Optimization Approach for Solving Problems in Signal Processing and Communications Systems

Presenter: Dr. Bingo Wing-Kuen Ling

Lecturer, Centre for Digital Signal Processing, Department of Electronic Engineering, King's College London.

Email address: wing-kuen.ling@kcl.ac.uk

Collaborators

- Internal personnel:
  - Dr. Zoran Cvetkovic
  - Dr. Wing-Kuen Ling
  - Dr. Mohammad Shikh-Bahaei
  - Mr. Zhi-Wei Chi

  (Welcome all PhD students and colleagues in EE department to join us to work on this area!)
- National personnel:
  - Miss Charlotte Yuk-Fan Ho
- International personnel:
  - Prof. Wan-Chi Siu
  - Dr. Ted Chi-Wah Kok
  - Prof. Kok-Lay Teo
  - Dr. Yan-Qun Liu

Outline

- Basic Concepts
- Examples of Smooth Functional Inequality Constrained Optimization Problems
- Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems
- Examples of Nonsmooth Optimization Problems
- Challenges and Some Solutions for Solving Nonsmooth Optimization Problems
- Examples of Nonsmooth Functional Inequality Constrained Optimization Problems
- Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems
- Conclusions
- Questions and Answers

Basic Concepts

- What are Optimization Problems?
  - Finding the value of $\mathbf{x}$ such that the functional value at $\mathbf{x}$ is either minimum or maximum.
  - Minimization problem

$$\min_{\mathbf{x}\in\Re^d} f(\mathbf{x})$$

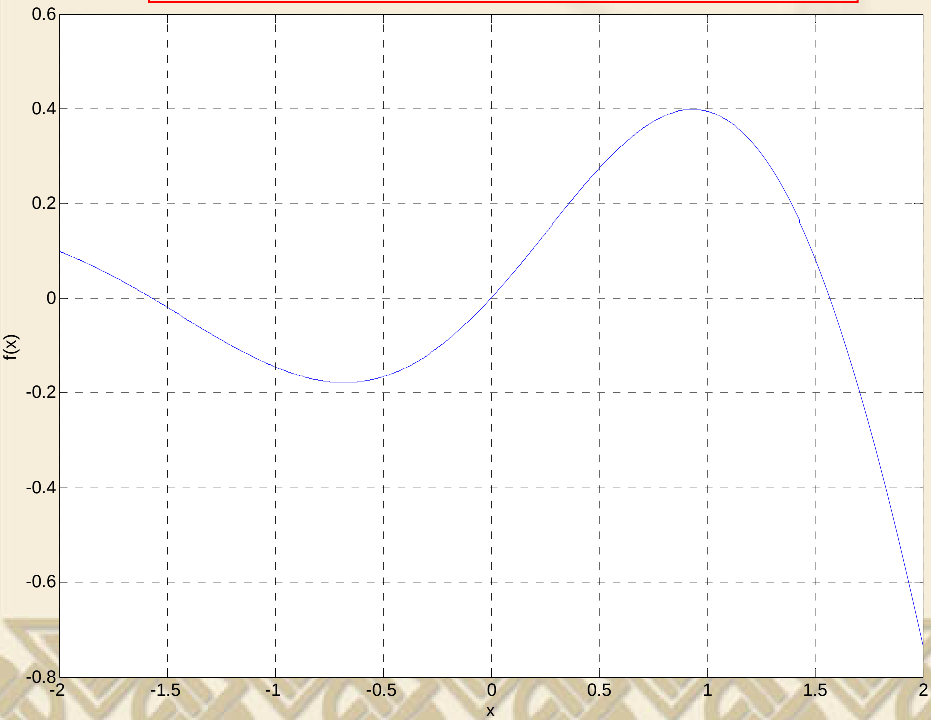
  - Maximization problem

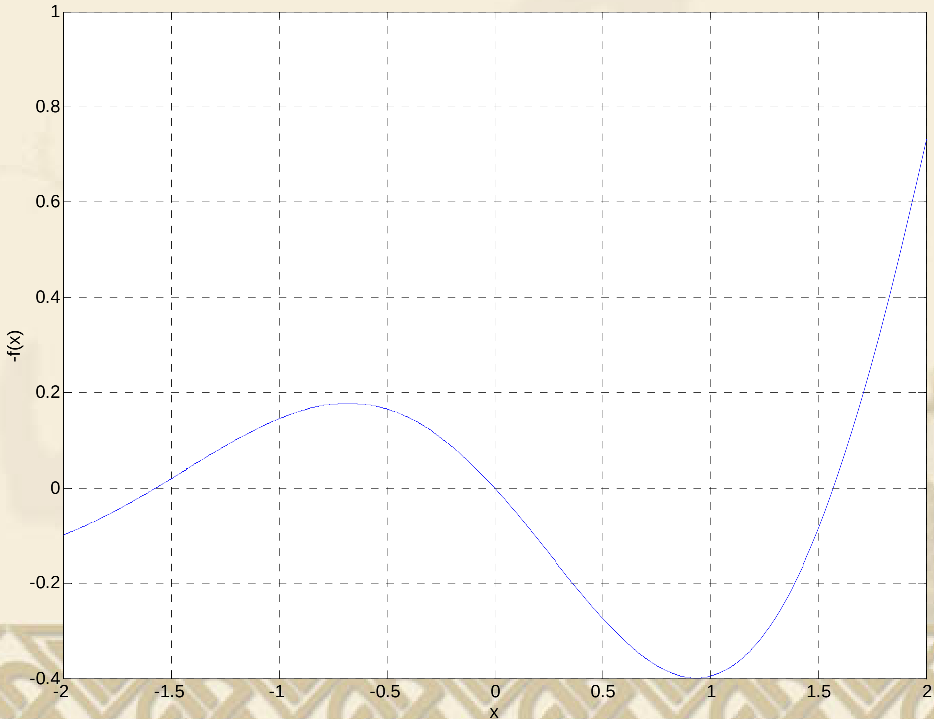
$$\max_{\mathbf{x}\in\Re^d} f(\mathbf{x})$$

Basic Concepts

- Relationship Between Maximization Problems and Minimization Problems

$$\max_{\mathbf{x}\in\Re^d} f(\mathbf{x}) \Leftrightarrow \min_{\mathbf{x}\in\Re^d} - f(\mathbf{x})$$





Basic Concepts

- Constrained and Unconstrained Optimization Problems
  - Unconstrained optimization problem

$$\min_{\mathbf{x}\in\Re^d} f(\mathbf{x})$$

  - Constrained optimization problem

$$\min_{\mathbf{x}\in\Re^d} f(\mathbf{x})$$

subject to $g_i(\mathbf{x})\le 0$ for $i=1,2,\cdots,M$ (inequality constraints)

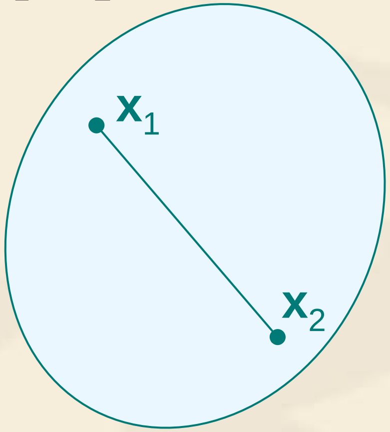
$h_i(\mathbf{x})=0$ for $i=1,2,\cdots,N$ (equality constraints)

$p_i(\mathbf{x},\omega)\le 0$ for $i=1,2,\cdots,K$ and $\forall\omega\in\Omega$ (functional inequality constraints)

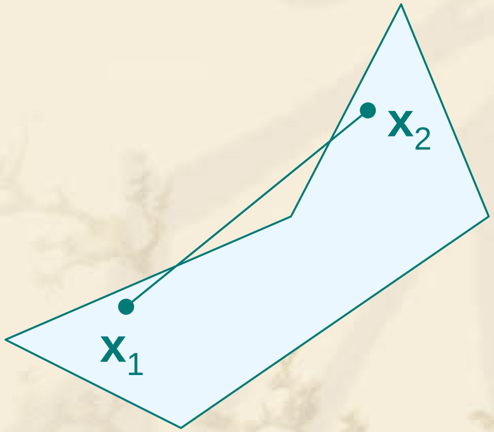
Basic Concepts

- Convex and Nonconvex Optimization Problems
  - Convex sets

$\forall \mathbf{x}_1, \mathbf{x}_2 \in S$ and $\forall \lambda \in [0,1]$, $\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2 \in S$



(a) Convex set



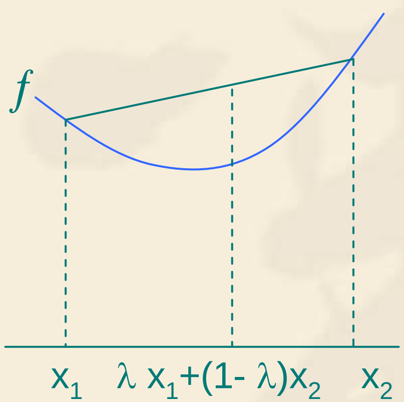
(b) Nonconvex set

Basic Concepts

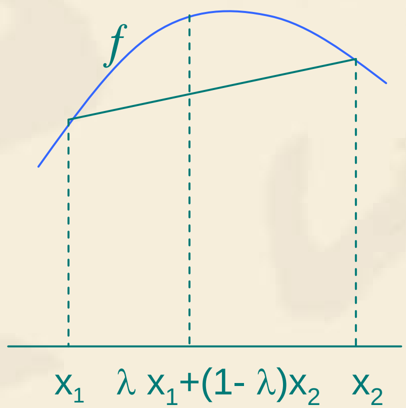
- Convex and Nonconvex Optimization Problems
  - Convex functions

Let $f : S \to \Re$, where S is a nonempty convex set. The function f is said to be convex on S if

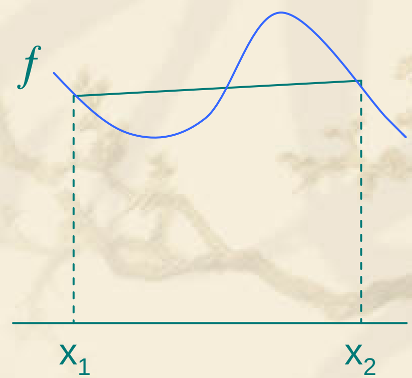
$\forall \mathbf{x}_1, \mathbf{x}_2 \in S$ and $\forall \lambda \in [0,1]$, $f(\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \le \lambda f(\mathbf{x}_1) + (1-\lambda) f(\mathbf{x}_2)$



convex function



concave function



neither convex nor concave

Basic Concepts

- Convex and Nonconvex Optimization Problems
  - Feasible set

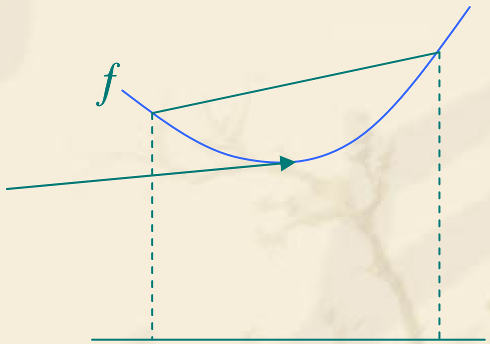
$$\Psi \equiv \left\{ \begin{array}{l} \mathbf{x} : g_i(\mathbf{x}) \le 0 \text{ for } i = 1,2,\cdots,M, \\ \quad h_i(\mathbf{x}) = 0 \text{ for } i = 1,2,\cdots,N, \\ \quad p_i(\mathbf{x},\omega) \le 0 \text{ for } i = 1,2,\cdots,K \text{ and } \forall \omega \in \Omega \end{array} \right\}$$

  - Convex optimization problem
    - Feasible set is convex and f is convex.
  - Nonconvex optimization problem
    - Feasible set is not convex, or f is not convex, or neither.

Basic Concepts

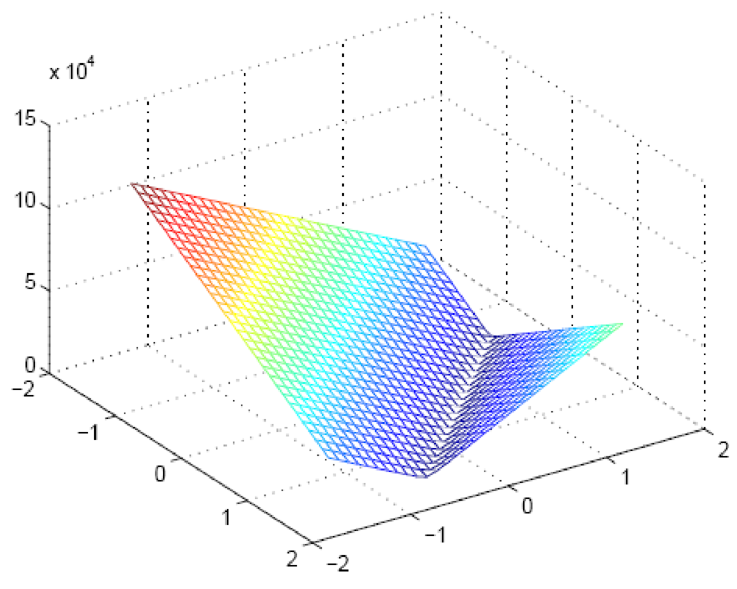
- Convex and Nonconvex Optimization Problems
  - If the optimization problem is convex, then any local minimum is a global minimum.

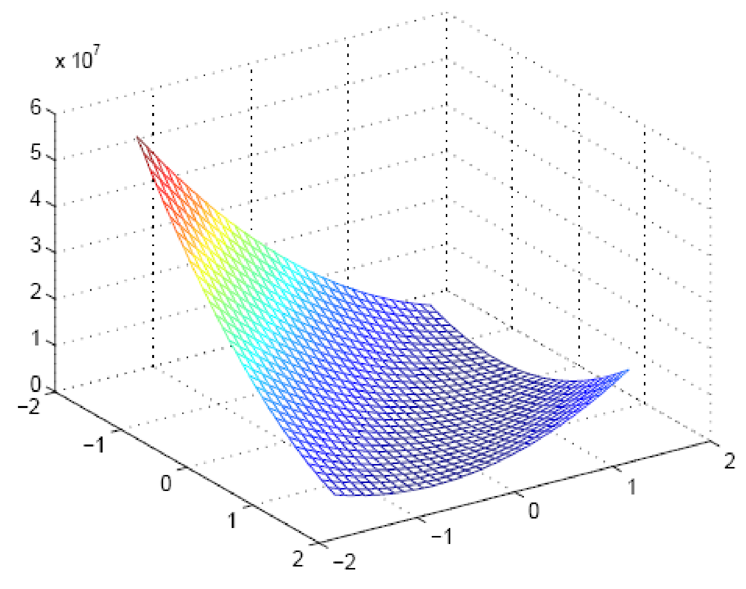
Local minimum = Global minimum



Basic Concepts

- Smooth and Nonsmooth Optimization Problems
  - Smooth optimization problems
    - f is differentiable.
  - Nonsmooth optimization problems
    - f is not differentiable.





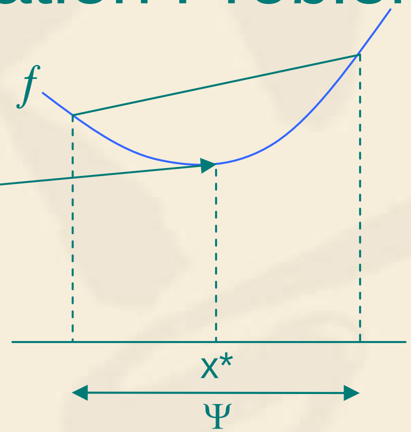
Basic Concepts

- Smooth and Nonsmooth Optimization Problems
  - For smooth optimization problems, if $\mathbf{x}^*$ is a local minimum of f and $\mathbf{x}^* \in \Psi$, then $\mathbf{x}^*$ is a stationary point. If $\mathbf{x}^*$ is a stationary point, $\mathbf{x}^* \in \Psi$ and the Hessian matrix evaluated at $\mathbf{x}^*$ is positive definite, then $\mathbf{x}^*$ is a local minimum.

