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# The Integration of Computational Thinking in Mathematics Education: The Current State of Practices in School, Outreach, and Public Educational Settings

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A thesis submitted in partial fulfillment of the requirements for the Master of Arts degree in Education

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## Abstract

This study investigates the integration of computational thinking (CT) in mathematics education by examining current CT practices in school, community outreach, and public educational settings to seek insight into further affordances of CT. A qualitative content analysis through a mix of inductive and deductive approaches is used to analyze online CT resources and computational artifacts. I interpreted the data through Kafai et al.'s (2020) framings of CT and a combination of constructionism, social constructivism, and critical literacy theories of learning. This study revealed that cognitive framing of CT (acquisition) receives greater attention compared to situated framing (participation), whereas the affordances of CT from critical framing (action) receive insufficient emphasis. The findings illustrate that using CT concepts and tools when incorporating real-world problems into mathematics instruction can improve the affordances associated with these three framings. These results will contribute to improvements in the integration of CT in mathematics teaching and learning.

## Keywords

Computational thinking, coding, mathematics curriculum, mathematics education, framings of computational thinking, real-world application

## Summary for Lay Audience

Computational thinking learning expectations are generally associated with problem-solving and thinking skills; however, they also improve many other skills, such as critical thinking, collaboration, communication, and citizenship. However, CT studies have mainly focused on a few of its benefits and mostly ignored its wide opportunities. This study sought a comprehensive understanding of these broader and deeper possibilities of using CT concepts and tools by investigating its current state in practices and real-life applications and implications (e.g., using CT concepts and tools to illustrate real-world problems) in mathematics teaching and learning. With this purpose, I ask what the current state of integration of computational thinking in mathematics education in school, outreach, and public educational settings is. Specifically, this study examines the online resources and the publicly shared computational artifacts used to illustrate real-world problems (e.g., simulating the spread of disease such as during a pandemic).

The findings of the study show that the most attention has been paid to understanding and using CT concepts and tools in improving understanding school mathematics curriculum. However, what has often been overlooked by practitioners is the potential for using CT concepts and tools to offer students a better understanding of real-world problems and provide wider opportunities for learning, such as enhancing mathematics knowledge and skills (e.g., problem-solving, reasoning), promoting social interaction, and fostering citizenship through mathematical modelling. By providing an understanding of the current state of CT in practices, this study provides new ideas to researchers, educators, and policymakers regarding the integration of CT in mathematics education. In this study, I explore possible ways of integrating CT into mathematics teaching and learning and draw attention to potential improvements for this integration.

## Co-Authorship Statement

This integrated article thesis consists of two research papers co-authored with Dr. Immaculate Kizito Namukasa. Chapter 3 has been published in the special issue “The COVID-19 Pandemic and Its Impact” of the Journal of Research in Innovative Teaching & Learning. A version of Chapter 2 has been submitted to the special issue “Integration of Technology to Advance Computational Thinking Education” in Educational Technology and Society (ET&S) and is currently under review.

During the submission of the manuscript, Dr. Immaculate Kizito Namukasa provided full support and assistance. The published research paper is as follows:

Chapter 3:

Sezer, H. B. & Namukasa, I. K. (2021). Real-world problems through computational thinking tools and concepts: the case of coronavirus disease (COVID-19). *Journal of Research in Innovative Teaching & Learning*, 14(1), 46-64. <https://doi.org/10.1108/JRIT-12-2020-0085>

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# Chapter 1

## 1 Introductory Chapter

This chapter presents an introduction to the study followed by a problem statement. The research purpose and questions also are provided along with a rationale for the study. The methodology of the study and the scholarly works related to the context are also presented in this chapter. The chapter concludes with an overview of the rest of the thesis chapters.

### 1.1 Introduction

The integration of computational thinking (CT) in K-12 education has become a global initiative (Bocconi et al., 2018; Gannon & Buteau, 2018). While integrating CT in the K-12 curriculum, the common approach is to put the emphasis on teaching programming and its applications like games, robots, and simulations (Kong, et al., 2019). Computing and computational ideas, however, can also play an important role in facilitating teaching and learning in other school disciplines like mathematics (Gadanidis, 2017b; Namukasa et al., 2017; Papert, 1980; Resnick et al., 2009) maintains that CT provides a variety of affordances, such as agency, access, abstraction, automation, and audience, in teaching and learning mathematics. The integration of CT concepts and tools in curriculum, therefore, specifically coding is gaining momentum. With this momentum, it has become necessary to explore the current understanding of integrating CT (i.e., coding) in school disciplines, such as mathematics, and to explore the further affordances of using CT concepts and tools in teaching and learning. It is, hence, essential to provide an overview of the current state of the integration of CT into school mathematics and an insight into the wide affordances of CT, which might be beyond the expectations of current curricula content and skills learning expectations.

This study asks what the current state of integration of computational thinking in mathematics education in school, outreach, and public educational settings is. To respond to this research question, this study examines the online resources on CT school and outreach practices and the publicly shared computational artifacts used to illustrate real-

world problems (e.g., simulating the spread of disease such as during a pandemic). This examination is helpful for understanding the aspects of CT depicted in this ecology, which support CT practices<sup>1</sup> in school, outreach, and public educational settings.

## 1.2 Problem Statement

*Mathematics is everywhere* has been used as an ongoing mantra in mathematics education (OME, 2018; OME, 2020). The meaning of this phrase has been discussed by many scholars (Copes, 2003; Guzmán, 2019; Klein, 2007). Copes (2003), for instance, states that “things all around us could be counted or added or measured or described through geometric shapes or Fibonacci numbers or even fractals. Is this really mathematics everywhere?” (p.43). Following Copes (2003), I want to highlight the question: Considering the role mathematics plays in every facet of life, what does this statement really mean?

According to Skovsmose (1994), the role mathematics plays in technological development demonstrates the diverse influences of its power on society. He also argues that the mathematics curriculum taught to learners plays a potential role in helping students to shape society by making it possible for them to interpret the world and to make changes in the world (Skovsmose, 2014). Given this perspective, mathematics can help them perceive the forces invisibly shaping their world and allow them to make more informed choices to impact the world (Fish, 2012). Given the way mathematics is currently taught, however, many students rarely see applications of their mathematical knowledge to tasks embedded in real-world contexts or reach the level of distinguishing “between using things in the world around us to do mathematics and using mathematics to understand the world around us” (Stocker, 2008, p. 11). Whereas mathematics plays a role in technological development such as in computational technologies, what is yet to be explored is the use of CT concepts and tools along with mathematics concepts and skills in computational settings either unplugged or virtual (e.g., through representation and decomposition) to help children and youth gain a deeper

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<sup>1</sup> In this study, CT Practice is used to define a practice that engages the learner with CT concepts and tools in teaching and learning.

understanding of the more complex real-world setting around us (Computer Science Teachers Association, 2011; Lee, 2012; Pöllänen & Pöllänen, 2019; Stocker, 2008; Wing, 2006).

CT, in essence, is a process of formulating problems and offering solutions through the utilization of computational steps (Aho, 2012; Wing, 2016), and it relates to mathematical thinking through the use of similar concepts, such as abstraction, algorithmic thinking, decomposition, pattern recognition, conditional logic (Bocconi et al., 2016; Gadanidis, 2015; Grover and Pea, 2013). Papert (1971) defines CT as something which students can “manipulate, extend, [and] apply” and by doing so acquire a thorough comprehension of the world (p.10). Expanding on CT, Kafai (2016) mentions computational participation, which incorporates “solving problems, designing systems, and understanding human behavior in the context of computing.” (p.26). Further, Tissenbaum et al. (2019) refer to computational action, and state that “while learning about computing, young people should also have opportunities to create with computing that have a direct impact on their lives and their communities,” in a way that makes “computing education more inclusive, motivating, and empowering for young learners” (p. 34).

Papert (1993) observes that the more learners take charge of their learning, the better learning takes place. Guzdial et al. (2019) also stress the substantial impact of computing on students’ lives. Guzdial et al. (2019) further stress the necessity of redesigning computing to empower students in making sense of the world as well as changing it. They further mention that “we might get expanded thinking if we follow along the lines of extending mathematics and systems organizations to model complex situations that go beyond our commonsense reasoning, as seen in many scientific, engineering, medical, mathematical, and literary fields. Computing simulations have already revolutionized many fields [especially in science and engineering]. We might significantly impact society if all fields used this expanded thinking” (p.29).

Many examples of CT concepts and tools are used to model and simulate mathematics in the real world. Many of these simulations modelling real-life events have been shared in



online communities centered on the use of coding or simulation apps. The Scratch<sup>2</sup> community, for instance, is a “vibrant online community, with people sharing, discussing, and remixing one another’s projects” (Resnick et al., 2009, p. 60). This community uses a block-based programming language, Scratch, which is designed to be accessible to users of all ages and backgrounds to create programs, such as interactive stories, animations, simulations, or even games, and share their work with others (Resnick et al., 2009, p.60). Scratch has been applied for modelling simple mathematics problems, such as dice rolling (Brickware, 2013), or more complex problems, such as modelling the spread of the COVID-19 pandemic (Iainbro<sup>3</sup>, 2020; Resnik, 2020).

Several examples that use coding tools (e.g., graphical illustration, simulations, and data maps) to illustrate real-world problems have also been shared in school contexts. For instance, the National Council of Teachers of Mathematics (NCTM) Association published a mathematical simulation modelling the spread of the COVID-19 pandemic to be used as part of the activities for Grades 5-12 (NCTM, 2020). This simulation<sup>4</sup>, which models the spread of a virus through social contact, was created based on four variables: the population size, the number of days contagious, the number of physical contacts between an infected person and others, and probability of the virus transmission from an infected person to a healthy person by physical contact. Through this activity, students have a chance to manipulate these four variables to see the outcomes of different scenarios. This NCTM (2020) mathematical simulation uses CT graphical illustration, simulations, and data maps to model a real-life situation. Through this simulation, learners may experiment by changing any of the four variables to see the effect of these variables on the rate of spread of the disease in the population, which is visualized in the data map. Although in this model, students may not add new variables, they can tinker with how the model works or modify it to create their own model. Hence, learners are

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<sup>2</sup> <https://scratch.mit.edu/>

<sup>3</sup> Scratch simulation authors usually use nicknames instead of their real full names.

<sup>4</sup> Page source: <https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Pandemics-How-Are-Viruses-Spread/> - Framesource: <https://www.nctm.org/tools/pandemic2020/index.html>

afforded the opportunity to use and learn mathematics and CT representations using a simulator of a real-life setting. There are also simulations that support classroom teaching and allow students to modify and create their own models on real-world issues. For instance, the *civilization: sustainable growth app*<sup>5</sup> lets students to code a sustainable system based on some elements of the system (e.g., fertilization, food chain). By using coding constructs (e.g., functions, conditional logic, parameters) and mathematical concepts (e.g., number sense, operations, geometry), this app allows students to create their own code, share this code, and hence remix one another's code. These actions empower students' communication and collaboration skills while improving their mathematical and CT knowledge and enhancing their awareness of critical environmental issues (Eduapps, n.d.).

Advocates for teaching and learning how CT concepts and tools (e.g., graphical illustration, simulations, and data maps) are used argue that focusing on these concepts and tools will contribute significantly to the development of a broader digital competency and literacy (diSessa, 2000, 2018). Therefore, there is a necessity to frame CT beyond a basic understanding of it as a process of expressing solutions to problems as computational steps or algorithms (Aho, 2012). Yet, within the curriculum, it is crucial to include a detailed understanding of the digital technologies together with the values, biases, and histories which shape them (Kafai et al., 2020). diSessa (2018) stresses the substantial influence of CT, including its socio-cultural influence, and views CT as a resource that is helpful for all disciplines. Wilson (2019) mentions that because students of all ages will acquire a variety of critical and transferable skills along with coding during the CT activities, it makes great sense to consider coding to enhance students' learning whenever and wherever possible. Similarly, Pöllänen and Pöllänen (2019) observe that using CT knowledge and skills students learn to choose the appropriate representation and to find the small steps to improve intellectual skills that are potentially transferable. As a result, CT would benefit society in all social, economic, and scientific fields.

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<sup>5</sup> <http://eduapps.ca/civilization/growth/>

Kafai et al. (2020) note the need to emphasize more than just one aspect of CT. Lodi and Martini (2021) also mention that the focus should not be only on CT's cognitive aspects, but also “on the immense educational value of social and affective involvement of the student into the construction of a (computational) artifact, and on the value of computation as an interdisciplinary tool for learning” (p. 904). Currently, however, existing scholarly works and school practices on the integration of CT concepts and tools into mathematics widely focus on only one aspect, namely, how to use CT concepts and tools, to foster the learning of mathematics concepts and skills (Barcelos et al., 2018; Hickmott et al., 2017; Rodríguez-Martínez et al., 2019). While CT concepts and skills have been emphasized in the literature, the importance of determining what students might do using these skills has unfortunately been ignored (Haduong, 2019). Resnick (1995) emphasizes that computational paradigms “can significantly influence not only what people do with computers, but also how they think about and make sense of the world” (p. 31). Considering the computational literacy (diSessa, 2018), computational participation (Kafai et al., 2020), and computational action (Tissenbaum et al., 2019) opportunities of CT, it is essential to encourage students, by using CT concepts and tools (Lee & Soep, 2016), to question the underlying purposes, values, goals, and impacts of what they do as well as to explore solutions together with the values, and histories which shape these solutions (diSessa, 2018; Kafai et al., 2020).

With the view that the integration of CT in mathematics education should aim beyond just fostering mathematical understanding of existing mathematics curricular expectations by using CT concepts and tools in mind, I draw on diverse framings of CT, and online resources on school and outreach practices of CT, and the publicly shared computational artifacts used to that illustrate real-world problems. This study intends to provide new ideas on how to improve the integration of CT in mathematics education by also focusing on the broader and deeper possibilities of this integration. In order to fulfill the wide goals of integrations of CT in learning and use of mathematics, I examine available professional and scholarly online resources (i.e., professional articles, events, projects, symposiums, blogs, documentaries, videos, etc.) informing and informed by the integration of CT in mathematics instruction in schools. I also sample online resources informing mathematics instruction in outreach contexts. Further, I examine selected

computational artifacts for understanding and illustrating real-world phenomena in public contexts.

In this study, main research question is: What is the current state of integration of computational thinking in mathematics education in school, outreach, and public educational settings? The following two sub-questions frame the research:

1. What is the understanding of integrating computational thinking in Grades 1-9 mathematics education in Ontario?
2. In what ways do computational thinking concepts and tools help students comprehend real-world problems?

### 1.3 Significance of the Study

This study provided an overview of the integration of CT into mathematics teaching and learning as presented in practice, in curricula, and in real-life applications. I examined the online resources on CT practices conducted in Ontario, as well as the sample computational simulations of mathematical models used in public setting.

This examination identified further affordances of CT that met and exceeded the expectations of curricula and outreach content and skills. From these affordances, I extrapolate possible recommendations to the improvements in the integration of CT in mathematics teaching and learning that would allow school curriculum to integrate all different framings of CT (as detailed in Chapter 4). Recently there has been attention paid to the integration of coding to new curricula in Ontario, across Canada, and internationally (Bocconi et al., 2018; Gannon & Buteau, 2018; Gadanidis et al., 2021); therefore, this study is useful to researchers, educators, and policymakers in improving their understanding of integrating CT into mathematics curricula.

### 1.4 Reflexivity of the Researcher

Reflexivity is defined as the introspective examination of how an individual's subjectivity may have influenced the research process (Palaganas et al., 2017, p. 427).

The values of researchers must be reflected in their research (Mertens, 2005), and my previous experiences in mathematics education and my accumulated knowledge on CT have shaped this research. As a former Grades 6-8 mathematics teacher, one of my main concerns has been engaging students with real-life problems and motivating them to provide solutions to these problems. However, mathematics in real life is more complex than school mathematics. To seek a solution to this challenge, I was committed to continuing my professional learning in graduate studies. During my first year at Western University, I became aware of the wide range of possibilities and opportunities in integrating CT into mathematics to help students to question, reflect, comprehend, analyze and interpret, and by doing so shape their understandings of the world around them (Sezer & Namukasa, 2021). The Computational Thinking in Mathematics and Science Education course I took sparked my interest to explore more about CT and consequently, I decided to conduct this study.

Initially, I was interested in broader and deeper affordances of CT depicted in scholarly and professional works, so I analyzed current CT practices by using Kafai's (2020) framings of CT (see Chapter 2). Since I conducted this research during the onset of the pandemic, I witnessed a huge increase in the number of COVID-19 simulations. Most of them employed mathematical models in which use computational tools. This sparked my interest in conducting further research applications which CT knowledge, concepts and tools, such as those involved in coding, can be used to help students comprehend the mathematical models of the outbreak that could embrace the different framings of CT (see Chapter 3).

## 1.5 Method of the Study

The focus of qualitative research is on humans' experiences, comprehension, and interpretations (Sandelowski, 2001). Qualitative research is "a form of social inquiry that tends to adopt a flexible and data-driven research design, to use relatively unstructured data, to emphasize the essential role of subjectivity in the research process, to study naturally occurring cases in detail, and to use verbal rather than statistical forms of approach" (Hammersley, 2013, p. 12). It seeks to offer an understanding of the world

through the artifacts of the people being studied, and the essence of this approach lies in the things people say and do (Morrison et al., 2012). Therefore, a qualitative research design is a primary way to gain a comprehensive understanding of the context of CT in mathematics teaching and learning through wide-angle lenses which include the perspectives and practices of researchers, educators, and policymakers, as well as the public (Johnson & Christensen, 2016).

A qualitative content analysis was used in this study to explore the practices of the integrating of CT in mathematics education. Content analysis, as a research technique, makes inferences from textual material to their context of use by converting the qualitative data into quantitative data through coding and then interpreting this data (Flick, 2009; Krippendorff, 2004). To understand the subject matter of this study better, qualitative data was collected from a variety of written and visual resources (e.g., articles, events, projects, symposiums, blogs, documentaries, videos). The analysis of online resources in the school and outreach settings, and computational artifacts aimed to provide a picture of CT applications and implications in mathematics teaching and learning.

My supervisor and thesis committee member were involved in data analysis to make sure that proper procedures were being carried out and reasonable conclusions had been made.

## 1.6 Limitations

The scope and the nature of the research questions have shaped this research project. Limitations of this study arise from two main areas: the scope of the study and the method of the study. First, this research focused on the existing understanding of practices and applications of CT in connection with Grades 1-9 mathematics curricula, specifically with a focus on Ontario. Despite this limited geographical context, its content can be applied more broadly to other disciplines and regions with similar curricular contexts. In other words, it has the potential to be generalized through transferability (Frey, 2018). Coding expectations, for instance, have been integrated under STEM strands in the new Ontario science and technology curriculum (OME, 2022); therefore, having the same scope and related context, this study's findings on mathematics

education are applicable to science education. Second, this study consulted only online written and visual resources, and artifacts. Although, this study does not include various data collection methods, the data set includes various materials (e.g., articles, events, projects, symposiums, blogs, documentaries, videos) and these materials should be considered an extension of the researchers, professionals, policymakers, and real-life users of CT who created them (Saldaña, & Omasta, 2017). Therefore, analyzing these online resources provided insights into the understandings of their creators. These two concerns were alleviated through a deep and thorough study of the resources to extract valuable insights and information, which helped to answer the research questions. The measures taken to maximize the trustworthiness of the study are detailed in the following section.

## 1.7 Trustworthiness of the Study

The validity of qualitative data requires attention to the following aspects: “the honesty, depth, richness, and scope of the data achieved, the extent of triangulation and the disinterestedness or objectivity of the researcher” (Cohen et al., 2018, p. 246). To ensure validity, this study used different data sources which include written documents, visual materials, artifacts used to capture multiple aspects of the integration of CT in mathematics teaching and learning.

In qualitative research, determining the level of rigour is challenging and counterintuitive; therefore, the essence of reliability for qualitative research depends on consistency (Carcary, 2009; Grossoehme, 2014; Leung, 2015). To address reliability and reduce research bias, I consulted other studies that used qualitative research design to explore CT perspectives and practices. Moreover, while interpreting the findings, I referred to the research questions for this study to ensure that these findings are consistent with what this study intended to investigate.

Additionally, member checking was used in this study, which helps in establishing “the credibility, transferability, dependability, and confirmability of the data and the findings” (Cohen et al., 2018, p. 645). Using member checking, which is also known as respondent or member validation, as a cross-checking strategy improves the validity of the findings

in qualitative research (Frey, 2018; Given, 2008). Member checking can be used while collecting data as well as afterwards when the analysis is completed (Hamilton & Corbet, 2013). In my study, following the grounded theory, I started my analysis with open coding. I recorded the initial codes that were obtained from the data set. Then, based on the linkages, axial codes were created which was followed by generating core categories. From the beginning to the end of this process, my supervisor and I conducted follow-up meetings during which we evaluated how accurate, correct, and comprehensive these coding steps and the interpretations of the data were. A coding agenda, based on Kafai et al.'s (2020) framings of CT, was employed to provide clear and explicit definitions, examples, and coding rules to use during member checking process. Afterwards, I invited the supervisor and thesis committee member who are to critically review the interpretations of the findings. Lastly, to address transferability, which is the ability to transfer the research design and results from this study into other situations, the research processes (collection and analysis of data as well as the presentation of results) are reported in detail in each chapter (Dick, 2014; Maxwell & Chmiel, 2014).

## 1.8 Ethical Considerations

Research ethics review is not required for this study since the information used is in the public domain. Besides, there is no reasonable privacy expectation in this information even if it may contain identifiable information (Social Sciences and Humanities Research Council of Canada, 2018).

## 1.9 Thesis Narrative: An Overview of Chapters

This research report is in the format of an integrated article thesis consisting of four chapters. This first chapter has been introductory and presented the problem statement, research purpose, and research questions. The methodology, trustworthiness of the study, ethical considerations, significance, limitations, reflexivity of the researcher of the study, and the literature review of the study were also presented in this chapter. Chapter two focuses on the first sub-question and reports the results of content analysis of online resources related to school and outreach practices of integrating CT in mathematics education. This chapter provides the findings from an examination of online



resources related to CT applications and implications in mathematics teaching and learning. Chapter three answers the second sub-question and studies the role of computational and mathematical tools in understanding and illustrating the pandemic. This chapter examines the use of computational simulation of mathematical models in public setting to respond to the current global health crisis. Chapter three has been published in the *Journal of Research in Innovative Teaching and Learning*. The fourth and final chapter is integrative and provides a reflection of Kafai et al.'s (2020) framings of CT on the basis of previous chapters. This chapter concludes with contributions and recommendations for practice and policy, suggestions for future research, and concluding remarks.

## 1.10 Literature Review

To lay a foundation for this study, this section focuses on perspectives and practices of integrating CT in mathematics teaching and learning.

### 1.10.1 Origin of CT

*Computational thinking* as a term was introduced by Papert (1971, 1980, 1996). Papert (1971) describes CT as “something children themselves will learn to manipulate, to extend, to apply to projects, thereby gaining a greater and more articulate mastery of the world, a sense of the power of applied knowledge and a self-confidently realistic image of themselves as intellectual agents” (p.1). According to Papert (1971), computation is the richest source for learning by doing, and the development of educational environments facilitating the use of the computer as a tool for computation should be promoted. He mentions that “[giving] children unprecedented power to invent and carry out exciting projects by providing them with access to computers, with a suitably clear and intelligible programming language and with peripheral devices capable of producing online real-time action” (Papert, 1971, p. 2). Bull et al. argue that there are two key ideas to Papert’s vision of CT. First, the act of programming helps to develop skills, such as breaking down problems into smaller units that are easier to handle, that facilitate learning of other subjects, such as mathematics. Second, there should be computer

languages specifically developed for the purpose of helping the learning process of students.

Although the integration of CT in school disciplines was previously researched following the work of Papert, it is Wing (2006), who popularized it by drawing attention to the value of “thinking like a computer scientist” (p. 34). Wing’s central focus is the thought process of a “computer-human or machine” (Wing, 2014, para. 5), whereas the focus of Papert (1980) is “not on the machine but the mind”. Papert (1980) argues that in the process of constructing things in the physical world, new ideas and theories will be developed in the minds of the children, which will inspire them to construct new things, and this process will repeat itself again and again (Papert, 1980). This difference between the focus on the machine and on the mind could be seen as the difference between the focus on CT in computer science disciplines and in other school disciplines. Wing’s CT sets the focus on the importance of teaching computer science, since CT exists as a result of computer science, and it provides method and unity to CT (Lodi and Martini, 2021). Lodi and Martini (2021) point out that Wing’s pure disciplinary approach would limit the potentialities of CT for learners. Wing (2006) claims that CT approaches are going to be fundamental for all disciplines and fields since with the help of computing technologies researchers can come up with new problem-solving strategies and can test the strategies in both real and virtual settings. Further, diSessa (2018) states that CT is influential precisely because it involves more social and cultural elements, hence it is useful for all disciplines and to society.

CT is broader than programming or coding as it includes a higher level of abstraction, the use of mathematical thinking to design and develop algorithms, and the evaluation of the effectiveness of solutions on different scales of problem complexity. Coding, in particular, defined as computer programming knowledge, stands at the core of all CT skills and abilities in learning (Mecca et. al., 2021). Stephens (2018) describes coding as a formal way of creating and running algorithms and programming as the process of using a logic-focused mindset to develop instructions for a computer to perform. In this study coding and programming could be seen as a way of discovering the fundamentals of CT (Nardelli, 2019). Burke et al. (2016) note that as an essential component of CT,

coding is the most powerful way to make use of a computer and to have a digital presence. For example, students can create their own applications in a coding environment like Scratch, and share them to interact with online communities, where the participants can investigate others' projects, comment on them, and more importantly use them to create their own project (Kafai, 2016).

### 1.10.2 The Integration of CT in Mathematics Teaching and Learning

Following the invention of computers, scholars in education have explored how to make use of coding and computer programming within CT to improve mathematics teaching and learning (diSessa, 1985; Feurzeig et al., 1970; Harel & Papert, 1990; Papert, 1971, 1980, 1996; Solomon, 1986). Logo, for instance, was designed as a computer programming language for use in teaching students mathematical thinking by using suitable activities to introduce them to programming ideas. Since the 1970s, research on Logo expanded to include both geometry and algebra concepts and problem-solving strategies (Battista & Clements, 1986; Clements, 1985; Feurzeig & Lukas, 1971; Hoyles & Noss, 1992; Hoyles, & Sutherland, 1989). Jones (2005) reports that the body of research suggests that students build connections between spatial and algebraic thinking when they work with Logo and interact with visible and quantifiable objects. Papert's legacy continues to grow at the Massachusetts Institute of Technology (MIT) Media Lab., where Mitchel Resnick developed the Scratch programming language, many of whose users, as indicated earlier, join the online Scratch community.

Today, increasingly sophisticated computational tools are commonly used to model processes ranging from science to policy and economics (Gadanidis & Caswell, 2018; Gadanidis & Cummings, 2018). Further computational tools have also extended the range of nonlinear phenomena (e.g., climate change, the spread of disease) that can be explored mathematically and simulated through models (Weintrop et al., 2016). Computational artifacts, that utilize recent tools for coding (such as block-based and text-based programming languages) have been used to illustrate mathematical models. This provides opportunities for users, in the public including students, to use, modify and create the models to experiment and understand the dynamics of the different scenarios such as sustainability and health care. (e.g., Eduapps, n.d; Resnick, 2020). The *Scratch epidemic*



*simulator*<sup>6</sup>, for instance, demonstrates the effect of staying at home on the rate and risk of the spread of disease based on the mathematical Susceptible-Infectious-Recovered (SIR) model. This simulation is not specific to mathematics but includes key mathematical concepts, namely:

- coordinate geometry for representing the location of the individuals and for the grapher (i.e., showing increases in the infection rate)
- angle measurements for illustrating the direction of the movement of individuals
- mathematics operations for counting increments in the steps
- the probability operation of picking a random angle for the direction of the turn and the number of people staying at home
- and an algebraic equation, which is a function of the sick individuals, for the path of the grapher


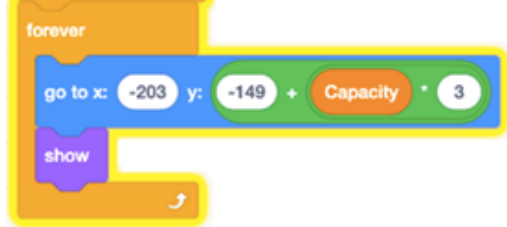
In Chapter 3, I elaborate on samples of CT concepts used to code mathematics concepts in this and in another Scratch simulator of an epidemic (Iainbro, 2020). The samples of the mathematical and computational concepts included in these simulations are given in Figure 1 and Figure 2. This model also integrates key CT concepts of block-based programming, which offer the potential for learners to learn and use these coding concepts and to modify the code so as to experiment with a different set of scenarios. For instance, I illustrate coding blocks for illustrating movements, transmissions, simulations, recovery, graphical representations, parameters, initialization steps, health care capacity, and recover steps (see in Chapter 3).

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<sup>6</sup> Source: <https://www.media.mit.edu/posts/scratch-epidemic-simulator/> (Resnick, 2020)

Probability and angle measurements	Algebraic equation and coordinate geometry
	
<p>Source:  <a href="https://scratch.mit.edu/projects/376656449/">https://scratch.mit.edu/projects/376656449/</a>            (Resnick, 2020)</p>	<p>Source: <a href="https://scratch.mit.edu/projects/376656449/">https://scratch.mit.edu/projects/376656449/</a>            (Resnick, 2020)</p>

**Figure 1: The samples of the mathematical concepts included in Scratch simulation of an epidemic**

Conditional logic (if-then-else)	Repetition (creating a loop)
	
<p>Source:  <a href="https://scratch.mit.edu/projects/376656449/">https://scratch.mit.edu/projects/376656449/</a>(Resnick,            2020)</p>	<p>Source:  <a href="https://scratch.mit.edu/projects/380357960/">https://scratch.mit.edu/projects/380357960/</a>            Iainbro, 2020</p>

**Figure 2: The samples of the CT concepts included in Scratch simulation of an epidemic**

Namukasa et al. (2021) state that the integration of CT and mathematics empowers mathematical learning through CT concepts, tools, and ideas, such as “iterative design, decomposition, simulation, algorithmic thinking, tinkering” (p. 287). In that regard, learners may be taught to use CT concepts (e.g., abstraction, decomposition, pattern recognition, algorithms) and tools (e.g., coding through text-based or block-based programming languages, robots, apps, games) to make simulations and to provide

solutions to mathematics problems (Nardeli, 2019; Sanford, 2013; Sanford & Naidu, 2017). In a school context, using CT concepts and tools, especially coding, can be used to model and visualize mathematics processes in the real world. CT can provide extensive support to build and test models as part of the solution of complex problems (Gadanidis, Hughes et al. 2017, p.78).

In the Ontario mathematics curriculum, for instance, coding is associated with the process of mathematical modelling under the algebra strand (OME, 2020). Coding experiences are exemplified as “from representing movements on a grid, to solving problems involving optimization, to manipulating models to find which one best fits the data they are working with in order to make predictions” (OME, 2020, p. 85). Further, coding can be helpful in all strands, as it provides opportunities for students to apply their knowledge and to improve their thinking and communicating skills. These opportunities are exemplified for Grade 3 in unplugged practice, which includes students moving in the classroom following a set of given directions. During this activity, students explore coding, improve their skills in other strands (i.e., spatial sense-location and movement and social-emotional learning-oral communication), and make connections to other disciplines (i.e., social studies map and globe skills continuum). This activity also includes a discussion of real-life applications such as reading a map, coordinates on a grid, Global Positioning System (GPS), dance routines, and quarter and half turns on an analog clock (Ontarimathsupport, n.d).

