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This dissertation, SECONDARY MATHEMATICS TEACHERS' PERSPECTIVES OF MATHEMATICAL MODELING, by SIMONE WELLS-HEARD, was prepared under the direction of the candidate's Dissertation Advisory Committee. It is accepted by the committee members in partial fulfillment of the requirements for the degree, Doctor of Philosophy, in the College of Education & Human Development, Georgia State University.

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SECONDARY MATHEMATICS TEACHERS' PERSPECTIVES OF MATHEMATICAL MODELING

by

SIMONE WELLS-HEARD

Under the Direction of Under the Direction of Dr. Christine D. Thomas

ABSTRACT

The inception of Common Core State Standards for Mathematics (CCSSM) has increased the focus on mathematical modeling in high school mathematics curriculum in the United States. While the expectation that students engage in mathematical modeling is established by the standards, the standards do not include a clear and consistent definition of a mathematical model (Cirillo et al., 2016). The absence of a common description of a mathematical model or the mathematical modeling process, a single goal for mathematical modeling, and a standard process for designing modeling tasks has resulted in Kaiser and Sriraman's (2008) conception of "perspectives of mathematical modeling." Using this conception as a frame, this study employed a qualitative case study design (Yin, 2018) to explore the research question, "In what ways are teachers' perspectives of mathematical modeling connected to the ways in which they plan learning experiences for students?"

The participants in this study were five experienced Algebra II teachers from a southeastern state in the United States which include the CCSSM demand for mathematical

modeling in the course curricula. Data were collected through a survey, two interviews, a teacher selected task, and task exemplar. The results of this study are framed by participants reporting limited learning experiences involving mathematical modeling. The learning described included: (a) an emphasis on using identified manipulatives to develop an understanding of content standards; (b) the use of representations to solve problems; and (c) the importance and impact of mathematical modeling as teacher practice, absent of a clear description, examples of classroom implementation, or opportunities for practice. The cross-case analysis uncovered two themes: (1) content mastery and connections to students which grounded participants' perspectives of mathematical modeling, and (2) the ways they planned to engage students. Three categories of descriptions of mathematical models and modeling were present: (a) mathematical models as concrete tools for the progression from concrete to abstract understanding, (b) mathematical models as representations transformed to solve mathematical and real-world problems, and (c) mathematical models as teacher models with the purpose of exposing students to replicable thinking useful in solving mathematical and real-world problems.

INDEX WORDS: mathematical models, mathematical modeling, teacher perspectives'

SECONDARY MATHEMATICS TEACHERS' PERSPECTIVES OF MATHEMATICAL
MODELING

by

SIMONE WELLS-HEARD

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in

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Atlanta, GA
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It is hard for me to recall exactly when I began to pursue a doctorate degree but yet in the official record there is a starting date, a date of completion, and a few "milestones" along the way. I could use those dates and easily determine the amount of time this pursuit has taken but doing so would not provide an adequate representation of the journey. If my Dissertation Committee ever questioned my ability to engage in and complete this process, they hid it beautifully! My experience with them was one of full confidence, of space and freedom to research, explore, analyze, reflect, doddle, sketch, rest, and write in repetitive cycles. Thank you, Dr. Christine D. Thomas, for "knocking the chain off my bicycle" and forcing me to stop and take a holistic look at my work.

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1 INTRODUCTION

The Common Core State Standards for Mathematics (CCSSM) were released in 2010 and to date have been implemented in 42 states. The standards explicitly highlight the inclusion of mathematical modeling in high school mathematics curricula. While the high school standards explicitly call for mathematical modeling, the standards document lacks clarity and consistency around the meaning of the term “model” (Cirillo et al., 2016). This lack of clarity and a common conception of a mathematical model and the modeling process is also persistent in the international body of research on mathematical modeling (Blum & Niss, 1991; Cirillo et al., 2016; Lesh & Doerr, 2003a; Lesh & Fennewald, 2010), leaving teachers to rely on their personal perspectives of mathematical modeling to plan for and deliver instruction. The purpose of this study is to explore the perspectives of mathematical modeling held by high school mathematics teachers.

The research question, “In what ways are teachers’ perspectives of mathematical modeling connected to the ways in which they plan learning experiences for students?” serves as the guide for this study. This question was addressed through an exploration and analysis of the ways in which teachers describe a mathematical model, the mathematical modeling process, the goal or purpose of mathematical modeling, and the process of designing tasks to support mathematical modeling. These factors aligned with Kaiser and Sriraman’s (2006) conception of perspectives of mathematical modeling which served as a frame for this study.

Why the Focus on Mathematical Modeling?

There has been an increased focus on the teaching and learning of mathematical modeling in mathematics education. In the United States this increased focus was sparked by the development and release of the Common Core State Standards (CCSS) in 2010. The motivation

behind the development of the standards for both English Language Arts and mathematics came from the recommendations of the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) report *Benchmarking for Student Success: Ensuring U.S. Students Receive a World-Class Education* (2008), this report found that there was a need to develop a common set of rigorous and internationally benchmarked standards. The report concluded that a common set of rigorous and internationally benchmarked standards would prepare students in the U.S. to be competitive in the global marketplace. The standards were developed, reviewed, and revised in process with input from the NGA, CCSSO, professional organizations such as the National Council of Teachers of Mathematics (NCTM), the National Council of Teachers of English (NCTE), and the National Education Association (NAE). Classroom teachers, school administrators, and public stakeholders were also included in the process. The resulting CCSS for English Language Arts and Mathematics were released in 2010 and states had the opportunity to present them at the local level and consider adoption.

The CCSSM are composed of content standards by grade band and eight Standards for Mathematical Practice (SMPs) intended to represent the skills of proficient mathematicians thus should be incorporated into the curriculum in grades K-12. The eight SMPs are: (1) make sense of problems and persevere in solving them, (2) reason abstractly and quantitatively, (3) construct viable arguments and critique the reasoning of others, (4) model with mathematics, (5) use appropriate tools strategically, (6) attend to precision, (7) look for and make sense of structure, and (8) look for and express regularity in repeated reasoning. The elementary and middle school content standards are organized in domains which represent the conceptual focus areas in each grade level. The high school standards represent a much larger collection of standards, and as such a larger number of domains. The high school focus areas, or domains are organized by

theme into larger categories called conceptual categories, and six of these conceptual categories are labeled to represent the grade 9-12 standards. The six high school conceptual categories are: Number and Quantity, Algebra, Functions, Modeling, Geometry, and Statistics and Probability (NGACBP & CCSSO, 2010).

The significance of mathematical modeling to the CCSSM is established by its inclusion as SMP 4 for grades K-12: Model with Mathematics and also as a conceptual category for grades 9-12. Each of the high school conceptual categories begins with an introduction or overview of its theme and with the exception of modeling, that theme is then followed by a listing of domains and content standards which compose the category. In the case of modeling, the overview includes a modeling cycle and a note that there is no prescribed set of standards assigned solely to modeling. Instead, there are a set of standards throughout the other conceptual categories denoted with a “*” to express the relationship between modeling and the content standards (NGACBP & CCSSO, 2010). The excerpt below from the CCSSM provides direction as to the intent SMP 4: Model with Mathematics and establishes the significance of mathematical models and mathematical modeling within the field of mathematics education:

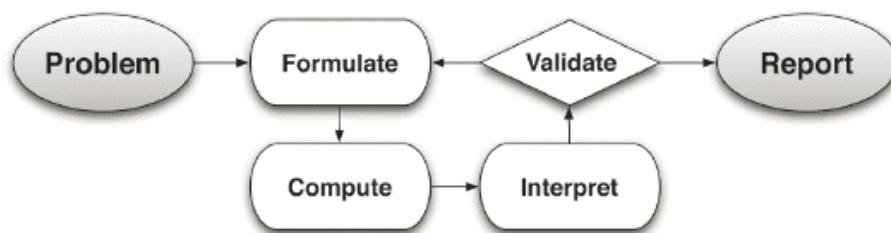
Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. (NGACBP & CCSSO, 2010, p.7)

Though explicitly included in the CCSSM, mathematical modeling as a component of effective mathematics instruction in the United States did not originate with the Common Core standards document. As a contributor to the best practices which informed the development of the CCSSM, NCTM had long echoed the recommendation that mathematical modeling be included in U.S. math curriculum. In 2000 the NCTM released *Principles and Standards for School Mathematics* in order to inform both the content and process standards that should be included in an effective mathematics curriculum. Many consider the NCTM Process Standards: problem solving, reasoning and proof, communication, connection, and representation as the foundation for the CCSSM Standards for Mathematical Practice. Though all five process standards bare some connection to mathematical modeling the clearest connection lies in its relationship with problem solving standard below. Instructional programs from prekindergarten through grades 12 should enable all students to:

- build new knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving. (NCTM 2000, p.52)

Figure 1

CCSSM High School Mathematical Modeling Process (NGACBP & CCSSO, 2010, p.72)



The characteristics of the NCTM process standard can be seen visibly in Figure 1, the mathematical modeling process for high school mathematics in CCSSM. The first three steps of the process involve students applying mathematics to solve problems, interpret, and validate steps demanding that students consider whether or not their solutions make sense in the context of the problem. It is through this consideration or reflection that students determine whether to report their solutions or re-engage with the problem to develop a more appropriate model. The connections between mathematical modeling and the NCTM process standards aligns the significance of mathematical modeling with almost a decade of commonly accepted best practices in mathematics instruction in the United States.

The second edition of *Guidelines Assessment and Instruction in Mathematical Modeling (GAIMME)* published in 2019 provides a more contemporary argument for the inclusion of mathematical modeling for students ranging in levels from pre-kindergarten to undergraduate. The revised version of the original 2015 report was constructed through a collaboration between the Consortium for Mathematics and its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM). The report is purposed with providing a rationale for the inclusion of mathematical modeling, clarity around mathematical modeling and the modeling process, and resources to support pre-kindergarten to undergraduate teachers in incorporating the practice of mathematical modeling into their classrooms. Though published after the release of the CCSSM the GAIMME report is not intended to provide clarity on the CCSSM expectations of mathematical modeling, nor are the resources included intended to serve as a curriculum at any grade level.

The inclusion of mathematical modeling in the CCSSM requires a new set of expectations from classroom teachers. Niss et al. (2007) asserted that in order to provide

effective instruction around mathematical modeling, teachers must possess an extensive understanding of mathematical modeling and best practice in instruction and assessment which support mathematical modeling. The CCSSM standards document includes a recommended modeling cycle, content standards connected to mathematical modeling, but does not include guidance on best practices in instruction and assessment to support the teaching of mathematical modeling. Grounded in Ball et al.'s (2008) conception of Mathematical Knowledge for Teachers (MKT), Bommero Ferri and Blum (2009) specifically identified a set of skills which required among effective teachers of mathematical modeling which include a knowledge of mathematical models, modeling cycles, the goals assigned to mathematical modeling, and tasks which support mathematical modeling.

Problem Statement

Tan and Ang (2012) stated, "Essentially the mathematical modeling process is characterized by the iterative negotiation of learning between the real and mathematical world" (p. 713); this essential understanding is common throughout the body of research on mathematical modeling. However, this essential understanding does not reflect a common definition or description of mathematical model, nor is there a common process or procedure associated with mathematical modeling (Blum & Niss, 1991; Cirillo et al., 2016; Lesh & Doerr, 2003a; Lesh & Fennewald, 2010; Zawojewski, 2013). As is the case with any other process, mathematical modeling is informed by its purpose or goal. The central idea of the varying perspectives of mathematical modeling is the distinction between the purpose or goal assigned to the process and the theory of learning accepted by the researcher (Ferri, 2013). Despite the lack of a common definition of mathematical modeling in the research base, the CCSSM explicitly requires that students receive mathematical modeling instruction using a prescribed modeling

cycle. While the modeling cycle within the CCSSM provides guidance as to the expected modeling process, the standards documents fail to provide a consistent description of a mathematical model. The term “model” is used to describe a fixed representation, such as a graph or physical object, as well as to represent the modeling process (Cirillo et al., 2016). The absence of a single description of mathematical models or the modeling process in the research base and the lack of clarity in the CCSSM is problematic, as it leaves high school teachers with the responsibility of delivering instruction based largely on their personal perspectives of mathematical modeling.

This problem is particularly applicable for Algebra II teachers in Georgia who are responsible for teaching a curriculum which includes the CCSSM demand for mathematical modeling. Georgia is one of the 42 states, which has adopted an adaptation of the CCSSM and named this adaptation the Georgia Standards of Excellence (GSE). It is important to note that modeling as a conceptual category is required in all high school courses, inclusive of the two required courses which precede Algebra II. Algebra II is the culminating course in a mandatory three-course sequence for all high school students. This study was intentionally focused on Algebra II because students are expected to have engaged in modeling in two prior courses and as a third-year course the state curriculum includes content standards related to modeling in six out of seven curriculum units.

Purpose and Research Questions

The purpose of this study is to explore high school mathematics teachers’ perspectives of mathematical modeling. All of the participating teachers taught the Algebra II course in Georgia during the 2019-2020 school year. The Algebra II course standards include mathematical modeling as a conceptual category and all of the SMPs, including SMP 4 model with

mathematics. I engaged in an exploration of the teachers' perspectives as they reflected on their perception of mathematical modeling and the ways in which they engaged students in mathematical modeling. This exploration is framed by Kaiser and Sriraman's (2006) "perspectives of mathematical modeling," this conception served as the foundation in the planning, conduction, and the data analysis procedures for this qualitative case study. Kaiser and Sriraman's conception of "perspectives of mathematical modeling" is composed of descriptions of a mathematical model and the modeling process, the purpose assigned to mathematical modeling, and the design of tasks used for mathematical modeling. This conception of "perspectives of mathematical modeling," along with two dimensions of four dimensions of MTK, (Ball et al., 2008) that Ferri and Blum (2009) identify as requirements for teachers of mathematical modeling, informed the development of the overarching research question stated earlier, "In what ways are teachers' perspectives of mathematical modeling connected to the ways in which they plan learning experiences for students?" and the sub-questions below:

- a. How do Algebra II teachers describe a mathematical model?
- b. How do Algebra II teachers describe the mathematical modeling process?
- c. What goals do Algebra II teachers assign to mathematical modeling?
- d. How do Algebra II teachers select and implement tasks that support mathematical modeling?

Overview of Conceptual Framework

Perspectives of Mathematical Modeling

Kaiser and Sriraman (2006) define a classification system composed of five broad perspectives of mathematical modeling and one meta-perspective: (a) realistic modeling, (b) contextual modeling, (c) educational modeling, (d) socio-critical modeling, (e) epistemological

modeling, and (f) cognitive modeling. This classification system is intended to represent the depth and width of the current research on mathematical modeling through commonalities in the goals assigned to mathematical modeling, the specific ways in which they describe a mathematical model and the mathematical modeling process, and the types of tasks they align to mathematical modeling. An overview of each of these perspectives of mathematical modeling is shown below in Table 1.

Table 1

Overview of the Perspectives of Mathematical Modeling

Name	Goals	Modeling Process/Cycle
Realistic Modeling	The development of mathematical modeling competencies	Pollak's 8 steps Blum and Leiß's 7- step cycle
Contextual Modeling	The development of mathematics content presented in context	MMP modeling cycle
Educational Modeling	The development of mathematics content and mathematical modeling competencies	Blum and Leiß's 7- step cycle Blum and Ferri's 4 step process for students
Socio-critical Modeling	The application of mathematics to support the development of critical citizenship	Process and cycle are undefined
Epistemological Modeling	The development of mathematical knowledge	Expansive definition of the term model prevents a defined process or model
Cognitive Modeling	Individual cognitive processes while engaged in modeling	Blum and Leiß's 7- step cycle

The development of CCSSM was research based but the conception of mathematical modeling and the modeling process contained in the standards is not directly connected to the body of research which contributes to Kaiser and Sriraman's (2006) classification system. The high school CCSSM define mathematical modeling as a conceptual category intended to be conceived through a relationship to a specified set of content standards and the real world.

Additionally, the standards include a modeling cycle to guide this process (NGACBP & CCSSO, 2010). The standards however do not include a reference to the research base that frames this view of modeling or the modeling cycle. The ambiguity in the perspective or perspectives which ground the CCSSM leaves us unable to make a direct connection between the existing body of research around mathematical modeling and serves as a rationale for studies which explore the relationship between the CCSSM and the larger body of research on mathematical modeling. The conceptual framework for this study is described in greater detail in chapter two.

Research Design

Based on the intent of the research questions, this study is a qualitative case study employing case study procedures to explore the perspectives of mathematical modeling held by five high school mathematics teachers in a southeastern state. The participants were recruited and selected using purposive sampling, in order to identify Algebra II teachers who had at least five years of experience teaching high school mathematics and had taught the Algebra II course for at least two years. Data collection began with a brief contextual survey to inquire as to the educational background and exposure to mathematical modeling for each participant. Next, I conducted two interviews with each participant to explore their perspective of mathematical modeling. Participants were asked to identify documents which represent evidence of lesson planning and implementation of mathematical modeling to support an in-depth exploration of their perspectives of mathematical modeling. Kaiser and Sriraman's (2006) "perspectives of mathematical modeling" was used to guide the analysis of data and both triangulation and the application of multiple case study procedures allowed me to attend to the validity, reliability, and integrity of this study. Each participant represents a single case, and then their individual case

study reports were analyzed to create a cross case report. A complete description of the design of this study can be found in chapter three.

Rationale and Significance

The teaching and learning of mathematical modeling in high school classrooms is established as a priority through its inclusion in the CCSSM and the adoption of the standards in 42 states. While included in the standards there is not a consistent and clear definition of a mathematical model in the standards documents (Cirillo et al., 2016) or in the international body of research (Blum & Niss, 1991; Cirillo et al., 2016; Lesh & Doer, 2003a; Lesh & Fennewald, 2010; Zawojewski, 2013). This lack of clarity leads to classroom teachers leaning on their own perspectives of mathematical modeling to design and deliver instruction which aims to meet the modeling demands of the CCSSM. The current study aims to explore these perspectives within the context of Georgia's Algebra II course.

The significance of this research study is that as we approach the close of the first decade of the implementation of CCSSM, there is a desire to consider how we are meeting the demand to develop "mathematically proficient students." Since Georgia has largely adopted the CCSSM with fidelity, the results of this study are relevant to that goal. Insight on the ways in which classroom teachers assign a goal or purpose to mathematical modeling, describe a mathematical model and the modeling process, and design tasks to engage students in mathematical modeling is useful in building an awareness among classroom teachers and school administrators. Such an awareness may prompt dialogue and reflection around teacher perspectives and practice. The perspectives of mathematical modeling held by teachers are of particular significance to teacher educators which support teachers in both pre-service and in-service settings. These perspectives can support the development of formal and informal coursework designed to support the

development of content and pedagogical knowledge of mathematical modeling. Additionally, the current study may also be influential for curriculum writers in public and private environments as they develop resources to support the implementation of state level adaptations of the CCSSM.

Summary

The imperative to engage in research around mathematical modeling has been set by international assessment reports, the CCSSM, and NCTM which have led to its inclusion in K-12 curriculum in the United States. The variation in the purpose and descriptions assigned to mathematical models and mathematical modeling within the international research base on mathematics education is represented within Kaiser and Sriraman's (2006) classification system. Additionally, the CCSSM and state adaptations of the CCSSM lack clarity in these concepts leaving teachers to lean on their own perspectives to plan and implement mathematical modeling instruction.

