# **Effective length of short Fabry-Perot cavity** formed by uniform fiber Bragg gratings

## Yuri O. Barmenkov

Centro de Investigaciones en Óptica, Loma del Bosque 115, Col. Lomas del Campestre, 37150, León, Gto, México yuri@cio.mx

#### Dobryna Zalvidea, Salvador Torres-Peiró, Jose L. Cruz, and Miguel V. Andrés

Departamento de Física Aplicada, Universidad de Valencia, Dr. Moliner 50, E46100 Burjassot (Valencia), Spain

Abstract: In this paper, we describe the properties of Fabry-Perot fiber cavity formed by two fiber Bragg gratings in terms of the grating effective length. We show that the grating effective length is determined by the group delay of the grating, which depends on its diffraction efficiency and physical length. We present a simple analytical formula for calculation of the effective length of the uniform fiber Bragg grating and the frequency separation between consecutive resonances of a Fabry-Perot cavity. Experimental results on the cavity transmission spectra for different values of the gratings' reflectivity support the presented theory.

© 2006 Optical Society of America

OCIS codes: (050.2230) Diffraction and gratings: Fabry-Perot; (140.3510) Lasers and Laser Optics: Lasers, fiber; (230.1480) Optical devices: Bragg reflectors.

## **References and links**

- J.S.Zyskind, V. Mizrahi, D.J. DiGiovanni and J.W. Sulhoff, "Short single-frequency erbium-doped fiber laser," 1 Electron. Lett. 28, 1385-1387 (1992).
- Ch. Spiegelberg, J. Geng, Y. Hu, Yu. Kaneda, S. Jiang and N. Peyghambarian, "Low-noise narrow-linewidth fiber laser at 1550 nm (June 2003)," J. Lightwave Techn. 22, 57-62 (2004). 2
- Yu.O. Barmenkov, A.V. Kir'yanov, J. Mora, J.L. Cruz and M.V. Andrés, The continuous-wave and giant-pulse 3. operations of a single-frequency Erbium-doped fiber laser. IEEE Photon. Technol. Lett. 17, 28-30 (2005).
- 4. Pavel Polynkin, Alexander Polynkin, Masud Mansuripur, Jerome Moloney and N. Peyghambarian, "Singlefrequency laser oscillator with watts-level output power at 1.5 µm by use of a twisted-mode technique," Opt. Lett. 30, 2745-2747 (2005).
- J. Canning, N. Groothoff, E. Buckley, T. Ryan, K. Lyytikainen and J. Digweed, "All-fibre photonic crystal distributed Bragg reflector (PC-DBR) fibre laser," Opt. Express **11**, 1995-2000 (2003). 5
- J.J. Zayhowski, "Limits imposed by spatial hole burning on the single-mode operation of standing-wave laser 6. cavities," Opt. Lett. 15, 431-433 (1990).
- J. Canning, M. Janos and M.G. Sceats, "Rayleigh longitudinal profile of optical resonances within waveguide 7 grating structures using sidescattered light," Opt. Lett. 21, 609-611 (1996).
- 8.
- Orazio Svelto, *Principles of Lasers*, (New York: Plenum Press, 1989), chapter 4. T. Erdogan, "Fiber grating spectra," J. Lightwave Technol. **15**, 1277-1294 (1997).
- 10. A. Othonos and K. Kalli, Fiber Bragg gratings: fundamentals and applications in telecommunications and sensing, (Boston: Artech House, 1999), chapter 5.
- 11. Raman Kashyap, "Fiber Bragg Gratings," San Diego: Academic Press, 1999, chapter 4.
- 12. H. Renner, "Effective-index increase, form birefringence and transition losses in UV-side-illuminated photosensitive fibers," Opt. Express 11, 546-560 (2001).
- K. Dossou, S. LaRochelle and M. Fontaine, "Numerical Analysis of the Contribution of the Transverse 13. Asymmetry in the Photo-Induced Index Change Profile to the Birefringence of Optical Fiber," J. Lightwave Technol. 20, 1463-1470 (2002).