### 1.10.3 CT Affordances in Mathematics Teaching and Learning

Incorporating CT affordances (e.g., abstraction, automation, dynamic modelling) and tools (e.g., Scratch, Sphero) allows mathematics learners to gain a tangible feel of abstraction to automate the process (e.g., creating code once to draw polygon through Scratch then using and modifying same code to draw other polygons) and to apply dynamic modelling to “[investigate] relationships, pose and test what-if questions” (Gadanidis, Brodie, et al., 2017, p.3; Wilson, 2019). Abstraction, which is at the heart of mathematics, facilitates dynamic modelling and helps students to understand the nature of a relationship they are interested in and to generalize their results to other patterns and relationships. Moreover, through these possibilities, students are provided opportunities

to explore different scenarios, test the conjectures they made, and improve their mathematical insight regardless of their initial level of knowledge (Gadanidis et al., 2018). This resonates with Papert's (1993) discussion which states that "the best learning takes place when the learner takes charge" (p. 25). Lee et al. (2011) suggest that youth engaging with the rich CT environment usually follow a three-stage progression, i.e., use, modify (edit), and create. They emphasize that key aspects of CT, abstraction, automation, and analysis, are instrumental especially in the "create" stage. Lee et al. (2011) identify these key aspects in the example drawn from modelling and simulation as follows:

- "Selecting features of real-world to incorporate in a model" (abstraction),
- "Time stepping using a model as an experimental testbed" (automation)
- "Reflective practice that refers to the validation of whether the abstractions made were correct" or "the model reflects reality" (analysis) (p.33).

When students use CT tools such as coding in mathematics, they are afforded greater student agency, engagement, and access to mathematics concepts that would otherwise be more advanced for their grade levels (Gadanidis, 2017; Gadanidis, Hughes, et al., 2017; Namukasa et al., 2017; Sanford & Naidu, 2016). For instance, Mulder (2017), a prospective mathematics teacher, mentions using programming to solve Buffon's Needle Problem (a problem in the field of geometrical probability) and how the freedom to choose the solution that makes the most sense to her instead of getting instructions from the professor made her feel a true sense of agency. The richness of mathematical problems allows for addressing the cases where specific programming commands are required, and the discussion of the mathematics that is explicitly or implicitly inherent to these programming commands enhances the understanding of both mathematical and programming concepts (Tepylo & Floyd, 2016). For instance, students use robots, generally made of Lego pieces, as a learning tool for representing geometry concepts (e.g., drawing or stamping geometrical shapes) and spatial recognition (e.g., defining a path). Besides, students get a better understanding of number concepts (e.g., multiplication facts) by coding the mathematics machines which are capable of

performing calculations and understanding patterns by using the coding environment to build mathematical art with repeating patterns (Namukasa, 2018). In addition to cultivating confidence in learners' mathematical ability and agency (Gadanidis, Brodie, et al., 2017), coding also increases collaboration and creates a sense of community and common purpose (Gadanidis & Caswell 2018). For instance, Scratch which has online libraries that enables sharing codes with others as well as using or remixing the codes of others, provides students "a low floor", so very little knowledge is required for engagement, "a high ceiling", so it provides opportunities that extend to high levels, and "wide walls", so it is possible to have multiple different types of projects and people can easily find one that fits their interests or learning style (Gadanidis, 2017a, p.136; Resnick, 2009).

Gadanidis et al. (2021) report that access to coding provides significant new opportunities "to remediate, reformulate, reorganize and revitalize mathematics education" (pp.1, 6), which they draw from diSessa (2018), who also argues that computer applications not only provide a simulation of all resources to satisfy the curiosity of students but also allow students to express themselves through these applications' interactive nature (diSessa, 2000). Many scholarly works focus on the possibilities of using CT concepts and tools at all levels of mathematics education. Harel and Papert (1990), for instance, observed that with the integration of computing into teaching fractions, students constructed better understanding in both mathematics and programming. Rodríguez-Martínez et al. (2019) show that the Scratch programming environment makes mathematical ideas accessible to primary school students through the use of basic CT concepts (e.g., use of sequences, iterations, conditionals, and events-handling for greatest common divisor and the least common multiple in solving word problems). Similarly, Sung et al. (2017) show that activities that involve a CT perspective of mathematical problem solving, improve students' mathematics and programming skills. Dunbar and Rich (2020) point out four benefits of using robots (i.e., Sphero) for teaching mathematics: "(1) reasonable time frames; (2) authentic purposes for mathematics; (3) visual and nonevaluative feedback; and (4) introduction to computational thinking" (p. 565). Pei et al. (2018) examine the opportunities provided



by the design of Lattice Land, interactive software that helps learners to uncover geometrical concepts and to build their habits of mathematical thinking, such as tinkering, experimenting, and recognizing the patterns through practices of CT. Psycharis et al., (2017) show the significant difference in the reasoning skills and the self-efficacy of students in their experimental study on the impact of computer programming at high school mathematics. In their study, students used the Matlab software tool to develop a code based on algorithmic thinking, which is used to solve mathematic problems and to generate the graphical representation that illustrates the accuracy of the code as well as the exercise's solution.

The wide opportunities of using CT concepts and tools in mathematics teaching and learning are also supported by professional publications. For instance, London District Catholic School Board educator Richard Annesley, a participant of the Collaborative Learning Community (CLC) project, asserts that learning coding has improved his students' confidence level in mathematics and helped them to find new ways to think. Moreover, they were very enthusiastic about sharing their projects and exchanging their thoughts about each other's projects to improve them (Teachontarioteam, 2016). Brodie (2015) has observed similar benefits of coding in his Grades 7 and 8 classrooms at St. Andrews Public School in the Toronto District School Board. He (2015) asserts that students have demonstrated their mathematical knowledge and understanding through the programs they write in Scratch. Students discovered a variety of solutions to coding challenges (such as using loops, using their own coded blocks), and they gained a much deeper understanding of the topics in geometry through coding challenges compared to using only paper and pencil to solve these challenges.

#### 1.10.4 The framings of CT for designing learning and teaching

Based on current literature on CT implications, Kafai et al. (2020) define three conceptual framings of CT, namely cognitive, situated, and critical, which are identified according to the focus of researchers, practitioners, or policymakers. Cognitive framing of CT focuses on “skill-building and competencies” and provides an “understanding of key computational concepts, practices, and perspectives”. Situated framing of CT emphasizes the importance of “personal creative expression” and “social engagement” of

the teaching and learning process of CT. Critical framing of CT, on the other hand, focuses on critical approaches to CT and stresses the necessity of providing an “analytical approach to the values, practices, and infrastructure underlying computation as part of a broader goal of education for social justice and ethics” (Kafai et al., 2020, p.102).

Cognitive CT (i.e., short version of cognitive framing of CT) is widely seen in the literature, and it commonly refers to using CT concepts and tools as a way of problem-solving (Grover and Pea, 2013; Sanford et al, 2017; Weintrop et al., 2016, Wing, 2006). Cognitive CT particularly emphasizes using computational concepts (e.g., iteration and recursion, modelling and simulation, tinkering, pattern recognition, decomposition, determining, defining algorithms, debugging) through programming tools (e.g., Scratch, Python, Latticeland, Math+C app, GeoGebra) to enhance mathematics knowledge and skills (e.g., problem-solving, analytical thinking, reasoning, and data analysis and modelling skills) (Aydeniz, 2018; Ghosh, 2019; Ho et al., 2019; Pei et al., 2018; Weintrop et al., 2016). For instance, Ghosh (2019) explains the iteration and recursion process through the construction of the Pythagorean tree using GeoGebra. He mentions that “[t]he initial steps of the iterative process were demonstrated to students using diagrams on the whiteboard. Subsequently, they constructed various stages of the tree on GeoGebra using the ‘create tool’ feature which enables recursive constructions” (p.6).

Situated CT (i.e., short version of situated framing of CT) emphasizes collaboration and participation (Deborah et al., 2015; diSessa, 2018; Wilkerson, 2014). Studies pertaining to this framing also focus on the cultivation of learners’ confidence in their mathematical ability or agency (Gadanidis, Brodie, et al., 2017), and the improvement of collaboration to build a sense of community, and the creation of a common purpose (Burke & Kafai, 2012; Fields et al., 2015). In that regard, Brenna and Resnick (2012) draw attention to situated CT’s affordances through using Scratch, which allows “young people to create their own interactive stories, games, and simulations, and then to share those creations in an online community with other young programmers from around the world” (p.1).

In the context of critical CT (i.e., short version of critical framing of CT) is about recognizing people's thoughts about situations and problems in the world and offering ideas and solutions to those problems and students learning both curriculum and disciplinary concepts (Kite & Park, 2018; Lee & Garcia, 2014). Margolis et al. (2012) maintains that activities should support students' agency in making creative and authentic computational artifacts that address critical issues which could be selected to be personally relevant to the students, such as social justice issues. Green Dot Public Schools, for instance, provides rich sources for the integration of critical CT into the curriculum. One of the activities they have designed for Grade 7 students is on racial bias in police traffic stops. In this activity, students learn about random sampling, population sampling, sampling variability, compound probability, and tree diagrams through social context (i.e., racial profiling by the police forces and individuals' legal rights pertaining during traffic stop situations). By making use of the primary CT concept of pattern recognition, students are given the opportunity to use real-world data to determine whether there is significant evidence of racial bias in data on traffic stop situations (CT Lessons and Projects from Green Dot Public Schools, n.d.).

The reflection of cognitive, situated, and critical framings of CT in mathematics teaching and learning is further discussed in Chapter 2. Chapter 3 provides the analysis of two computational simulations to present a way of engaging students with real-life mathematics with the inclusion of these framings' aspects.

## 1.11 Chapter references

- Aho, A. (2012). Computation and computational thinking. *The Computer Journal*, 55(7), 832-835. <https://doi.org/10.1093/comjnl/bxs074>
- Aydeniz M. (2018). Integrating computational thinking in school curriculum. In: Khine M. (Ed.) *Computational thinking in the STEM disciplines*. Springer. [https://doi.org/10.1007/978-3-319-93566-9\\_13](https://doi.org/10.1007/978-3-319-93566-9_13)
- Baker, S. C., Wentz, R. K., & Woods, M. M. (2010). Using virtual worlds in education: Second life as an educational tool. *Teaching of Psychology*, 36(1), 59-64. <https://doi.org/10.1080/00986280802529079>
- Barcelos, T., Munoz, R., Villarroel, R., Merino, E., & Silveira, I. (2018). Mathematics learning through computational thinking activities: A systematic literature review. *Journal of Universal Computer Science*, 24(7), 815-845.

[http://jucs.org/jucs\\_24\\_7/mathematics\\_learning\\_through\\_computational/jucs\\_24\\_07\\_0815\\_0845\\_barcelos.pdf](http://jucs.org/jucs_24_7/mathematics_learning_through_computational/jucs_24_07_0815_0845_barcelos.pdf)

- Battista, M., & Clements, D. H. (1986). The effects of Logo and CAI problem-solving environments on problem-solving abilities and mathematics achievement. *Computers in Human Behavior*, 2, 183-193. [https://doi.org/10.1016/0747-5632\(86\)90002-6](https://doi.org/10.1016/0747-5632(86)90002-6)
- Burke, Q. & Kafai Y. B. (2012). The writers' workshop for youth programmers: digital storytelling with scratch in middle school classrooms. In *SIGCSE '12: Proceedings of the 43rd ACM technical symposium on Computer Science Education*, 433–438. <https://doi.org/10.1145/2157136.2157264>
- Bocconi, S., Chiocciariello, A., Dettori, G., Ferrari, A., Engelhardt, K., Kampylis, P., & Punie, Y. (2016). *Developing computational thinking in compulsory education: Implications for policy and practice*. EUR 28295 EN. <https://doi.org/10.2791/792158>
- Bocconi, S., Chiocciariello, A., & Earp, J. (2018). *The Nordic approach to introducing Computational Thinking and programming in compulsory education*. Report prepared for the Nordic@BETT2018 Steering Group. <https://doi.org/10.17471/54007>
- Brickware (2013 October 4). *Dice roll - on line*. Scratch MIT. <https://scratch.mit.edu/projects/12817068/>
- Brodie, I. (2015). A coding story. *Math + Code 'Zine – Exploring math through code*, 1(1). <http://researchideas.ca/mc/article-1-title-recent-issue/a-coding-story/>
- Bull, G., Garofalo, J., & Hguyen, N. R. (2020). Thinking about computational thinking. *Journal of Digital Learning in Teacher Education*, 36(1), 6-18. <https://doi.org/10.1080/21532974.2019.1694381>
- Burke, Q., O’Byrne, W. I., & Kafai, Y. B. (2016). Computational participation: Understanding coding as an extension of literacy instruction. *The Journal of Adolescent & Adult Literacy*, 59(4), 371-375. <https://doi.org/10.1002/jaal.496>
- Carcary, M. (2009). The research audit trail - Enhancing trustworthiness in qualitative inquiry. *Electronic Journal on Business Research Methods*, 7(1), 11-24. <https://www.semanticscholar.org/paper/The-Research-Audit-Trial-Enhancing-Trustworthiness-Carcary/43d95239aca1f8e187e7bff40cdeb2bd80e8b3ad>
- Clements, D. H. (1985). Research on Logo in education: Is the turtle slow but steady, or not even in the race? *Computers in the Schools*, 2(2&3), 55-71. [https://doi.org/10.1300/J025v02n02\\_07](https://doi.org/10.1300/J025v02n02_07)
- Cohen, L., Manion, L., & Morrison, K. (2018). *Research methods in education* (8th ed.). Routledge.
- Computer Science Teachers Association [CSTA]. (2011). *K-12 computer science standards*. CSteachers. <http://csta.acm.org/Curriculum/sub/K12Standards.html>
- Copes, L. (2003). Problems in the real world of mathematics education. *For the Learning of Mathematics*, 23(1), 43-46. <https://eric.ed.gov/?q=%22+mathematics+is+everywhere%22&id=EJ678239>

- CT Lessons and Projects from Green Dot Public Schools (n.d.). *Ct. excelwa*  
<http://ct.excelwa.org/math/racial-bias-in-traffic-stops/>
- Dick, B. (2014). Transferability. In D. Coghlan, & M. Brydon-Miller (Eds.), *The SAGE encyclopedia of action research* (pp. 786-788). SAGE Publications. <https://www-doi-org.proxy1.lib.uwo.ca/10.4135/9781446294406.n347>
- diSessa, A. A. (1985). Learning about knowing. *New Directions for Child Development*, 28, 97–124. <https://doi.org/10.1002/cd.23219852808>
- diSessa, A. A. (2000). *Changing minds: Computers, learning, and literacy*. MIT Press.
- diSessa, A. A. (2018). Computational literacy and “the big picture” concerning computers in mathematics education. *Mathematical Thinking and Learning*, 20(1), 3-31. <https://doi.org/10.1080/10986065.2018.1403544>
- Dunbar, K. M., & Rich, K. M. (2020). Mathematics makes robots roll. *Mathematics Teacher: Learning and Teaching PK-12*, 113(7), 565-572. <https://doi.org/10.5951/MTLT.2019.0225>
- Eduapps. (n.d.). *Civilization: sustainable growth*. <http://eduapps.ca/civilization/growth/>
- Feurzeig, W., & Lukas, G. (1972). LOGO – A programming language for teaching mathematics. *Educational Technology*, 12(3), 39-46. <https://www.learntechlib.org/p/162609/>.
- Feurzeig, W., Papert, S., Bloom, M., Grant, R., & Solomon, C. (1970). Programming-languages as a conceptual framework for teaching mathematics. *ACM SIGCUE Outlook*, 4(2), 13–17. <https://doi.org/10.1145/965754.965757>
- Fields, D., Vasudevan, V., & Kafai, B. Y. (2015). The programmers’ collective: fostering participatory culture by making music videos in a high school Scratch coding workshop. *Interactive Learning Environments* 23(5), 613–633. <https://doi.org/10.1080/10494820.2015.1065892>
- Fish, M. C. (2012). Promoting critical mathematics literacy in secondary mathematics teacher education (3506126) [Doctoral Dissertation, The University of Wisconsin – Madison]. ProQuest Dissertations Publishing.
- Frey, B. B. (2018). *The SAGE encyclopedia of educational research, measurement, and evaluation* (Vols. 1-4). SAGE Publications, Inc. <http://dx.doi.org/10.4135/9781506326139>
- Gadanidis, G. (2015). Young children, mathematics, and coding: a low floor, high ceiling, wide walls environment. In D. Polly (Ed.), *Cases on technology integration in mathematics education*, (pp. 308-329). IGI Global. <http://doi:10.4018/978-1-4666-64>
- Gadanidis, G. (2017a). Artificial intelligence, computational thinking, and mathematics education. *The International Journal of Information and Learning Technology*, 34(2), 133-139. <https://doi.org/10.1108/IJILT-09-2016-0048>

- Gadanidis, G. (2017b). Five affordances of computational thinking to support elementary mathematics. *Education Journal of Computers in Mathematics and Science Teaching*, 36(2), 143-151. <https://eric.ed.gov/?id=EJ1154750>
- Gadanidis, G., Brodie, I., Minniti, L., & Silver, B. (2017). Computer coding in the K-8 mathematics curriculum? What Works? Research into Practice: Research Monograph #69. [https://thelearningexchange.ca/wp-content/uploads/2017/09/Computer\\_Coding\\_K8\\_en.pdf](https://thelearningexchange.ca/wp-content/uploads/2017/09/Computer_Coding_K8_en.pdf)
- Gadanidis, G., & Caswell, B. (2018). Computational modelling in elementary mathematics education: Making sense of coding in elementary classrooms. *KNAER Mathematics Knowledge Network, White Paper*. <http://mkn-rcm.ca/wp-content/uploads/2018/05/MKN-white-paper2-May-2018.pdf>
- Gadanidis, G., Clements, E., & You, C. (2018). Group theory, computational thinking, and young mathematicians. *Mathematical thinking and learning*, 20(1), 32–53. <https://doi.org/10.1080/10986065.2018.1403542>
- Gadanidis, G., & Cummings, J. (2018). Integrated mathematics + computer studies-Gr. 10: Reforming secondary school mathematics education. *KNAER Mathematics Knowledge Network*. <http://mkn-rcm.ca/wp-content/uploads/2018/04/MKN-white-paper-April-2018.pdf>
- Gadanidis, G., Floyd, S., Hughes, J.M., Namukasa, I.K., & Scucuglia, R. (2021). Coding in the Ontario mathematics curriculum, 1-8: Might it be transformational? Math Knowledge Network. <http://mknrcm.ca/coding-in-the-ontario-mathematics-curriculum-1-8- might-it-be-transformational/>
- Gadanidis, G., Hughes, J.M., Minniti, L., & White, B. J. G. (2017). Computational thinking, Grade 1 students and the binomial theorem. *Digital Experiences Mathematics Education* 3, 77–96. <https://doi.org/10.1007/s40751-016-0019-3>
- Gannon, S. & Buteau, C. (2018). Integration of computational thinking in Canadian provinces. In Online *Proceedings of the computational thinking in mathematics education symposium*, UOIT (Scarborough).
- Given, L. M. (2008). *The SAGE encyclopedia of qualitative research methods* (Vols. 1-0). SAGE Publications, Inc. <https://dx.doi.org/10.4135/9781412963909>
- Grossoehme, D. H. (2014). Overview of qualitative research. *Journal of Health Care Chaplain*, 20(3), 109-122. <https://doi.org/10.1080/08854726.2014.925660>
- Grover, S. & Pea, R. (2013). Computational thinking in K–12: a review of the state of the field. *Educational Researcher*, 42(1), 38–43. <https://doi.org/10.3102/0013189X12463051>
- Guzdial, M., Kay, A., Norris, C., & Soloway, E. (2019). Computational thinking should just be good thinking. *Communications of the ACM*, 62(11), 28–30. <https://doi.org/10.1145/3363181>
- Guzmán, L. D. (2019). Complex and contradictory conversations: Prospective teachers interrogating dominant narratives within mathematics education discourse.

- Mathematics Teacher Educator*, 8(1), 7-21.  
<https://doi.org/10.5951/mathteaceduc.8.1.0007>
- Haduong, P. (2019). I like computers. I hate coding: a portrait of two teens' experiences. *Information and Learning Sciences*, 120 (5/6), 349-365.  
<https://doi.org/10.1108/ILS-05-2018-0037>
- Hamilton, L., & Corbett-Whittier, C. (2013). Approaches to data analysis. In *Using case study in education research* (pp. 134-146). SAGE Publications. <https://www-doi-org.proxy1.lib.uwo.ca/10.4135/9781473913851>
- Hammersley, M. (2013). *What is qualitative research?* Bloomsbury Academic.
- Harel, I., & Papert, S. (1990). Software design as a learning environment. *Interactive Learning Environments*, 1(1), 1–32. <https://doi.org/10.1080/1049482900010102>
- Hickmott, D., Prieto-Rodriguez, E., & Holmes, K. (2017). A scoping review of studies on computational thinking in k–12 mathematics classrooms. *Digital Experiences in Mathematics Education*. 4, 48-69. <https://doi.org/10.1007/s40751-017-0038-8>
- Ho, W.K., Looi, C., Huang, W., Seow, P., & Wu, L. (2019). Realizing computational thinking in the mathematics classroom: Bridging the theory practice gap. In *Proceedings of the 24th Asian technology conference in mathematics*, National Institute of Education Nanyang Technological University Singapore.  
<https://www.semanticscholar.org/paper/Realizing-Computational-Thinking-in-the-Mathematics-Ho-Looi/7a5ebe29113b4bff2fbc546c3cb4c575526b91ccFPEI>
- Hoyles, C., & Noss, R. (1992). *Learning mathematics and Logo*. MIT Press.
- Hoyles, C., & Sutherland, R. (1989). *Logo mathematics in the classroom*. Routledge.
- Iainbro. (2020). Infectious disease simulator with healthcare capacity.  
<https://scratch.mit.edu/projects/380357960/>. Accessed 2020 May 10.
- Jones, K. (2005). Using Logo in the teaching and learning of mathematics: a research bibliography. *MicroMath*, 21(3), 34-36.  
<https://www.semanticscholar.org/paper/Using-spreadsheets-in-the-teaching-and-learning-of-Jones/c368b11170711eb361459e70084e82eedbcc6a9c>
- Johnson, B., & Christensen, L. (2016). *Educational research: quantitative, qualitative, and mixed approaches*. (6th ed.). SAGE Publications.
- Kafai, Y. B. (2016). From computational thinking to computational participation in K-12 education. *Communications of the ACM*, 59(8), 26–27.  
<https://doi.org/10.1145/2955114>
- Kafai, Y. B., Proctor, C., & Lui, D. (2020). From theory bias to theory dialogue: Embracing cognitive, situated, and critical framings of computational thinking in k-12 CS education. *ACMInroads*, 11(1). <https://doi.org/10.1145/3381887>
- Kite, V., & Park, S. (2018). Boom. Bust. Build: Teaching computational thinking and content through urban redevelopment. *The Science Teacher*, 85(3), 22-28.  
<https://www.proquest.com/docview/2010664205?pq-origsite=gscholar&fromopenview=true>

- Klein, R. (2007). Educating in place: Mathematics and technology. *Philosophical Studies in Education*, 38, 119-130. <https://eric.ed.gov/?id=EJ1072406>
- Kong, S. C., Abelson, H., & Lai, M. (2019). Computational thinking education. In Kong SC., Abelson H. (Eds.). *Introduction to computational thinking education*. Springer. [https://doi.org/10.1007/978-981-13-6528-7\\_1](https://doi.org/10.1007/978-981-13-6528-7_1)
- Krippendorff, K. (2004). *Content analysis: An introduction to its methodology* (2nd ed.). SAGE Publications.
- Layzej (2014, February 1). *Global warming*. Scratch. <https://scratch.mit.edu/projects/17364672/>
- Lee, C. H. (2012). *Re-mastering the master's tools: Recognizing and affirming the life experiences and cultural practices of urban youth in critical computational literacy through a video game project*. [Unpublished doctoral dissertation]. University of California.
- Lee, C. H., & Garcia, A. D. (2014). I want them to feel the fear...: critical computational literacy as the new multimodal composition. In Ferdig, R. E., & Pytash, K. E. (Ed.), *Exploring multimodal composition and digital writing* (pp. 364-378). IGI Global. <http://doi:10.4018/978-1-4666-4345-1.ch022>
- Lee, C. H., & Soep, E. (2016). None but ourselves can free our minds: Critical computational literacy as a pedagogy of resistance. *Equity & Excellence in Education*, 49 (4), 480-492. <https://doi.org/10.1080/10665684.2016.1227157>
- Lee, I., Martin, F., Denner, J., Coulter, B., Allan, W., Erickson, J., Malyn-Smith, J., & Werner, L. (2011). Computational thinking for youth in practice. *ACM Inroads*, 2(1), 32-37. <https://doi.org/10.1145/1929887.1929902>
- Leung, L. (2015). Validity, reliability, and generalizability in qualitative research. *Journal of Family Medicine Primary Care*, 4(3), 324-327. <https://doi.org/10.4103/2249-4863.161306>
- Lodi, M., & Martini, S. (2021). Computational thinking, between Papert and Wing. *Science & Education*, 30, 883-908. <https://doi.org/10.1007/s11191-021-00202-5>
- Margolis, J., Ryoo, J.J., Sandoval, C.D., Lee, C., Goode, J., & Chapman, G. (2012). Beyond access: Broadening participation in high school computer science. *Inroads*, 3, 72-78. <https://doi.org/10.1145/2381083.2381102>
- Maxwell, J., & Chmiel, M. (2014). Generalization in and from qualitative analysis. In U. Flick (Ed.), *The SAGE handbook of qualitative data analysis* (pp. 540-553). SAGE Publications, Inc. <https://www-doi-org.proxy1.lib.uwo.ca/10.4135/9781446282243.n37>
- Mecca, G., Santoro, D., Sileno, N., & Veltri, E. (2021). Diogene-CT: Tools and methodologies for teaching and learning coding. *International Journal of Educational Technology in Higher Education*, 18(12). <https://doi.org/10.1186/s41239-021-00246-1>



- Mertens, D. M. (2005). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative and mixed methods* (2nd edition). SAGE Publications.
- Morrison, M., Haley, E., Sheehan, K., & Taylor, R.E. (2012). Using qualitative research in advertising; strategies, techniques, and applications, 2d ed. *Reference & Research Book News*, 27(1). Ringgold, Inc.
- Mulder, A. (2017). Needles, pi(e) and coding. *Math + Code 'Zine – Exploring math through code*, 2(3). <https://researchideas.ca/mc/needles-pie-and-coding/>
- Namukasa, I. K. (2018). Enacting computational thinking concepts at different levels. *Math + Code 'Zine – Exploring math through code*, 3(1). <https://researchideas.ca/mc/computational-thinking-activities-enacting-concepts-at-different-levels/>
- Namukasa, I. K., Gadanidis, G., Hughes, J. M., & Scucuglia, R. (2021). In U. Bakan & S. Berkeley (Eds.), *Gamification and Social Networks in Education* (pp. 281-314). MacroWorld Pub. Ltd. [https://doi.org/10.15340/978-625-00-0106-6\\_14](https://doi.org/10.15340/978-625-00-0106-6_14)
- Namukasa, I. K., Minakshi, P., & Miller, M. (2017). Introductory CT and math in the middle grades: Tools and resources. *Math + Code 'Zine— Exploring math through code*, 2 (2). <http://researchideas.ca/mc/ct-and-math-in-middle-grades/>
- Nardelli, E. (2019). Do we really need computational thinking? *Communications of the ACM*, 62(2), 32-35. <https://dl.acm.org/toc/cacm/2019/62/2>
- National Council of Teachers of Mathematics (NCTM). (2020, March). #KnowTheMath: *impact of social distancing*. <https://www.nctm.org/blog/pandemicapp/>
- News Ontario. (2020, June 23). *Ontario introduces new math curriculum for elementary students: Focus on the fundamentals will better prepare students for jobs of the future*. <https://news.ontario.ca/opo/en/2020/6/ontario-introduces-new-math-curriculum-for-elementary-students.html>
- News Ontario (2021). *Modernized math course prepares students for jobs of the future and life skills*. Queen's Printer for Ontario
- Ontarimathsupport (n.d.). *Coding resources: Grade 3- Coding Lesson- Location and Movement- Coding Ourselves*. <https://ontariomath.support/viewResource.php?id=46&lang=EN&type=Codingresource&cat=lesson>
- Ontario Ministry of Education (2022). *The Ontario curriculum: Science and Technology, Grades 1-8*. Queen's Printer for Ontario
- Palaganas, E. C., Sanchez, M. C., Molintas, M. P., & Caricativo, R. D. (2017). Reflexivity in qualitative research: a journey of learning. *The Qualitative Report*, 22(2), 426-438. <https://nsuworks.nova.edu/tqr/vol22/iss2/5>
- Papert, S. (1971). *Teaching children thinking*. MIT Artificial Intelligence Laboratory Memo no. 2247, Logo Memo no. 2. <http://hdl.handle.net/1721.1/5835>.
- Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. Basic Books.