2 REVIEW OF LITERATURE

In this literature review I explore the research base that seeks to describe mathematical models, mathematical modeling, and Kasier and Sriraman's (2006) conception of perspectives of mathematical modeling. Additionally, I examine the connections between that conception, the CCSSM, and the existing research base on the perspectives of mathematical modeling held by teachers. This chapter is composed of six sections and launches with a general description of a mathematical model, the goals of mathematical modeling, and the mathematical modeling cycles which are prominent in the field. In the next section I provide a detailed exploration of the conceptual framework which guides the research questions and methods of this study. The third and fourth sections make connections between the CCSSM and the concept of teacher perspectives of mathematical modeling to the existing research. The last section is a summary of the rationale for the use of Kasier and Sriraman's perspectives of mathematical modeling as an appropriate conceptual framework for this study and the gaps in the existing research which support the possible contributions of the study.

Mathematical Models and the Modeling Process

Before we can explore the perspectives of mathematical modeling held by secondary teachers there is a need to explore the defining goals and characteristics of both a mathematical model and the mathematical modeling process as currently presented in the larger body of research. The absence of a common definition and process to describe mathematical modeling has implications for grades nine through twelve instruction around mathematical modeling. Blum and Niss (1991) write that there are six broadly accepted approaches to mathematical modeling instruction: (a) two separate courses, one for mathematics and another for mathematical modeling, (b) one course with two-compartments, (c) small islands of modeling in a pure math

class, (d) mixing where the math is given but activated through modeling, (e) integrated mathematics curriculum where math is developed through real world problems, and (f) interdisciplinary integrated. These six approaches are informed by the variance in the assigned purpose of mathematical modeling and the aligned mathematical modeling process. While these characteristics only differ slightly among the major works in the field, discussing those variations can inform teacher perspectives of mathematical modeling. The participants in the study are responsible for providing instruction in alignment with the mathematical modeling process as defined in the CCSSM and adopted by the Georgia Standards of Excellence (GSE) for Algebra II, in order to provide context, it is critical that this definition be situated within the larger context of the research field.

What is a Mathematical Model?

Certainly, one's definition of a mathematical model is shaped by how they contextualize the term "model." The Cambridge Dictionary (2010) assigns the parts of speech noun, adjective, and verb to the term "model" and makes distinctions based on the context in which the term is used. Upon hearing the word "model" I immediately begin to consider and then to regret my meal choices and not "making it" to the gym this morning. Lesh and Doer (2003) write that in many cases a teachers' conception of the term model is linked to perfection or an example such as a "model classroom" (p. 9). In these cases, a model is contextualized as a fixed construct such as an equation or diagram. Consider instead the definition provided by Lesh and Fennewald (2010), "A model is a system for describing (or explaining or designing) another system(s) for some clearly specified purpose" (p. 7). This definition, though hard to connect to our social experiences, is foundational to the description of a "mathematical model." If I were to simply layer my understanding of a mathematical model over the Lesh and Fennewald (2010) definition,

I would understand a mathematical model as a mathematical system which serves to represent another system in order to meet some desired outcome. That understanding though not an exact replica is very closely aligned to the definition Pollak (2003) gives when he describes a mathematical model as a mathematical representation of a real-world situation, a representation that can itself offer insight by providing a description, underlying causes, or a path to predict future outcomes. Real world situations are composed of an infinite number of constantly changing entities or variables and in order to manageably represent a real-world situation with mathematics the situation must undergo a stabilizing process. Pollak (2003) accounted for this stabilization by specifying that a mathematical model displays only an “idealized” version of the real world.

While the Pollak definition explicitly connects mathematics and the real world, Niss et al. (2007) provided a more generalizable description of a mathematical model as the picture of a path between an “extra-mathematical domain” D , and a mathematical domain M . Their description does not specify that the “extra-mathematical domain” be limited to real world situations.

The Goals of Mathematical Modeling

When we move from the concept of a mathematical model to a description of mathematical modeling all of the discussion about the appropriate part of speech disappears, clearly adding the “ing” implies that mathematical modeling requires action or a process. There is no single description of the mathematical modeling process (Blum & Niss, 1991; Cirillo et al, 2016; Lesh & Doerr, 2003a; Lesh & Fennewald, 2010; Zawojewski, 2013). This variation is present in the many processes and/or cycles used to represent mathematical modeling in the prevailing body of research. As is the case with any action, the process of mathematical

modeling is interconnected with the goal or purpose assigned to that action, the differences between the descriptions of the mathematical modeling process can be attributed to varying purposes. There are two goals which serve as an umbrella within the research base on mathematical modeling, mathematical modeling to explore advanced mathematics, and mathematical modeling as content (Julie & Mudaly, 2007; Niss et al., 2007). Julie and Mudaly (2007) portrayed, “Central to the debate is whether mathematical modeling should be used as a vehicle for the development of mathematics or treated as content in and of itself” (p.504), while Niss et al. (2007) described the differing goals as “modeling as a means” and “modeling as an end.” These two goals do serve as two ends of a segment with each of the major perspectives falling somewhere in between or on each endpoint. Related to the goals of mathematical modeling are five categories or arguments for including mathematical modeling as a component of a curriculum (Blum & Niss, 1991):

- i. Formative argument- Modeling should be included in curriculum because in developing modeling competencies students also develop creative problem-solving centered attitudes. These attitudes benefit the whole child and support students in becoming more open-minded, self-confident, and self-reliant.
- ii. Critical competence argument- Students develop the skills for critical and active citizenship through their engagement with mathematics in the real world. They then are inclined to apply mathematical modeling competencies to all areas of citizenship.
- iii. Utility argument- Through mathematical modeling students develop an appreciation for the usefulness of mathematics content in the real world.
- iv. Picture of mathematics argument- Expects that a quality mathematics education provides students with a complete understanding of mathematics theory and application.

- v. Promoting mathematics learning argument- Experiences with mathematical modeling and applications of mathematics increase student interest and motivation in mathematics content.

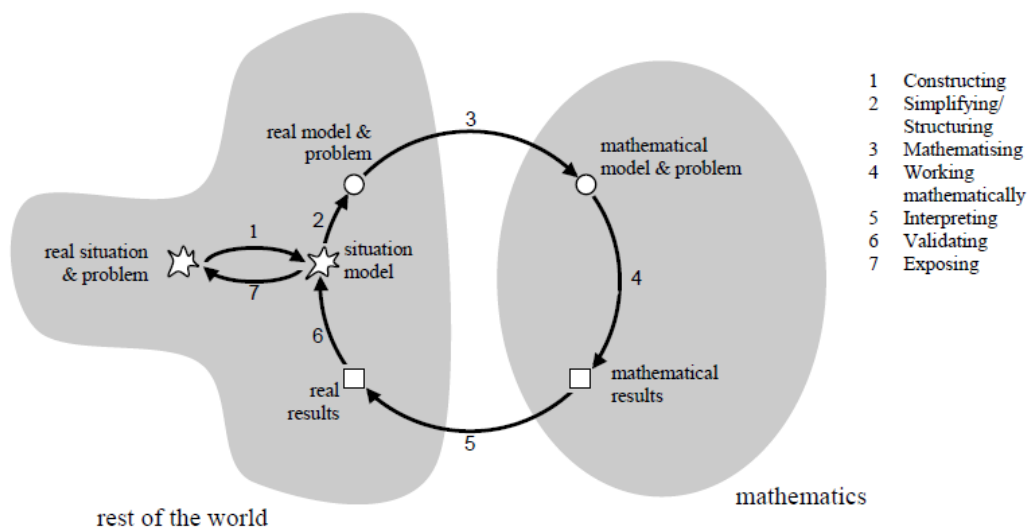
These arguments for including mathematical modeling in mathematics curriculum provide further detail on the spectrum of goals assigned to mathematical modeling.

The Mathematical Modeling Cycle

It follows that mathematical modeling cycles have been developed in order to serve the goals and arguments for modeling held by various researchers. Though not universal, the modeling cycle below in Figure 2 (Blum & Leiß, 2007) is commonly used to represent the mathematical modeling process.

Figure 2

Modeling Process (Blum & Leiß, 2007)



It is composed of both a cyclical diagram and seven steps which represent the cognitive processes of the modeler/student as they engage in the modeling process and move the modeling task from the real world to the realm of mathematics, and then back into the real world. The diagram displays the path of a modeling problem as it begins in the real world and through the

cognitive processes of constructing, simplifying, and mathematicising the problem enters the realm of mathematics; it is in this realm that a mathematical model is constructed and used to approach the problem. In order for the problem to be returned to the real world the modeler/student must interpret the meaning of the mathematical solution in a real-world context and validate that the model was appropriate. In the event the model is deemed inappropriate the modeler/student begins a new cycle with additional context to inform the development of a new model. Based on their goals, researchers may be focused on particular components of this process, it is used by researchers with a variety of goals including, the development of mathematical modeling competencies, the development of mathematics content and competencies, and examinations of the cognitive processes required in the use of mathematical modeling competencies.

Conceptual Framework

Description of Perspectives of Mathematical Modeling

The research purpose of exploring the perspectives of mathematical modeling held by high school mathematics teachers is both massive and uncertain without a clearly defined conception of the term “perspectives of mathematical modeling.” Kaiser and Sriraman (2006) propose a classification system to support the understanding of the perspectives of mathematical modeling represented in the current international body of research on mathematical modeling. In this classification system perspectives are distinguished by the goals or purpose assigned to mathematical modeling, the description of a mathematical model and the modeling process, and lastly by the design of instructional tasks determined appropriate for mathematical modeling. It is this conception of “perspectives of mathematical modeling” which frames this study and guides the research question and sub-questions. The research questions and sub-questions are in direct

alignment with this conception as they are purposed to explore mathematics teacher perspectives of mathematical modeling through the goals high school teachers assign to mathematical modeling, the ways in which high school mathematics teachers describe both a mathematical model and the mathematical modeling process, and the tasks these teachers select to engage students in mathematical modeling.

Classification of Perspectives of Mathematical Modeling

Kaiser and Sriraman (2006) define a classification system composed of five broad perspectives of mathematical modeling and one meta-perspective: (a) realistic modeling, (b) contextual modeling, (c) educational modeling, (d) socio-critical modeling, (e) epistemological modeling, and (f) cognitive modeling.

Realistic Modeling

The realistic modeling perspective is grounded in the work of Pollak (2003) and the idea that the goal of mathematical modeling is to understand the real world by engaging in the process of finding the solutions to real world problems. These real-world problems require the application of mathematics (Kaiser and Sriraman, 2006). According to Blomhøj (2009) “it is essential that the students work with realistic and authentic real-life modeling” (p.3), to emphasize this critical characteristic the realistic perspective of mathematical modeling requires that students engage with problems which are rich with the complexity of the real world and that demand to be approached through the modeling cycle. In this perspective, problems begin in the real world, are approached through the modeling cycle, and then are returned to the real-world context; through this process students are developing mathematical competencies which support a greater understanding of and engagement with the real world.

Given the complexity that exists in authentic real-world problems there is not a single mathematical modeling process or cycle assigned to the realistic modeling perspective. Instead of a defined modeling process or cycle, Pollak (2003) asserted that the distinguishing characteristics between mathematical modeling and other types of applied mathematics is that problems begin outside of mathematics, move to mathematical representations, and must be reconciled in the real world. Pollak (2003) described this process in eight steps:

- i. The identification of a question from the real world which needs to be understood
- ii. The identification of the important factors in the real-world scenario and the relationships that exist between them
- iii. The determination of the most important aspects and relationships in order to idealize the real-world scenario
- iv. The translation of the real-world scenario into a mathematical description, or mathematical model
- v. The identification of the relevant mathematics to work with in the mathematical model
- vi. The application of the relevant mathematics in order to obtain a mathematical solution
- vii. The translation of the mathematical solution back into the real-world, which results in a theory
- viii. The examination of the theory within the real-world in order to determine if it is appropriate

The modeling cycle presented earlier in Figure 2 (Blum & Leiß, 2007) was developed based on the work of Pollak (2003), it transforms the eight steps into a cycle with six locations and aligns seven steps to describe the cognitive processes required to move throughout the cycle. In contrast, Blum and Ferri (2009) assert that the application of the seven-step modeling cycle is not

appropriate for work with students, instead they propose an abbreviated four step process to support students in the application of the modeling process to solve problems. They argue that their four-step process better supports students receiving explicit instruction around the modeling process and a multitude of experiences which require the application of that process in order to support students in the development of “modeling competencies.” From this focus it follows that research conducted under the realistic perspective is typically focused on the development of those “modeling competencies,” which Blum and Ferri (2009) defined “as the ability to construct models by carrying out those various steps appropriately as well as to analyse or compare given models” (p.47).

Contextual Modeling

Kaiser and Sriraman (2006) align psychological goals, the idea that engaging in modeling will increase student motivation and attitudes towards mathematics subject-related goals, as well as the teaching of new mathematics concepts and structures to the contextual modeling perspective. Julie and Mudaly (2007) write that two central goals dominate the research base on mathematical modeling, “modeling as a vehicle” is the term they use to represent subject matter goals and “modeling as content” is used to represent goals assigned with teaching students modeling competencies. Contextual modeling embraces both of these goals simultaneously and is often confused with problem solving. Like problem solving, the conceptual perspective calls for mathematical modeling to serve as an approach to engage students in the learning and application of new mathematics. The distinction is that not all problem-solving situations require mathematical modeling. The Models and Modeling Perspective (MMP) was developed based on these dual goals and MMP is often used interchangeably with the term “contextual modeling.”

MMP begins with two basic assumptions regarding the nature of models: (a) people interpret their experiences using models, (b) these models consist of conceptual systems that are expressed using a variety of interacting media (concrete materials, written symbols, spoken language) for constructing, describing, explaining, manipulating or controlling systems that occur in the world (Lesh & Doer, 2003, p.536).

The idea that a model is a conceptual system or representation of a specific situation differs from the realistic perspective where a model was intended to be a mathematical representation of a real-world situation. The realistic perspective and MMP both assert that the modeling process begins in the real world. Lesh and Doer's (2003) modeling process is prevalent in MMP and this process begins with the description of the real-world situation, creating a mapping to a conceptual system or model which is then manipulated in order to generate predictions which must be verified in the real-world, this process often requires multiple iterations in order to ensure the predictions are applicable and appropriate from the real-world situation.

In order to ensure alignment between the goals, description of a mathematical model, and the mathematical modeling process of the MMP, model eliciting activities (MEAs) are a critical component of this perspective. MEAs are specifically designed activities to ensure the development of mathematical models which serve as conceptual systems or tools. Six principles guide the development for MEAs: model construction, reality, self-assessment, model documentation, reusability, and effective prototype (Lesh et al., 2000). The model construction principle requires that students be able to explicitly describe not the real-world situation but the process which led to the development of their mathematical model. The reality principle refers to the setting of the activity in the real-world. These problems, unlike the problems presented in the

realistic perspective, do not have to be authentic problems that occur in the real world. The real-world context breeds opportunities for students to self-assess throughout the modeling process. The model documentation principle simply requires that students be able to chart or document their work. Through the reusability and effective prototype principles MEAs lead students towards generalizations and models which can be used to address similar problems involving significant mathematical concepts.

Given that MEAs are central to the MMP, the perspective is often confused with traditional or applied problem solving. Mathematical modeling requires the competencies of traditional problem solving but the term problem solving does not fully represent the breadth and depth of activities designed to support students in the development of conceptual tools that can be used to simplify more complex problems. Lesh and Yoon (2007) simply offer that the distinction between problem solving and MEAs is that “Rather than being interested in ‘problem solving’ for its own sake, models and modeling perspectives are interested in the development of meaning and usefulness for powerful mathematical concepts or conceptual systems” (p.166). In alignment with that assertion, it follows that research conducted under the contextual perspective is focused on uncovering student thinking as they engage with those powerful concepts and develop conceptual systems (Doer & Lesh, 2011).

Educational Modeling

Educational modeling is centered almost equally around subject-related and pedagogical goals, creating a perspective that encourages the use of a modeling cycle and the development of modeling competencies to introduce and develop mathematics content (Kaiser and Sriraman, 2006). Since this perspective is focused on students developing modeling competencies it has strong connections to the realistic perspective, the two perspectives differ in the role or purpose

of mathematics learning and the design of modeling tasks. Blum and Niss (1991) write that there are two prevailing counterarguments to critics who argue that applied mathematics and modeling should not be explicitly included in K-12 mathematics curriculum; first that mathematics now has an essential function and role in the daily lives of people and secondly that mastery in mathematics cannot be obtained absent of the ability to apply mathematics processes, i.e., modeling competencies. The dual focus on the development of mathematics competencies and mathematics content development creates a necessity for modeling tasks which lend themselves to the development of specific mathematics concepts and for that reason those tasks while placed in a real-world context cannot be classified as “authentic” but are instead an “idealized” version of the real-world problems that live within the realistic perspective. With its dual purposes research conducted under the educational perspective is concerned with the development of modeling competencies, teachers’ understandings of mathematics or mathematical modeling, student learning of mathematics content, or a combination of any of the preceding goals.

Socio-critical Modeling

The socio-critical perspective is concerned with the application of mathematics in order to serve an emancipatory purpose and support students in the development of a critical lens in order to understand their world (Kaiser and Sriraman, 2006). This perspective includes a minute focus on mathematics content development but finds its true purpose in developing students’ competencies around critical citizenship. Underpinning this perspective is an assumption that “The extensive use of mathematical modeling in society contributes to establishing mathematics as a language of power” (Blomhøj, 2009, p.11). Engaging students in modeling therefore empowers them to impact the world in which they live. Barbosa (2006) described that under this perspective:

In specific terms, I have established the boundaries of modeling as a learning milieu where students are invited to take a problem and investigate it with reference to reality via mathematics. This notion is quite removed from the characterization of modelling as involving diagrammatic representations. It refers to modeling as a school activity, which may be informed by a pragmatic, scientific or socio-critical perspective. (p. 294)

The tasks used to engage students in this perspective are authentic and specific to student contexts as such there is no prescribed modeling cycle or process and thus in contrast to the realistic perspective there is no focus on the development of mathematical modeling competencies. Research conducted under this perspective investigates students as critical citizens, their abilities to understand and critique the power assigned to mathematics in society (Blomhøj, 2009).

Epistemological Modeling

The last of the five perspectives holds closely to theory related goals in the teaching and learning of mathematics, specifically how people develop relationships between mathematics and the real world (Kaiser and Sriraman, 2006). These theory related goals are not explicitly connected to the “modeling as content” or the “modeling as a vehicle” approaches.

Epistemological modeling differs from the other perspectives in its assumption that modeling is not solely a relationship between the real world and mathematics, this perspective instead also accepts that a mathematical model can also represent the relationship between concepts within mathematics, labeled as “intra-mathematical modeling” (Kaiser and Sriraman, 2006). Since research conducted under this perspective is centered on the teaching and learning of mathematics it is fueled by a theory of how that learning occurs. There are two prominent

theories within this perspective Realistic Mathematics Education theory (RME) and the anthropological theory of didactics (Kaiser and Sriraman, 2006).

