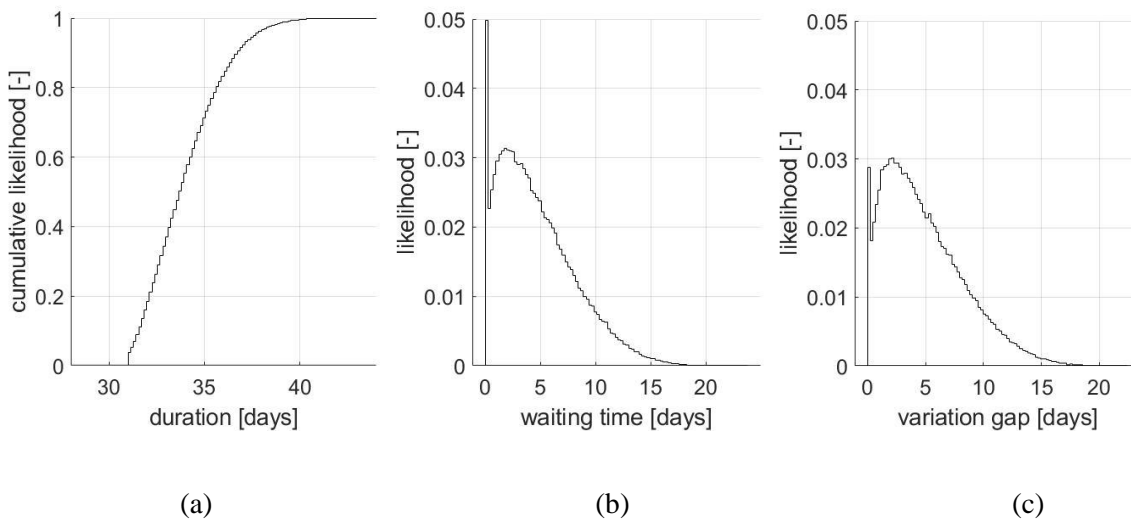


Fig. 3. Project Performance: (a) The cumulative distribution function of the work effort needed to complete the project. (b) The probability distribution related to waiting time. (c) The probability distribution related to the variation gap.

The critical path method is considered an approach that helps create a deeper understanding of what activities cause the risk of delay and waste in the project. The initial network included five different paths as shown in Table 2. Based on the simulation, the likelihood of time overrun caused by each path is calculated. The simulation reveal that path P3 is the path most likely to cause delay with a likelihood of 50.13%. This is expectable, since P3 in Table 1 has the longest mean duration and is thus considered to be

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the critical path. More interestingly, P3 is found to be the critical path only in 60.8% of the simulations, while P1 is found critical in 3.7% and P5 is found critical in 35.5%. The likelihood of a path to overrun the deadline is shown in Fig. 4 together with the relative number of critical overruns per path.

As a project manager (PM), it is important to realize that, although P3 is considered the critical path, other paths might actually be critical. Thus, P3 is only the most probable critical path. Surely, the PM needs to be aware of activities on the critical path, but keeping a narrow focus only on P3 could, in approximately one out of three cases, have fatal consequences on the project's performance. Of course, the PM could use managerial strategies such as adding more men or using overtime to avoid project delays, but this is costly and it affects both the economic aspect and quality performance of the project (Noyce and Hanna, 1998; Li et. al, 2000).