- Papert, S. (1993). *The Children's Machine. Rethinking School in the Age of the Computer*. Basic Books.
- Papert, S. (1996). An exploration in the space of mathematics educations. *International Journal of Computers for Mathematical Learning*, 1(1), 95-123.
- Papert, S. (2005). Teaching children thinking. *Contemporary Issues in Technology and Teacher Education*, 5(3), 353-365.  
<https://www.learntechlib.org/primary/p/21844/>.
- Pei, C., Weintrop, D., & Wilensky, U. (2018). Cultivating computational thinking practices and mathematical habits of mind in Lattice Land. *Mathematical Thinking and Learning*, 20(1), 75-89.  
<https://doi.org/10.1080/10986065.2018.1403543>
- Price, J. H. and Murnan, J. (2004). Research Limitations and the Necessity of Reporting Them. *American Journal of Health Education*, 35, 66-67.  
<https://doi.org/10.1080/19325037.2004.10603611>
- Psycharis, S., & Kallia, M. (2017). The effects of computer programming on high school students' reasoning skills and mathematical self-efficacy and problem solving. *Instructional Science*, 45, 583–602. <https://doi.org/10.1007/s11251-017-9421-5>
- Pöllänen, S., & Pöllänen, K. (2019). Beyond programming and crafts: towards computational thinking in basic education. *Design and Technology Education*, 24(1). <https://eric.ed.gov/?id=EJ1211980>
- Resnick, M. (1995). New Paradigms for Computing, New Paradigms for Thinking. In: diSessa A.A., Hoyles C., Noss R., Edwards L.D. (eds) *Computers and Exploratory Learning*. NATO ASI Series (Series F: Computer and Systems Sciences), 146. Springer. [https://doi.org/10.1007/978-3-642-57799-4\\_3](https://doi.org/10.1007/978-3-642-57799-4_3)
- Resnick, M. (2020, March 17). *Scratch epidemic simulator*. MIT Media Lab. <https://www.media.mit.edu/posts/scratch-epidemic-simulator/>
- Resnick, M. (2020, March 27). *Cultivating creativity during the coronavirus crisis*. MIT Media Lab. <https://medium.com/mit-media-lab/cultivating-creativity-during-the-coronavirus-crisis-fedfca3b6036>
- Resnick, M., Maloney, J., Monroy-Hernandez, A., Rusk, N., Eastmond, E., Brennan, K., Millner, A., Rosenbaum, E., Silver, J., Silverman, B., & Kafai, Y. (2009). Scratch: Programming for all. *Communications of the ACM*, 52(11), 60-67.  
<https://doi.org/10.1145/1592761.1592779>
- Rich, K. M., Yadav, A., & Larimore, R. A. (2020). Teacher implementation profiles for integrating computational thinking into elementary mathematics and science instruction. *Education and Information Technologies*, 25(4), 3161-3188.  
<https://doi.org/10.1007/s10639-020-10115-5>
- Rodríguez-Martínez, J. A., González-Calero, J. A., & Sáez-López, J. M. (2019). Computational thinking and mathematics using Scratch: an experiment with sixth-grade students. *Interactive Learning Environments*, 28(3), 316-327.  
<https://doi.org/10.1080/10494820.2019.1612448>

- Saldaña, J., & Omasta, M. (2017). *Qualitative Research: Analyzing Life*. SAGE Publications.
- Sandelowski, M. (2001). Real qualitative researchers do not count: the use of numbers in qualitative research. *Research in Nursing and Health*, 24 (3), 230-240. <https://doi.org/10.1002/nur.1025>
- Sanford J. F. (2013). Core concepts of computational thinking. *International Journal of Teaching and Case Studies*, 4(1), 1–12. <https://doi.org/10.1504/IJTCS.2013.053383>
- Sanford, J. F., & Naidu, J. T. (2016). Computational thinking concepts for grade school. *Contemporary Issues in Education Research (Online)*, 9(1), 23-32. <https://doi.org/10.19030/cier.v9i1.9547>
- Sanford J. F., & Naidu, J. T. (2017). Mathematical modeling and computational thinking. *Contemporary Issues in Education Research*, 10(2), 158-168. <https://files.eric.ed.gov/fulltext/EJ1137705.pdf>
- Sezer, H. B. & Namukasa, I. K. (2021). Real-world problems through computational thinking tools and concepts: the case of coronavirus disease (COVID-19). *Journal of Research in Innovative Teaching & Learning*, 14(1), 46-64. <https://doi.org/10.1108/JRIT-12-2020-0085>
- Skovsmose, O. (1994). Towards a philosophy of critical mathematics education. *Educational Studies in Mathematics*, 27(1), 35-57. <https://www.jstor.org/stable/3482665>
- Skovsmose, O. (2014). Critical Mathematics Education. In Lerman S. (Ed.) *Encyclopedia of Mathematics Education*. Springer, Dordrecht. [https://doi-org.proxy1.lib.uwo.ca/10.1007/978-94-007-4978-8\\_34](https://doi-org.proxy1.lib.uwo.ca/10.1007/978-94-007-4978-8_34)
- Social Sciences and Humanities Research Council of Canada, Tri-Council Policy Statement: Ethical Conduct for Research Involving Humans, December 2018.
- Solomon, C. (1986). *Computer environments for children: A reflection on theories of learning and education*. MIT Press.
- Stephens, M., & Kadijevich, D. M. (2020). Computational/algorithmic thinking. In: Lerman S. (Ed.) *Encyclopedia of Mathematics Education*. Springer. [https://doi.org/10.1007/978-3-030-15789-0\\_100044](https://doi.org/10.1007/978-3-030-15789-0_100044)
- Sung, W., Ahn, J., & Black, J. B. (2017). Introducing computational thinking to young learners: practicing computational perspectives through embodiment in mathematics education. *Technology, Knowledge and Learning*, 22(3), 443-463. <https://doi.org/10.1007/s10758-017-9328-x>
- Tepylo, D. H., & Floyd, L. (2016). Learning math through coding. *Math + Code 'Zine – Exploring math through code*, 1(3). <https://researchideas.ca/mc/learning-math-through-coding/>
- Tissenbaum, M., Sheldon, J., & Abelson, H. (2019). From computational thinking to computational action. *Communications of the ACM*, 62(3), 34–36. <https://doi.org/10.1145/3265747>

- Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K. Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal Science Education and Technology*, 25, 127–147. <https://doi.org/10.1007/s10956-015-9581-5>
- Wills, B. (2016, August 25). *The United States of coding*. Newamerica. <https://www.newamerica.org/weekly/united-states-coding/>
- Wilkerson-Jerde, M. H. (2014). Construction, categorization, and consensus: Student generated computational artifacts as a context for disciplinary reflection. *Educational Technology Research and Development*, 62(1), 99-121. <https://eric.ed.gov/?id=EJ1040722>
- Wilson, L. (2019). Coding in K-8 education. In R. Power (Ed.) *Technology and the curriculum: Summer 2019*. Pressbooks.
- Wing J. M. (2006). Computational thinking. *Communications of the ACM*, 49(3), 33–35. <https://doi.org/10.1145/1118178.1118215>
- Wing, J.M. (2014). Computational thinking benefits society. *40th Anniversary Blog of Social Issues in Computing*. Academic Press. <http://socialissues.cs.toronto.edu/index.html%3Fp=279.html>

## Chapter 2

### 2 A Content Analysis on School and Community Practices of Computational Thinking in Mathematics Teaching and Learning

National initiatives all around the world propose the teaching of core components of CT both inside and outside of formal curricula (e.g., Australian Computing Academy, n.d.; Components in Electronics, 2018; European Commission, 2019; Farnell, n.d.; Hour of Code Activities, n.d.) and in the school curricular provision of opportunities (e.g., British Columbia, BC, K-12 curriculum; England national curriculum, United States of America, U.S. A., Common Core Standards). In Canada, there has also been significant attention paid recently to incorporating CT into the curriculum in the context of coding; however, the extent of the integration differs among provinces, from the mandatory inclusion of coding as part of the curriculum to providing online resources and offering elective courses in computing (Gannon & Buteau, 2018). Starting with British Columbia in 2016 and followed by Nova Scotia, and New Brunswick as of 2017, coding started to become a mandatory part of school curricula (Julie, 2017). Ahead of the national curriculum developments, many schools, teachers, students, and communities across Canada independently appeared to develop systematic approaches to teaching coding skills to school-aged children. For example, The Learning Partnership, a charity that focuses on innovating education programs for early learners in schools across Canada, developed a program for teaching coding to Grades 1 to 8 children (The Learning Partnership, n.d.).

In the case of Ontario, the Ontario Ministry of Education (OME) released a new mathematics curriculum for Grades 1-8 in June 2020 making coding mandatory in the algebra strand of mathematics education. Under this new curriculum, schools have started incorporating coding skills in Grade 1 (OME, 2020). Subsequently, a new Grade 9 mathematics curriculum was released in June 2021 and Ontario became the first province in Canada to mandate coding as a course expectation in Grade 9 mathematics (OME, 2021). In addition to the coding, financial literacy was also added to the curriculum starting in Grade 1 with an emphasis on real-life applications and implications (OME,

2020; OME, 2021). Although the focus is on the fundamental mathematics concepts and skills, students can also develop social-emotional learning skills while using mathematics processes. Changes in the new curriculum mainly aimed to provide students with the necessary skills to be successful in school and help them to become future-ready (OME, 2020; OME, 2021). The curriculum expects students “to solve problems and create computational representations of mathematical situations” (OME, 2020) and “to represent mathematical concepts and relationships dynamically, and to solve problems” in learning the algebra strand in mathematics (OME, 2021). This is in line with the work of Feurzeig and Papert (2011) on Logo programming where they chose to explore first and foremost introductory algebra, saying they were being mindful of the heavy load of formal concepts and problem-solving in the algebra courses. Coding concepts and skills are accepted as a component of the curriculum, and they progress through the grades, “from representing movements on a grid, to solving problems involving optimization, to manipulating models to find which one best fits the data they are working with in order to make predictions”. Moreover, they can be used across all strands and can provide opportunities for students “to apply and extend their math thinking, reasoning, and communicating” (OME, 2020, p. 85).

Prior to the release of the new mathematics curriculum, the OME has promoted integrating coding across disciplines in K-12 classrooms since 2016 (OME, 2016). These promotions involved the development of new teaching resources, provision of more opportunities for hands-on learning, and more board-level support for teachers and students (OME, 2016). As part of these initiatives, lesson plans for coding based on the mathematics curriculum were provided on Teach Ontario and TELO websites. Similarly, some district programs emphasized the importance of coding. For instance, Wellington Catholic District School Board, for instance, offered Grade 10 students a two-credit package that primarily focuses on the promotion of coding and computer science education in connection to mathematics education (Gadanidis & Cummings, 2018).

In addition to the efforts of the OME and of the school boards, there are other networks and initiatives, such as The Mathematics Knowledge Network (MKN) and Computational Thinking in Mathematics Education research group, which have also

acted independently and developed systematic approaches to the use of CT knowledge, concepts, skills, and tools, particularly coding in mathematics teaching and learning. These initiatives hosted school and community outreach events intending to explore how to integrate CT in the context of coding in all levels of mathematics education, from pre-school to undergraduate as well as in mathematics teacher education. Cases of events included webinars on computational literacy in mathematics education featuring researchers such as Andy diSessa (MKN, 2019), professional development seminars to develop knowledge and learning tasks of CT (Ctmath, 2016-18; MKN, 2016-17), and classroom projects on integrating computational thinking in mathematics classrooms (Gadanidis & Caswell, 2018; Gadanidis & Cummin, 2018).

Consequent to the increasing the integration of coding in educational practice and curriculum policy, research on the practices of using CT knowledge and skills through coding has gained more significance (Lavigne et al., 2020; Lee & Malyn-Smith, 2020; Rich et al., 2020; Weintrop et al., 2016). As mentioned by Gadanidis, Cendros et al. (2017), there is a need for further research to gain a better understanding of this phenomenon along with what it suggests for mathematics teaching and learning, as well as for the training of mathematics educators. In Ontario where this study took place, one of the most convenient ways to gain a better understanding of CT practices in mathematics teaching and learning was to employ a sampling of outreach initiatives that collaborated with school boards and other education partners. With a focus on the understanding of the integration of CT into mathematics teaching and learning, I examined the online resources for school and outreach practices.

Using qualitative content analysis, I intended to respond to the first research sub-question of this thesis: What is the understanding of integrating computational thinking in Grades 1-9 mathematics education in Ontario? To respond to this first sub-question, I asked the following two data collection questions:

1. What is the nature of CT practices in school and outreach settings in Grades 1-9 mathematics education in Ontario?

2. How are CT practices framed in mathematics education in Ontario in Grades 1-9 mathematics education in Ontario?

## 2.1 Problem Statement

The rationale behind the integration of CT concepts and tools into the Ontario mathematics curriculum in the context of coding is to increase students' fluency with technological tools, improve their problem-solving skills, and help them get ready for the future. (News Ontario, 2020, 2021; OME 2020, 2021). Hence, according to Kafai et al.'s (2020) framework, this rationale is associated with cognitive framing of CT which refers to enhancing computational or mathematical understanding. Concerning this framing, Ghosh (2019) focuses on the CT-based activities in the mathematics classroom and embraces CT as one of the important skills to pursue a mathematics-related career. Likewise, Aydeniz (2018) mentions that the main motivation for the integration of CT into the curriculum is the importance of CT for the popular professions of the twenty-first century, such as scientists, mathematicians, and engineers.

In addition to cognitive framing, Kafai et al. (2020) state two more framing of CT: the first one is situated framing which includes “creating personally meaningful applications, building communities, supporting social interactions, [and] play”, and the second one is critical framing which comprises “understanding and critique of existing computational infrastructures, creating applications to promote thriving, awareness, and activism” (p. 105). Meyers (2019) maintains that this integrated framing of CT provides a broader way of thinking and strengthens students' thinking skills in both social and technical terms. Hence, coding should not be disentangled from computational implications in form of the social contexts of use, such as social interaction and social justice (Kafai et al., 2020). In that regard, CT helps in creating “communities in which design sharing and collaboration with others are paramount” and offers “a context for making applications of significance for others” (Kafai, 2016, p. 26). Here it is important to recall what Kafai and Burke (2013) point out about bringing CT into classrooms: “we must first understand what computational thinking is, how we can teach it, and why the computational participation



of online communities and traditional schools together offer new opportunities to engage students” (p. 62).

Many contemporary national initiatives, courses, and curricula put most of the emphasis on cognitive CT (i.e., teaching and learning computational concepts). They appear to ignore the use of CT affordances and opportunities for collaboration with others to provide solutions to the challenges of the world (Kafai et al., 2020). Moreover, only a few scholarly works include situated and critical framings of CT (Haduong, 2019; Hostetler et al., 2018; Lee & Garcia, 2014; Lee & Soep, 2018; Proctor & Blikstein, 2019; Przybylski, 2018; Sengupta et al., 2018; Tissenbaum et al., 2019; Weidler-Lewis et al., 2019), and none of these are primarily focused on mathematics education. As one of the few exceptions with an emphasis on STEM, Veeragoudar-Harrell’s (2009) study focuses on how to foster the mathematical and computational agency of high school STEM learners. Further, Sengupta et al. (2018) provide an argument for deepening and broadening the focus on the lived experiences of CT in K-12 STEM Education.

Kafai et al. (2020) assert that it is necessary to move beyond cognitive goal of CT. When designing mathematics curriculum and pedagogy, it is essential to broaden (e.g., students’ social-emotional skills and personal interest) and deepen (e.g., questioning the purpose of coding, improving social and political awareness, and addressing the critical issues in the communities they live in) the integration of CT into mathematics on a larger scale (Lee & Soep, 2018). Sengupta et al. (2018) also call for an appropriate reconceptualization of CT as a social endeavor, as opposed to putting all the focus on its objective and subjective aspects. To achieve both breadth and depth in the integration of CT into mathematics, it is imperative that researchers, professionals, and policymakers understand the perspectives of both CT applications and CT implications in mathematics teaching and learning.

## 2.2 Theoretical Framework

De Freitas & Walshaw (2016) mention that “Every theory is simply a lens and cannot bring everything into focus all at once. None of these theories is absolutely independent” (p. 8). Hence, instead of sticking to a single theory, I preferred to seek relations among

theories, perspectives, metaphors, and how to make use of them to help students to thrive in life (Jablonka et al., 2013). Therefore, this study adopted three theoretical frameworks of learning (i.e., constructionism, social constructivism, and critical literacy) in connection with Kafai et al.'s (2020) framings of CT (i.e., cognitive, situated, and critical) to provide a comprehensive theoretical background for ongoing research and professional practices in integrating CT into mathematics education. In this study I adopted Kafai et al.'s framings as a conceptual framework, the epistemological commitments and their correspondence to the framings of CT is provided in Table 1.

**Table 1: Overview of Learning Perspectives in Framing CT**

Frame	Unit of Analysis	Epistemology	Priorities	Computational Thinking
Cognitive	Individual learners	Skills, competencies, knowledge of a particular discipline	Measurable, transferrable skills, economic opportunity	Computational concepts (algorithms, abstraction) and practices (remixing, iteration)
Situated	Communities of practice, activity systems, learning ecologies	Practices, participation, preparation for future learning	Equity, interest, identity development, creativity	Creating personally-meaningful applications, building communities, supporting social interactions, play
Critical	Society at-large: existing structures of power, privilege, and opportunity (race, gender, social class, ability)	Awareness of ideologies, strategies for social action	Justice, critical understanding, enacting social change	Understanding and critique of existing computational infrastructures, creating applications to promote thriving, awareness, and activism

*Note.* Overview of Learning Perspectives in Framing CT From “From Theory Bias to Theory Dialogue: Embracing Cognitive, Situated, and Critical Framings of Computational Thinking in K-12 CS Education” (p. 105), by Y. Kafai, C. Proctor and E., and. D. Lui, 2019, Association for Computing Machinery (ACM), Ink. (<https://doi.org/10.1145/3291279.3339400>). Copyright 2020 by Association for Computing Machinery (ACM), Ink. Reprinted with permission.

The overview of the connection between the three frameworks of learning (constructionism, social constructivism, and critical literacy) and Kafai et al.'s (2020) framings of CT (cognitive, situated, and critical) is elaborated on Table 2.

**Table 2: The connection between the three theoretical frameworks of learning and Kafai et al.'s framings of CT**

Theoretical frameworks of learning	The framings of CT
<p><b>Constructionism:</b></p> <p>Constructionism has investigated <i>cognitive dimension of the creation of computational artifacts by learners themselves</i> and explored the context provided by the</p>	<p><b>In a cognitive framing context,</b></p> <p>CT is seen as “an assessment and evaluation of students’ programming ability and comprehension of basic and advanced coding constructs (e.g., variables,</p>

computational medium for learning as well as expression (Papert, 1980).	conditional logic), through activities such as think-aloud interviews, <i>creating functional open-ended projects, and engaging with design scenarios</i> ” (Kafai et al., 2020, p. 103)
<p><b>Social constructivism:</b></p> <p>According to social constructivist theory, <i>the construction of the understanding and the knowledge of learners take place through social interactions</i>; environmental, cultural, and ethnic conditions influence this construction process; and the role of the teachers is to facilitate the learning process through guidance for the active involvement of learners (Amineh &amp; Asl, 2015).</p>	<p><b>In a situated framing context,</b></p> <p>CT is seen as “a vehicle for personal expression and <i>connecting with others alongside</i> and intersecting a plurality of other literacy practices” (Kafai et al., 2020, p. 103).</p>
<p><b>Critical literacy:</b></p> <p>Lee and Soep (2016) point out that through critical literacy, students will be allowed to <i>create projects which are more likely to have positive impacts by breaking silences and revealing truths</i> while building software design and coding skills.</p>	<p><b>In a critical framing context,</b></p> <p>CT is seen as “<i>a potential channel for engaging with the political, moral, and ethical challenges of the world</i>” (Kafai et al., 2020, p. 103).</p>

## 2.2.1 Constructionism

Seymour Papert's theory of constructionism can be defined as learning-by-making (Papert & Harel, 1991). Papert (1980) emphasized using the computer (e.g., coding) as a tool to think with. He builds it upon constructivist theory and develops it further through the inclusion of the actions that take place in the creation of a meaningful product to improve or reinforce student learning (Martinez & Stager, 2013). As in constructivism, it aims to build knowledge structures regardless of learning conditions and claims that a context for active engagement in building public goods helps this process enormously (Papert & Harel, 1991). Constructionism, unlike constructivism, emphasizes the importance of diving deep into problems instead of examining from a distance and of connectedness instead of separation to attain a better understanding (Ackermann, 2001).

Constructionism has investigated cognitive dimension of the creation of computational artifacts by learners themselves and explored the context provided by the computational medium for learning (Papert, 1980) as well for expressing this learning (Resnick, 1996). Based on Papert's (1980) approach, Martinez and Stager (2013) summarize constructionism as the action of creating a product that is personally meaningful or that is visible and shareable, which makes learning more relevant to learners.

## 2.2.2 Social Constructivism

Constructivism, which goes beyond behaviourism and cognitivism, is the philosophy of learning from one's own experiences (i.e., knowledge is created by individuals in the process of their interactions with other people and their environment; Draper, 2002). To understand the learning process and learner's appropriation of the cultural tools which transform the relationships of individuals with others and the world, Vygotsky's learning stages approach is very useful. This approach asserts that cultural development and learning take place at two levels, the social and the individual, in that cultural development and learning first occur between people (interpsychological), and then become internalized (intrapsychological) (Vygotsky, 1978). Thus, this approach provides an understanding of the construction of knowledge by individuals through interactions (Vygotsky, 1978).

According to social constructivist theory, the construction of the understanding and the knowledge of learners take place through social interactions. Environmental, cultural, and ethnic conditions influence this construction process; and the role of the teachers is to facilitate the learning process through guidance for the active involvement of learners (Amineh & Asl, 2015). Kotsopoulos et al. (2017) explain the social constructivist approach in CT by stating that pedagogical experiences of CT (i.e., unplugging, tinkering, making, and remixing) reflect a “developmental continuum or zones of proximal learning” with each experience demanding a higher level of cognitive demand functioning than the previous one (p.158).

## 2.2.3 Critical Literacy

Drawing on Freire's (2005) theory of Pedagogy of the Oppressed, scholars have put forward many theories and studies to determine the forces causing oppression in education and suffering in the larger world. In a school context, critical literacy could be viewed “as a concept, as a framework, or as a perspective for teaching and learning; a way of being in the classroom; and a stance or attitude toward literacy work in schools at all levels, and irrespective of whether students are working in the languages they are

fluent in or languages that they are adding to their linguistic repertoires” (Vasquez et al., 2019, p. 302).

Blending the critical literacy and CT concepts and skills, Lee (2012) defined critical computational literacy which provides “the real-world application of critical literacy in tools that are revolutionizing our present and future worlds” (p. 126). The integration of critical literacy with the technical skills in CT sustains and enhances students' literacy and thinking skills in both arenas (Lee, 2012). Building connections across fields becomes easier when a CT approach is used for critical topics (Sheldon, 2018). Many recent examples make use of this critical computational literacy approach in school or outreach contexts, including a Grade 7 mathematics classroom activity on racial bias in traffic stops through which students learn statistics and probability along with practicing pattern recognition (CT Lessons and Projects from Green Dot Public Schools, n.d.), a Grade 8 algebra class projects on real-world issues (e.g., an incidence of breast cancer, a percentage of women versus men in Congress since 1970, the effects of gendered bathrooms) (Snelling, 2018), mobile apps created by teenage girls that provide solutions for the problems in their local communities (e.g., sexual harassment, water pollution, and inadequate and poor education facilities) through the utilization of MIT App Inventor (Joshi, 2016), and a covid-19 case tracker as well as a protest tracker coded by a high school student Avi Schiffmann (Basu, 2020). In these examples, students analyze data using CT tools, including mathematics-specific tools such as Desmos (an online graphing calculator), to bring awareness and to offer solutions (Snelling, 2018).

Merging critical literacy with CT knowledge in a classroom setup is expected to improve students' motivation to acquire this knowledge and help them recognize how to use it to pursue their own social learning objectives. At the same time, this recognition could inspire them to become more responsible, accountable, and caring individuals in social contexts (Noguerón-Liu, 2017). Lee and Soep (2016) point out that through critical literacy, students while building software design and coding skills may create projects which are more likely to have a positive impact on social aspects of their lives, such as by breaking silences and revealing truths. Their findings demonstrate that using a critical literacy approach in the organization of learning environments encourages students to

contend with more dimensions of their work (e.g., engaging in the complex process of problem-solving, design, and understanding human behavior in the creation of gentrification app). Eventually merging critical literacy with CT knowledge will compel students to develop as critical problem-solvers, who are competent enough with technical know-how to leverage CT tools to explore mathematical solutions to complex problems and to potentially contribute to ideas needed to positively influence their communities. (Lee & Soep, 2016, p. 489).

#### 2.2.4 Connection Between Theoretical Frameworks and the Conceptual Framework (Framings of CT) in the Context of the Study

Cognitive framing of CT emphasizes student understanding of key computational concepts and applications. It focuses on building skills and competencies for their school performance and future careers. Considering the individual level of learning and cognitive dimension of creation, constructivist theory relates to cognitive framing (Papert, 1980). Kafai et al. (2020), explain cognitive CT in the context of cognitive research traditions. Cognitive theories emphasize the mental process of learning, whereas learning and teaching CT concepts and skills also include a physical process and have *learning by making* aspect (which relates to constructionism) (Papert & Harel, 1991); therefore, I employ constructionism to interpret cognitive framing of CT in mathematics.

Situated framing of CT (Kafai et al., 2020) involves “solving problems, designing systems, and understanding human behavior in the context of computing” (Kafai, 2016, p. 26). Situated CT focuses on social interactions such as collaboration and participation, and it is in line with the social constructivist theory, which emphasizes interpersonal interactions (Iyioko, 2020).

From a critical framing, CT is seen “as a potential channel for engaging with the political, moral and ethical challenges of the world” (Kafai et al., 2020, p. 103). This framing carries the goal of integrating CT in education beyond providing a basis of CT for competence in computational action and gives the students a deeper purpose for computing. For example, they might strive to make a positive impact on themselves, their

communities, and even the whole world (Tissenbaum et al., 2019). Critical framing is drawn from critical literacy, which explores the concept of power, privilege, and oppression and fosters critical consciousness to search for equity and justice (Stevens & Bean, 2007). Whereas critical literacy is employed as a theoretical background for critical framing of CT, critical computational literacy provides pedagogical and conceptual aspects; for instance, interpreting critical framing as real-world application of critical literacy (Lee, 2012).

Sfard (1998) suggests two metaphors, *acquisition* and *participation*, to identify cognitive and situated framings of CT, and mentions that the educational practice becomes more powerful as it stands on more metaphorical legs. Kafai et al. (2020) suggest a third metaphor, *action*, to identify critical CT and draw attention to these three metaphors:

- What is learned: acquisition [cognitive CT]
- How it is learned: participation [situated CT]
- How it is valued (how it reflects the particular norms, values, and power structures of society): action [critical CT]

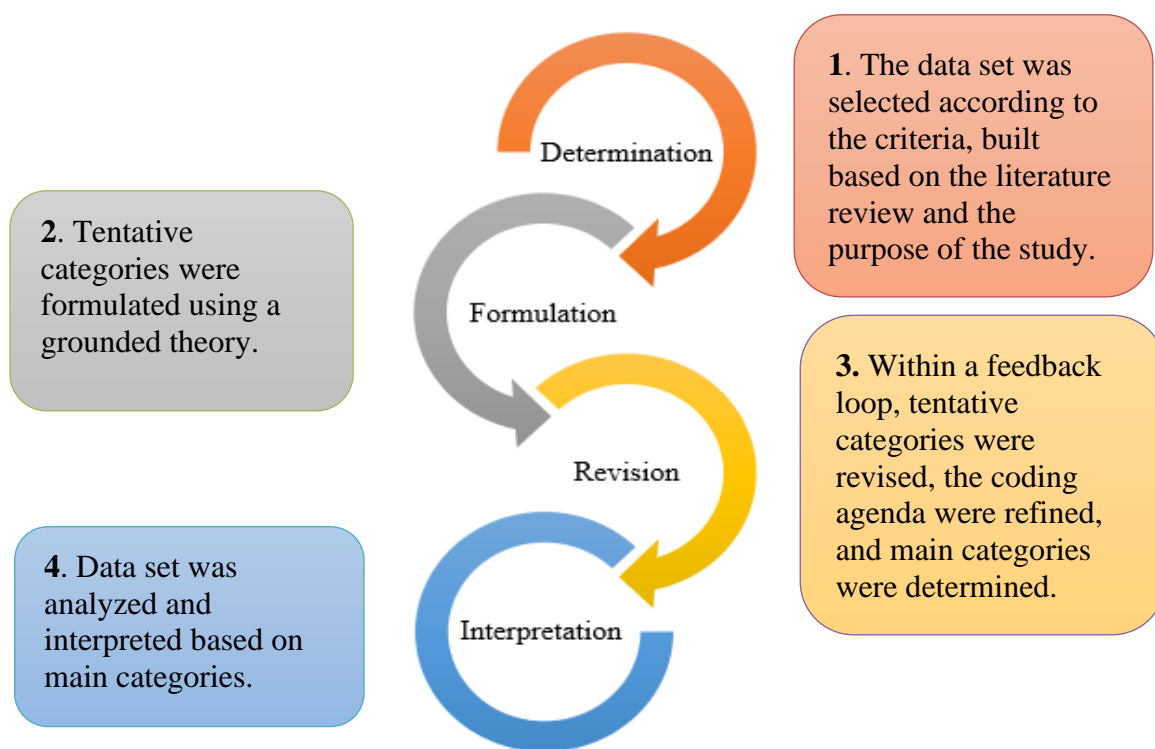
## 2.3 Methodology

In this study, I used qualitative research methods to investigate and explain the content and context of CT practices in mathematics teaching and learning to respond the first sub-research question, which is concerned with the analysis of the online resources, including written and visual sources (e.g., articles, events, projects, symposiums, blogs, documentaries, videos), drawn from a sample of three Ontario websites.

I employed content analysis to analyze these online resources. The intent was to gain a more detailed understanding of current practices concerning the integration of CT into the school mathematics curriculum. According to Krippendorff, (2004), content analysis is “a research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use” (p. 18), where texts refer to a variety of sources of information, such as images, audio, videos, all sorts of artworks, content

shared on social media, and numerical data (Flick, 2009). To conduct content analysis, I coded the categories found in the selected online resources based on their content and the data collection research questions. Following coding, I interpreted the presence and relationship of the themes and concepts extracted from the content analysis.

In this study, I adopted Mayring's (2000) deductive category application and inductive category development to analyze the selected data set (shown in Appendix A). With this approach, the analysis includes the following four main steps of determination, formulation, revision, and interpretation which are shown in Figure 3.



**Figure 3: The steps of data selection and data analyses**

### 2.3.1 Determination Step

#### Sampling Process

Sampling is the selection process of a subset of data from within a larger data set (Neuendorf, 2017). The purpose of sampling is to estimate the characteristics of a large set by using the selected representative data.



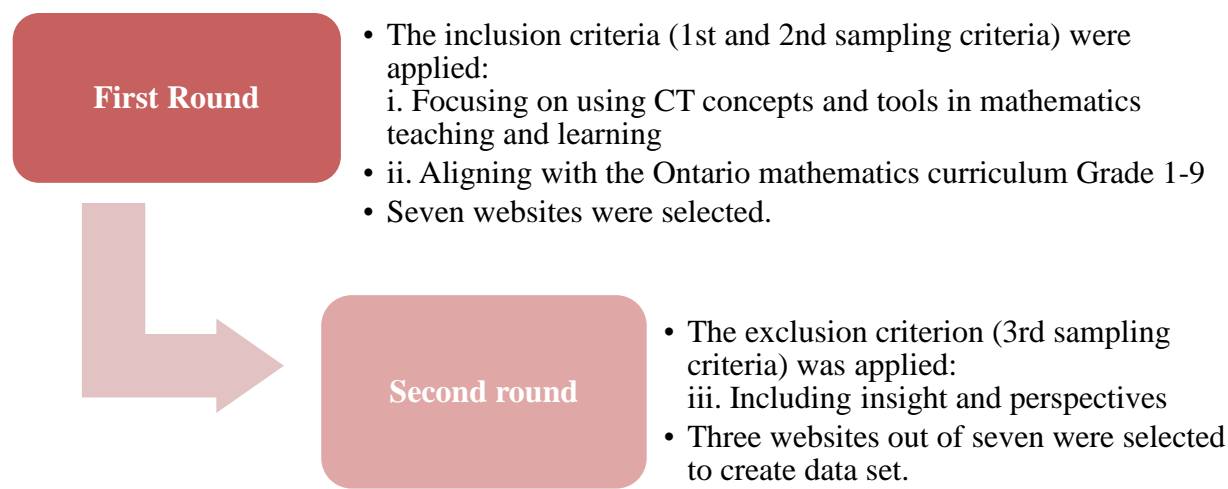
In this study, I used a purposeful (purposive) sampling method to offer an in-depth analysis of “rich cases that provide a great deal of important information aligned with the purpose of the research” (Patton, 1990, p. 169). As such, samples “must be selected to fit the purpose of the study, the resources available, the questions being asked, and constraints being faced” (Pattison, 1990, p. 181–182). The purpose of this study is to present an in-depth understanding of the perspectives of using CT concepts and tools in mathematics education in Ontario and their connection to the two curricular frameworks relevant to integrating CT in mathematics instruction (i.e., the new Ontario mathematics curricula Grades 1-8 and 9). The goal was to examine the CT practices in mathematics teaching and learning, which were depicted in the selected resources.