## **1. Introduction**

Single-frequency fiber lasers are promising light sources for applications in optical communications, sensors, spectroscopy and interferometry. Such lasers with fiber Bragg gratings (FBGs) as output couplers, or distributed Bragg reflector (DBR) lasers, are

Received 22 May 2006; revised 22 June 2006; accepted 28 June 2006 #71062 - \$15.00 USD (C) 2006 OSA 10 July 2006 / Vol. 14, No. 14 / OPTICS EXPRESS 6394 potentially high-stable, low-noise and narrow-bandwidth devices [1-4] and have been extensively treated during the last decade. Recent results on rare-earth-doped photonic crystal fibers demonstrate new promising opportunities for this type of lasers [5]. DBR fiber lasers offer some important advantage, with respect to distributed feed-back lasers, as, for instance, possibility to utilize very high doped fibers inside the Fabry-Perot cavity or longer cavities. The output power achieved directly from DBR fiber lasers has been from hundreds of mW [2] to watts [4], which is limited by spatial hole burning [6] or/and short active fiber length inside the cavity.

A short fiber cavity is characterized by a high spacing between the cavity modes and may support a single-frequency regime when the FBGs reflectors are selective enough. For reaching single-frequency lasing one needs to estimate the optimal cavity length, at which the output laser power is maximized and only one longitudinal mode is supported at the same time. Since the FBG reflector is distributed along its length, knowledge of the effective FBG's length is needed for a proper estimate of the cavity length. In fact, one can use side-scattered light to measure the power distribution along FBGs [7], but this type of measurements does not provide a precise value for gratings' effective length.

In this paper we study the properties of a Fabry-Perot cavity in terms of FBG's effective length. We demonstrate that mode-spacing increases at an increase of the grating's reflectivity owing to the grating effective length shortening. We show that the effective length of FBG is determined by the group delay of light reflected from the grating and depends on its diffraction efficiency. It varies from a half of the physical grating length, when reflectivity is low, to zero, when reflectivity is high. We present a simple analytical formula for calculation of the effective length of the uniform fiber Bragg grating. Theoretical results are compared with an experimental study of transmission of a Fabry-Perot fiber cavity formed by two uniform FBGs.

## 2. Effective length of uniform FBGs forming a Fabry-Perot fiber cavity

Following to the calculations for a Fabry-Perot cavity (see, e.g., Ref. [8]), the power transmittance of the cavity with two FBGs is written in the form

$$T = \frac{(1 - R_1)(1 - R_2)}{\left(1 - \sqrt{R_1 R_2}\right)^2 + 4\sqrt{R_1 R_2} \sin^2\left(\beta L_0 + \frac{\varphi_1 + \varphi_2}{2}\right)},$$
(1)

where  $R_{1,2}$  are the reflection coefficients for the corresponding gratings,  $L_0$  is the length of fiber between FBGs (see Fig.1),  $\beta$  is the propagation constant of the fundamental mode LP<sub>01</sub>, and  $\varphi_{1,2}$  are the phases of the FBGs' reflection coefficients.



Fig. 1. Fabry-Perot fiber cavity formed by two FBGs (FBG1 and FBG2).  $L_{1,2}$  are the physical lengths of FBGs and  $L_0$  is the distance between them.

The transmittance maxima corresponding to the Fabry-Perot resonances are observed when the argument of sinus in Eq. (1) is multiple by  $\pi$ . From this, taking into account dispersions of the fiber and FBGs [9,10], the free spectral range (mode-spacing) of the Fabry-Perot cavity,  $\Delta\lambda$ , can be expressed as

#71062 - \$15.00 USD	Received 22 May 2006; revised 22 June 2006; accepted 28 June 2006
(C) 2006 OSA	10 July 2006 / Vol. 14, No. 14 / OPTICS EXPRESS 6395

$$\Delta \lambda = \frac{\lambda^2}{2n_g \left( L_0 + L_{eff1} + L_{eff2} \right)},\tag{2}$$

where  $\lambda$  is the free-space wavelength,  $n_g$  is the group refractive index for LP<sub>01</sub> fiber mode, and the effective lengths of the FBGs,  $L_{eff1,2}$ , are defined as

$$L_{eff} = \frac{v_g \tau_{1,2}}{2},\tag{3}$$

where  $v_g = c/n_g$  is the group velocity,  $\tau_{1,2} = \partial \varphi_{1,2}/\partial \omega$  are the group delays for corresponding FBGs, and  $\omega$  is the angular optical frequency. Note that for conventional optical fibers  $n_g \approx n_{eff}$ , where  $n_{eff}$  is the effective refractive index (the difference between  $n_g$  and  $n_{eff}$  is  $\sim 1\%$  for fused silica at 1.5 µm), thus  $v_g$  in Eq. (3) can be replaced with the fundamental mode velocity  $v = c/n_{eff}$ .

