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Reliability Modeling of Multistate Degraded Power Electronic Converters With Simultaneous Exposure to Dependent Competing Failure Processes

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ABSTRACT Depending on the application in which power electronic converters (PECs) are deployed, failure processes may endanger the desirable performance of PECs. This paper offers holistic insights on reliability modeling of PECs considering dependencies in two simultaneous failure processes, namely gradual wearing-out degradation and vibration sudden degradation. While sudden and gradual degradation processes may individually affect the useful lifetime of PECs, their mutual interdependencies could significantly accelerate the aging mechanisms. A new analytical model for reliability assessment of PECs is proposed that can capture such mutual interdependence of simultaneous failure processes. The proposed analytics are applied to a DC-DC boost converter in a hybrid electric vehicle exposed to both gradual wearing-out and vibration sudden degradations. Monte Carlo-aided numerical results will demonstrate how critical it is to take into account the mutual interdependence in the failure processes when assessing the reliability performance of PECs, failure to do which may lead to inaccurate useful lifetime estimations and the corresponding maintenance and replacement decisions.

INDEX TERMS Lifetime estimation, multistate degraded systems, power electronic converters (PECs), reliability.

NOMENCLATURE

a, b, c, e, f	Constant coefficients of the deterministic gradual degradation trend.
D	Sudden failure threshold.
E_1/E_2	Sudden/Total reliability event.
$F(w; t)$	Cumulative distribution function of W_i.
$f_Y^{<m>}$	Convolution of the probability density functions of Y_i variables.
$f_Y(y_i)$	Probability distribution function of Y_i.
$f_{\boldsymbol{\theta}}(x)$	Probability distribution function of $\boldsymbol{\theta}$.
$G(r; t)$	Cumulative distribution function of $\mathscr{R}_s$.
G_{rms}	Root mean square acceleration.
H	Gradual failure threshold.
i, j	Arbitrary distinct shock loads ($i < j$).
m	Number of shock loads before the failure.
$N(t)$	Number of shock loads prior to time t.
$o(\Delta)$	Higher order terms.
$P_{i,j}(s,t)$	Strictly increasing transition probability of the Markov point counting process.
$\mathcal{R}(t; \boldsymbol{\theta})$	Gradual degradation trend.
$\mathcal{R}_s(t; \boldsymbol{\theta})$	Total degradation trend.
s, t	Arbitrary distinct time points ($s < t$) corresponding to the shock loads i and j.
$S(t;\mathbf{y})$	Cumulative sudden damage magnitude.
$t_{G\text{-}Failure}$	Gradual failure time.
$t_{S\text{-}Failure}$	Sudden failure time.
u, v	Dummy variables.
W_i	Magnitude of the shock load i.

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Y_i	Damage magnitude caused by shock load i.
β	Gradual-sudden degradation dependence factor.
$\boldsymbol{\theta}$	Vector of uncertainties in the gradual degradation process.
$\lambda_0(t)$	Initial occurrence rate of vibration random shocks.
$\Lambda_0(t)$	Averaged initial occurrence rate over $[0, t]$.
Δ	Negligible time period.
λ_0	Constant initial occurrence rate of vibration random shocks.
$\lambda_i(t)$	Occurrence rate of the vibration random shock i.
$\Lambda_j(s, t)$	Average occurrence rate between $[s, t]$.
ξ	Facilitation factor.
μ_x,	Mean value of x normal distribution.
σ_x^2	Variance of x normal distribution.
ψ	Normal distribution of the gradual reliability model.
φ	Normal probability distribution function.
Φ	Normal cumulative distribution function.

I. INTRODUCTION

Power electronic converters (PECs) are frequently exposed to a wide range of simultaneous failure processes emerging from a variety of mechanisms in real-world applications. Most PECs may experience a number of individual failure processes (e.g., wear, fatigue, corrosion, etc.) or a combination thereof [1]–[3]. Generally, such failures can be divided into two categories of *sudden failures* and *gradual failures*. While gradual failures are caused by internal continual degradations due to the wearing-out mechanisms, sudden failures are triggered by external random shocks such as vibrations, drops, etc. [4]. These degradations are competing in nature and may have degrees of interdependencies depending on the environment in which PECs are deployed [5].

Competition among failures may be readily considered by any priority algorithm for useful lifetime estimation (ULE) of PECs [6], [7]. However, a yawning gap exists in considering the dependencies among diverse failure processes (either gradual or sudden or a combination thereof) when evaluating the PECs' reliability performance. Mutual interdependences, i.e., from gradual to sudden degradation and vice versa, are highly present in PECs (see Fig. 1) [8]. These dependencies may intensify the individual effects of the gradual and sudden degradations on PECs' ULE, leading to accelerated aging mechanisms in PECs and potentially inaccurate reinforcement decisions [8], [9].

In power electronic systems, several reliability models have been employed for analyzing the PECs' ULE [10]–[15]. Such reliability models are in structure commonly based on the failure mode, mechanism and effect analysis (FMMEA) in which the critical components are found according to the mission profile and the environmental conditions under which the system is working. These studies have broadly focused on power semiconductors, which are deemed to be the most vulnerable elements in power electronic systems

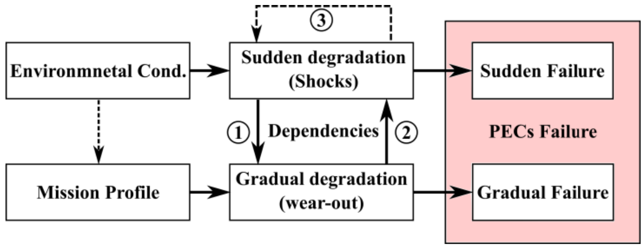

FIGURE 1. Dependent competing failure processes in PECs.

from the reliability perspective [10], [11]. Former studies have focused mostly on the gradual degradation of power MOSFETs [10] and power IGBT modules [11] in different applications [16]–[18], where PECs' reliability assessments were only based on one failure process (gradual failure) in the critical components. In other words, the failure process diversity and subsequently their dependencies were not considered in the previous literature. Low cycle fatigue reliability assessment of random mechanical shocks (e.g., vibration) was investigated in [14], [15]. These studies focused on the mechanical shocks and the sudden failures in power discrete chips, where similar to [19], [20], sudden failure of the power components were experimentally analyzed with no consideration to either gradual degradation or degradation dependencies. Gradual and sudden degradations are often significant in PECs due to highly strained and intense connection among their components [9], [21]. As demonstrated in Fig. 2, the gradual degradation caused by a normal operating of PEC may lead to internal damages in either PEC or its subcomponents [8]. Accordingly, it may also weaken the strength of either PEC or its components against sudden degradations caused by random external shocks. Such random shocks also weaken the strength of either PEC or its components against gradual degradation. These mutual dependencies play a key role on the accurate PECs' ULE. However, the dependencies between both degradations have been rarely studied in the PECs' reliability modeling and assessments in the literature.

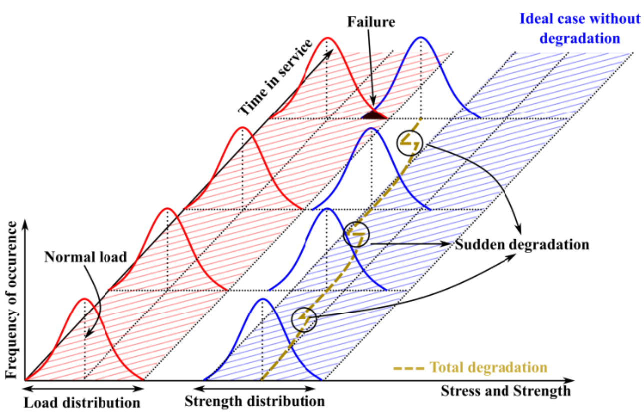

FIGURE 2. Load-strength analysis to capture the failure dependencies.