According to the purpose of the study, the following sampling criteria are applied:

- i. The resources on the website should focus on using CT concepts and tools in mathematics teaching and learning; therefore, sources should focus on coding, programming, computational modelling, and computational thinking, the last of which is the umbrella term for the key terms of this research.
- ii. The resources on the website should align with the Ontario mathematics curricula Grades 1-8 and 9.
- iii. The resources on the website should reflect insights and perspectives of researchers, educators (including prospective teachers), and students on CT practices.

Based on the sampling criteria, I narrowed the scope to available websites about practices for integrating CT into mathematics education and curricula in Ontario school and outreach setting. I also limited the research to the resources published until and including 2021.

The sampling process took place in two rounds. A flowchart of this process is presented in Figure 4:



**Figure 4: Sampling process**

In the first round, I searched for websites that include resources on the integration of CT (i.e., coding) into mathematics within Ontario. The inclusion criteria were as follows: (i) The resources on the website should focus on using CT concepts and tools in mathematics teaching and learning; therefore, sources should focus on coding, programming, computational modelling, and computational thinking, the last of which is the umbrella term for the key terms of this research and (ii) The resources on the website should align with the Ontario mathematics curricula Grades 1-8 and 9.

Based on these criteria, seven websites selected that align with the focus of the study: the Learning Partnership, which provides a Coding Quest program that emphasizes “experiential STEAM-based education that incorporates global competencies” (the Learning Partnership, n.d.); Edugains and OAME/AFEMO Elementary Math Curriculum Resource Project, which include resources for educators on coding in elementary education (Edugains, n.d.; Ontarimathsupport, n.d.); TVO Digital Learning Outreach, which includes research ideas for coding implementation (TVO Digital Learning Outreach, n.d.); the Mathematics Knowledge Network (MKN), which provides evidence-based practices for mathematics instruction to support improved educational achievement in partnership with educators, researchers and organizations across Ontario (Mkn-rcm,

n.d.); Computational Thinking in Mathematics Education, which focuses on the use of CT in mathematics teaching and learning (Ctmath, n.d.), from pre-school to undergraduate mathematics and in mathematics teacher education; and Math+Code ‘Zine, which includes professional publications providing learning opportunities of using coding for students in mathematics. The selected websites in the first round and their emphases are presented in Figure 5:

Coding Quest program (The Learning Partnership)	lesson plans and resources for educators
Edugains	resources on coding in elementary education
Ontario math support (OAME/AFEM)	resources on coding in elementary education
TVO Digital Learning Outreach	research ideas for coding implementation
The Mathematics Knowledge Network (MKN)	practices for mathematics instruction in partnership with educators, researchers, and organizations across Ontario
Computational Thinking in Mathematics Education	projects on the use of CT in mathematics teaching and learning
Math+Code ‘Zine	professional publications of using coding for students in mathematics

**Figure 5. The selected websites in the first round of the sampling process**

In the second round, I eliminated the websites that do not provide any insights and perspectives about the focus of my research. This exclusion criterion is associated with the third sampling criteria: (iii) The resources on the website should reflect insights and perspectives of researchers, educators, (including prospective teachers), and students. Accordingly, I excluded the websites that provide resources about CT integration into mathematics education without reflecting any insights or perspectives. I narrowed the websites that I selected in the first round to three websites, Computational Thinking in

Mathematics Education<sup>7</sup>, Math+Code ‘Zine<sup>8</sup> and Math Knowledge Network<sup>9</sup>, to create the data set.

### **Data Set Selection Process**

The selection criteria provided a heterogeneous sample of pertinent CT practices without redundancy. Selection criteria is based on inclusion and exclusion criteria, and it ensures the consistency of the selected sample, decreases the impact of confounding factors, and increases the probability of finding an accurate correlation between inputs and outcomes (Salkind, 2010).

Based on selection criteria, specific resources were selected from three websites (i.e., Computational Thinking in Mathematics Education, and Math +Code ‘Zine and Math Knowledge Network), for in-depth study, if they:

- focused only on mathematics education; therefore, any content that focused on integrated disciplines, such as STEM or STEAM, were excluded from this study.

Since the inclusion of different disciplines might change the result of the study so affect the trustworthiness and creditability; any disciplines outside mathematics excluded from this study.

and if they:

- targeted the Grades 1-9 level of education in Ontario; therefore, any content that is out of the regional (Ontario) and educational scope (Grades 1-9) were excluded from this study.

This study limited to the Grades 1-9 level of education since the interpretation of the findings were conducted in connection to the two curricular frameworks relevant to integrating CT in mathematics instruction in Ontario (i.e., the new Ontario mathematics curricula Grades 1-8 and 9). Although the post-secondary level of education

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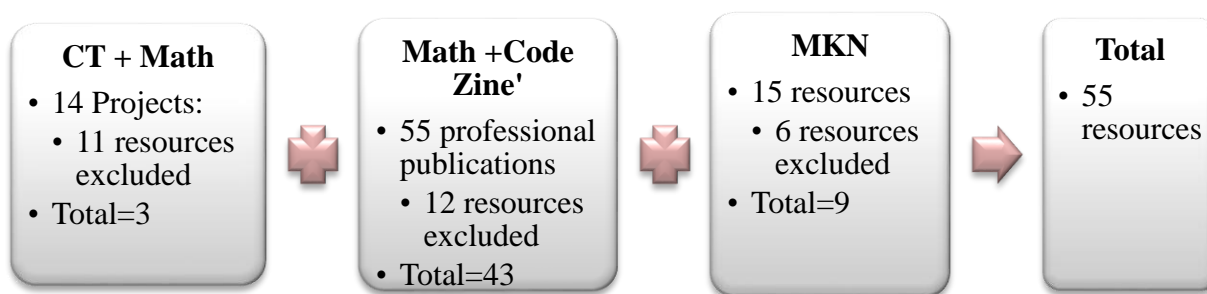
<sup>7</sup> <http://ctmath.ca>

<sup>8</sup> <https://researchideas.ca/mc/>

<sup>9</sup> <http://mkn-rcm.ca>

was excluded from this study, the practices that include mathematics teacher candidates' experiences on integrating CT into Grades 1-9 mathematics classrooms were included in the study.

Given the selection criteria, 55 resources were retained from the selected websites (shown in Appendix B). The resources were written and visual sources (e.g., articles, events, projects, symposiums, blogs, documentaries, videos, etc.). The data was manually organized and coded using Microsoft Word and Excel. A flowchart of this process is presented in Figure 6:



**Figure 6: Flowchart of data selection process**

The Computational Thinking in Mathematics Education website provides 14 projects focusing on CT and mathematics. Among these 14 projects 11 of them were excluded for which details are provided in Appendix C. After 55 publications were examined, 12 of them were excluded due to not meeting the selection criteria, which are detailed in Appendix C. Math Knowledge Network website focuses on different concepts in mathematics education, which are critical transitions, indigenous knowledge, mathematics leadership, and CT. Therefore, I searched the website for resources related to CT concepts. I selected 15 resources under the four sections of the website, which are MKN Quarterly, Communities of practice, Events, and MKN resources. After checking these 15 resources based on the selection criteria, I excluded 6 of them, which are detailed in Appendix C. Additionally, I identified overlapping resources and excluded them. In total, I selected 55 resources, which reflect the insights and perspectives of researchers, educators, teacher candidates, and students on the implications of CT in Ontario mathematics education.

### 2.3.2 Formulation Step

Inductive and deductive approaches have vast differences, but they can also complement each other (Blackstone, 2019). As DeCarlo (2018) mentions, to make use of this complementarity, some researchers organize their research to include one inductive and one deductive component (e.g., Blackstone et al., 2006; Uggen & Blackstone, 2004). In some other cases, even though the research is initially planned to use only one of these approaches, the findings along the way require the help of the other approach (e.g., Berk et al., 1992; Pate & Hamilton, 1992). Following Blackstone's (2019) perspective, I conducted the analysis based on both inductive and deductive approaches to provide comprehensive and more complete results. I started the analysis inductively. Following the category development step, I used a deductive approach and classified these categories based on Kafai et al.'s (2020) framing of CT given in Appendix C.

#### **Inductive Procedure**

In this inductive procedure, the main idea is to determine a criterion of definition, emanating from the research question and the existing theoretical background that will address the relevant aspects of the textual material (Mayring, 2000). In the induction process, I worked through the data, determined tentative categories, and initiated the revision process. The categories were gradually reduced step-by-step to the main categories and the reliability was checked by my supervisor.

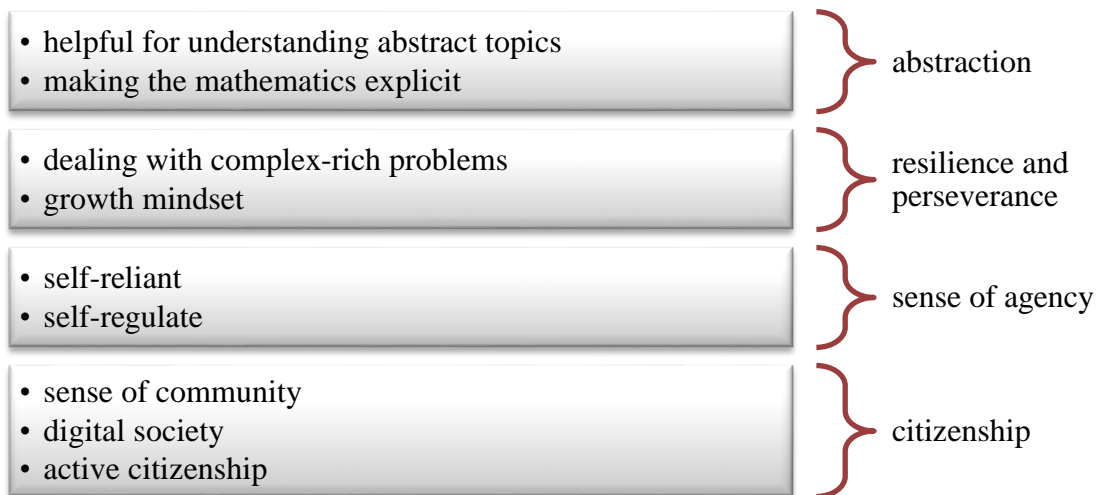
While conducting inductive analysis of data, I employed grounded theory to analyze and interpret the collected data. Because grounded theory discovers and constructs theory from the data, it is inherently flexible (i.e., there is no unique set of sequential steps of grounded theory) (Strauss, 1987). Strauss and Corbin (1998) maintain that developing a grounded theory from given data requires three kinds of coding procedures: open (to develop categories of information), axial (to interconnect the categories), and selective coding (to build a storyline connecting the categories). These procedures, which constitute the coding process, are described as follows.

### *Open Coding*

Open coding was the first step. It included breaking down, examining, comparing, conceptualizing, and categorizing data (Strauss & Corbin, 1998). In this step, I recorded the initial codes on understandings of current practices concerning the integration of CT into the school mathematics curriculum that were obtained from the data set. Some of these open codes are as follows: helpful for understanding abstract topics, making the mathematics explicit, dealing with complex-rich problems, growth mindset, self-reliant, self-regulate, sense of community, digital society, and active citizenship.

### *Axial Coding*

Axial coding, which is the second step, involved relating categories to subcategories and constructing linkages between data (Belgrave & Kapriskie, 2019). Both inductive and deductive reasoning was used to relate the open codes identified in the first step (Edwards & Jones, 2009), and a list of axial codes were created (Sample is provided in Figure 7). This list was refined until there are no more repetitive and overlapping codes. Some of the axial codes from the data are as follows: abstraction, resilience and perseverance, sense of agency, and citizenship.



**Figure 7. Axial codes based on the open codes**

### *Selective Coding*

Selective coding was the third step and was conducted according to the results of the axial coding step. It included “settling on one's core category, relating it to other categories, validating these relationships and fleshing out any categories that are incomplete” (Belgrave & Kapriskie, 2019, p.176.) Hence, one category was chosen as the “core category” and the relationship between other categories and this “core category” was identified to unify them around it. Moreover, descriptive detail is provided for the categories that require further explanation (Corbin & Strauss, 2008). In this step, I attempted to integrate and organize the codes. Some of the selective codes I considered from the data were as follows: the learning benefits of the integration of CT into mathematics, the skill sets improving through using CT concepts and tools. However, rather than generating core categories inductively, I employed a deductive approach and categorized codes into cognitive, situated, and critical categories. Details of this process are given below under the deductive procedure.

### **Deductive Procedure**

In this procedure, I attempted to interpret the themes that were obtained from the data analysis based on Kafai et al.'s (2020) framings of CT. Following Mayring's (2000) deductive approach, I used a coding agenda to determine the circumstances under which a theme can be related to the cognitive, situated and critical categories. Kafai et al.'s definitions for framings of CT were used to define these categories. A sample of the coding agenda is in Table 3.

**Table 3: Sample of coding agenda for deductive analysis**

<b>Category</b>	<b>Definition</b>	<b>Examples</b>	<b>Coding rules</b>
<i><b>Cognitive</b></i>	Computational concepts (algorithms, abstraction) and practices (remixing, iteration) (Kafai et al., 2020, p.105).	“Probability & Scratch is a task created with the goal of teaching probability by using computational thinking concepts to help students grasp a new and <i>abstract math topic concretely</i> ” (Source 5).	One or more concepts in the code/theme should match with the definition.



<i>Situated</i>	Creating personally meaningful applications, building communities, <i>supporting social interactions</i> , play (Kafai et al., 2020, p.105).	“This activity was helping to create opportunities for the students to take ownership of their learning experiences and <i>promote collaboration and peer learning</i> ” (Source 50).	
<i>Critical</i>	Understanding and critique of existing computational infrastructures, creating applications to <i>promote thriving, awareness, and activism</i> (Kafai et al., 2020, p.105).	“Helps to develop resilient, thriving, and successful learners who will become active and contributing members of society!” (Source 13).	

### 2.3.3 Revision Step

Within a feedback loop of revision, the tentative categories that were created in the previous steps were revised, and the main categories were determined. This step also included refinement of the coding agenda to create the final scheme of the categories.

The revision process consisted of multiple rounds of member checking with my supervisor. Member checking, which is sometimes called participant or respondent validation, helps researchers to check the accuracy of their findings, to check whether these findings resonate with the experiences of the participants, and to establish the credibility of these findings (Birt et al., 2016; Creswell, 2012). To improve the accuracy of the findings, I conducted follow-up meetings with my supervisor to evaluate the correctness and comprehensiveness of the coding steps and the interpretations I made of the data.

Corbin & Strauss (2008) maintain that there are overlapping parts of open, axial, and selective coding that occur throughout the research process. Therefore, I visited the data several times during the revision process, examined the content in detail, and even focused on the single words which were significant to the focus of the study.

### 2.3.4 Interpretation Step

The data set was analyzed and interpreted based on main categories. The results are presented under research findings.

## 2.4 Research Findings

In this section, I present the findings from the data analysis to answer the following main research question: What is the understanding of integrating computational thinking in Grades 1-9 mathematics education in Ontario? This research question is addressed by two data collection questions and the corresponding sections below:

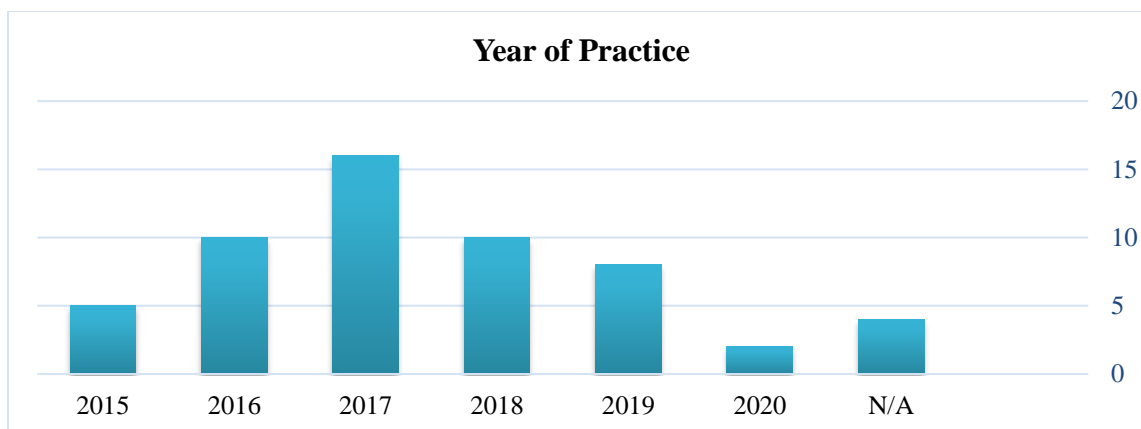
1. What is the nature of CT practices in school and outreach settings in Grades 1-9 mathematics education in Ontario?
2. How are CT practices framed in mathematics education in Ontario in Grades 1-9 mathematics education in Ontario?

### 2.4.1 Bibliographic Information and Context of Data Set

In this section, I respond to the first data collection question what the nature of CT practices in school and outreach settings in Grades 1-9 mathematics education in Ontario is. With this aim, bibliographic information and context obtained from the data are presented under three categories: (1) Year, (2) Level, and (3) Tool.

#### **Year**

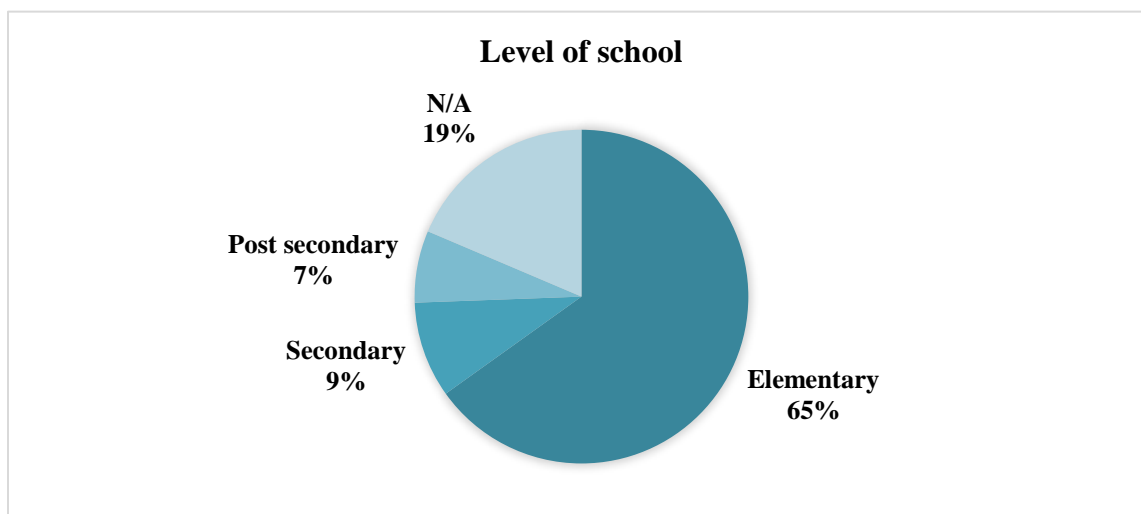
Practices related to integrating CT into mathematics teaching and learning in the data set are from 2015 and on, and the number of the practices fluctuates over time (see in Figure 8). Initially, there is a gradual increase in the number of practices with a peak in 2017, and then a gradual decrease is observed.



**Figure 8: The distributions of the practices by year**

### School level

The practices are categorized under three levels: Elementary, secondary, post-secondary. As seen in Figure 9, the elementary level was, by far, the most common level, in which practices were conducted.



**Figure 9: The distribution of the practices by school level**

### Tool

Based on content analysis, the ways of integrating CT into mathematics teaching and learning are categorized under four main categories based on content analysis: (1) Block/ visual/ text-based coding language, (2) Digital tangibles, (3) Apps and games and, (4)

Unplugged. Tools specified under these categories are shown in Figure 10. The majority of the practices have been conducted using block-based coding languages, and among them, Scratch is the most commonly used in K-8 practices

<b>Block/ visual/ text-based coding Language</b>	<b>Digital Tangibles</b>	<b>Apps and games</b>	<b>Unplugged</b>
<ul style="list-style-type: none"> <li>•Scratch</li> <li>•Scratch Jr.</li> <li>•Tynker</li> <li>•Python</li> </ul>	<ul style="list-style-type: none"> <li>•Robotics</li> <li>•Sphero, Dash, Beebots, Edison robots called Edblocks</li> <li>•Micro:bits</li> </ul>	<ul style="list-style-type: none"> <li>•Web Sketchpad</li> <li>•Video games</li> <li>•Lightbot and Kodable (iPad apps)</li> <li>•Tickle app</li> <li>•Microsoft MakeCode</li> <li>•Eduapps</li> </ul>	<ul style="list-style-type: none"> <li>•TUI Tools</li> <li>•Chibitronics</li> <li>•Bare Conductive electric paint</li> <li>•Kibo</li> <li>•Cubetto</li> <li>•Connected foam floor tiles</li> </ul>

**Figure 10. The categories of tools used in practices**

### Summary

In the section above, I presented the results of the data based on the year, school level, and tool used in practices. There are three main findings based on the analysis of the bibliographic information and context of CT practices:

- The number of CT practices in mathematics education made a peak in 2017.
- CT practices were mostly conducted in elementary schools.
- The most common tool used for CT practices in mathematics classrooms is Scratch.

### 2.4.2 The framings of CT practices in Mathematics Education

In this section, I respond to the second data collection question how CT practices framed in mathematics education in Ontario in Grades 1-9 mathematics education in Ontario are.

As mentioned earlier, I employed Kafai et al.'s (2000) CT framings, cognitive, situated, and critical, in the analyses with two objectives in mind:

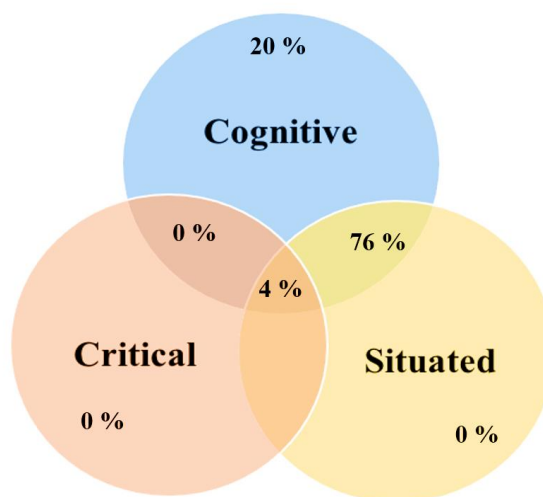
1. To categorize each practice based on the framings of CT.

2. To organize the codes obtained from the analyses in their relation to the framings of CT.

The results referring to these two objectives are presented below.

### 1. Categorization of Practices Based on the Framings of CT

As can be seen in Figure 11, the most common perspective embraced in CT applications in mathematics education fits under *cognitive* framing, and it is followed by *situated* framing. Since situated CT is mostly embedded into practice along with cognitive CT, *cognitive and situated* category has the biggest percentage of all. No practice falls solely under situated or critical framings. These findings are in line with the preliminary results of the study on scholarly works related to the integration of CT into Grades 1-9 mathematics education, for which the details can be found in Appendix E.



**Figure 11: Categorization of the practices based on the framings of CT**

### 2. Organization of the Codes Based on the Framings of CT

The codes obtained from the data are presented below under the corresponding categories.

### *Cognitive Framing*

All the sources I analyzed are aligned with cognitive CT (100 %). In the analysis, I formed five codes for practices under cognitive framing, namely, problem-solving, abstraction, critical thinking, analytical thinking, and imagination. These are presented in Table 4 along with the samples from the data set.

**Table 4: Cognitive framing’s codes and related samples**

<b>Code</b>	<b>Sample from the data set</b>
<b>Problem Solving</b>	“Integrating coding into your classroom or your school helps to support students in their development of both <i>problem solving</i> and collaboration skills” (Source 38).
<b>Critical Thinking</b>	“We have found that open coding tasks not only support our students learning to be creative through problem solving, but they also allow for everyone to engage at their own level, and most importantly to <i>think critically</i> ” (Source 15).
<b>Abstraction</b>	“Probability & Scratch is a task created with the goal of teaching probability by using computational thinking concepts to help students grasp a new and <i>abstract math topic concretely</i> ” (Source 5).
<b>Analytical Thinking</b>	“Debugging a program is a <i>powerful exercise in analytic thinking</i> , trial and error, and just plain perseverance” (Source 46).
<b>Imagination</b>	“Interesting ideas for young children, that <i>capture their imagination</i> and get them thinking” (Source 40).

### *Situated Framing*

Situated CT is also widely embedded in the implications of CT in mathematics classrooms along with cognitive framing (76%). In the analysis, I formed seven codes to describe the practices that correspond with situated CT, namely resilience and perseverance, student agency, creativity, engagement/participation, collaboration, communication, and fun. These are presented in Table 5 along with the samples from the data set. Certain sources, such as the quote from Sources 31 fell under more than one

code (i.e., creativity and having fun) and others such as the quote from Source 26 under only one category (i.e., resilience and perseverance).

**Table 5: Situated framing’s codes and related samples**

<b>Code</b>	<b>Sample from the data set</b>
<b>Resilience and Perseverance</b>	“Coding and computational thinking allow us to have students <i>practice being resilient</i> through dealing with complex problems that need time to solve” (Source 23).
<b>Sense of agency</b>	“I was not told by my professor how to solve the problem, but instead was given free reign [sic] to approach a solution that made most sense to me. Since I had this level of control over the approach and design of the program, there was a true <i>sense of agency</i> in the design process” (Source 32).
<b>Creativity</b>	“While they are being creative and having fun, the students are still learning about coding and variables” (Source 31).
<b>Engagement/ participation</b>	“Through my observation, I was able to see that the students were <i>productively engaged in the activity</i> and each student was <i>actively participating</i> ” (Source 50).
<b>Collaboration</b>	“What we’re finding the most interesting is how much problem solving and <i>collaboration</i> we saw in our coding club. In this aspect, integrating coding into your classroom or your school helps to support students in their development of both problem solving and <i>collaboration skills</i> ” (Source 38).
<b>Communication</b>	“Another thing that we do regularly is to have the students <i>talk through their code to highlight what they have done</i> . This allows us to ask and probe the student’s thinking and to make sure that they understand the ideas they have used.” (Source 15).
<b>Fun</b>	“While they are being creative and <i>having fun</i> , the students are still learning about coding and variables” (Source 39).

### *Critical Framing*

Critical CT has been embedded in practices by very few educators and researchers (4%). Under this framing, I formed one code, citizenship, which is presented in Table 6 along with the sample from the data set.

**Table 6: Critical framing’s codes and related samples**

Code	Sample from the data set
<b>Citizenship</b>	“This lesson seemed like an early springboard for teaching children how to use technology to solve problems, so that by Grade 5 they can look at the world around them and reach out into the world and think about how they can use these skills to <i>become compassionate, active citizens</i> ” (Source 6).

### **Summary**

In the section above, I presented the results based on the analysis of CT practices concerning their framings of CT. The most significant finding revealed by these results is that these perspectives of CT (in the context of cognitive, situated, and critical framings) are not completely distinct; in fact, they are closely connected. Here are a few examples from the data:

“Coding and computational thinking allow us to have students *practice being resilient* [situated] through *dealing with complex problems* [cognitive] that need time to solve” (Source 15).

“What we’re finding the most interesting is how much *problem solving* [cognitive] and *collaboration* [situated] we saw in our coding club. In this aspect, integrating coding into your classroom or your school helps to support students in their development of both problem solving and collaboration skills” (Source 30).

“While they are being *creative and having fun* [situated], the students are still *learning about coding and variables* [cognitive]” (Source 31).



### 2.4.3 Other Findings

Four more codes were obtained based on content analysis: integrated learning, social emotional skills, real world applications, and 21st century skill. These codes do not necessarily fall under the aforementioned categories of framings; however, they are related to the curricular frameworks for the integration of CT in mathematics education. I presented these codes along with the samples from the data set in Table 7.

**Table 7: Other codes and related samples**

Code	Sample from the data set
<b>21<sup>st</sup> century skill</b>	“Another factor that contributed to our desire to start this club was belief that <i>coding is a valuable 21st century learning skill.</i> ” (Source 38).
<b>Real-world applications</b>	“The [Mathematics Integrated with Computers and Applications] MICA program truly <i>deepened my understanding of mathematics in the current world.</i> This understanding has been beneficial to me in my work with secondary students as I can explain how <i>mathematical modeling is applicable to the world</i> around them and even help them with the basics of <i>modeling real-world phenomena using computer-based applications</i> ” (Source 26).
<b>Integrated learning</b>	“Here we had wide walls that let us learn from different directions, but also broadened our learning horizons to <i>include complex mathematics, interesting visual arts, as well as the aesthetics of efficient code</i> ” (Source 23).
<b>Intergenerational learning</b>	“I am determined that <i>the students at my school and I will learn coding together. I am an intergenerational learner,</i> humbly willing to be the oldest learner in the room” (Source 41).

## 2.5 Discussion

In this section, I discussed three main topics revealed by the research findings: discussions on bibliographic information and context of CT practices, the intersection of

the CT framings, and connection of the findings with the integration of CT into mathematics curriculum.

### 2.5.1 Discussions on Bibliographic Information and Context of CT Practices

Based on the analysis of the bibliographic information and context of CT practices, three points are noted among other results:

First, most of the CT practices in mathematics education are from 2017. This result is consistent with the timing of scholarly works, and the support and promotion provided by the Ontario Ministry of Education (Gadanidis et al., 2017; OME, 2016). The majority of CT practices I analyzed are pilot projects in the outreach context. Following these outreach initiatives, school boards (e.g., Coding activities-TVSDB<sup>10</sup>) and government funded (e.g., Coding in the classroom-TVO Digital Learning Outreach<sup>11</sup>) initiatives have taken the lead to share resources in the school context to support educators, learners and parents. This could explain the drop in the outreach initiatives after 2017 in the graphic.

Second, most of the CT practices were conducted in elementary schools which is in line with the developments of the integration of CT into the mathematics curriculum, starting with lower grades (curriculum for Grades 1-8) and followed by the higher grades (Grade 9 math curriculum) (OME 2020; OME, 2021).