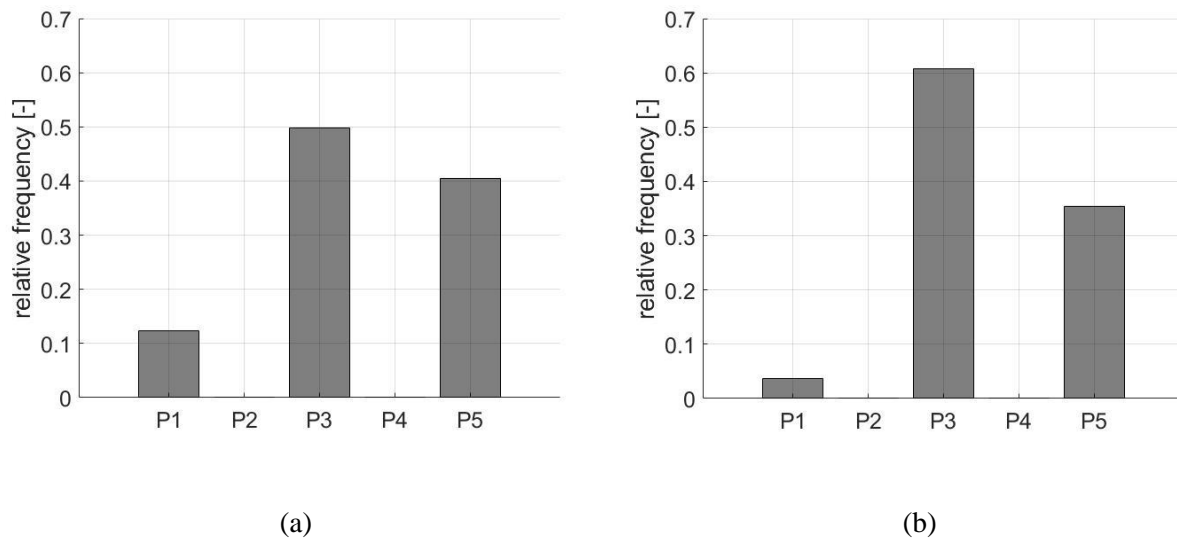


Fig. 4. Path analysis: (a) The likelihood of path exceeding deadline. (b) The number of critical overruns per path.

One approach to reduce the risk of overrun is to reduce variability (Tommelein et. al, 1999; Thomas et. al, 2002; Thomas et. al, 2003). Reduced variability will increase labor productivity, and it can be achieved in a number of different ways. This could be done through using more skilled and experienced workers, putting more effort in introducing and explaining the task and in improving supervision, improving the quality of tools and material, and reducing the risk of breakdowns (Thomas et. al, 1986; Thomas and Yiakoumls, 1987; Tsehayae and Fayek, 2016). Due to limited resources, the PM often cannot remove variability from all activities; instead, he/she needs to identify the activities that are most decisive in relation to schedule performance.

A strategic reduction of variance

In the following, the study examines which activities in the network diagram variability have the highest impact on schedule performance. Based on the initial simulation, the Spearman coefficient of each activity's impact on the total duration is calculated. Thus, the duration of each activity is compared to the

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project's duration, and by carrying out multiple simulation runs, a correlation between the two can be calculated. The Spearman correlation coefficients range from -1 to +1, where 0 indicates no association and -1 or +1 indicates a perfect correlation. The Spearman coefficients are presented in Table 3.

Table 3. The Spearman coefficients showing each activity's impact on total duration

A	B	C	D	E	F	G	H	I	J	K
0.48	0.00	0.00	0.10	0.46	0.02	0.11	0.03	0.43	0.02	0.26

As the table above shows, the activities having the largest correlation coefficients and correlating best with total duration are activities A, E, I and K. This implies that reducing the variability in these activities leads to reduced variability in the overall schedule. Therefore, these activities are examined in the simulation. Here, the best-case and worst-case duration estimates of the identified activities are reduced percentwise, meaning that the ranges of the associated distributions are confined to smaller intervals, thus reducing the related variances.

A,E,I, and K are central activities in paths P3 and P5. The two paths are the ones with the highest likelihood of exceeding the deadline. They share several similarities; both paths contain an activity with high variance (either A or E), and they share two activities with lower variance (I and K). The analysis takes outset in two different scenarios that are simulated and compared. In the two scenarios, the variability is only reduced in two activities. In the first scenario, the variability is reduced in activities A and E. Activity A lies on P3 while activity E lies on P5. The two activities are the ones with the highest amount of variability. In the second scenario, the variability is reduced in activities I and K. Activities I and K both lie on P3 and on P5, but the variability in these activities is much smaller than this in A and E.

In both scenarios, the variability is reduced in four steps from no reduction to a reduction by 20, 40, 60, and 80 %. The effect on the duration is showed in Fig. 5 below that plots the cumulative distribution function. With an 80% reduction, the mean duration in scenario 1 is reduced by 2.7%, while the likelihood of completing the project on schedule is increased to 51.9%. In scenario 2, the mean duration is reduced by 1.8%, while the likelihood of completing the project on schedule is also increased to 47.7%. Thus, in this specific case, the biggest effect is achieved by reducing variability in activities A and E. It is important to note that reducing the variability in activity A also has a small double effect. Besides of affecting path P3, a reduction in activity A leads to a reduction in the duration of path P1 which is a near critical path. In the initial simulation, P1 is found critical in 3.7% of the simulations.