There exist several reliability models in other engineering domains to evaluate the effects of dependent competing failure processes [22]–[26]. Sudden-gradual degradation dependencies are studied in [24]–[26] assuming an abrupt increase in the accumulated degradation or the degradation rate. In [24], [25], each external shock was assumed to result in abrupt degradation in each component's damage model and a cumulative shock model was also employed to assess the total degradation. A reliability model for a multi-component system exposed to multiple dependent competing failure processes was proposed in [26]. The double effects of random shocks (sudden degradation) [27]–[29] and the shock intensity division making [30] were taken into account to capture the sudden-gradual dependencies. In all aforementioned studies available in the literature, the sudden-gradual failure dependencies were solely analyzed with no or minimum consideration to the gradual-sudden degradation dependencies.

While the gradual-sudden degradation dependencies have been captured via the doubly stochastic Poisson processes, non-homogeneous Poisson processes, and the multistate physics models [31]–[33], to the authors' best knowledge, simultaneous consideration of both gradual-sudden and sudden-gradual degradation dependencies has been remained a challenge from the reliability perspective and has been scarcely investigated in complex engineering systems.

This paper focuses on PECs' reliability assessment and investigates modeling and analyzing simultaneous exposure to competing failure processes and their dependencies. A PEC in a hybrid electric vehicle exposed to simultaneous random vibration shocks and gradual internal wearing out owing to its mission profile is taken as the studied test case. The thrust of our argument is to introduce a new reliability model for PEC that accurately assesses its useful lifetime considering vibration wear-out and wear-out-vibration mutual interdependencies.

The remainder of this paper is structured as follows. Section II presents the PEC system description under simultaneous exposure to dependent competing failure processes and expresses the corresponding assumptions. Section III introduces the proposed reliability model for PECs. Section IV presents a case study to evaluate the performance of the proposed reliability model for PECs. Section V expresses the numerical results, Monte Carlo simulations for further validations, and additional discussions. The concluding remarks are eventually drawn in Section VI.

II. SYSTEMS WITH SIMULTANEOUS EXPOSURE TO DEPENDENT COMPETING FAILURE PROCESSES

In this section, different aspects of the studied PEC system with the corresponding assumptions are expressed.

A. PEC SYSTEM DESCRIPTION

As illustrated in Fig. 3, a PEC with two simultaneous dependent competing failure processes (i.e., a gradual degradation due to internal wearing out and a sudden degradation due to vibration random shocks) is here taken as the test case. A PEC may fail due to either wear-out ($t_{G\text{-}Failure}$) or vibration random shocks ($t_{S\text{-}Failure}$). It can be readily observed that abrupt degradations contribute to the system's total degradation path $\mathscr{R}_s(\cdot)$. The PEC may fail whenever either the overall degradation $\mathscr{R}_s(t)$ or the magnitude of the shock load $W(t)$ exceeds the gradual failure threshold (H) or the sudden failure threshold (D), respectively. In fact, these two failure processes are competing with each other in forming the failure process. As depicted in Fig. 1, line ① demonstrates the sudden-gradual degradation dependence where the vibration shocks will cause abrupt and accelerated degradation in PEC. Cumulative damage models have often been utilized to consider abrupt degradations due to sudden loads. Line ② expresses the (overall) gradual-sudden degradation dependence, reflecting the fact that the overall gradual degradation weakens the PEC's strength as shown in Fig. 2. In addition to these two dependencies, one can also consider the effects of arrival random shocks on the next arriving shocks (line ③), hereafter called *facilitation* effects.

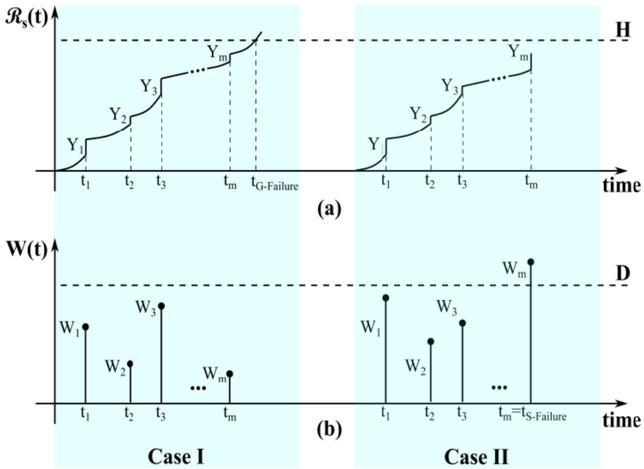

FIGURE 3. Simultaneous dependent competing failure processes: (a) total degradation trend and (b) magnitude of shock loads. Case I: Gradual failure process and Case II: Sudden failure processes.

This paper proposes a model to capture such mutual dependencies. Accordingly, random vibration shocks can cause a sudden degradation (line ①) and the gradual degradation originated jointly from the internal wear out and the abrupt degradation ($\mathscr{R}_s(\cdot)$) can increase the intensity of random vibration shocks (line ②). Additionally, the arrival random vibration shock can also exacerbate the intensity of random vibration shocks (line ③).

B. MODEL ASSUMPTIONS

To put forward a practical solution tackling the aforementioned challenges, the following assumptions are considered in the proposed reliability model:

1- PEC may fail whenever either the overall degradation or magnitude of shock loads exceeds H or D, respectively.
2- The gradual degradation is a strictly non-decreasing function, $\mathscr{R}(t) = \mathscr{R}(t; \boldsymbol{\theta})$ where vector $\boldsymbol{\theta}$ is a parameter reflecting the uncertainties. This function may be characterized through accelerated aging tests [21].
3- Vibration random shocks are assumed to have an occurrence rate of $\lambda_i\ (t) = (1 + \xi i)\lambda_0(t)$ and

$\lambda_0(t) = \lambda_0 + \beta\mathscr{R}(t;\boldsymbol{\theta})$ [22], where $\lambda_0(t)$ and $\lambda_i(t)$ are the initial and the i^{th} occurrence rates of the vibration random shocks, respectively. ξ and β are the facilitation and the gradual-sudden dependence factors, respectively.

4- Vibration shocks are assumed to be independent and identically distributed (I.I.D) random variables and their corresponding cumulative distribution function (CDF) is denoted by $F(w; t)$ [22], [23]. The total degradation CDF is also declared by $G(r; t)$.

5- The system is assumed to be non-repairable. Therefore, the failed component is replaced with a new one.

III. THE PROPOSED RELIABILITY MODEL FOR PECS

A. SUDDEN AND GRADUAL RELIABILITY MODEL

With the CDF of vibration shock loads defined as $F(w, t)$, one can assess the probability that no sudden failure occurs:

$$P(W_i < D) = F(D; t) \quad for\ i = 1, 2, \ldots, m \tag{1}$$

As mentioned earlier, the total degradation $\mathscr{R}_s(\cdot)$ denotes the sum of the gradual and abrupt degradation of internal wearing out and vibration shock loads, respectively. While the gradual degradation path follows a strictly increasing function ($\mathscr{R}(\cdot)$), the random shock loads follow discrete jumped degradations (Y_i for $i = 1, 2, \ldots, m$) as shown in Fig. 3. Referring to analytics proposed in the literature [15], [21]–[23], sudden degradations may cumulatively be measured. Accordingly,

$$S(t;\mathbf{y}) = \begin{cases} \sum_{i=1}^{N(t)} Y_i & if\ N(t) > 0 \\ 0 & if\ N(t) = 0 \end{cases} \tag{2}$$

According to the cumulative sudden damage and gradual degradation models, one can readily conclude that

$$\mathscr{R}_s(t) = \mathscr{R}(t;\boldsymbol{\theta}) + S(t;\mathbf{y}) = \mathscr{R}(t;\boldsymbol{\theta}) + \sum_{i=1}^{N(t)} Y_i \tag{3}$$

The probability that no failure occurs at time t can be attained as follows:

$$P(V_s(t) < H) = G(H; t) \tag{4}$$

B. VIBRATION RANDOM SHOCK MODEL

There are a number of diverse counting processes to model the random shocks, including the homogenous Poisson, non-homogenous Poisson, reward and Markov point processes, etc. [34], [35]. In the proposed reliability model, a Markov Point Process (MPP) is utilized to model the vibration random shocks [34] that considers the facilitation factor and captures the mutual dependence of the total degradation on the subsequent arrival shocks.

