Eigenstate Equivalent Circuit for Bi-Periodic Surfaces: Application to Three Parallel Dipoles

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***Abstract*—An equivalent circuit for the unit cell of bi-periodic surfaces is proposed. The equivalent circuit is general and simple. It is based on the decomposition of the eigenmodes of the structure and it takes into account the polarization of the incident wave. It consists of the interconnection of two different admittances and two transformers. The model of a unit cell composed of three parallel dipoles is extracted. The transformer of the proposed equivalent circuit models the rotation of the dipoles, while the admittances model their frequency behaviour (resonances). This equivalent circuit provides physical insight and, at the same time, it models in separate elements the contribution of the rotation and the dipole trio resonances. The model is able to replicate the full-wave simulation result with great accuracy.**

***Index Terms*—bi-periodic scatterers, eigenstates, equivalent circuits, lumped elements, polarizing reflector.**

I. Introduction

There has been an increased interest in bi-periodic scatterers, such as transmitarrays, reflectarrays, polarization rotators, or rasorbers. The analysis and synthesis of such structures are usually performed by FEM or FDTD full-wave electromagnetic simulation [1]. Another approach is by the means of an equivalent circuit [2], which has obvious advantages, but usually needs a more in-depth study of the structure and is only manageable for simple geometries.

The authors proposed in [3] a general equivalent-circuit topology for asymmetric two-ports that allowed the decomposition in eigenstates. The circuit consists of two parallel networks made of a transformer with an admittance at each side. The values of the admittances are the eigenvalues of the admittance matrix of the two-port, while the turns ratio of the transformer is the ratio between the components of its eigenvectors. The topology has very interesting properties but it has not been applied to many different structures yet.

The proposal of this work is to use this eigenstate-based equivalent circuit to elaborate a simple model with enhanced capabilities for the unit cells of bi-periodic scatterers. To provide an example of application and illustrate the benefits of the use of the topology, the model will be applied to the structure of [1] used to design a linear-to-circular polarization reflector.

The proposed approach consists in extracting information from the computational effort of one or a few electromagnetic simulations, to obtain a circuit with a greater field of application than the simulated geometry from which it was obtained. Compared with other equivalent circuits that simply seek to model the results of the simulations [2], [4], [5], this approach leads to much faster results that also provide physical insight. There are also those that seek to provide information on the behavior of the unit cell by analytical methods, avoiding simulations [6], [7]. Although their subsequent use is much more efficient, their scope of application is limited to geometries which, although interesting, must be relatively simple.

II. Equivalent Circuit Topology

A usual way of analyzing bi-periodic surfaces consists in simulating a single unit cell surrounded by walls of periodic boundary conditions. This allows for a fast simulation with the assumption that it would be infinitely large and with the same periodicity along its surface, a good approximated result for bi-periodic surfaces made of a large number of unit cells. In order to simplify the analysis even more, other assumptions can be made, such as considering only normal incidence, zero-thickness metallization, and the cut-off of higher-order Floquet modes. To consider normal incidence under any angle, it is possible to decompose the two orthogonal polarizations incident to the surface (vertical and horizontal polarizations) and treating each of these polarizations as separate ports in the structure. These all simplifications do not compromise the accuracy of the analysis significantly and allow the behavior of the unit cell to be reproduced by using a four-port circuit made of the connection of a single two-port as shown in [2].

An eigenstate-based equivalent circuit can be used for modeling the two-port network. Specifically, the proposed topology is extensively analyzed in [3] and is shown in Fig. 1. The parameters of the model, two different admittances and two transformers, are obtained from the eigenvalues and eigenvectors of the admittance matrix of the two-port. λ_1 and λ_2 are each of the eigenvalues, and $\tan(\phi)$ (denoted as p in [3]) is the ratio of the components of the eigenvectors. This circuit topology guarantees that the real parts of the admittances are always positive or zero, as shown in [3]. In this case, $\tan(\phi)$ is used instead of p because the transformer gives information about the relation of the eigenexcitations,

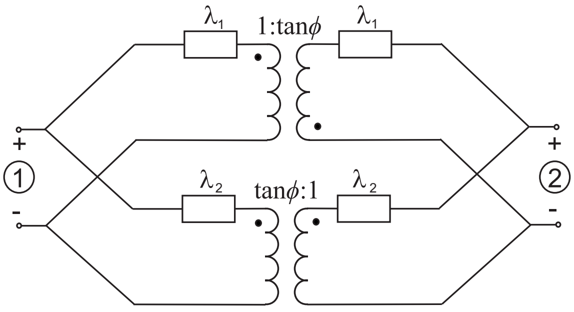

Fig. 1. Proposed topology for the equivalent circuit.

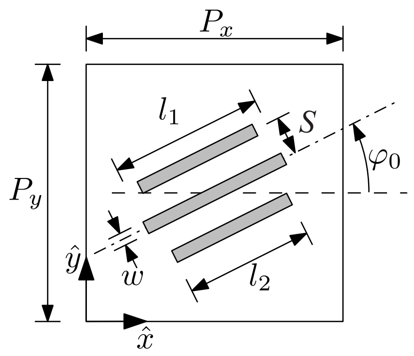

Fig. 2. Geometry of the unit cell under study ($P_x = P_y = 10$ mm, $l_1 = 7$ mm, $l_2 = 5.6$ mm, $w_1 = 0.5$ mm, $w_2 = 0.4$ mm, and $S = 1.1$ mm).