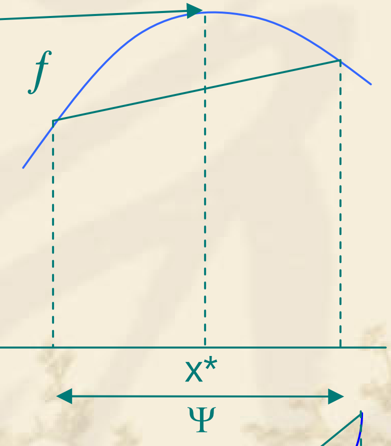
Basic Concepts

- Smooth and Nonsmooth Optimization Problems

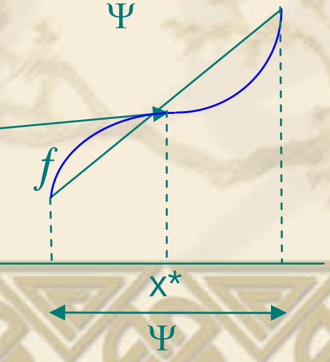
Local minimum ⇒ stationary point
A stationary point and convex ⇒ local minimum



Local maximum ⇒ stationary point
A stationary point and concave ⇒ local maximum

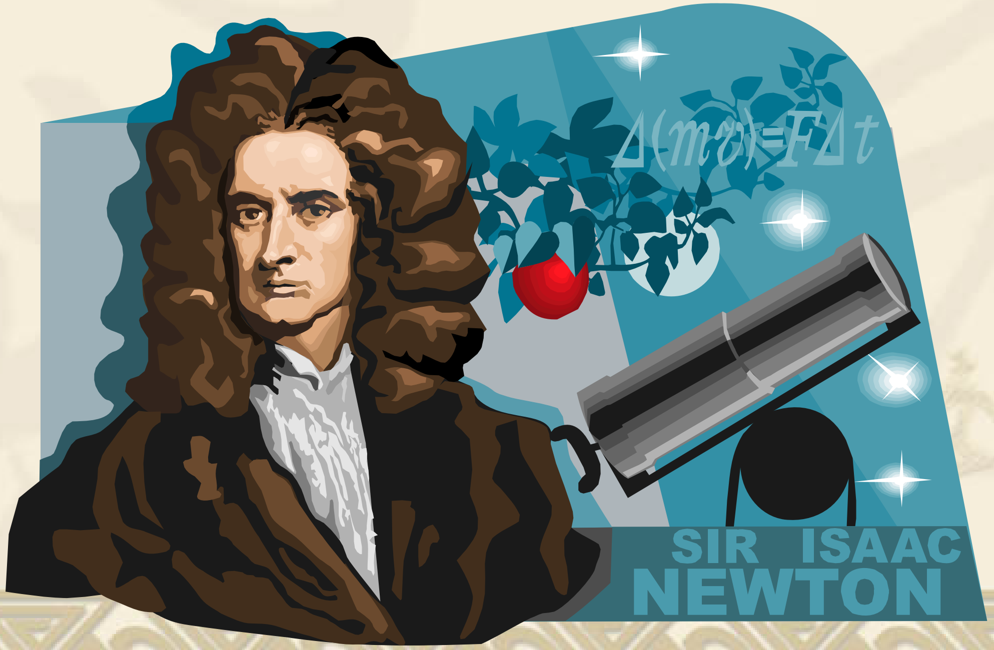


Point of reflection ⇒ stationary point
A stationary point with twice differentiable, but neither convex nor concave ⇒ point of reflection



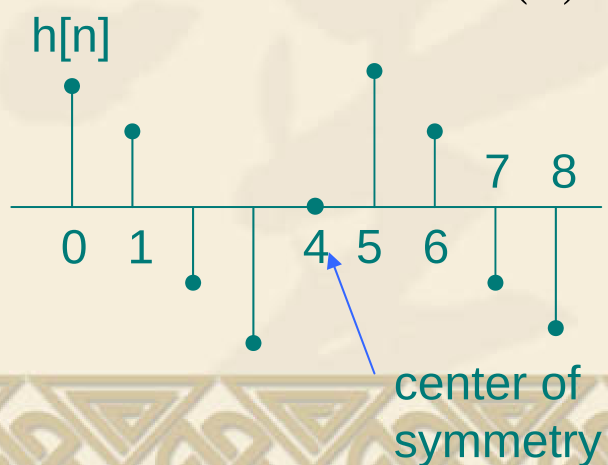
Basic Concepts

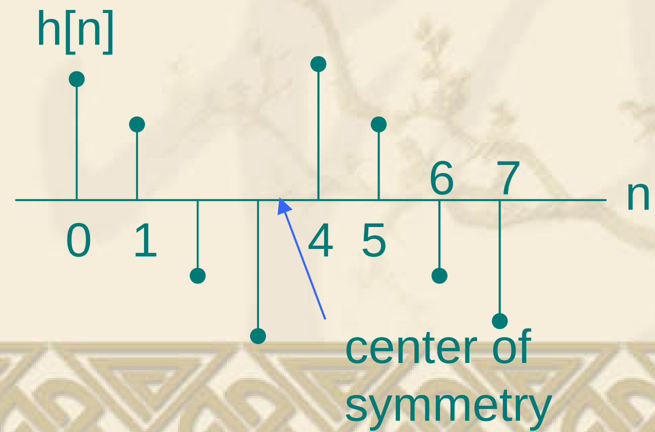
- Smooth and Nonsmooth Optimization Problems
  - Local optimal solution of smooth problems could be found by Newton's method, steepest decent method, etc.



Examples of Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Anti-symmetric Filter Design Problems
  - For N is odd, $\begin{cases} h(k) = -h(N-1-k), & k = 0,1,2,\cdots,\dfrac{N-3}{2} \\ h\left(\dfrac{N-1}{2}\right) = 0 & \end{cases}$
  - For N is even, $h(k) = -h(N-1-k)$ for $k = 0,1,2,\cdots,\dfrac{N}{2}-1$





Examples of Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Anti-symmetric Filter Design Problems
  - Denote

$$\mathbf{x} \equiv \begin{cases} \begin{bmatrix} a_1, & a_2, & \cdots, & a_{\frac{N-1}{2}} \end{bmatrix}^T, & N \text{ is odd} \\ \begin{bmatrix} a_1, & a_2, & \cdots, & a_{\frac{N}{2}} \end{bmatrix}^T, & N \text{ is even} \end{cases}$$

where

$$a_n \equiv \begin{cases} 2h\left(\dfrac{N-1}{2} - n\right), & N \text{ is odd and } n = 1,2,\cdots,\dfrac{N-1}{2} \\ 2h\left(\dfrac{N}{2} - n\right), & N \text{ is even and } n = 1,2,\cdots,\dfrac{N}{2} \end{cases}$$

Examples of Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Anti-symmetric Filter Design Problems
  - Denote

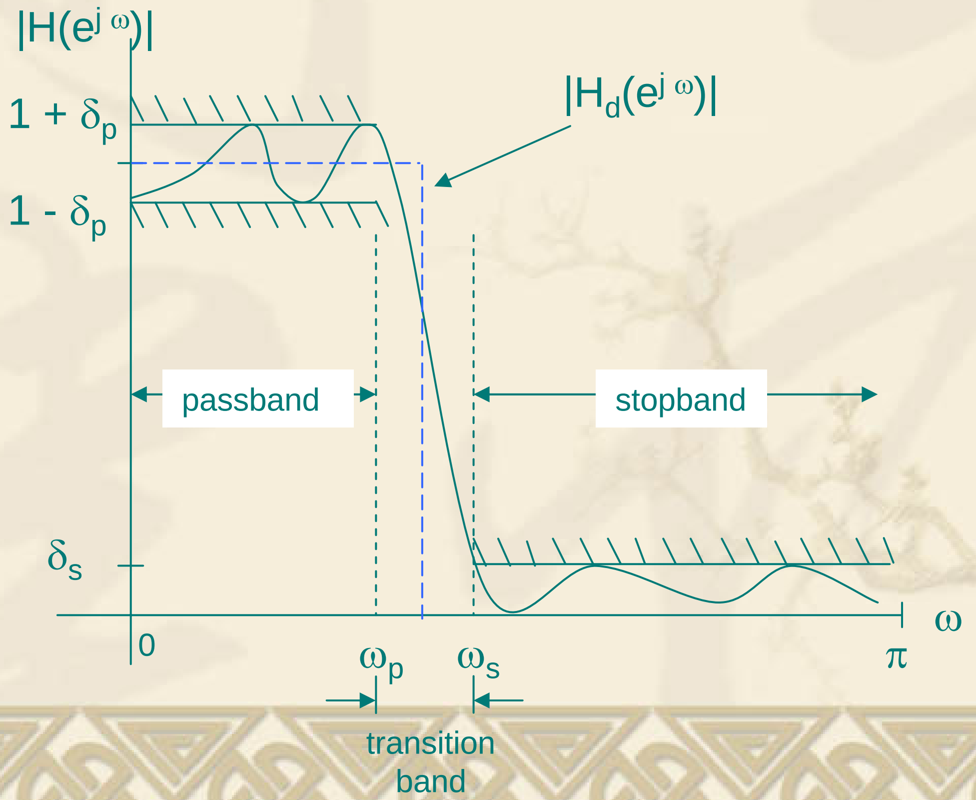
$$\boldsymbol{\eta}(\omega) \equiv \begin{cases} \left[\sin\omega, \quad \sin 2\omega, \quad \cdots, \quad \sin\left(\left(\dfrac{N-1}{2}\right)\omega\right)\right]^T, & N \text{ is odd} \\ \left[\sin\dfrac{\omega}{2}, \quad \sin\dfrac{3\omega}{2}, \quad \cdots, \quad \sin\left(\left(\dfrac{N-1}{2}\right)\omega\right)\right]^T, & N \text{ is even} \end{cases}$$

$$H_0(\omega) \equiv \left(\boldsymbol{\eta}(\omega)\right)^T \mathbf{x}$$

  - Then $H(\omega) = \sum_{k=0}^{N-1} h(n) e^{-jn\omega} = je^{-j\omega\left(\frac{N-1}{2}\right)} H_0(\omega)$

Examples of Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Anti-symmetric Filter Design Problems
  - Objective: Minimize the weighted total ripple energy subject to the weighted peak constraint.



Examples of Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Anti-symmetric Filter Design Problems

$$J(\mathbf{x}) \equiv \int_{B_d} W(\omega)\left|H_0(\omega) - D(\omega)\right|^2 d\omega = \frac{1}{2}\mathbf{x}^T \mathbf{Q}\,\mathbf{x} + \mathbf{b}^T \mathbf{x} + p$$

where

$$\mathbf{Q} = 2\int_{B_d} W(\omega)\boldsymbol{\eta}(\omega)\left(\boldsymbol{\eta}(\omega)\right)^T d\omega$$

$$\mathbf{b} = -2\int_{B_d} W(\omega) D(\omega)\boldsymbol{\eta}(\omega) d\omega$$

$$p = \int_{B_d} W(\omega)\left(D(\omega)\right)^2 d\omega$$

$$W(\omega) > 0 \;\; \forall \omega \in B_d$$

Examples of Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Anti-symmetric Filter Design Problems

$$W(\omega)\left| H_0(\omega) - D(\omega) \right| \le \delta \quad \forall \omega \in B_d$$

$$\mathbf{A}(\omega)\,\mathbf{x} \le \mathbf{c}(\omega) \quad \forall \omega \in B_d$$

where $\mathbf{A}(\omega) = W(\omega)\left[\boldsymbol{\eta}(\omega), \quad -\boldsymbol{\eta}(\omega)\right]^T \qquad \forall \omega \in B_d$

$$\mathbf{c}(\omega) = \left[D(\omega)W(\omega) + \delta, \quad \delta - D(\omega)W(\omega)\right]^T \quad \forall \omega \in B_d$$

Problem (**P**) $\min\limits_{\mathbf{x}} \quad J(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\,\mathbf{x} + \mathbf{b}^T\mathbf{x} + p$

Subject to $\mathbf{g}(\mathbf{x}, \omega) = \mathbf{A}(\omega)\,\mathbf{x} - \mathbf{c}(\omega) \le \mathbf{0} \quad \forall \omega \in B_d$

Examples of Smooth Functional Inequality Constrained Optimization Problems

- PAM Signal Design Problems
  - Suppose there are N transmitters and the transmitted signals are denoted as $X_i(\omega)$ for $i = 0,1,\cdots,N-1$. Suppose that there are M receivers and the received signals are denoted as $Y_i(\omega)$ for $i = 0,1,\cdots,M-1$.

    Denote $\mathbf{X}(\omega) \equiv [X_0(\omega) \quad \cdots \quad X_{N-1}(\omega)]^T$ and $\mathbf{Y}(\omega) \equiv [Y_0(\omega) \quad \cdots \quad Y_{M-1}(\omega)]^T$.

    Assume that the fading characteristics of the channel is governed by $\mathbf{H}(\omega)$. Denote $\boldsymbol{\eta}(\omega)$ as an AWGN noise. Then

    $\mathbf{Y}(\omega) = \mathbf{H}(\omega)\mathbf{X}(\omega) + \boldsymbol{\eta}(\omega)$

Examples of Smooth Functional Inequality Constrained Optimization Problems

- PAM Signal Design Problems
  - Suppose these transmitted signals are generated by a set of symbols s_i for $i = 0,1,\cdots,L-1$ via a kernel function $\boldsymbol{\xi}(\omega)$. That is $\mathbf{X}(\omega) = \boldsymbol{\xi}(\omega)\mathbf{s}$ where $\mathbf{s} = \begin{bmatrix} s_0 & \cdots & s_{L-1} \end{bmatrix}^T$. Then $\mathbf{Y}(\omega) = \mathbf{H}(\omega)\boldsymbol{\xi}(\omega)\mathbf{s} + \boldsymbol{\eta}(\omega)$
  - By using ML detection, the objective is to minimize $\int_{-\pi}^{\pi} \left\| \mathbf{Y}(\omega) - \mathbf{H}(\omega)\boldsymbol{\xi}(\omega)\mathbf{s} \right\|^2 d\omega$ subject to $\left| \boldsymbol{\xi}(\omega)\mathbf{s} - \tilde{\mathbf{X}}_d(\omega) \right| \le \boldsymbol{\delta}(\omega) \;\; \forall \omega \in B_p \cup B_s$

Examples of Smooth Functional Inequality Constrained Optimization Problems

- PAM Signal Design Problems
  - This is equivalent to

$$\min_{\mathbf{s}} \quad J(\mathbf{s}) = \frac{1}{2}\mathbf{s}^T \mathbf{Q}\,\mathbf{s} + \mathbf{b}^T \mathbf{s} + p$$

subject to $\mathbf{g}(\mathbf{s},\omega) = \mathbf{A}(\omega)\,\mathbf{x} - \mathbf{c}(\omega) \le \mathbf{0} \;\forall \omega \in B_p \bigcup B_s$

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Challenges of Functional Inequality Constrained Optimization Problems
  - The domain of functional inequalities is $\Re^d \times \Omega$.
  - $\Rightarrow$infinite number of constraints.
  - How to guarantee that these infinite number of constraints are satisfied?
  - How to solve these problems efficiently?
  - For the FIR linear phase anti-symmetric filter design problem, the specifications contain the maximum passband ripple magnitude and the maximum stopband ripple magnitude. How to determine these specifications?

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Some Solutions for Solving Functional Inequality Constrained Optimization Problems
  - Dual parameterization approach
    - For smooth and convex optimization problems, by discretizing the index set Ω with finite number of elements, denoted as ω_i for $i=1,2,\cdots,k$, and introducing parameters λ_i for $i=1,2,\cdots,k$, then the following two optimization problems are equivalent:

$$\min_{\mathbf{x}\in\Re^d} \quad f(\mathbf{x})$$

subject to $p(\mathbf{x},\omega)\le 0 \quad \forall\omega\in\Omega$

$$\max_{(\mathbf{x},\boldsymbol{\omega},\boldsymbol{\lambda})\in\Re^{d\times k\times k}} \quad f(\mathbf{x})+\sum_{i=1}^{k}\lambda_i g(\mathbf{x},\omega_i)$$

subject to $\lambda_i \ge 0$ for $i=1,2,\cdots,k$

$\omega_i\in\Omega$ for $i=1,2,\cdots,k$

$$\nabla_{\mathbf{x}} f(\mathbf{x})+\sum_{i=1}^{k}\lambda_i\nabla_{\mathbf{x}} g(\mathbf{x},\omega_i)=\mathbf{0}$$

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

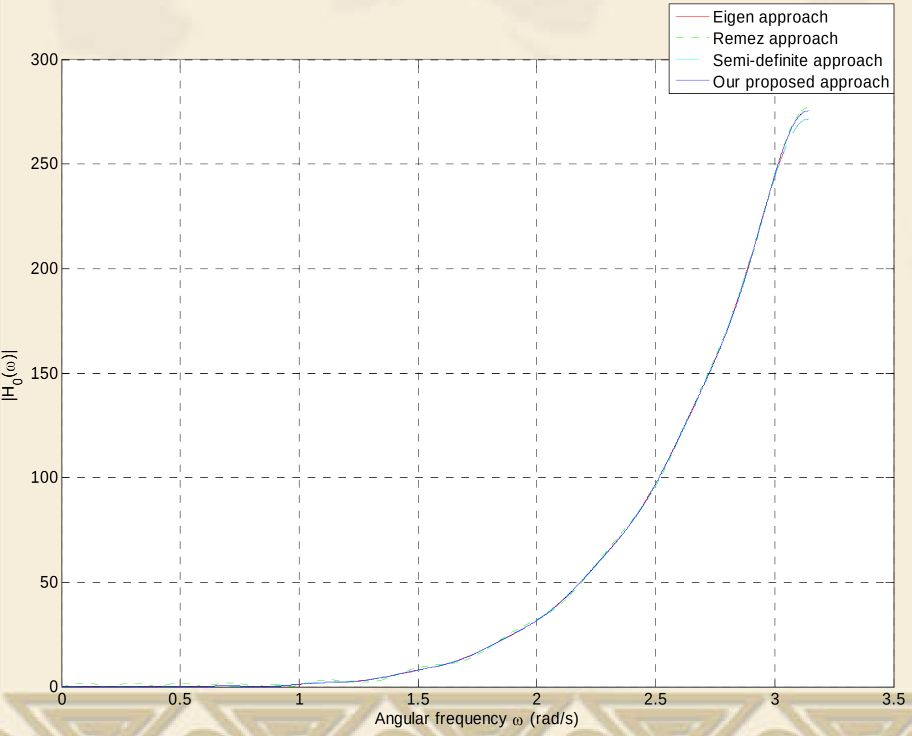
- Some Solutions for Solving Functional Inequality Constrained Optimization Problems
  - Dual parameterization approach
    - guarantees that the obtained global minimum solution satisfies the required functional inequality constraint if the feasible set is non-empty.
    - The maximum number of discretization points is less than or equal to $d+2$. Hence, this optimization problem can be solved efficiently.

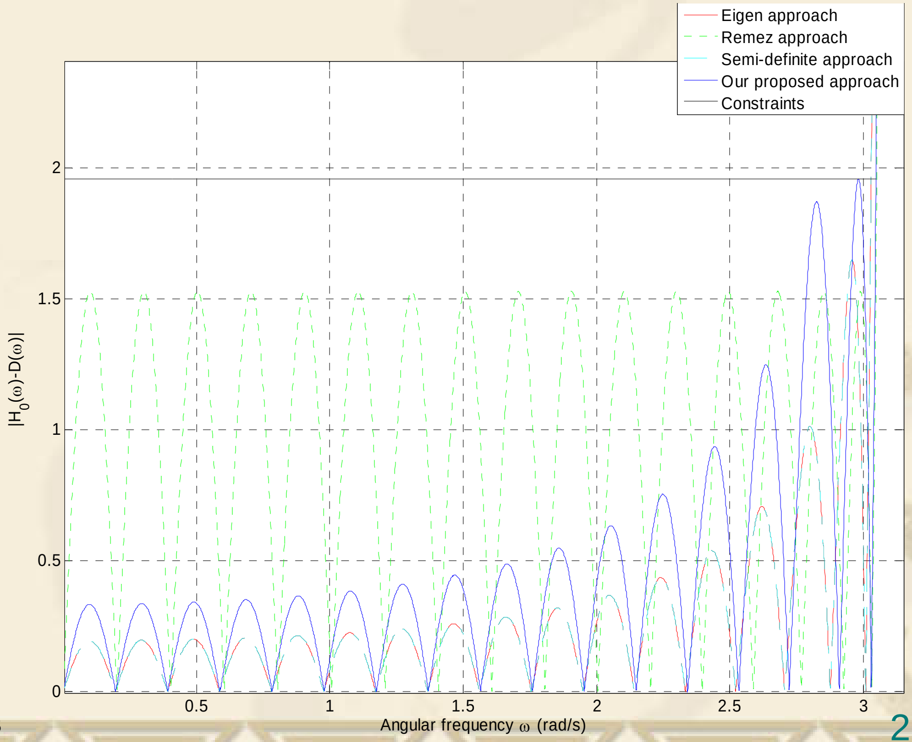
Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Some Solutions for Solving Functional Inequality Constrained Optimization Problems
  - Conventional discretization approach
    - The discretization points are uniformly disturbed in the index set.
    - It is not guaranteed that the original functional inequality constraint is satisfied.
    - The number of discretization points are usually more than $d+2$. Hence, it is not efficient.