Third, Scratch is the most common tool used in implications of CT in mathematics classrooms. Brendan explains Scratch's success in Grades 4-8 classes as follows: "While creating shapes in Scratch works as a tremendous introduction to coding, the potential in Scratch extends much further than simple movements and drawings. It's easy to be tricked by its simple, colorful, block-based user interface, but the fact of the matter is that Scratch is a powerful tool with endless possibilities" (Source 39). Text-based languages,

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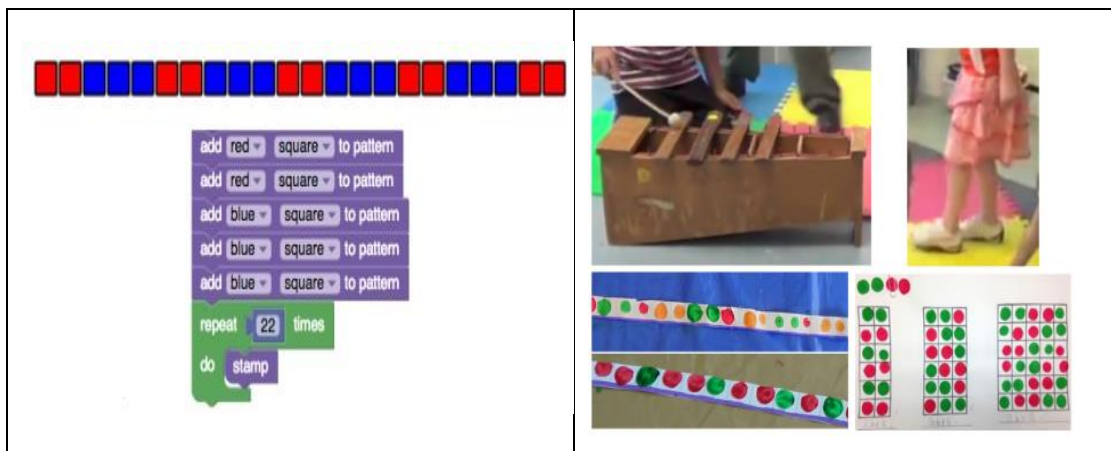
<sup>10</sup> <https://emilycarr.tvdsb.ca/en/our-school/coding-activities.aspx#GRADE-6>

<sup>11</sup> <https://outreach.tvolearn.com/codingintheclassroom/>

such as Python, are mostly preferred in Grades 9-12 practices and are generally used because of their rich libraries and their advanced coding capability (Source 17).

## 2.5.2 The Intersection of Cognitive, Situated, and Critical Framings

Cognitive framing of CT in mathematics teaching and learning maintains that the understanding of CT concepts and their application helps build and enrich mathematical knowledge and skills for children and youth. The common purpose of the CT practices, in this framing then, focuses on methods of using computational concepts and programming tools to promote mathematics learning (e.g., Source 5, 6, 7, 15, 32, 35, and 44). For instance, Source 6 reflects that when coding is used to dynamically model mathematics (i.e., experimenting with different variables to see what would happen). It helps students bring mathematics concepts to life and helps students to automate that process and develop other mathematical skills. Modelling color patterns with block-based programming or through unplugged coding (e.g., playing and singing repeating patterns on a xylophone, dancing them on color mats, and stamping them with bingo dabber) allows students to develop algebra, patterning, and numeracy skills as seen in Figure 12.



**Figure 12: Modelling color patterns with coding**

Situated framing of CT aims to create “personally-meaningful applications”, build “communities”, and “support social interactions and play” (Kafai et al., 2020, p.105). An intermediate teacher, Derek approaches the use of CT from a situated framing and mentions that coding helps create “a culture of learning which transcends the walls of the classroom” and allows young people to explore their imaginations and to create their own

meaningful content (Source 22). It is also reflected in the context of situated framing of CT practices that coding increases collaboration and creates a sense of community and common purpose (e.g., Source 6 and 46). Grades 2/3 teacher, for instance, narrates that “Even though they may not be working together, they are observing what’s going on in others and then choosing which of those that they see that they wish to put into play. I had kids get up and go over and say, “How did you do that?” and then go back and do it.” (Source 6).

According to the analysis, while some practices fall into a distinct category, some others fall into more than one category. In these practices, situated CT is mostly embedded into practice along with cognitive CT (76%). In their classroom activity with Grades 7 and 8 students, Rita and Rachel reflect on the coding club they started as a part of their practicum and express that they are most impressed with students’ *engagement, persistence, and collaboration* [situated framing] while observing the improvement of their *problem-solving skills* [cognitive framing] through coding using Tynker or Scratch plugged activities, or unplugged activities (using conditional (if/else) statements to state the rules of simple card games such as clapping if the card is lower than 5, or else saying “awwww” if the card is 5 or greater) (Source 38). Hence, these three framings are not strictly distinct from each other; but are connected and complementary. This is particularly true of cognitive and situated framings of CT. Thus, my view differs from Kafai et al.’s (2020) argument that “situated framing is an alternative proposition to cognitive emphasis” (p.103). In actual practice, for instance, Matthew mentions that “debugging a program is a powerful exercise in analytic thinking, trial and error, and just plain perseverance” and refers to both cognitive (analytic thinking) and situated (perseverance) framings (Source 42). Similarly, Donna states that coding provides an opportunity for students to improve their “perseverance, mathematical reasoning, and problem-solving skills” (Source 41). As reflected in Source 23, because “coding and computational thinking [require] dealing with complex problems that need time to solve”, which is in an aspect of cognitive framing, this “allows us to have students practice being resilient”, which is a situated framing. In line with these professional reflections, Gadanidis (2017b) draws attention to the five affordances of CT that support elementary mathematics education: “agency, access, abstraction, automation, and audience” which

are embraced as parts of cognitive (e.g., abstraction and automation) and situated framings (e.g., agency and audience).

Critical CT is also embedded in professional publications along with cognitive CT and or situated CT. With the inclusion of critical framing, students not only “understand and critique computational infrastructures”, but also have the opportunity to create “applications to promote thriving, awareness, and activism” (Kafai, et al., 2020, p. 105). Source13, for instance, reflects that while the goal of the project is to “teach the Grade 6 Tech Buddies how to code using Microsoft MakeCode and Micro:bits, so that they could facilitate the *learning of coding* [cognitive framing]”, the results of this project show that it also “helps to develop *resilient* [situated framing], thriving, and successful learners who will become *active and contributing members of society* [critical framing]”. A junior grade teacher also reflects that using CT skills to solve problems and to make use of these skills in becoming compassionate and active citizens “building CT skills early empowers the children to not just being participants on that digital stage” and “by Grade 5 they can look at the world around them and reach out into the world and think about how they can use these skills to become compassionate, active citizens.” (Source 6); therefore, cognitive, situated, and critical framings are connected and have the potential to promote one another.

### 2.5.3 Connection of the Findings with the Integration of CT into the Mathematics Curriculum

The Ontario Ministry of Education has made five years of substantial progress in integrating coding and computational skills into teaching initiatives since 2016. However, according to the findings of this study on school and outreach practices the integration of CT in mathematics education is in its early stages. In this section, I discuss the connection between CT framings and the two curricular frameworks for the integration of CT in mathematics education: the new Ontario mathematics curriculum Grades 1-8 (OME, 2020) and revised Grade 9 mathematics curriculum (OME, 2021).

In the new mathematics curriculum, computational thinking is defined as “the thought process involved in expressing problems in such a way that their solutions can be reached

using computational steps and algorithms” (OME, 2020, p. 513) and coding is identified as “solve problems and create computational representations of mathematical situations using coding concepts and skills” (OME, 2020, p. 419). From these expressions, it is clear that cognitive CT is the primary focus within the curriculum. Hence it is used in the integration of coding into the mathematics curriculum. However, the new curriculum also provides a link to situated and critical framings by mentioning the possibility of using coding “to solve future, more ambiguous real-life problems” (OME, 2020, p. 419). There are other connections between the framings of CT and other aspects (e.g., social-emotional learning skills, transferrable skills) of the curriculum. Below I discuss other connections as shown by the results of the content analyses and the aspects of the curricular frameworks (i.e., the new Ontario mathematics curriculum Grades 1-8 and 9).

### **Social Emotional Learning Skills (SEL)**

As one of the new developments in the Ontario mathematics curriculum, building and promoting social emotional learning (SEL) skills has been added as a strand “to support [students’] learning of math concepts and skills” and “foster their overall well-being and ability to learn” while helping them “build resilience and thrive as math learners” (OME, 2020, p. 110). This addition shows the intersection of cognitive and situated framings of the integration of CT in mathematics teaching and learning, as it refers to supporting mathematical knowledge and skills while promoting SEL skills. SEL skills are defined in the context of situated CT (Kafai et al., 2020). SEL skills link to social context according to situated view of CT, however SEL skills could also be defined in the context of efforts addressing systemic oppression and racism, which refers to critical CT, and this linkage is formalized explicitly in the updated curriculum expectations of Grade 9 (OME, 2021).

### **Transferrable Skills**

Transferable skills (OME, 2020) or 21st century skills (also referred to as global competencies (Council of Ministers of Education Canada (CMEC), n.d.) are prioritized in education so that students can succeed in today's modern world (Barr, et al., 2011). Seven categories of transferable skills, which are aligned with the 21st century competencies

(OME, 2016) and six pan-Canadian global competencies (CMEC, n.d.), are (1) critical thinking and problem solving, (2) innovation, creativity, and entrepreneurship, (3) self-directed learning, (4) communication, (5) collaboration, (6) global citizenship and sustainability, and (7) digital literacy and applicable to all curriculum documents, Grades 1-12 including mathematics curriculum. For instance, self-directed learning skill is defined in the mathematics Grades 1-8 curriculum as follows: “In mathematics, they [students] initiate new learning, monitor their thinking and their emotions when solving problems and apply strategies to overcome challenges” (OME, 2020., p. 103). It is identified that transferable skills “are developed through students’ cognitive, social, emotional, and physical engagement in learning” and “through a variety of teaching and learning methods, models, and approaches” (Transferrable skills, 2020-22). As an effective teaching and learning method, using CT concepts and tools, such as coding, in mathematics education also promotes these skills (Eguchi, 2013; Gretter & Yadav, 2016; Wong & Cheung, 2020). Enzo reflects on how coding promotes self-directed learning skill based on his classroom experience with 6<sup>th</sup> grades and says: “I have witnessed the persistence students have in developing the perfect code. When their code works, they exhibit such happiness. They want to share their code and end up explaining their mathematical processes” (Source 52). Source 13 refers to the innovation, creativity, and entrepreneurship skills by stating that “[b]y tapping into the students’ interests and passions, we are creating an environment where children are more willing to take greater risks in their own learning”.

In mathematics curriculum document coding is linked to three transferrable skills: critical thinking and problem solving, communication and digital literacy (OME, 2020; OME, 2021). However, based on the analysis, six of the above seven transferrable skills overlap with the codes under the framings of CT, cognitive, situated, and critical. The connections are shown in Table 8.

**Table 8: Overlap of transferable skills and codes under the framings of CT**

<b>Transferrable skill (OME, 2020)</b>	<b>Code from content analysis</b>	<b>Framing of CT</b>
<b>critical thinking and problem solving</b>	problem solving critical thinking	cognitive
<b>innovation, creativity, and entrepreneurship</b>	creativity	situated
<b>self-directed learning</b>	resilience and perseverance	situated
<b>communication</b>	Communication/participation	situated
<b>collaboration</b>	collaboration	situated
<b>global citizenship and sustainability</b>	citizenship	critical

The seventh OME (2020) transferrable skill, digital literacy, intersects with all three framings since it includes cognitive, social, and critical processes and is connected to other skills, such as self-directed learning, communication, collaboration, and citizenship (Hughes, 2017; Hughes & Burke, 2014). In today's digital era, students need higher-order reasoning skills, dynamic thinking, and the ability to engage in an inquiry process and design practical prototype solutions for ill-defined problems, which may lack most of the important information required for a complete solution (Canadians for 21st Century Learning and Innovation, 2016).

In mathematics, students and teachers learn how to make use of technology to model real-life issues, and to use these models to obtain a comprehensive understanding of these issues, to solve problems related to them, to predict outcomes for different scenarios, and to assess the validity of their results (OME, 2020). Hughes and Morrison (2014) also highlight that "When students gain understanding of others using contemporary media texts, they are afforded the opportunity for agency and change in their own lives within school, the community, and beyond" (36).



## **Real-world Applications**

“Being integrated with the world beyond the classroom” and “making connections to the world” through real-life applications are the principles related to real-world applications underlined in the Ontario mathematics curriculum (OME, 2020, p.65). In the revised curriculum for the Grade 9 mathematics course students are expected to make use of the processes, of modelling and coding to make sense of what they are learning and to deepen the knowledge and understanding they acquire through applying these skills to relevant real-life situations that are culturally responsive (OME, 2021). Algebra strands of Grade 9 also emphasize real-life applications in addition to coding, “[s]tudents develop an understanding of the constant rate of change and initial values of linear relations and solve related real-life problems” (OME, 2021). These principles are embraced by many educators and researchers in their practices. A high school mathematics teacher, for instance, reflects that how developing and teaching mathematics integrated coding courses deepened their understanding of mathematics in the real world: “This understanding has been beneficial to me in my work with secondary students as I can explain how mathematical modelling is applicable to the world around them and even help them with the basics of modelling real-world phenomena using computer-based applications” (Source 26).

## **Intergenerational Learning**

Intergenerational learning is defined by Ropes (2013) as “an interactive process that takes place between different generations resulting in the acquisition of new knowledge, skills, and values” (p. 714). Based on their experience at JK/SK classroom through the coding buddies’ program, Brandon et al. report that while there is a benefit for young students in working with older students, older students who are more knowledgeable about the programs they teach also benefit by gaining new perspectives from their younger peers (Source 18). Further, Donna also mentions that teaching coding provides a good setting for intergenerational learning. She added that she has become more of a learner and less of a teacher and has gained experience in intergenerational learning through being taught by students (Source 41).

Although intergenerational learning is not openly mentioned in the mathematics curriculum, the skill of “*building healthy relationships and communicating effectively in mathematics*,” which is under SEL skills, refers to the purpose of intergenerational learning as it includes expressing students’ own thinking considering other ideas and perspectives and practicing inclusivity (OME, 2020; OME, 2021).

### **Integrated Learning**

Integrated learning “engages students in a rich learning experience that helps them make connections across subjects and brings the learning to life” and helps students “to develop their ability to think and reason and to transfer knowledge and skills from one subject area to another” while they are learning “specific knowledge and skills from the curriculum” (OME, 2020, p. 28). Hence, each exercise or activity in integrated learning emphasizes more than one subject matter.

As pointed out in the mathematics curriculum, CT also allows students to explore across subjects such as visual arts and music while they are learning computational and mathematical concepts (Gadanidis, Cendros, et al., 2017). Grades 2/3 teacher, for instance, reflects on the classroom activity that students wrote and performed the symmetry song while learning symmetry in a coding environment (Source 7). Related to this integrated learning context of coding, many resources on how to bridge mathematics, code, and art (e.g., generating spiral triangles by coding in Tynker) at the mathematics knowledge network website (Source 8).

## **2.6 Conclusion**

Each of cognitive, situated, and critical framings is valuable, as each of them offers different insights to understand the learning and teaching of CT (Kafai et al., 2020). Although they seem theoretically different, as shown by Kafai et al. (2020) they are connected and mutually reinforcing in practice. According to the content analysis, cognitive and situated framings are the most common perspectives embedded in the context of purpose and outcome. However, critical CT should also be adequately considered while designing these practices for teaching CT in schools and community

settings. The finding from this content analysis is in line with the statement that the attention paid to understanding and applying CT concepts to improve mathematical understanding and skills can too easily allow practitioners to miss the ability of critical CT “to promote thriving, awareness, and activism” (Kafai et al., 2020, p. 105).

It is possible to approach CT from a cognitive framing and interpret it as solving problems through computing. However, situated and critical framings recognize the interaction between people and technology and highlight how deeply computational skills are intertwined with other societal concerns, such as collaboration (e.g., Source 38 and 50) and citizenship (e.g., Source 6 and 13). Students have the capacity to learn to code and create computational products and learn ideas that can potentially make a positive impact in their lives and the lives of their families and communities. This claim which has been demonstrated in the analyzed practices, such as the reflection in which a 5th Grade teacher said: “I am always thinking of using technology, coding and CT to prepare children for active citizenship so they will have the skills and the heart to make positive changes in the world” (Source 6). To realize such an impact, all that is needed is greater awareness of the affordances of CT.

In this study, I presented how different framings of CT reflect on school and outreach practices in connection with the current curriculum policy. Using cognitive, situated, and critical framings as a conceptual framework, this study highlighted the wide range of possibilities of CT in mathematics education to not only provide useful skills and competencies for students’ curriculum learning and future career paths, but also to contribute to their personal and social lives as well as improve their understandings of broader societal settings which are meet and exceed the expectations of curricula content and skills.

## 2.7 Chapter references

Ackermann, E. (2001). Piaget’s Constructivism, Papert’s Constructionism: What’s the Difference. *Future of Learning Group Publication*, 5, 438.  
[https://learning.media.mit.edu/content/publications/EA.Piaget%20\\_%20Papert.pdf](https://learning.media.mit.edu/content/publications/EA.Piaget%20_%20Papert.pdf)

- Amineh, R. J., & Asl, H. D. (2015). Review of constructivism and social constructivism. *Journal of Social Sciences, Literature, and Languages*, 1(1), 9-16. <https://www.semanticscholar.org/paper/Review-of-Constructivism-and-Social-Constructivism-Amineh-Asl/38903f4a7255496f75124d639e14e9b810c17370>
- Australian Computing Academy. (n.d.). *Helping teachers implement the Digital Technologies Curriculum in classrooms around the country*. <https://aca.edu.au/>
- Aydeniz M. (2018). Integrating computational thinking in school curriculum. In: Khine M. (Ed.) *Computational thinking in the stem disciplines*. Springer, Cham. [https://doi.org/10.1007/978-3-319-93566-9\\_13](https://doi.org/10.1007/978-3-319-93566-9_13)
- Barr, D., Harrison, J., & Conery, L. (2011). Computational thinking: A digital age skill for everyone. *Learning & Leading with Technology*, 38(6), 20–23. <https://eric.ed.gov/?id=EJ918910>
- Basu, T. (2020, June 12). A teenager’s guide to building the world’s best pandemic and protest trackers. *MIT Technology Review*. <https://www.technologyreview.com/2020/06/12/1002838/avi-schiffmann-17-year-old-guide-building-pandemic-protest-tracker/>
- Belgrave L.L., & Seide K. (2019). Coding in grounded theory. In A. Bryant & K. Charmaz (Eds.) *SAGE handbook of current developments in grounded theory* (pp. 167-185). SAGE Publications.
- Berk, R., Campbell, A., Klap, R., & Western, B. (1992). The deterrent effect of arrest in incidents of domestic violence: A Bayesian analysis of four field experiments. *American Sociological Review*, 57, 698–708. <https://doi.org/10.2307/2095923>
- Birt, L., Scott, S., Cavers, D., Campbell, C., & Walter, F. (2016). Member checking: A tool to enhance trustworthiness or merely a nod to validation? *Qualitative Health Research*, 26(13), 1802-1811. <https://doi.org/10.1177%2F1049732316654870>
- Blackstone, Amy. (2019). *Social research: Qualitative and quantitative methods (Version 2.0)*. Flat World Knowledge.
- Blackstone, A., Houle, J., & Uggen, C. (2006). At the time I thought it was great: Age, experience, and workers’ perceptions of sexual harassment. Presented at the 2006 meetings of the American Sociological Association.
- Brennan, K., & Resnick, M. (2012). *Using artifact-based interviews to study the development of computational thinking in interactive media design*. Paper presented at annual American Educational Research Association meeting, Vancouver, BC, Canada.
- Canadians for 21st Century Learning and Innovation (C21 Canada). (2016). *Shifting minds 4.0: Redefining the learning landscape in Canada*. [https://www.sunwestpebl.ca/uploads/1/0/5/2/105217071/c21\\_canada\\_learning\\_and\\_innovation\\_inventory\\_index\\_part\\_1\\_and\\_2\\_and\\_cover.pdf](https://www.sunwestpebl.ca/uploads/1/0/5/2/105217071/c21_canada_learning_and_innovation_inventory_index_part_1_and_2_and_cover.pdf)
- Caputo-Rostami, S. (2019, June 6). *TLLP-cardinal newman CES cracking the math code: Engaging students and fostering learning through coding and robotics*.

- TeachOntario.  
<https://www.teachontario.ca/community/explore/tlppke/projects/cracking-the-math-code/blog/2019/06/06/tlp-cardinal-newman-ces-cracking-the-math-code-engaging-students-and-fostering-learning-through-coding-and-robotics>
- Components in Electronics (CIE). (2018, December 13). *Premier Farnell brings physical computing into the classroom with Leeds-based Binary Bots*. Cieonline.  
<https://www.cieonline.co.uk/premier-farnell-brings-physical-computing-into-the-classroom-with-leeds-based-binarybots/>
- Corbin, J., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). SAGE Publications.
- Council of Ministers of Education, Canada. (n.d.). *Global competencies*.  
[https://www.cmec.ca/682/Global\\_Competencies.html](https://www.cmec.ca/682/Global_Competencies.html)
- Creswell, J. W. (2012). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (4th ed.). Pearson.
- CT in kindergarten. (2016-18). Ctmath.ca. <http://ctmath.ca/projects/ct-in-kindergarten/>
- CT Lessons and Projects from Green Dot Public Schools (n.d.). *Ct. excelwa*  
<http://ct.excelwa.org/math/racial-bias-in-traffic-stops/>
- DeCarlo, M. (2018). *Scientific inquiry in social work*. Open Social Work Education.
- De Freitas, E., & Walshaw, M. (2016). *Alternative theoretical frameworks for mathematics education research: Theory meets data*. Springer.  
<https://doi.org/10.1007/978-3-319-33961-0>
- diSessa, A. A. (2000). *Changing minds: Computers, learning, and literacy*. MIT Press.
- Draper, R. J. (2002). School mathematics reform, constructivism, and literacy: A case for literacy instruction in the reform-oriented math classroom. *Journal of Adolescent & Adult Literacy*, 45(6), 520-529. <https://www.jstor.org/stable/40014740?seq=1>
- Edugains (n.d.) Coding in elementary a professional learning resource for Ontario educators.  
<https://www.edugains.ca/resourcesTELO/CE/CodingInterface/interface.html>.  
 Accessed 2020 June 20.
- Edwards, K.E., & Jones, K. (2009). Putting my man face on: A grounded theory of college men's gender identity development. *Journal of College Student Development*, 50, 210 - 228. <http://hdl.handle.net/1903/6862>
- Eguchi, A. (2014). Educational robotics for promoting 21st century skills. *Journal of Automation, Mobile Robotics & Intelligent Systems*, 8(1), 5–11. [https://doi.org/10.14313/JAMRIS\\_1-2014/1](https://doi.org/10.14313/JAMRIS_1-2014/1)
- Ertmer, P. A., & Newby, T. J. (2013). Behaviorism, cognitivism, constructivism: Comparing critical features from an instructional design perspective. *Performance Improvement Quarterly*, 26(2), 43-71.
- European Commission. (2019). *EU code week - Bringing ideas to life*.  
<https://ec.europa.eu/digital-single-market/en/eu-code-week>

- Farnell. (n.d.). *Coding in the national curriculum*. <https://uk.farnell.com/coding-national-curriculum>
- Feurzeig, W., & Papert, S. A. (2011). Programming-languages as a conceptual framework for teaching mathematics. *Interactive Learning Environments*, 19(5), 487-501, <https://doi.org/10.1080/10494820903520040>
- Flick, U. (2009). *An introduction to qualitative research* (4th ed.). SAGE Publications.
- Floyd, S & Floyd, L. A. (2019, May 22). *Computational literacy and mathematics education: A webinar with Dr. Andy diSessa*. Mathematics Knowledge Network. <http://mkn-rcm.ca/computational-literacy-and-mathematics-education-a-webinar-with-dr-andy-disessa/>
- Freire, P. (2005). *Pedagogy of the oppressed* (M. B. Ramos, Trans.) Continuum. (Original work published in 1970).
- Gadanidis, G. (2017). Five affordances of computational thinking to support elementary mathematics. *Education Journal of Computers in Mathematics and Science Teaching*, 36(2), 143-151. <https://eric.ed.gov/?id=EJ1154750>
- Gadanidis, G., Brodie, I., Minniti, L., & Silver, B. (2017). Computer coding in the K-8 mathematics curriculum? What Works? Research into Practice: Research Monograph #69. [http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Computer\\_Coding\\_K8\\_en.pdf](http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Computer_Coding_K8_en.pdf)
- Gadanidis, G., & Caswell, B. (2018). Computational modelling in elementary mathematics education: Making sense of coding in elementary classrooms. *KNAER Mathematics Knowledge Network, White Paper*. <http://mkn-rcm.ca/wp-content/uploads/2018/05/MKN-white-paper2-May-2018.pdf>
- Gadanidis, G., Cendros, R., Floyd, L., & Namukasa, I. (2017). Computational thinking in mathematics teacher education. *Contemporary Issues in Technology and Teacher Education*, 17(4), 458-477. <https://citejournal.org/volume-17/issue-4-17/mathematics/computational-thinking-in-mathematics-teacher-education>
- Gadanidis, G., & Cummings, J. (2018). Integrated mathematics + computer studies: Reforming secondary school mathematics education. *KNAER Mathematics Knowledge Network*. <http://mkn-rcm.ca/wp-content/uploads/2018/04/MKN-white-paper-April-2018.pdf>
- Gannon, S. & Buteau, C. (2018). Integration of computational thinking in Canadian provinces. In *Online Proceedings of the computational thinking in mathematics education symposium*, UOIT (Scarborough).
- Ghosh, J. (2019). Some opportunities for computational thinking in the mathematics classroom. In *Proceedings of the 24th Asian technology conference in mathematics*. Leshan Vocational and Technical College, Leshan, China.
- Gretter, S., & Yadav, A. (2016). Computational thinking and media & information literacy: An integrated approach to teaching twenty-first century skills. *TechTrends*, 60, 510-516.

- Haduong, P. (2019). I like computers. I hate coding: a portrait of two teens' experiences. *Information and Learning Sciences*, 120(5/6), 349-365. <https://doi.org/10.1108/ILS-05-2018-0037>
- Ho, W.K., Looi, C., Huang, W., Seow, P., & Wu, L. (2019). Realizing computational thinking in the mathematics classroom: Bridging the theory practice gap. In *Proceedings of the 24th Asian technology conference in mathematics*, national institute of education Nanyang technological university Singapore. <https://www.semanticscholar.org/paper/Realizing-Computational-Thinking-in-the-Mathematics-Ho-Looi/7a5ebe29113b4bff2fbc546c3cb4c575526b91cc>
- Hostetler, A, Sengupta, P, & Hollett, T. (2018). Unsilencing critical conversations in social-studies teacher education using agent-based modeling. *Cognition & Instruction*, 36(2), 139 – 170. <https://doi.org/10.1080/07370008.2017.1420653>
- Hour of Code Activities. (n.d.). *Hour of code*. <https://hourofcode.com/ca/learn>
- Hughes, J. (2017). Digital making with “at-risk” youth. *The International Journal of Information and Learning Technology*, 34(2), 102-113. <https://doi.org/10.1108/IJILT-08-2016-0037>
- Hughes, J., & Burke, A. (2014). *The digital principal*. Pembroke Publishers Limited.
- Hughes, J., & Morrison, L. (2014). At the intersection of critical digital literacies, YAL and literature circles. *ALAN Review*, 42(1), 35–43. <https://scholar.lib.vt.edu/ejournals/ALAN/v42n1/hughes.html>
- Iyioke, I. C. (2020). Cognitive vs. social constructivist learning for research and training on the angoff method. In Ş. Orakçı (Eds.), *Paradigm shifts in 21st century teaching and learning* (pp. 181-201). IGI Global. <http://doi:10.4018/978-1-7998-3146-4.ch012>
- Jablonka E., Wagner D., & Walshaw M. (2012). Theories for studying social, political and cultural dimensions of mathematics education. In: Clements M., Bishop A., Keitel C., Kilpatrick J., Leung F. (Eds.) *Third international handbook of mathematics education. Springer international handbooks of education*, 27. Springer. [https://doi.org/10.1007/978-1-4614-4684-2\\_2](https://doi.org/10.1007/978-1-4614-4684-2_2)
- Joshi, S. (2016, March 29). Teenage girl coders from Mumbai slum are building apps to solve local problems. *Mashable*. <https://mashable.com/2016/03/29/mumbai-dharavi-girls-coding-apps/>
- Julie, A. (2017, August 28). *Teaching coding in Canadian schools: How do the provinces measure up?* GlobalNews. <https://globalnews.ca/news/3693932/teaching-coding-in-canadian-schools-how-do-the-provinces-measure-up/>
- Kafai, Y. B. (2016). Education from computational thinking to computational participation in K-12 education: Seeking to reframe computational thinking as computational participation. *Communications of the ACM*, 59(8), 26-27. <https://doi.org/10.1145/2955114>
- Kafai Y. B., & Burke, Q. (2013). Computer programming goes back to school. *Phi Delta Kappan*, 95(1). 61-65. <https://doi.org/10.1177/003172171309500111>

- Kafai, Y. B., Proctor, C., & Lui, D. (2020). From theory bias to theory dialogue: embracing cognitive, situated, and critical framings of computational thinking in K-12 cs education. *ACM Inroads*, *11*(1). <https://doi.org/10.1145/3381887>
- Kotsopoulos, D., Floyd, L., Khan, S., Namukasa, I. K., Somanath, S., Weber, J., & Yiu, C. (2017). A pedagogical framework for computational thinking. *Digital Experiences Mathematics Education*, *3*, 154–171. <https://doi.org/10.1007/s40751-017-0031-2>
- Krippendorff, K. (2004). *Content analysis: An introduction to its methodology* (2nd ed.). SAGE Publications.
- Lavigne, H. J., Lewis-Presser, A., & Rosenfeld, D. (2020). An exploratory approach for investigating the integration of computational thinking and mathematics for preschool children. *Journal of Digital Learning in Teacher Education*, *36*(1), 63–77. <https://doi.org/10.1080/21532974.2019.1693940>
- Lee, C. H. (2012). *Re-mastering the master's tools: Recognizing and affirming the life experiences and cultural practices of urban youth in critical computational literacy through a video game project*. [Unpublished doctoral dissertation]. University of California.
- Lee, C. H., & Garcia, A. D. (2014). I Want Them to Feel the Fear...: Critical computational literacy as the new multimodal composition. In Ferdig, R. E., & Pytash, K. E. (Ed.), *Exploring multimodal composition and digital writing* (pp. 364–378). IGI Global. <http://doi:10.4018/978-1-4666-4345-1.ch022>
- Lee, I., & Malyn-Smith, J. (2020). Computational thinking integration patterns along the framework defining computational thinking from a disciplinary perspective. *Journal of Science Education and Technology*, *29*, 9–18. <https://doi.org/10.1007/s10956-019-09802-x>
- Lee, C. H., & Soep, E. (2016). None but ourselves can free our minds: Critical computational literacy as a pedagogy of resistance. *Equity & Excellence in Education*, *49*(4), 480–492. <https://doi.org/10.1080/10665684.2016.1227157>
- Lee, C. H., & Soep, E. (2018). Beyond coding: using critical computational literacy to transform tech. *Texas Education Review*, *6*(1). <http://hdl.handle.net/2152/64975>
- Margolis, J., Ryoo, J. J., Sandoval, C. D. M., Lee, C., Goode, J., & Chapman, G. (2012). Beyond access: Broadening participation in high school computer science. *ACM Inroads*, *3*(4) <https://doi.org/10.1145/2381083.2381102>
- Martinez, S. L., & Stager, G. (2013). *Invent to learn: Making, tinkering, and engineering in the classroom*. Constructing Modern Knowledge Press.
- Mathematics Knowledge Network (MKN). (2016–17). *Math + Coding in Wellington CDSB*. <http://mkn-rcm.ca/wcdsb1/>
- Mayring, P. (2000). Qualitative content analysis. *Forum: Qualitative Social Research*, *1*(2). <https://www.qualitative-research.net/index.php/fqs/article/view/1089/2386>



- Mayring, Philipp (2000). Qualitative Content Analysis [28 paragraphs]. Forum Qualitative Sozialforschung / Forum: Qualitative Social Research, 1(2), Art. 20, <http://nbnresolving.de/urn:nbn:de:0114-fqs0002204>.
- Meyers, E.M. (2019). Learning to code, coding to learn: Youth and computational thinking "Guest editorial", *Information and Learning Sciences*, 120(5/6), 254-265. <https://doi.org/10.1108/ILS-05-2019-139>
- Nardelli, E. (2019). Do we really need computational thinking? *Communications of the ACM*, 62(2), 32-35. <https://dl.acm.org/toc/cacm/2019/62/2>
- Neuendorf, K. A. (2017). *The content analysis guidebook (2nd ed.)*. SAGE Publications.
- News Ontario. (2020, June 23). *Ontario introduces new math curriculum for elementary students: Focus on the fundamentals will better prepare students for jobs of the future*. <https://news.ontario.ca/opo/en/2020/6/ontario-introduces-new-math-curriculum-for-elementary-students.html>
- News Ontario (2021). *Modernized math course prepares students for jobs of the future and life skills*. Queen's Printer for Ontario.
- Noguerón-Liu, S. (2017). Expanding notions of digital access: parents' negotiation of school-based technology initiatives in new immigrant communities. *Equity & Excellence in Education*, 50(4), 387–399. <https://doi.org/10.1080/10665684.2017.1395301>
- Ontarimathsupport (n.d.). *Coding resources: Grade 3- Coding lesson- location and movement- coding ourselves*. <https://ontariomath.support/viewResource.php?id=46&lang=EN&type=Codingresource&cat=lesson>
- Ontario Ministry of Education (2016). *21st century competencies: Foundation document for discussion*. Queen's Printer for Ontario.
- Ontario Ministry of Education (2020). *The Ontario curriculum Grades 1–8 mathematics curriculum context*. Queen's Printer for Ontario.
- Ontario Ministry of Education (2021). *The Ontario curriculum: Mathematics, Grade 9*. Queen's Printer for Ontario.
- Ontario Ministry of Education (2020-22). *Transferable skills*. Queen's Printer for Ontario. <https://www.dcp.edu.gov.on.ca/en/transferable-skills/introduction>
- Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. Basic Books.
- Papert, S., & Harel, I. (1991). *Constructionism*. Ablex Publishing Corporation.
- Pate, A., & Hamilton, E. (1992). Formal and informal deterrents to domestic violence: The Dade County spouse assault experiment. *American Sociological Review*, 57, 691–697. <https://doi.org/10.2307/2095922>
- Patton, M. (1990). Qualitative evaluation and research methods (pp. 169-186). SAGE Publications.