Meta-perspective Cognitive Modeling

Kaiser and Sriraman (2006) distinguished cognitive modeling from the other five perspectives and labeled it a meta-perspective due to an overwhelming overlap with the other perspectives; additionally, they asserted that research under only this perspective is relatively new. As the name suggests research conducted under the cognitive perspective is concerned with examining the cognitive processes of the individual during the modeling process. Blomhøj (2009) connected research conducted under the cognitive meta- perspective to educational modeling and the development of modeling competencies but distinguishes the two by referring to one as “basic research” and the latter as an “applied science.” It could be argued that the connection between the cognitive meta-perspective and the educational perspective is no different than the overlap that exists between many of the other perspectives. This argument is the basis for a stance held by Cai et al. (2014) that the cognitive meta-perspective be given equal billing to the five perspectives.

Summary of the Five Perspectives and One Meta-Perspective

A summary of the distinguishing characteristics of the five perspectives and one meta-perspective is shown below in Table 2.

Table 2*Characteristics of the Perspectives of Mathematical Modeling*

Name	Goals	Modeling Process/Cycle	Tasks	Prominent Researchers
Realistic Modeling	The development of mathematical modeling competencies	Pollak's 8 steps Blum and Leiß's 7- step cycle	Authentic real-world tasks	Pollak, Blum, Ferri
Contextual Modeling	The development of mathematics content presented in context	MMP modeling cycle	Model eliciting activities (MEAs)	Lesh, Doerr
Educational Modeling	The development of mathematics content and mathematical modeling competencies	Blum and Leiß's 7- step cycle Blum and Ferri's 4 step process for students	Can be authentic but they are often simplified to meet mathematical goals	Niss, Blum
Socio-critical Modeling	The application of mathematics to support the development of critical citizenship	Process and cycle are undefined	Real world tasks that impact the learner, often connected to mathematical goals	D'Ambrosio, Barbosa
Epistemological Modeling	The development of mathematical knowledge	Expansive definition of the term model prevents a defined process or model	Not specified	Freudenthal
Cognitive Modeling	Individual cognitive processes while engaged in modeling	Blum and Leiß's 7- step cycle	Can be authentic but they are often simplified to meet mathematical goals	Ferri

While classified into six categories these perspectives are very closely related. Educational modeling is situated on a balance beam between realistic modeling, focused on the development

of modeling competencies and epistemological modeling with its goals purely centered about the development of mathematics content. Both cognitive and contextual modeling place mathematics within a context in order to promote the content, while the socio-critical perspective seeks to promote content as well as develop participatory citizens. In the case of each perspective, the one meta-perspective modeling tasks are directly influenced by the goals of the perspective and the research aims of the prominent scientists in the field. The width and depth of the research bases on mathematical modeling supports the importance of mathematical modeling in mathematics education research. Additionally, the relationship between perspectives, goals, and tasks supports the exploration of the perspectives of mathematical modeling held by high school mathematics teachers.

Perspectives of Mathematical Modeling in the CCSSM

Absent from Kaiser and Sriraman's perspectives of mathematical modeling is a mention of the CCSSM, this absence is due to the classification system being developed prior to the writing and adoption of the standards. The high school CCSSM define mathematical modeling as a conceptual category intended to be conceived through a relationship to a specified set of content standards and the real world; the standards include a modeling cycle to guide this process (NGACBP & CCSSO, 2010). The standards however do not include a reference to the research base that frames this view of modeling or the modeling cycle. Without this reference we can only lean on the goals of mathematical modeling, the description of a mathematical model and the mathematical modeling process directly from the standards documents; in order to determine the perspective of mathematical modeling referenced in the standards.

Common Core State Standards Writing Team (2013) clearly states that the goal of mathematical modeling in the High School CCSSM is to support students in understanding the

world and to support students in understanding the importance of mathematics and its usefulness in the world. The *CCSSM Progressions for Modeling* (2013), specifies that for the purposes of the standards a model can be thought of as a noun or a verb; some distinction between cases is assigned based on the ages of students which informs the complexity of the model. For example, in early grades a model may be limited to a pictorial representation of a numerical expression but as students move to middle school, they begin to use linear graphs and statistical software. Within the high school standards, the term “model” is used interchangeably to describe a fixed representation and the process of mathematical modeling. This lack of clarity within the standards is problematic for the implementation of the standards in classrooms.

The Common Core Standards Writing Team (2013) asserted that there are a variety of process and cycles which are used for mathematical modeling, and they present the Lesh and Doer (2003) cycle from the contextual perspective as an “example” of a modeling cycle concerned with the reasoning processes the modeler/student engages in. This example of a modeling cycle was not adopted as the representation of mathematical modeling in the CCSSM. Instead, the CCSSM (2010) includes the mathematical modeling cycle described below:

Basic modeling cycle is summarized in the diagram. It involves (i) identifying variables in the situation and selecting those that represent essential features, (ii) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (iii) analyzing and performing operations on these relationships to draw conclusions, (iv) interpreting the results of the mathematics in terms of the original situation, (v) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is

acceptable, (vi) reporting on the conclusions and the reasoning behind them. (NGACBP & CCSSO, 2010, p.72)

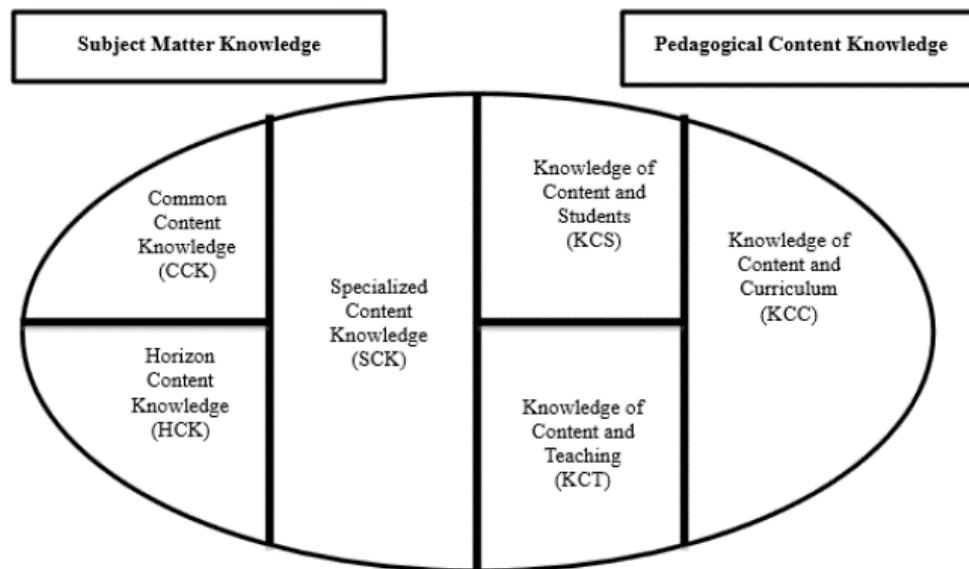
Given the goal of connecting mathematics content and the real world through the application of a modeling cycle one could infer that the CCSSM's conception of mathematical modeling lives within the contextual or educational perspective. However, the standards do not include a set of criteria for modeling tasks; we can omit contextual modeling from our schema and conclude the intent is aligned with the educational perspective. Contrary to this conclusion one could refer to the claim in the standards that "Models can also shed light on mathematical structures themselves..." (NGACBP & CCSSO, 2010, pg. 72) and make an argument for an epistemological perspective. The ambiguity in the perspective or perspectives which ground the CCSSM leaves us unable to make a direct connection between the existing body of research around mathematical modeling and serves as a rationale for studies which explore the relationship between the CCSSM and the larger body of research on mathematical modeling.

Teachers' Perspectives of Mathematical Modeling

Ball et al. (2008) offered the construct of Mathematical Knowledge for Teachers (MKT) as a content specific refinement to Shulman's (1987) Pedagogical Content Knowledge (PCK). The term MKT is defined as follows: "By this phrase, we mean the *mathematical knowledge that teachers need to carry out their work as teachers of mathematics*" (Ball et al, 2008, p.4). Figure 3 displays a model of MKT, and it is important to note that all concepts involving content are specific to the grade level and or course in which the teacher is responsible for planning and providing instruction.

Figure 3

Model of Teachers' Mathematical Knowledge for Teaching (Ball et al., 2008, p.403)



Borromeo Ferri and Blum (2009) involved 25 pre-service teachers in a 14-week college level course on mathematical modeling, as a result of their work with pre-service teachers they have determined that there are four dimensions of MKT which are specifically required among teachers of mathematical modeling:

- (a) Theoretical competency (knowledge about modeling cycles, about goals and perspectives for modeling, and about types of modelling tasks),
- (b) Task related competency (ability to solve, analyse, and create modelling tasks),
- (c) Teaching competency (ability to plan and perform modelling lessons and knowledge of appropriate interventions during the pupil's modelling processes),
- (d) Diagnostic competency (ability to identify phases in pupils' modelling processes and to diagnose pupils' difficulties during such processes). (p. 2047)

Kaiser and Sriraman's (2006) conception of "perspective of mathematical modeling" is composed of (a) goals of mathematical modeling, (b) description of a mathematical model and mathematical modeling, and (c) task design. Using this conception for this study, is supported by the alignment between the conception perspectives of mathematical modeling and the description of the theoretical competency dimension of MKT for mathematical modeling which includes knowledge around the goals of mathematical modeling, mathematical modeling cycles, and mathematical modeling tasks.

In recent years there have been a number of studies conducted under the realistic, contextual, and educational perspectives of mathematical modeling which focus on the engagement of pre-service and/or in-service teachers in the learning of mathematical competencies, or the learning of mathematics through mathematical modeling. However, explorations of teachers' (both preservice and in-service) perspectives of mathematical modeling are scarcer in the body of research. The adoption of Kaiser and Sriraman's (2006) conception of perspectives of mathematical modeling left me unable to locate studies which explored the goals teachers assigned to mathematical modeling, the ways in which teachers describe a mathematical model and the modeling process, and the design of tasks teachers use to engage students in mathematical modeling, the review below addresses some of the major work around each component.

Research on Teachers' Descriptions of the Goals of Mathematical Modeling

Paramount to a person's perspective on any construct is the purpose or goal they assign to it; in considering teachers' perspectives of mathematical modeling, one would expect that at the surface a part of the goal lies in the inclusion of mathematical modeling in the CCSSM. Anhalt and Cortez (2015) conducted a qualitative case study with 11 preservice secondary mathematics

teachers centered specifically on their understanding of mathematical modeling as defined as a high school conceptual category in the CCSSM during a graduate level course. Through a collaborative examination of the CCSSM standards and their engagement in mathematical modeling activities the preservice teachers expressed that a primary purpose of the mathematical modeling conceptual category was to make a connection between the CCSSM content standards and the SMPs, specifically SMP 1 (Make sense of problems and persevere in solving them), SMP 2 (Reason abstractly and quantitatively), and SMP 6 (Attend to precision). From the assignment of a dual purpose of mathematics content and competencies, it can be inferred that the participants in this study lean towards an educational perspective of mathematical modeling. This inference cannot be confirmed though because while Anhalt and Cortez explored teachers' definitions of mathematical modeling, they did not include work around their conceptions of mathematical modeling tasks.

Also centered around teachers' conceptions of mathematical modeling in the context of the CCSSM and its purpose, Gould (2013) found in her mixed methods study that preservice and in-service mathematics teachers assigned the following goals to mathematical modeling: (a) students learning to use mathematics in their daily lives, (b) student application of mathematics, (c) students learning to "think mathematically," and (d) students' exploration and understanding of phenomena in other disciplines.

Akgum (2015), Bautista et al. (2014), and Girant and Eichler (2011) all studied the goals teachers assigned to mathematical modeling outside of the context of the CCSSM and found that teachers strongly believe that one of the major goals of mathematical modeling is to connect mathematics to the real world and to provide applications for mathematics content. The in-service secondary teachers in the Girant and Eichler (2011) case study asserted that there is a relationship

between the mathematical content goals and the appropriateness of mathematical modeling. Specifically, they felt that there was not a place for mathematical modeling in an introductory geometry course. Though not explicitly mentioned in either of the three studies these findings suggest that these teachers assign mathematical modeling goals which are aligned to the contextual and epistemological perspectives.

Research on Teachers' Descriptions of Mathematical Models and Modeling

An additional focus of Gould (2013) was the ways in which both in-service and preservice teachers describe mathematical models and the mathematical modeling process. In order to collect this information, the researcher employed quantitative methods through a 20-question online survey which was completed by 274 in-service and preservice teachers from 35 states and one U.S. territory. The survey was created by Gould for the purpose of this study and there was no information included as to how reliability was established. Participants responded to six statements developed to uncover their conceptions and misconceptions of a mathematical model and eight statements with the same purpose around the mathematical modeling process. The findings support that teachers and teacher candidates equate the term “mathematical model” with visual models, physical objects such as manipulatives, and other representations. Additionally, the participants saw traditional problem solving and mathematical modeling as interchangeable and rejected the notion that mathematical modeling required making choices, making assumptions, or multiple iterations.

Bautista et al. (2014) also found a strong connection between teachers' conceptions of mathematical models and multiple representations in the study of 56 US in-service mathematics teachers, grades five through nine. The teachers in the study were presented the scenario of Famous Amos and his cricket thermometer displayed as a set of data points, a scatter plot, a line of

best fit, in tabular form, and as an equation which was identified as the “model” within the context of the scenario. Despite this identification 64% of the participants declared that the line of best fit was the most appropriate model. The researchers found distinctions among teacher views of mathematical modeling based on their educational backgrounds; among teachers with mathematics and mathematics education backgrounds there tended to be a view that there was a single model for scenarios and that model was an idealization of the real world. Teachers with educational backgrounds in the natural sciences and technology held a broader view of mathematical models and cleaved to the idea that multiple models were possible for a given scenario and thus should be considered and explored.

In contrast to the data collection methods in the two preceding studies Anhalt and Cortez (2015) collected survey data from preservice teachers on two occasions, at the start, and then again at the conclusion of a modeling module in a graduate course in an effort to collect data around the evolution of their conceptions of mathematical modeling. The data collected from the initial questionnaire reflected the findings of Bautista et al. (2014) and Gould (2013) where participants described models as visual models and physical representations and saw mathematical modeling as problem solving. The results collected after engaging in the modeling process demonstrated what the researchers described as a reasonably accurate understanding of mathematical modeling and reflect that the distinction between modeling and problem solving is the process of making assumptions, attempting to validate the model in a real-world context, and engaging in multiple iterations as needed.

Research on Teachers’ Descriptions of Mathematical Modeling Tasks

The contextual perspective provides a very robust criteria for tasks which support mathematical modeling through six principles of MEAs: (i) model construction, (ii) reality, (iii)

self-assessment, (iv) model documentation, (v) reusability, and (vi) effective prototype (Lesh et al, 2000). From this it follows that there is a body of research around the views of MEAs held by both in-service and pre-service teachers. Yu and Chang (2011) used quantitative methods to investigate the views of MEAs held by secondary mathematics teachers in Taiwan and found that teachers regarded the connection between MEAs and the real-world as a strength to support both students' learning of mathematics and the development of modeling competencies. However, the secondary mathematics teachers in this study believed MEAs to be outside of the school mathematics curriculum and applicable as supplements to the curriculum.

Altay et al. (2013) and Thomas and Hart (2010) used qualitative methods to explore the views of MEAs held by elementary pre-service teachers in the context of professional learning intended to improve their modeling competence. In the more recent study, elementary pre-service teachers were found to hold positive views around MEAs and the connections they made between the real world and mathematics. Teachers described MEAs as “activities including real life problem situations they face in real life” (Altay et al., 2013, p.347), which allowed for multiple solution paths. The Thomas and Hart study also found that preservice elementary teachers held positive views of MEAs and saw them as a way to make connections between mathematics and the real world. The study centered on preservice teachers' perceptions of MEAs, specifically the use of MEAs to help teachers learn mathematics, the role of MEAs in developing teacher modeling competencies, and the use of MEAs with students with disabilities. Participants communicated that there is a degree of ambiguity involved with MEAs which allowed them to accept ownership and generate their own mathematical ideas with multiple solution paths. While the teachers saw this ownership as a strength to facilitate their own learning, they believed that students, particularly students with disability would require more

guidance and direction. In each of the preceding studies teachers completed MEAs and were asked about their learning experiences and the implementation of MEAs in the classroom setting, all presented MEAs as the vehicle for mathematical modeling. That focus led to findings which included a limited attention to teachers' descriptions of mathematical modeling tasks.

Kuntze (2011) characterized teachers' views of modeling tasks as the exploration of which types of tasks teachers viewed as most representative of mathematical modeling. He engaged 230 pre-service and 79 in-service teachers in a quantitative study which employed a self-created survey instrument using a four-point Likert scale to distinguish tasks with lower to higher modeling requirements; the reliability of the survey instrument was established through positive Cronbach scores. In this study tasks with low modeling requirements consisted of tasks where there was not a requirement that the learner translate between the mathematical and the real-world because the model was provided and there was a single solution, on the opposite end of the spectrum tasks with higher modeling requirements include "tasks that require at least one translation step between a given situational context and a mathematical model, and that allow different solutions" (Kuntze, 2011, p. 280). The findings of this work reported that while in-service teachers showed a preference for tasks with higher modeling requirements their pre-service counterparts saw greater learning opportunities in tasks with lower modeling requirements.

Summary

The lack of consensus around the description of a mathematical model or the mathematical modeling process in the research base necessitated the development of Kaiser and Sriraman's (2006) perspectives of mathematical modeling. There is not a direct connection made between the standards and the body of research which underpin their development, this

combined with the lack of clarity around the description of a mathematical model within the standards requires teachers to lean on their perspectives of mathematical modeling to guide instruction. The use of Kaiser and Sriraman's conception to frame this exploration of teachers' perspectives of mathematical modeling is supported by the theoretical competency dimension of MKT for mathematical modeling which also includes the goals of mathematical modeling, the modeling process, and types of modeling tasks. While this framework has not been widely applied in the research on perspectives held by teachers, findings which include components of the framework suggests that further study could have implications for curriculum writers and professional development.

3 METHODOLOGY

Mathematical Modeling is an explicit expectation of the CCSSM expressed as a Standard for Mathematical Practice: Model with Mathematics and as one of the six conceptual categories which frame the high school standards, Modeling. In this study, I ground the exploration of the perspectives of mathematical modeling held by Algebra II teachers with the research question, “In what ways are teachers’ perspectives of mathematical modeling connected to the ways in which they plan learning experiences for students?” Kaiser and Sriraman’s (2006) conception of “perspectives of mathematical modeling” serves as the conceptual framework for the study and was directly influential to the development of the following four sub-questions: (a) How do Algebra II teachers describe a mathematical model? (b) How do Algebra II teachers describe the mathematical modeling process? (c) What goals do Algebra II teachers assign to mathematical modeling? (d) How do Algebra II teachers select and implement tasks that support mathematical modeling?

This chapter provides a description of the methodological components of the study and is composed of five sections. The opening section is to provide a description and rationale for the methodological orientation. The next section provides the research context and information about the participants. The third and fourth sections discuss the techniques for data collection and then data analysis. The culminating section addresses credibility, including the role of the researcher, confidentiality and ethics, prospective limitations, and a summary.

