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Estimation of Longevity Risk and Mortality Modelling

TABI ROSY CHRISTY ATEMNKENG

Dissertation presented as partial requirement for obtaining the master's degree in Statistics and Information Management Risk, with a specialization in Analysis and Risk Management

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Abstract

Previous mortality models failed to account for improvements in human mortality rates thus in general, human life expectancy was underestimate. Declining mortality and increasing life expectancy (longevity) profoundly alter the population age distribution. This demographic transition has received considerable attention on pension and annuity providers.

Concerns have been expressed about the implications of increased life expectancy for government spending on old-age support. The goal of this paper is to lay out a framework for measuring, understanding, and analyzing longevity risk, with a focus on defined pension plans. Lee-Carter proposed a widely used mortality forecasting model in 1992. The study looks at how well the Lee-Carter model performed for female and male populations in the selected country (France) from 1816 to 2018. The Singular Value Decomposition (SVD) method is used to estimate the parameters of the LC model.

The mortality table then assesses future improvements in mortality and life expectancy, taking into account mortality assumptions, to see if pension funds and annuity providers are exposed to longevity risk. Mortality assumptions are predicted death rates based on a mortality table. The two types of mortality are mortality at birth and mortality in old age. Longevity risk must be effectively managed by pension and annuity providers. To mitigate this risk, pension providers must factor in future improvements in mortality and life expectancy, as mortality rates tend to decrease over time.

The findings show that failing to account for future improvements in mortality results in an expected provision shortfall. Protection mechanisms and policy recommendations to manage longevity risk can help to mitigate the financial impact of an unexpected increase in longevity.

Keywords: Lee-Carter (LC) model, Mortality modeling, Forecasting, Life expectancy, Singular value decomposition (SVD),

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1. INTRODUCTION

1.1. Background of the study

The continuous improvements in longevity bring new problems and challenges at different levels of political, social, economic, and regulatory. However, one of the most observed effects of this improvements in longevity is on pensions. In recent decades, most high-income countries have responded to continuous life expectancy increases, below replacement-level fertility, an upward trend in old-age dependency ratios, low productivity gains and economic growth, a rapidly shifting labor market and declining financial market returns with systemic (e.g., the switch towards a Non-Financial Defined Contribution (NDC) scheme in Sweden, Italy, Poland, Latvia and Norway; pension financialization, i.e., the expansion of private complementary occupational and personal pre-funded defined-contribution (DC) pensions) and/or gradual parametric reforms in national public pension schemes (e.g., updates in the early and normal retirement ages, modifications in the defined benefit (DB) pension formula) as part of their efforts to reduce or eliminate short-term and long-term imbalances between revenues and expenditures, alleviating the pressure on public finances, together with efforts to preserve minimum pension adequacy (OECD, 2019; Bravo & Herce, 2020).

For national public pension schemes, a common denominator of most reforms has been to introduce automatic adjustment or stabilization mechanisms specifically designed to correct for the financial imbalance of the pension system, mechanically updating the scheme's parameters to demographic and/or economic developments. A common denominator in most pension reforms adopted in developed countries has been to automatically link pension benefits to life expectancy developments observed at retirement ages. The link has been established and reinforced in multiple ways (Ayuso, Bravo & Holzmann, 2021b; Bravo & Ayuso, 2020, 2021): i) by indexing normal and early retirement ages to life expectancy (e.g., Denmark, The Netherlands, Portugal, UK); (ii) by linking entry pensions to sustainability factors (e.g., Finland, Portugal), (iii) by indexing the eligibility requirements to the contribution length (e.g., France); (iv) by conditioning the annual pension indexation (e.g., The Netherlands, Luxembourg); (v) by introducing longevity-linked risk-sharing life annuities in public and private pension schemes (Bravo & El Mekkaoui, 2018; Bravo, 2019, 2020, 2021a).

In 2009, most companies in developing countries closed the defined benefit retirement plans (such as 401(K) plans in the United States) offer to their employees. The pension plans provided to the employer can either be defined contribution or defined benefit. These plans ensure employees receive a certain amount at retirement. In addition, defined benefit pension plans have been replaced by defined contribution plans.

A defined-benefit program is a promise of lifetime retirement benefits and the most vital risk for retirement resulting from longevity. Longevity risk is the risk that insurance companies or pension funds faced when assumptions about life expectancies and mortality rates are inaccurate. Mortality rates and longevity trend risk are the main indicator considered when attempting to transfer longevity risk.

The insurance sectors faced the risk arising from increased longevity i.e., the trend of longevity improvement will significantly change in the future. To face this long-term risk, more capital must be set asides. Hence it has become more important for life office (insurance companies, pension funds) to find efficient and suitable method to transfer part of longevity risk to capital or financial market. However, longevity risk cannot be transferred so easily since it is difficult to understand and manage due to its long-term nature, precisely projections for longevity are sensitive and the modeling of integrated interest rate risk remains challenging. Two main factors when transferring the longevity risk for a particular pension plan or insurer must be considered. The first is the current mortality levels, which can be observed but vary considerably between socio-economic and health categories. The second is the risk of longevity, which is the risk trajectory for the ageing population and is systemic. Systematic mortality trend risk can be offset directly by keeping exposure to increased mortality. One reason for ceding the risk is the uncertainty concerning the longevity risk, especially because of the systematic nature, in a pension plan or an insurance company.