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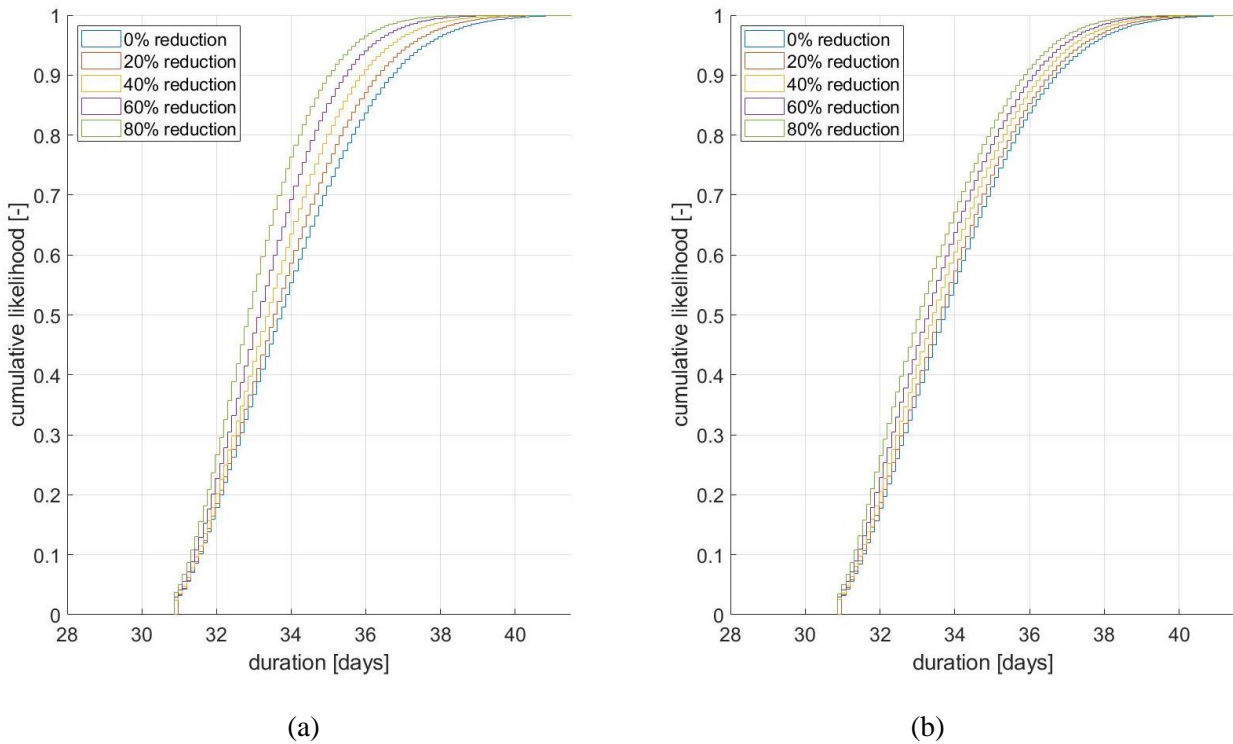


Fig. 5. Effect of reducing variability on the cumulative density function of the work effort needed to complete the project: (a) The effects of reducing variability of activities A and E. (b) The effects of reducing variability of activities I and K.

Fig. 6 shows the waiting time caused by the delayed activities. With an 80% reduction, the mean waiting time in scenario 1 is reduced by 30.3% while in scenario 2 is reduced by 6.5%. The difference in the effect on waiting time is significant. Activities I and K have a doubled effect on duration because they affect both paths P3 and P5, while the waiting time after activities I and K only has an effect on the following activity. Therefore, the result is simply given by the fact that when reducing variability percentwise, more variability is removed from activity A and E.