In the vibration random shock process $N(t)$, $\lambda_i(t)$ is employed as the shock occurrence rate including the facilitation factor. The transition probability can be defined as follows:

$$P_{i,j}(s, t) = P(N(t) = j|N(s) = i) \quad for\ i \le j,\ s \le t \tag{5}$$

where, $P_{i,j}(s, t)$ is the strictly increasing transition probability of MPP. According to the counting process properties, the following expression is valid for any negligible period of time (Δ) and for any state i that [34]

$$\begin{aligned} P(N(t, t+\Delta) = 0|N(t) = i) &= 1 - \lambda_i(t)\Delta + o(\Delta) \\ P(N(t, t+\Delta) = 1|N(t) = i) &= \lambda_i(t)\Delta + o(\Delta) \end{aligned} \tag{6}$$

where, $N(t, t+\Delta)$ expresses the number of vibration random shocks between time t and time $t + \Delta$. Therefore,

$$P_{i,i+1}(t, t+\Delta) = \lambda_i(t)\Delta + o(\Delta) \tag{7}$$

Accordingly, one can obtain that

$$\begin{aligned} P_{i,j}(s, t+\Delta) &= P_{i,j-1}(s, t)P_{j-1,j}(t, t+\Delta) \\ &\quad + P_{i,j}(s, t)P_{j,j}(t, t+\Delta) \\ &= P_{i,j-1}(s, t)(\lambda_{j-1}(t)\Delta + o(\Delta)) \\ &\quad + P_{i,j}(s, t)(1 - \lambda_j(t)\Delta + o(\Delta)) \end{aligned} \tag{8}$$

By expanding (8), ignoring higher orders term, and re-arranging the above equation, it yields:

$$\frac{\partial P_{i,j}(s,t)}{\partial t} + P_{i,j}(s, t)\lambda_j(t) = P_{i,j-1}(s, t)\lambda_{j-1}(t) \tag{9}$$

The general solution of the achieved linear first-order differential equations is expressed as follows

$$P_{i,j}(s, t) = \frac{\int_s^t exp(\Lambda_j(s, t))P_{i,j-1}(s, t)\lambda_{j-1}(t)dt}{exp(\Lambda_j(s, t))} \tag{10}$$

where,

$$\Lambda_j(s, t) = \int_s^t \lambda_j(u)du \tag{11}$$

Rearranging (10), and defining $Q_j(t) = P_{i,j}(s, t).exp(\Lambda_j(s, t))$ for $i \le j$ and $s \le t$, one can yield the following recursive equation:

$$Q_j(t) = \int_s^t Q_{j-1}(v)\lambda_{j-1}(v)exp\left(\int_s^v (\lambda_j(u) - \lambda_{j-1}(u))\, du\right) dv \tag{12}$$

Generally, as counting process (vibration random shock) is considered a transition from state $N(0) = 0$ to $N(t) = i$. Thus, equation (12) can be rewritten as follows:

$$Q_i(t) = \int_0^t Q_{i-1}(v)\lambda_{i-1}(v)exp\left(\int_0^v (\lambda_i(u) - \lambda_{i-1}(u))\, du\right) dv \tag{13}$$

where, $Q_i(t)$ can be expressed as $P_i(t).exp(\Lambda_i(t))$ in which $P_i(t)$ is the probability of arriving i shocks by time t and the statement $\Lambda_t(t) = \int_0^t \lambda_i(u)du$ always holds. Since $\lambda_i\,(t) = (1 + \xi i)\lambda_0(t)$ was assumed valid, therefore,

$$\begin{aligned} \int_0^v (\lambda_i(u) - \lambda_{i-1}(u))\, du &= \int_0^v \begin{pmatrix} (1+\xi i)\, \lambda_0\,(u) \\ -\ (1+\xi i - \xi)\, \lambda_0\,(u) \end{pmatrix} du \\ &= \int_0^v \xi\lambda_0\,(u)\, du = \xi\,\Lambda_0(v) \end{aligned} \tag{14}$$

Using equations (13) and (14), it yields (for $i > 0$):

$$Q_i(t) = (1+\xi(i-1)) \int_0^t Q_{i-1}(v)\lambda_0(v)\, exp(\xi\Lambda_0(v))\, dv = \left(\xi^{-1}+i-1\right) \int_0^t Q_{i-1}(v) \left(\frac{d}{dv} exp(\xi\Lambda_0(v))\right) dv \quad (15)$$

Based on the above equations and defining $z^{(k)} = z(z-1)\ldots(z-k+1)$, one can obtain $Q_i(t)$ as follows

$$Q_i(t) = \frac{\left(\xi^{-1}+i-1\right)^{(i)} (exp(\xi\Lambda_0(t))-1)^i}{i!} \quad (16)$$

Consequently [22],

$$P_i(t) = Q_i(t) exp(-\Lambda_i(t)) = C^i_{\xi^{-1}+i-1}(1-exp(-\xi\Lambda_0(t)))^i (exp(-\xi\Lambda_0(t)))^{\xi^{-1}} \quad (17)$$

where C is the combination terms.

C. SYSTEM RELIABILITY MODEL

The system/item is assumed to be exposed to simultaneous competing failure processes, which have mutual effects on the strength of the system/item. Gradual-sudden dependence has been characterized by the β factor and reflected in the occurrence rate of the counting process. Meanwhile, sudden-gradual dependence has been directly reflected by applying its resulting jumped damage into the gradual degradation path (see eq. (3)). We have also taken into account the facilitation factor ξ demonstrating the effects of arrived vibration shock loads on the next arriving load. It has been assumed that facilitation can linearly intensify the coming vibration random loads. With regards to Fig. 1, both gradual and sudden degradations may lead to a system/item failure (e.g., PEC failure). Therefore, their contributions and stiff competitions have to be accounted for in the global reliability model [22]. Since the failure criteria have been considered as D and H for the sudden and global gradual degradations, respectively, one can obtain the reliability function of the system as follows

$$R(t) = \sum_{m=0}^{\infty} P\left(\begin{matrix} W_1 < D, W_2 < D, \cdots, W_m < D, \\ \mathscr{R}_s(t) < H | N(t) = m \end{matrix}\right) \times P(N(t) = m) \quad (18)$$

Hereafter, for the sake of brevity, let E_1 and E_2 signify the event $W_1 < D, W_2 < D, \ldots, W_m < D$ and $\mathscr{R}_s(t) < H$. Although E_1 and E_2 are inherently dependent (owing to the mathematical correlations), they can be conditionally assumed to be independent for given values of $\boldsymbol{\theta}$, Y_i and $N(t)$ [22], [36]. Thus,

$$R(t) = \sum_{m=0}^{\infty} \int_\theta \int_{Y_1} \cdots \int_{Y_m} P\left(\begin{matrix} E_1, E_2 | N(t) = m, \boldsymbol{\theta} = x, \\ Y_1 = y_1, \ldots, Y_m = y_m \end{matrix}\right) \times P(N(t) = m | \boldsymbol{\theta} = \boldsymbol{x}, Y_1 = y_1, \ldots, Y_m = y_m) \times f_\theta(x) dx f_Y(y_1) \cdots f_Y(y_m) dy_1 \cdots dy_m \quad (19)$$

where, $f_\theta(x)$ and $f_Y(y_i)$ are the probability density function (PDF) of θ and Y_i for $i = 1, 2, \ldots, m$. Using convolution integral theorem in the summation of the independent and identically distributed PDFs and the assumption that Y_i elements are independent and identically distributed, we have