To manage the risk of longevity better, Individuals and life offices needs to fully understand longevity risk, or consider its implications, when they come to plan their retirement income. Three causes for this are uncertainty, underestimation, and complexity. To help us better understand these terms, (Yinglu Deng et al., 2012a) assert that

> *"Longevity risk describes the risk that an individual or group will live longer life than expected thus their mortality rate will be lower than expected, while mortality risk describes the risk that an individual or group will live a shorter life than expected thus their mortality rate will be higher than expected."*, pp. 697,2012.

Longevity risk for pensioners refers to "*the possibility that they will live to such an advanced age that they will deplete their retirement savings and have to rely solely on Social Security and Medicare for their expenses*".

Longevity risk in retirement planning can be defined as "*the risk that members of some reference population might live longer on average than anticipated*" (Stamp duty and land tax for non-resident owners of Australian property, n.d.). Longevity risk for individuals with DC pension savings can have significant implications when they retire. The risk of people outliving their retirement savings or the risk of people underspending their savings leads to lower pension incomes. For pension plans, longevity risk refers to the increase in retirement pension duties because of longer lifespans. For individuals, longevity risks mean a person's possibility of outliving on their pension assets. In the first place, the longevity risk is due to the fact that people are now living longer due to various factors like medical and health. This means that one can reasonably expect to add another

10 to 20 years after he retires at age 65 that is an arbitrary age of reference for exposure(Antolin, 2007).

The possible shortfall on the retirement system or pension arises from the uncertainty around mortality rates, life expectancy, and future improvement. Life expectancy at birth has doubled and life expectancy for individuals aged 65 has increased by average over recent decades. Thus, improvement in mortality needs to be considered when establishing mortality assumptions to determine how long payments are expected to be made by pension providers (Pablo Antolin, 2014, p.16).

Almost every calculation entail making a prediction about the future. The Lee-Carter model is one of the most commonly used models for predicting future mortality (Lee & Carter, 1992). Although it was originally designed to predict life expectancy, it is now also used to predict mortality rates at various ages. (Richards & Currie, 2009). Despite a substantial amount of research on mortality risk, the impact of all three mortality risk components and basic risk resulting from adverse selection on the risk level of a life insurance company, as well as the effectiveness of different risk management strategies in terms of reaching a desired risk level and hedging against unexpected changes in mortality, has not been studied. As a result, mortality risk is comprehensively modelled in order to gain a better understanding of the interaction between the various types of risk and risk management instruments.

Recently, there has been a rising interest to study mortality risk and its management. Mortality risk is the key indicator for modeling longevity risk for life offices and arises when an individual lives shorter than anticipated. This possesses a serious problem for annuity providers and pension funds because they need to take into consideration the increased life expectancy and declining mortality. Life expectancy estimates are a key variable in national public pension schemes.

Forecasts of age-specific mortality and survival rates are also central for the pricing of novel capital market solutions for longevity risk management such as longevity bonds, longevity swaps, q-forwards, S-forwards, K-forwards, longevity caps & floors (Coughlan et al., 2007; Bravo & Silva, 2006; Blake et al., 2019; Bravo, 2021b, Bravo & Nunes, 2021).

We need to understand longevity risk to avoid inaccurate estimates of life expectancy, by decomposing the different aspects of longevity risk and discussing a framework in which longevity risk can be controlled. It´s crucial for policy makers and the private sector to understand and use the best method to estimate an individual’s ‘remaining life expectancy. This is important to determine the initial benefit or pricing of retirement income products.

To estimate life expectancy two main approaches are used: cohort life tables and period life tables (Ayuso et al., 2021). Methods differentiate also according to whether they consider simultaneously the dimensions of age, period, and cohort (or year of birth). These three variables are a natural way to analyze how mortality rates change for people as they age, how medical and social progress over time, and what effects people experience from birth on a life-long lifetime. Through the projection of the effects of period and cohort, we can gain insights into how mortality rates might be in the future death rates. In addition, the magnitude of longevity risk impacts of annuity payments not only depends on the type of annuity guarantees but on how pensions funds account for mortality and life expectancy improvements. It is complicated by the lack of a common methodology to account for long life risk to assess the best way to address the improvements in mortality and life expectancy.

1.2. Statement of the Problem

Longevity may be one of the most important economic developments for mankind, this trend began about 250 years ago. As development progresses, longevity becomes a challenge for all institutions in society, especially those that provide income for retirement, health care, and long-term care. Nevertheless, longevity is a potential risk for societal development, income, and consumption per capita. However as with the income per capita, the growth in longevity (measured by a decrease in mortality rates or, via, an increase in life expectancy) is not uniform across socioeconomic groups. Current international evidence available to developed countries shows that the diversity of living conditions is manifested in many aspects of the socioeconomic system and is usually very large. Soon, there will be a slight increase in signal intensity in the nearest future (Ayuso et al., 2016).