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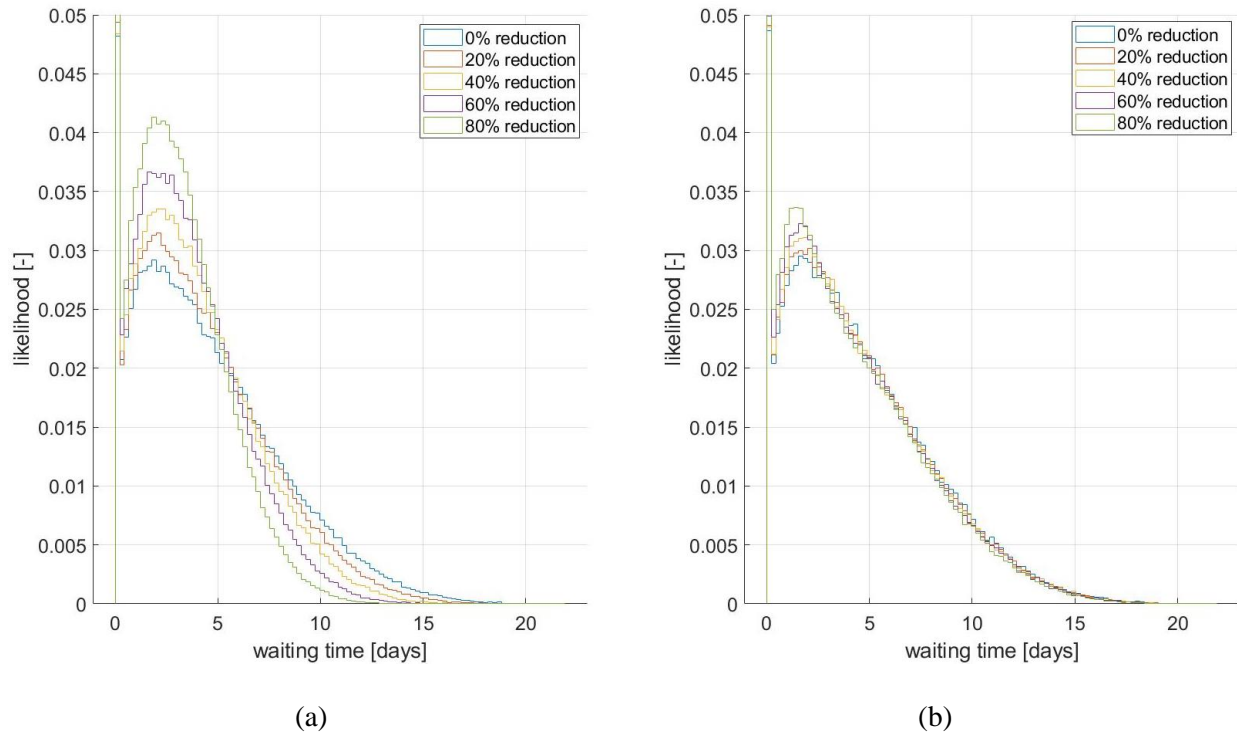


Fig. 6. Probability density function related to waiting time: (a) The effects of reducing variability of activities A and E. (b) The effects of reducing variability of activities I and K.

In Fig. 7, the variation gap is depicted. With an 80 % reduction, the mean variation gap in scenario 1 is reduced by 28.6% while in scenario 2 is reduced by 6.4%. The comparison shows a significant difference between reductions in variation gaps in the two scenarios. Once again, the difference is occurring because the variation gap after activities I and K only has an effect on the following activities. Therefore, the result is due to the fact that more variation is removed from activity A and E.

The different paths can help identify what happens in the network. The likelihood of deadline overrun per path is shown in Fig. 8. In scenario 1, the effect of the reduced variances is a reduced likelihood of delay in paths P1 (75.9 % reduction), P3 (22.1 % reduction), and P5 (31.4 % reduction). In scenario 2, the effect of the reduced variances is a reduced likelihood of delay in paths P3 (32.2 % reduction) and P5 (29.2 % reduction).

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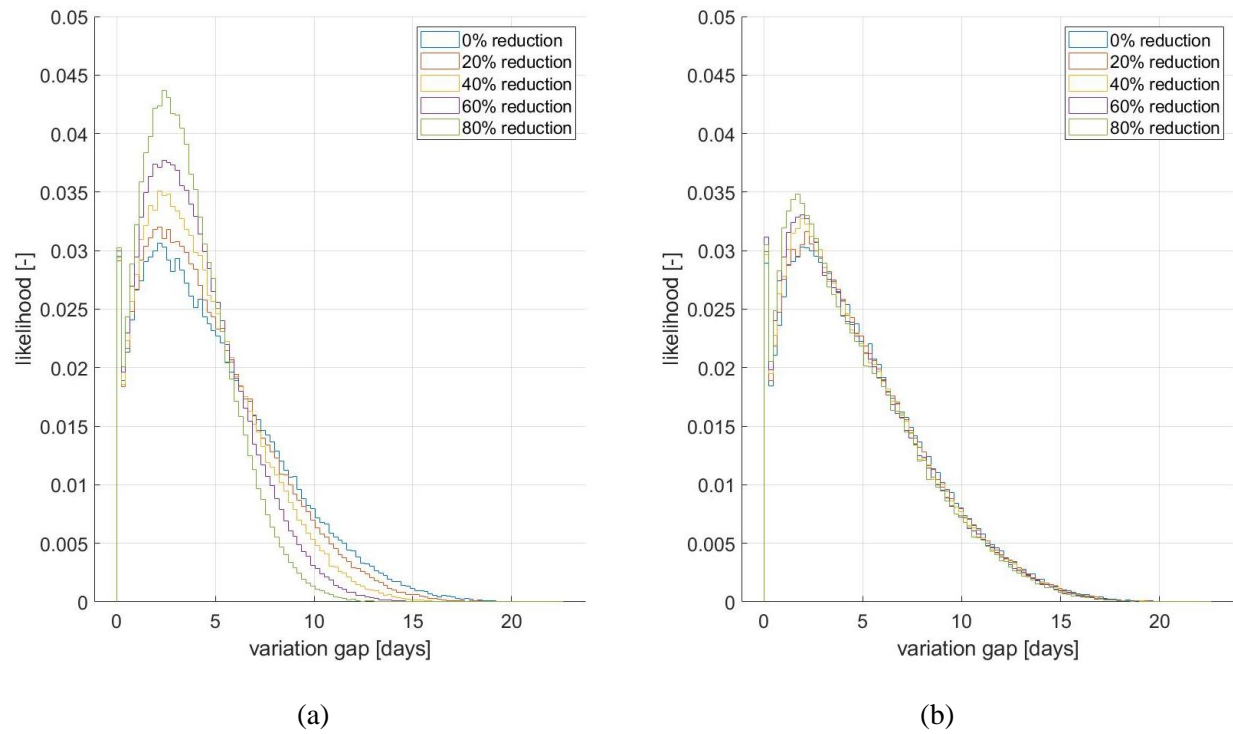