$$\int_{Y_1} \cdots \int_{Y_m} f_Y(y_1) \cdots f_Y(y_m) dy_1 \cdots dy_m = \int_Y f_Y^{<m>}(u) du \quad (20)$$

where $u = \sum_{i=1}^m Y_i$ and $f_Y^{<m>}$ is the convolution of the probability density functions of Y_i for $i = 1, 2, \ldots, m$ ($f_Y^{<m>} = ((f_{Y1} * f_{Y2}) * f_{Y3}) \ldots * f_{Ym}$). Inserting (20) into (19), it yields

$$R(t) = \sum_{m=0}^{\infty} \int_\theta \int_Y P\left(E_1 | N(t) = m,\ \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right) \times P\left(E_2 | N(t) = m,\ \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right) \times P\left(N(t) = m | \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right) \times f_\theta(x) dx f_Y^{<m>}(u) du \quad (21)$$

where, $P\left(E_1 | N(t) = m,\ \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right)$ actually denotes the conditional probability that no sudden failure occurs at time t and its given conditions. One can find that the probability of sudden failure occurrence is directly governed by $N(t)$ owing to the taken assumptions, namely independence of W_i and Y_i and the type of W_i that is assumed to be i.i.d random variable. Thus

$$P\left(E_1 | N(t) = m,\ \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right) = F(D; t)^m \quad (22)$$

$P\left(E_2 | N(t) = m,\ \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right)$ reflects the conditional probability that no gradual failure occurs at time t and its given conditions. One may find that this probability is *0-1* distributed. In other words, the following statement holds:

$$P\left(E_2 | N(t) = m,\ \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right) = \begin{cases} 1 & if\ \mathscr{R}_s(t) < H \\ 0 & if\ \mathscr{R}_s(t) \geq H \end{cases} \quad (23)$$

where, $P\left(N(t) = m | \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right)$ denotes the conditional probability of arriving m vibration loads at time t and its given conditions. According to (12), one can have:

$$P\left(N(t) = m | \boldsymbol{\theta} = x, \sum_{i=1}^m Y_i = u\right) = C^m_{\xi^{-1}+m-1} \times (1 - exp(-\xi\Lambda_0(t; x)))^m \times (exp(-\xi\Lambda_0(t; x)))^{\xi^{-1}} \quad (24)$$

where

$$\Lambda_0(t; x) = \int_0^t (\lambda_0 + \beta(\mathscr{R}_s(v; \boldsymbol{\theta}))) dv \quad (25)$$

Consequently, the reliability model proposed in (18) may be re-written as follows:

$$
\begin{aligned}
R(t) = \sum_{m=0}^{\infty} F(D;t)^m \int_{\theta}\int_{Y} P & \left(\begin{matrix}\mathscr{R}(t;\boldsymbol{\theta}+u) < H|N(t)=m,\\ \boldsymbol{\theta}=x, \sum_{i=1}^{m} Y_i = u\end{matrix}\right) \\
& \times C^{m}_{\xi^{-1}+m-1} \times (1-exp(-\xi\,\Lambda_0(t;x)))^m \\
& \times (exp(-\xi\,\Lambda_0(t;x)))^{\xi^{-1}} \times f_{\theta}(x)dx.f_Y^{<m>}(u)du \qquad (26)
\end{aligned}
$$

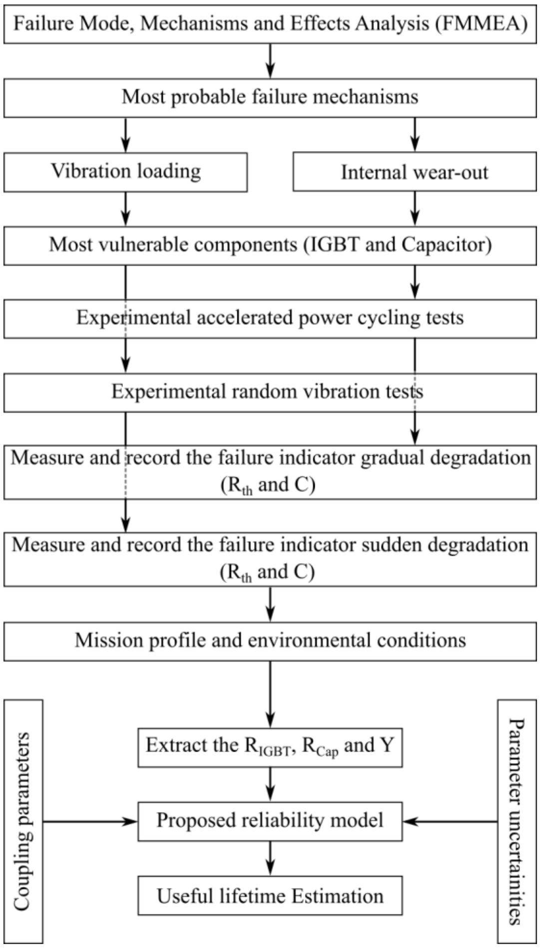

FIGURE 4. Procedure of the proposed reliability model considering failure interdependences.

IV. CASE STUDY

A. METHODOLOGY AND EXPERIMENTAL PROCEDURE

Fig. 4 demonstrates the methodology and experimental procedure of the proposed reliability model. Failure modes, mechanisms and effects analysis (FMMEA) is employed to characterize the most critical failure mechanisms and components in the considered PECs based on the literature review and the field experiences. Regarding the automotive application, vibration loading and internal wear-out are found to be the most prevailing failure mechanisms in PECs [21], [37]–[41]. The analysis has also revealed that IGBT and power capacitors are the most critical components in PECs. Thus, the primary focus was shifted towards the IGBT and power capacitor reliability assessment under internal wear-out and vibration loading. The main failure modes in the discrete IGBTs and power capacitors are found to be the thermal resistance and capacitance deviations, respectively.

Following a determination of the critical failure modes, mechanisms and components, the next stage is to experimentally measure the characteristics of the failure indicators (thermal resistance and capacitance) under several accelerated power cycling and random vibration tests considering the mission profile. The results illustrate the degradation trends ($\mathscr{R}_{IGBT}$ and $\mathscr{R}_{Cap}$) of the failure indicators under power cycling tests. The tests were performed for 64 IGBTs under different temperature swings as listed in Table 1. For each temperature swing scenario, four IGBTs were tested. Regarding failure site and failure mechanism, junction-case thermal resistance is the most reliable failure indicator to monitor the aging status of the power IGBTs. Since it cannot be directly measured, the Thermo sensitive electrical parameter, i.e. collector emitter voltage, was used in order to estimate the junction-case thermal resistance. In each power cycle, the junction-case thermal resistance was estimated. The test continued until the junction-case thermal resistance passed through its failure criteria (*20%* increment). The same procedure was followed for the power capacitor. Several power capacitors were placed under test and their capacitances were directly measured. The test continued until the capacitance passed through its failure criteria (*5%* decrement). The procedure of the power cycling accelerated tests of IGBTs and capacitors are provided in details by the authors in [21], [37]–[39]. The aging trend of power IGBTs and power capacitor are expressed in (27) and (29), with the corresponding coefficient presented in Section IV.C.