In the last two hundred years, the life expectancy of developing countries has been steadily increasing. As an example, Oeppen and Vaupel (2002) estimate that the global record for women's life expectancy at birth has increased at a stable rate of approximately 3 months per year in the last 160 years. Despite a sign of social progress, increase in life expectancy is challenging to governments, private pension schemes and life insurers as it affects pension costs and healthcare costs. The actuaries and demographers have recognized the problems of aging and longevity and have therefore given considerable attention to the development of statistical methods to model and project mortality rates.

According to (Sexauer et al., 2015), Longevity risk is the risk to which an individual life span exceeds his or her life expectancy resulting in higher-than-expected payouts for insurance companies. Longevity risks have significant consequences on the individual level when they come to retire, as the individual turns to outlive their retirement savings. Pension funds and annuity providers make payments for the lifetime of the individual, and so accept and insure longevity risk. Reserve or funds are set aside to meet future payments. Under DC pension arrangement, the financial cost of longevity risk rests with individuals outlive their retirement savings.

Annuity providers, pension funds and insurance institutions with potential liabilities on longevity risks need to set aside funds or reserves for which is driven by two factors: the accumulated return on asset and the length of period of payment to be made to meet future obligation. Mortality rates must be considered to determine how long payment will be made until the death of the individual, on account of the time value of money. Individual living longer than anticipated, the reserve amount provisioned leads to insufficient funds for the plan provider (Iulian MIRCEA, 2011).

Looking forward, future improvement to life expectancy is unpredictable. When dealing with the area of health insurance, this particular risk should be considered carefully. Longevity risk affects long-term care (LTC) annuities and sickness benefits for the elderly. Since the 20th century, there is a significant increase in human being life expectancy resulting in a strong change in mortality, fertility, and the age structure of most populations. Understanding the dynamics of future mortality is important for pricing, reserving. Lee and Carter (1992) suggested modeling and forecasting mortality risk. Loeys et al., 2007, developed a parametric model that captures the evolution of the mortality curve (Cairns et al., 2011).

Until recently, longevity risk was never securitized and there were no longevity derivatives that plan sponsors and life companies could use to hedge their exposure to longevity risk. However, improvement was made and markets for longevity derivatives are starting to develop. Longevity derivatives are securities that provide a hedge against longevity risks and come in the form of bonds (LBs). December 203, Swiss Re issued the first bond to link payments to mortality risk: catastrophic mortality hazard. Now with a capital market, life offices and pension funds may hedge their long-term longevity exposure. Annuity bonds and longevity bonds make coupon payments that are not fixed over time, each year the coupon payments received is by a percentage of the surviving population. The payment is intended to provide a steady income for the lifetime of the annuitant which starts in the future.

1.3. Objective

The continuing increase in life expectancy have brought a critical importance of mortality forecasting due to rising developments of longevity and mortality improvement. Hence the overall objective is to find out the best method to estimate life expectancy associated with longevity and mortality, describe a set of modeling approaches, quantified the inherited risk.

1.3.1. Specific Objective

i. What are the best approaches to best model longevity risk?

ii. Develop models of mortality rates

iii. Decompose the more effective ways for life office (pension funds, annuity, insurance companies) to better assess and manage the inherited risks.

1.4. Justification of the Study

Almost every country witnesses a demographic shift towards an ageing population where the number of retired people is rapidly catching up to the workforce population putting a strain on existing retirement systems. Pension funds and annuity providers use suitable mortality tables to account and make provisions for expected future improvements. They do this by setting up clear guiding principles for the development of mortality tables used for reserving annuity and pension liabilities.

Previous studies address mortality assumptions as a key variable in estimating life expectancies, which in turn determines the cost of insurance for an insurer and the long-term obligations of a pension fund. Because of the critical importance of underlying mortality assumption, actuaries as well as government and schemes follow guideline set by pension and insurance regulators in deciding on an appropriate assumption. Mortality assumption are the modelled projections of expected death rates to estimate pension obligations. This is based on mortality tables which are statistical tables of expected annual mortality rate.

Longevity risk is a risk often overlook; people live longer than expected would mean increases average length of time over which benefits are provided and, given that the size of global pension promises is substantial, means the potential costs of longevity risk are worthy of serious examination. The findings of this studies will directly benefit actuaries, insurance company expose to longevity risk. we use data provided by the HMD. This dataset contains detailed information on various actuarial assumptions, including the mortality table, which allows us to estimate the effect of life expectancy assumptions on the value of pension liabilities.

To measure the effect of mortality rates on the stability of insurance and pension provider's financial risk, longevity risk should be considered. To investigate the future mortality and longevity risk with different age structure, Lee–Carter mortality model is used on the historical census data to forecast future mortality rates. Additionally, the result will set policy options to account for future improvement in mortality and life expectancy as well as ways to manage longevity risk.

2. LITERATURE REVIEW

Longevity risk is one of the extreme and least understood risk for insurance and reinsurance companies, pension plan sponsors and the government. The exposure to unexpected decreases of mortality from an insurance company or a defined benefit plan sponsor is longevity risk. This is the opposite of the mortality risk, which is increase mortality risk exposure. This chapter is a literature review of presentations on the topic of longevity risk and mortality risk intended to be resourceful for actuaries, students, and other professionals.