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

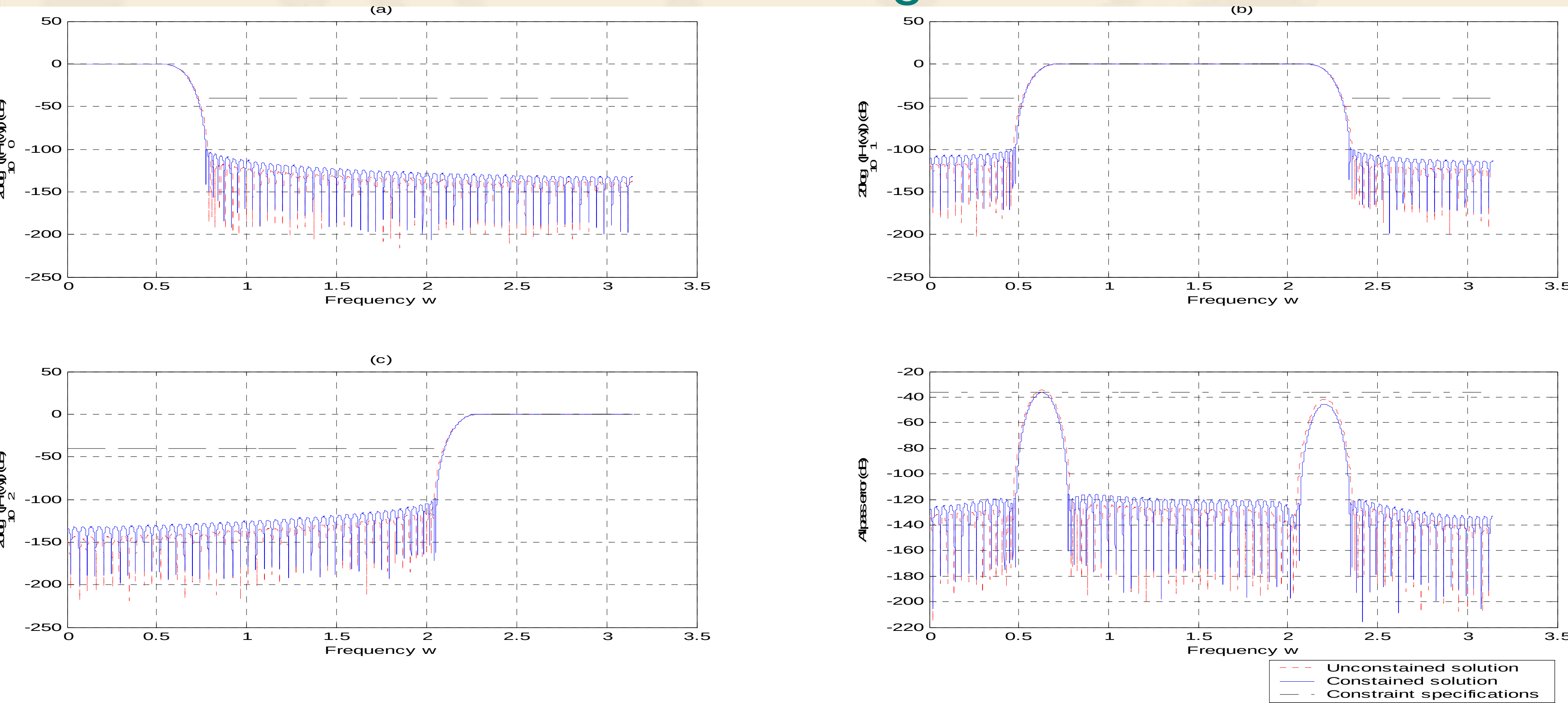
- FIR Linear Phase Anti-symmetric Fifth Order Differentiator Design Problem





Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Near Allpass Complementary Nonuniform Filter Bank Design Problem



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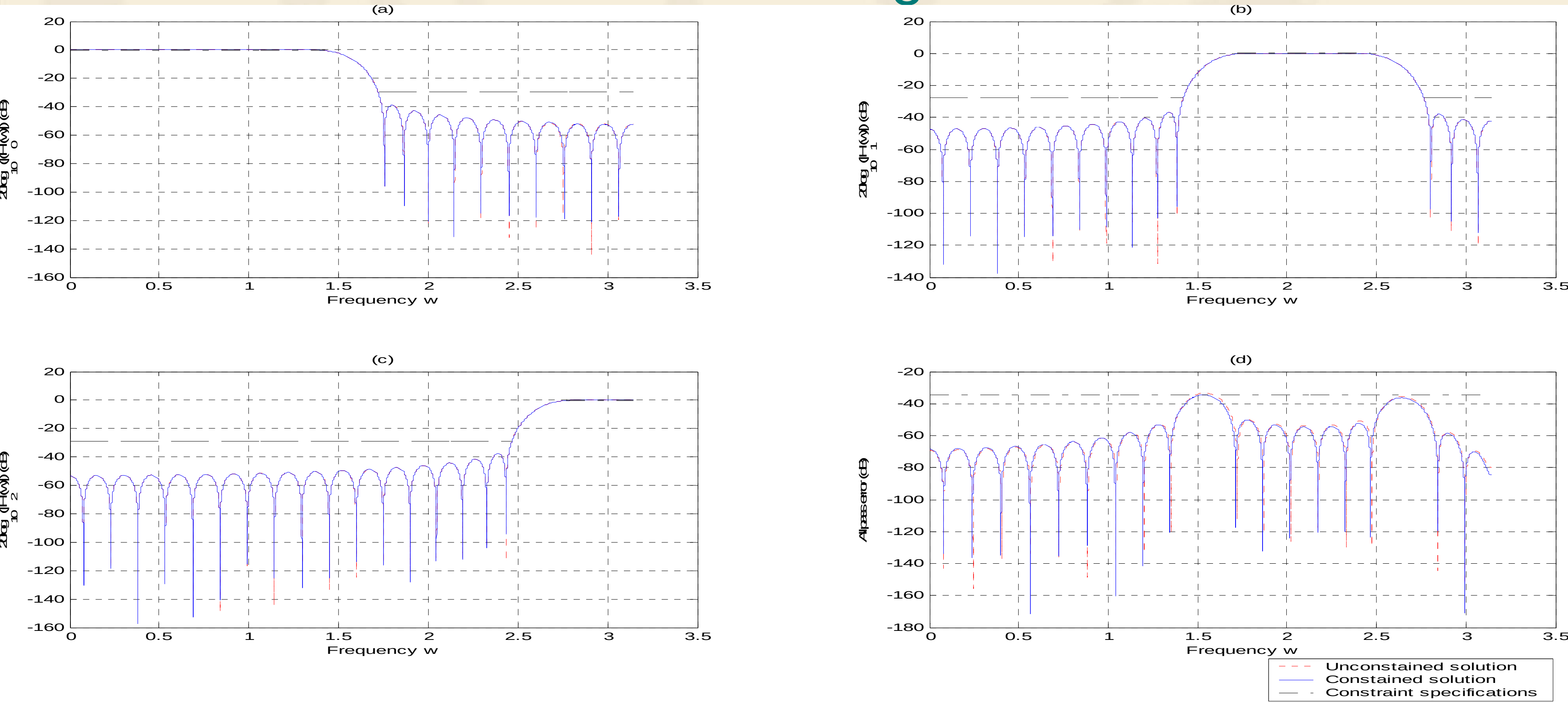
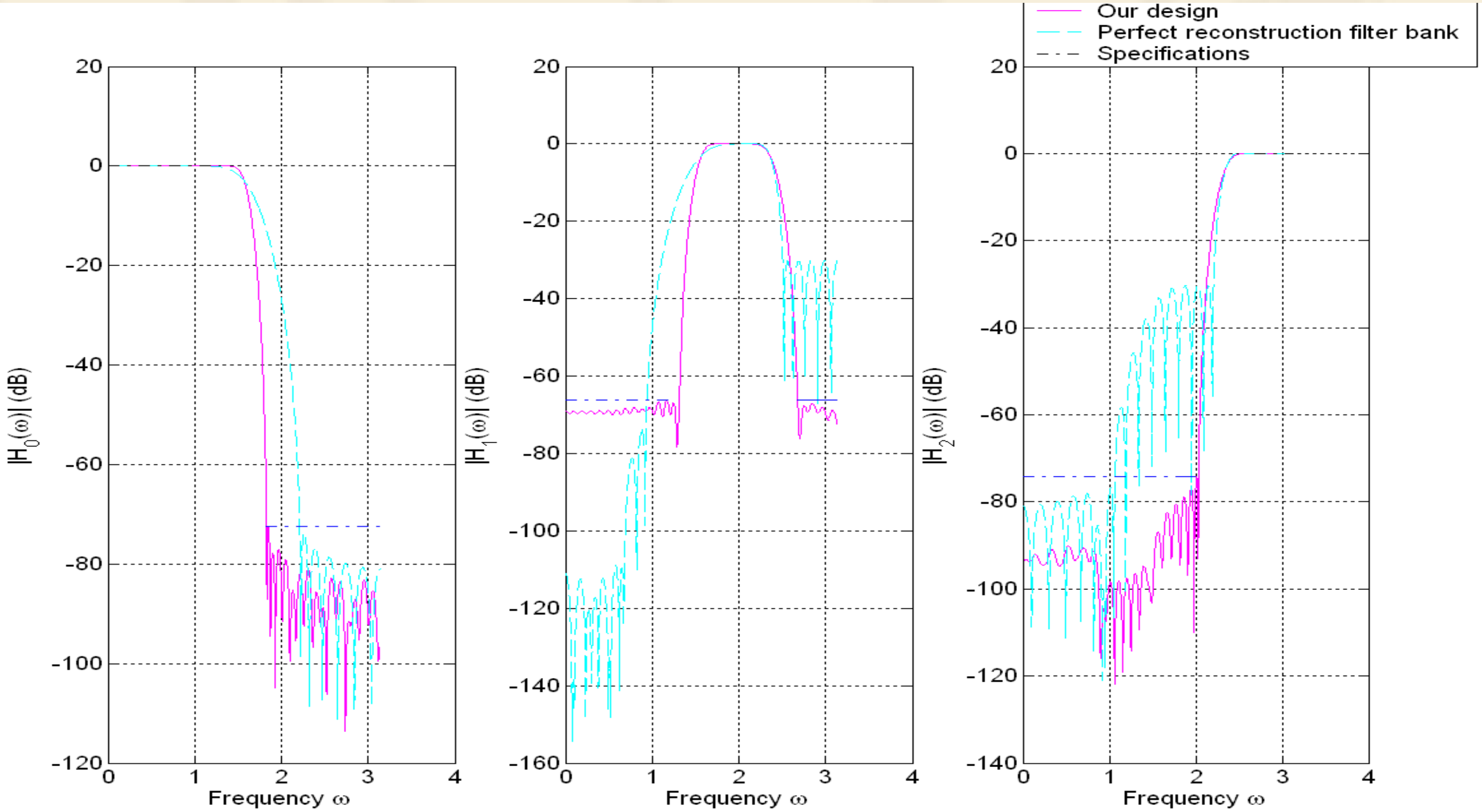


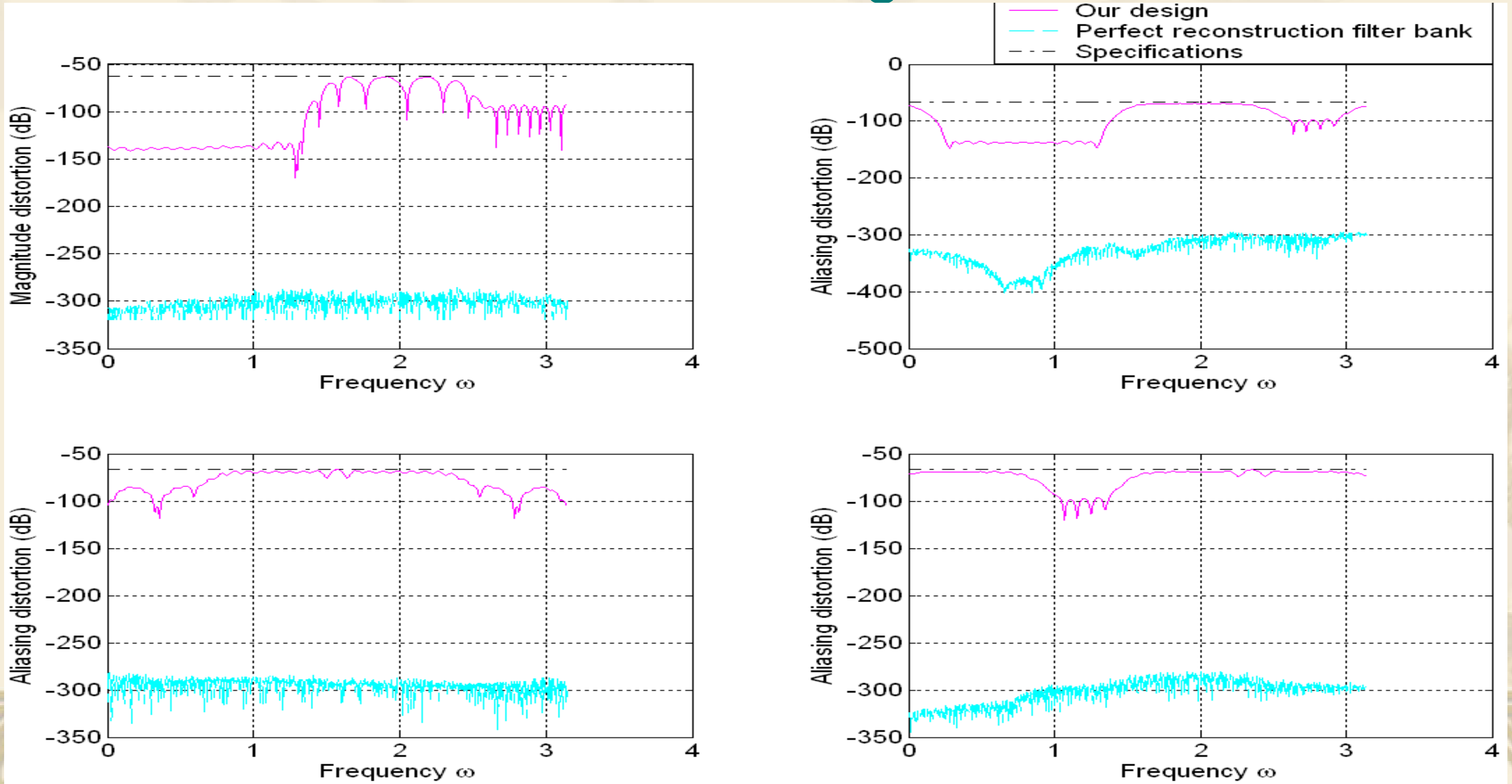
Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Near Perfect Reconstruction Nonuniform Filter Bank Design Problem



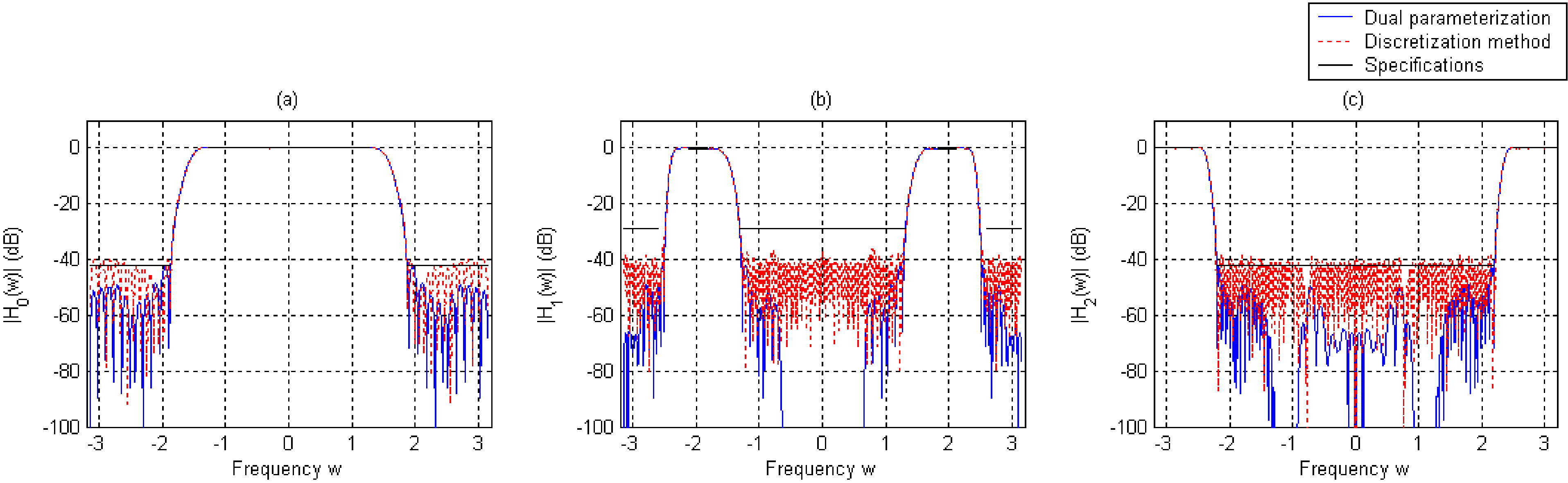
Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Near Perfect Reconstruction Nonuniform Filter Bank Design Problem



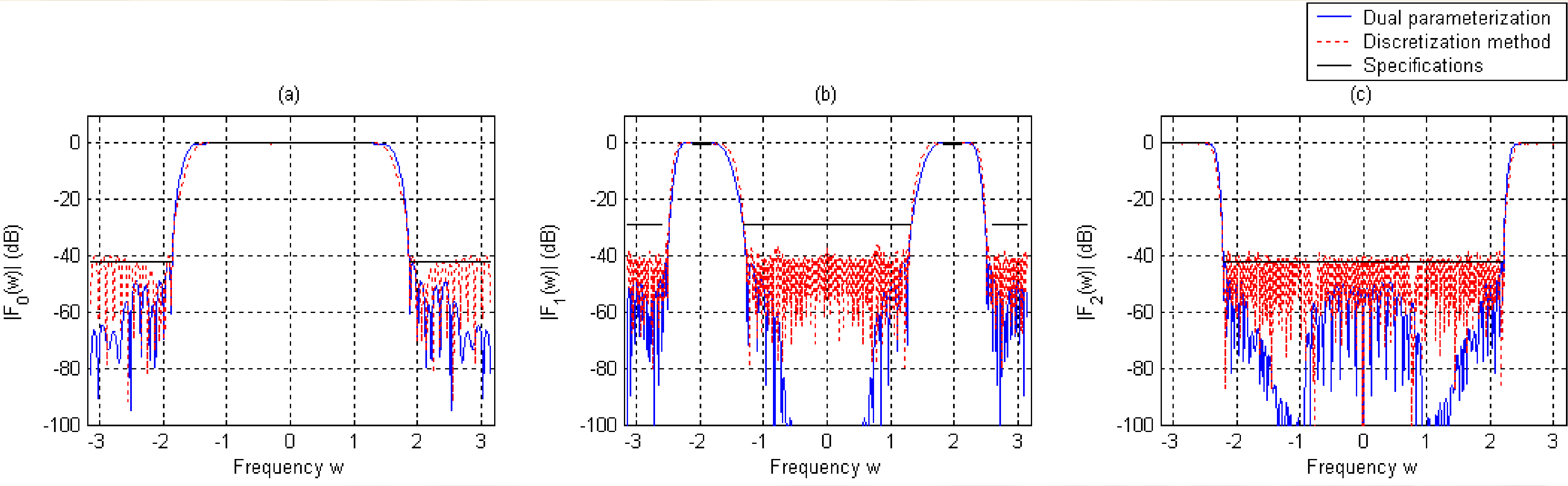
Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Near Perfect Reconstruction Nonuniform Transmultiplexer Design Problem