Vibration shock loads may have two effects on the PEC's performance. First, it may accelerate the PEC's global degradation by accumulating its resulted damages (Y_i) into gradual degradation ($\mathscr{R}$). Second, it may directly cause a sudden failure if the input vibration magnitude exceeds its failure threshold (D). Vibration levels in HEV are directly correlated to the HEV speed, road conditions, and the load level [15], [42]. Half cycle car mathematical modelling [43] was used for effective assessment of the dynamic interactions between the vehicle (at the surface on which PEC was mounted) and the road roughness profile and consequently, vibrations generated by the road were extracted. All the critical parameters such as sprung mass, unsprung mass, suspension stiffness, suspension damping, and tire stiffness were considered. Road roughness was considered with a random profile while the vehicle speed followed the worldwide harmonized light vehicles test procedure (WLTP) pattern. The fast Fourier transform analysis was then employed to extract the frequency expressed as power spectral density (PSD). Accordingly, the measured input PSD applied to the considered PEC [15] and the response root mean square acceleration of PEC (G_{rms})—a commonly-used indicator for vibration loads [42]—were extracted using a low-mass accelerometer. In this regard, an electrodynamic shaker was employed as shown in Fig. 5 in order to record the acceleration response of input random vibration. Two ultra-tiny accelerometers (1.45g) were mounted on the center of the considered PEC (including capacitors and IGBTs) in order to measure the input and response acceleration. The data recording and test continued until the root mean square acceleration of the PEC passed through its failure criteria (threshold). The PCB module was vertically bolted into the vibration fixture via its corner holes. In-detail data of the test implementation and measurements are available in [44].

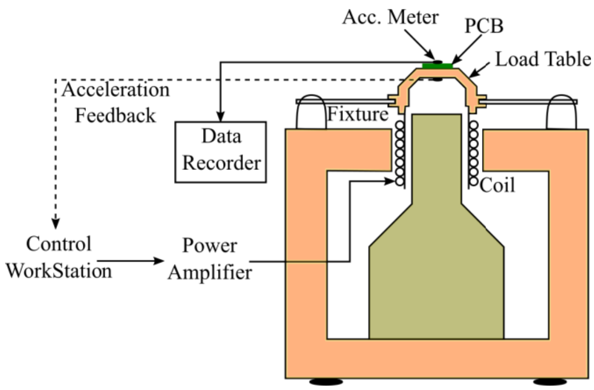

FIGURE 5. Block diagram of the experimental test.

Now, the required data was achieved and can be inserted to the proposed reliability model. For generalizing the approach, parameters uncertainties were also considered. For example, for the power cycling tests conducted on 64 IGBTs, we found that considering 5% deviation in R_{IGBT} (thermal resistance) might cover all the data extracted from various tests. Therefore, a normal distribution with 5% deviation for all parameters in the reliability evaluation was considered.

In the reliability tests, it is common to perform accelerated tests for individual components and then apply the results to the reliability model in order to evaluate the useful lifetime of the global system. It is very time consuming (taking several years) and costly to perform reliability tests on the global system under its normal conditions. Thus, accelerated tests were carried out on the individual components, namely IGBTs and capacitors, and then the associated results were applied to the proposed reliability model for global system reliability assessment.

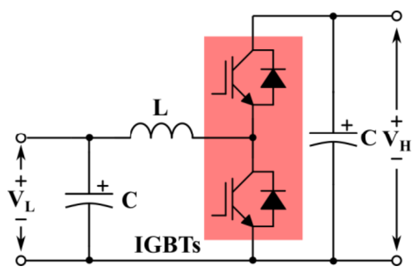

FIGURE 6. Conventional DC-DC boost power converter.

B. RELIABILITY MODEL SIMPLIFICATIONS

In this Section, a 3-kW DC-DC boost power electronic converter (see Fig. 6) utilized in a hybrid electric vehicle (HEV) is considered as the PEC test case. The studied HEV was exposed to the WLTP driving cycle. The most vulnerable components in the considered PEC are recognized and reported as the IGBT power module and power capacitor in the system [21]. IGBT internal wearing out was its junction-case thermal resistance and its degradation path function was statistically modeled through the following expression:

$$\mathscr{R}_{IGBT}(t) = \mathscr{R}_{IGBT}(t; a, b, c) = at^2 + bt + c \tag{27}$$

All the aforementioned coefficients are assumed to be normally distributed, denoted as $a \sim N(\mu_a, \sigma_a^2)$, $b \sim N(\mu_b, \sigma_b^2)$ and $c \sim N(\mu_c, \sigma_c^2)$. Therefore,

$$\begin{aligned} \mathscr{R}_{IGBT} &\sim N\left(\mu_a t^2 + \mu_b t + \mu_c, \sigma_a^2 t^4 + \sigma_b^2 t^2 + \sigma_c^2\right) \\ &= \psi_{IGBT} \sim N(\mu_{\psi_{IGBT}}, \sigma_{\psi_{IGBT}}) \end{aligned} \tag{28}$$

Power capacitor internal wearing out was its capacitance and its degradation path function was statistically modeled through the following expression [45]:

$$\mathscr{R}_{Cap}(t) = \mathscr{R}_{Cap}(t; e, f) = et + f \tag{29}$$

All the aforementioned coefficients are assumed to be normally distributed, denoted as $e \sim N(\mu_e, \sigma_e^2)$ and $f \sim N(\mu_f, \sigma_f^2)$. Therefore,

$$\begin{aligned} \mathscr{R}_{Cap} &\sim N\left(\mu_e t + \mu_f, \sigma_e^2 t^2 + \sigma_f^2\right) \\ &= \psi_{Cap} \sim N(\mu_{\psi_{Cap}}, \sigma_{\psi_{Cap}}) \end{aligned} \tag{30}$$

Furthermore, the magnitude of vibration loads and the corresponding damages are also assumed to be normally distributed as $W_i \sim N(\mu_W, \sigma_W^2)$ and $Y_i \sim N(\mu_Y, \sigma_Y^2)$ [15], [21], although generic enough to accommodate any other probability distribution such as Weibull, etc. With these assumptions taken, one can re-evaluate equations (22)-(24), re-written as (A1)-(A3), as shown at the bottom of page 10, for power IGBT module. The last unspecified terms, namely $f_Y^{<m>}(u)du$ and $f_{\theta}(x)dx$, in eq. (26) may be evaluated as equations (A4) and (A5), as shown at the bottom of page 10. By inserting (A1)-(A5) into (26), one can eventually obtain the reliability function of the system as expressed in (A6), as shown at the bottom of page 11. One can also recalculate all the equations for power capacitors as written in equations (A7) to (A12), as shown at the bottom of page 11. The main differences between this set of equations for power IGBT module and power capacitors are their degradation paths—see equations (27) and (29)—as well as their gradual-sudden degradation dependence factors defined as β_{IGBT} and β_{Cap}.

C. NUMERICAL EXAMPLE

From a reliability point of view, the critical components in the studied PEC system are IGBT power module and power capacitor exposed to simultaneous gradual and sudden degradation processes. The gradual degradations have been

TABLE 1. IGBT aging results for power cycling accelerated test [37]–[39].

Temperature swing (°C)	T_{men} (°C)	ΔT (°C)	Mean number of Cycles to Failure
40-170	105	130	6853
50-160		110	8303
60-150		90	10212
70-140		70	13820
40-150	95	110	9825
50-140		90	12233
60-130		70	16102
70-120		50	NA

monitored and indicated by junction-case thermal resistance increase ($\mathscr{R}_{IGBT}$) and capacitance ($\mathscr{R}_{Cap}$) as the aging precursors for IGBT and capacitor, respectively. The nominal value of $\mathscr{R}_{IGBT}$ is $1.15°C/W$ and the failure criterion (threshold) is set as 20% increment, namely $1.38°C/W(H_{IGBT})$. The nominal value of $\mathscr{R}_{Cap}$ is $50\mu F$ and the failure criterion (threshold) is set as 5% decrease, namely $47.5\mu F(H_{Cap})$.

As expressed in (27) and (29), the gradual degradation paths may be statistically modelled by a second-order and a first-order polynomial functions. In order to fit the extracted data from the thermal cycling tests, MATLAB fitting tool was employed. Using Bisquare algorithm, the coefficients a, b, c, e and f are 9.25×10^{-12}, 1.181×10^{-7}, 1.15, -1.52×10^{-8} and 50, respectively (and t is in hours) [21]. In order to accurately capture the uncertainties in the accelerated power cycling tests performed, all these five coefficients are assumed to be normally distributed with 5% deviations from their mean values.