2.1. Theoretical Background: Longevity Risk and Mortality Risk

Longevity risk is becoming increasingly significant because of its consistent increase in life expectancy. Longevity risk is crucial in estimating future benefit made to register members of a defined-benefit program. Pension funds, Annuity provider and insurance companies faced longevity risk when mortality assumption to estimate life expectancy are inaccurate. A promise lifetime retirement benefits is made by the plan sponsor throughout the life of the individual until death. Increasing life expectancy became a major issue for insurance companies and pension providers. Individuals underestimate longevity risk, most retires do not expect to live past 85 in USA (half of 65-years old men will live to 85 or more, as well as women). The most significant risks faced by life officers have made these institutions specialized in understanding and managing mortality and longevity risk.

Human mortality decreased during the twentieth century. Life expectancy has risen at an astonishing rate. In most industrialized countries, mortality rates at adult and old ages show decreasing annual death probabilities (McDonald, Cairns, Gwilt, and Millers, 1998). The main factor driving continued gains in life expectancy in industrialized countries since 1970 has been a decrease in death rates among the elderly. These mortality improvements present a challenge for the planning of public retirement systems as well as the private life annuities business. When it comes to long-term living benefits, calculating expected present values (for pricing or reserving) requires an appropriate mortality projection to avoid underestimating future costs. Actuaries must therefore rely on lifetables that include a forecast of future mortality trends (the so-called projected tables). As a result, a new risk emerges the risk that the mortality projections are incorrect and that the annuitants live longer than predicted by the projected lifetables. This is referred to as the longevity risk.

Longevity risk, or the risk that the individual might live longer than average, may be reduced by the purchase of annuities at retirement age. However, whereas the purchase of annuities at retirement age provides insurance against longevity risk as of this age, a young individual saving for retirement faces substantial uncertainty as to the level of aggregate life expectancy, and consequently annuity prices, that she will face when she retires. Furthermore, markets may be incomplete in the sense that they may lack the financial assets that would allow individuals to insure against this risk studied by (Cocco & Gomes, 2009).

Since the 1960s, life expectancy for 65-year-olds in Europe and North America has increased by about one year every decade (Loeys et al. 2007). Longevity improvement has emerged as a high-profile risk for defined benefit (DB) pension plans and insurance companies with significant annuity policies. It is estimated that each additional year of life expectancy increases the value of pension liabilities in the United Kingdom by 3–4%. Lower investment returns, IFRS reporting requirements, regulatory changes (e.g., Solvency II capital requirement), and a lower fertility rate have all contributed to the growing importance of longevity risk.

Traditional longevity market participants include DB pension funds, insurers, and re-insurers. Because the value of their liabilities increases with life expectancy, pension funds have a negative exposure to longevity risk. The longevity risk exposure of life insurance companies is relatively flat, with annuity portfolios offsetting insurance policies (Loeys et al. 2007). As a result, the market has a net negative exposure to longevity improvements. Re-insurers lack the capacity and are unwilling to accept such a large risk (Wadsworth 2005). With their depth, capacity, and experience in risk hedging, capital markets have the potential to effectively hedge longevity risk (Blake et al. 2009).

It seems clear that trends in mortality will continue, at least in the near term, but in complex ways, mortality changes, and are influenced by socio-economic factors, bio-variables, public policies, environmental influences, conditions of health and health behaviors. With time and experts' views on how and how these trends are oriented, not all these factors differ greatly. Although mortality trends are ongoing, disruptions can be caused by many sources: epidemics; pandemics; war and terrorism; natural disasters. Although the likelihood of these events is low, it is not zero.

Some argue that a biological limit exists for how much time a person can live before the body wears out. Supported by this view, improvements to date are due to better health and diet, which are unlikely to be repeated constantly. This study distinguishes between age and age-related illnesses by suggesting that medical research focuses exclusively on diseases linked to age. Research shows that elimination of the three main death causes in elderly people (heart disease, cancer, and cerebrovascular disorder) would only increase life span by 17 years is one of the supporting mathematical arguments. Furthermore, future life expectancy can be reduced or even reduced by factors such as obesity and a reduction in food derived.

In 2002, the British Continuous Mortality Investigation Bureau has selected three projections of the future mortality of the UK, in order to assess future rates of mortality for assured lives more accurately: short, medium and long-cohort projections. These three projections reflect the duration of the cohort's better mortality than the insured population in its entirety. The projections also differ in the magnitude of the improvements, with the short cohort showing lower levels of improvement. This is part of a deliberate shift away from the false certainty of a single projection, and a step toward explicit recognition of the uncertainty surrounding the path of future improvements.