- Proctor, C., & Blikstein, P. (2019). Unfold studio: supporting critical literacies of text and code. *Information and Learning Sciences*, 120 (5/6), 285-307. <https://doi.org/10.1108/ILS-05-2018-0039>
- Rich, K. M., Yadav, A., & Larimore, R. A. (2020). Teacher implementation profiles for integrating computational thinking into elementary mathematics and science instruction. *Education and Information Technologies*, 25(4), 3161-3188. <https://doi.org/10.1007/s10639-020-10115-5>
- Resnick, M. (1996). Distributed Constructionism. In *Proceedings of the 1996 international conference on Learning sciences (ICLS '96)*. International Society of the Learning Sciences, 280–284. <https://dl.acm.org/doi/10.5555/1161135.1161173>
- Ropes, D. (2013). Intergenerational learning in organizations. *European Journal of Training and Development*, 37(8), 713-727. <https://doi.org/10.1108/EJTD-11-2012-0081>
- Salkind, N. J. (2010). Inclusion criteria. In *Encyclopedia of research design* (Vol. 1, pp. 629-591). SAGE Publications, Inc., <https://dx.doi.org/10.4135/9781412961288.n183>
- Sengupta, P., Dickes, A., & Farris, A.V. (2018). Toward a phenomenology of computational thinking in STEM. In: Khine, M.S. (Ed.): *Computational thinking in STEM: research highlights* (pp. 49-72). Springer. [https://doi.org/10.1007/978-3-319-93566-9\\_4](https://doi.org/10.1007/978-3-319-93566-9_4)
- Sfard, A. (1998). On two metaphors for learning and the dangers of choosing just one. *Educational Researcher*, 27(2), 4-13 <https://doi.org/10.3102/0013189X027002004>
- Sheldon, E. (2018, June 15). *The flexibility of computational thinking*. Edutopia. <https://www.edutopia.org/article/flexibility-computational-thinking>
- Snelling, J. (2018, March 27). Students use computational thinking to analyze real problems. *ISTE*. <https://www.iste.org/explore/Computational-Thinking/Students-use-computational-thinking-to-analyze-real-problems>
- Stevens, L. P., & Bean, T. W. (2007). *Redefining literacy*. In *Critical literacy: Context, research, and practice in the K-12 classroom* (pp. 1-14). SAGE Publications, Inc., <https://www-doi-org.proxy1.lib.uwo.ca/10.4135/9781452204062.n1>
- Strauss, A. (1987). *Qualitative analysis for social scientists*. Cambridge University Press.
- Strauss A. L., & Corbin J. M. (1998). *Basics of qualitative research: techniques and procedures for developing grounded theory* (2nd ed.), SAGE Publications.
- The Learning Partnership. (n.d.). *Teach kids to code with the new and improved Coding Quest!* <https://www.thelearningpartnership.ca/programs/coding-quest>. Accessed 2020 May 15.
- Tissenbaum, M., Sheldon, J., & Abelson, H. (2019). From computational thinking to computational action. *Communications of the ACM*, 62 (3), 34-36. <https://doi.org/10.1145/3265747>

- TVO Digital Learning Outreach. (n. d.). Coding in the classroom.  
<https://outreach.tvolearn.com/codingintheclassroom/>
- Uggen, C., & Blackstone, A. (2004). Sexual harassment as a gendered expression of power. *American Sociological Review*, 69, 64–92.  
<https://doi.org/10.1177/000312240406900105>
- Vasquez, V. M., Janks, H., & Comber, B. (2019). Critical literacy as a way of being and doing. *Language Arts*, 96 (5), 300-311.  
[https://www.academia.edu/39049513/Critical\\_Literacy\\_as\\_a\\_Way\\_of\\_Being\\_and\\_Doing?auto=download](https://www.academia.edu/39049513/Critical_Literacy_as_a_Way_of_Being_and_Doing?auto=download)
- Veeragoudar-Harrell, S. (2009). *Second chance at first life: Fostering the mathematical and computational agency of at-risk youth*. [Dissertation, University of California, Berkeley].
- Vygotsky, L. S. (1978). *Mind in society: the development of higher psychological processes*. Cole, M., Jolm-Steiner, V., Scribner, S., & Souberman, E., (Eds.). Harvard University Press.
- Weidler-Lewis, J., DuBow, W., Kaminsky, A., & Weston, T. (2019). Supporting women’s persistence in computing and technology: A case for compulsory critical coding? *Information and Learning Sciences*, 120(5/6), 366-382. <https://doi.org/10.1108/ILS-08-2018-0083>
- Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K. Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms. *Journal Science Education and Technology*, 25, 127–147.  
<https://doi.org/10.1007/s10956-015-9581-5>
- Wong, G. K. W., & Cheung, H. Y. (2020). Exploring children’s perceptions of developing twenty-first century skills through computational thinking and programming. *Interactive Learning Environments*, 28(4), 438–450. <https://doi.org/10.1080/10494820.2018.1534245>

## Chapter 3

### 3 Understanding Real-World Problems through Computational Thinking Tools and Concepts

When the novel coronavirus was first identified, countries and the World Health Organization (WHO) could not largely understand the risk and rate at which this disease would culminate into a global crisis. The following questions need to be answered rapidly by experts when responding to the outbreak: At what rate was the infection going to spread in different populations? How were experts, health officials, and policymakers going to effectively convey information that would help in understanding the nature of the outbreak? How were they to demonstrate how the different recommended collective control protocols would alter the spread of the outbreak? As Shepherd (2020) indicates, it became very crucial to make the public comprehend the severity of the health risks of this novel virus, and the need to critically interpret and implement the precautions recommended by the public authorities.

Mathematics, along with other disciplines, is essential in helping to understand several aspects of an outbreak. For instance, health officials and policymakers may communicate about the rates, trends, and parameters of the outbreak. Kucharski (2020) mentions that mathematics can also help with determining what needs to be done to help control the rates of morbidity and mortality. By providing tools for assessment, analysis, and predictions, mathematical modelling has been very vital in efforts by experts from a variety of fields who have investigated the dynamics of both emerging and re-emerging infectious diseases, and insights drawn from them help policymakers to determine and debate courses of action that may prevent high mortality rates (Siettos & Russo, 2013; Wang et al., 2020; Yates, 2020).

Many models are about the risks associated with a pandemic, the probability, and rate of spread in a population, and the effects of possible interventions to reduce morbidity and mortality (Rodrigues, 2016; Walters et al., 2018). Those mathematical models, however, use sophisticated mathematics that is normally only understood by experts in fields that make or use mathematics. Complex mathematics equations, data displays, and graphs

make it difficult to comprehend the data and the dynamics behind these mathematics artifacts for non-experts. Computational and programming tools used to compute the equations or illustrate the visualizations are equally complex as they require an understanding of the tools and languages used; however, interactive illustrations of the mathematical models of the outbreak, which utilized recent and less sophisticated tools for computational programming such as block-based and text-based programming languages, make it easy for the mathematical models of the outbreak to be read and understood by the public (Froese, 2020; Resnick, 2020; Yeghikyan, 2020). Many of these models are designed to afford opportunities to experiment with different scenarios, and users may view, study, and modify the code for the simulations. Recent school activities also show that students might be able to better understand real-world problems by using programming languages. The principal of the Oklahoma School of Science and Math, Dr. Frank Wang, for instance, claimed that offering students chances to play with the tools including the mathematical models, similar to those that real epidemiologists were using, enabled students to acquire a better idea of the pandemic on their own (Skarky, 2020).

In the light of above observations and arguments, this study asks in what ways computational thinking concepts and tools help students comprehend real-world problems. In this research paper, I focus on computational simulations of an outbreak based on the Susceptible – Infectious – Recovered (SIR) model which commonly has been used to illustrate the spread of the COVID-19 disease (Ciarochi, 2020). I selected and analyzed two sample simulations of the SIR model designed using a block-based programming language, Scratch. These simulations, in addition to being dynamic and interactive, have visible code, hence they are modifiable, which helps students who wish to experiment with them. I specifically investigated: a) the ways in which Scratch simulations were accessible by students to comprehend the dynamics of the outbreaks and the response needed to slow down the rate of the outbreak; and b) the extent to which these simple simulations illustrate the impacts of variations in precautions and policies implemented during the pandemic crisis, including social/physical distancing, reduced mobility (through staying at home, isolation, or quarantine), and regular handwashing. This study aims to shed light on learning opportunities for using CT concept and tools in mathematics instruction during the current global health crisis and how CT concepts and

tools might help or not help students understand key mathematical and computational concepts of infectious disease dynamics.

### 3.1 Literature Review: Computational Thinking and Its Integration with Real-World Problems

Haudong (2019) maintains that the increasing relationship between power and technology in today's world makes it even more essential to use digital tools to examine children's and youth's experiences, goals, and expectations which influence both their current and future lives. Hoyles et al. (2002) and Wilkerson-Jerde (2014) designed environments of computational programming coupled with computational modelling of mathematics and science concepts, processes, and systems, for example, population dynamics (e.g., Stroup & Wilensky, 2014; Wilkerson-Jerde, Gravel, et al., 2015; Wilkerson-Jerde, Wagh et al., 2015). While focusing on CT, these researchers engaged students in the practices of mathematicians (Wilkerson-Jerde, 2014) and of STEM professionals (Wilkerson-Jerde et al., 2018).

To Skovsmose, (1994) the role mathematics plays in technological development shows its formatting power on society and shows the potential role of mathematics in helping to positively shape society. The reality is, however, given the way mathematics is currently taught and the nature of the mathematics curriculum, many students are unable to see the applications of their mathematical knowledge and skills to tasks embedded in real-world contexts. This is because real-world mathematics looks messier and more complex than school mathematics, and is often hidden or invisible (Taut, 2014).

Furthermore, Lee (2012) asserts that mathematical expressions are not always broad enough to address all the issues encountered in the real world. Lee is among the researchers who argue that integrating computational thinking in mathematics teaching offers help to explore and gain a deeper understanding of certain real-world challenges. Wilkerson-Jerde and Fenwick (2016), for example, observe that by focusing on processes, CT concepts facilitate using CT tools and algorithms to make data manipulation, simulation, and analysis easier and more manageable. When students use CT concepts in mathematics teaching and learning as a tool to learn with, they are

afforded greater student agency, engagement, and access to mathematics concepts that would otherwise be more advanced for their grade levels (Gadanidis, 2017; Namukasa et al., 2017; Sanford & Naidu, 2016). Further, Wilkerson-Jerde et al. (2015) identified the complementary role of representations (such as drawings) that emphasize components and relationships with representations (such as animations and simulations) that emphasize change across space and time.

In school settings, coding and modelling are some of the ways in which CT is used to model and visualize real-world problems to students. For instance, Sanford, (2013) and Sanford and Naidu (2017) indicated that learners may be taught to make use of CT concepts and tools to make simulations and to model solutions to mathematics problems. Many countries are integrating learning expectations on mathematical modelling and coding in their new curricula. In Ontario's new mathematics curriculum Grades 1-9, for example, it is asserted that the models and simulations may use algebraic or probabilistic reasoning in analyzing, representing, and modelling data, and provide different levels of experiences that are aligned across different grade levels (OME, 2020; OME, 2021). When CT tools are used to help understand and represent a disease outbreak, for instance, the emphasis is on conveying the patterns, relations, and processes of the outbreak, students may be guided to estimate the dynamics of the pandemic, to investigate the effects of specific control measures, and to analyze and synthesize the available real data.

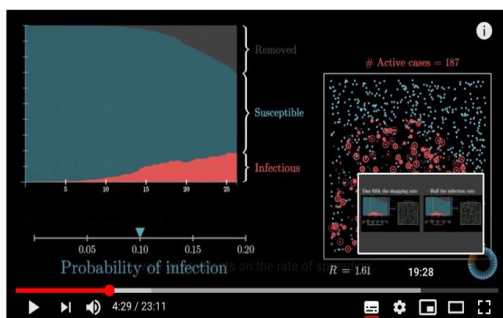
### 3.2 Context: The Current Contexts of Computational Tools for Understanding Outbreak Models

With the outbreak, several computational artifacts (e.g., graphical illustration, simulations, and data maps) have been shared in the news and on the internet, particularly on social media. (Adam, 2020; Tahiralli & Ho, 2020; Yates, 2020). Many of these attempted to illustrate to the public audience the spread of the disease (Ciarochi, 2020; Stevens, 2020) and to explain what needed to be done to slow the spread. Examples of these artifacts of the pandemic include a simulation published by Stevens (2020) and translated in 12 languages; a video illustration of exponential growth by Sanderson

[3blue1brown] (2020) in which he explains the probability of infection (transmission),  $R$ , and the effectiveness of responses that lower this rate; and a graphical illustration on flattening the curve of the spread of the disease<sup>12</sup>, which when tweeted by former President Barack Obama was retweeted over 180, 000 times (Barclay & Scott, 2020; Obama, 2020). Whereas certain simulations and graphs only visually illustrate the dynamics of the disease spread, its containment measures, and the different possible scenarios, which are dependent on how countries responded, a few others, which can be seen in Figure 13, are interactive simulations that offer manipulable and modifiable interfaces (Namukasa et al., 2016). Simler (2020) explains that the model he offered through utilizing JavaScript<sup>13</sup> included:

"[p]layable simulations of a disease outbreak. Playable" means you'll get to tweak parameters (like transmission and mortality rates) and watch how the epidemic unfolds. By the end of this article, I hope you'll have a better understanding — perhaps better intuition — of what it takes to contain this thing. But first!... [He cautioned] This is not an attempt to model COVID-19. What follows is a simplified model of a disease process. The goal is to learn how epidemics unfold in general”.

#### Exponential growth and epidemics

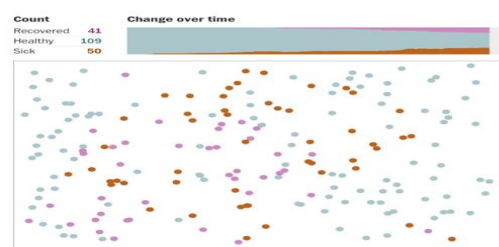


(Sanderson, 2020)

Source: <https://www.youtube.com/watch?v=Kas0tIxDvrg&feature=youtu.be>

#### Simulation of social distancing (Stevens, 2020)

To simulate more social distancing, instead of allowing a quarter of the population to move, we will see what happens when we let just one of every eight people move.



Source: <https://www.washingtonpost.com/graphics/2020/world/corona-simulator/>

**Figure 13: Interactive simulations examples**

<sup>12</sup> Source: <https://www.vox.com/2020/3/10/21171481/coronavirus-us-cases-quarantine-cancellation>

<sup>13</sup> Source of code: <https://github.com/kevinsimler/outbreak>



In addition to stand-alone models shared in public media, certain models have been offered by other institutions to provide opportunities for students to improve their comprehension of the pandemic, and hence to lead to their informed participation when taking precautions during the pandemic. Further, certain coding communities and initiatives have offered opportunities (e.g., webinars, courses, classes, projects) to promote understanding of the facts and predictions of the disease for students and the public. One example is a course offered at MIT that claimed to apply data science, artificial intelligence, mathematical models, and a programming language, Julia, which was repurposed to study the Covid-19 pandemic in the 2020 spring semester. According to Raj Movva, a sophomore who took the course, the course provided opportunities for using computation to better comprehend the pandemic and helped in identifying the misinformation about the coronavirus (cited in Miller, 2020).

Another set of examples are simulations shared in online forums such as the National Council of Teachers of Mathematics (NCTM) website and the Scratch online community<sup>14</sup>, which is a “vibrant online community of people sharing, discussing, and remixing one another’s projects” (Resnick et al., 2009, p. 60). For instance, the NCTM association published a mathematics simulation<sup>15</sup> on the spread of COVID-19 through social contact to be used as part of the activities for Grades 5-12 (NCTM, 2020). The NCTM simulation is interactive and modifiable and shows a dynamic simulation, a graph of cases over time, four parameters in the parameter pane, and a pane for data. Learners are able to ask “what happens if” questions and carry out experiments to answer the questions they are wondering about using the NCTM simulation.

### 3.3 Method

I used qualitative research design, specifically content analysis, to analyze two interactive simulations, which are selected from the publicly shared computational simulations of the current global health crisis. The simulations are designed using Scratch that students

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<sup>14</sup> <https://scratch.mit.edu/>

<sup>15</sup> <https://www.nctm.org/blog/pandemicapp/>

might have learned in schools and are modifiable to experiment with different scenarios of reality. This research responds to the second sub-research question: In what ways do computational thinking concepts and tools help students comprehend real-world problems? To address this question, I examined how the Susceptible-Infectious-Recovered (SIR) model are potentially helpful (or not helpful) to make key mathematical and computational concepts of these model understandable to students.

Through content analysis, I began by examining the dimensions of the simulations and their connections to computational thinking and mathematical concepts. I coded the dimensions found in the selected simulations based on the steps of these simulations using inductive category development (Mayring, 2000). Following coding, I interpreted the presence of the mathematical and computational concepts extracted from the content analysis.

### 3.3.1 The Design of a Basic SIR Model and Its Extensions

Epidemiologists use a family of mathematical techniques, called compartmental models, to model infectious diseases. In a compartmental model, people are classified into separate groups of people who share the same characteristics and mathematical equations are used to model the processes that affect the movement of people from one classification to another. The SIR model is one of the basic compartmental models, in which the population is split into three types of people: the susceptible type individuals,  $S$ , are the ones who are not currently infected but could get infected; the infected type individuals,  $I$ , are the ones who have the disease and henceforth can transmit it to the susceptible; and the removed (recovered, immune or dead) individuals,  $R$ , are the ones who cannot get infected and cannot transmit the disease to others (Capitanelli, 2020).

Two processes are simultaneously at work in this model. First, as a result of contact with an infected individual, a susceptible individual may get infected and move to the infective type. In the SIR model, the number of these movements is proportional to the number of infected and healthy people. Hence the change in susceptible population type is given by the following differential equation:

$$S' = -\beta I \frac{S}{N}$$

where the parameter beta,  $\beta$ , stands for the rate at which infected individuals contact and infect other people, and  $S'$  stands for change in  $S$ .

The second simultaneous process is that the infected people can enter the removed class, and the number of these movements is proportional to the number of infected people. Hence, we have the following differential equations for the changes in infected and removed:

$$I' = \beta I \frac{S}{N} - \gamma I, \text{ and}$$

$$R' = \gamma I,$$

where the parameter gamma,  $\gamma$ , is the rate at which people who are infected either recover or die.  $\beta/\gamma$  is  $R_0$ , the total number of people an infected person infects.

Adam (2020) mentions that by grouping individuals into compartments and using mathematical equations to model the interactions between the compartments, the compartmental models do not require an understanding of complex computations. These models, however, are criticized as they make several assumptions that have not necessarily been observed in real pandemics (Daughton et al., 2017), and the subtleties of many pandemics cannot be comprehensively captured in the simplicity of the SIR models (Yates, 2020). Even though SIR models seem simple, they appear to have been very useful in explaining the needed precautions and responses to the global pandemic crisis (Rodrigues, 2016). Moreover, simple SIR models could be extended to more sophisticated models with four or more compartments, such as by introducing an exposed compartment between  $S$  and  $I$ , or by introducing two simultaneous compartments, recovered and fatal, after  $I$  instead of  $R$  and several scientists are currently working on these complex models to come closer to modelling the reality of pandemic (Froese, 2020). For instance, Giordano et al.'s (2020) model consists of eight compartments for modelling the COVID-19 pandemic in Italy: susceptible ( $S$ ), infected ( $I$ ), diagnosed ( $D$ ), ailing ( $A$ ), recognized ( $R$ ), threatened ( $T$ ), healed ( $H$ ) and extinct ( $E$ ). They referred to this model as SIDARTHE. Their model offers a detailed classification of infected individuals depending on how severe their symptoms are. Extensions of SIR-type models may be further extended to more closely model reality by incorporating demographics

and taking diffusion and migration effects along with possible genetic mutations into consideration (Siettos & Russo, 2013).

### 3.4 Results of the content analysis of the simulations of the SIR model

I analyzed two different simulations to ascertain the ways in which they are helpful (or not helpful) in illustrating, understanding, and responding during the current global health crisis.

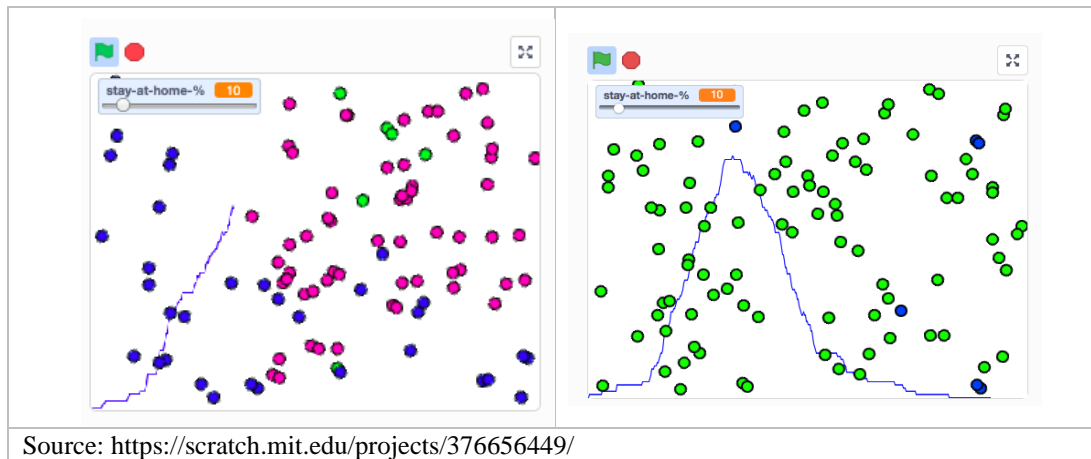
The results of the analysis show that the simulations have the following common dimensions: (i) initialization (the addition of a group people, their initial location, and their initial condition in terms of infection), (ii) movements (the direction of people's movement, which is reduced if they choose not to stay at home), (iii) transmission of infection (the probability of getting infected when in contact with an infected person, which may depend on immunity and on variations in preventative measures and policies such as social/physical distancing, reduced mobility (through staying at home, isolation, or quarantine), regular handwashing, wearing a mask, sneezing in the elbow rather than in open space or hands, avoiding meetings with large numbers of people, etc., and (iv) recovery process (either a certain time for recovery or a probability of recovery/death at each period, which may depend on the capacity to care for the infected, the timing of infection, and the availability of possible treatments).

Based on these dimensions, the analysis of the Epidemic simulation sample 1 and Epidemic simulation sample 2 are presented below. In the figures, the visual illustrations and the assembled code are shown in the simulation and the code construction panes, simultaneously.

#### 3.4.1 Epidemic simulation sample 1

This model demonstrates the effect of staying at home on the rate and risk of the spread of the disease in an SIR model. The interactive simulation of the model shows the three classes of people: pink for the infected (sick and contagious), blue for the susceptible (healthy), and green for the recovered. The user may modify the parameter of the process

of staying at home to visualize different scenarios. One scenario of 10% of the population staying at home is shown in Figure 14. The left image is for an earlier time but with the same parameters and initializations.



**Figure 14: Epidemic simulation (Resnick, 2020)**

### Simulation 1's specifications

The simulation involves four parameters: (i) sick, (ii) health, (iii) time sick, and (iv) time to recover (shown in Figure 15).



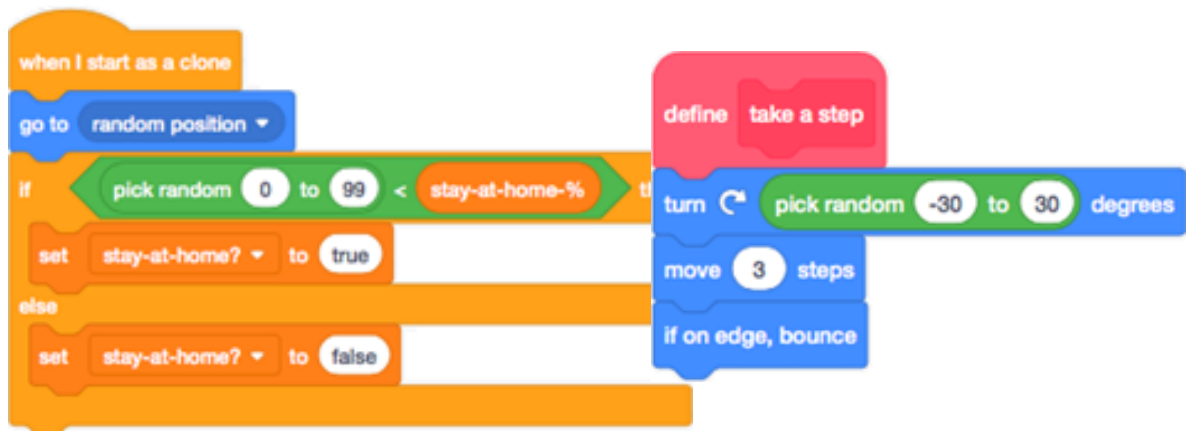
**Figure 15: The parameters in simulation 1**

The simulation starts with 1 sick individual and 100 healthy individuals. The initial locations of people are randomly determined (shown in Figure 16).



**Figure 16: Initialization of the parameters in simulation 1**

Initially, the directions of the people who choose not to stay at home are randomly determined (between  $-30^\circ$  and  $+30^\circ$ ), they move 3 steps in these directions, and their directions change randomly after every 3 steps (where the changes are between  $-30^\circ$  and  $+30^\circ$ ). Moreover, if they reach one of the edges while moving, their path bounces from the edge (shown in Figure 17).



**Figure 17: Coding blocks for illustrating movements in simulation 1**

When there is contact between an infected person and a healthy person, the healthy person gets infected. This means the chance of contracting the disease per contagious contact is 1 (shown in Figure 18).



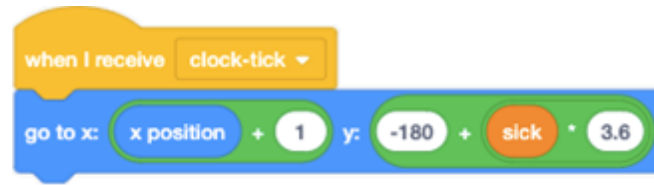
**Figure 18: Coding blocks for illustrating transmission in simulation 1**

As shown in Figure 19, the infected people recover in a fixed amount of time (100 periods).



**Figure 19: Coding blocks for illustrating the recovery in simulation 1**

The number of cases over time, which is instantly updated according to the running simulation, is illustrated by a path traced by a character, referred to as the grapher in the code, whose motion is coded as an equation on X-Y coordinate axes (shown in Figure 20).



**Figure 20: Coding blocks for the equation of the grapher character in simulation 1**

### **Simulation 1's connection to mathematical concepts**

Simulation 1 uses the following mathematical concepts: coordinate geometry for the location for the individuals and for the grapher; angles for the direction of the movement of individuals; mathematics operations of counting increments in the steps; the probability operation of picking a random angle for the direction of the turn and the number of people staying at home; and an algebraic equation, which is a function of the sick individuals, for the path of the grapher.

### **Simulation 1's connection to CT concepts**

Simulation 1 uses the following CT concepts: repetition and conditional logic, defining a block of parameters, and changing looks of characters. By showing both a pictorial simulation and a graph of the variation in the number of people infected depending on people staying at home, it appears helpful with the basic understanding of the SIR models of growth in an outbreak and the effect of actions that may slow down the rate of spread. A student, especially if they have experience with coding simple games in Scratch, may choose to modify the code, for instance, to change the initialization such as time to recover to obtain rates closer to the real-life data of a pandemic.