Methodological Orientation

Qualitative Study Design

The research questions guiding this study are aligned with the purpose and intent of qualitative research. Qualitative research is interpretive research (Bogdan & Biklen, 2007; Cohen

et al., 2018; Creswell, 1994; Yin, 2011). The methodological design of any study should be driven by its declared purpose and research questions, in contrast to quantitative designs the qualitative researcher is not driven by proving or disproving a declared hypothesis (Creswell, 1994; Guba & Lincoln, 1994). Cohen et al. (2018) establish that within the field of education qualitative designs are applicable when the purpose is to describe, explain, explore, report, create new concepts, generate new theories, or to test an existing theory. Given the varied purposes of qualitative research designs and the variations in qualitative methods Yin (2011) establishes 5 features unique to qualitative research: (a) involves the study of meaning assigned by people in their lives, (b) represents the perspectives of people, (c) incorporates the context of people's lives, (d) contributes insight to existing or emerging concepts that may help to explain human behavior, and (e) includes multiple sources of data.

This study employs a qualitative study design since its purpose is to explore the perspectives of mathematical modeling held by Algebra II teachers through the connections between their descriptions of mathematical modeling, the modeling process, modeling tasks, the purpose of mathematical modeling, and the ways in which they plan learning experiences for students. In addition to aligning with the purpose of qualitative design, there is evidence of each of Yin's (2011) five features of qualitative research in this study's design. The participants' perspectives of mathematical modeling are intended to explore the meaning they assign to mathematical modeling within the context of the Algebra II course. The study design includes data collected from a survey, two interviews, and documents in the form of a modeling task and teacher exemplar intended to provide insight on lesson planning and implementation assigned to mathematical modeling instruction.

Case Study

Yin (2011) categorizes case study as an “illustrative variation” of qualitative research that “Studies a phenomenon (the “case”) in its real-world context.” (p. 17). As a variation of qualitative research case studies seek to describe, explore, or explain and according to Yin (2018) have three distinctive features:

- copes with the technically distinctive situation in which there will be many more variables of interest than data points, and as one result
- benefits from the prior development of theoretical propositions to guide design, data collection, and analysis, and as another result
- relies on multiple sources of evidence, with data needing to converge in a triangulating fashion. (p.15)

These features help to distinguish case studies within the larger field of qualitative research.

When determining whether case study is an appropriate research method Yin (2018) suggests that I, as the researcher consider three conditions:

1. Research questions that specifically seek to explore how and why. In this study, I explore the perspectives Algebra II teachers hold about mathematical modeling, how they describe a model, the modeling process, modeling tasks, and how those descriptions are connected to the ways in which they plan for student learning experiences.
2. Research that does not require control over the behavior of participants. In this study, I explore the perspectives of teachers within the context of the Algebra II course; the context of the course requires participants to engage in thinking and planning around mathematical modeling.

3. The focus of the study is contemporary and not a historical event or phenomenon.

The study meets this condition; it aims to explore the perspectives of mathematical modeling held by practicing Algebra II teachers.

A critical component in case study design is clearly defining and creating a boundary for the case (Yin, 2018). In this study, the case is defined as the perspective of mathematical modeling held by each research participant. The term “perspective” is both elusive and expansive making it a difficult subject to explore through a single study without a set of conditions or boundaries. Kaiser and Sriraman’s (2006) conception of “perspectives of mathematical modeling” establishes a boundary for the study; this conception includes the description of- a mathematical model, the modeling process, and modeling tasks; as well as the goals assigned to mathematical modeling instruction.

This study includes five participants, each representing a single case, and employs multiple-case study procedures. Limiting the number of participants to five teachers ensures that the amount of data that was collected and analyzed is manageable. Multiple case study procedures require that data be analyzed in at least two cycles, first to write an individual case study report for each case, and then to draw cross-case conclusions, make theory modifications, and then write a cross-case report (Yin, 2018).

Research Context and Participants

Research Context

The CCSSM explicitly require mathematical modeling as a focus for teaching and learning through Standard for Mathematical Practice 4: Model with Mathematics for students in grades K-12, and as a one of six conceptual categories for high school mathematics. The number of high school standards dictates that domains be organized into conceptual categories or

conceptual themes, with the exception of modeling each conceptual category is composed of an overview and a discrete set of content standards. Distinctly, as a conceptual category modeling is composed of an overview, a modeling cycle, and an expectation that modeling is connected to the other conceptual categories. This connection is denoted with an “*” to express the relationship between modeling and the content standards with the other five conceptual categories: Number and Quantity, Algebra, Functions, Geometry, and Statistics and Probability (NGACBP & CCSSO, 2010).

The Algebra II course is labeled a culminating course in a sequence of three required mathematics courses. Modeling as a conceptual category is required throughout the course sequence which sets an expectation that students have been engaged in modeling in two courses before Algebra II. The course description incorporates the CCSSM modeling cycle, and six of seven curricular units include standards denoted with an * and therefore directly connected to the modeling conceptual category (Georgia Department of Education, 2015). While not required the Georgia Department of Education (2015), provides a curriculum framework for each unit of study which includes mathematics tasks suggested to support the full intent of the CCSSM content standards and the standards for mathematical practice. This study was focused on the perspectives of mathematical modeling held by Algebra II teachers because as a third-year course there is an expectation that students have experienced mathematics content across multiple domains inclusive of algebra, geometry, and statistics along with modeling prior to beginning the Algebra II course.

Participant Selection and Description

Given the context of the course, participants were recruited from several school districts in a large metropolitan area in the southeastern United States. Research participants were all

teachers of at least one section of Algebra II during the spring semester of 2020. A call for potential participants was distributed via email to professional contacts including a P-12 mathematics organization which is an affiliate of National Council of Teachers of Mathematics (NCTM) affiliate and social media. Yin (2011) asserts that purposive sampling is an approach in qualitative research which allows the researcher to narrow the focus based on the research purpose to increase and enrich the data collected. I employed purposive sampling to seek and select research participants who: (a) had 5 or more years of experience teaching high school mathematics, (b) had at least 2 years of experience teaching Algebra II, and (c) were responsible for teaching the Algebra II at the time of data collection. The aforementioned criteria resulted in my appeal for participant referral being extended from the fall of 2019 and into the spring of 2020. As I received referrals of potential participants, I contacted them via email and included a short questionnaire to screen for the criteria. The only criteria that participants were aware of was that they were Algebra II teachers. Chapter four includes a description of each of the five research participants.

Plan for Data Collection and Management

Data collection plays a critical role in all research, and it is essential that the data collection methods chosen by a researcher are consistent with both the research purpose and methodology. Yin (2018) asserts that 4 principles should guide the collection of data for case studies: (a) the use of data from multiple sources, (b) the existence of a research database which separates data collected from the researchers' reports of that data, (c) the critical need to maintain a chain of evidence and maintain a visible connection between the research purpose, questions, data collected, and reported findings, and (d) the requirement that data collected from social media sources, inclusive of web-based interviews be handled with great care and caution. The

multiple sources of data for case study research typically include but are not limited to archival records, documents, direct observation, interviews, participant observation, and physical artifacts (Cohen et al., 2018; Yin, 2018). In this study, I data were collected from an initial contextual survey, two interviews, and documents in the form of a modeling task and teacher exemplar intended to provide insight on lesson planning and implementation assigned to mathematical modeling instruction.

It is important to note that while surveys traditionally serve as a primary data source in quantitative research, quantitative data can be useful in qualitative research, specifically in supporting descriptions and suggesting trends present in a setting or context (Bogdan & Biklen, 2007; Miles et al., 2014;). The survey instrument for this study is composed of questions regarding the context or background of the participants, with special attention given to information about their level and area of education and teacher certification. This information alone is not directly connected to the research questions, the data collected from the survey was used to enrich the descriptions of the participants and cross-case trends among participants. The survey instrument did not serve as the only data source around the educational background of participants; their educational experiences, specifically those involving mathematical modeling were also discussed during the interviews and thus included in the interview protocol (Appendix C).

Yin (2018) writes that “One of the most important sources of case study evidence is the interview” (p.118). In seeking to understand the perspectives of mathematical modeling held by mathematics teachers it is essential that their voices be heard. The purposive sampling of teachers with experience teaching Algebra II was intended to support their ability to speak about their lived experiences, understandings, and perspectives of mathematical modeling. Roulston

(2010) categorizes interviewing under these conditions as phenomenological interviewing and advises that multiple interviews are needed to fully garner data aligned to these goals. I engaged each participant in two, 75-minute interviews, each one in alignment with the interview protocols included in Appendix C. The intent of the first interview was to explore teachers' descriptions of mathematical modeling and the mathematical modeling process. Ahead of the second interview participants were asked to bring a task and a teacher exemplar to support an exploration of the tasks they align with mathematical modeling and the ways in which their descriptions of a mathematical model and the modeling process are evident in the ways in which they plan to engage students. When I communicated the request for artifacts to participants, I defined a "teacher exemplar" as an example of a successful engagement with the task that the teacher created and/or modified from an instructional resource. Jacob and Ferguson (2012) suggest that interview protocols serve as a "procedural guide" for conducting interviews and their application along with the appropriate interview setting support the effective collection of data from interviews.

Prior (2003) asserts that the discussion around what constitutes a "document" in qualitative research is expansive and that the discussion can be simplified if we remember that a document is a product. The strengths of using documents in case study research include that they are stable, specific, and unobtrusive, meaning that they exist absent from the case study (Yin, 2018). Documents were collected in the form of a mathematical modeling task and a teacher exemplar of that mathematical modeling task. Additionally, participants were encouraged to bring lesson and/or unit plans aligned to the selected tasks. These documents were a component of the interview protocol for the second interview and were intended to broaden the data set to include: a sample of how teachers engage in the mathematical modeling process, a concrete

example of the task's teachers align with mathematical modeling, and the ways in which teachers' descriptions of a mathematical model and the mathematical modeling process are evident in the ways in which they plan to engage students. A summary of the data collection methods and their connection to the research questions which guided my study is included below in Table 3.

Table 3

Summary of Data Collection Methods and their Connection to the Research Questions

Research Questions	Data/Evidence Collected
<p>Overarching Research Question: In what ways are teachers' perspectives of mathematical modeling connected to the ways in which they plan learning experiences for students?</p>	<p>Survey Interviews Task and Teacher Exemplar</p>
<p>Sub-questions:</p>	
<p>How do Algebra II teachers describe a mathematical model?</p>	<p>Interviews Task and Teacher Exemplar</p>
<p>How do Algebra II teachers describe the mathematical modeling process?</p>	<p>Interviews Task and Teacher Exemplar</p>
<p>What goals do Algebra II teachers assign to mathematical modeling?</p>	<p>Interviews Task and Teacher Exemplar</p>
<p>How do Algebra II teachers select and implement tasks that support mathematical modeling?</p>	<p>Interviews Task and Teacher Exemplar</p>

Procedures for Data Analysis

In describing qualitative data analysis Cohen, Manion, and Morrison (2018) write that: It includes, among other matters, organizing, describing, understanding, accounting for, and making

sense of data in terms of the participants' definitions of the situation (of which the researcher is one), noting patterns, themes, categories, and regularities, all of which are the tasks of the qualitative (p. 643)

In case studies the process of data analysis begins almost simultaneously with data collection, is ongoing, and includes the researcher's memos as a method of searching for emerging patterns, insights, and concepts (Yin, 2018). All of the participant interviews were audio-recorded so that I could be fully present in each interview where I took very limited notes. Immediately following each interview, I played the audio recording and captured my initial thoughts in my research journal. That first listen to each recorded interview was the beginning of data analysis for this study, those initial thoughts, and my first research memos. All audio recorded interviews were initially transcribed by a service and then I reviewed them against the original audio recordings to ensure they accurately represented the participants' shared experiences. I then engaged in multiple reads and reviews of the data from both interview transcripts and the documents to become familiar with the data for exploration, I continued to capture research memos in my journal during that engagement.

Once I became familiar and entrenched in the data, I began the process of coding. The research questions, conceptual framework, and primarily the data itself supported the development of codes. Roulston (2010) advises that researchers "stay close to the data" (p.152) when selecting and refining codes through the data analysis process. At its core coding, at each iteration is the act of pattern-seeking, looking for words, and/or phrases to support the development of broad categories which will launch the process of coding. Those patterns were then examined more deeply to support an emergence of perspectives, categorical, thematic, conceptual, and a theoretical organization of the data (Saldana, 2013). In order to fully

understand and describe the data set multiple methods and cycles of coding were used in accordance with the “Generic” coding methods recommended by Saldana (2013). I depended on data displays and organizational tools such as charts, tables, and poster size mappings to support the organization, summarization, and the identification of patterns within a data set (Kohn, 1997). In the analysis for each single-case and then the cross-case I used wall size nested tables to visually display the mapping between words, sentences, phrases, passages, and artifacts, to codes, groups, subgroups, and emergent themes. I would then record the tables and mapping in my journal, break down the life size pieces and repeat the process with fresh eyes to look for changes in my perception. I was careful to move from the coding methods described above to the initial stages of writing process for each participant case as a discrete process with the aim of exploring individual teacher perspectives. In each of those single cases my early coding groups were largely driven by the characteristics of perspectives of mathematical modeling which were the foundation for the study’s sub questions. As I moved thru the coding process I begin to notice that there were multiple places where my codes were beginning to overlap and where there were simultaneous codes and thus emergent themes developing from a single phrase or component of an artifact. Through the data analysis process, specifically as I begin the second cycle coding methods one of the sub-questions guiding this study shifted from “How do Algebra II teachers describe and implement tasks that support mathematical modeling” to “How do Algebra II teachers select and implement tasks that support mathematical modeling?”; this shift was motivated by the voices of participants, their values and beliefs visible in those simultaneous coded and nested subgroups. I have provided a summary of the first and second cycle coding methods I employed in Table 4.

Table 4*First and Second Cycle Coding Methods*

Coding Method	Description
First Cycle Coding Methods: Attribute Coding	Context coding used to begin analysis and organization of the data set to include data format (survey, interview transcript, participant artifacts, my field notes), participant characteristics, setting, time and duration.
Holistic Coding	Interview transcripts and participant artifacts were reviewed in chunks with the intent of beginning to uncover themes.
Descriptive Coding	Interview transcripts and participant artifacts were reviewed in passages and summarized with a word or phrase that captured the topic of each passage.
Values Coding	Interview transcripts, participant artifacts, and my field notes were examined in discrete parts or topics identified through my descriptive coding cycle and codes were applied to reflect participants' values, attitudes, and beliefs. The codes for each participant's values, attitudes, and beliefs were then compiled and categorized. I reflected on them collectively to begin to develop assertions.
Second Cycle Coding Methods: Eclectic Coding	Interview transcripts, participant artifacts, and my field notes were examined in discrete parts or topics identified through my descriptive coding cycle. I used a combination of first cycle coding methods (initial coding, emotional coding, and versus coding) to look for similarities and differences, label participants' emotions, and look for incidences of dichotomous or binary terms or relationships.
Pattern Coding	For each discrete topic I used inferential codes to holistically capture the emergent themes and construct final assertions by looking for patterns or groupings between the product of both values and eclectic coding.

The writing process for each single case and then the cross case analysis happened almost in tandem with the conclusion of the second cycling coding methods, beginning as emergent themes in the form of words, phrases, and then sentences in my research journal which captured the emergent themes and evidence from my large scale mappings. The products of that process, my findings, are presented in chapter four.

Confidentiality and Ethics

Role of the Researcher

As a mathematics educator I consciously entered this study with not only my own perspective of mathematical modeling but also with some assumptions of commonalities between my perspective and the perspective generally held by experienced teachers. Roulston (2010) speaks to the importance that researchers “critically examine their perspectives and assumptions about the key elements of the research project” (p.20). Additionally, she recommends the use of research journals during the data collection process and the inclusion of a subjectivity statement in the reporting of research. I leaned into the practice of reflective journaling during data collection, data analysis, and the process of writing the findings of this study. Capturing my reflections created an explicit awareness of my assumptions of subjectivities. During data collection, I engaged participants in two reflective interviews, one which included a discussion of selected tasks and a teacher exemplar. It was imperative that during this process I continuously examined and took note of my subjectivities. Though all participants electronically submitted their task and teacher exemplar ahead of time I did not review the files prior to our interview. I did this in order to ensure I did not enter the second interview with preconceptions regarding the task or its implementation.

It is important to note that all but one of the interviews for this study took place between March and May of 2020, in the midst of the COVID-19 Global Pandemic that left schools around the world physically closed and educators to adopt virtual instructional methods with little preparation time or training. Given this context, I took a great deal of care to ensure that all phone interviews took place at a time that was comfortable and convenient for each research participant. As you can imagine there were many instances where interviews had to be

rescheduled as teachers navigated a constantly changing landscape. I constantly and consistently communicated to participants that their physical and mental health, families, and students were the priority.

I journaled feverishly during data analysis and the writing process! Through that I became aware of how my personal unrest with research which centers around “teacher deficits” and “misconceptions” influenced the development of the purpose and research questions which guided this study. That unrest became an invaluable tool as I sought to ensure that the data analysis process remained centered in participant voices, descriptions, and eventually perspectives. In mapping between artifacts and interview transcripts, codes, code groups, categories, and the themes presented in chapter four I was able to regularly check for the presence of my assumptions and my perspectives. In my journaling I became conscious of how things like my scheduled work activities and personal interactions could impact and/or break the connections within those maps. I learned to refrain from data analysis or writing after a day spent observing classrooms or coaching teachers one on one. I was able to identify patterns that supported me not just in the process of data analysis but also the “when” of data analysis, teaching me to check in with myself and the energy I was bringing into the room before I posted maps on my walls. I took great care to not include my reflections or thoughts alongside the perspectives of participants in chapter four.

Ethics

Bogdan and Biklen (2007) recommend seven guidelines to prevent ethical issues in qualitative research: (a) avoid research sites where participants may feel that they must participate, (b) protect the privacy of potential participants, (c) honestly and clearly communicate the participant's time commitment, (d) protect the identity of participants, (e) show respect for participants and act honestly, (f) honor your commitments to participants, and (g) honestly report

the findings of your research. I adhered to each of these guidelines during the recruitment, data collection, data analysis, and reporting stage of this study. I dedicated particular attention to the first guideline articulated by Bogdan and Biklen since I have served as a school and central office high school mathematics leader in the past. My position in the three years immediately preceding and during data collection was not content specific, and my work was limited to interactions with leaders and teachers at the K-8 level. Additionally, my recruitment efforts were not limited to school systems or school sites where I have previously been employed.

This study was conducted under the full approval, guidelines, and supervision of the Instructional Review Board (IRB) with the intent of fully protecting the participants. The requirements of the IRB encompass the guidance from Bogdan and Biklen (2007). In accordance with IRB requirements, I took great care to protect participants by obtaining their informed consent, ensuring they were fully aware of the time commitment of the project, and again impacted by the global context of the Spring of 2020 that they were free to withdraw from the study at any time. Lastly, throughout the project, I maintained participant confidentiality by using numeric codes, pseudonyms, and applying appropriate data management procedures.

Limitations

Limitations exist for all research, and this study is not an exception. The proposed study is a qualitative case study involving five secondary mathematics teacher participants. The results of this study should not be generalized as a representation of all secondary mathematics teachers. An additional limitation is that the participating teachers selected artifacts from a single planned or implemented lesson in order to discuss their perspectives on mathematical modeling. It cannot be assumed that these lesson artifacts are a comprehensive representation of mathematical modeling instruction in their classrooms.