2.1.1. Products With Longevity Risk Exposure

Longevity risk exists in any product in which the issuer is exposed to financial losses if policyholders live longer than expected. This is common when payments from the issuer are contingent on the policyholder's survival. Traditionally, these products have been issued by insurance companies and used to hedge against an individual outliving their assets. In recent years, the number and variety of products exposed to longevity risk has grown. This can happen even if transferring longevity risk is not the primary goal of the transaction. We examine some of the products on the market that are vulnerable to longevity risk. We also take into account the other risks that these products face, such as financial risk, pricing risk, and regulatory risk. Conversely, longevity risk is generally defined as the exposure of a company to lower-than-expected mortality(Owusu et al., 2016).

i. Immediate annuities

An immediate annuity is a product that usually provides payments for life in exchange for a lump sum. The frequency and payment amount may vary over the course of the contract. They can be designed to provide a fixed level payment, a stream of payments that increase at a predetermined rate, or a stream of payments that is linked to an underlying equity index.

Immediate annuities can be purchased as either single life or joint-and-survivor policies. In the latter case, annuity payments continue as long as one of the two lives is alive, though the size of the annuity payment may decrease if the primary insured dies. Immediate annuities are also subject to pricing risk. Companies that set prices for their products that are inconsistent with best estimate assumptions face a greater risk that the actual experience will differ from what was expected. Because annuity rates are simple to understand and compare for insurers, pricing for longevity risk is competitive.

ii. Enhanced and impaired life annuities

Impaired or enhanced annuities provide higher annuity payments to people who can demonstrate that they are in poor health or are terminally ill. For the insurer, there is a greater risk of medical breakthroughs in a single condition extending an individual's life, which necessitates that enhanced products be priced at a higher margin than standard annuities. This also has implications for estimating future mortality improvements.

The risks associated with enhanced and impaired life annuities are similar to those associated with standard immediate annuities. However, given the higher expected mortality rates assumed for these policies, the longevity risk may be exacerbated, as there is likely to be less data on the mortality experience of subgroups of the population.

iii. Deferred annuities Traditional

Deferred annuities are primarily used to accumulate tax-deferred savings, which can then be distributed as an immediate annuity or as a lump sum payment. Fixed, variable, and equity-indexed annuities are the three types of deferred annuities available in the United States. As a result, they are less vulnerable to the risks associated with aging. The addition of guarantees to product offerings has introduced longevity risk as the market has developed and become more competitive.

When a deferred annuity is annuitized at maturity, it is subject to a number of risks that are not present when it is distributed in lump sum. These products have been in place for a long time, and it is difficult to protect the cash flows due to a scarcity of assets with the appropriate duration. As a result, deferred annuities are subject to reinvestment risk.

iv. Advanced Life Delayed Annuities

ALDAs (advanced-life delayed annuities) are a type of longevity insurance. ALDAs are inflation-linked annuities sold to people in their early twenties that begin paying out at the age of 80, 85, or 90. There is no cash value, and no mortality insurance benefits that can be repaid at any time. ALDAs are designed to mimic a defined benefit pension benefit at advanced ages for people who do not have access to this type of protection. Traditional deferred annuities may be better suited to protecting against catastrophic longevity (Owusu et al., 2016).

v. Corporate pensions

There are two types of corporate pension plans: defined benefit (DB) and defined contribution (DC). The employee receives a fixed income stream based on his or her salary, years of service, retirement age, and other factors under a DB plan. Typically, the benefit stream is set. Contributions are made into individual accounts by each employee under a DC plan, and the employer may make a matching contribution. When you retire, you can take a lump sum equal to the value of your current account. The lump sum can be used to supplement retirement income.

vi. Structured settlements

Structured settlements are payments made as the result of a general insurance liability involving human life (e.g., serious injury, medical negligence, or occupational injury). Payments are sometimes made in the form of a lump sum for the injured party's lost earnings and/or the cost of care if they are seriously injured. Annuities payable for life, on the other hand, have recently been used as a type of settlement.

vii. Life settlements

Purchasers of life settlements face longevity risk because lower mortality means they must pay insurance premiums for a longer period of time and receive the death benefit later than expected. Most buyers of this type of contract are not in the business of profiting from mortality. Life Settlements are a way for an investment bank or hedge fund to diversify risk while potentially achieving a high rate of return, as has historically been the case with these portfolios.

2.2. Decomposition of Mortality Risk

Mortality risk is generally defined as a company's exposure to greater-than-expected mortality. The International Actuarial Association divides mortality and longevity risk into four categories: level, trend, volatility, and catastrophe. Risk can be classified into two types: systematic risk and specific risk. The term "systematic risk" refers to incorrect base assumptions (level and trend), whereas "specific risk" refers to volatility that surrounds the base assumptions (volatility and catastrophe). Specific risk is decreasing, but the systematic risk cannot be diversified as the number of lives covered increases. There are considerable and increasing costs of systematic risk for pension plans and insurers.

Mortality risk is a vital risk factor for insurance companies and mortality risk is broken up into subcategories, systemic risk, unsystematic risk, and adverse remedies.