In our application, sudden degradation has been originated from vibration random shock loads. It is obvious that riding on the rough roads may lead to several vibration loads that have to be taken into account while assessing the reliability performance of the PEC. The magnitude of the vibration shock loads and the sudden damage are assumed to be I.I.D with the normal probability distribution of $W \sim N(0.3, 0.1^2) m/s^2$ and $Y \sim N(0.03, 0.01^2)°C/W$, respectively. The sudden failure threshold is considered $0.5 m/s^2$ (D) [14], [15]. The initial occurrence rate λ_0 is assumed to be 25×10^{-6} [46]. The gradual-sudden dependence factor β_{IGBT}, β_{Cap} and the facilitation factor ξ are considered 0.0003, 0.00018 and 0.05, respectively [15], [45], [47].

V. RESULTS AND DISCUSSIONS

In this section, the proposed analytical model for reliability assessment of PECs including both gradual-sudden and sudden- gradual degradation dependencies as well as the facilitation factor is applied to the studied PEC. The analytical results are compared with simulation results as well as the analytical reliability model proposed in [21] in which no degradation dependencies were considered. Fig. 7 demonstrates the PEC's ULE under different conditions.

The reliability functions for IGBT and capacitor are those illustrated in eq. (A6) and eq. (A12). Blue expressions in (A6) and (A12) represent the probability of sudden failure caused by vibration either in the capacitor or in IGBT. Red expressions in (A6) and (A12) represent the probability of gradual failure caused by internal wear-out either in capacitor or in IGBT. Magenta expressions in (A6) and (A12) represent the coupling effects (mutual dependencies) of the gradual and sudden failures either in capacitor or in IGBT. For cases with no dependencies in PEC's ULE, we set $\beta = 0$ and $Y = 0$. In order to take into account the sudden-gradual and gradual-sudden dependencies, we set $\beta = 0, Y \sim N(0.03, 0.01^2)°C/W$ and $\beta \neq 0, Y = 0$, respectively. $\beta \neq 0$ and $Y \sim N(0.03, 0.01^2)°C/W$ were considered to capture the mutual dependence in PEC's ULE. One can observe, from Fig. 7 and Table 2, that B_{10}'s reliability of the studied PEC (i.e., the time when the PEC achieves a 90% probability of survival) reveals a declining trend while different types of dependencies have been taken into account. As reported in [21], [39], the B_{10}'s reliability is found ~58,040 and ~86420 hours which reflects the PEC's ULE with no consideration to any dependencies for IGBT and capacitor, respectively.

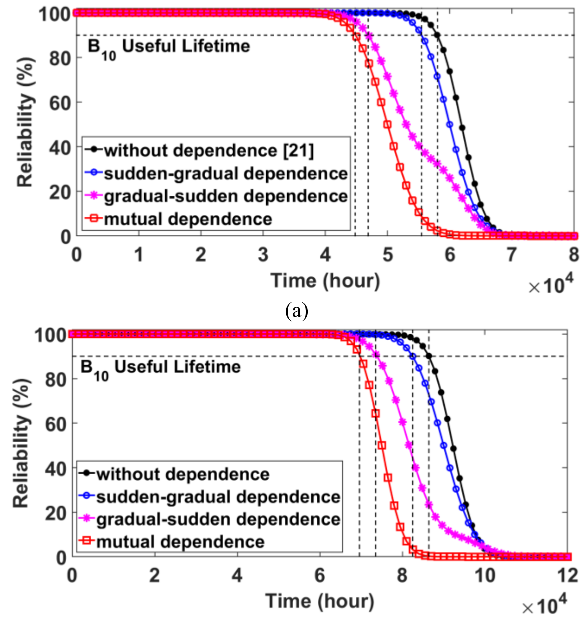

FIGURE 7. PEC components' Reliability functions in four different conditions. a) IGBT power module, b) power capacitor.

TABLE 2. Comparison of B_{10} reliability of different models for power IGBT and power capacitor.

Reliability Model	Useful Lifetime (h) IGBT	Useful Lifetime (h) Capacitor
Complete Proposed Reliability model	44800	69620
Proposed Reliability with only Gradual-sudden dependence consideration	46923	73500
Proposed Reliability with only Sudden-gradual dependence consideration	55508	82504
Proposed Reliability model in [21] and [36]	58040	86420

For IGBT power module, the ULE reliability metric is found about 55,508, 46,923 and 44,800 hours for sudden-gradual-only degradation dependence, gradual-sudden-only degradation dependence and the mutual dependencies, respectively. For power capacitor, the ULE reliability metric is found about 82504, 73500 and 69620 hours for sudden-gradual-only degradation dependence, gradual-sudden-only degradation dependence and the mutual dependencies, respectively. The results reveal a significant difference on the expected reliability performance in scenarios with and without considering the dependencies in competing failure processes in the PECs.

A multicomponent reliability evaluation of the converter needs to be used as two critical components, namely IGBT power module and power capacitor, have been presumed. Following the reliability (or surviving function) of individual power electronics components under the specified mission profiles, global PEC reliability can be evaluated based on the components' reliability block diagram model. Since these two components are logically in series, i.e., occurrence of a failure either in the IGBT or in the capacitor leads to PEC's failure, and the following reliability model holds:

$$R_{PEC\text{-}Series}(t) = \prod_{x=1}^{2} R_x(t) \tag{31}$$

where $R_{PEC\text{-}Series}$ is the system overall reliability and R_x is the x^{th} component's reliability function, namely IGBT and capacitor. The global PEC's reliability is shown in Fig. 8. From this figure, the B_{10}'s reliability is found ~42300 hours which reflects the PEC's ULE when considerating to all kinds of dependencies. It also shows that the PEC's ULE without considering the competing failure processes and their dependencies results in optimistic reliability metrics with 37.2% increase in ULE.

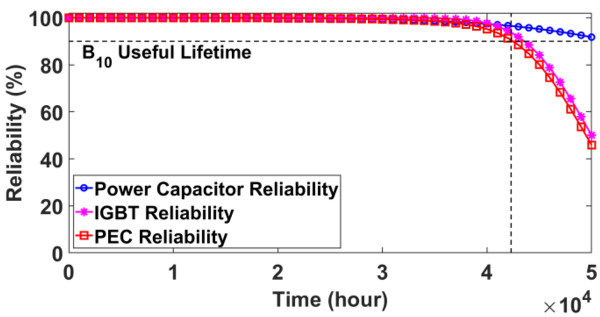

FIGURE 8. PEC's reliability function with consideration of all dependencies.

In order to validate the performance of the proposed reliability model for PECs, Monte Carlo Simulations were performed for 120,000 samples, through which the reliability of the PEC's IGBT power module with gradual-sudden and sudden-gradual dependencies was evaluated. The proposed algorithm for validations is illustrated in Fig. 9. As shown in this figure, a set of samples to capture the uncertainties in the constant coefficients of degradation paths expressed in

Set M (Maximum number of arrival vibration loads)
Set N (Number of Samples)
For k=1:N
Sample a set of $\{a_k,b_k,c_k,e_k,f_k\}$ from their normal distributions
Calculate $\mathcal{R}_{IGBT}(t;a,b,c)$ Eq. (27)
Calculate $\mathcal{R}_{Cap}(t;e,f)$ Eq. (29)
Calculate $P(E_1|\ldots)$ Eq. (A1) and Eq. (A7)
For m=0:M
Calculate $P(N(t)=m|\ldots)$ Eq. (A3) and Eq. (A9)
Sample a u_k from $f_y^{<m>}(y)$
Calculate $P(E_2|\ldots)$ Eq. (A2) and Eq. (A8)
End for
Calculate $R_k(t)$ Eq. (A12) and Eq. (A6)
End for
Calculate average of $R_k(t)$

FIGURE 9. Performed Monte Carlo procedure for algorithm validation.

equations (27) and (29) was applied in order to validate the performance of the proposed model with respect to degradation paths' variations. This simulation was performed for every time step of 1 hour. Since there existed 120,000 input data samples, there also existed 120,000 values of reliability measures. For each time step, the average reliability value extracted from the Monte-Carlo simulations is considered for the reliability function. The numerical results are demonstrated in Fig. 10, where it can be seen that the proposed analytical approach closely follows the survival function achieved from the simulations with a large number of samples. The relative error between the analytical and simulation results was averagely evaluated 0.52%, negligible enough to reveal that the proposed analytical model is sufficiently valid and accurate for real-world scenarios.