Fig. 7. Probability density function related to variation gaps: (a) The effects of reducing variability of activities A and E. (b) The effects of reducing variability of activities I and K.

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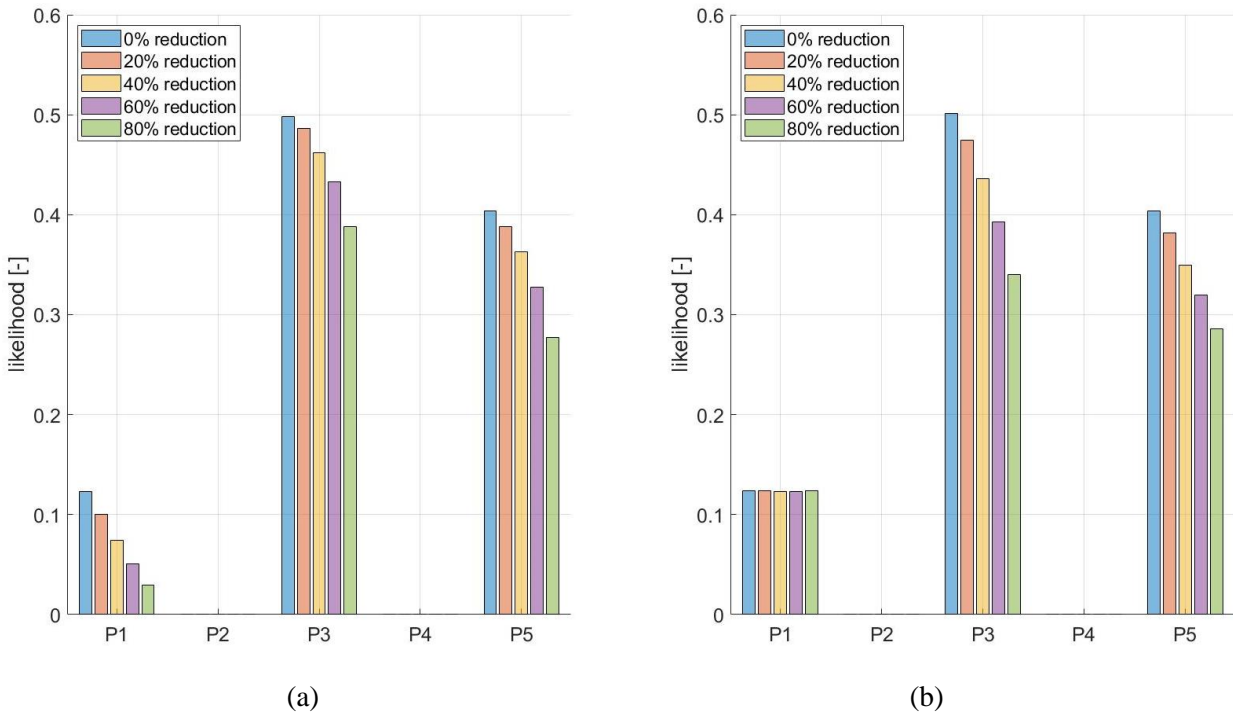


Fig. 8. The paths' likelihood of exceeding deadline: (a) The effects of reducing variability of activities A and E. (b) The effects of reducing variability of activities I and K.