The risk of mortality refers to the risk of a person living for a shorter life than expected and is, therefore, higher than expected. The interest of life insurers and pensioners in longevity risk to the design of a defined benefit plan has increased (Gatzert & Wesker, 2014).

i. Unsystematic Mortality risk:

The risk of individual deaths is a random variable with a certain probability (see Biffiss, Denuit, and Devolder, 2010). Thus, it may be diversified through natural hedges, or transfers through mortality to the capital market, Contingent bonds (MCBs).

ii. Systematic Mortality risk:

The risk of systematic mortality is the risk of sudden changes to underlying population mortality, for example as a result of common factors affecting deaths of the entire population that trigger life dependencies and cannot be diversified by broadening the portfolio (see Wills and Sherris, 2010).

iii. Adverse Selection:

This referred to the fact that, for various populations of assured persons, for example, life insurers and pensioners, the probability distribution differs in age level and trend (see Brouhns, Denuit and Vermunt, 2002a). In addition, adverse selection is a major source of risk when hedging longevity risk via MCB or other capital markets instruments, because of individual mortality heterogeneity and information asymmetries between the insurance company and Insured (see, e.g., Sweeting, 2007).

iv. Basic Risk:

This occurs when hedge population mortality does not coincide with the portfolio hedge mortality. This means that there is a base risk in longevity hedges in the differences in population mortality and mortality of the insured pensioners caused by adverse selection. In this analysis, we explicitly consider the fundamental risk in hedges and models every kind of mortality risk in order to analyze its impact on the risk situation of the life insurer.

2.3. Management And Quantification of Longevity Risk

To ensure that insurers' exposure to longevity risk is effectively managed, actuaries must first be aware of the current methods for quantifying and managing this risk. Only then can they take an active part in identifying and building additional risk management techniques that are more effective in addressing longevity risks.

Companies are required to maintain a certain percentage of their net risk or reserves to cover the risk that death is different from expected. As a result, most companies continue to quantify the risk of longevity with relatively fundamental methodologies. Because the risk of long life for insurers is increasing, major annuity authors and reinsurers look for ways to manage their costs effectively. To date, product design, contracting, natural hedging, and reinsurance are the conventional methods that direct authors use. Furthermore, companies have started to use their longevity risk exposure solutions to the financial markets.

• Buy-ins, A pension scheme's liabilities such as pensioners' in-payment, are covered by buy-in. The policy pays an income equivalent to the members' benefits, removing the danger of insufficient assets to fund future commitments.

• Bulk Annuity and reinsurance transactions to transfer rents between insurers and reinsurers. • These solutions are also insurance.

• Longevity bonds which transfer a long-term risk to another party in the form of a security from a pension plan or annuity portfolio. These are solutions to the capital markets.

• Longevity swaps to transfer longevity only to another party from a pension scheme or annuity portfolio. These can either be insurance solutions or solutions for the capital markets.

• Mortality catastrophe and swapping, transferring from life insurer or reinsurer to other parties, the risk of devastating (catastrophe) increases in mortality due, e.g., to a pandemic or natural disaster. These are solutions to the capital markets.

• Life securitizations that transfer risks related to a specific block of insurance undertakings, as a security, to capital markets. These are solutions to the capital markets.

• US life settlements transactions transferring to investors small portfolios of U.S. life insurance policies. These are solutions to the capital markets.

• Pensions buy-outs that transfer pension obligations and all associated risks and obligations to insurers (also known as pension plan terminals). These are the solutions for insurance.

The hedging instrument is the third feature of risk transactions with a pure longevity. The longevity swap for survivors has previously been the most common structure.

Mortality forward (q-forward)

A forward mortality contract is often known as a forward, as the letter 'q' stands for actuarial mortality rate symbols. It is the simplest type of longevity (and mortality) risk transfer instrument (Coughlan et al. 2007b) and was the first type of capital markets that were used for longevity hedges. This was an agreement between UK Lucida and J.P. Morgan pension insurers and is described in the next section. The importance of q-forwards is that they form fundamental blocks from which other life-related derivatives can be built.

A q-forwards portfolio can be used, if appropriately designed, to replicate and safeguard a lifetime exposure or to protect a life insurance book or a pension liability. A q-forwards shall be defined as an agreement between two parties in which a sum proportional to the actual mortality rates performed for a given population (or subpopulation) is exchanged in exchange for the sum proportional to a fixed death rate agreed upon at the outset to be payable in the future (the maturity of the contract). If there is a fair price of the q-forward, there is no change in payment hands at the start of the trade, but at maturity one of the two counterparties makes a net payment (unless the fixed and actual mortality rates happen to be the same). The maturity payment is based on the net amount payable and is proportional to the difference between the fixed mortality rate (the forward rate transacted) and the reference rate realized. If in the reference year the rate is lower than the fixed rate (that is, a lower death rate), the settlement is positive, and the settlement payment is received by the pension plan to make up for the increase in its liability value. Where, on the other hand, the reference rate is higher than the fixed rate (ie. higher mortality), the repayment is negative, and the pension plan pays the hedge provider the settlement payment, which is offset by the decline in the value of the payment. The net liability value is therefore locked with regards to the mortality rates. The scheme is protected against unexpected mortality rate changes.