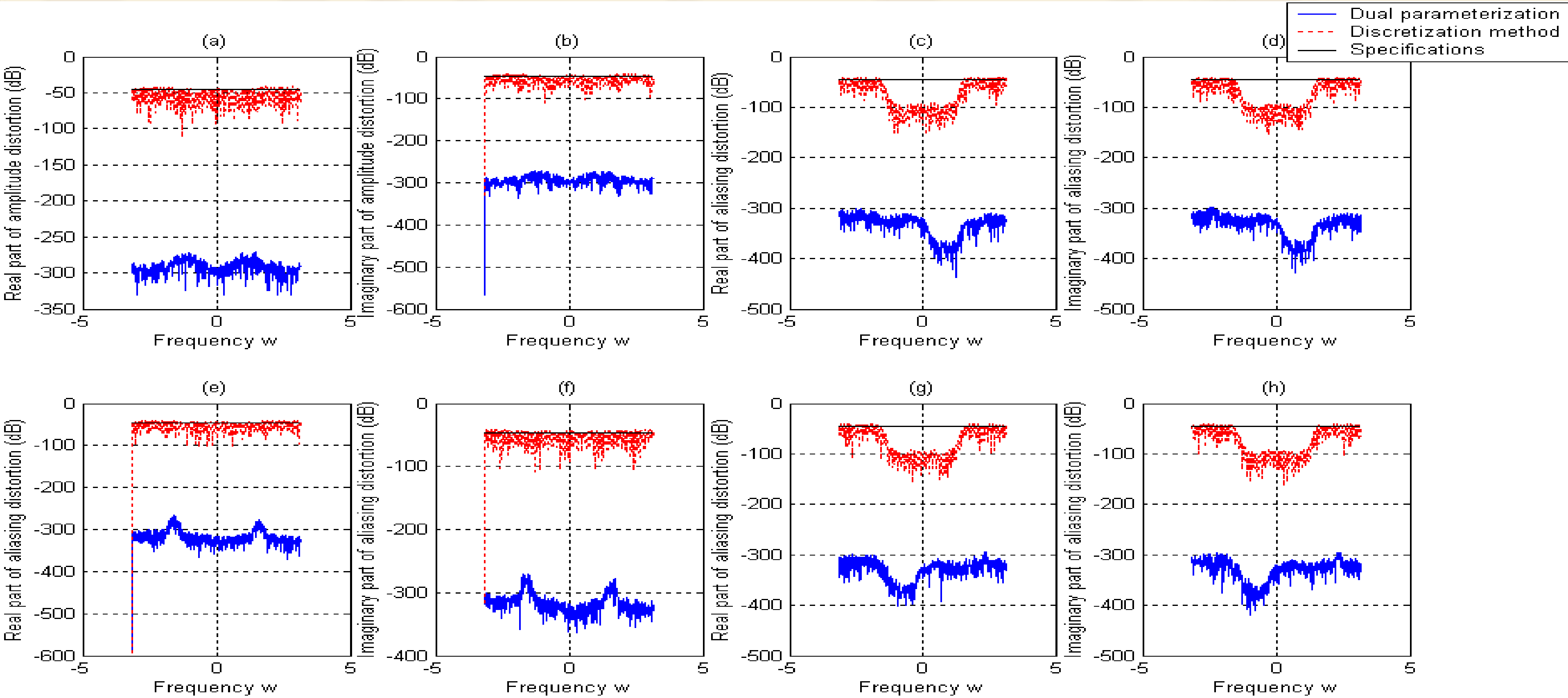
Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Near Perfect Reconstruction Nonuniform Transmultiplexer Design Problem



Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- FIR Linear Phase Near Perfect Reconstruction Nonuniform Transmultiplexer Design Problem



Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Specifications for designing FIR linear phase anti-symmetric filters include:
    - Filter length
    - Transition band bandwidth
    - Centre frequency
    - Maximum passband ripple magnitude
    - Maximum stopband ripple magnitude
  - The performance of FIR linear phase anti-symmetric filters is measured by the total ripple energy.

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

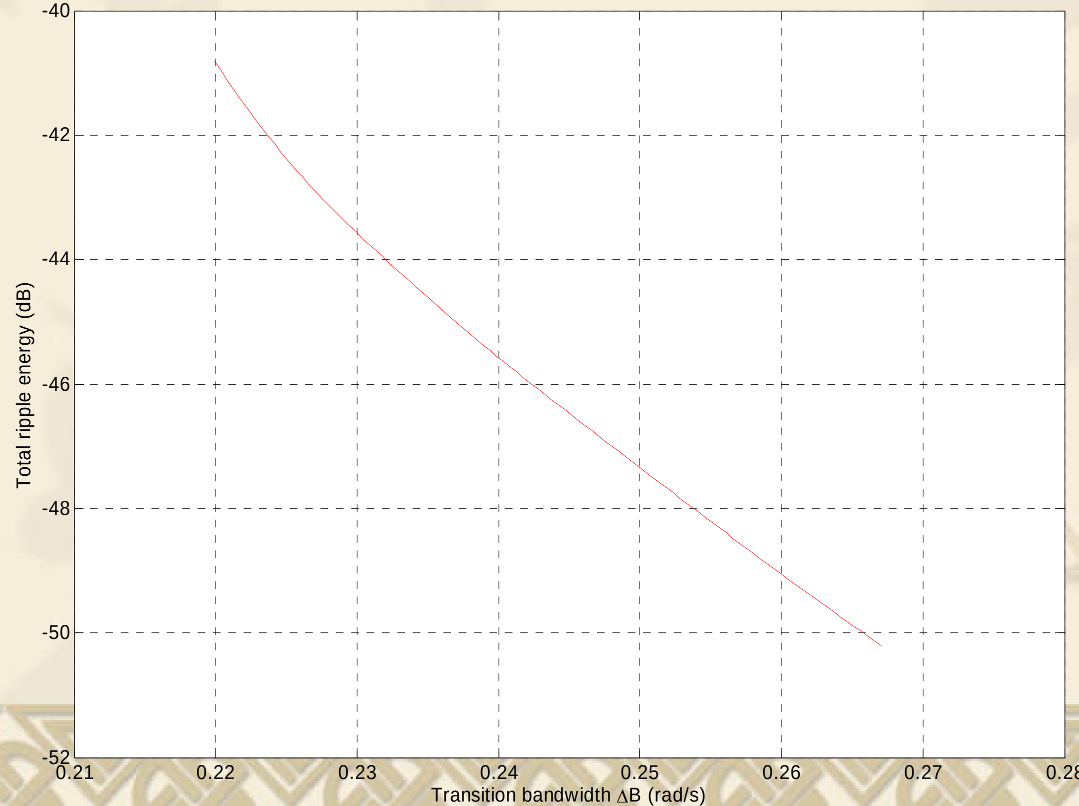
- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of filter length on total ripple energy

$$10\log_{10} J\left(\delta_p,\delta_s, f_c,\Delta B, N\right)= a_{1,1}\left(f_c,\Delta B,\delta_p,\delta_s\right) N + a_{1,2}\left(f_c,\Delta B,\delta_p,\delta_s\right)$$

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of transition band bandwidth on total ripple energy

$$10\log_{10} J(\delta_p,\delta_s, f_c,\Delta B, N) = a_{2,1}(f_c, N,\delta_p,\delta_s)\Delta B + a_{2,2}(f_c, N,\delta_p,\delta_s)$$



Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

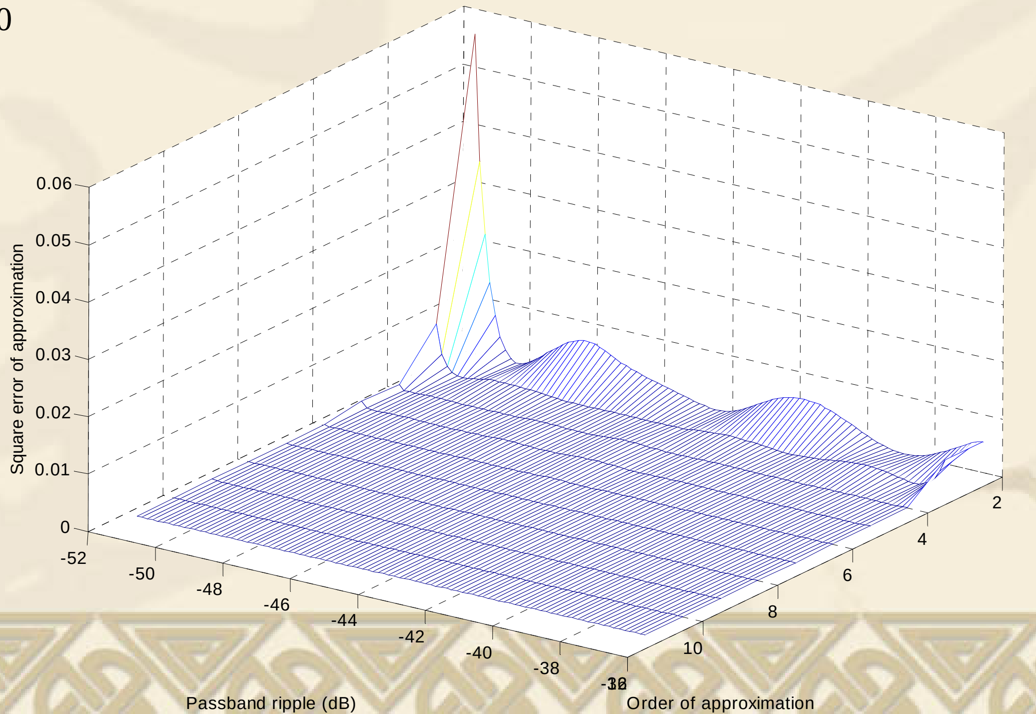
- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of maximum passband ripple magnitude on total ripple energy

$$10\log_{10} J\left(\delta_p, \delta_s, f_c, \Delta B, N\right) = \sum_{k=0}^{M_p} a_{3,k}\left(f_c, \Delta B, N, \delta_s\right)\left(20\log_{10}\delta_p\right)^k$$

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of maximum passband ripple magnitude on total ripple energy

$$E_p\left(\delta_p, M_p\right) \equiv \left(\sum_{k=0}^{M_p} a_{3,k}\left(f_c, \Delta B, N, \delta_s\right)\left(20\log_{10}\delta_p\right)^k - 10\log_{10} J\left(\delta_p, \delta_s, f_c, \Delta B, N\right)\right)^2$$



Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

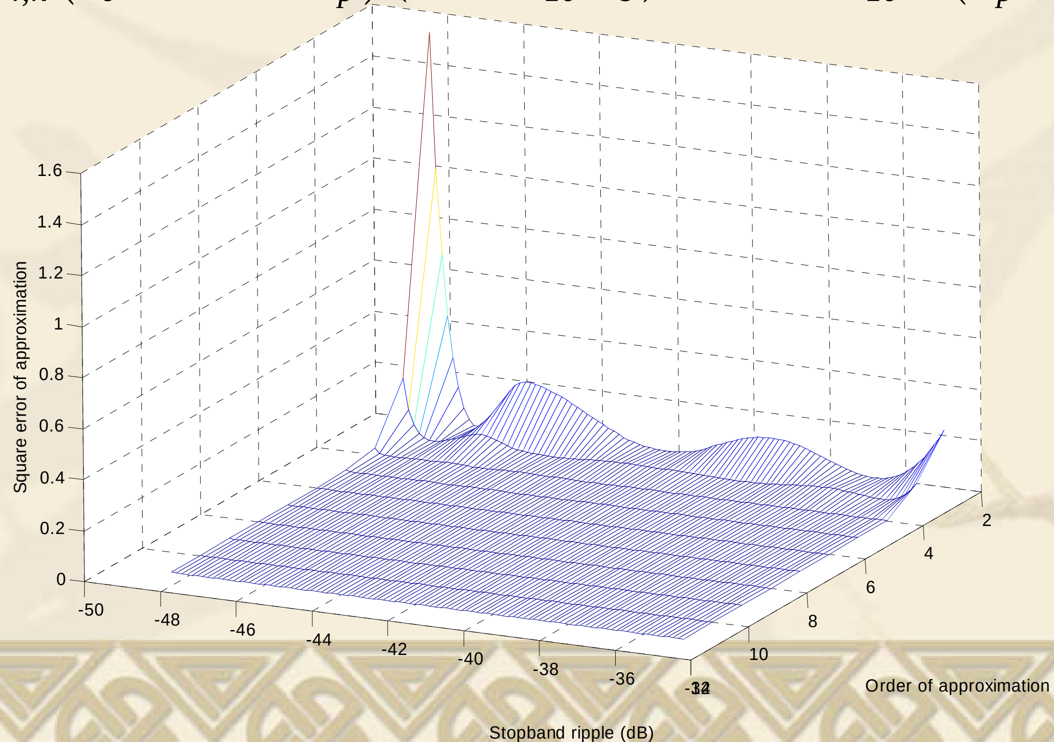
- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of maximum stopband ripple magnitude on total ripple energy

$$10\log_{10} J\left(\delta_p,\delta_s,f_c,\Delta B,N\right)=\sum_{k=0}^{M_s} a_{4,k}\left(f_c,\Delta B,N,\delta_p\right)\left(20\log_{10}\delta_s\right)^k$$

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of maximum stopband ripple magnitude on total ripple energy

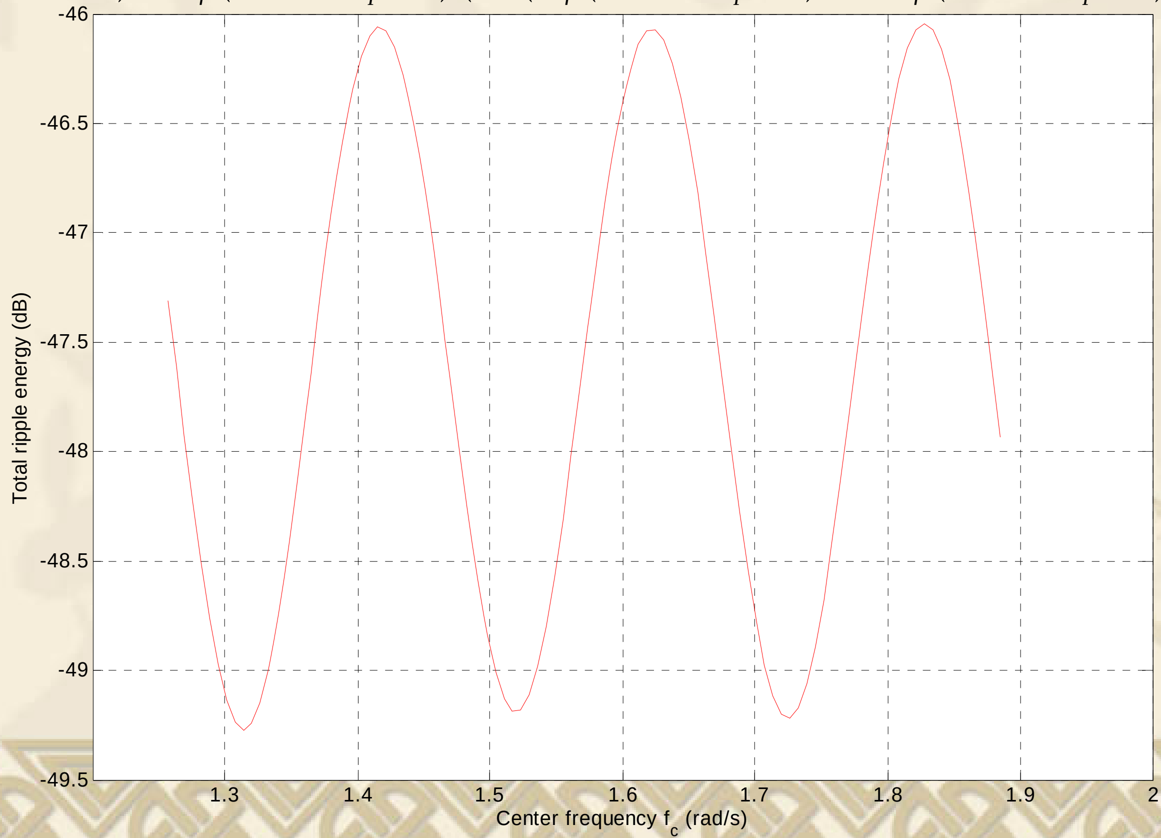
$$E_s(\delta_s, M_s) \equiv \left(\sum_{k=0}^{M_s} a_{4,k}\left(f_c, \Delta B, N, \delta_p\right)\left(20\log_{10}\delta_s\right)^k - 10\log_{10} J\left(\delta_p, \delta_s, f_c, \Delta B, N\right) \right)^2$$



Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of centre frequency on total ripple energy