Summary

This study is a qualitative case study employing multiple case study procedures to explore the perspectives of mathematical modeling held by high school mathematics teachers. I used purposive sampling to select five Algebra II teachers as participants, each representing a single case. Data were collected from a contextual survey, interviews, and documents participants identified as evidence of lesson planning and implementation of mathematical modeling. Kaiser and Sriraman's (2006) "perspectives of mathematical modeling" and the voices of participants guided the analysis of data, and the triangulation data from multiple sources and the application of multiple case study procedures will allow me to attend to the validity, reliability, and integrity of the study. The findings of this study are detailed in chapter 4.

4 RESULTS

In this study, I explored the perspectives of mathematical modeling held by Algebra II teachers. The primary research question, “In what ways are teachers’ perspectives of mathematical modeling connected to the ways in which they plan learning experiences for students?” served as the guide for this exploration. The exploration was framed by Kaiser and Sriraman’s (2006) conception of “perspectives of mathematical modeling” which was used directly to develop the sub-questions listed below:

- a. How do Algebra II teachers describe a mathematical model?
- b. How do Algebra II teachers describe the mathematical modeling process?
- c. What goals do Algebra II teachers assign to mathematical modeling?
- d. How do Algebra II teachers select and implement tasks that support mathematical modeling?

This chapter presents the results of the study by revisiting the study’s context, providing an overview of the Algebra II course, and acknowledging the social environmental context at the time the data were collected for this study. Participants shared their perspectives through two descriptive interviews regarding both a task and task exemplar they align with mathematical modeling. They provided rich descriptions about their planning and the instructional practices they employed to implement the modeling task with students. Additionally, each participant completed a brief survey to provide context on their educational background and areas of certification. The next section of this chapter is dedicated to presenting the findings of the single case analysis, replicated, and then organized into three sections for each participant:

- (a) descriptions of a mathematical model and the modeling process,
- (b) the goals of mathematical modeling, and

(c) the selection and implementation of tasks that support mathematical modeling.

Each case was analyzed and is intentionally presented separately in order to honor and protect the integrity of each participant as an individual with a unique perspective. For this reason, I introduce each participant at the onset of their story or case. The chapter concludes with an exploration of the emergent themes which resulted from the cross-case analysis and a summary.

Research Context

The five participants in this study are five experienced mathematics teachers, David, Bianca, Evan, Sarah, and Brian (pseudonyms). The participants were employed by two similarly sized school districts, each in a metropolitan area. David, Bianca, Evan, and Brian were teachers in School District A, and Sarah was teaching in School District B. It is important to note that Bianca, Evan, and Brian were teaching at the same school during the collection of data for this study. Participation in this study was confidential and I did not share the participant list or discuss other participants during or after the study. While the School District A serving approximately 58,000 students and School District B serving approximately 44,000 students are close in size there are significant differences among the demographic composition of the communities and students that each district serves. In School District A 74.6% of students and families identify as Black, 14.5% as white, 7.2% as Hispanic, 2.3% as multi-racial, and approximately 1.3% identify as Asian/Pacific Islander, Native American or Alaskan Native. In contrast in School District B 7.9% of students and families identify as Black, 66% as white, 19.6% as Hispanic, 4.4% as multi-racial, and approximately 2% identify as Asian/Pacific Islander, Native American or Alaskan Native. The school districts have similar sized populations among students with disabilities 12.7% in School District A and 13.3% in School District B.

Significant differences exist between the socio-economic compositions of each school district community. Title I is a U.S. Department of Education classification used to describe schools where at least 40% of the student population is from low-income homes. In School District A about 81% of schools are designated as Title I schools and approximately 77% of students are designated as Economically Disadvantaged. In contrast just 13.9% of schools in District B hold the Title I designation and near 29% of students are designated as Economically Disadvantaged.

The five participants in this study, David, Bianca, Evan, Sarah, and Brian were all teaching Algebra II in the spring of 2020. The Algebra II course is one of three mathematics courses required by the state for graduation. The state department of education incorporates mathematical modeling in the frameworks for five out of six curriculum units. Modeling is included in the state expectations for Algebra II through a defined set of related content standards. I will share again, as a reminder that all but one of the interviews for this study took place during March and April of 2020. The COVID-19 pandemic during that time resulted in the closure of traditional face to face schooling and a spur of the moment transition to virtual teaching and learning. The interview protocols were not edited to probe into teaching and learning in this new context, but while it was not discussed in our interviews, it is highly likely that it had some impact on the well-being and the workload of the participants. I constantly and consistently communicated to participants that their physical and mental health, families, and students were the priority via email and phone. We worked together to schedule and reschedule interviews in response to sudden conflicts. An introduction to each of the five participants, David, Bianca, Evan, Sarah, and Brian based on the information provided in the contextual study is included at the opening of each of their stories. Table 4 below includes a summary of each participant.

Table 5*Summary Introductions of Research Participants*

Name	Years of 9-12 teaching experience	Years of Algebra II experience	Minor, major, or formal coursework in mathematical modeling or applied mathematics
David	12	5	No
Bianca	8	5	No
Evan	11	9	No
Sarah	16	6	No
Brian	8	6	Coursework

Teacher Perspectives*David*

David has twelve years of high school mathematics teaching experience and has taught Algebra II for five of those twelve years. He holds a bachelor's degree and teaching certification in the areas of secondary mathematics and high school Physics. Despite his areas of certification David reports that he did not minor or major in mathematics during his undergraduate studies and took no coursework in mathematical modeling or applied mathematics.

David's Descriptions of a Mathematical Model and Mathematical Modeling. During the launch of our initial interview, I probed David about his engagement with mathematical modeling during college coursework or in-service professional development. His response to my question was to ask, "How I defined "mathematical modeling." It is important to note that David was the only research participant to pose that question. I responded by sharing that I did not hold a fixed definition and thus I was interested in hearing about the perspective he and other high school mathematics teachers held. My response was quick, but natural and that appeared to put him at ease.

"I'm kind of lost in what exactly we are going to discuss because a mathematical model, when you're doing research or trying to describe a phenomenon, so I can

think of a mathematical model that way. So, is that the way that you want me to talk about, or do you want me to talk about from the perspective of when you're teaching and . . . ?" (Interview 1)

The quote above captures David's belief that the description of a mathematical model is defined by the context or place where the model exists; from his perspective mathematical models exist within research and within the classroom. In both settings David sees a mathematical model as a relationship between mathematics and the real world. In research, he shares that mathematical model, in the form of equations are created to measure or predict phenomena, such as weather using mathematics, science, and technology. In the classroom, mathematical models are built when the teacher successfully connects mathematics to the real world, and "models," as a verb, the application of mathematics concepts.

When asked about the difference or distinction between a mathematical model and the process of mathematical modeling David shares that the difference is characterized by the two contexts he just finished describing, a mathematical model describing the research setting and mathematical modeling being the process teachers use to engage and make connections between the real world and mathematics for students. This process, he shared, begins with an important mathematics concept that can be "easily connected to the real world," and then the teacher uses gradual release "I do, we do, you do." When this process results in students understanding the concept, successful mathematical modeling has occurred. David echoes this perspective in interview two when he describes how he implements a task to engage students in mathematical modeling. His description includes teaching the content without context first, modeling problems that mirror the task next, and then supporting students as they work to solve similar application problems with peers and independently.

The Goals David Aligns to Mathematical Modeling. There are two constant themes when David speaks about the goals of mathematical modeling: connections and thinking. Though seemingly simple David uses the term “connections” to describe a cycle that consists of connecting his students to mathematics concepts, connecting those mathematics concepts to real world situations which can exist in a variety of domains, and lastly the connection that occurs when students build an understanding of the mathematics and how it can be applied in the real world. From this cycle of connections one can infer that David assigns content related goals to mathematical modeling and while this is the case the second theme, thinking also plays a critical role in his motivations for engaging students in tasks that he aligns to mathematical modeling. In his view, these new situations and applications of mathematics, and student experiences can have a lasting impression on the ways in which students’ process new information. He emphasized that “It might not be about the math itself. It might be about expanding their vision and their thinking” (Interview 1). David assigns a student’s ability to communicate or explain reasoning as evidence that “thinking” has occurred.

When asked why he believes mathematical models and mathematical modeling were included in the CCSSM and his state standards David replied as follows:

“Well, I think it has to be included because it’s how the real world works. We cannot teach or see things isolated in our connections and the world is so complex. It’s not one process, one phenomenon, one law. There’s a group of things happening at the same time that are interconnected, so I think that process, of thinking, involves, definitely mathematical modeling” (Interview 1)

His response communicated that the two themes: connection and thinking, are not discrete and instead intertwined and interdependent. The connections students make in understanding mathematics, connections from mathematics to the real world and among situations and contexts, and these social experiences are the vehicle for thinking.

David's Process for Selecting and Implementing Tasks that Support Mathematical Modeling. When asked to describe the ideal task to engage students in mathematical modeling, David's response outlined two types of tasks, general and specific, that aligned to his two descriptions of mathematical modeling. In general, he described the ideal task as one that consists of a problem or situation, a context with some background information, and a fixed set of questions. When probed for a specific example his description of an ideal task was much more open ended. He presented an example connected to the current (as it was in April of 2020) Coronavirus pandemic. In the specific example, he provided, students would be presented with a variety of data connected to the pandemic and given the opportunity to look for patterns that could be modeled with functions to make predictions about questions that were of interest to students. In this process, students would have the opportunity to fit types of functions to the data, validate or reason about their appropriateness, and then make adjustments. When selecting tasks to engage students in mathematical modeling, David's selections are driven by content or standard alignment, his content knowledge, and his experience as a teacher. He uses these competencies to critically review state and district curriculum resources as a starting point and then expands his search to additional curriculum resources as needed. While David shared three characteristics, his primary goal is to find tasks that align to the content standards and that "cover the skills of the standards." He mentioned the state assessment system as a driver for ensuring tasks and all classroom instruction is aligned to content standards; it is important to note that Algebra II does not have a correlated state assessment. In Interview 1, he concludes with the following about an ideal task: "And you want to think a good task of course, covers the objectives of that standard."

In order to support and guide the discussion during Interview 2, David created and submitted a teacher exemplar for a task that he has used to engage students in mathematical modeling. The exemplar he submitted is a task entitled “Half-Life” and it was sourced from the state curriculum frameworks for Algebra II. The state curriculum frameworks label this task as a “Scaffolding Task: Tasks the build up to the learning task,” learning tasks are designed to provide students the opportunity to construct an understanding of mathematical concepts through problem solving in context (Georgia Department of Education, 2019). The objectives lead with the assigned mathematical goals of representing a real-life situation with an exponential function, solving exponential functions both graphically and algebraically, and report that the task addresses all eight SMPs (Georgia Department of Education, 2019). David explained that he selected this task because modeling involves functions, and exponential functions are important because they connect to a number of things in the real world, “I really like that they have real life, and it relates to chemistry and physics” (Interview 2).

In order to plan for the implementation of the task with students David first defines the goals for the task:

Through the task, I want the students to be able to first make connections between the math, let's say in this case, exponential function. I want them to see the application and how they can use the math to model real life situations. I want them to understand. I want them to be able to graph. I want them to be able to do all that, but really apply it to real life. So, the goal is that they are able to build functions in the context of a real-world problem. (Interview 2)

In planning a lesson around this task, David intends to begin with a video hook that shows a real life application of exponential functions. He expects the task to surface a student misconception that the relationship between the variables is linear and not exponential. In order to avoid that misconception, he would then go over the structure of exponential functions absent of context. After this introduction, he would begin the modeling he described in the classroom context,

using gradual release he would model similar problems in context and then release students to work on the task in groups. While students are working David plans to “actively monitor” student work by making a pathway through the classroom listening to students work, posing probing questions, redirecting groups, and providing additional examples in small groups as needed. Ideally, David would like to conclude the lesson with a student debrief and exit ticket that allows students to explain their thinking about exponential functions; however, the reality of time constraints in the classroom often do not allow this to happen. Instead, he is often left to depend on formal quizzes and assessments to measure student mastery of content.

Bianca

Bianca has eight years of high school mathematics teaching experience, and she has taught Algebra II for five the last years. She holds a bachelor’s degree in mathematics, a graduate degree in mathematics education, and teaching certification in the areas of secondary mathematics. She reports that she had no coursework in mathematical modeling or applied mathematics in her undergraduate or graduate programs.

Bianca’s Descriptions of a Mathematical Model and Mathematical Modeling. Bianca did not report having taken any college coursework in mathematical modeling, but shared that as her school worked to obtain Science, Technology, Engineering, Arts and Mathematics (STEAM) certification her Professional Learning Community (PLC) has engaged in a great deal of study around modeling mathematics, connecting it to students and real-world applications. When asked to describe mathematical models and the modeling process she stated, “Applications come to mind, how you actually use math in the real world, and how it is actually applied to concrete things” (Interview 1). Bianca distinguishes mathematical modeling by the process of solving application problems. Her verbal descriptions of a mathematical model and mathematical

modeling but present when she describes the goals of modeling and tasks that support modeling is a connection between mathematical modeling and multiple representations, particularly pictorial or graphical representations. She spoke specifically about drawing pictures or graphs (Interview 1), number lines (Interview 1), and area models (Interview 2).

The Goals Bianca Aligns to Mathematical Modeling. When asked about the goals she assigned to mathematical modeling and why mathematical modeling was explicitly included in the CCSSM Bianca shared an experience with her sister.

My sister teaches elementary school, so we always talk about the new math, the new math problems. I went to one of her math nights and actually kind of sat to see how they're dividing fractions on a number line now and it's more of a conceptual thought process. So they're doing the same things, but it's not necessarily what my memory is. It's do you consistently understand what the math is doing
So I think in the common core we've added the modeling piece so that they can conceptually understand what the math means. This is why this is such. This is how we utilize it. This is what it looks like. (Interview 1)

This experience is evidence of the value Bianca assigns to mathematical modeling to support students in building a conceptual understanding of mathematics, understanding the structure of mathematics, and being able to apply mathematical concepts. When probed to connect her experiences with her sister to the goals of engaging her Algebra II students in mathematical modeling she saw the retention of mathematical concepts as the benefit of conceptual understanding but also connected mathematical modeling to the development of the whole child. She consistently spoke about the mathematical deficits her students often enter high school with and the compound and complex relationship between those deficits and her students' sociocultural experiences, with these things in mind mathematical modeling for Bianca was almost equally purposed with content related goals and exposure she believes will impact her students' confidence and expand their post-secondary options.

“So, I think it helps one with the retention of information and then it also helps with broadening their horizons. I think about my students and how their whole lives are very small. It might not even be outside of 285, 20 (these numbers denote highways in close proximity to her school), but you know how, it just brings a whole other avenue of information. Things that, maybe somebody might go and research something else about and they might one day, they may major in it or work in that field. So, I think it broadens the horizon for a lot of our students. (Interview 1)

Bianca’s Process for Selecting and Implementing Tasks that Support Mathematical

Modeling. Bianca’s description of the ideal task to engage students in mathematical modeling is connected to her desire to “build students up” and impact student’s confidence. Her first words were, “For me and my students, the task will be sequential. And what I mean by that is it’s almost like a stair-step that leads them to a door, and the door just slips wide open” (Interview 1). Her process for selecting tasks is guided by her commitment to building student confidence and content standards. Her perspective around the types of tasks that will build student confidence is centered in accessibility and the content “deficits” she has observed among the majority of her students.

Well, and I’ll say that a lot of our students come with deficits in the basic skills and deficits in math coursework, here and there. So, once we build those up we can kind of get most of them to a point where they feel confident. So, if we take you from where you are and then we step, take a step and then we take a step, understand, take a step, ok, do you understand? Take a step. And then once you get to the end, you’re like “Oh wow, look, I built a home!” (Interview 1)

When selecting a task, she uses her knowledge of her students and seeks the collective input of her school based Professional Learning Community. She has the support of a school-based mathematics coach that brings possible tasks and resource sites to their planning meeting and creates opportunities for teachers to explore resources and work through the tasks together, and then collectively decide which tasks they will use to engage students.

The task, titled “Polynomial Farm” and exemplar Bianca selected to support our discussion during our second interview, was found through an internet search based on “junior

standards” (Interview 2), selected, planned for, and implemented during the 2019-2020 school year, with the support of her PLC. The task was originally developed through a project at Radford University with the Secondary Mathematics Professional Development Center and is credited to a Virginia high school teacher, Emily O’Rourke. Though developed through the project the task is no longer housed on the center’s website, likely due to a shift in content standards and course expectations. The task was originally written for an Algebra I course with the mathematical goals performing all four basic operations with polynomials and the ability to factor first and second-degree binomials. It presents a scenario of a farmer determining how to distribute crops in his garden and assigns polynomial expressions to the dimensions of his plot, see an excerpt from the task below.

Figure 4.

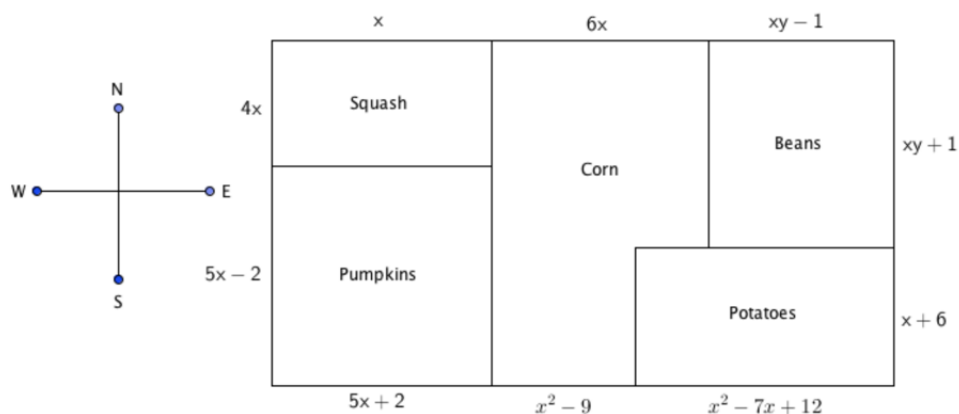
Performance Task-Expressions & Operations

Performance Task – Expressions & Operations
A.2bc Polynomial Farm

Names: _____

Date: _____ Block: _____

Directions: Farmer Bob is planting a garden this spring. He wants to plant squash, pumpkins, corn, beans, and potatoes. His plan for the field layout in feet is shown in the figure below. Use the figure and your knowledge of polynomials, perimeter, and area to solve the following:



1. Write an expression that represents the length of the south side of the field.
2. Simplify the polynomial expression that represents the south side of the field.
3. Write a polynomial expression that represents the perimeter of the pumpkin field.
4. Simplify the polynomial expression that represents the perimeter of the pumpkin field. State one reason why the perimeter would be useful to Farmer Bob.
5. Write a polynomial expression that represents the area of the potato field.

The PLC selected this task to engage students in mathematical modeling because it presents polynomial operations in a “real-world context” and requires students to apply their prior knowledge of area and perimeter to a new situation with the support of a visual model. The

rationale for the selection of this task is directly aligned to Bianca's description of mathematical modeling being centered in the application of mathematical skills and visual models.