Survivor forward (S-forward)

A survivor forward, also known as a “S-forward,” is similar to a q-forward in concept but uses survival rates rather than mortality rates. It is an agreement between two parties to exchange an amount proportional to the actual, realized survival rate of a given population (or subpopulation) in exchange for an amount proportional to a fixed survival rate that has been mutually agreed upon at the contract's inception to be payable at the contract's maturity. As such, it entails exchanging a notional amount multiplied by a pre-agreed-upon fixed survival rate for the same notional amount multiplied by the realized survival rate for a specified cohort over a specified time period (Coughlan et al., 2008b; Dawson et al., 2010). If the contract has a one-year maturity, a survivor forward is the inverse of a mortality forward. However, if the contract maturity exceeds a year, this simple relationship no longer exists because survival rates over longer time periods are non-linear functions of annual mortality rates. Because it is a function of several mortality rates at different ages and times, a survivor forward is more complex than a q-forward. In some situations, it can nevertheless be a useful building block.

Longevity swaps

A longevity swap can be classified as either a capital markets derivative or an insurance contract. In either case, it is a financial instrument that involves exchanging actual pension payments for a series of pre-agreed-upon fixed payments (Dowd et al., 2006; Bravo & Nunes, 2021). Each payment is based on an amount weighted survival rate. In any longevity swap, the hedger of longevity risk (for example, a pension plan) receives the actual payments it must make to pensioners from the longevity swap provider and, in exchange, makes a series of fixed payments to the hedge provider. As a result, if retirees live longer than expected, the higher pension amounts that the pension plan must pay are offset by the higher payments received from the longevity swap provider. As a result, the swap offers the pension plan a long maturity, customized cash flow hedge of its longevity risk. The July 2008 Canada Life-J.P. Morgan transaction (Trading Risk 2008; Life & Pensions 2008).

Variants on longevity swaps

The transaction carried out by Aegon and Deutsche Bank in January 2012 is one variant of the standard longevity swap. This was an "out-of-the-money" longevity swap because it only transferred the longevity risk associated with a significant increase in life expectancy (or equivalently, a very large and sustained fall in mortality rates). Aegon, the hedger, receives no incremental payment for modest increases in life expectancy until a certain threshold, or "attachment point," is crossed. Aegon will then be paid for which the life span increases until a certain maximum level of protection is attained when life expectancy rises to a very extreme level. This swap is indeed a standard long-life swap, except that it has floating caps and floors. The swap in capital markets was based on indexes over 20 years and the index matched the national population data of the Netherlands. This swap also included, like the Aviva-RBS transaction, a swap payment at maturity to protect the longevity of any responsibility cash flow that exceeds the maturity date.

Longevity bonds

Since the start of this market, longevity bonds have been widely spoken to prevent the risks of longevity. A longevity bond (or a survivor bond as it was originally called) is a bond that pays coupons that proportionally correspond to the number of survivors still living on the coupon payment date in the population cohort specified. (Wolff, 2001; Blake et al., 2006a, 2006a; Dowd, 2003). The cash flows of a single longevity vanilla bond are the same as those of a longevity swap floated bearing. However, longevity bonds with different structures have recently been proposed. The cash flows of the bond are indexed to the mortality experienced in the United Kingdom by 65-year-old men. There is a 10-year deferment period before the start of payment and a terminal switching payment at 105 years is made to cover the risk of a long life after 105 years. If more people survive at each age, then the bond pays more; if fewer people survive, then the bond pays less (similar to the floating leg of the RBS-Aviva longevity swap).

3. METHODOLOGY

Life expectancy is the most common statistical indicator of the average remaining lifespan an individual is expected to live (Ayuso et al., 2021).

3.1. Basic Mortality Functions

Let (x) denote a life that survives to the age x. The life (x) is called a life-age-x.

Let D_{xt} be a random variable, in a population who die at aged (x) last birthday during a calendar year t.

d_{xt} denote the observed number of persons who die between ages (x) and $(x + t)$

l_x denote number of persons who attain age x according to the mortality table.

q_x denote the probability that (x) will die within 1 year

p_x denote the probability that (x) will live 1 year.

d_x denote

3.1.1. Initial Mortality Rate

q_x is called the mortality rate at age x, in actuarial terminology q_x is the probability that (x) dies before age $(x + 1)$. We can also subscribe a (t) to get ${}_tq_x$ which is the probability that (x) dies before age $x + t$,

$$q_x = \frac{d_x}{l_x} \tag{1}$$

3.1.2. Probability of Survival

The survival function of T_x is denoted by ${}_tp_x$. It is the probability that a life aged x survives t more years or is the probability that an age (x) survives to at least age $(x + t)$. In simplicity removing (t), we get.

$$p_x = \frac{l_{x+1}}{l_x} \tag{2}$$

3.1.3. Central Death Rate

The number of people who died during the year divided by the total number of people who were alive during the year. The Central death Rate (m_x) denotes as the central death rate for the year of age (x) to $(x+1)$.