In Fig. 9, the critical overruns per path is depicted. The critical overruns refers to the longest overrun, therefore, there is an interrelationship between the paths. If the duration of P3 is reduced, the likelihood of P1 or P5 to become critical increases. In scenario 1, the likelihood of P3 becoming critical is increased while this of P5 is decreased. The reason is that the variability of activity A is smaller than the variability of activity E. Thus, an 80% reduction in variability of activity E has a higher impact on duration. To get the optimal effect of reducing variability, the reduced variability on both path should be close to identical. In scenario 2, the likelihood of P1 becoming critical is increased, while this of P3 is decreased. Because a reduction in the variance of activities I and K reduces the durations of paths P3 and P5, consequently, the likelihood of activity P1 to become critical increases.

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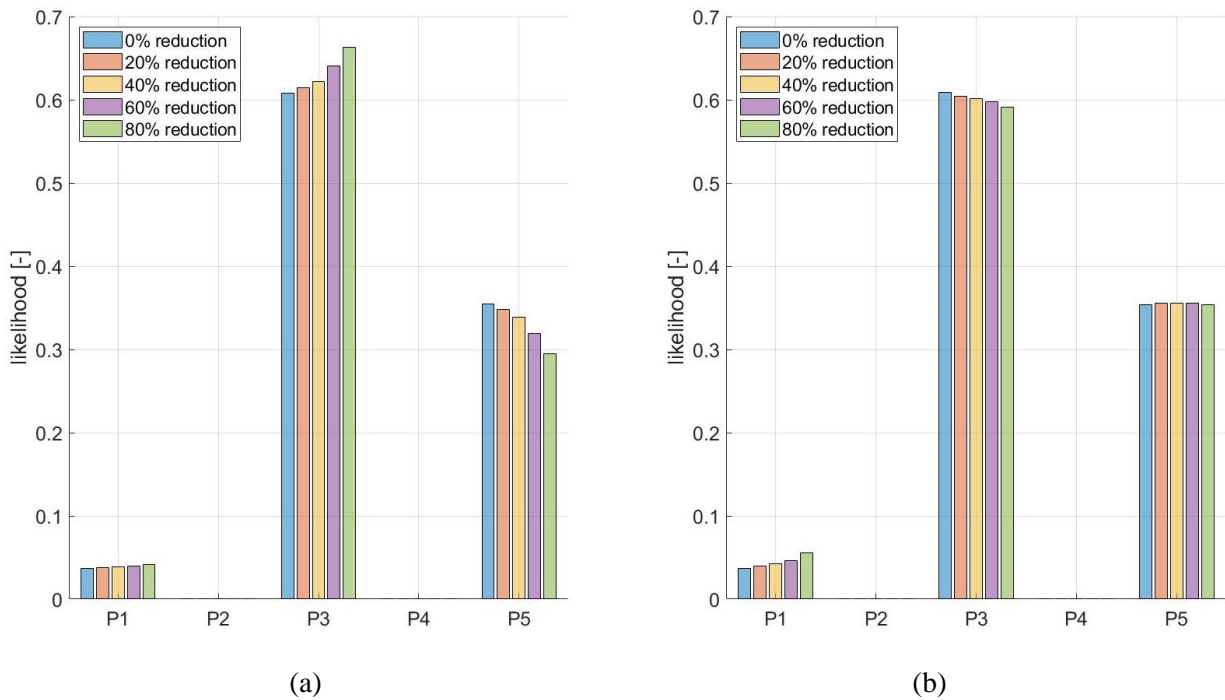


Fig. 9. The relative number of critical overruns per path: (a) The effects of reducing variability of activities A and E. (b) The effects of reducing variability of activities I and K.

STUDY IMPLICATIONS

It is important to decide where to focus the effort of reducing variability. It could be either on the activities that have high variability and that belong to the critical/near critical paths, or on the activities belonging to multiple paths. To begin with, although some activities may have the highest variation in duration, they might not be important to study in case they don't lay on the critical path. Moreover, the simulation results confirm that the Spearman coefficient is a suitable metric to predict where to focus the effort of reducing variability.

It is important to note that the Spearman coefficient cannot stand alone; it should be looked at along with the network characteristics. For instance, activity A has the highest Spearman coefficient, but reducing variability in activity A alone will have very little impact on reducing the project duration because path P5 will directly become critical. Thus, variability in Activity A should be reduced together with activity E's in order to reduce the overall variability. Optimal results are achieved through reducing variability in all the activities lying on the critical and near critical paths. On the other hand, if the PM should choose a single activity to focus on, it should be activity I. Although both activities I and K belong to paths P3 and P5, activity I has the largest Spearman coefficient; thus, it has the highest impact on duration. The combined effect on duration of reducing variability in either activities A and E, or in I and K, is case specific; it depends on the summed variation, and it can be identified by looking at which activities have

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the highest Spearman coefficient. Finally, waiting time, variation gaps, and other forms of waste are transferred only to the successive activities.