From Eq. (2) one can conclude that the effective cavity length,  $L_c$ , is a sum of the effective lengths of both the FBGs forming the cavity and the distance between them is  $L_c = L_0 + L_{eff1} + L_{eff2}$ .

Reflectivity of FBG is usually calculated using the coupled-mode theory for two counterpropagating waves [9,10,11]. From this theory the reflection coefficient for the field amplitude,  $\rho$ , is:

$$\rho = -\frac{\kappa \sinh(\varepsilon L)}{\xi \sinh(\varepsilon L) + i\varepsilon \cosh(\varepsilon L)},\tag{4}$$

where  $\kappa = \pi n_1/\lambda$  is the coupling coefficient  $(n_1$  is the grating amplitude and  $\lambda$  is the free-space wavelength),  $\varepsilon = \sqrt{\kappa^2 - \xi^2}$  (here  $\xi = 2\pi n_{eff} (\lambda^{-1} - \lambda_0^{-1})$  is the detuning from the Bragg wavelength,  $\lambda_0$ ) and *L* is the grating physical length. The group delay of the light reflected by a FBG,  $\tau$ , can be found from the reflection coefficient phase:

$$\tau = -\frac{\lambda^2}{2\pi c} \frac{d\varphi}{d\lambda},\tag{5}$$

where  $\varphi$  is obtained from Eq. (4) as:

$$\varphi = -\operatorname{atan}\left(\frac{\varepsilon}{\xi}\operatorname{cotanh}(\varepsilon L)\right)$$
(6)

From Eqs. (3,5,6) the grating effective length at the wavelength  $\lambda_0$  is found as:

$$L_{eff} = \frac{\lambda_0}{2\pi n_1} \tanh\left(\frac{\pi n_1}{\lambda_0}L\right).$$
(7)

Taking into account the dependence of the grating diffraction efficiency,  $R = |\rho(\lambda_0)|^2$ , on the grating amplitude  $n_1$  ( $R = \tanh^2(\pi n_1 L/\lambda_0)$ ), the final formula for the grating effective length at the Bragg wavelength is written

#71062 - \$15.00 USD	Received 22 May 2006; revised 22 June 2006; accepted 28 June 2006
(C) 2006 OSA	10 July 2006 / Vol. 14, No. 14 / OPTICS EXPRESS 6396

$$L_{eff} = L \frac{\sqrt{R}}{2 \operatorname{atanh}(\sqrt{R})}.$$
(8)

Figure 2 shows the dependence of the relative effective length,  $L_{eff}/L$ , on the grating peak reflectivity *R* for various values of detuning from the peak wavelength  $\lambda_0$ . As it is seen, at relatively low values of the grating diffraction efficiency the grating effective length is near a half of its physical length; at high diffraction efficiencies the effective length approaches zero. Physically, this can be explained by the evident fact that a weak FBG ( $\kappa L \ll 1$ ) reflects light power along its length practically homogeneously, but a very intensive FBG ( $\kappa L \gg 1$ ) reflects the most of light power from its initial part.



Fig. 2. Relative effective length  $L_{eff}/L$  of a uniform 4-cm fiber Bragg grating versus its diffraction efficiency for  $\lambda_0 = 1530$  nm. The labels near the curves correspond to detuning from the peak wavelength.



Fig. 3. Group delay calculated for uniform 4-cm fiber Bragg grating versus detuning from the grating peak wavelength (1530 nm). The labels near the curves correspond to the diffraction efficiency values.

At wavelengths slightly different from  $\lambda_0$ , the effective length is bigger owing to the group

#71062 - \$15.00 USD (C) 2006 OSA Received 22 May 2006; revised 22 June 2006; accepted 28 June 2006 10 July 2006 / Vol. 14, No. 14 / OPTICS EXPRESS 6397 delay increase (Fig. 3). The curve for zero-detuning was calculated using Eq. (8), and curves for the detuning values different from zero were calculated using Eqs. (3,5,6). Since in singlefrequency DBR lasers the distance between FBGs  $L_0$  lies usually in the range of several cm, giving mode-spacing of the order of 10 pm ( $\lambda_0 \pm 5$  pm), Eq. (8) can be used to estimate a minimal FBG effective length.