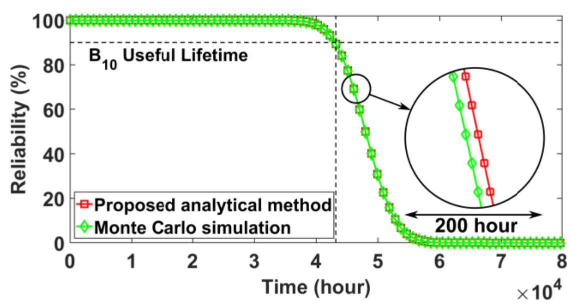

FIGURE 10. PEC's reliability function resulted from the Monte Carlo simulations and the proposed analytical model.

To investigate the significance of vibration loading intensity on the global system reliability, Fig. 11 demonstrates the survival functions of the studied IGBT power module under four different initial occurrence rates (λ_0). The initial occurrence rate or initial intensity λ_0 represents the physical intensity of the sudden-gradual dependence on the global system reliability as illustrated in eq. (A6) and eq. (A12). As expected, the reliability functions are characterized with an overall declining trend that increases initially when the occurrence rate varies from 25×10^{-6} to 40×10^{-6}. The severity of the vibration process will increase as the

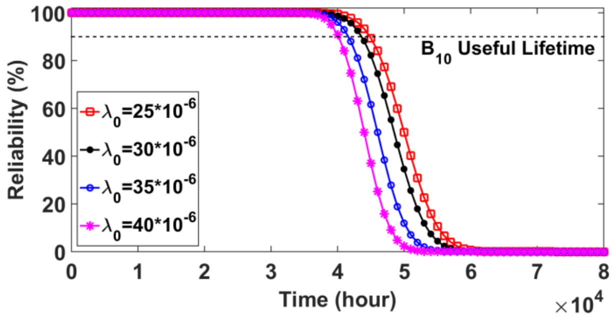

FIGURE 11. PEC's reliability functions in four different occurrence rates.

degradation process proceeds by increasing the initial occurrence rate (λ_0). It will result in more vibration loads, which will lead to more abrupt failures. Due to the positive degradation-sudden dependency, the high risk of the vehicle vibration may cause a sudden failure. Thus, $R(t)$ reduces rapidly over time. In this case, four different Monte Carlo simulations were also performed in order to validate the performance of the proposed analytical model in different cases. The validation results are presented in Table 3. B_{10}'s ULE has been decreasing from 44,800 to 40,150 hours as the initial occurrence rate increases. From Table 3, one can see that the proposed analytical model for reliability assessment of PECs is satisfactorily valid and accurate owing to the negligible errors resulted from the Monte Carlo simulations. Sensitivity of PEC's IGBT reliability function under four different gradual-sudden dependence factors is presented in Fig. 12. As expected, the greater the dependencies, the lower the system ULE. B_{10} reliability of the considered PEC is declining from 44,800 to 38,040 hours as the gradual-sudden

TABLE 3. B_{10} reliability in four different initial occurrence rates for IGBT power module.

	$\lambda_0=25\times10^{-6}$	$\lambda_0=30\times10^{-6}$	$\lambda_0=35\times10^{-6}$	$\lambda_0=40\times10^{-6}$
B_{10} (hour)	44800	43630	41640	40150
Error	0.52%	0.48%	0.51%	0.49%

dependence factor, β, increases from 0.0003 to 0.0012. There is essentially no degradation-sudden dependency when β is relatively low. Since degradation of IGBT power module at the beginning of the PEC operation is comparatively low, a gradual failure rarely happens. Therefore, the dominant failure is the sudden failure due to vibration shocks. As β increases, the reliance on degradation-sudden is considered, the vibration amplitude will increase as the degradation processes proceeds.

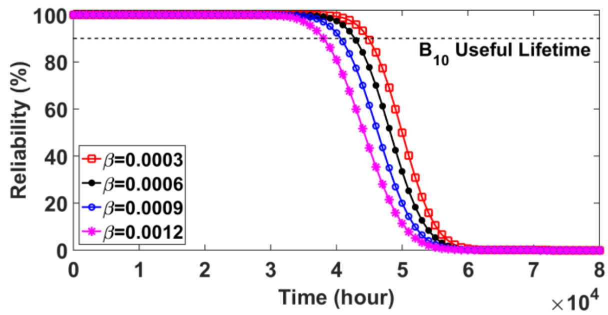

FIGURE 12. Sensitivity of PEC's reliability functions with four different gradual-sudden dependence factor.

Therefore, additional shocks will occur, resulting in more serious sudden failures. Due to the positive degradation-sudden dependency, sudden failure may happen even at the

$$P\left(E_1|N(t)=m,\ \boldsymbol{\theta}=x,\ \sum_{i=1}^{m}Y_i=u\right)=F(D;t)^m=\Phi\left(\frac{D-\mu_W}{\sigma_W}\right)^m\quad for\ m=0,1,2,\ldots,\infty \tag{A1}$$

$$P\left(E_2|N(t)=m,\ \boldsymbol{\theta}=x,\ \sum\nolimits_{i=1}^{m}Y_i=u\right)=\left\{\begin{matrix}1\ if\ at^2+bt+c+u<H_{IGBT}\\0\ if\ at^2+bt+c+u\geq H_{IGBT}\end{matrix}\right\}=\Phi\left(\frac{H_{IGBT}-(at^2+bt+c+u)+\mu_\varepsilon}{\sigma_\varepsilon}\right) \tag{A2}$$

$$\begin{aligned}P\left(N(t)=m|\,\boldsymbol{\theta}=x,\ \sum\nolimits_{i=1}^{m}Y_i=u\right)&=C^m_{\xi^{-1}+m-1}\times(1-exp(-\xi\,\Lambda_0(t;x)))^m\times(exp(-\xi\,\Lambda_0(t;x)))^{\xi^{-1}}\\&=C^m_{\xi^{-1}+m-1}\left(1-exp\left(-\xi\int_0^t\left(\lambda_0+\beta_{IGBT}\left(av^2+bv+c\right)\right)dv\right)\right)^m\\&\quad\times\left(exp\left(-\xi\int_0^t\left(\lambda_0+\beta_{IGBT}\left(av^2+bv+c\right)\right)dv\right)\right)^{\xi^{-1}}\end{aligned} \tag{A3}$$

$$f_Y^{<m>}(u)du=\varphi\left(\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y}\right)d\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y}=\varphi\left(\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y}\right)\frac{du}{\sqrt{m}\sigma_Y} \tag{A4}$$

$$f_\theta(x)dx=f_\theta(\psi_{IGBT})dx=\varphi\left(\frac{\psi_{IGBT}-\mu_{\psi_{IGBT}}}{\sigma_{\psi_{IGBT}}}\right)d\frac{\psi_{IGBT}-\mu_{\psi_{IGBT}}}{\sigma_{\psi_{IGBT}}}=\varphi\left(\frac{\psi_{IGBT}-\mu_{\psi_{IGBT}}}{\sigma_{\psi_{IGBT}}}\right)\frac{d\psi_{IGBT}}{\sigma_{\psi_{IGBT}}} \tag{A5}$$

beginning of the PEC operation, owing to the high probability of vibration shock ($\lambda_i(t) = (1+\xi i)(\lambda_0 + \beta \mathscr{R}_{IGBT}(t;\boldsymbol{\theta}))$). Regarding the PEC's specification and its mission profile, one can find that an increase in β may result in a 15.1% decrease in system's ULE. Therefore, the coupling parameters, β, have to be wisely chosen based on the experimental tests or the collected data in real-world applications the PEC is operated in.