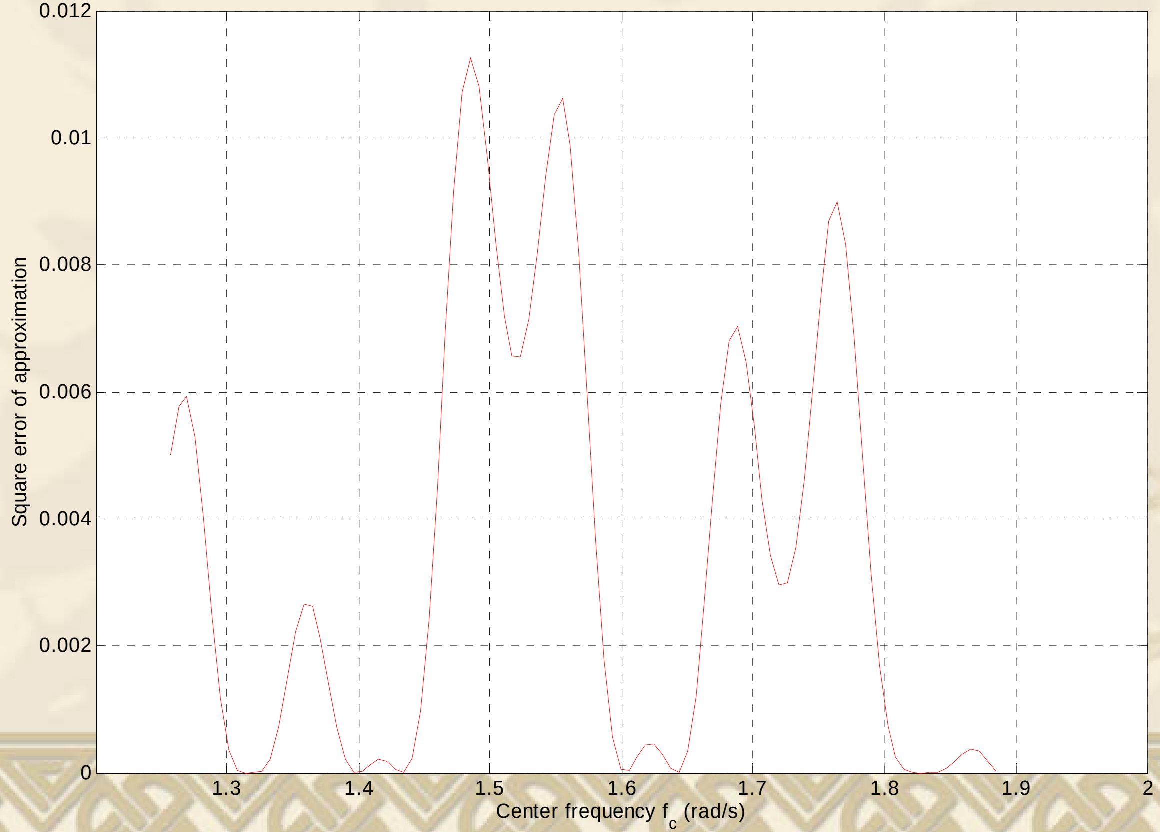
$$10\log_{10} J(\delta_p,\delta_s, f_c,\Delta B, N)= A_f(\Delta B, N,\delta_p,\delta_s)(\sin(\varpi_f(\Delta B, N,\delta_p,\delta_s)f_c+\phi_f(\Delta B, N,\delta_p,\delta_s)))+c_f(\Delta B, N,\delta_p,\delta_s)$$



Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Effect of centre frequency on total ripple energy

$$E_f(f_c) \equiv \left(A_f(\Delta B, N, \delta_p, \delta_s)\left(\sin\left(\varpi_f(\Delta B, N, \delta_p, \delta_s)f_c + \phi_f(\Delta B, N, \delta_p, \delta_s)\right)\right) + c_f(\Delta B, N, \delta_p, \delta_s) - 10\log_{10} J(\delta_p, \delta_s, f_c, \Delta B, N)\right)^2$$



Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Empirical formulae for designing FIR linear phase anti-symmetric filters

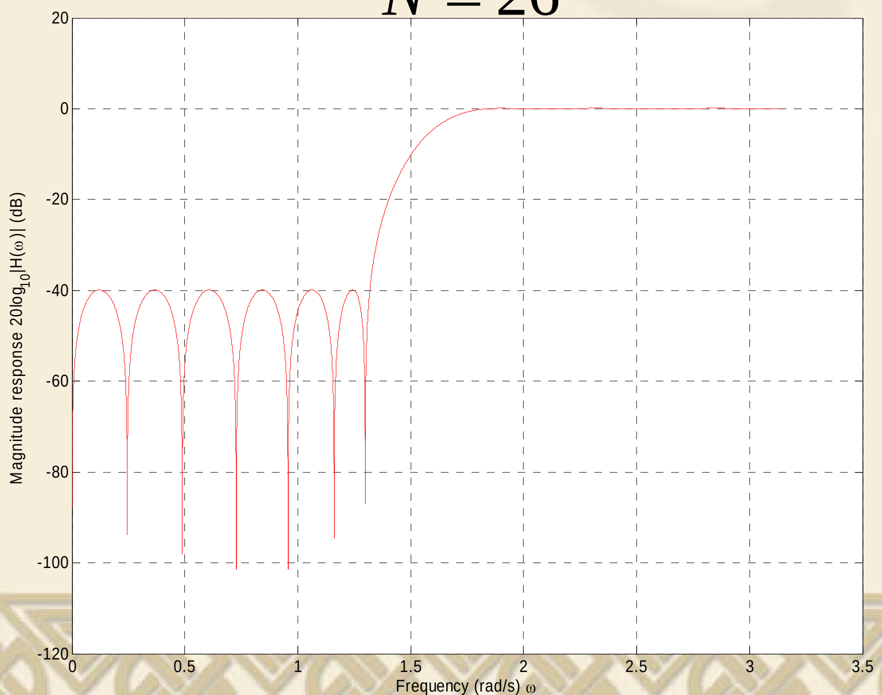
$$10\log_{10} J(\delta_p,\delta_s,f_c,\Delta B,N)=\left(A'_f\sin(\varpi_f f_c+\phi_f)+1\right)\sum_{m=0}^{M_p}\sum_{n=0}^{M_s}\left(a^1_{m,n}N\Delta B+a^2_{m,n}N+a^3_{m,n}\Delta B+a^4_{m,n}\right)(20\log_{10}\delta_p)^m(20\log_{10}\delta_s)^n$$

Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

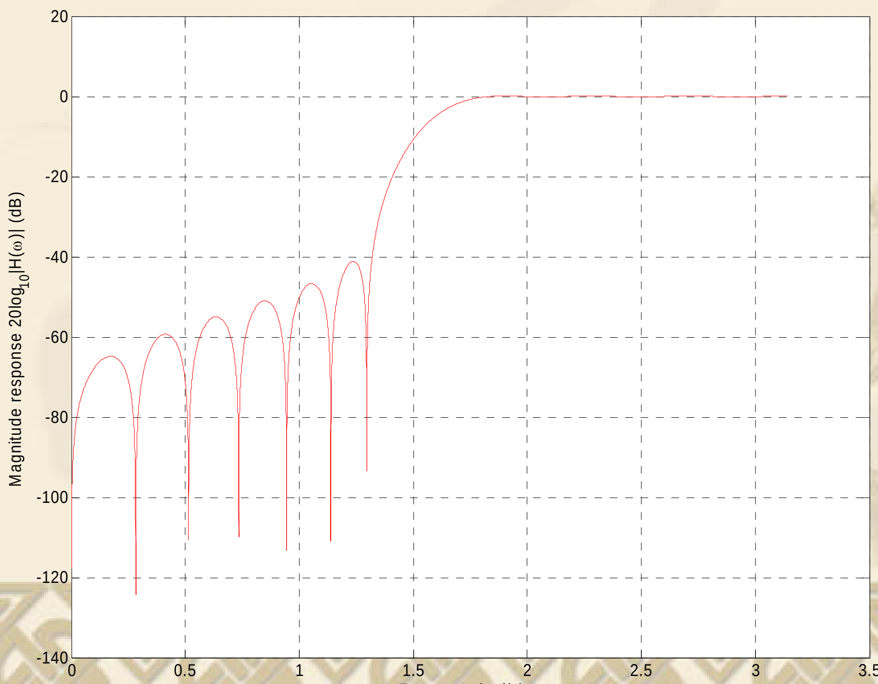
- Specification Design for Functional Inequality Constrained Optimization Problems
  - Estimation of minimum filter length with $\delta_p = 0.01$, $\delta_s = 0.01$, $\Delta B = 0.08\pi$, $f_c = \frac{\pi}{2}$ and $J \le -40\text{dB}$

    $\Rightarrow N = 28$

$N = 26$



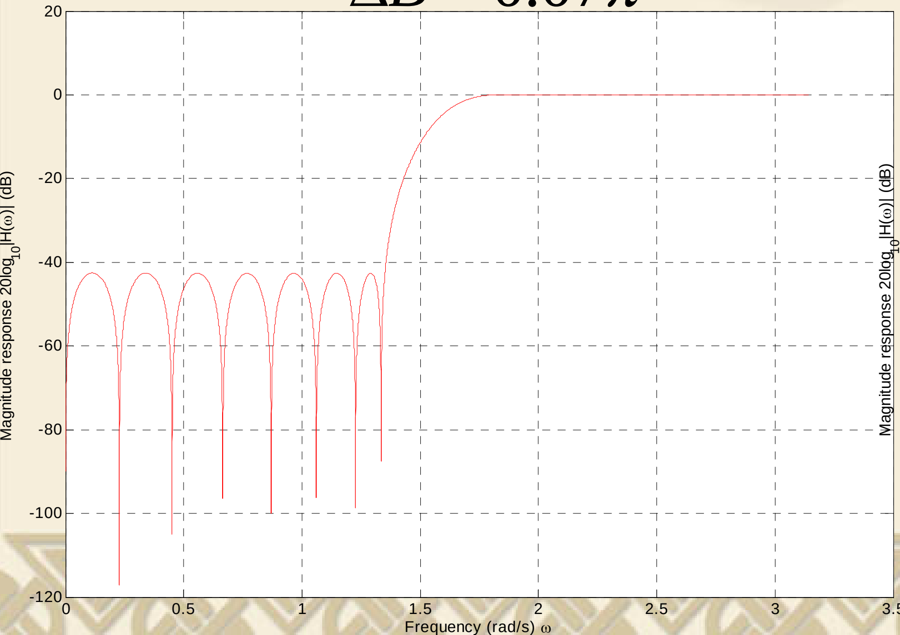
$N = 28$



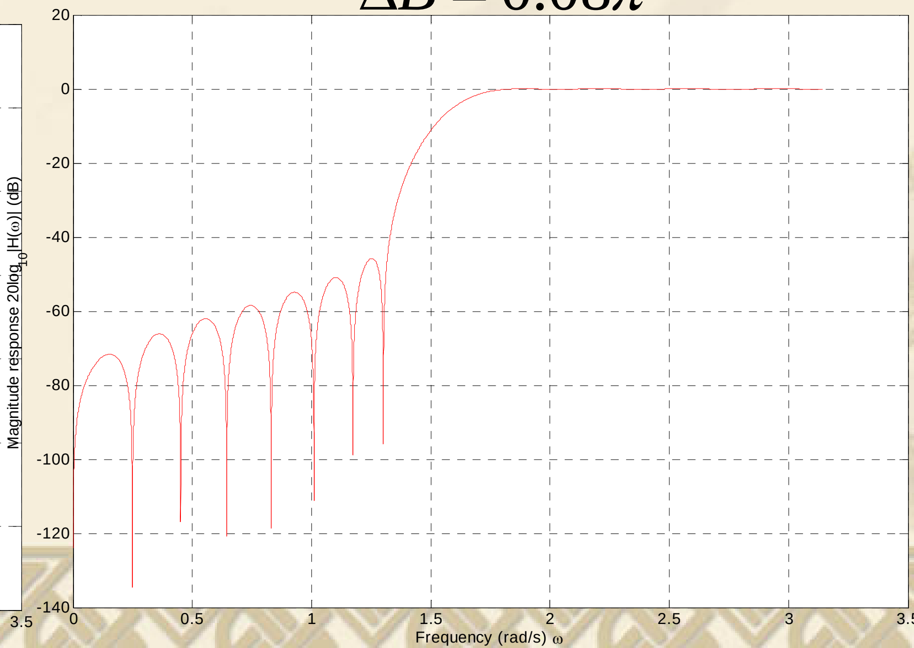
Challenges and Some Solutions for Solving Smooth Functional Inequality Constrained Optimization Problems

- Specification Design for Functional Inequality Constrained Optimization Problems
  - Estimation of minimum transition band bandwidth with $\delta_p = 0.006$, $\delta_s = 0.006$, $N = 32$, $f_c = \dfrac{\pi}{2}$ and $J \leq -45\text{dB}$
  $\Rightarrow \Delta B = 0.08\pi$

$\Delta B = 0.07\pi$



$\Delta B = 0.08\pi$



Examples of Nonsmooth Optimization Problems

- Integer-pixel, Half-pixel, Quarter-pixel, Fractional Pixel and Irrational Pixel Search in Motion Estimations
  - Objective: Find a motion vector such that the mean absolute difference between a block of an image in the current shifted frame and that in the next frame is minimized.

Examples of Nonsmooth Optimization Problems

- Integer-pixel, Half-pixel, Quarter-pixel, Fractional Pixel and Irrational Pixel Search in Motion Estimations

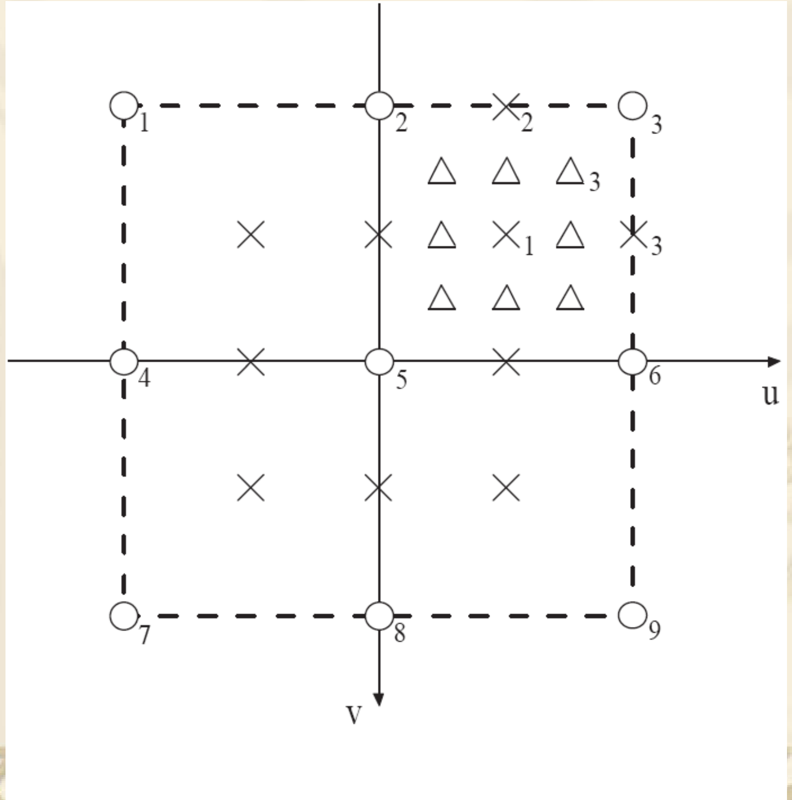


Fig. 3: Conventional fractional-pixel search.

Examples of Nonsmooth Optimization Problems

- Integer-pixel, Half-pixel, Quarter-pixel, Fractional Pixel and Irrational Pixel Search in Motion Estimations

$$x_1 = \frac{1}{4}(o_2 + o_3 + o_5 + o_6)$$

$$\Delta_3 = \frac{1}{4}(x_1 + x_2 + x_3 + o_3)$$

$$x_2 = \frac{1}{2}(o_2 + o_3)$$

$$x_3 = \frac{1}{2}(o_3 + o_6)$$

$$\Delta_3 = \frac{3}{16}o_2 + \frac{9}{16}o_3 + \frac{1}{16}o_5 + \frac{3}{16}o_6$$

Examples of Nonsmooth Optimization Problems

- Integer-pixel, Half-pixel, Quarter-pixel, Fractional Pixel and Irrational Pixel Search in Motion Estimations

$$h_2 = \frac{3}{16}, h_3 = \frac{9}{16}, h_5 = \frac{1}{16}, h_6 = \frac{3}{16}$$

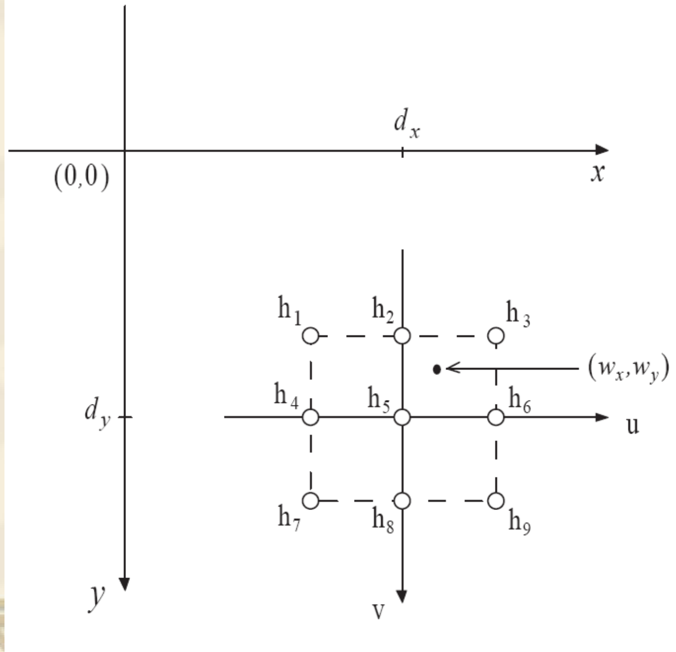


Fig. 1: Proposed 2D FIR filter structure.

Examples of Nonsmooth Optimization Problems

- Integer-pixel, Half-pixel, Quarter-pixel, Fractional Pixel and Irrational Pixel Search in Motion Estimations

$$MAD = \frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left|B_{k+1}(x,y) - B_k\left(x+d_x+w_x, y+d_y+w_y\right)\right|$$

$$B_k\left(x+d_x+w_x, y+d_y+w_y\right) = \boldsymbol{\alpha}_{Nx+y}^T \mathbf{x}$$

where $\mathbf{x} \equiv \begin{bmatrix} h_1 & h_2 & \cdots & h_{(2L+1)^2} \end{bmatrix}^T$

Examples of Nonsmooth Optimization Problems

- Integer-pixel, Half-pixel, Quarter-pixel, Fractional Pixel and Irrational Pixel Search in Motion Estimations

$$
\boldsymbol{\alpha}_{Nx+y} \equiv \begin{bmatrix} B_k\left(x+d_x-L, y+d_y-L\right) \\ \vdots \\ B_k\left(x+d_x+L, y+d_y-L\right) \\ B_k\left(x+d_x-L, y+d_y-L+1\right) \\ \vdots \\ B_k\left(x+d_x+L, y+d_y-L+1\right) \\ \vdots \\ B_k\left(x+d_x-L, y+d_y+L\right) \\ \vdots \\ B_k\left(x+d_x+L, y+d_y+L\right) \end{bmatrix}
$$

Examples of Nonsmooth Optimization Problems

- Integer-pixel, Half-pixel, Quarter-pixel, Fractional Pixel and Irrational Pixel Search in Motion Estimations

Problem (**R**) $\min_{\mathbf{x}} SAD = \sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\left|B_{k+1}(x,y) - \boldsymbol{\alpha}_{Nx+y}^{T}\mathbf{x}\right|$

subject to $0 \le x_i \le 1$ for $i = 1,2,\cdots,(2L+1)^2$

Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

- Challenge of Nonsmooth Optimization Problems
  - How to solve nonsmooth optimization problems?

Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

- Some Solution for Solving Nonsmooth Optimization Problems
- Denote $z_{Np+q} \equiv B_{k+1}(p,q) - \boldsymbol{\alpha}_{Np+q}^T \mathbf{x}$

$$s_{Np+q} = \operatorname{sgn}(z_{Np+q})$$

where $\operatorname{sgn}(z_{Np+q}) = \begin{cases} 1 & z_{Np+q} \geq 0 \\ -1 & z_{Np+q} < 0 \end{cases}$

$$SAD = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1}\left|B_{k+1}(p,q) - \boldsymbol{\alpha}_{Np+q}^T \mathbf{x}\right| = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1}\left|z_{Np+q}\right| = \sum_{p=0}^{N-1}\sum_{q=0}^{N-1} s_{Np+q} z_{Np+q}$$

- Denote $\mathbf{z} \equiv \begin{bmatrix} z_0 & \cdots & z_{N-1} & \cdots & z_{N(N-1)} & \cdots & z_{N^2-1} \end{bmatrix}^T$

$$\mathbf{s} \equiv \begin{bmatrix} s_0 & \cdots & s_{N-1} & \cdots & s_{N(N-1)} & \cdots & s_{N^2-1} \end{bmatrix}^T$$

$$\mathbf{y} \equiv \begin{bmatrix} \mathbf{z} \\ \mathbf{s} \end{bmatrix}$$

Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

$$\mathbf{A} \equiv \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$$

then $SAD = \frac{1}{2}\mathbf{y}^T\mathbf{A}\mathbf{y}$

- Denote $\mathbf{B} \equiv [\mathbf{I} \quad \mathbf{0}]$

$$\boldsymbol{\beta} \equiv [B_{k+1}(0,0) \quad \cdots \quad B_{k+1}(0,N-1) \quad \cdots \quad B_{k+1}(N-1,0) \quad \cdots \quad B_{k+1}(N-1,N-1)]^T$$

$$\boldsymbol{\alpha} \equiv [\boldsymbol{\alpha}_0 \quad \cdots \quad \boldsymbol{\alpha}_{N-1} \quad \cdots \quad \boldsymbol{\alpha}_{N(N-1)} \quad \cdots \quad \boldsymbol{\alpha}_{N^2-1}]$$

then $\boldsymbol{\alpha}^T\mathbf{x} = \boldsymbol{\beta} - \mathbf{B}\mathbf{y}$ and $\mathbf{x} = (\boldsymbol{\alpha}\boldsymbol{\alpha}^T)^{-1}(\boldsymbol{\alpha}\boldsymbol{\beta} - \boldsymbol{\alpha}\mathbf{B}\mathbf{y})$

- Denote $\mathbf{C}_i \equiv [\mathbf{0} \quad 1 \quad \mathbf{0}]$ such that $\mathbf{C}_i\mathbf{y} = s_i$ for $i = 0,1,\cdots,N^2-1$

  $\mathbf{D}_i \equiv [\mathbf{0} \quad 1 \quad \mathbf{0}]$ such that $\mathbf{D}_i\mathbf{y} \equiv z_i$ for $i = 0,1,\cdots,N^2-1$

  and $\mathbf{E}_i \equiv [\mathbf{0} \quad 1 \quad \mathbf{0}]$ such that $\mathbf{E}_i\mathbf{x} \equiv x_i$ for $i = 1,2,\cdots,(2L+1)^2$

Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

$s_i^2 = 1$ for $i = 0,1,\cdots,N^2 - 1$ implies $\mathbf{y}^T\mathbf{C}_i^T\mathbf{C}_i\mathbf{y} = 1$ for $i = 0,1,\cdots,N^2 - 1$

$s_i z_i \ge 0$ for $i = 0,1,\cdots,N^2 - 1$ implies $\mathbf{y}\mathbf{C}_i^T\mathbf{D}_i\mathbf{y} \ge 0$ for $i = 0,1,\cdots,N^2 - 1$

$0 \le x_i \le 1$ for $i = 1,2,\cdots,(2L+1)^2$ implies

$0 \le \mathbf{E}_i\left(\boldsymbol{\alpha}\boldsymbol{\alpha}^T\right)^{-1}\left(\boldsymbol{\alpha}\boldsymbol{\beta} - \boldsymbol{\alpha}\mathbf{B}\mathbf{y}\right) \le 1$ for $i = 1,2,\cdots,(2L+1)^2$

$$\min_{\mathbf{y}} \quad SAD = \frac{1}{2}\mathbf{y}^T\mathbf{A}\mathbf{y}$$

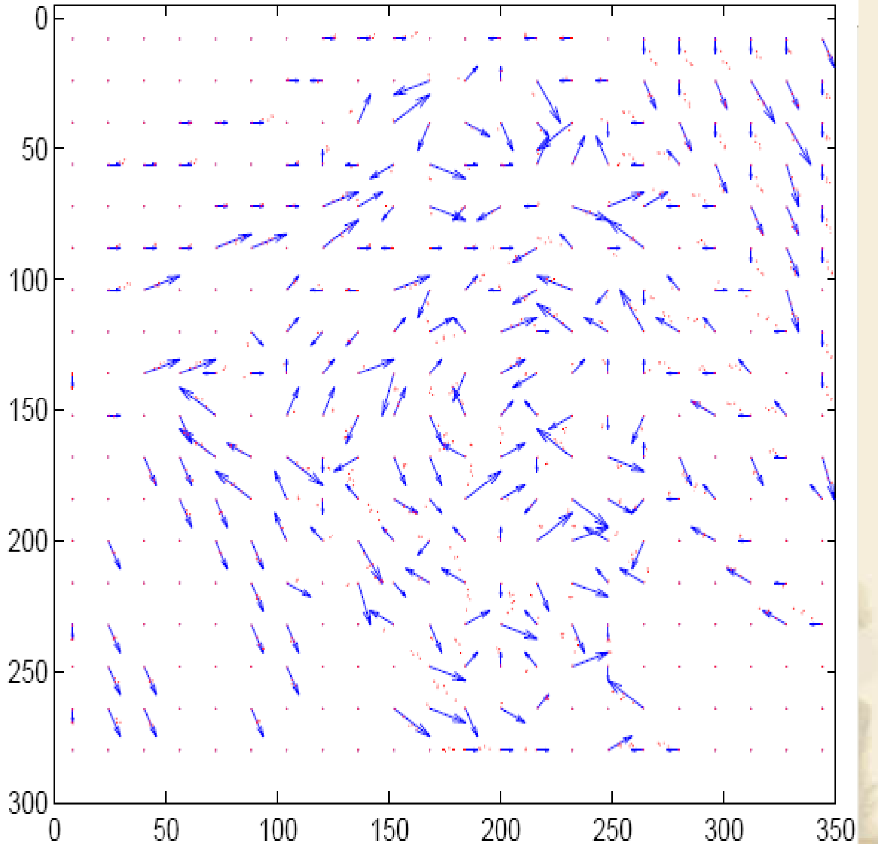
subject to $\mathbf{y}^T\mathbf{C}_i^T\mathbf{C}_i\mathbf{y} = 1$ for $i = 0,1,\cdots,N^2 - 1$

$\mathbf{y}\mathbf{C}_i^T\mathbf{D}_i\mathbf{y} \ge 0$ for $i = 0,1,\cdots,N^2 - 1$

$0 \le \mathbf{E}_i\left(\boldsymbol{\alpha}\boldsymbol{\alpha}^T\right)^{-1}\left(\boldsymbol{\alpha}\boldsymbol{\beta} - \boldsymbol{\alpha}\mathbf{B}\mathbf{y}\right) \le 1$ for $i = 1,2,\cdots,(2L+1)^2$

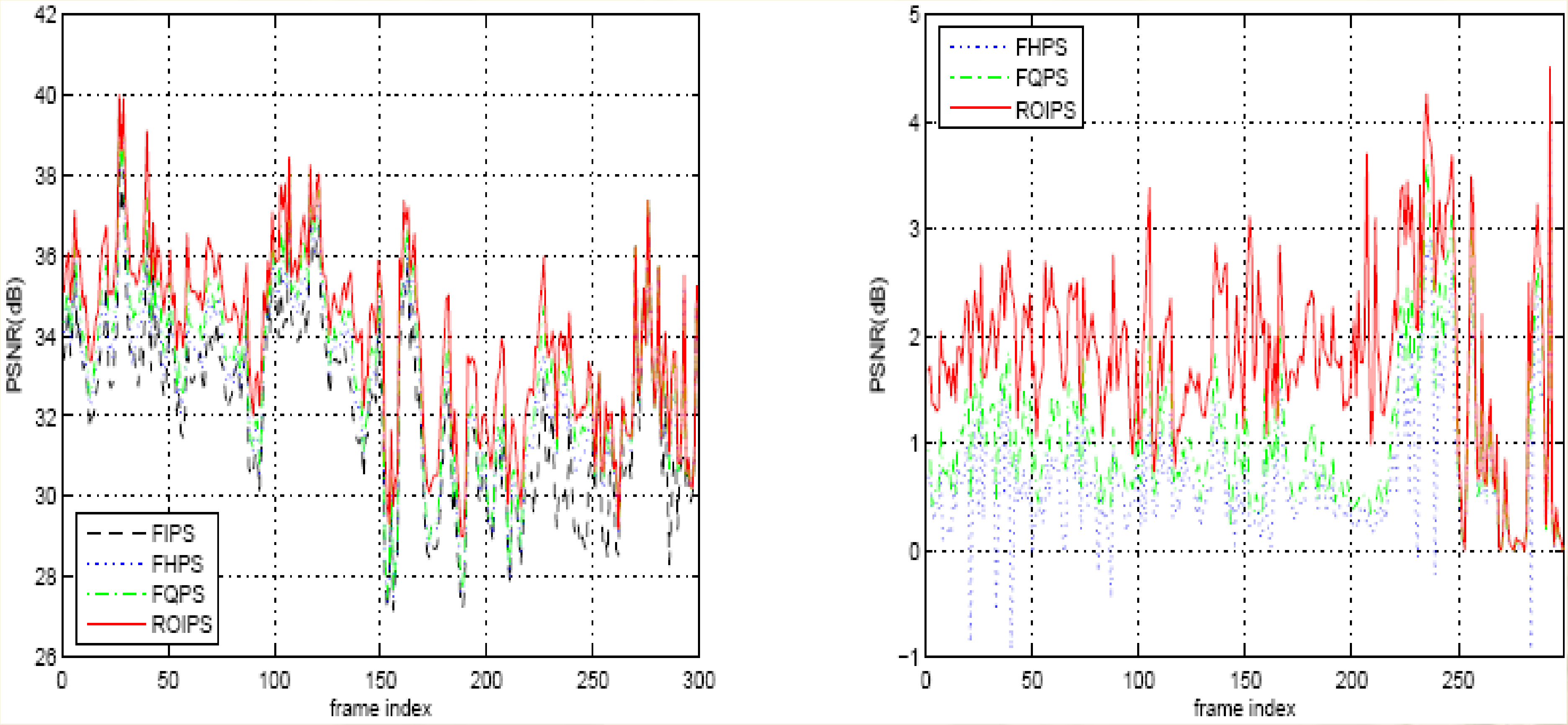
Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

- Motion Vector of “Foreman”



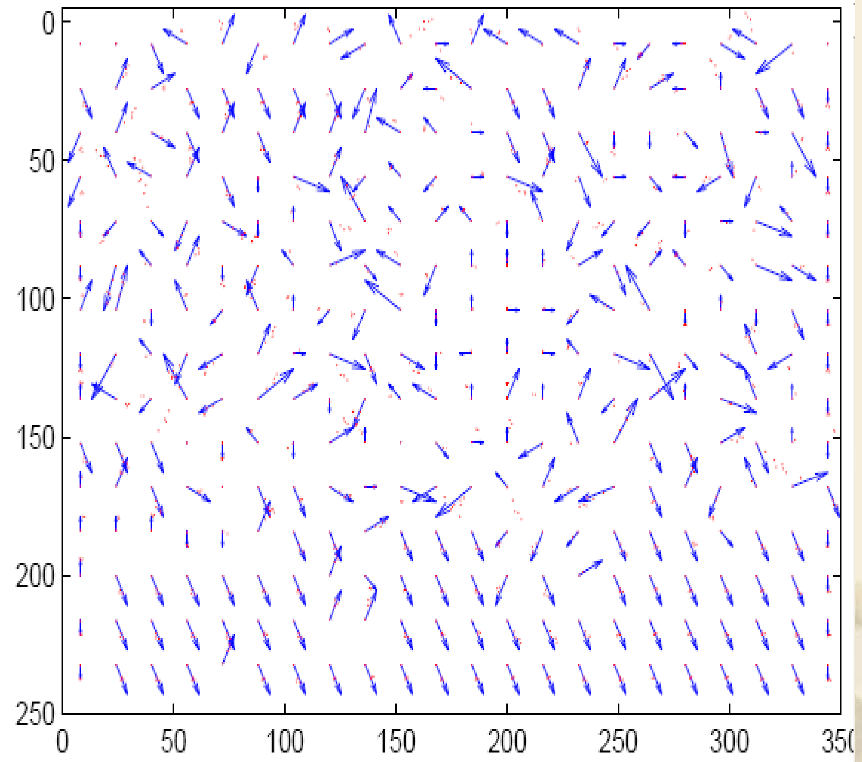
Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

- PSNR of "Foreman"



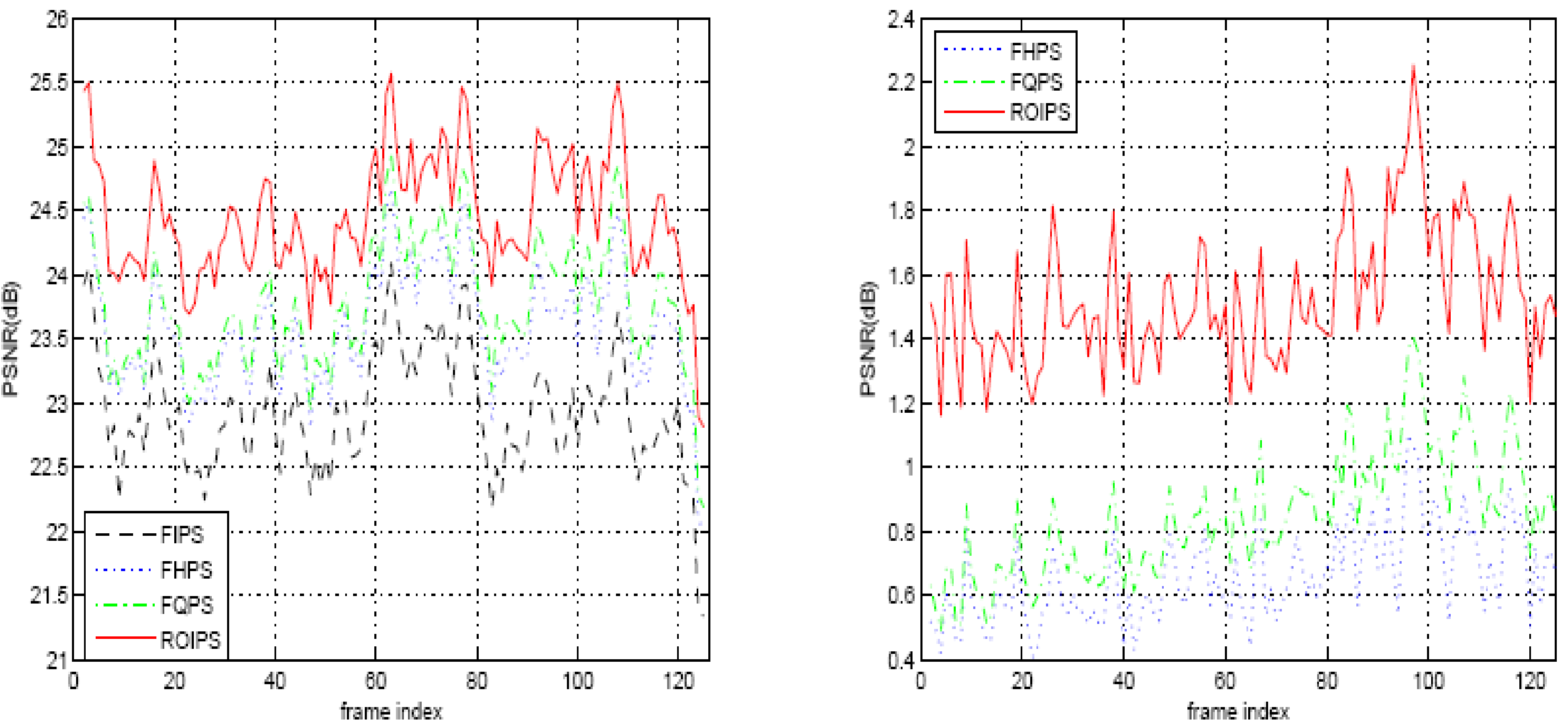
Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

- Motion Vector of "Football"



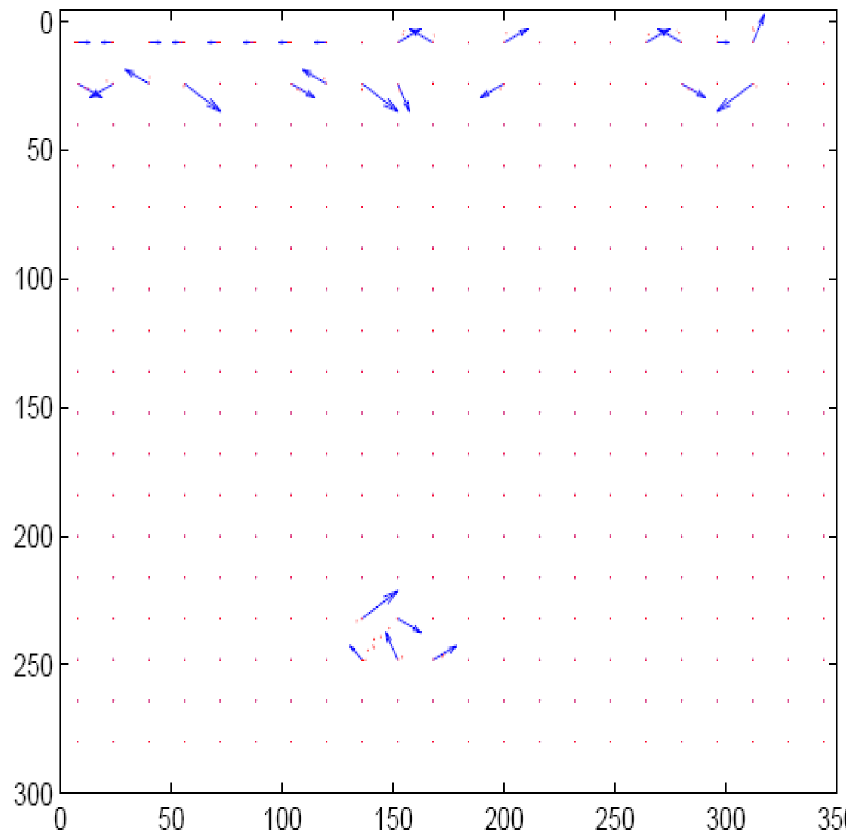
Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