During our second interview, I asked Bianca to walk me through her most recent implementation of this task with students and she shared that the task was used as a culminating task at the end of a week focused on operations with polynomials. The class period opened with a few warm-up problems where students were asked to find the area and perimeter of rectangles with numerical values for length and width. The intention was to activate students' prior knowledge around calculating area and perimeter and clear up any misconceptions from middle school. Next, Bianca modeled a few problems on multiplying polynomials and combining like terms. "Because for some students, they've never seen it. You've never been to a farm, you know so you want to see what farming is" (Interview 2). In order to expose students to the context of farming, she showed a short video on farming, so that they were able to see crops on a plot of land and hear a farmer talk about his day-to-day work. Students worked to complete the task with a partner or in small groups that Bianca referred to as "families." While they were working on the task her aim was to be both an encourager and a guide by listening to students, posing questions, and if needed modeling parts of the task. She used her observations to determine whether students had misconceptions about polynomial operations before giving a quiz on the concept on the next day.

Evan

In the spring of 2020, Evan was completing his eleventh year as a high school mathematics teacher. He holds an undergraduate degree in mathematics and a provisional teaching certificate granted by his current state of residence. He reports having taken classes in

mathematics, a few courses in mathematics education, and no coursework in mathematical modeling or applied mathematics.

Evan's Descriptions of a Mathematical Model and Mathematical Modeling. While Evan shared that he has had no coursework in mathematical modeling he shared that there was some time in his mathematics education courses that was focused on researching the impact of mathematical modeling in classroom instruction. His coursework did not provide the opportunity for him to engage in practices he defined as mathematical modeling. That experience, the absence of engagement, has been replicated in limited in-service professional learning engagements with mathematical modeling as a practicing teacher. He described those opportunities:

“I think that we've had a look. We have had professional learning on it. Most of it is telling us and not modeling or doing it. Unfortunately, that's not supposed to be, not even just with that (mathematical modeling). I'm talking nationwide. That's how most of them are, so you don't get a chance. It's (mathematical modeling) one of those things you have to sort of just do it in the classroom. Just try it out.” (Interview 1).

Evan describes mathematical modeling as “making the math make sense” and believes that happens when a mathematical model is used to make something abstract, concrete. He referenced physical objects and manipulatives (mirrors, rulers, Algebra Tiles), pictures, graphs, and technological tools (Geometers' Sketchpad, graphing calculators, and calculator software) as examples of mathematics models that when used by students to solve problems constitute mathematical modeling. Simultaneously he defined his practice, as teacher, as a “mathematical model,” when referencing a model of the mathematics.

The Goals Evan Aligns to Mathematical Modeling. When asked why mathematical modeling was explicitly included in the CCSSM, he shared that he believes politics and “Corporate America”, the relationship between politics, education, and the economy led to

mathematical modeling being called out in the standards. Leaning on his conception that mathematical modeling is “making the math make sense,” Evan assigns content related goals to mathematical modeling with the idea that mathematical modeling serves to support students in mastering the abstract nature he believes to be inherent in mathematics.

Because math a lot of times is very abstract. Well, most math at the high school level is very abstract and so it's not as concrete as people think it should be or people want it to be. You have to try to make the abstract concrete for some people. We're not just abstract learners so what modeling does is it takes what is abstract and starts to bridge the gap between the abstract and the concrete, the concrete that people want and the abstract that math is. You know what I'm saying? (Interview 1)

In middle school algebra I and geometry, Evan believes the content standards allow students to interact with concrete objects and manipulate visual representations which support the progression from concrete to abstract. However, as students move through the progression of mathematics topics those objects and the manipulation of representations becomes less applicable and the intent of mathematical modeling is to support students in developing logical patterns of thinking and approaching mathematical problems.

When engaging in mathematical problems, and if I'm doing it, I'm actually modeling the math. I'm thinking about what they know and what I'm trying to get them to understand and I'm bridging. I'm saying here is the point in the middle where we can have that “ah ha moment”. I'm taking my idea of our knowledge of mathematics. I go do a little model and at that point I'm thinking what they know which may not be what they need to know, or they may not have an interest in mathematics. They may not have the knowledge of thinking of mathematics because math is a logical thinking type of thing, and everyone is not a logical thinker. Some people are more. English, English is not what I would consider a logical thinking subject. So, some people don't have that logic part. That's not their gift. (Interview 1)

Embedded in his conception of developing logical patterns of thinking is the goal of making mathematics accessible to all students by providing a structure for them to access.

Evan's Process for Selecting and Implementing Tasks that Support Mathematical Modeling. In the middle grades, Evan gives an example of a mathematical modeling task as

taking a group of students outside with mirrors and yard sticks to explore ratios and proportions within similar triangles. He believes Geometry is the ideal space to engage students in mathematical modeling because of the opportunities Geometers' Sketchpad provides for students to manipulate figures and validate theorems. He believes that in the upper grades the mathematics becomes more abstract and any instructional activity that calls for students to make sense of the math is a modeling task.

I think that everything is a task, and I say that I think when we use repeated math problems, it could be considered a task as well. And because we do that (repeated math problems) in math. When we just think about it, we do that (repeated math problems) in math. I'm a math major, I'm a pure math major and we do that. (Interview 1)

In alignment with this description of a task Evan selected a teacher guided PowerPoint lesson titled "Radical Functions", a foldable template, and an assignment titled "Graphing Transformations" as an example of a task he has used previously to engage his students in mathematical modeling. The assignment consists of 18 problems, eight problems which require students to graph transformations of $f(x) = x$ and $f(x) = 3x$, six problems which provide the equation for a transformation of $f(x) = x$ and ask students to describe the transformation, and four problems that provide a verbal description of a transformation of $f(x) = x$ and ask students to use the description to write a function.

The modeling engagement Evan described was a two-day lesson with the learning goal of students being able to complete the "Graphing Transformations" assignment successfully and independently at home with the possibility of some class time for students to begin working on it. He selected this task because prior to this lesson students have already had experience with graphing radicals, so the transformations are an extension. The PowerPoint presentation and the foldable serve as the guide for the student experience in class. The lesson would launch through the PowerPoint which begins with a definition of a radical function and a teacher modeled

example of how to create the graph of the parent functions $f(x) = x$ and $f(x) = 3x$. He connects the graphs of these two functions to “shooting a basketball”, $f(x) = x$ and a “freestyle swim” ($f(x) = 3x$). He provides these connections in order to “Give some type of relatability to the students. So, now they know to think we’re looking at shooting a basketball or swimming.” (Interview 2). Next, he provides the function $f(x) = a b(x - h) + k$, where a , b , h , and k are color coded, the color-coded notation is present throughout the lesson also to help students make connections. During the first day of the lesson students are given a set of rules for the values of one value of the function to add to their foldable, a teacher worked example, and then a set of problems for the class to complete, this process is repeated for values a , b , h , and k . Evan describes this eb and flow as him moving from a teacher providing a model, to a facilitator of student learning, “Slide 12 is where I turn into a facilitator, that’s a break for them to work and to see what they are able to complete at that point” (Interview 2). The same methodology is applied to the two remaining concepts in the PowerPoint: writing the equation to represent a verbal description of a transformation and creating a graph of a transformation on the coordinate plane. While this is taking place it is important to Evan that students feel comfortable in the classroom and have the ability to change their seats and collaborate with peers. As a teacher he is focused on students being able to justify their answers using the vocabulary and real-world connections he modeled during the lesson. He shared, “Kind of that back and forth between the two. So, here I am teaching you or explaining to you a new concept, and then I want to allow you the opportunity to think about it, talk about it, kind of work through it. And then, let’s come back together and add to it” (Interview 2). Hence, this is integral to students mastering the content and being able to complete assignments independently.

In addition to the artifacts mentioned above he provided a teacher exemplar of the “Graphing Transformation” assignment which is color coded to align with the guidance in the PowerPoint and that incorporates the real-world connections he made during the lesson to “shooting a basketball” and a “freestyle swim”. In successfully completed student work Evan is looking for work that is mathematically accurate, that references the informal real-world connections, and uses the appropriate mathematics vocabulary from the PowerPoint and foldable.

Sarah

Sarah has been teaching high school mathematics for 16 years, for the last 6 years she has taught Algebra 2. She holds a professional teaching certificate in both mathematics and special education. In addition to graduate coursework in mathematics and mathematics education, Sarah also holds a graduate degree in instructional technology. She has had no college coursework in mathematical modeling or applied mathematics.

Sarah’s Descriptions of a Mathematical Model and Mathematical Modeling. Sarah shared that she has participated in in-service professional learning on mathematical modeling but that those sessions have been limited to the use of traditional manipulatives such as Algebra Tiles and area models for quadratics. When asked to describe a mathematical model Sarah stated, “For me, mathematical models are going to be things that help children take abstract concepts and put them into something they can lay hands on, something they can relate to, something that makes sense” (Interview1). The examples of mathematical models she discussed are physical tools and manipulatives, and technology applications such as Desmos and Geogebra but she expressed some hesitation when talking about some of the activities within these applications “I’m not sure if that (Desmos and Geogebra) fits the bill for mathematical modeling because in

some of it, they've gamified some of the models to make it interactive and fun" (Interview 1).

Sarah believes that mathematical modeling as expected in the standards is to use the models she described to represent and solve things in the real world. This expectation presents a conflict with the progression of learning she feels is supportive of students making connections.

We're trying to pull the math from the real world, where you're actually using those models. I don't honestly find those as applicable to the kids, as taking it back to things they learned in elementary or middle school. I generally try to figure out where the kids have seen something similar to what I'm about to teach and use that. I will eventually get to the real-world applications of math, but they(students) don't find that as interesting.... Like right now we're doing rationals, I keep a basic fraction problem on the board so that they can see this ugly Algebra 2 thing, I call it an ugly cousin, it just maps back to this thing you know. (Interview 1)

The Goals Sarah Aligns to Mathematical Modeling. The goals Sarah assigned to mathematical modeling as directly connected to her initial description of a mathematical model, students in mastering content through support centered on making connections between abstract concepts through concrete representations or prior experiences.

Well for our kids to have something that's abstract and connect it to something tangible and sometimes that tangible thing doesn't necessarily mean something mathematical. It's just some way they can connect it and they've used it enough so that they've created their own knowledge, and basically it sticks. (Interview 1)

She believes mathematical modeling was explicitly included by the writers of the CCSSM because when students are able to discover those connections themselves and construct their own understanding, students accept ownership and retain concepts. In sharing that belief, she again shared what she believes to be a contradiction within the expectations of the standards and the reality of school structures. Her claim, "Our curriculum is still a mile wide and an inch deep and they say, 'Oh no, it's not, not anymore,' but, yes, it is. You never get the time!" (Interview 1), speaks to that conflict and that the opportunities for students to construct their own knowledge is

limited due to the breadth of the standards and the amount of instructional time provided for students to master them.

Sarah's Process for Selecting and Implementing Tasks that Support Mathematical Modeling. In selecting mathematical modeling tasks to use with her students Sarah is driven by content goals, and tools that are both engaging and allow students to manipulate representations.

I think that modeling is difficult for kids trying to make a connection. I use things like Desmos because you can move the applet around, the game around. You get a little competition. It opens up conversations and dialogues. To me, it's about sparking curiosity and an easy way for the kids, a fun way for the kids to make a connection."
(Interview 2)

In order to discuss the implementation of a modeling task in her classroom, she selected the task "Will It Hit the Hoop?" from Desmos, an interactive application that uses the context of attempting basketball shots to model the graphs of quadratic functions. The task is designed to engage students in what the writers describe as a cycle consisting of predict-analyze-verify. Students are shown seven video clips, one at a time of the release of basketball shot after each shot, they are asked to predict if the shot will be successful. Next, in the "analyze" component students are presented with the same seven short clips but they have the ability to manipulate an image of a parabola to sketch a line of best fit to model the completed shot. After each manipulation students have the opportunity to reconsider their original predictions. In the "verify" component students are shown the completed shot for each of the seven scenarios and learn whether their results are accurate.

Sarah has implemented this task on many occasions in both a block and traditional 55-minute class period, in each situation she has used this task as a culminating task at the end of a week dedicated to exploring quadratic functions with the goal of connecting with students who are struggling with the mathematics concepts. She launches the lesson by showing one of the

video clips and engaging students by asking that they use a raised hand to indicate whether or not they believe the shop will be successful, students are then asked to defend their predictions. To complete the lesson introduction Sarah explains the goals of the task and asks that students pull their cell phones out. The application is teacher paced and students watch each clip and make predictions individually using their cell phones. During the “analyze” component of the task students have the opportunity to continue to work individually or to work with a pre-established “buddy” in the classroom. She encourages students to ask questions, not just of her but also of their peers to spark academic conversations.

I encourage students to ask a question. My mantra is “Be Brave” because the bravest thing you can do is ask a question. Then we move to “The most brave thing you can do is to ask me out loud”. The second is to ask a peer, because even I don’t like to look stupid in front of my peers, but you’re not looking stupid. Stupid is not asking. In my classroom, by the second month. Kids will get up from one side of the classroom and go ask or go help someone on the other side. (Interview 2)

Once students have completed this process and had the opportunity to revise their initial predictions Sarah shows the “verify” clips asking that students use mathematical vocabulary terms to describe the characteristics of each graph. Sarah would close the lesson with a two-step Exit Ticket: 1) What did you learn today? What clicked? and 2) Given the graph of a parabola identify the vertex and the y-intercept. Write an equation to represent the graph. She shared how exciting it is to read student responses to question 1 and that there are additional activities in Desmos to support students who are still struggling with the second question, but she does not usually have the time to engage students in additional opportunities to explore.

Generally, the next day we go into the boring part. I mean, you know the next day is, “Let’s grasp this. Let’s talk about this. Let’s look at this. What are these points? What do we call them mathematically?” The general I do, you do, I do, you do, “Cool! High-fives!” Just continuing the math part of it. (Interview 2)

Brian

Brian just completed his eighth year as a high school mathematics teacher and has taught Algebra II for the majority, six years of that time. He has a Masters' degree in Mathematics Education and was completing an Educational Specialist degree in Educational Leadership at the time of our interviews. During his master's program he recalls a calculus class that was based on mathematical modeling and about two in-service professional learning sessions that incorporated mathematical modeling. He shared that both of those experiences were very specific and focused on the content specific scenarios and problems.

Brian's Descriptions of a Mathematical Model and Mathematical Modeling. Using his college and in-service professional learning experience as a launching point Brian speaks of mathematical modeling as a translation between the real world and mathematics concepts, "...formulating models and trying to solve things" (Interview 1). He distinguishes between the two experiences by highlighting that in his college coursework he felt like he was engaged in creating mathematical models to solve problems. In contrast, his experiences with in-service professional learning sessions were centered on the teacher practices to make the thinking required to create a model more explicit for students, essentially teacher modeling. When asked to describe a mathematical model in his own words he stated, "I would say that it is a tool, tool of pedagogy or teaching instruction to make the content for the class more relevant to exact applications in the real world and cycling between both (the real world and mathematics content)" (Interview 1). He lifts Algebra Tiles, a physical manipulative as an example of a model, a tool that can be used to make quadratics and polynomials concrete for students.

"So, for example, they can start by figuring out what the actual problem is and instead of going straight for a solution of straight for an answer, they can come up with maybe assumptions, maybe try some prior knowledge type math and then go through whatever model or set of steps or process that's set up for them and the kind of discover, "Hey, I

knew some of these things or I did not know any of those things”, kind of deal. So, I think it’s a more inclusive, it provides more thinking opportunities and allows them to kind of navigate the learning process because it’s just following strategic steps.”
(Interview 1)

The quote above captures Brian’s description of the ideal student engagement with mathematical modeling and implies that mathematical modeling is a process. For Brian, in alignment with the CCSSM SMP modeling with mathematics begins with students accessing prior experiences to make sense of the problem, and then make some assumptions that allow them to apply a model. The important distinction is that Brian speaks of the “model” or “steps” being provided for students so that they are able “navigate” their learning.

The Goals Brian Aligns to Mathematical Modeling. In the quote where he describes the process of mathematical modeling Brian also alludes to one of the goals he aligns with mathematical modeling, “...more thinking opportunities” or exposure for students. Through mathematical modeling he believes that students are also exposed to applications of mathematics and possible careers. For example, during Interview 2 when Brian describes the implementation of the task he selected “exposure” is the first goal he shares. It is his desire that the engagement with the “How will you invest task?” will expose students to context and the possibility of investing money, and investment related careers. To foster exposure, he plans to launch the task by exploring investment exchanges with students, invite investment professionals into the classroom to speak with students, and require that students interview an adult regarding investments.

Content mastery is the second goal he aligns with mathematical modeling and believes the engagement with modeling tasks should positively impact content mastery.

They should walk away with a better understanding of whatever standard was being covered or a portion of the standard being covered. But they should walk away with a better understanding of what or how their knowledge connects to that was being learned

through the task. They should walk away with a specific mastery level or understanding of some misconceptions to look out for. (Interview 1)

Brian believes that modeling tasks should improve content mastery by connecting prior knowledge, a real-world scenario, and current mathematical concepts. When asked why he believes mathematical models and mathematical modeling were included in the common core standards, Brian shared that he “hopes” they were included because through research, in a variety of settings it was found to be successful in improving student performance in mathematics. He fears that it was included, “Probably because it showed some signs of working in certain scenarios and certain environments and someone decided to implement it across the entire curriculum” (Interview 1). Additionally, he assigns value to the goal of providing students with opportunities to communicate their mathematical understandings verbally and in writing. He connects this value to the SMPs and stated, “The thing is, how to present an idea. So, in our practice we talk about standards for mathematical practice and modeling and making things from the real world make sense to the students” (Interview 2). While we will more thoroughly explore the connection between the perspectives shared by the study participants and the SMPS in Chapter 5, I feel it important to lift here that the SMPs are intended to provide the observable behaviors of “mathematically proficient students,” and SMP 1 is that students “Make sense of problems and persevere in solving them.” During the second interview, I asked Brian to share the components of an ideal student response to “his” task he resurfaced connections to the SMPs, “I mean precision and accuracy (SMP 6) are important, but I think the communication piece could probably be the standard for measurement” (Interview 2). The task he selected requires students to create a presentation board or PowerPoint to display their work, write a written reflection, and present their results to the class.

Brian's Process for Selecting and Implementing Tasks that Support Mathematical Modeling. When Brian is looking for tasks to engage students in mathematical modeling he is looking for “structured” tasks that are not ambiguous and that are relatable for his students. His first filter when selecting tasks is to seek out sources that are reliable and “research based,” he then tries on the work of the task himself. “A second thing I do is I do the task myself, go through the task and try to think like the students. What questions would they ask? And if I’m creating too many questions, it’s probably not as good of a task because the kids will get bogged down with the questions of trying to figure out what’s the next thing to do” (Interview 2). The idea of structured tasks that are not ambiguous connects to his description of mathematical modeling, and the modeling process consisting of steps that need to be followed. When selecting tasks Brian is looking for tasks from reliable sources, with content related goals, that provide students with the steps or process for completion.