As shown, the critical path is not critical in all cases, and the project duration is affected by network complexity and near critical paths. Also, the likelihood of a path becoming critical is affected by variability and can be moderated by reducing variability in both the critical path itself and in other near critical paths.

DISCUSSION

Variability is a chronic disease in construction. Deterministic planning alone prior to starting construction is not enough to mitigate variability. Indeed, deterministic scheduling techniques and modelling tools such as CPM seem to have acquired wide acceptance in the construction industry; nevertheless, it is apparent that there exists a need for developing better tools to be used by construction practitioners in managing production and work flow (Tommelein et al., 1999). Aiming at providing a solid foundation for an accurate planning approach, this research presents a thorough discussion on different lean concepts to be integrated with this study in future work. Put simply, this study suggests bridging between CPM and Lean Construction through attacking variability and eliminating waste. A study done by Huber and Reiser (2003) demonstrates how LPS and CPM scheduling can be two complementary processes enhancing work and crew flows in a lean management approach. Because CPM scheduling continues to be a craft skill with a vast variety of utilizations and methodologies, there is enough flexibility to twist it into a system that improves and supports the Lean Construction process (Huber and Reiser, 2003). Lean Construction strategies have been developed to reduce workflow variability and increase predictability of various workloads in the overall production systems through better coordination of work between participants (Howell et. al, 2004). More precisely, reducing variability is one of the primary objectives of Lean Construction that is achieved through implementing the Last Planner System (LPS) (Ballard, 2000). For instance, a study conducted by Erol et. al (2017) indicates that integrating lean principles helps decrease the variability; in particular, a decrease in the standard deviation of the total project by 20.18 percent was achieved.

LPS is a construction production planning and control system that seeks easing variations in workflow, reducing uncertainties in operations, and providing planning foresight (Hamzeh et al., 2012). LPS plays a vital role in addressing variability via production control, where it extends the analysis of constraints throughout various stages of project lifecycle and on different levels of project hierarchy (Brodetskaia et al., 2013). Four different chronological spans are addressed which are master scheduling, pull/phase scheduling, lookahead planning, and weekly work planning. Tasks would be filtered into a workable backlog, which contributes to stabilizing the flow with regard to planning predictability and reliability (Brodetskaia et al., 2013). Therefore, integrating LPS with this study provides a comprehensive containment of variability. Results from this study identify highly variable activities that could be studied by LPS in more detail and assigned higher priorities when removing constraints. Correct implementation of look ahead planning comprises planning in greater detail as approaching activity execution, including those who actually perform the work in the planning process, identifying and removing constraints for upcoming activities, and making reliable promises among project participants (Hamzeh et. al, 2012).

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Involving the people who possess the skills and the practical know-how in the planning process is way more effective than relying on the theoretical knowledge of the planner alone. Both types of waste, waiting time and variation gaps, could be addressed through LPS. Variation gaps imply a wasted opportunity for enhancing the project schedule. Even though no delay occurred, this denotes a hidden waste in the production system. Also, waiting time is one of the seven identified waste types that are detrimental in the production system (Brodetskaia et al., 2013). Unfavorable consequences resulting from such waste include interruptions in flow of resources that lead to productivity loss and rework. Therefore, injecting diverse expertise into the planning process during the make-ready phase implies a more realistic schedule that is characterized by less flow variability and lower wasted time. This shall also lead to potential decrease in process time variability by means of improving the work process. An integrated planning process, combining different aspects of knowledge and levels of expertise, provides much clarity on how the work will be carried out, resulting in less variability in process time.

In summary, attempting to reduce variability by employing LPS is seeking the increase of control over production. A different approach would be to inject some flexibility into the production system. This can be achieved through buffering in general, e.g. increasing capacity or overtime. Whilst more working hours imply opportunity to achieve more work, the use of overtime should be carefully carried out. This is because performance is negatively affected during overtime due to physical and mental fatigue (Alvanchi et. al, 2012). To mitigate such issues, proper overtime planning such as optimal scheduling of overtime might be adopted.

It is worth noting that the integration of CPM and LPS happens when they inform each other of the planning reliability (percentage of completed tasks out of those planned) and pull intensity (total float) as the project progresses through planning and re-planning in periodic cycles (Huber and Reiser, 2003). The planning process would be crew-centric and focused on ensuring a smooth and stable crew flow in a made ready space.