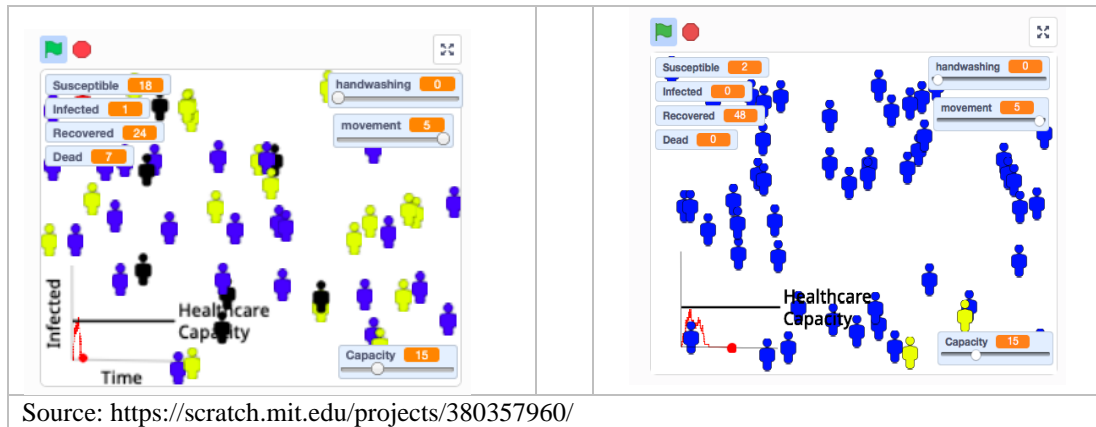
### **3.4.2 Epidemic simulation sample 2**

Similar to Simulation 1, Simulation 2 demonstrates the effect of decreased movement (staying at home), handwashing, and hospital capacity on the rate and risk of spread in an SIR model. The interactive simulation of the model shows four compartments of the population: yellow for the susceptible, red for the Infected, blue for the recovered (no longer contagious), and black for the removed (dead). It graphs the number of infected alongside the capacity of hospitals. To visualize the different scenarios, the user may



change the parameters of four processes. One scenario for zero hand washing, maximum movement, and a moderate hospital capacity is shown in Figure 21 at two different times.

### Simulation 2's specifications



**Figure 21: Infectious disease simulation with healthcare capacity (Iainbro, 2020)**

Simulation 2's specifications are as follows:

The simulation involves six parameters: (i) population, (ii) initial rate, (iii) recovery rate, (iv) handwashing, (v) movement, and (vi) capacity. Users may adjust the handwashing and movement sliders to see how they affect the spread of infection, or the capacity slider to change the healthcare capacity. Users may also press the spacebar to start/stop movement to mimic social distancing (shown in Figure 22).



**Figure 22: Coding blocks for illustrating parameters in simulation 2**

The simulation starts with the cloning of individuals one by one, their initial locations are randomly determined, and each individual once cloned starts to move. A total of 50 individuals are created, and each of them has a positive probability of being sick (shown in Figure 23).



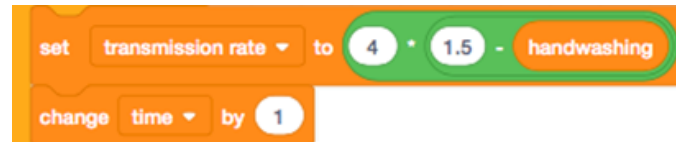
**Figure 23: Coding blocks for illustrating the initialization step in simulation 2**

The directions of the people are randomly chosen, they move in these directions, and the number of steps they move every period is determined by the chosen movement parameter. If people reach one of the edges while moving, their path bounces from it, and this is the only source of change in moving directions (shown in Figure 24).



**Figure 24: Coding blocks for illustrating the movements step in simulation 2**

When there is contact between an infected person and a healthy person, the healthy person gets infected. The probability of infection is equal to  $4(1.5 - \textit{handwashing})/100$ , which decreases proportionally to the chosen handwashing parameter (shown in Figure 25).



**Figure 25: Coding blocks for illustrating the transmission equation in simulation 2**

The infected people recover with a probability of 0.01 every period, and once the capacity of hospitals is exceeded, the infected people start to die with a probability of 0.01 every period (even if the number of infected people decreases below the capacity afterward). The simulation never stops, however, when there is no infected individual anymore, there will be no more change in the data (shown in Figure 26).



**Figure 26: Coding blocks for illustrating the recovery step in simulation**

The number of cases over time is illustrated and this makes following the running simulation in real-time possible (shown in Figure 27).



**Figure 27: Coding blocks for the equation of the marker on the grapher in simulation 2**

The healthcare capacity is also illustrated, and it is assumed that infected people will not die as long as capacity is not satiated (shown in Figure 28).



**Figure 28: Coding blocks for the equation of the healthcare capacity sprite in simulation 2**

### **Simulation 2's connection to mathematical concepts**

Simulation 2 uses the following mathematical concepts: coordinate geometry for the location of the people pictographs and for the grapher; angles for the direction of the movement; mathematics operations of counting increments in the steps of the infected, susceptible and time; the probability operation of picking each of the initial rates of recovery and death; and an algebraic equation for the path of the grapher and of the health care capacity.

### **Simulation 2's connection to CT concepts**

In simulation 2 the following CT concepts are used: repeat and conditional logic, defining a block of parameters, and changing looks of characters. This simulation shows the varied rates of infection hand, washing, movement, and capacity parameters. A student may choose to change the initialization steps, transmission equation, or the movements of the people to closely mimic the data from a real pandemic.

### **3.4.3 The comparison of the two simulations**

The basic dynamics of both simulations, Simulation 1 and Simulation 2, are based on the SIR model, and when these discrete models converge towards continuous time models of large populations, they appear to behave like an SIR model (Kaplan, 2020). The first model is simple as it only simulates the effect of staying at home, whereas the second model simulates the effect of three different factors, decreased movement, handwashing, and capacity of hospitals. The second model also displays dynamic data of the parameter on the top left corner of the simulation pane. However, the movements are closer to

reality in the first simulation, as the directions of people's movements are dynamically and randomly changing, unlike the movement of people always going in the same direction, except the bouncing at the edges, in the second simulation. Overall, both simulations illustrate the spread of disease and the effect of precaution on the process. In addition to changes that a user or reader of the simulation may make on the parameters that are modifiable, a user may change the initial states, steps, or rates in the simulation to experiment with different scenarios.

### 3.5 Discussion

In this study, I pondered the ways in which certain presentations of SIR models have been helpful (or not helpful) in illustrating the dynamics of the outbreak, in demonstrating the responses needed to slow down the rate of the outbreak, and in explaining the effects of certain responses to the public including students during the current global health crisis. Additionally, I reflected on opportunities to use real-world problems experienced during the COVID-19 outbreak to promote elementary and middle-grade students' computational and mathematical knowledge and skills.

Mathematical models used to explain real-world problems are not always transparent, and some level of mathematical literacy is required to comprehend them, however, CT tools would offer students a better understanding of the problem and exploration of varied scenarios in the simulations. As demonstrated in our analysis, while differential equations used in the SIR model, such as  $S' = -\beta IS/N$ , require an understanding of high school academic and university mathematics, observing simulations, and changing parameters in the Scratch simulations might require only a basic understanding of algebraic and computational concepts. Readers or users of the simulations, who may read and wish to remix the code, would require an understanding of assembling probability and algebraic expressions blocks in block-based programming as well as of computational thinking concepts of conditional statements, such as “if-then”, “if-else-if”, and the use of data block codes used to initialize parameters and to change them.

The two block-based simulations that I analyzed appear to provide some understanding of the SIR model of diseases, specifically the effect of precautions on the spread of the

diseases. Given the less sophisticated mathematics required to manipulate these simulations, it appears that users might be motivated to re-examine the code to understand it, and, if necessary, remix it. Advantages of the block-based programming languages are readability, ease of use, not requiring an advanced level of technical coding knowledge, and not encountering syntax errors (Abraham, 2019; Chumpia, 2018). The simplicity of Scratch simulations, however, makes it hard to model the reality of the pandemic and possible complex scenarios, and it is not possible to feed a Scratch simulation with real-life data, which is essential for accurate predictions. Significantly, these simple models could be useful as a basis for advanced simulations in text-based programming languages; once students are familiar with block-based programming, they quickly transition to understanding and building complex programs in text-based programming languages (Weintrop & Wilensky, 2015). After coding in Scratch, for example, students might pursue how to code similar and more advanced simulations in text-based languages, such as Python and Julia.

Text-based programming languages would ideally provide the complexity necessary for modelling and analyzing an outbreak based on scientific and statistical real-life data. Python, as a text-based programming language, allows its wide community of programmers and a growing number of student coders to do more complex computations, some of which may use real data to feed the model. Yeghikyan (2020), for instance, uses Wesolowski et al.'s (2017) SIR model to simulate the spread of COVID-19 in Yerevan, Armenia, using the Python programming language. He is able to model the effect of mobility patterns on the disease spread, by adding one more type of transition between the compartments of S and I that takes place due to the mobility of infected people from other locations to the location of interest. Similarly, Sargent and Stachurski (2020) use Atkeson's (2020) SIR model and publish a Python version of their code. Their simulations additionally show the parameters – including transmission rates, physical distances among people from different homes, and lockdown of non-essential services – which may help slow down the spread, lower the peak, or slow the peak of the outbreak. Julia is another programming language, which has been noted to be faster than Python in compiling big and complex codes, although its libraries are not as equally well maintained as python. Vahdati (2019) provides a package in Julia for agent-based

modelling (i.e., modelling of phenomena as dynamic systems of interacting autonomous agents) and its application on an SIR model for COVID-19. He simulates both the exponential growth that takes place if there is no intervention during the spread of COVID-19 and the growth with the effect of social distancing on "flattening the curve", by making some agents simply not move, which he claims is a good approximation of reality.

## 3.6 Conclusion

Mathematics is an inevitable part of almost everything humans in today's world do, and the most important thing about teaching and learning mathematics is acquiring the ability to read mathematics in our immediate environment. Following the COVID-19 outbreak, politicians and policymakers, as well as society, have been bombarded with mathematical artifacts based on mathematical models of the outbreak, which explain the rates and probabilities of spread, track progress of the outbreak, and report the effects of the interventions on the pandemic. However, the complexity of mathematics used in many artifacts makes it hard to comprehend the data and the dynamics behind these artifacts for non-experts. Computational concepts when integrated with mathematics concepts in programming tools, such as Scratch, not only aid to illustrate the dynamics (e.g., parameters, the rates) of the outbreak, but also demonstrate the effects of responses or controls needed to slow down the rate of the outbreak (e.g., the effects of certain precautions). The simulations include dynamic pictorial, numerical, and graphical displays, and offer easy access to the code, which provides an opportunity to understand the basis of the recommended actions and policies during a pandemic while promoting the mathematical and computational skills of learners.

As Resnick (2020) mentions, the coronavirus crisis presents many unprecedented challenges, but also some unexpected opportunities in classroom settings. Implementing computational tools during this period of global health crisis not only offers a better understanding of the current health crisis but also has been helpful for understanding other interrelated crises in the post-pandemic world. Teachers may use the computational tools related to the COVID-19 pandemic to demonstrate to students the specific

applications of their mathematical and computational knowledge and skills to tasks embedded in real-world contexts. The general context of disease-spreading to teach mathematical modelling and computational thinking can be used as a springboard for empowering students to engage in more advanced simulations in text-based languages. Moreover, coupling mathematical models and computational thinking concepts could be helpful for students to realize the use of mathematics in reading, understanding, and experimenting with simulations of magnitude, dynamics, and recommended responses to facilitate informed decision-making during crises.

### 3.7 Chapter references

- Abraham, R. (2019, June 17). *Coding, programming, block-based, text-based...what does it all mean?* Teq. <https://www.teq.com/news/coding-programming-what-does-it-all-mean/>
- Adam, D. (2020). Special report: The simulations driving the world's response to COVID-19. *Nature*, 580, 316-318. <https://www.nature.com/articles/d41586-020-01003-6>
- Albrecht, J. R., & Karabenick, S. A. (2018). Relevance for learning and motivation in education. *The Journal of Experimental Education*, 86(1), 1-10. <https://doi.org/10.1080/00220973.2017.1380593>
- Aslan-Tutak, F., Bondy, E., & Adams, T. L. (2011). Critical pedagogy for critical mathematics education. *International Journal of Mathematical Education in Science and Technology*, 42(1), 65-74. <https://doi.org/10.1080/0020739X.2010.510221>
- Atkeson, A. G. (2020). *What will be the economic impact of COVID-19 in the US? Rough estimates of disease scenarios*. (NBER Working Papers No. 26867), National Bureau of Economic Research.
- Barclay, E., & Scott, D. (2020, March 10). *How canceled events and self-quarantines save lives, in one chart*. Vox. <https://www.vox.com/2020/3/10/21171481/coronavirus-us-cases-quarantine-cancellation>
- Capitanelli, A. (2020, March 8). *Modeling the spread of diseases: A simulation exercise with SIR models*. Towards Data Science. <https://towardsdatascience.com/modeling-the-spread-of-diseases-821fc728990f>
- Chumpia, E. (2018, May 12). *Visual-based vs. Text based programming languages*. Coding Kids. <https://www.codingkids.com.au/for-teachers/visual-based-vs-text-based-programming-languages/>



- Ciarochi, J. (2020, March 12). *How COVID-19 and other infectious diseases spread: Mathematical modeling*. Triple Byte. <https://triplebyte.com/blog/modeling-infectious-diseases>
- Daughton, A. R., Generous, N., Priedhorsky, R., & Deshpande, A. (2017). An approach to and web-based tool for infectious disease outbreak intervention analysis. *Scientific Reports*, 7, 4607. <https://doi.org/10.1038/srep46076>
- Froese, H. (2020, April 11). *Infectious disease modelling: beyond the basic SIR model*. Towards Data Science. <https://towardsdatascience.com/infectious-disease-modelling-beyond-the-basic-sir-model-216369c584c4>
- Gadanidis, G. (2017). Five affordances of computational thinking to support elementary mathematics. *Education Journal of Computers in Mathematics and Science Teaching*, 36(2), p. 143-151. <https://eric.ed.gov/?id=EJ1154750>
- Giordano, G., Blanchini, F., Bruno, R., Colaneri, P., Di Phillo, A., Matteo, A., & Colaneri, M. (2020). Modelling the COVID-19 epidemic and implementation of population-wide interventions in Italy. *Nature Medicine*. <https://doi.org/10.1038/s41591-020-0883-7>
- Hoyles, C., Noss, R., & Adamson, R. (2002). Rethinking the Microworld idea. *Journal of Educational Computing Research*, 27(1&2), 29-53. <https://doi.org/10.2190/U6X9-0M6H-MU1Q-V36X>
- Iainbro. (2020). Infectious disease simulator with healthcare capacity. <https://scratch.mit.edu/projects/380357960/>. Accessed 2020 May 10.
- Kaplan, Edward H. (2020). COVID-19 scratch models to support local decisions. *Forthcoming, Manufacturing & Services Operations Management*, 22(4), 645-867. <http://dx.doi.org/10.2139/ssrn.3577867>
- Kucharski, A. (2020, February 13). *Coronavirus: How maths is helping to answer crucial COVID-19 questions*. New Scientist. <https://www.newscientist.com/article/2233386-coronavirus-how-maths-is-helping-to-answer-crucial-covid-19-questions/#ixzz6MH6VRGr4>
- Lee, C. H. (2012). *Re-mastering the master's tools: Recognizing and affirming the life experiences and cultural practices of urban youth in critical computational literacy through a video game project*. [Unpublished doctoral dissertation]. University of California.
- Miller, S. (2020, April 7). *Computational thinking class enables students to engage in COVID-19 response*. MIT News. <http://news.mit.edu/2020/computational-thinking-class-enables-students-engage-covid-19-response-0407>. Accessed 2020 May 15.
- Namukasa I. K., Gadanidis, G., Sarina, V., Scucuglia, R., & Aryee K. (2016). Selection of apps for teaching difficult math topics: concrete props, virtual applets and touch pad apps. In P. Moyer-Packenham (Eds.), *International Perspectives on*

*Teaching and Learning Mathematics with Virtual Manipulatives* (pp. 275-300). Dordrecht: Springer.

- Namukasa, I. K., Minakshi, P., & Miller, M. (2017). Introductory CT and math in the middle grades: tools and resources. *Math + Code 'Zine*, 2 (2). <http://researchideas.ca/mc/ct-and-math-in-middle-grades/>
- National Council of Teachers of Mathematics (NCTM). (2020, March). *#KnowTheMath: impact of social distancing*. NCTM. <https://www.nctm.org/blog/pandemicapp/>
- Obama, B. [@BarackObama]. (2020, March 12). If you're wondering whether it's an overreaction to cancel large gatherings and public events (and I love basketball), here's a useful primer as to why these measures can slow the spread of the virus and save lives. We have to. [Tweet]. <https://twitter.com/BarackObama/status/1238238576141352966>
- Ontario Ministry of Education (2020). *The Ontario curriculum grades 1–8 mathematics curriculum context*. Queen's Printer for Ontario
- Resnick, M. (2020, March 17). *Scratch epidemic simulator*. MIT Media Lab. <https://www.media.mit.edu/posts/scratch-epidemic-simulator/>
- Resnick, M. (2020, March 27). *Cultivating creativity during the coronavirus crisis*. MIT Media Lab. <https://medium.com/mit-media-lab/cultivating-creativity-during-the-coronavirus-crisis-fedfca3b6036>
- Resnick, M., Maloney, J., Monroy-Hernandez, A., Rusk, N., Eastmond, E., Brennan, K., Millner, A., Rosenbaum, E., Silver, J., Silverman, B., & Kafai, Y. (2009). Scratch: Programming for all. *Communications of the ACM*, 52(11), 60-67. <https://doi.org/10.1145/1592761.1592779>
- Rodrigues, H. S. (2016). Application of SIR epidemiological model: new trends. *International Journal of Applied Mathematics and Informatics*, 10, 92-97. <https://arxiv.org/abs/1611.02565>
- Sanderson, G. [3blue1brown]. (2020, March 8). *Exponential growth and epidemics*. YouTube. <https://www.youtube.com/watch?v=Kas0tIxDvrg&feature=youtu.be>
- Sanford J.F. (2013). Core concepts of computational thinking. *International Journal of Teaching and Case Studies*, 4(1), 1–12. <https://doi.org/10.1504/IJTCS.2013.053383>
- Sanford J. F., & Naidu, J. T. (2017). Mathematical modeling and computational thinking. *Contemporary Issues in Education Research*, 10(2). <https://files.eric.ed.gov/fulltext/EJ1137705.pdf>
- Sargent, T. J., & Stachurski, J. (2020, May 26). *Modeling COVID 19*. QuantEcon. [https://python.quantecon.org/sir\\_model.html](https://python.quantecon.org/sir_model.html). Accessed 2020 May 18.

- Schor, J. (n.d). *Teaching kids to code: Text-based vs block-based programming*. Fractus Learning.  
<https://www.fractuslearning.com/teaching-kids-code-text-based-vs-block-based/>
- Shepherd, M. (2020, March 3). *Poor math literacy making it harder for people to understand COVID-19 coronavirus*. Forbes.  
<https://www.forbes.com/sites/marshallshepherd/2020/03/23/is-the-math-too-hard-for-people-to-understand-covid-19-coronavirus/#344a229a6a9c>
- Siettos, C. I. & Russo, I. (2013). Mathematical modeling of infectious disease dynamics. *Virulence*, 4(4), 295-306. <https://doi.org/10.4161/viru.24041>
- Simler, K. (2020, March 16). *Outbreak*. Melting Asphalt.  
<https://meltingasphalt.com/interactive/outbreak/>
- Skarky, B. (2020, March 27). *Math teacher uses COVID-19 to keep students learning during pandemic*. Robot LAB. <https://www.robotlab.com/blog/math-teacher-uses-covid-19-to-keep-students-learning-during-pandemic>
- Skovsmose, O. (1994). Towards a philosophy of critical mathematics education. *Educational Studies in Mathematics*, 27 (1), 35-57.  
<https://www.jstor.org/stable/3482665>
- Stevens, H. (2020, March 14). *Why outbreaks like coronavirus spread exponentially, and how to “flatten the curve”*. The Washington Post.  
<https://www.washingtonpost.com/graphics/2020/world/corona-simulator/>.  
 Accessed 2020 May 18.
- Stroup, W. M. & Wilensky, U. (2014). On the embedded complementarity of agent-based and aggregate reasoning in students’ developing understanding of dynamic systems. *Technology, Knowledge and Learning*, 19(1), 19-52.  
<https://doi.org/10.1007/s10758-014-9218-4>
- Tahiralli, J. & Ho, S. (2020, April 3). *COVID-19 curves: Compare Canada and other key nations*. CTV News. <https://www.ctvnews.ca/health/coronavirus/covid-19-curves-compare-canada-and-other-key-nations-1.4881500?cache=%2Fbuild-a-work-wardrobe-from-scratch-without-going-broke-1.2361542%3FclipId%3D89925>.  
 Accessed May 10.
- Taut, D. (2014, August 15). *Connecting maths to the real world*. Australian Council for Educational Research (ACER).  
<https://www.teachermagazine.com.au/articles/connecting-maths-to-the-real-world>
- Vahdati, A. R. (2019). Agents.jl: agent-based modeling framework in Julia. *The Journal of Open Source Software*, 4(42), 1611, <https://doi.org/10.21105/joss.01611>
- Vos, P. (2018). How real people really need mathematics in the real world—authenticity in mathematics education. *Education Sciences*, 8(4), 1-14.  
<https://doi.org/10.3390/educsci8040195>

- Walters C. E., Meslé M. M. I., & Hall I. M. (2018). Modelling the global spread of diseases: A review of current practice and capability. *Epidemics*, 25, 1–8.  
<https://doi.org/10.1016/j.epidem.2018.05.007>
- Wang, N., Fu, Y., Zhang, H., & Shi, H. (2020). An evaluation of mathematical models for the outbreak of COVID-19. *Precision Clinical Medicine*, 3(2), 85–93,  
<https://doi.org/10.1093/pcmedi/pbaa016>
- Weintrop D., & Wilensky, U. (2005). To block or not to block, that is the question: students' perceptions of blocks-based programming. *Proceedings of the 14th International Conference on Interaction Design and Children*, 199-208.  
<https://doi.org/10.1145/2771839.2771860>
- Wesolowski, A., zu Erbach-Schoenberg, E., Tatem, A.J., Lourenço, C., Viboud, C., Charu, V., Eagle, N., Engø-Monsen, K., Qureshi, T., Buckee, C. O., & Metcalf, C. J. E. (2017). Multinational patterns of seasonal asymmetry in human movement influence infectious disease dynamics. *Nature Communications*, 8(2069). <https://doi.org/10.1038/s41467-017-02064-4>
- Wiedemann, K., Chao, J., Galluzzo, B., & Simoneau, E. (2020). Mathematical modeling with R: embedding computational thinking into high school math classes. *ACM Inroads*, 11(1) 33–42. <https://doi.org/10.1145/3380956>
- Wilkerson-Jerde, M. H. (2014). Construction, categorization, and consensus: Student generated computational artifacts as a context for disciplinary reflection. *Educational Technology Research and Development*, 62(1), 99-121.  
<https://www.learntechlib.org/p/153777/>.
- Wilkerson-Jerde, M., & Fenwick, M. (2016). The practice of using mathematics and computational thinking. In C. V. Schwarz, C. Passmore, & B. J. Reiser (Eds.), *Helping students make sense of the world using next generation science and engineering practices*. National Science Teachers' Association Press.  
[https://www.researchgate.net/publication/316158531\\_The\\_practice\\_of\\_using\\_mathematics\\_and\\_computational\\_thinking](https://www.researchgate.net/publication/316158531_The_practice_of_using_mathematics_and_computational_thinking)
- Wilkerson-Jerde, M., Gravel, B., & Macrander, C. (2015). Exploring shifts in middle school learners' modeling activity while generating drawings, animations, and computational simulations of molecular diffusion. *Journal of Science Education and Technology*, 24(2-3), 396-415. <https://doi.org/10.1007/s10956-014-9497-5>
- Wilkerson-Jerde, M., Wagh, A., & Wilensky, U. (2015). Balancing curricular and pedagogical needs in computational construction kits: Lessons from the Delta Tick project. *Science Education*, 99(3), 465-499.  
<https://doi.org/10.1002/sce.21157>
- Yates, C. (2020, March 25). *How to model a pandemic*. The Conversation.  
<https://theconversation.com/how-to-model-a-pandemic-134187>

Yeghikyan, G. (2020, February 4). *Modelling the coronavirus epidemic in a city with python*. Towards Data Science. <https://towardsdatascience.com/modelling-the-coronavirus-epidemic-spreading-in-a-city-with-python-babd14d82fa2>

## Chapter 4

### 4 Integrative Chapter

This integrative chapter includes a reflection on previous chapters. Research contributions and recommendations, and future research suggestions are also presented in this chapter along with concluding remarks.

#### 4.1 Reflection on Previous Chapters

In this section, I provide an overview of the research questions in connection with the synthesis of each study presented in Chapter 2 and Chapter 3. Further, I clarify these chapters' interconnections by highlighting the focus, method, main findings, and integrated findings of the thesis.

##### 4.1.1 Reflection on Chapter 1: A Literature Review of CT

Papert originally introduced *computational thinking* as a term (1971, 1980, 1996) and described CT as “something children themselves will learn to manipulate, to extend, to apply to projects, thereby gaining a greater and more articulate mastery of the world, a sense of the power of applied knowledge and a self-confidently realistic image of themselves as intellectual agents” (Papert, 1971, p.1).

Based on the literature review in Chapter 1 on CT origins, integration, affordances, and framings, mathematics curricula follow Papert's ideas in that CT concepts and tools are incorporated into the curricula not only to teach coding but rather to enhance mathematics teaching and learning (diSessa, 1985; Feurzeig et al., 1970; Harel & Papert, 1990; Papert, 1971, 1980, 1996; Solomon, 1986).

Kafai et al. (2020) note that teaching CT or using CT to promote other learning, such as mathematics learning, takes advantage of only the perspective of cognitive framing. However, considering the computational literacy (diSessa, 2018), computational participation (Kafai et al., 2020), and computational action (Tissenbaum et al., 2019) opportunities provided by CT, it is crucial to encourage students to think about the underlying purposes and effects of the things they do and seek for solutions keeping

in mind that their values and histories shape these solutions (diSessa, 2018; Kafai et al., 2020; Lee & Soep, 2016).

Based on these views about the need to broaden and deepen using of CT concepts and tools in mathematics curriculum and pedagogy, it is therefore necessary to emphasize other framings (i.e., situated and critical) which have been explored in Chapter 2.

In a school context, CT concepts and tools can be used to model mathematical processes and to solve real-world problems through coding. Dynamical modelling, for instance, allows students to investigate relationships and test what-if questions (Gadanidis, Brodie, et al., 2017; Weintrop et al., 2016; Wilson, 2019). Through utilizing coding tools (e.g., graphical illustration, simulations, and data maps), students can illustrate real world problems and manipulate the models to experiment and understand the dynamics of the different scenarios as part of the solution to complex problems, so students can create their own models on real world issues (e.g., Eduapps, n.d; Gadanidis & Caswell, 2018; Gadanidis & Cummings, 2018; Gadanidis, Hughes et al. 2017; Resnick, 2020). These wide possibilities of integrating coding in mathematics teaching and learning have been examined in Chapter 3.

With an aim to examine the broader and deeper possibilities of CT by exploring various practices of the integration of CT in mathematics education, I asked what the current state of integration of computational thinking in mathematics education in school, outreach, and public educational settings is.

To respond to this question, I sought answers to the following sub-questions through two different but connected studies:

1. What is the understanding of integrating computational thinking in Grades 1-9 mathematics education in Ontario?

While seeking answer to this sub-question 1, I asked the following data collection questions:

- a. What is the nature of CT practices in school and outreach settings in Grades 1-9 mathematics education in Ontario?
  - b. How are CT practices framed in mathematics education in Ontario in Grades 1-9 mathematics education in Ontario?
2. In what ways do computational thinking concepts and tools help students comprehend real-world problems?

Sub-question 1 is associated with Chapter 2 and sub-question 2 is associated with Chapter 3. I summarized the main components of these studies in Chapter 2 and Chapter 3 in Table 9.

**Table 9: Main components of Chapter 2 and Chapter 3**

	<b>Chapter 2:</b>	<b>Chapter 3:</b>
<b>Research Purpose:</b> Providing an overview of the current state of the integration of CT in mathematics education and an insight into the wide affordances of CT	This chapter explores the nature of school and outreach practices related to the integration of CT in mathematics education and how these CT practices are framed in the context of framings of CT.	This study examines the ways CT, concepts and tools are used to help students comprehend the real-world problems.
<b>Focus:</b> Written and visual resources and computational artifacts of mathematical models	Examination of the online resources for school and community outreach practices regarding CT integration in mathematics education in Ontario.	Analysis of computational simulations of the outbreak based on the SIR model to investigate the learning opportunities real-life uses of CT concept and tools.
<b>Method:</b> Qualitative content analyses (Deductive and inductive)	Qualitative content analyses: Deductive category application and inductive category development (Mayring, 2000).	Qualitative content analyses: Inductive category development (Mayring, 2000).
<b>Main Findings:</b> Using CT concepts and tools when incorporating real-	<ul style="list-style-type: none"> <li>• The number of CT practices in mathematics education made a peak in 2017.</li> </ul>	<ul style="list-style-type: none"> <li>• The simulations have the following common dimensions: (i) initialization (ii) movements</li> </ul>



<p>world problems into mathematics instruction improve the affordances associated with the framings of CT</p>	<ul style="list-style-type: none"> <li>• CT practices were mostly conducted in elementary schools.</li> <li>• The most common tool used for CT practices in mathematics classrooms is Scratch.</li> <li>• The most common perspective used in CT practices in mathematics education is cognitive CT and only 3 % of them include the emphasis of critical CT.</li> <li>• Other findings connected with the mathematics curricula aspects include 21<sup>st</sup> century competencies, real-world applications, integrated learning, and intergenerational learning.</li> </ul>	<p>(iii) transmission of infection (iv) recovery process.</p> <ul style="list-style-type: none"> <li>• The simulations include the following mathematical concepts: coordinate geometry, angles, mathematics operations, probability operation, and algebraic equation.</li> <li>• The simulations include the following CT concepts: repetition and conditional logic, defining a block of parameters, and changing looks of characters.</li> <li>• Coding allows students to dynamically model mathematical problems. For instance, users may change the initial states, steps, or rates in the simulation to experiment with different scenarios.</li> </ul>
<p><b>Integrative Findings:</b> Applying the framings of CT to the analysis of the computational simulations of the mathematical model</p>	<ul style="list-style-type: none"> <li>• The computational simulations analyzed in Chapter 3 include possible affordances of three framings of CT outlined in Chapter 2:</li> </ul> <p><b><i>In the cognitive context:</i></b> Learning mathematical concepts such as probability, coordinate geometry and computational concepts such as repetition and conditional logic, defining parameters.</p> <p><b><i>In the situated context:</i></b> Social interaction, such as collaboration and communication, through remixing one another project.</p> <p><b><i>In the critical context:</i></b> Engaging with the political, moral, and ethical challenges of the world, such as creating applications to promote awareness of preventing the spread of COVID-19</p>	

#### 4.1.2 Reflection on Chapter 2: Deductive and Inductive Analyses of Online Resources

The first study presented in Chapter 2, was carried out through analyzing online resources at 3 selected websites related to school and outreach practices of integrating CT in mathematics education. I decided to analyze the educational research products to understand, as closely as possible, the perspectives and intentions of their creators, the researchers, educators, teachers, and resource designers. These products included research projects and reports on classroom activities published online by researchers and practitioners as a resource for others in the field. The analysis of these resources aimed to provide insight into the understanding of practitioners and researchers.