As an exemplar mathematical modeling task, Brian selected “How Will You Invest? Project.” I asked Brian to share why he selected the task and he responded, “So, my main reason for selecting the task is the relevance or potential relevance to student interest. So, because the task is about, it’s an introduction which will usually lead to modeling interest formulas or investment formulas and things like that. It’s a good engaging piece for students who seriously consider all the time. How much money they plan to make or have?” (Interview 2). Like the task Bianca selected, the task was developed through a project at Radford University with the Secondary Mathematics Professional Development Center. “How Will You Invest?” written by a Virginia high school teacher, Maggie Hughes, is part of a larger unit titled “Population Project.” The mathematical goals of the task are centered around students making a connection between exponential functions and investment opportunities, in order to make a decision regarding the

best investment. Brian describes the task as open, relevant, and divergent from the typical mathematics classroom experience, “I thought the task was very open to spark an interest and creativity. This is something we missed as teachers, sometimes as math teachers. To allow students to create a hands-on real- world. We focus on functions and solutions and formulas, and you know accuracy” (Interview 2). In the task students are presented with a scenario which claims they have just inherited \$20,000 and are trying to determine the best investment for their money given five investment options. For each option students are expected to: represent a situation with an exponential equation, graph the equation, and then use the equation to determine the potential profit. In addition to calculating the potential profit, students are to interview “a trusted adult” to present their findings, inquire about their investments, and solicit their advice in making a decision regarding the hypothetical \$20,000. The final product of the task is a PowerPoint presentation or poster board, along with a written reflection based on the task and the required interview. The task includes notes on exponential growth, a project handout with the scenario and each investment option, a sample PowerPoint presentation, a sample written reflection, and a scoring rubric.

“How Will You Invest?” is not a task that Brian has implemented with his students, during our second interview he shared his plan for implementing it with students in the future.

Those plans were informed by his experiences with a similar task.

Well first of all, they should see a model example. Either, maybe a previous student’s work. Something where a student has gone through and done the exact same thing to give them kind of an idea or the teacher could create one with the class, kind of walk them through the process. These are the things that I’m looking for. I know for myself with a very similar project I showed the students which standard of mathematical practice I use, which standards I was pulling from for the content. I created a specific example on the poster because they did it in class, like I did all the things if I was doing the project so they could see. And then I went through the rubric to say okay this meets, this meets this meets here, what do you think? So, they can get an idea for the whole spectrum of the

project, and I let them repeat the process with different problems. So, I think that will be beneficial.” (Interview 2)

The quote above captures the ways in which Brian’s perception of mathematical modeling and teacher modeling are connected. When planning for his students to access the modeling task he selected he feels it essential that he model the thinking required and the desired finished product. To build engagement and relevance he would launch the lesson by visiting investment exchange sites and possibly showing a video with the intent of students understanding that work they will be doing, is something that is used “on a regular basis” He estimates that his lesson involving the “How Will You Invest?” task will require about four class periods, with students working in mixed ability pairs based on a quiz or test on the mathematical content, representing and solving exponential equations. During our first interview Brian described mathematical modeling as a process where students would be called on to make assumptions, access prior knowledge, and then apply a model. In the four-day lesson outlined below he used the term “modeling” on several occasions, each time it is used to describe the model he plans to provide for students to ensure that they are successful in completing the project.

I don’t know about four class periods and that could just be for me and my students. But the first class period would be mostly modeling. You know, it might be with a little inquiry or student input but mostly modeling the expectation and all of this of course depends on the timeframe of the class, 45 minutes for everything. But we will be doing modeling to make sure everyone understood. The second class would be more guided or facilitated. Okay, so this is what we talked about in the first class. This is what I’m expecting you to get done in two days. And then three would be finalizing their projects or finishing their projects. The teacher would be checking for misunderstandings, maybe helping with their technology usage. You know, guidance still and the fourth class would probably be presentations with the rubric. I tend to like to have students do a kind of peer evaluation as well so it might be something that you would see with these students evaluating each other based on the original rubric. And me (the teacher) asking questions so they can communicate their ideas, their solutions. But something along those lines for four days. (Interview 2)

When asked what his next steps were, after students completed their presentations, Brian lifted that no task is perfect and that there are always changes that can and should be made. He believes that other students and teachers should examine student work to inform those changes and believes in the value of posting student work in a designated area for review, “I would like to display the work preferably in one location and have other teachers come and review the work. It offers insight for the teacher, for reflection as well as additional insight for students” (Interview 2).

Cross-Case Analysis

The findings of the cross-case analysis represent my attempt to “turn off” my personal perspective and experiences in order to “listen” intently and uncover the themes, in relation to the research questions which emerged across the perspectives of mathematical modeling shared by participants. In the preceding chapters I established that this study was framed using Kaiser and Sriraman’s (2006) conception of “perspectives of mathematical modeling,” composed of the descriptions of mathematical modeling and mathematical modeling, the goals of mathematical modeling, and the tasks aligned with mathematical modeling. The cross-case analysis begins with a section detailing the professional learning and coursework experiences of the study’s participants intended to provide a frame for the two themes: content mastery and connections to students, which bridge the relationship between participant perspectives of mathematical modeling and the ways in which participants plan learning experiences for students. In connection with these two themes the categories of descriptions of mathematical models and mathematical modeling below are visible in participant responses:

- Mathematical models as physical or technological tools which support students in the mathematical modeling, the progression of understanding from concrete to abstract for mathematics concepts.
- Mathematical models as representations which support students in mathematical modeling, described as students “making sense” of the structure of mathematics concepts, connections between mathematics concepts, and connections between mathematics and the real world. This is done through the translation between multiple representations and can be aided by technological tools.
- Mathematical models as teacher models of procedures and operations, where mathematical modeling is the practice teachers employ to model content expectations for students.

The categories among descriptions of mathematical models and mathematical modeling and the themes of content mastery and connection were interwoven in the perspectives of mathematical modeling among participants. The final three sections of the cross-case analysis contain support for the connection between the categories of mathematical models and mathematical modeling, each theme and the research questions that served as a guide for this study.

Professional Learning and Coursework

As a component of the study’s survey and at the launch of the first interview I asked that participants share some information regarding their exposure to formal coursework and professional learning experiences in connection to mathematical modeling. I am beginning the cross-case analysis with an overview of those experiences as a frame to the three themes that emerged when exploring the connections between the participants’ descriptions of mathematical models and mathematical modeling, the goals of mathematical modeling, and the ways they

select and implement mathematical modeling tasks. All of the participants in this study are experienced mathematics teachers with eight to sixteen years of teaching experience. All but one possesses a clear and renewable certification in high school mathematics from the state's certification board which establishes that in addition to their teaching experience they possess a strong grasp of mathematics content knowledge and pedagogy. Noteworthy, is that given their experience and certification all of the participants shared that their experience with mathematical modeling in both formal college coursework and in-service professional learning has been limited. Brian and Evan were the only participants to describe college coursework connected to mathematical modeling or applied mathematics. Brian recalled that in his college mathematics courses he often was required to create models to solve problem. Evan's response to the survey question noted no formal coursework in mathematical modeling or applied mathematics however during the interviews he described an exposure to mathematical modeling in one of his mathematics education courses. In that course he was required to read about the impact of mathematical modeling as an instructional practice. When asked to recount professional learning experiences all but one participant shared that the opportunities rarely included opportunities for them to understand mathematical modeling, practice, and prepare to engage students. Evan expressed that mathematical modeling has been presented as something teachers should be doing but that those sessions did not include a clear definition of what mathematical modeling is or how to incorporate it in their classrooms. Sarah and Brian spoke of professional learning sessions designed to encourage teachers to use specific manipulatives to support students in mastering specific concepts like linear and quadratic equations. Bianca was the one participant who shared that she had participated in a lot of professional learning grounded mathematical modeling as real-world applications of mathematics concepts. These sessions were a component of her

school's quest to obtain STEM certification. Because all of the participants are experienced teachers it is unlikely that their perspectives of mathematical modeling can be fully attributed to college coursework or in-service professional learning.

Theme 1: Content Mastery

During data analysis content mastery, the ability to understand, perform operations, and apply mathematics content included in their course standards consistently emerged as a theme connected to the goals participants assigned to mathematical modeling, their descriptions of mathematical models and mathematical modeling, and the ways in which they selected and implemented the identified tasks with students. Each participant included content mastery either as one of the goals they associated with mathematical modeling or as they described both the benefits of engaging students in mathematical modeling tasks and the success criteria, they assigned to the modeling tasks they selected to share. However, there was a distinct variation in the conception of content mastery when analyzing the data sources connected to interview one, focused more on theoretical beliefs and interview two, centered in their classroom practice and experience.

Content Mastery as Conceptual Understanding and Application. During the first interview participants were asked to share the goals they assigned to mathematical modeling and why they believed it was explicitly included in the standards. In response to those questions and threaded throughout their responses during that interview was the intent of content mastery centered that encompasses understanding, operations, and application. During that interview participant responses were inclusive of each of three categories of descriptions of mathematical models and mathematical modeling: mathematical models as tools in support of modeling as the progression from concrete to abstract understanding, mathematical models as representations

with modeling being the process that facilitates connections between mathematics and the real-world, the structure of mathematics and sense making, and lastly the conception of mathematical models as teacher practices and mathematical modeling as the implementation of these practices to support students in meeting content standard expectations. Bianca, Brian, Sarah, and Evan provided descriptions of mathematical models and mathematical modeling as physical or technological tools leveraged to support conceptual understanding. All five participants provided descriptions of mathematical models and mathematical modeling as representations useful in “making sense” of mathematics and both the connections, and applications between mathematics to the real world. David and Brian were the only participants that described mathematical models and mathematical modeling as teacher models to reinforce procedures and operations.

Content Mastery Centered in Classroom Experience. During the second interview participant descriptions of mathematical models and mathematical modeling centered in their classroom experience, and both their interview responses and artifacts again established content mastery as a primary goal of mathematical modeling; however, content mastery was limited to accuracy in operations, procedures, and applications of mathematics concepts. In preparation for the second interview participants were asked to share a mathematical modeling task they had implemented or planned to implement along with a teacher prepared exemplar of the task. During the interview participants shared their process for selecting the identified task, the goals of the task, and to describe their implementation, actual or planned of the task with students. As was the case during interview one content mastery was strongly connected to the goals participants assigned to the task and the criteria used to select the task. A document analysis on the submitted tasks found that all participants submitted a task centered on the real-world applications of procedures with mathematics concepts. David, Bianca, Brian, and Sarah all

selected real-world applications and Evan's task was a set of mathematics problems absent of context. The tasks required that students leverage mathematical models described as representations (context, equation, graphs, pictures) to engage in mathematical modeling, moving between representations to solve problems and write some level of explanation for their solutions. Absent from the tasks selected by participants was the development of conceptual understanding through concrete experiences, "sense making" of connections among mathematical concepts and in the real-world. All of the tasks included a statement of the mathematics concept and aligned function, students were given information about the structure of that function. The expectations of the selected tasks were that students use the structure provided, and the provided solution paths, repetitively allowing students multiple opportunities to practice.

When discussing the implementation of the selected tasks participants again leaned into the goal of content mastery as the ability to accurately perform a set of procedures or operations to solve mathematics problems with and without a real-world context. Each participant described their task and mathematical modeling as a culminating activity after several days of direct instruction on a mathematics concept which began without a real-world context. While only David and Brian described mathematical models as teacher models of procedures and operations during interview one, all five participants described a teacher directed review or model of a similar task or a component of the task as a teacher action prior to releasing students to engage with the task. Students were then expected to collaborate with a partner or group to replicate the teacher model. Aligned with the goal of content mastery, success was described as students being able to accurately replicate the teacher model to solve concept centered problems on upcoming assessments (homework, exit ticket, quiz, test).

Theme 2: Connections to Students

Similar to the theme of content mastery, the second theme, connections to students, was evident in the goals participants assign to mathematical modeling, their descriptions of mathematical models and mathematical modeling, and the ways they select and implement mathematical modeling tasks. In addition to the goal of content mastery among all five teachers, the data analysis also reflected not just a goal but more deeply a belief regarding the importance of connections to students. When describing the goals of their selected tasks Evan and Sarah felt the exposure to mathematical modeling would build investment and interest, and therefore perseverance for students who were struggling to show mastery of the concepts. David, Brian, and Bianca all spoke about how mathematical modeling enlarged students' conception of the world outside of school and the relevance between the things they were learning, ways of thinking and that world. In this larger world view Bianca and Brian were intent on leveraging mathematical modeling, experiences with real world applications to introduce additional post-secondary options to students in the form of careers and college majors.

Teachers leaned into two of the categories of descriptions of mathematical models and mathematical modeling with differing intensities in service to the theme of connections to students. Sarah and Bianca spoke of mathematical models as common representations that connect current mathematical concepts with concepts and experiences from previous grade levels. Sarah used the example of fraction models to support rational expressions, and Bianca noted that elementary and middle school experiences with area models were used to build a bridge to the area models with polynomials that were included in the task she selected. Each of the teachers referenced experiences where students were lacking prerequisite skills. The

description of mathematical models as teacher models and thus mathematical modeling is the teacher practice of modeling procedures and operations, was riddled with a need to support and connect to students. Participants spoke of teacher modeling as a way to make connections both for and to students, as a method to encourage students to attempt the mathematics and provide a path that would lead to successful experiences.

Classrooms are the space where teachers directly interact with students and as such the theme of connections to students undergirds the tasks participants selected and the ways in which they planned or implemented those tasks. The four participants that selected tasks involving a real-world context considered student experience, interest with that context, and the value they personally assigned to the context. Sarah selected a task that involved multiple scenarios of shooting a basketball to model quadratic functions she believed all of her students had experience and possibly, an interest in playing or watching basketball. The multiple scenarios and ability to engage with the task using their cell phones were intentional to provide access and a way for students to “Be Brave” (Interview 2) without their names being assigned to their attempts. Bianca and Brian believed that the contexts of agriculture and investments were of value for students in the eleventh grade because they would be entering the employment market. David knew that his students had experience with growth and decay from their science courses making the context of the task familiar for students. While Evan’s task did not include real-world context, he believed students had experience with swimming and could use that experience to build a connection between his students and transformations of parent functions. Under the theme of content mastery, I discussed the document analysis of the tasks selected by participants and identified the common characteristics of identified solution paths, structure, and repetition. The participants saw those characteristics as scaffolds in support of students, the repetition as an

additional opportunity for students to experience success. Each of the participants' selected tasks they identified as mathematical modeling tasks that they felt would allow a connection to students in the areas of experience, interest or value, and access.

As participants described their actual or planned implementation of their tasks with students, they spoke of creating an environment where students felt capable and comfortable. Ahead of the tasks students had participating in days of instruction aligned to the mathematics concept they would be using. In each of these classroom spaces instruction began with a teacher model of a connection to the context, often involving a video, the mathematics concept, and the procedures they would need to follow to successfully complete the task. Also, a component of the classroom environment participants described was student choice in seating and grouping to foster collaboration and communication. After the teacher model, participants shifted their focus to supporting and encouraging students. That support and encouragement consisted of monitoring student progress and affirming their effort and work, asking questions to support students in recalling procedures, or providing an additional teacher model for a small group or individual students.

Summary

This chapter began with the research questions that analyzed perspectives of mathematical modeling held by five Algebra II teachers. Next, I included the context in which this research took place. The single case reports represent the perspectives of mathematical modeling, and ways in which the selected and planning to implement mathematical modeling tasks for each participant. Serving as a frame for the cross-case analysis was that each participant described a limited learning experiences centered on mathematical modeling in both college coursework and in-service professional learning. The learning they described was specific to

using concrete tools and representations to support the conceptual understanding and application of identified mathematics concepts. Their learning also established mathematical modeling as an important teacher practice but did not address what a mathematical model was or how to engage students in mathematical modeling. The cross-case analysis uncovered two themes, (1) content mastery and (2) connections to students as the foundation for the ways participants described mathematical models and the modeling process, the goals they assigned to mathematical modeling, the criteria they used for selecting task, and the ways they planned to engage students in mathematical modeling. The descriptions of mathematical models and mathematical modeling provided by participants could be placed into three categories: (a) mathematical models as concrete tools or manipulatives to support the progression from concrete to abstract understanding; (b) a mathematical model as a representation, where mathematical modeling is the movement between representations to solve mathematical and real-world problems; and (c) a mathematical model as a teacher model. Thus, mathematical modeling is a teacher practice with the purpose of exposing students to the replicable thinking steps required to solve mathematical and real-world problems. A discussion of these findings in the context of relevant research is contained in Chapter 5.

5 DISCUSSION

In this qualitative case study, I explored the perspectives of mathematical modeling held by high school mathematics teachers. This exploration was guided by the overarching research question, “In what ways are teachers’ perspectives of mathematical modeling connected to the ways in which they plan learning experiences for students?” The overarching question for this study demanded that I first leverage relevant research to frame this study with an accepted and scholarly conception of “perspectives of mathematical modeling.” In response to that demand Kaiser and Sriraman’s (2006) “perspectives of mathematical modeling” served as the conceptual framework for this study and the development of four sub questions to undergird the overarching question: (a) How do Algebra II teachers describe a mathematical model? (b) How do Algebra II teachers describe the mathematical modeling process? (c) What goals do Algebra II teachers assign to mathematical modeling? (d) How do Algebra II teachers select and implement tasks that support mathematical modeling?

The participants in this study were five experienced mathematics teachers, employed in school districts within a metropolitan area. Three of the participants were from the same school but I did not disclose or discuss their participation in this study. Data were collected through a survey, two interviews conducted during the spring of 2020, and documents. The analysis of this data defined each participant as a single case and employed multiple case study procedures. The results of this study, which are presented in chapter four begin with single case reports to richly explore the perspective of mathematical modeling held by David, Bianca, Evan, Sarah, and Brian (pseudonyms) through the lens of the research questions that guided this study. These teachers, each having between eight and sixteen years of experience all shared that their preparation to engage students in mathematical modeling, through college coursework and in-service

professional learning has been limited. The professional learning, they described included: (a) an emphasis on using identified manipulatives to support conceptual understanding of specific content standards, (b) using mathematical models in the form of representations to solve problems, and (c) messaging about the importance and impact of mathematical modeling as teacher practice, absent of a clear description, examples of classroom implementation, and opportunities for teachers to practice. Among their descriptions of mathematical models and mathematical modeling there were three categories operating simultaneously. The first was that mathematical models are tools to support mathematical modeling as the progression from concrete to abstract understanding. The second category was the conception of a mathematical model as a representation, where mathematical modeling is the movement between representations to solve mathematical problems, often involving real-world contexts and applications. The final category was the description of a mathematical model as a teacher model and mathematical modeling as a teacher action that consisted of modeling, as a verb, the types of problems described in category two. The cross-case analysis revealed the emergence of two themes: (a) Content Mastery and (b) Connections to Students; deeply interwoven through the participants' perspectives of mathematical modeling and the ways the participants planned to engage students in mathematical modeling tasks. These two themes were at the core of the goals each participant assigned to their roles as mathematics teachers, and of mathematical modeling.

In this final chapter, I offer a discussion of the results of the study in the context of CCSSM and relevant research. The discussion of the results serves as a foundation for the implications of the findings from this study. As stated in the introduction and throughout this study modeling is included as a conceptual category and thus an expectation within the CCSSM in grades 9-12. While this study was centered within the context of Algebra II the implications of

its' finding and recommendations for future research are not limited to the Algebra II course.

This dissertation closes with recommendations for future research and a personal statement from the researcher.