- PSNR of "Football"



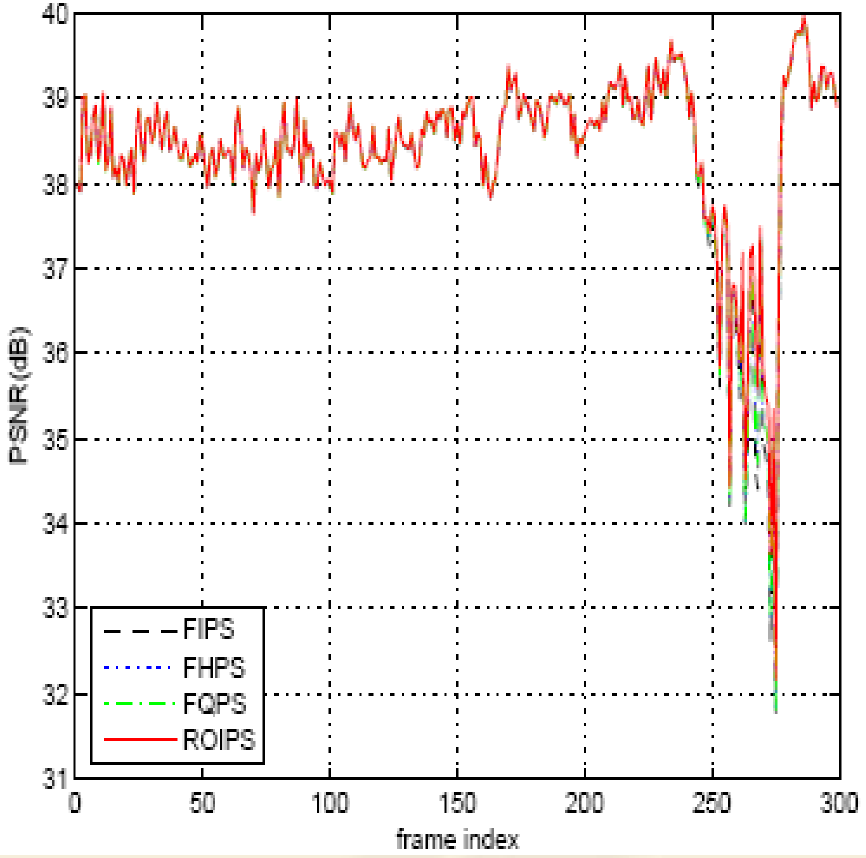
Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

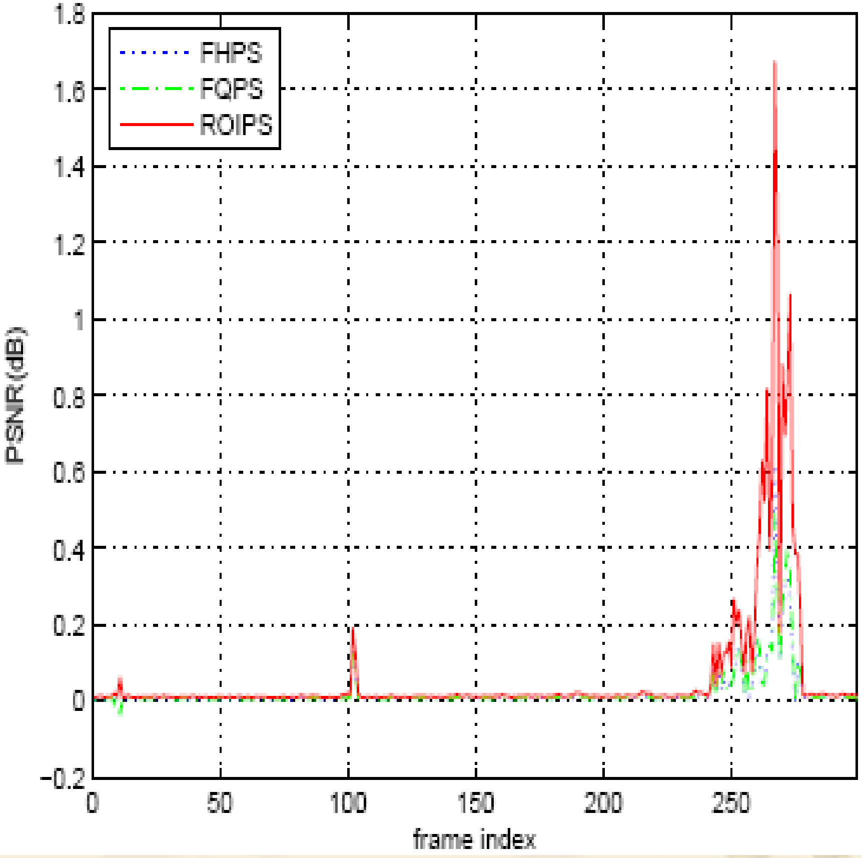
- Motion Vector of “Container”



Challenges and Some Solutions for Solving Nonsmooth Optimization Problems

- PSNR of “Container”





Examples of Nonsmooth Functional Inequality Constrained Optimization Problems

- IIR Filter Design Problems
  - Objective: Minimize the weighted total ripple energy subject to the weighted peak constraint.

$$H(\omega)=\frac{e^{-jD\omega}\sum_{m=0}^{M}b_m e^{-jm\omega}}{1+\sum_{n=1}^{N}a_n e^{-jn\omega}}$$

$$\left|\frac{e^{-jD\omega}\sum_{m=0}^{M}b_m e^{-jm\omega}}{1+\sum_{n=1}^{N}a_n e^{-jn\omega}}\right|^2 \approx \left(\tilde{H}(\omega)\right)^2$$

$$E(\omega)\equiv\left|e^{-jD\omega}\sum_{m=0}^{M}b_m e^{-jm\omega}\right|^2-\left(\tilde{H}(\omega)\right)^2\left|1+\sum_{n=1}^{N}a_n e^{-jn\omega}\right|^2$$

Examples of Nonsmooth Functional Inequality Constrained Optimization Problems

- IIR Filter Design Problems

$$\mathbf{x}_n \equiv [b_0, \quad b_1, \quad \cdots, \quad b_M]^T$$

$$\mathbf{x}_d \equiv [a_1, \quad a_2, \quad \cdots, \quad a_N]^T$$

$$\boldsymbol{\eta}_n(\omega) \equiv [1, \quad e^{-j\omega}, \quad \cdots, \quad e^{-jM\omega}]^T$$

$$\boldsymbol{\eta}_d(\omega) \equiv [e^{-j\omega}, \quad e^{-j2\omega}, \quad \cdots, \quad e^{-jN\omega}]^T$$

$$E(\omega) = \left|(\boldsymbol{\eta}_n(\omega))^T \mathbf{x}_n\right|^2 - \left(\tilde{H}(\omega)\right)^2 \left|1 + (\boldsymbol{\eta}_d(\omega))^T \mathbf{x}_d\right|^2$$

$$\tilde{J}(\mathbf{x}_n, \mathbf{x}_d) \equiv \int_{B_P \cup B_S} W(\omega) \left|E(\omega)\right| d\omega$$

where $W(\omega) > 0 \quad \forall \omega \in B_P \cup B_S$

Examples of Nonsmooth Functional Inequality Constrained Optimization Problems

- IIR Filter Design Problems

$$\mathrm{Re}\left(1+(\boldsymbol{\eta}_d(\omega))^T\mathbf{x}_d\right)>0 \quad \forall\omega\in[-\pi,\pi]$$

$$\tilde{W}(\omega)\left|E(\omega)\right|\le\tilde{\delta}(\omega) \quad \forall\omega\in B_P\bigcup B_S$$

where $\tilde{W}(\omega)>0 \quad \forall\omega\in B_P\bigcup B_S$

$$\tilde{W}(\omega)E(\omega)\le\tilde{\delta}(\omega) \quad \forall\omega\in B_P\bigcup B_S$$

and $-\tilde{\delta}(\omega)\le\tilde{W}(\omega)E(\omega) \quad \forall\omega\in B_P\bigcup B_S$

Problem (**Q**) $\min_{(\mathbf{x}_n,\mathbf{x}_d)} \tilde{J}(\mathbf{x}_n,\mathbf{x}_d)\equiv\int_{B_P\bigcup B_S}W(\omega)\left|E(\omega)\right|d\omega$

subject to

$$\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)\equiv\tilde{W}(\omega)E(\omega)-\tilde{\delta}(\omega)\le 0 \quad \forall\omega\in B_P\bigcup B_S$$
$$\tilde{g}_2(\mathbf{x}_n,\mathbf{x}_d,\omega)\equiv-\tilde{W}(\omega)E(\omega)-\tilde{\delta}(\omega)\le 0 \quad \forall\omega\in B_P\bigcup B_S$$
$$\tilde{g}_3(\mathbf{x}_d,\omega)\equiv\mathrm{Re}\left(1+(\boldsymbol{\eta}_d(\omega))^T\mathbf{x}_d\right)>0 \quad \forall\omega\in[-\pi,\pi]$$

Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Challenge of Nonsmooth Functional Inequality Constrained Optimization Problems
  - How to solve nonsmooth functional inequality constrained optimization problems?

Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Some Solution for Solving Nonsmooth Optimization Problems
  - Consider the following IIR filter design problem with the error function

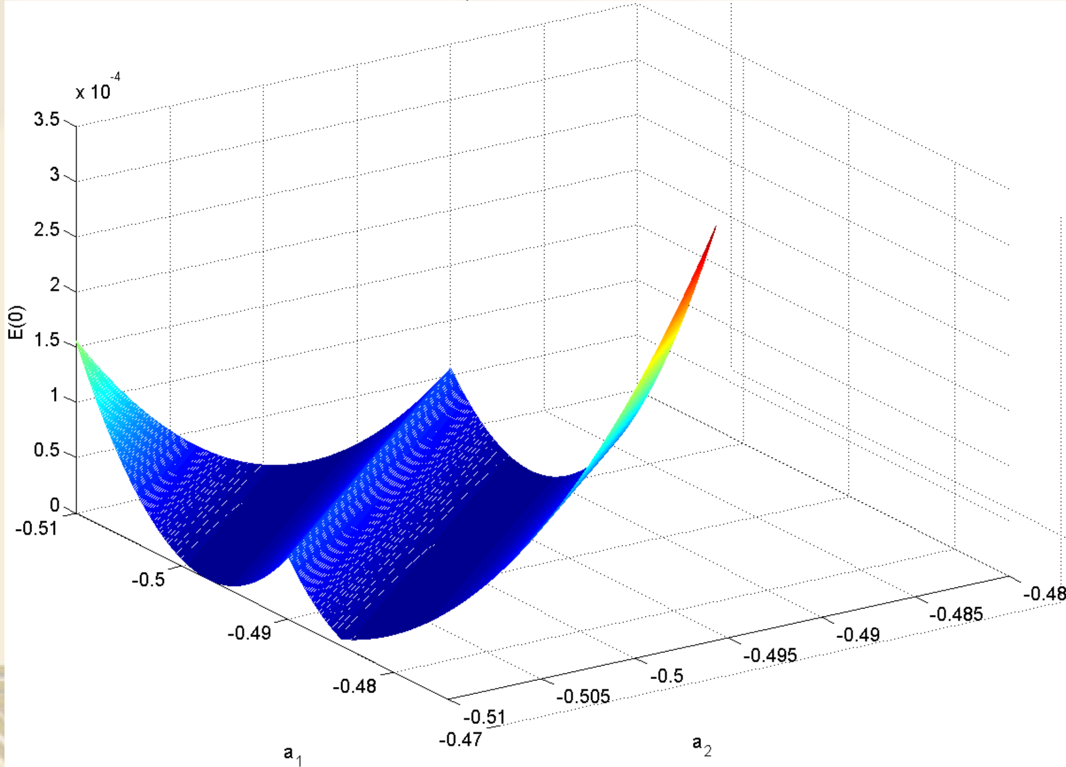
$$E(\omega) \equiv \left(\left| \sum_{m=0}^{2} b_m e^{-jm\omega} \right| - \left| 1 + \sum_{n=1}^{2} a_n e^{-jn\omega} \right| \right)^2$$

where

$$b_0 = 2.816335701763035 \times 10^{-3}$$

$$b_1 = 1.87755713450866 2 \times 10^{-3}$$

$$b_2 = 2.816335701763063 \times 10^{-3}$$



Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Some Solution for Solving Nonsmooth Optimization Problems
  - Since $\max\{\tilde{g}_1(\mathbf{x}_n, \mathbf{x}_d, \omega), 0\} = \begin{cases} 0 & \tilde{g}_1(\mathbf{x}_n, \mathbf{x}_d, \omega) \le 0 \\ \text{positive value} & \tilde{g}_1(\mathbf{x}_n, \mathbf{x}_d, \omega) > 0 \end{cases}$
  - By defining
$$\hat{g}_1(\mathbf{x}_n, \mathbf{x}_d) \equiv \int_{B_P \cup B_S} \left(\max\{\tilde{g}_1(\mathbf{x}_n, \mathbf{x}_d, \omega), 0\}\right)^2 d\omega$$
  - We have
$$\hat{g}_1(\mathbf{x}_n, \mathbf{x}_d) = \begin{cases} 0 & \forall \omega \in B_P \cup B_S, \tilde{g}_1(\mathbf{x}_n, \mathbf{x}_d, \omega) \le 0 \\ \text{positive value} & \exists \omega \in B_P \cup B_S, \tilde{g}_1(\mathbf{x}_n, \mathbf{x}_d, \omega) > 0 \end{cases}$$
$$\tilde{g}_1(\mathbf{x}_n, \mathbf{x}_d, \omega) \le 0 \ \forall \omega \in B_P \cup B_S \Leftrightarrow \hat{g}_1(\mathbf{x}_n, \mathbf{x}_d) = 0$$

Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Some Solution for Solving Nonsmooth Optimization Problems

$$\left(\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\right)^2 = \begin{cases} 0 & \tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)\le 0 \\ \left(\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)\right)^2 & \tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)> 0 \end{cases}$$

$$\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\left(\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\right)^2 = \begin{cases} \mathbf{0} & \tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)< 0 \\ 2\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega) & \tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)> 0 \end{cases}$$

  - As $2\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)=\mathbf{0}$ when $\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)=0$. $\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\left(\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\right)^2$ is continuous at $\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)=0$.
  - As $2\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)=\mathbf{0}$ when $\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)<0$ and $2\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)=2\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)$ when $\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)>0$

    We have, $\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\left(\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\right)^2 = 2\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)$

Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Some Solution for Solving Nonsmooth Optimization Problems
  - Consequently, we have

$$\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\hat{g}_1(\mathbf{x}_n,\mathbf{x}_d)=2\int_{B_P\cup B_S}\max\{\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_1(\mathbf{x}_n,\mathbf{x}_d,\omega)d\omega$$

  - Similarly, define $\hat{g}_2(\mathbf{x}_n,\mathbf{x}_d)\equiv\int_{B_P\cup B_S}\left(\max\{\tilde{g}_2(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\right)^2 d\omega$

$$\hat{g}_3(\mathbf{x}_d)\equiv\int_{[-\pi,\pi]}\left(\max\{\tilde{g}_3(\mathbf{x}_d,\omega),0\}\right)^2 d\omega$$

  - We have

$$\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\hat{g}_2(\mathbf{x}_n,\mathbf{x}_d)=2\int_{B_P\cup B_S}\max\{\tilde{g}_2(\mathbf{x}_n,\mathbf{x}_d,\omega),0\}\nabla_{(\mathbf{x}_n,\mathbf{x}_d)}\tilde{g}_2(\mathbf{x}_n,\mathbf{x}_d,\omega)d\omega$$

$$\nabla_{\mathbf{x}_d}\hat{g}_3(\mathbf{x}_d)=2\int_{[-\pi,\pi]}\max\{\tilde{g}_3(\mathbf{x}_d,\omega),0\}\nabla_{\mathbf{x}_d}\tilde{g}_3(\mathbf{x}_d,\omega)d\omega$$

Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Some Solution for Solving Nonsmooth Optimization Problems
  - Now the problem become the following equality constrained optimization problem.

$$\min_{(\mathbf{x}_n,\mathbf{x}_d)} \quad \tilde{J}(\mathbf{x}_n,\mathbf{x}_d) \equiv \int_{B_P \cup B_S} W(\omega)\left|E(\omega)\right| d\omega$$

subject to

$$\hat{g}_1(\mathbf{x}_n,\mathbf{x}_d) = 0$$

$$\hat{g}_2(\mathbf{x}_n,\mathbf{x}_d) = 0$$

$$\hat{g}_3(\mathbf{x}_d) = 0$$

$\forall \omega \in B_P \cup B_S$ and $\forall \varepsilon > 0$, define $E_\varepsilon(\omega) \equiv \begin{cases} \left|E(\omega)\right| & \left|E(\omega)\right| \geq \frac{\varepsilon}{2} \\ \frac{(E(\omega))^2}{\varepsilon} + \frac{\varepsilon}{4} & \left|E(\omega)\right| < \frac{\varepsilon}{2} \end{cases}$

and $J_\varepsilon(\mathbf{x}_n,\mathbf{x}_d) \equiv \int_{B_P \cup B_S} W(\omega) E_\varepsilon(\omega) d\omega$

Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Some Solution for Solving Nonsmooth Optimization Problems
  - Now we approximate the problem as the following smooth optimization problem:

$$\min_{(\mathbf{x}_n,\mathbf{x}_d)} \quad J_\varepsilon(\mathbf{x}_n,\mathbf{x}_d) \equiv \int_{B_P \cup B_S} W(\omega) E_\varepsilon(\omega) d\omega$$