The overall results show that the dominant perspective embedded in CT practices aligned with the context of cognitive framing of CT. Cognitive framing of CT is embedded in most of the sources analyzed in Chapter 2, which emphasizes using computational concepts and programming tools to promote mathematics learning (e.g., Source 5, 6, 8, 18, 36, 39, 40, 38, and 50). For instance, Source 8 shows that when computer coding is dynamically used to model mathematics, it helps students bring mathematics concepts to life. Writing code to model a pattern or a relationship helps students to automate that process (as detailed in Chapter 2).

Situated framing is also widely embraced by educators in designing classroom activities of CT in mathematics which commonly emphasize that using CT concepts and tools promote collaboration, student agency, sense of community, resilience, perseverance (e.g., Source 23, 32, 31, 50, 38, 15 and 39). As discussed in the intersection of the framings section in Chapter 2, most of the Ct practices are designed for learning in cognitive framing such as problem-solving actually involve learning in the context of situated framing, which aims to promote students' social-emotional skills, such as collaboration, and resilience (e.g., Source 15 and 30).

The possibilities of CT from a perspective of critical framing, unlike cognitive and situated framings, have not been adequately stressed. It can be seen from the analysis that

only 2 of the 63 resources analyzed in Chapter 2 referred to critical CT and both of them put emphasis on citizenship (i.e., Source 6 and 13).

#### 4.1.3 Reflection on Chapter 3: Inductive Analysis on Computational Simulations of SIR Mathematical Model

The study presented in Chapter 3 was carried out to ascertain the ways in which two computational simulations based on the Susceptible – Infectious – Recovered (SIR) mathematical model by considering how these applications were accessible to help students comprehend the dynamics of the outbreaks and the responses needed to slow down the rates of the outbreaks. Because the simulations, in addition to being dynamic and interactive, had open-source code, which was modifiable, they afforded students experimenting with different scenarios to help comprehend the impacts of variations in human behavior. In the analysis, I also considered how each of the simulations, through its specifications, code, and simulations were connected to both learning and using mathematical and computational concepts. Thus, while the intention is to raise awareness of the impacts of variations in precautions (e.g., social/physical distancing, reduced mobility through staying at home, isolation, or quarantine, and regular handwashing) and policies implemented during the pandemic crisis, the dynamics of the disease and measures taken to contain it illustrate that tinkering with these models of a real-life application could help learners develop a broader and deeper understanding of using mathematical and computational concepts and tools. As I mentioned earlier, I chose to focus on these simulations because they include additional aspects of incorporating CT concepts and tools beyond cognitive framing.

This study aimed to shed light on learning opportunities using CT concepts and tools during the current global health crisis and, to determine how CT might be potentially helpful (or not helpful) in making key mathematical and computational concepts of infectious disease dynamics (i.e., (i) initialization, (ii) movements, (iii) transmission of infection, and (iv) recovery process) understandable to students. Thus, I presented a way of engaging students with real-life mathematics through embracing different framings of CT.

#### 4.1.4 Reflection on Integrative Findings: Applying the Framings of CT to the Analysis of the Computational Simulations

Interpreted through the framings of CT presented in Chapter 2, computational simulations modeling a real-life pandemic analyzed in Chapter 3 show the possibilities of using CT concepts and tools including all three framings on use and learning of CT:

- i. cognitive framing, such as *learning/using mathematical concepts*, namely, probability and coordinate geometry, and *learning/using computational concepts*, namely, repetition, conditional logic, and defining parameters,
- ii. situated framing, such as social interaction through remixing or modifying others' codes,
- iii. critical framing on understanding real-life problems and offering solutions (e.g., tinkering with applications to see different scenarios which in turn) to experiment the implications (e.g., impacts) of variations in precautions and policies in preventing the spread of the disease).

The study's findings illustrate that using CT concepts and tools when incorporating real-world problems into mathematics instruction can both improve and increase the affordances associated with different framings of CT. Therefore, CT studies need to move beyond mainly focusing on cognitive benefits of improving understanding of school mathematics curriculum to focus on wider opportunities. Utilizing real-world problems including critical issues (e.g., health care, sustainability, social justice, etc.) to design CT could boost students' capacities to learn to code and create computational ideas and artifacts which can have a positive impact on their lives and the lives of their families and communities.

## 4.2 Contributions and Recommendations for Practice and Policy

In the context of this study, I expanded the knowledge base on the integration of CT in mathematics education by considering the applications and implications of CT that

influence educational practice and policy. In the light of the findings of the study, I outline the following recommendations for practitioners, policymakers, and parents.

- i. The OME has made five years of substantial progress for integrating coding and computational skills into teaching since 2016 and the support for recently adding coding to the mathematics curriculum in 2020-2021. While OME has made progress in this integration, I draw attention to possible improvements for integrating CT into mathematics teaching and learning in ways that focus on the different aspects of using CT knowledge, concepts and tools. Policymakers could strengthen the coding component in curriculum documents by including situated and critical aspects of CT framings. This inclusion allows educators to teach both CT concepts and tools as well as the deeper opportunities afforded by using coding in mathematics. For instance, students may engage in CT for cognitive skills in solving problems through computing with technology, in interaction with other learners for situated and critical understandings, and in the exploration of further applications of the concepts and tools for intertwining skills such as collaboration and citizenship, which transcend disciplines.
- ii. As illustrated in the findings using CT concepts and tools while working with real-world problems in mathematics teaching and learning can improve the affordances associated with all three framings of CT (cognitive, situated, critical). Accordingly, practitioners might design classroom activities that incorporate these framings to offer students opportunities in mathematical learning and improved SEL skills while enhancing their understanding of real-world problems.
- iii. Teacher education and professional development programs may include more coding applications and implications that refer to different framings of CT. For example, in the context of situated framing educators may use CT concepts and tools for their social and affective value when students are involved in the construction of computational artifacts.

- iv. Parents may promote the affordances of incorporating coding in mathematics teaching and learning to enhance the cognitive, social, and critical skills of their kids by leading them to engage in rich coding activities at home or outdoor, which embrace multiple aspects of CT. They may also help their kids experiment and understand real-world problems.

### 4.3 Suggestions for Future Research

The results of this integrated thesis could serve as a basis for future research in the field. There are several lines of research that could be built on this study.

First, in this study, I focused on mathematics education, and I analyzed teaching and outreach resources and artifacts for the Ontario region to analyze practices that integrate CT into mathematics teaching and learning. In this regard, future studies may apply the theoretical and methodological approach of this study in two different ways: (1) by focusing on different core or integrated disciplines such as science, STEM, and STEAM, studied by scholars interested in CT implications (Lee et al., 2020; Leonard et al., 2016; Miller, 2019), and (2) by focusing on other regions of Canada and other countries, which are each integrating CT in curriculum and outreach at various paces and with distinct emphasis. A comparison of the perspectives and practices in different disciplines and regions could make valuable contributions to the field.

Second, a different methodology may be applied. I limited this study to the content analysis of online resources; however, to provide an in-depth insight for perspectives and practices of integration of CT in mathematics education, case studies using different kinds of data, such as interviews, surveys, and classroom observations should be applied (Schoch, 2016).

Third, I focused on practice and its connection to the policy of integration of CT in mathematics education; however, the curriculum policy itself could potentially be the focus of a future research project. In that regard, future research might use the method of document analysis on curriculum policy documents (e.g., policies, strategies, frameworks, guidelines, reports, resource guides, and related policy web pages) to

provide a greater understanding of the integration of CT in the mathematics curriculum. Additionally, comparative document analysis might be used to investigate the mathematics curriculum approaches of different regions or countries.

Finally, in Chapter 3, I presented two applications of CT which utilized Scratch block programming language. Considering the wide possibilities of CT, I suggest further research to explore other applications, such as using unplugged activities, digital tangibles, or different coding languages.

#### 4.4 Concluding Remarks

Within this study, I aimed to examine the current state of CT integration in mathematics education. I sought insight into the further affordances of CT that meet and exceed the expectations of curricula and outreach content and skills. With this purpose in mind, I framed the study with cognitive, situated, and critical theoretical framings, which also offered a lens through which it was possible to deductively analyze the CT resources and artifacts available for use in classroom and outreach practices and in real-life applications and implications.

The exploration of real-world applications in Chapter 3 is an example of using CT concepts and tools through critical framing along with cognitive and situated framings of CT to foster mathematical learning, social interaction, and citizenship. Additionally, as Kafai et al. (2020) assert, considering these three framings concurrently will help to conceive other framings, which are yet to be explored or even developed, regarding the integration of CT in mathematics education and in different disciplines. These new framings of integrating CT with an aim to understand real-world crises are not limited to the current pandemic. Students might also learn about other critical issues, such as environmental crises, food insecurities in certain regions, and widening income and standards of living gaps for marginalized groups (Bakker & Wagner, 2020; Yaro et al., 2020).

By provoking conversation on new perspectives to implement better practices of CT in mathematics as well as other disciplines, this thesis can be helpful to researchers,

practitioners, and policymakers. It goes beyond the commonly exposed cognitive framing and makes use of examples and artifacts to highlight the potential of considering situated and critical framings.

The knowledge and experiences I gained during this research do not end here, as they will continue to transcend my personal and professional life and will help me further explore avenues for promoting positive changes through the use of curricular and integrated CT concepts, applications, tools, and skill sets in the teaching and learning of mathematics in schools and in outreach contexts and beyond.

## 4.5 Chapter References

- Bakker, A., & Wagner, D. (2020). Pandemic: lessons for today and tomorrow? *Educational Studies in Mathematics*, *104*, 1–4. <https://doi.org/10.1007/s10649-020-09946-3>
- diSessa, A. A. (1985). Learning about knowing. *New Directions for Child Development*, *28*, 97–124. <https://doi.org/10.1002/cd.23219852808>
- diSessa, A. A. (2018). Computational literacy and “the big picture” concerning computers in mathematics education. *Mathematical Thinking and Learning*, *20*(1), 3-31. <https://doi.org/10.1080/10986065.2018.1403544>
- Eduapps. (n.d.). *Civilization: sustainable growth*. <http://eduapps.ca/civilization/growth/>
- Feurzeig, W., Papert, S., Bloom, M., Grant, R., & Solomon, C. (1970). Programming-languages as a conceptual framework for teaching mathematics. *ACM SIGCUE Outlook*, *4*(2), 13–17. <https://doi.org/10.1145/965754.965757>
- Gadanidis, G., Brodie, I., Minniti, L., & Silver, B. (2017). Computer coding in the K-8 mathematics curriculum? What Works? Research into Practice: Research Monograph #69. [http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Computer\\_Coding\\_K8\\_en.pdf](http://www.edu.gov.on.ca/eng/literacynumeracy/inspire/research/Computer_Coding_K8_en.pdf)
- Gadanidis, G., & Caswell, B. (2018). Computational modelling in elementary mathematics education: making sense of coding in elementary classrooms. *KNAER Mathematics Knowledge Network, White Paper*. <http://mkn-rcm.ca/wp-content/uploads/2018/05/MKN-white-paper2-May-2018.pdf>
- Gadanidis, G., & Cummings, J. (2018). Integrated mathematics + computer studies. reforming secondary school mathematics education. *KNAER Mathematics Knowledge Network*. <http://mkn-rcm.ca/wp-content/uploads/2018/04/MKN-white-paper-April-2018.pdf>



- Gadanidis, G., Hughes, J.M., Minniti, L., & White, B. J. G. (2017). Computational thinking, Grade 1 students and the binomial theorem. *Digital Experiences Mathematics Education* 3, 77–96. <https://doi.org/10.1007/s40751-016-0019-3>
- Harel, I., & Papert, S. (1990). Software design as a learning environment. *Interactive Learning Environments*, 1(1), 1–32. <https://doi.org/10.1080/1049482900010102>
- Kafai, Y. B., Proctor, C., & Lui, D. (2020). From theory bias to theory dialogue: embracing cognitive, situated, and critical framings of computational thinking in K-12 CS education. *ACM Inroads*, 11(1). <https://doi.org/10.1145/3381887>
- Lee, C. H., & Soep, E. (2016). None but ourselves can free our minds: Critical computational literacy as a pedagogy of resistance. *Equity & Excellence in Education*, 49 (4), 480-492. <https://doi.org/10.1080/10665684.2016.1227157>
- Lee, I., Grover, S., Martin, F. Pillai, S., & Malyn-Smith, J. (2020). Computational thinking from a disciplinary perspective: integrating computational thinking in K-12 science, technology, engineering, and mathematics education. *Journal of Science Education and Technology*, 29, 1–8. <https://doi.org/10.1007/s10956-016-9628-2>
- Leonard, J., Buss, A., Gamboa, R., Mitchell, M., Fashola, O. S., Hubert, T., & Almughyrah, S. (2016). Using robotics and game design to enhance children’s self-efficacy, STEM attitudes, and computational thinking skills. *Journal of Science Education and Technology*, 25, 860–876. <https://doi.org/10.1007/s10956-016-9628-2>
- Miller, J. (2019). STEM education in the primary years to support mathematical thinking: Using coding to identify mathematical structures and patterns. *ZDM Mathematics Education* 51, 915–927. <https://doi.org/10.1007/s11858-019-01096-y>
- Papert, S. (1971). *Teaching children thinking*. MIT Artificial Intelligence Laboratory Memo no. 2247, Logo Memo no. 2. <http://hdl.handle.net/1721.1/5835>.
- Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. Basic Books.
- Papert, S. (1996). An exploration in the space of mathematics educations. *International Journal of Computers for Mathematical Learning*, 1(1), 95-123.
- Resnick, M. (2020, March 17). *Scratch epidemic simulator*. MIT Media Lab. <https://www.media.mit.edu/posts/scratch-epidemic-simulator/>
- Schoch, K. W. (2016). Case study research. In Burkholder, Gary J., Cox, K. A., Crawford, L. M. (Eds.), *The scholar-practitioner’s guide to research design, 1st Edition* (pp. 227 – 241).
- Solomon, C. (1986). *Computer environments for children: A reflection on theories of learning and education*. MIT Press.
- Tissenbaum, M., Sheldon, J., & Abelson, H. (2019). From computational thinking to computational action. *Communications of the ACM*, 62(3), 34–36. <https://doi.org/10.1145/3265747>
- Weintrop, D., Beheshti, E., Horn, M., Orton, K., Jona, K. Trouille, L., & Wilensky, U. (2016). Defining computational thinking for mathematics and science classrooms.

*Journal Science Education and Technology*, 25, 127–147.  
<https://doi.org/10.1007/s10956-015-9581-5>

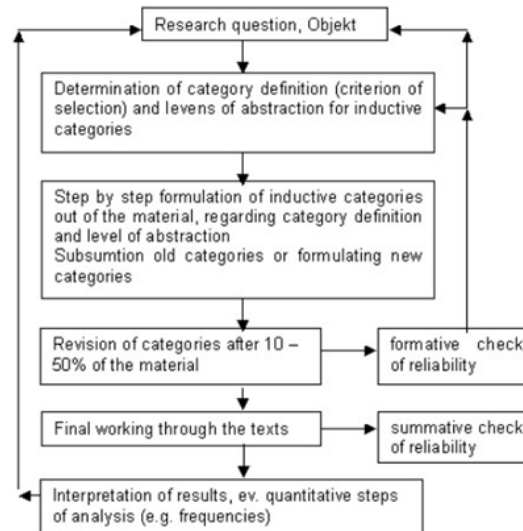
Wilson, L. (2019). Coding in K-8 education. In R. Power (Ed.) *Technology and the curriculum: Summer 2019*. Pressbooks

Wing J. M. (2006). Computational thinking. *Communications of the ACM*, 49(3), 33–35.  
<https://doi.org/10.1145/1118178.1118215>

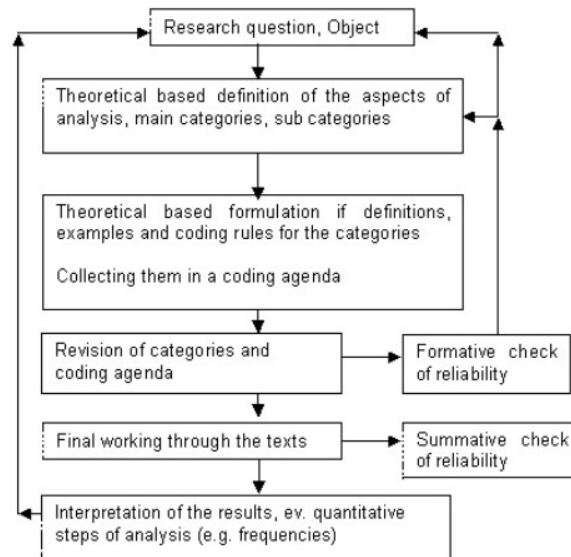
Yaro, K., Amoah, E., & Wagner, D. (2020). Situated perspectives on creating mathematics tasks for peace and sustainability. *Canadian Journal of Science, Mathematics and Technology Education*, 20, 218–229.  
<https://doi.org/10.1007/s42330-020-00083-w>

## Appendices

### Appendix A: Mayring's (2000) inductive category development and deductive category application



**Step model of inductive category development. Reprinted from *Qualitative Content Analysis* by Mayring, 2000.**



**Step model of deductive category application. Reprinted from *Qualitative Content Analysis* by Mayring, 2000.**

### Appendix B: The list of the samples from data set given in the context

The code of the source	Website	The title of the source with the hyperlink
1	Math Knowledge Network	Computational Literacy and Mathematics Education: A Webinar with Dr. Andy diSessa
2	Math Knowledge Network	Math & Computational Thinking in the Niagara Catholic Classroom
3	Math Knowledge Network	Modelling Civilization at St Andrews PS, TDSB: Coding, Making, Math
4	Math Knowledge Network	Repeating Patterns + coding
5	Math Knowledge Network	Brock U-NCDSB CT + Math Tasks
6	Math Knowledge Network	Computational Modelling in Elementary Mathematics Education – Making Sense of Coding in Elementary Classrooms
7	Math Knowledge Network	Symmetry as a transformation + coding
8	Math Knowledge Network	math, art, code
9	Math Knowledge Network	Back to the Future – Hour of Math + Code
10	Computational Thinking in Mathematics Education	CT + K-6 Teacher Candidates
11	Computational Thinking in Mathematics Education	Undergraduate’ perception of CT for/in mathematics (learning)
12	Computational Thinking in Mathematics Education	CT + Geometry
13	Math+Code Zine’	Social Emotional Learning in an Innovative, Inclusive Classroom
14	Math+Code Zine’	Integrated curricular and computational thinking concepts.
15	Math+Code Zine’	Open and Creative Coding
16	Math+Code Zine’	How should teachers respond to student questions when engaging in coding activities?
17	Math+Code Zine’	The Overlooked Value of “Use” in “Use-Edit-Create”
18	Math+Code Zine’	Coding Buddies and Intergenerational Thinking
19	Math+Code Zine’	Discerning Decomposition and computational disposition with Archelino: : A dialogue
20	Math+Code Zine’	Enacting Computational Thinking Concepts at Different levels
21	Math+Code Zine’	Review Article for Edison Edblocks
22	Math+Code Zine’	Computational Thinking and Design Thinking
23	Math+Code Zine’	Coding: Where Art AND Math > (Math + Art)
24	Math+Code Zine’	End of Year Math
25	Math+Code Zine’	5 Straight A’s for Coding + Math
26	Math+Code Zine’	Just like mathematicians do it!
27	Math+Code Zine’	Fractions + infinity in Grades 3-4

28	Math+Code Zine'	21C knowledge construction
29	Math+Code Zine'	Coding formula
30	Math+Code Zine'	Movement on a grid
31	Math+Code Zine'	Long distance coding
32	Math+Code Zine'	Needles, pi(e) and coding
33	Math+Code Zine'	Tools for integrating CT and mathematics in the middle grades
34	Math+Code Zine'	Symmetry + code
35	Math+Code Zine'	Scratching the surface
36	Math+Code Zine'	Coding in Kindergarten
37	Math+Code Zine'	March Break Maker Camp
38	Math+Code Zine'	Starting a coding club
39	Math+Code Zine'	Beyond shapes – Creativity with video games
40	Math+Code Zine'	Density, buoyancy, code + art
41	Math+Code Zine'	Teacher and Learner Roles are Ageless
42	Math+Code Zine'	Computational Thinking – The Journey from Skepticism
43	Math+Code Zine'	Learning Math Through Coding
44	Math+Code Zine'	Try this: Plotting Points in Scratch
45	Math+Code Zine'	Inspired by CLC, Teacher Inspires others with Coding
46	Math+Code Zine'	Perspectives on Teaching Code in Elementary Schools
47	Math+Code Zine'	To Code or Not to Code?
48	Math+Code Zine'	Learning on the Fly
49	Math+Code Zine'	Coding Shapes with Primary School Children
50	Math+Code Zine'	Western U's Teacher Candidates Reflect on a Coding Task
51	Math+Code Zine'	A Coding Story
52	Math+Code Zine'	Coding: Not an “Add on” to Math Instruction
53	Math+Code Zine'	Math + Coding Community Events
54	Math+Code Zine'	Why Math + Code?
55	Math+Code Zine'	Dr. Seymour Papert – Mathematician, Computer Scientist, Educator

### Appendix C: Excluded resources with the explanation of exclusion reasons

Name of the website	Excluded resource with hyperlink	Reason for exclusion
Computational Thinking in Mathematics Education ( <a href="http://ctmath.ca">http://ctmath.ca</a> )	<a href="#"><u>CT + Math Module</u></a>	Does not include any insight of perspectives about CT practices
	<a href="#"><u>CT + Young Mathematicians</u></a>	Focuses on STEAM
	<a href="#"><u>Interactions Between Mathematics and Programming at a Tertiary Level</u></a>	Targets undergrad level education
	<a href="#"><u>CT + Math undergraduate course</u></a>	Redundancy -This project links to <a href="#"><u>Needles, pi(e) and coding   Math + Code 'Zine (researchideas.ca)</u></a>
	<a href="#"><u>CT + Assessment (in practice)</u></a>	Targets undergrad level education
	<a href="#"><u>Math + Coding 'Zine</u></a>	Included in the data set with its' entire professional publications
	<a href="#"><u>Math + Coding Events</u></a>	Redundancy -This project links to <a href="#"><u>Math + Coding Community Events   Math + Code 'Zine (researchideas.ca)</u></a>
	<a href="#"><u>Project Math 9-12</u></a>	Focusing on mathematics but not CT
	<a href="#"><u>CT and Math Grades 4-8</u></a>	Redundancy -This project links to <a href="#"><u>Tools for Integrating Computational Thinking and Mathematics in the Middle Grades</u></a>
	<a href="#"><u>CT + University Math</u></a>	Focuses on high-school mathematics curriculum development
<a href="#"><u>CT in Kindergarten</u></a>	Out of the education level (Grade 1-9) scope	
Math+Code Zine' ( <a href="https://researchideas.ca/mc/">https://researchideas.ca/mc/</a> )	<a href="#"><u>Computational Learning in Nunavut</u></a>	Out of Ontario scope
	<a href="#"><u>The case for DIY STE(A)M</u></a>	Focuses on STEAM
	<a href="#"><u>Funding for Math + Coding Community Event</u></a>	Does not include any insight of perspectives about CT practices

	<u><a href="#">Creating a Video Game on Scratch and Building a Makey Makey Game Controller</a></u>	Out of Ontario scope
	<u><a href="#">Developing a math + computer science cohort in grade 10</a></u>	Out of the education level (Grade 1-9) scope
	<u><a href="#">The case for DIY STE(A)M</a></u>	Focuses on STEAM
	<u><a href="#">A Children's Story – Ada Lovelace – Countess of Coding</a></u>	Does not include any insight of perspectives about CT practices
	<u><a href="#">Developing Math Skills &amp; the 4C's Through CT</a></u>	Out of Ontario scope
	<u><a href="#">Creating a Cash Register Program to Learn about Percents</a></u>	Out of Ontario scope
	<u><a href="#">Digital Making: The UOIT STEAM-3D Maker Lab –</a></u>	Focuses on STEAM
	<u><a href="#">Arduino – Coding a Bicolour LED Grid to Create Math Patterns</a></u>	Does not include any insight of perspectives about CT practices
	<u><a href="#">Drawing Squares and Circles with Scratch</a></u>	Does not include any insight of perspectives about CT practices
Math Knowledge Network ( <a href="http://mkn-rcm.ca">http://mkn-rcm.ca</a> )	<u><a href="#">Cast a Wide Net at St Andrews PS, TDSB: AI, robotics, coding, math, social studies</a></u>	Focuses on IA
	<u><a href="#">Steven Floyd: The History of the Computer Science Curriculum in Ontario</a></u>	Does not include any insight of perspectives about CT practices
	<u><a href="#">Computational Modelling: Impact 2016 to 2019</a></u>	Redundancy -This report links to <u><a href="#">Computational Modelling in Elementary Mathematics Education – Making Sense of Coding in Elementary Classrooms</a></u>
	<u><a href="#">Computational Modelling Resources Report</a></u>	Redundancy -This report links to <u><a href="#">Computational modelling in Social Studies and in Mathematics</a></u>

	<u>Computational modelling in Social Studies and in Mathematics</u>	Does not include any insight of perspectives about CT practices
	<u>Integrated Mathematics + Computer Science – Grade 10: Reforming Secondary School Mathematics Education</u>	Out of the education level (Grade 1-9) scope



**Appendix D: The preliminary results of a systematic literature review: the current state of the integration of computational thinking into school mathematics**

**Title: Current State of the Integration of Computational Thinking into School**

**Mathematics**

**Purpose of the study:** Aim to provide an examination of existing literature, on the affordances and opportunities of integrating CT into school mathematics beyond the current state

**Research Question:** What is the understanding of the integrating computational thinking into school mathematics in the current literature?

**Method:** Systematic literature review

**Data selection**

**Data set:** 32 Articles (Education Database, Apa Psycinfo, ERIC, JSTOR)

The data set consists of classroom experiments, online course module evaluation, curriculum analyses, teacher-student interviews

***Inclusion criteria:***

- focuses only on mathematics education
- targets K-12 level of education or mathematics teacher education

***Exclusion criteria:***

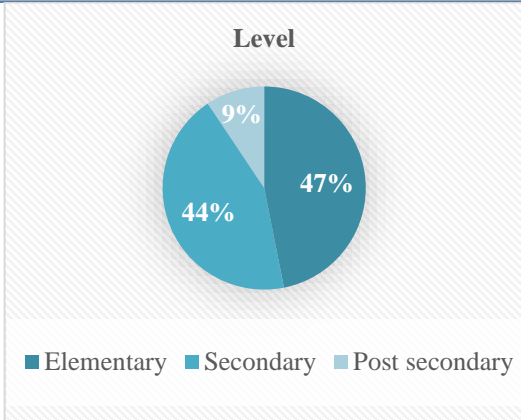
- focus on integrated disciplines such as STEM, STEAM

***The search key terms:***

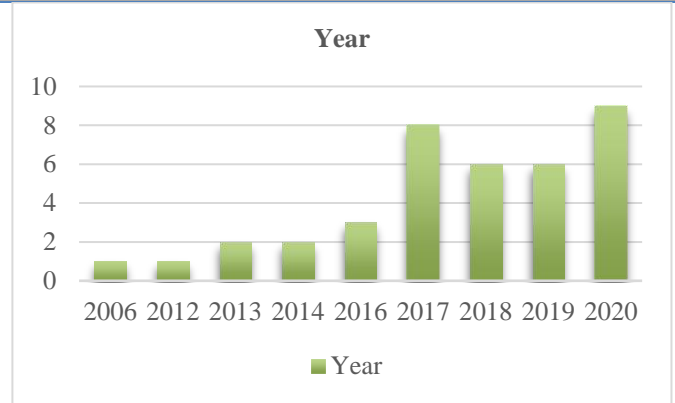
- computational thinking
- coding
- mathematics education
- mathematics learning
- school mathematics
- mathematics curriculum

**Findings:**

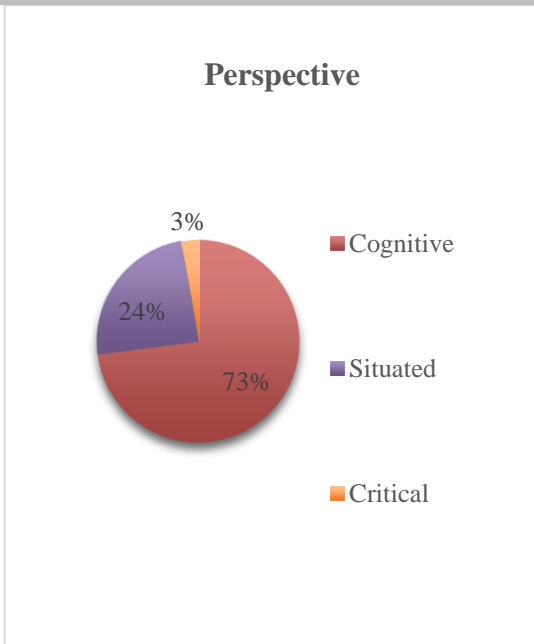
**Distribution of the publication focus by school level**



**Distribution of the publication by year**



**Distribution of the publication by CT framings**



**Codes and themes generated on CT framings**

Cognitive	Situated	Critical
<ul style="list-style-type: none"> <li>• abstraction</li> <li>• mathematical thinking</li> <li>• problem-solving</li> <li>• algebraic thinking and learning</li> <li>• reasoning skills</li> <li>• mathematical modeling</li> <li>• logical thinking</li> <li>• estimation skills</li> <li>• understanding mathematical concepts</li> <li>• understanding of basic computing concepts</li> <li>• creating algorithms</li> <li>• programming skills</li> </ul>	<ul style="list-style-type: none"> <li>• motivation</li> <li>• self-efficacy</li> <li>• social interaction</li> <li>• relevance</li> <li>• fun</li> <li>• engagement</li> <li>• student agency</li> <li>• collaboration</li> </ul>	<ul style="list-style-type: none"> <li>• social movement</li> <li>• sustainability</li> </ul>

## Curriculum Vitae

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**Post-secondary Education and Degrees:**

Baskent University  
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<https://doi.org/10.1108/JRIT-12-2020-0085>
- Sezer, H. B., & Taşpınar, M. (2015). *Recognition of Prior Learning Process in Turkey*. In Proceedings of Congress on Research of Bologna Process, 158-165, Ankara, Turkey.  
[http://www.bolognakongresi2015.hacettepe.edu.tr/belge/BSAK15\\_bildirikitabi.pdf](http://www.bolognakongresi2015.hacettepe.edu.tr/belge/BSAK15_bildirikitabi.pdf)

**Conferences:**

- Sezer, H. B. (2021). The Current State of the Integration of Computational Thinking into School Mathematics. Presented at *Canadian Society for the Study of Education (CSSE)*, 29 May - 3 June, Edmonton, Alberta, Canada.
- Sezer, H. B. (2021). Relationship between teacher candidates' mathematical knowledge and pedagogical beliefs. Presented at *Robert Macmillan Symposium in Education (RMSE)*, 25-27 Mar, Western University, London, Ontario, Canada.
- Sezer, H. B. (2021). The integration of computational thinking into school mathematics: Perspectives, practices, and applications. Presented at *Western Research Forum (WRF)*, 17-19 Mar, Western University, London, Ontario, Canada.
- Sezer, H. B. (2021). Understanding Mathematics in Real Life Context Through Coding: COVID-19 Simulation with Scratch. Presented at *MathEd Research Day*, 30 Jan, The Fields Institute, Toronto, Ontario, Canada.