On the other hand, several approaches could be followed to reduce process time variability. For instance, using a pull system, which is closely associated with Just-In-Time (JIT), is one method to reduce variability in cycle time (Hamzeh, 2009). JIT could be simply defined as moving the material to the necessary place at the necessary time. Pull planning is carried out in recurrent sessions called "pull sessions" that aim at rendering activities ready by eliminating constraints and making sure that prerequisites are available according to the actual site demands (Hamzeh et al., 2012). This is achieved through the Kanban signaling mechanism of JIT. This mechanism is used to notify the production when to produce in order to meet actual consumption. Using cards called Kanban cards, different types of information including pickup, transfer, and production information are communicated within the production system. Another factor that indirectly affects process time durations is site layout. Site layout refers to locations of temporary offices, worker rest areas, crane locations, storage and workshop areas, access points and access roads, and other critical features (Small and Baqer, 2016). Poor planning of site layouts eases the opportunities for variability to emerge. Adversely, proper and efficient site layout implies reduction in time wasted on excess movement of material, increase in labour productivity, improvement in worker safety, and positively impacting construction quality (Small and Baqer, 2016).

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CONCLUSIONS, LIMITATIONS, AND RECOMMENDATIONS

Variability, which stems from many resource input flows, should be well studied in construction projects as it can lead to reduced performance including productivity loss and project delays. The existing scheduling techniques fall short of addressing variability and rely on deterministic approaches. This study demonstrated how simulation could be employed in scheduling to pinpoint activities with high variability and that strongly affect the overall schedule. A Discrete Event Simulation model is built to investigate the possible variability in activity durations which are modeled using a Beta distribution and PERT assumptions. Two waste types are identified which are waiting time and variation gaps; the former results from running behind the scheduled finish date and the latter results when the activity has the potential of finishing ahead of schedule but still be completed on schedule. Both types of waste are detrimental to the production system.

The study aims at providing practitioners with an approach for managing waste in a production system by assessing the effect of reducing variability on project performance from the aspects of meeting the deadline, avoiding trade idling, and exploring potential opportunities for enhancing the project. Results reveal that reducing variability in certain activities has a significant impact on increasing the potential of meeting the deadline, minimizing waiting time, and reducing variation gaps. Mitigating waiting time implies minimizing the disruption in the flow of work, which has several negative impacts on the project. Also, variation gaps denote hidden waste in the production system through missing an opportunity to enhance the project performance. Since the resources on construction projects are usually limited, it would be impractical to mitigate variability in all the activities, hence there is need for determining the activities that are more decisive when it comes to schedule performance. The study contribution lies in (1) investigating through simulation the effects of variability on overlooked two types of waste, namely waiting times and variation gaps and (2) providing practitioners with a tool that integrates the network characteristics with simulation to determine where to reduce variability.. It was shown that there is a probability for the non-critical paths to become critical based on the amount of variability embodied in the activities lying on such paths, and that reducing the activities with the highest variability does not necessarily reduce most waste. Moreover, reducing variability in a critical activity not only affects the duration of the critical path, but also the duration of the near-critical path it belongs to, if any. Through tackling variability properly, productivity and project performance could be safeguarded. The output helps define which activities have the highest impact on project performance and the results show how reducing variability in such activities leads to a reduced variability in the overall schedule. The study also presents several approaches to be adopted in order to reduce process-time and flow variability. For instance, the Lean Construction concepts of look ahead and make-ready planning are presented as a solution to reduce variability in general and to improve the predictability of the upcoming tasks. In particular, the results from the study, summarized in identifying highly variable activities, can be used by last planners to prioritize such activities when removing constraints. Results suggest that the focus should be on the right activities in order to improve the schedule and the time performance and to increase the likelihood of delivering the project on time. Also, employment of more resources could be applied but this imposes additional costs and affects the budget performance.

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One limitation of the study lies in getting precise three-point estimates to accurately represent the variability of the involved activities. Such data could be challenging to obtain, and for big projects where the network can be complex, this might increase the uncertainties. Another limitation is that, as mentioned earlier, if right skewed distributions were applied, the effect of reducing variability will be doubled due to reduced extremes and a small reduction in mean. Determining the precise effect needs to be studied in future studies. Moreover, the effect of incorporating the suggested Lean approaches is not studied quantitatively. Thus, examining and simulating such effect on the variability inherited in the CPM networks and overall schedule is recommended to be carried in a future study. Finally, resources might affect the results; still, they have not been taken into consideration. Resource allocation, resource leveling, and continuous updating of resource usage throughout the project might result in more accurate and realistic outcomes. Therefore, addressing resources is recommended to be tackled in future studies.

DATA AVAILABILITY STATEMENT

Data that support the findings of this study are available from the corresponding author upon reasonable request.

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