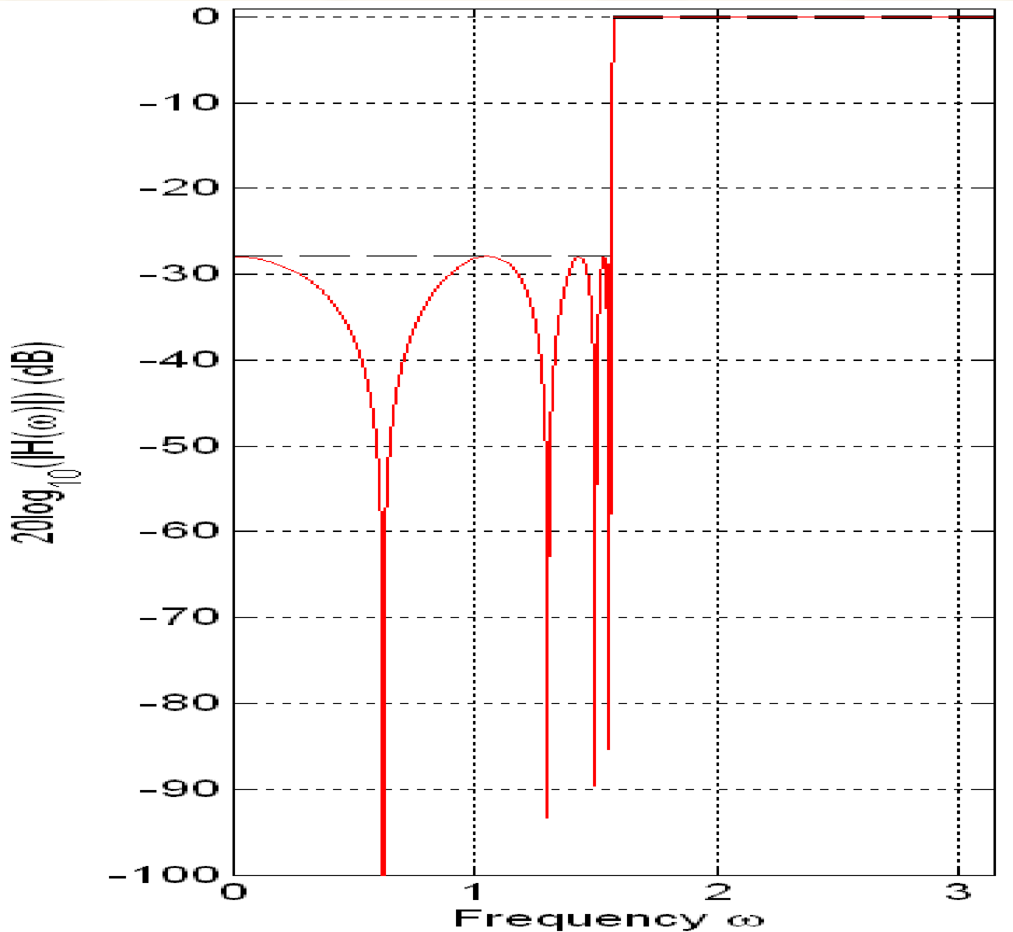
subject to $\hat{g}_1(\mathbf{x}_n,\mathbf{x}_d)=0$

$$\hat{g}_2(\mathbf{x}_n,\mathbf{x}_d)=0$$

$$\hat{g}_3(\mathbf{x}_d)=0$$

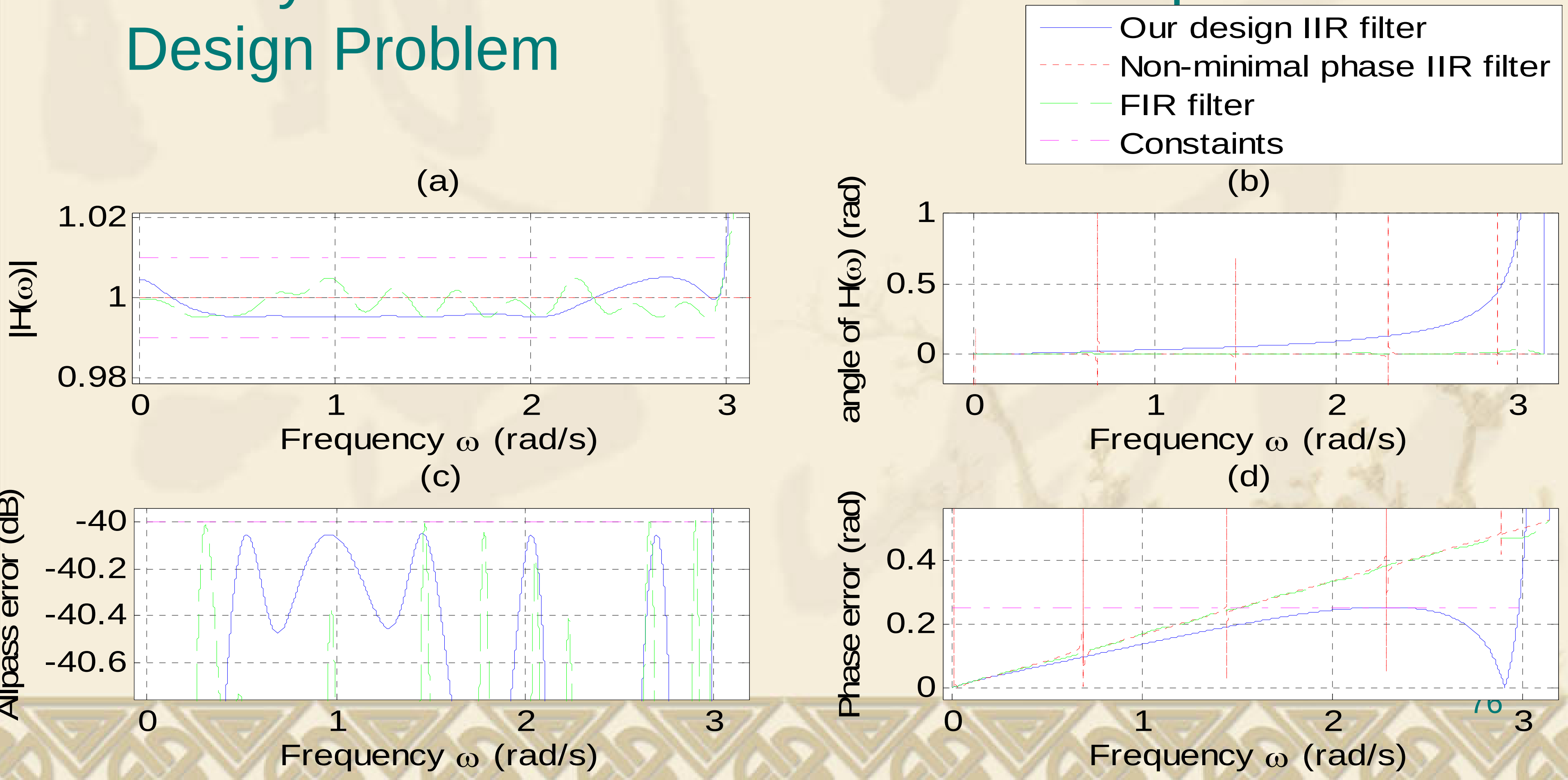
Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- IIR Filter Design Problem



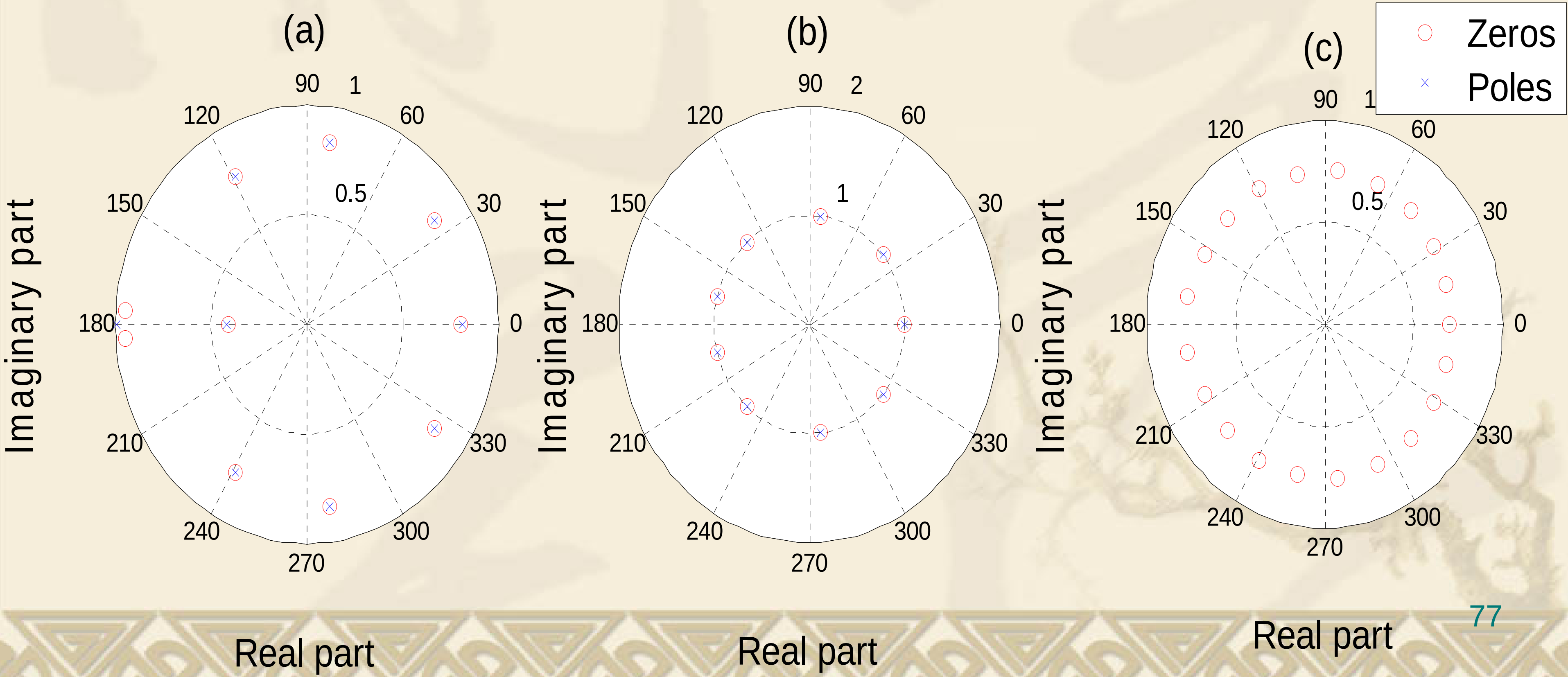
Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Strictly Stable Minimal Phase Near Allpass IIR Filter Design Problem



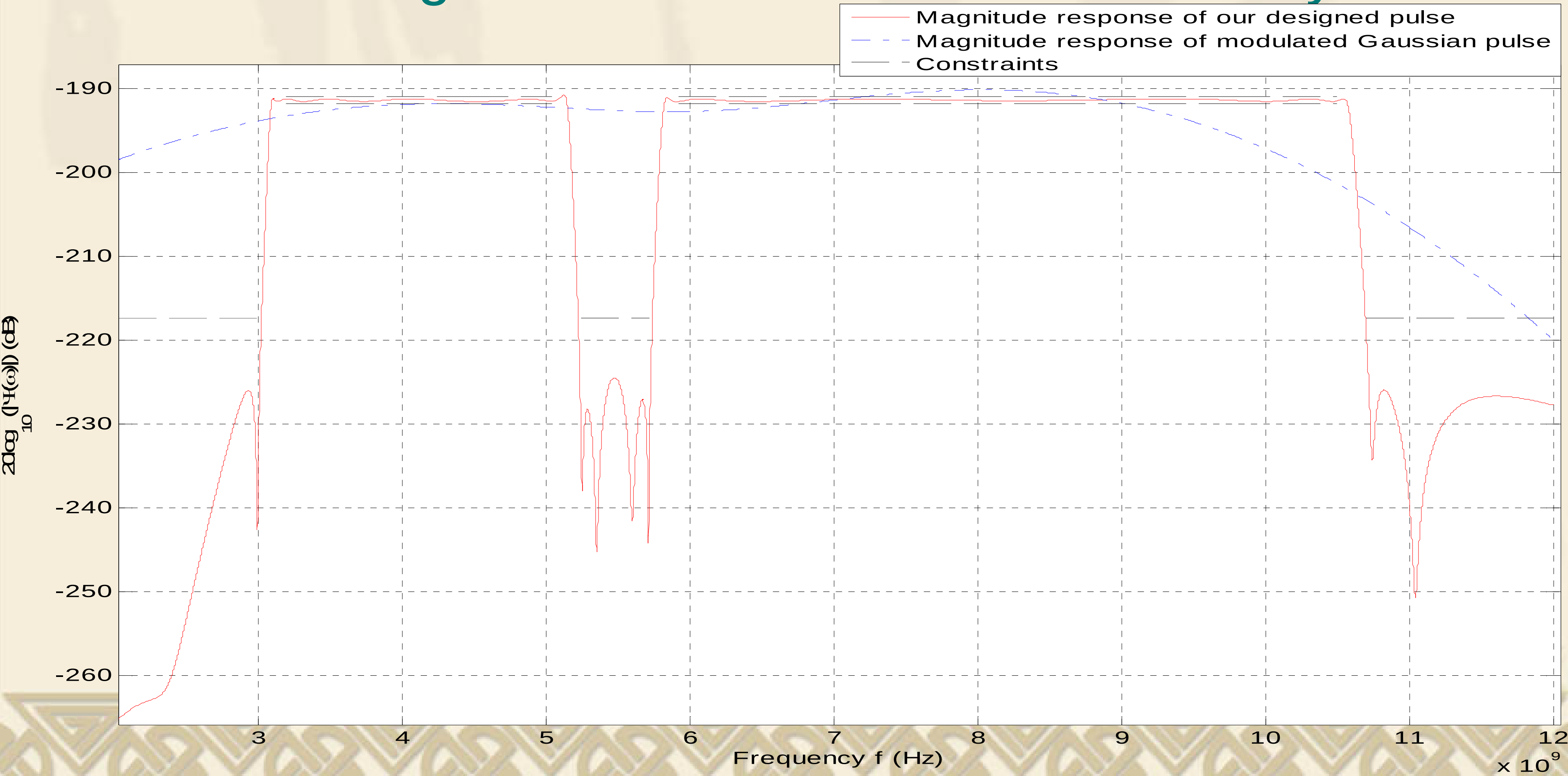
Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Strictly Stable Minimal Phase Near Allpass IIR Filter Design Problem



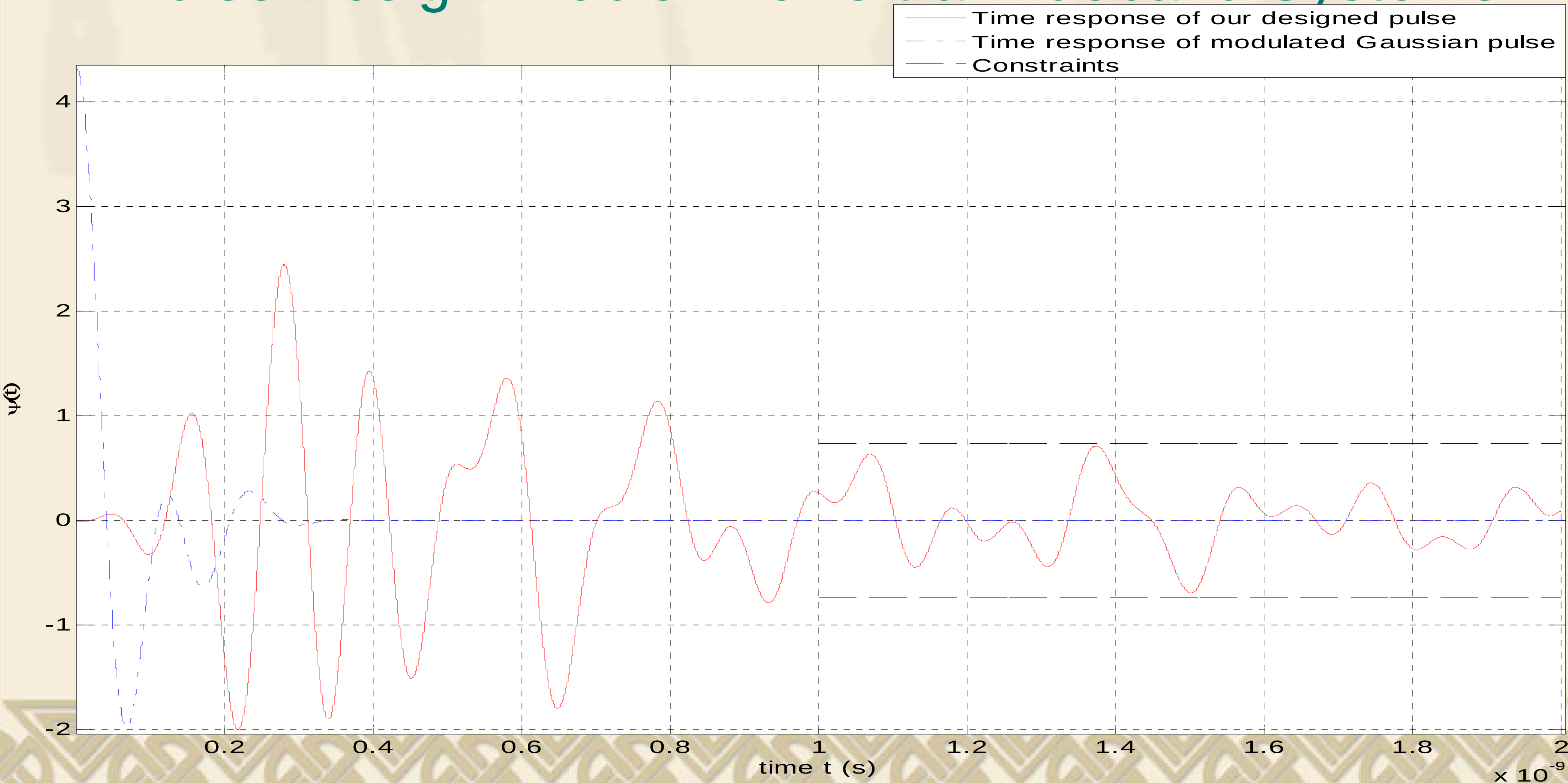
Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Pulse Design Problem for Ultra-wideband Systems



Challenges and Some Solutions for Solving Nonsmooth Functional Inequality Constrained Optimization Problems

- Pulse Design Problem for Ultra-wideband Systems



Conclusions

- Many practical problems in signal processing and communications systems can be formulated as optimization problems.
- A solution has been proposed to guarantee the obtained solution satisfying the functional inequality constraint.
- Efficient approach has been proposed to solve a functional inequality constrained optimization problem.
- Empirical formulae has been proposed for specification design of FIR linear phase anti-symmetric peak constrained least squares filter.
- A nonsmooth optimization problem is transformed to a smooth optimization problem.
- A nonsmooth functional inequality constrained optimization problem is approximated by a smooth equality constrained optimization problem.

Questions and Answers

Thank you!

Let me think…