Discussion of the Findings

This study opened with the inception of the CCSSM (2010) which as the catalyst for expectations for the inclusion of mathematical models and modeling in secondary mathematics. Model with Mathematics is included as SMP 4 and in the grades nine through twelve standards modeling is a conceptual category. This section will discuss the findings of this study in the context of the SMP Model with Mathematics, modeling as a conceptual category in grades nine through twelve, and commonly accepted best practices for teaching and learning. The cross-case analysis was framed by the learning experiences participants described in connection to mathematical modeling.

Model with Mathematics

The connection between mathematically proficient students and modeling with mathematics is well established as a best practice in mathematics instruction that predates the inception of the CCSSM. The premise can be traced back to Bruner's (1966) stages of cognitive development consisting of: (a) Enactive stage, involving actions, (b) Iconic stage, involving representations, and (c) Symbolic stage where symbols are used to describe learning. The first theme that emerged from the cross-case analysis, content mastery is connected to a belief that mathematics instruction is purposed with supporting learners to move from concrete approaches to abstract thinking and representations to solve problems (Miller & Mercer, 1993). This theme is supported by the participants' descriptions of mathematical models and mathematical modeling as both physical tools to support the progression from concrete to abstract understandings and

mathematical models as representations, leveraged in modeling the process of moving between representations to solve mathematical and real-world problems. Witzel et al. (2008) are credited with the acronym CRA to describe the mathematics instructional sequence from concrete, to representational, to abstract in service of content mastery. As a result of the long-standing practice of CRA and modeling with mathematics in the K-12 school setting there are a plethora of resources available to teachers which included teacher preparation program coursework, in-service professional learning, physical manipulatives, and technological tools as components of curriculum resources. The participants described learning experiences connected to mathematical modeling which focused on the use of manipulatives to support students in understanding mathematics concepts and to solve problems. Those learning experiences reinforced the perception that the SMP, model with mathematics and mathematical modeling are the same.

Modeling as a Conceptual Category

The CCSSM (2010) organizes standards into domains, and domains into conceptual categories or themes. Modeling is unique among the six conceptual categories in high school mathematics courses because in stark contrast with the other conceptual categories there are no standards or modeling competencies discretely aligned to modeling. Instead, the standards documents denote a set of standards from the other five conceptual categories as “related” standards. This distinct feature of the standards documents reinforces theme 1, content mastery. The tasks participants selected as evidence of mathematical modeling are centered in mathematics concepts from “related” standards in the Algebra II curriculum. In addition to the related standards the documents contain an iterative modeling cycling absent of an example of its application. The standards documents do not include a description of a mathematical model and use the term “model” to represent; an object, a representation such as a graph or an equation, and

as a competent model of the modeling process causing a lack of clarity around the expectation (Cirillo et al., 2016). Again, the lack of clarity recenters mathematical modeling in teacher experiences and resources designed to support the SMP.

Non-content Specific Instructional Best Practices

The second theme that emerged from the cross-case analysis was a positive classroom culture and instructional that were centered in a connection to students inclusive of their prior experiences, with-in and outside of mathematics content, interests. This theme was pivotal to the goals that participants assigned to mathematical modeling, their descriptions of mathematical models as both representations and teacher tools, categories two and three, along with the selection of tasks and plan for classroom interactions. Again, since the standards documents omit a description or goal for mathematical modeling the emergence of this theme in relationship to mathematical modeling suggests that participants leaned into their experience in effective instructional practices. Ladson-Billings (1994) lays the foundation for Culturally Relevant Teaching (CRT) in her observations of the following among successful teachers of African American children: (a) Students that are treated as competent often meet that expectation, (b) Instructional scaffolding creates a bridge from what students know to what they need to know, (c) Classrooms and teachers should be focused on instruction, (d) Education is about extending student thinking, and (e) Effective teachers know both their content and their students, deeply. These observations are alive in the beliefs and goals the participants in this study assigned to mathematical modeling. The description of mathematical models, as representations to support students in drawing connections between Algebra II mathematics concepts from mathematics concepts from previous grade levels. Their desire to select mathematical modeling tasks that connect to student experience, interest, and post-secondary options through their real-world

contexts and that include scaffolds to content mastery are also included in these five observations which have long been a focus of teacher preparation programs, in-service professional learning, and school reform movements.

The third and final category among the descriptions of mathematical models and mathematical modeling shared by participants was a mathematical model as a teacher model, and modeling as the practice of teachers demonstrating the movement between representations to solve real-world and mathematical problems. Gradual Release of Responsibility (GRR) is an approach for structured teaching in four phases: (a) Focused instruction, (b) Guided instruction, (c) Collaborative learning, and (d) Independent learning often quoted as “I (the teacher) do, We (teacher and students, students with peers) do, You (each student) do” (Fisher & Frey, 2008). David was the only participant to directly describe mathematical modeling as “I do, you do, we do” (Interview 1), but this framework is present in ways each participant described their planned or past implementation of mathematical modeling tasks.

Implications

One of the major findings of this study was that though mathematical modeling has been included in the CCSSM (2010) and is an expectation of in all high school mathematics courses, the participants have not been provided learning opportunities to support the knowledge needed for teaching mathematical modeling. In addition to participants describing limited learning opportunities, the expectations for mathematical models and mathematical modeling are not clearly defined in the CCSSM (2010). Beginning in the construct of MKT (Ball et al., 2008), there are four dimensions of knowledge that Ferri and Blum (2009) found to be essential for teachers of mathematical modeling: (a) knowledge of modeling cycles, goals, task types, and perspectives of mathematical modeling, (b) the ability to solve and create tasks, (c) the ability to

plan and implement modeling lessons, and (d) the ability to identify and address student misconceptions related to modeling. The findings of this study along with these four dimensions have implications for: state departments of education as they adopt policies which impact instructional requirements, teacher preparation programs and faculty, district mathematics leaders, and school based leaders. Each of these stakeholders has a level of responsibility in ensuring pre-service and in-service teachers have what they need to support student learning. At the conclusion of the first interview, I asked Sarah if there was anything else she wanted to share about mathematical models, modeling, task that she wanted to share. Here is her response, in part:

“Just because you put it in a standard does not mean that it’s going to translate down into the classroom. There’s a lot of focus of this aspect of teacher in math, but it’s just causing more frustration than anything else because you can call for something, demand it but unless you give us the tools, give them (students) the tools it can’t happen.” (Sarah, Interview 1)

This quote stamps the responsibility of each group of stakeholders and implores them to take an inventory of their environments to determine whether teachers that are charged with providing mathematical modeling instruction have the following resources:

- Clear Expectations- A consistent description of a mathematical model, the modeling process along with its’ goals and examples of modeling tasks.
- Professional Learning Opportunities- Access to learning opportunities in pre-service and in-service settings that reinforces the student expectations for mathematical modeling but is centered supporting teachers in the development of the competencies in dimensions three and four.
- Time for Collaboration- The competencies in dimensions three and four require a collaborative time and space for teachers to “try on the work,” practice elements

of delivery, analyze student work and refine lesson plans, receive feedback, reflect, and refine their practice.

In addition to the implications above the findings of this study, specifically the absence of learning experiences connected to a curriculum expectation implore those charged with crafting and adopting educational policy to consider the impact on both educators and students when policy is not accompanied with explicit and on-going professional learning and support.

Recommendations for Future Research

In this study I explored the perspectives of mathematical modeling held by secondary mathematics teachers and the connection between those perspectives and the ways they plan for learning experiences for students. This study did not consider the actual implementation of mathematical modeling tasks or the opportunity to observe teacher practices or student thinking. Research centered on teacher practices and student behaviors during the implementation of mathematical modeling tasks may provide additional insight on the ways in which teachers engage students in mathematical modeling.

The findings of this study show an alignment between participant descriptions of mathematical models and mathematical modeling and SMP5 Model with Mathematics. Participants described in-service professional learning experiences that reinforce that modeling with mathematics involves the use of tools, and representations to move from concrete to abstract understandings of mathematics concepts. Research that explores the perspectives of mathematical modeling among district and school mathematics leaders and the professional learning experiences they facilitate for teachers could be useful in informing the development of state and district professional learning plans around mathematical modeling.

Also connected to the learning experiences described by the participants in this study was the absence of learning connected to the four dimensions of MKT Ferri and Blum (2009) identified as necessary for teachers of mathematical modeling in their college coursework or in-service professional learning. Given that this study involved a small sample in the same metropolitan area one wondering I have is whether or how this experience and exposure varies among a larger sample and other geographic locations. Particularly since states and districts have elected to communicate and interpret the expectations of the CCSSM in different ways.

Similar to the role of prior knowledge for students learning, the prior knowledge of teachers, particularly experienced teachers such as the ones in this study, should be considered in planning learning opportunities. If mathematical modeling remains an expectation in secondary classrooms and that the implication that professional learning on mathematical modeling is needed exists beyond this study, teacher voices should inform the planning of those learning experiences. As a final recommendation for future study, I offer the suggestion of research that clearly defines the expectations of mathematical modeling and seeks to identify the types of professional learning and instructional tools teachers would find useful in preparing students to meet those expectations.

Personal Statement from the Researcher

I began this study aware of the discussion in the research base regarding the lack of consensus regarding mathematical modeling and the lack of clarity in the CCSSM. I was largely convinced that those were discussions that were only occurring among in academic settings and that the small nuanced in the descriptions of mathematical models and the modeling process, the goals of mathematical modeling, and which tasks support mathematical modeling would be inconsequential among experienced high school teachers, teachers I considered to be my peers. I

expected that at the core of participants' descriptions of mathematical modeling would be a cyclical and iterative process between the real-world and mathematics. In order to maintain the integrity of data collection and data analysis I was incredibly cautious not to allow that expectation to filter into the data I collected or the analysis, constantly checking and double checking, playing, and replaying interview audio recordings, reading transcripts, repeating coding cycles. That process and this study forced me to confront that my expectation was based on privilege awarded to me from two sources. The first is that my original undergraduate major was applied mathematics. I submitted a "change of major" form just three semesters before graduation to avoid a geometry course that was notoriously difficult but before that change I'd taken lab courses dedicated to "running" mathematical models, making changes some small, some large to get closer and closer to real-world behaviors. The second source of privilege was the hardest for me to confront because as I shared, though I have been outside of a full-time teaching position since 2008 I still consider, or I should say considered, experienced secondary mathematics teachers my peers. As proof of that claim, I held tightly to the fact that while I did have a "district title" my office was inside of a school and I was compensated on a teacher pay scale until 2013 when I accepted a position with a larger district, and "almost all of my friends are math teachers." While my first position outside of the classroom didn't allow me to purchase a new car, it did provide access to resources that were not available to me as a classroom teacher: (a) time to unpack and internalize the CCSSM and (b) the ability to participate in professional learning and professional learning communities with educators all over the country because doing so didn't require substitute coverage, and the costs could be absorbed by the grant that allowed the district to ability to fund by "teacher salary." I expected experienced secondary

teachers to share my perspective of mathematical modeling, leverage that perspective to plan for instruction without any of my experiences.

In addition to confronting my privilege, this study and the participants caused me to consider, and re-consider my professional practice as I engage with teachers. As school districts and education agencies adopt mission and vision statements grounded in “students first” I know that the realization of that vision cannot occur absent of a parallel mission or commitment to creating learning spaces for teachers. Years ago, my supervisor also supervised school principals, and we would visit classrooms along with the school principal every Monday. As soon as we exited the classroom my supervisor would turn to the school principal and say, “Tell me who sat with that teacher last week and how did they make sure she/he/they were ready to be in front of students today?” The first few times it happened most of the principals were stunned, in general the answers varied from silence to talk about professional learning communities, instructional coaches, school activities, personnel matters, and the weather. Honestly, I did not always listen carefully to the responses, but I understood that in asking it he was setting an expectation for teacher support at his assigned schools. I have also felt that my role, my responsibility was also to support teachers. The lasting impression of this study on my practice, on my conscious is that in addition to having someone to sit with them and offer support, ensuring teachers have what they need to promote the realization of our ambitious student-centered missions and visions for optimal learning environments that support lifelong learners, teachers also need advocates. They need spaces to be heard and valued, for their voices to have an impact on policy, on standards and curriculum, on professional learning, and on all the factors connected to their practice. The last line of most job descriptions includes some variation of “other duties as assigned,” and this

study, these participants have given me a new assignment: to put my perspective and experiences aside, listen, and fiercely advocate for schools to be optimal learning environments for teachers.

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APPENDICES

Appendix A

INFORMED CONSENT
Georgia State University
Department of Middle and Secondary Education
Informed Consent

Title: Secondary Mathematics Teacher Perspectives of Mathematical Modeling
Principal Investigator: Dr. Christine Thomas
Simone Wells-Heard

I. Purpose:

You are invited to participate in a research study. The purpose of this study is to explore secondary mathematics teachers' perspectives of mathematical modeling. You are invited to participate because you are a high school mathematics teacher, currently teaching the Georgia Standards of Excellence (GSE) Algebra II course. A total of 5 participants will be recruited for this study. Your participation in this study will require 15 hours of your time over the course of four months during the Fall of 2019.

II. Procedures

If you decide to participate, your participation in this research project will require approximately 15 hours of your time during the Fall of 2019 and will include completing a survey, two 75-minute interviews, one 40-minute interview, and lesson artifacts from your classroom. The details of your participation are given below:

1. You will be asked to complete an electronic survey on your education and professional background.
2. You will be asked to participate in a 75-minute interview about mathematical models and mathematical modeling. This interview will be auto recorded.
3. You will be asked to select and submit a single mathematical modeling task that you have used or plan to use with students; along with supporting artifacts such as unit plans, lesson plans, teacher exemplars, and/or anonymous student work samples.
4. You will be asked to participate in a second 75-minute interview where we discuss your selected task and artifacts. This interview will be audio recorded.
5. You will be asked to participate in a 40-minute interview in order to bring clarification to the researchers' understandings. This interview will be audio recorded.

III. Risks:

In this study, you will not have any more risks than you would in a normal day of life.

IV. Benefits:

Participation in this study may give you the satisfaction of having participated in a study that has contributed to the field of teaching and learning mathematics.

V. Voluntary Participation and Withdrawal:

Participation in research is voluntary. You do not have to participate in this study. If you decide to participate in the study and change your mind, you may do so at any time. You may also choose to skip questions or stop participating during the interviews. You will not lose any benefits to which you are otherwise entitled.

VI. Confidentiality:

We will keep your records private to the extent allowed by law. Dr. Thomas and I will have access to the information you provide. Information may also be shared with those who make sure the study is done correctly (GSU Institutional Review Board, the Office for Human Research Protection (OHRP)). We will use an assigned numeric code rather than your name on all study records. Your name and other facts that might point to you will not appear when we present this study or publish its results. You will not be personally identified. The information you provide will be stored in a locked cabinet, and a password and firewall protected computer in my home office for a period of 5 years.

VII. Contact Persons:

Contact Dr. Christine Thomas at 404-413-8065 or cthomas11@gsu.edu or Simone Wells-Heard at 770-548-7544 or swellsheard1@student.gsu.edu if you have any questions about this study. If you have questions or concerns about your rights as a participant in this research study, you may contact Susan Vogtner in the Office of Research Integrity at 404-413-3513 or svogtner1@gsu.edu.

VIII. Copy of Consent Form to Subject:

We will give you a copy of this consent form to keep. If you are willing to volunteer for this research, and you are willing to be auto recorded, please sign below.

Participant

Date

Principal Investigator or Researcher Obtaining Consent

Date

Appendix B**PARTICIPANT SURVEY**

1. Name:
2. Including this year, how many years of teaching experience do you have?
3. Including this year, how many years have you taught high school mathematics?
4. Including this year, how many years have you taught Algebra 2?
5. What type of teaching certification do you hold? (Select one response only)
 - Standard clear and renewable professional certificate
 - Provisional professional certificate (requires additional college coursework, student teacher, and/or the completion of an alternative certification program)
6. Do you hold National Board Certification in Adolescence and Young Adulthood Mathematics?
 yes no
7. What area(s) do you hold a teaching certificate in? (Select all that apply)
 - Elementary Education
 - Middle Grades Education
 - Secondary Education
 - Special Education
 - Mathematics
 - Natural Sciences and/or Technology Education
 - Social Sciences and/or Humanities
8. What is the highest level of education you have completed? (Select one response only)
 - Undergraduate degree

- Master's Degree
- Education Specialist or other professional degree
- Doctorate

9. As an undergraduate did you have a major, minor, or emphasis in any of the following areas? (Select one on each row)

	Major	Minor	Emphasis	No
Mathematics				
Mathematics Education				
Natural Sciences and/or Technology				
Social Sciences and Humanities				

10. If applicable, as graduate did you have a major, minor, or coursework in any of the following areas? (Select one on each row)

	Major	Minor	Coursework	No
Mathematics				
Mathematical Modeling or Applied Mathematics				
Mathematics Education				
Natural Sciences and/or Technology				
Social Sciences and Humanities				

Appendix C

INTERVIEW PROTOCOLS

Thank you for agreeing to be interviewed. The interviews will be audio recorded. I am conducting a study on high school mathematics teachers' perspectives of mathematical modeling. I am interested in knowing how high school mathematics teachers use their knowledge of mathematical modeling to plan for instruction. Your participation in research is completely voluntary, & you have the right to end it at any time. During these interviews you may skip questions or stop participating at any time. Your decision to skip questions or end the interview will not impact any benefits to which you are otherwise entitled. The information you provide will be kept confidential pursuant to the law.

How would you describe a mathematical model?

Considering your description of a mathematical model, how would you describe the mathematical modeling process?

Did you engage in college coursework and/or in-service professional learning on mathematical modeling? Tell me about your experiences with mathematical modeling.

What do you see as the goals of mathematical modeling?

Why do you believe mathematical models and modeling are explicitly included in the CCSSM?

How would you describe the ideal mathematical modeling task?

Is there anything else you'd like to share about mathematical models, mathematical modeling, or modeling tasks?

Thank you and that concludes the interview portion. I will review your responses and schedule a second interview to discuss your planned or implemented lesson. If you should have any questions and/or concerns, you may contact me at 770-548-7544 or

swellsheard1@student.gsu.edu.

Thank you again for agreeing to be interviewed. The interviews will be audio recorded. As a reminder, I am conducting a study on high school mathematics teachers' perspectives of mathematical modeling. I am interested in knowing how high school mathematics teachers use their knowledge of mathematical modeling to plan for instruction. Your participation in research is completely voluntary, & you have the right to end it at any time. During these interviews you may skip questions or stop participating at any time. Your decision to skip questions or end the interview will not impact any benefits to which you are otherwise entitled. The information you provide will be kept confidential pursuant to the law.

Describe your process for selecting mathematical modeling tasks.

Tell me about your reasons for selecting this task.

What were/are the goals of this task? How will you know if they have been achieved?

What prior knowledge will students need to attempt this task?

What will be the setting for this task (whole group, individual, pairs, groups, or a combination)?

Explain.

How do/did you plan to ignite student engagement with this task?

What will your role be during this task?

What feedback will students be given on their work?

Tell me about your next steps after this lesson.

Is there anything else you'd like to share about this lesson/task?

Thank you and that concludes the interview portion. I will review your responses and schedule a final interview to clarify my understandings. If you should have any questions and/or concerns, you may contact me at 770-548-7544 or swellsheard1@student.gsu.edu.