VI. CONCLUSION

In this paper, a new reliability model for PECs has been proposed, based on which, the dependencies between the

$$
\begin{aligned}
R_{IGBT}(t) = &\sum_{m=0}^{\infty} F(D;t)^m \int_\theta \int_Y P\left(\begin{matrix}(\mathscr{R}_{IGBT}(t;\boldsymbol{\theta})+u) < H_{IGBT}|N(t)=m, \\ \boldsymbol{\theta}=x, \sum_{i=1}^m Y_i = u\end{matrix}\right) \times C^m_{\xi^{-1}+m-1} \\
&\times (1-exp(-\xi\Lambda_0(t;x)))^m \times (exp(-\xi\Lambda_0(t;x)))^{\xi^{-1}} \times f_\theta(x)dxf_Y^{<m>}(u)du \\
= &\sum_{i=1}^{m} \Phi\left(\frac{D-\mu_W}{\sigma_W}\right)^m \int_0^\infty \int_0^\infty \Phi\left(\frac{H_{IGBT}-(at^2+bt+c+u)+\mu_\varepsilon}{\sigma_\varepsilon}\right) \\
&\times C^m_{\xi^{-1}+m-1}\left(1-exp\left(-\xi\int_0^t\left(\lambda_0+\beta_{IGBT}\left(av^2+bv+c\right)\right)dv\right)\right)^m \\
&\times \left(exp\left(-\xi\int_0^t\left(\lambda_0+\beta_{IGBT}\left(av^2+bv+c\right)\right)dv\right)\right)^{\xi^{-1}} \varphi\left(\frac{\psi_{IGBT}-\mu_{\psi_{IGBT}}}{\sigma_{\psi_{IGBT}}}\right)\frac{d\psi_{IGBT}}{\sigma_{\psi_{IGBT}}} \times \varphi\left(\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y}\right)\frac{du}{\sqrt{m}\sigma_Y}
\end{aligned} \tag{A6}
$$

$$
\begin{aligned}
&P\left(E_1|N(t)=m,\ \boldsymbol{\theta}=x,\ \sum_{i=1}^m Y_i = u\right) \\
&= F(D;t)^m = \Phi\left(\frac{D-\mu_W}{\sigma_W}\right)^m \quad for\ m=0,1,2,\ldots,\infty
\end{aligned} \tag{A7}
$$

$$
\begin{aligned}
&P\left(E_2|N(t)=m,\ \boldsymbol{\theta}=x,\ \sum_{i=1}^m Y_i = u\right) \\
&= \left\{\begin{matrix} 1\ if\ et+f+u<H_{Cap} \\ 0\ if\ et+f+u\geq H_{Cap}\end{matrix}\right\} = \Phi\left(\frac{H_{Cap}-(et+f+u)+\mu_\varepsilon}{\sigma_\varepsilon}\right)
\end{aligned} \tag{A8}
$$

$$
\begin{aligned}
&P\left(N(t)=m|\,\boldsymbol{\theta}=x,\ \sum_{i=1}^m Y_i = u\right) \\
&= C^m_{\xi^{-1}+m-1} \times (1-exp(-\xi\Lambda_0(t;x)))^m \times (exp(-\xi\Lambda_0(t;x)))^{\xi^{-1}} \\
&= C^m_{\xi^{-1}+m-1}\left(1-exp\left(-\xi\int_0^t(\lambda_0+\beta_{Cap}(ev+f))\,dv\right)\right)^m \times \left(exp\left(-\xi\int_0^t(\lambda_0+\beta_{Cap}(ev+f))\,dv\right)\right)^{\xi^{-1}}
\end{aligned}
$$

$$
f_Y^{<m>}(u)du = \varphi\left(\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y}\right)d\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y} = \varphi\left(\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y}\right)\frac{du}{\sqrt{m}\sigma_Y} \tag{A9}
$$

$$
f_\theta(x)dx = f_\theta(\psi_{Cap})dx = \varphi\left(\frac{\psi_{Cap}-\mu_{\psi_{Cap}}}{\sigma_{\psi_{Cap}}}\right)d\frac{\psi_{Cap}-\mu_{\psi_{Cap}}}{\sigma_{\psi_{Cap}}} = \varphi\left(\frac{\psi_{Cap}-\mu_{\psi_{Cap}}}{\sigma_{\psi_{Cap}}}\right)\frac{d\psi_{Cap}}{\sigma_{\psi_{Cap}}} \tag{A10}
$$

$$
\begin{aligned}
R_{Cap}(t) = &\sum_{m=0}^{\infty} F(D;t)^m \int_\theta \int_Y P\left(\begin{matrix}(\mathscr{R}_{Cap}(t;\boldsymbol{\theta})+u) < H_{Cap}|N(t)=m, \\ \boldsymbol{\theta}=x, \sum_{i=1}^m Y_i = u\end{matrix}\right) \times C^m_{\xi^{-1}+m-1} \\
&\times (1-exp(-\xi\Lambda_0(t;x)))^m \times (exp(-\xi\Lambda_0(t;x)))^{\xi^{-1}} \times f_\theta(x)dxf_Y^{<m>}(u)du
\end{aligned} \tag{A11}
$$

$$
\begin{aligned}
= &\sum_{i=1}^{m} \Phi\left(\frac{D-\mu_W}{\sigma_W}\right)^m \int_0^\infty \int_0^\infty \Phi\left(\frac{H_{Cap}-(et+f+u)+\mu_\varepsilon}{\sigma_\varepsilon}\right) \\
&\times C^m_{\xi^{-1}+m-1}\left(1-exp\left(-\xi\int_0^t(\lambda_0+\beta_{Cap}(ev+f))\,dv\right)\right)^m \\
&\times \left(exp\left(-\xi\int_0^t(\lambda_0+\beta_{Cap}(ev+f))\,dv\right)\right)^{\xi^{-1}} \varphi\left(\frac{\psi_{Cap}-\mu_{\psi_{Cap}}}{\sigma_{\psi_{Cap}}}\right)\frac{d\psi_{Cap}}{\sigma_{\psi_{Cap}}} \times \varphi\left(\frac{u-m\mu_Y}{\sqrt{m}\sigma_Y}\right)\frac{du}{\sqrt{m}\sigma_Y}
\end{aligned} \tag{A12}
$$

simultaneous failures processes have been accurately taken into account. The proposed research presented in this paper reveals that not only does the gradual-sudden degradation dependance, but also the sudden-gradual degradation dependence plays signifcant roles on an accurate PEC's ULE. The results reported in this paper indicated that there was a 37.2% difference in estimating the PEC's ULE with and without capturing such mutual dependencies. This research was concerned with the reliability modeling and assessment of a DC-DC boost converter in a hybrid electric vehicle; however, the proposed analytics, presented results, and the drawn conclusions are generic, valid and applicable to any other power electronic converters exposed to dependent competing failure processes.

APPENDIX

With the assumptions that the magnitude of vibration loads and the corresponding damages are also normally distributed as $W_i \sim N(\mu_W, \sigma_W^2)$ and $Y_i \sim N(\mu_Y, \sigma_Y^2)$, one can re-evaluate equations (22)-(24) and (26) re-written as (A1)-(A5), shown at the bottom of the page 67105 and (A6)-(A12), shown at the bottom of the page 67106 for power IGBT module and power capacitor respectively ($\varepsilon \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$, $\mu_\varepsilon = 0$ and $\sigma_\varepsilon^2 \to 0$).

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