## 3. Transmittance of a Fabry-Perot cavity formed by two FBGs

Figure 4 (a) shows the fiber cavity transmittance curves calculated using Eqs. (1,4,6) for the case of two equal uniform FBGs,  $L_1 = L_2 = 4$  cm, and a separation between them,  $L_0 = 5$  cm. As it is seen the mode spacing grows with an increase of the FBG's diffraction efficiency owing to a decrease of their effective lengths (see Fig. 2).



Fig. 4. (a) Theoretical and (b) experimental transmittance spectra of the Fabry-Perot fiber cavity formed by two equal uniform 4-cm FBGs separated by 5 cm. Gratings are centered at 1531.08 nm. The diffraction efficiency values are labeled near the corresponding curves.

We studied experimentally the transmittance of a Fabry-Perot fiber cavity with two equal uniform gratings written in a standard photosensitive single-mode fiber similar to SMF-28 for several values of diffraction efficiency. The gratings were of 4 cm in length and the distance between them was 5 cm. The peak wavelengths of the FBGs were centered at 1531.08 nm. To insure the accuracy of the device geometry, the gratings were written in a single piece of fiber using a 15 cm long phase mask. After writing the first 4-cm grating, the UV laser was blocked along 5 cm and then a second 4-cm grating was written. The displacement of the UV beam was controlled with a high precision translation stage. A fiber-optic polarization controller was used to adjust the input polarization to match one of the eigen states of the Fabry-Perot cavity. In absence of polarization controlling, there was observed small splitting (~1 pm) in peaks of the cavity transmission coefficient owing to the photo-induced birefringence in strong FBGs [12,13] and birefringence of the photosensitive fiber.

The cavity transmission spectra (see Fig. 4(b)) were obtained using a tunable singlefrequency diode laser with an optical bandwidth of 150 kHz. It is seen that the distance between the transmittance peaks varies with the gratings' reflectivity. The less the diffraction efficiency, the less the mode-spacing and vice versa. The mode-spacing grows as the diffraction efficiency increases. For the values of the diffraction efficiency used in the experiment, the change in the mode-spacing was ~ 4 pm. Figure 5 presents the theoretical and experimental data for the mode-spacing when the central resonance is at the Bragg wavelength of the gratings (the theoretical curves was calculated using Eqs. (2) and (8)). It is seen that the values of the measured mode-spacing are close to the calculated ones. Thus we

#71062 - \$15.00 USD (C) 2006 OSA Received 22 May 2006; revised 22 June 2006; accepted 28 June 2006 10 July 2006 / Vol. 14, No. 14 / OPTICS EXPRESS 6398 can conclude that the Fabry-Perot fiber cavity formed by two FBGs can be correctly described in terms of the effective lengths of FBGs and the overall effective cavity length.



Fig. 5. Comparison between theoretical (solid line) and experimental (stars) mode-spacing values.

#### 4. Conclusion

In summary, we have presented the results of a theoretical and experimental study of the free spacing between the longitudinal modes of a Fabry-Perot fiber cavity formed by two FBGs. It has been shown that when the cavity is composed of two FBGs, the group delay time of light reflected on them is the key parameter determining the cavity free spectral range. The properties of the cavity can be described in terms of the effective cavity length, in which the FBGs' effective lengths should be accounted for. Since the grating effective length depends on its diffraction efficiency, the mode-spacing depends on it as well.

We provide a simple analytical formula for calculating the effective length of a uniform FBG near its peak wavelength, which takes into account the grating diffraction efficiency and its physical length. It is shown that the FBG effective length changes from a half of the physical length to zero when the diffraction efficiency varies from low values to ~100%.

The experimental data on the transmission spectra of the Fabry-Perot fiber cavity with FBGs is described well by the presented theory. The results of this work can be applied for optimization of single-frequency DBR fiber lasers.

## Acknowledgments

This work has been supported by the CONACyT (grant # 47029) and the CONCyTEG (grant # 05-04-K117-113), Mexico, and the Ministerio de Educación y Ciencia (grants # TEC2005-07336-C02-01 and SB2003-0097), Spain.