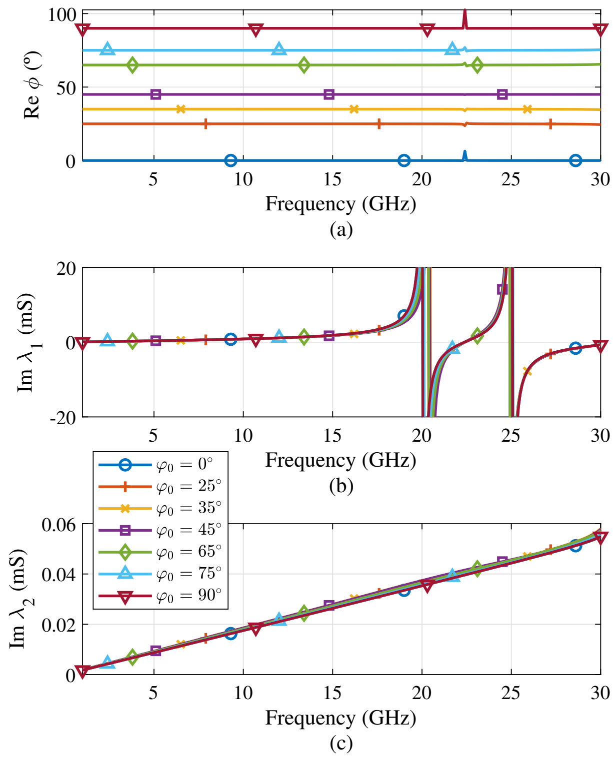

Fig. 3. Computed parameters of the equivalent circuit for the unit cell under study.

which have a relationship with the orientation of the unit cell, as will be shown in Section III with a practical example.

III. Application to Three Dipoles

In order to illustrate the properties of the proposed equivalent circuit with a simple and practical example, a unit cell with the same topology as that used in [1] will be analyzed and its model extracted using the approach of Section II. The structure consists of three parallel flat dipoles made of perfect electric conductor suspended into air, with the side dipoles being slightly shorter than the one in the center, and all rotated an angle φ_0 with respect to the x axis. Fig. 2 shows the geometry of the unit cell and its dimensions. The bi-periodic surface has been simulated using HFSS for different values of φ_0 (0°, 25°, 35°, 45°, 65°, 75°, and 90°) up to 30 GHz (cut-off frequency of the higher-order Floquet modes), the eigenvalues and the eigenvectors of the two-port are obtained, and the extracted circuit parameters are plotted in Fig. 3.

As proved in [3], since the structure is lossless, the parameter ϕ is real and λ_1 and λ_2 are imaginary. It can be seen that ϕ corresponds with the rotation of the structure, φ_0, at every single frequency except for 22.5 GHz. This is due to its value being indeterminate when λ_1 is 0. Also, two resonances are illustrated by λ_1, each for each dipole's length. Since the dipoles are relatively thin, the value of λ_2 is low and negligible, even though it clearly shows a capacitive behavior. An important result is the fact that the values of the admittances, λ_1 and λ_2, have almost no change when the dipoles are rotated. From this information, a simple and accurate equivalent circuit can be extracted and is shown in Fig. 4(a). The effect of λ_2 has been neglected in order to simplify even more the circuit (leaving only one transformer and two identical admittances). The turns ratio of the transformer is 1:tan(φ_0) and the admittance, λ_1, is modeled by three series LC resonators in parallel, as shown in Fig. 4(b). The first LC resonator resonates at 20 GHz, the second at 25 GHz, and the third one at 39 GHz (the last resonator could be removed if performance in the 27-30 GHz band is not a concern). Note that all the information about the rotation is included in the transformer and, thus, the same admittances can be used for any value of φ_0.

To prove the performance of the equivalent circuit proposed in Fig. 4, the S parameters of the four-port obtained from it are compared to those obtained from simulation in Fig. 5 for $\varphi = 25°$. Only the relevant three parameters, S_{11}, S_{21}, and S_{22}, are shown because, as explained in [2], the rest of the parameters can be extracted from these three due to the assumptions made in Section II. It can be seen that the S parameters are reproduced by the equivalent circuit with a great level of accuracy over the band, from 1 to 30 GHz. The same level of precision can be obtained for other values of φ by just modifying the value of the transformer accordingly.

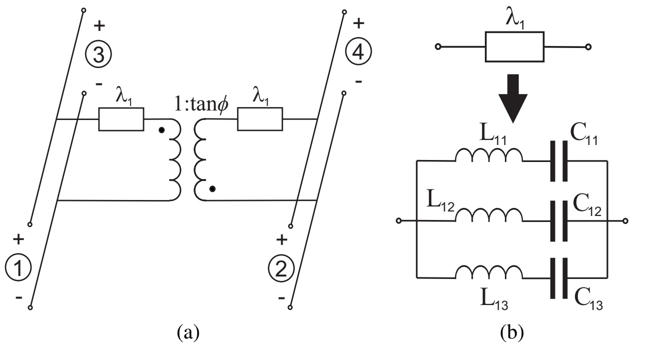

Fig. 4. Proposed equivalent circuit for the unit-cell, with $\phi = \varphi_0^\circ$. (a) General topology. (b) Equivalent circuit for the admittance ($L_{11} = 12.8$ nH, $C_{11} = 4.9$ fF, $L_{12} = 12.7$ nH, $C_{12} = 3.2$ fF, $L_{13} = 9$ nH, and $C_{13} = 2.8$ fF).

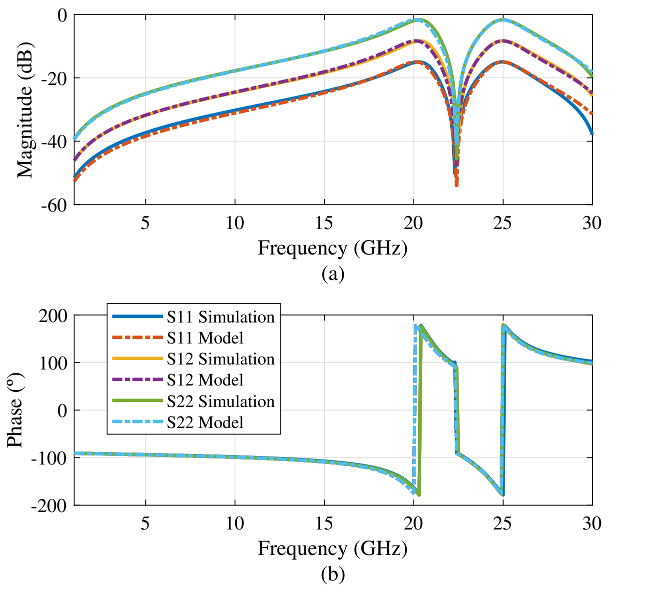

Fig. 5. S parameters of the periodic structure from simulation and using the proposed equivalent circuit for $\varphi_0 = 25°$.

IV. Conclusion

The proposed equivalent-circuit model for unit cells of biperiodic surfaces shows exceptional results. It allows the easy decomposition of the vertical and horizontal polarizations with meaningful results. In the practical case shown in this communication, three parallel dipoles are modeled. The proposed equivalent circuit is very simple, provides physical insight, and is especially useful if the rotation of the dipoles is to be parametrized. The model agrees with the simulation results.

This circuit can be used to analyze the unit cell of transmitarrays, reflectarrays, rasorbers, or polarizing surfaces. Although not explicitly shown in this paper, the model also works for more realistic surfaces printed on a dielectric substrate, ended in a ground plane, or made of stacked structures. Proof of this and the application to other unit-cell geometries are expected to be shown at the date of the conference.

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References

[1] E. Martínez-de Rioja, J. A. Encinar, A. Pino, and Y. Rodríguez-Vaqueiro, "Broadband linear-to-circular polarizing reflector for space applications in ka-band," *IEEE Trans. Antennas Propag.*, vol. 68, no. 9, pp. 6826–6831, Sep. 2020.

[2] G. Pérez-Palomino and J. E. Page, "Bimode Foster's equivalent circuit of arbitrary planar periodic structures and its application to design polarization controller devices," *IEEE Trans. Antennas Propag.*, vol. 68, no. 7, pp. 5308–5321, 2020.

[3] A. Hernández-Escobar, E. Abdo-Sánchez, P. Mateos-Ruiz, J. Esteban, T. M. Martín-Guerrero, and C. Camacho-Peñalosa, "An equivalent-circuit topology for lossy non-symmetric reciprocal two-ports," *IEEE J. Microw.*, vol. 1, no. 3, pp. 810–820, 2021.

[4] S. Maci, M. Caiazzo, A. Cucini, and M. Casaletti, "A pole-zero matching method for EBG surfaces composed of a dipole FSS printed on a grounded dielectric slab," *IEEE Trans. Antennas Propag.*, vol. 53, no. 1, pp. 70–81, Jan 2005.

[5] F. Costa and A. Monorchio, "Closed-form analysis of reflection losses in microstrip reflectarray antennas," *IEEE Trans. Antennas Propag.*, vol. 60, no. 10, pp. 4650–4660, Oct 2012.

[6] F. Mesa, R. Rodriguez-Berral, and F. Medina, "Unlocking complexity using the eca: The equivalent circuit model as an efficient and physically insightful tool for microwave engineering," *IEEE Microw. Mag.*, vol. 19, no. 4, pp. 44–65, June 2018.

[7] C. Molero and M. García-Vigueras, "Circuit modeling of 3-D cells to design versatile full-metal polarizers," *IEEE Trans. Microw. Theory Techn.*, vol. 67, no. 4, pp. 1357–1369, April 2019.