$$m_x = \frac{d_x}{l_x} \quad (3)$$

In the actuarial modeling literature, we use the following standard definitions (Dickson et al. (2013; 2009); Pitacco et al. (1998)). Let T_x denote the remaining life expectancy of an individual of age x. The cumulative function of distribution and survival of T_x is written as $\tau\, q_x$ = $P(T_x \leq \tau)$ and τp_x = $P(T_x > \tau)$ respectively. For an individual aged x, the force of mortality at age $x + \tau$ is defined as

$$\mu_{x+\tau} := \lim_{h \to 0} \frac{1}{h} P(T_x < \tau + h | T_x > \tau) = -\frac{d}{d_\tau} \ln \tau\rho_x$$

Let $f_x(t)$ be the density function of T_x, then from (1) we have.

$$\tau q_x = \int_0^\tau f_x(s)\, d_s = \int_0^\tau s\rho_x\, \mu_{x+s}\, d_s$$

The central death rate for x-year-old, where $x \epsilon \mathbb{N}$, is defined as

$$m_x := \frac{q_x}{\int_0^1 s^p x\, d_s} = \frac{\int_0^1 sP_x \mu_{x+s}\, d_s}{\int_0^1 sP_x\, ds}$$

which is a weighted average of mortality force ($q_x := {}_1q_x$). Taking account, the so-called constant force of mortality assumption, μ_{x+s} = μ_x where $0 \leq s < 1$ and $x \in \mathbb{N}$,

from (2), we have $m_x = \mu_x$.

If a Poisson assumption is denoting of the actual number of deaths, then the maximum likelihood estimates of the force of mortality $\hat{\mu}_x$ is given by $\hat{\mu}_x = D_x/E_x = \widehat{m}_x$ where D_x denotes the recorder number of deaths at age x last birthday and exposure to risk E_x is the average number of individuals in the observation year who were x years old on their last birthday. Notice that E_x is based on a population estimate of people who were x years old on their last birthday in the middle of the observation year.

E^{c}_{xt} represent the central exposed to risk at age x in year t, and E^{o}_{x} denotes the initial exposed to risk for all arrays of x-age and t-year comprising ages (on the rows) $x = x_1, x_2, x_3 \ldots,\ x_k$, and calendar years (on the columns) $t = t_1, t_2, t_3 \ldots, t_n$,

3.1.4. The force of mortality $(\mu_{x,t})$

$\mu_{x,t}$ represents the hazard rate for mortality for an individual at exactly age x and dies at the exact t years.

The force of mortality related to the death probability as

$$\mu_{x,t} = \lim_{d_{x \to 0^+}} \frac{P_r[T_0 \leq x + dx | T_0 > x]}{d_x}$$

$$\mu_x\, d_x \approx \lim_{d_{x \to 0^+}} P_r[T_0 \leq x + dx | T_0 > x]$$

$$\mu_x = \frac{-\frac{d}{d_x} S_0(x)}{S_0(x)} \tag{4}$$

$$Gompertz: \mu_x = B_C{}^x, 0 < B < 1, c > 0$$

3.1.5. Life expectancy $(e_{x,t})$

$e_{x,t}$ means that an individual of the given age x can expect to live with time t an additional number of years on average. Life expectancy, which is equivalent to the total life span, is most common at birth.

$$e_x = \frac{T_x}{L_x} \tag{5}$$

But life expectancy for a given age in which the age plus life expectancy is equal to the total life expectancy.

Consider the mortality model that represents the model which examines the structure of probability of death or central mortality rates across ages and or years.

Such Models are “Static laws of mortality such as Gompers law ($\mu_x = exp(A + Bx)$) which have no time dependence.

The Makeham Law $\left(\mu_x = A + B \exp(Cx)\right)$ which do not have any time dependence.

These models form modern dynamic mortality models by allowing the parameters to vary with time. A dynamic mortality model can be deterministic or stochastic, depending on whether future mortality is projected with certainty or according to an underlying distribution.

Advantages and Disadvantages of each model

Majority of the models examined here do not account for cohort effects, which cause mortality patterns to change from generation to generation. In many different populations, cohort effects were observed.

Fitting cohort effects can be difficult because more recent cohorts have only been observed for a limited number of years, making parameter estimates unstable. The lack of additional trends for the period can result in improbable longevity projections, extrapolated towards the future, and include adapted cohort effect in these birth years. The cohort effect intuitively refers to certain elements of a cohort that differ from the average mortality.

Another challenge with these models is predicting cohort effects. If we have correctly separated any period and cohort trends, then cohort features should revert around zero. However, be extremely consistent, should not vary significantly from year to year thus fitting a time series structure becomes extremely difficult because of this.

3.2. Modelling Structure and Specification

New models recently emerged seek to model mortality in various populations by modifying single-population models. Li and Lee (2005), Cairns et al. (2011b), Li and Hardy (2011), Jarner and Kryger (2011), and Dowd et al (2011a). However, a deeper knowledge of multi-population dynamics is central to the development of a vibrant longevity market.

As the field of modeling mortality has grown in recent years, the models used to analyze and project mortality rates have grown considerably more sophisticated. However, the number and importance of identifiability issues within mortality models have also grown in parallel with this increased sophistication. This has led both to robustness problems and to difficulties in making projections of future mortality rates.

Data Reliability

Generally, model fitting of the exposure data makes assumptions to be accurate. However, many national datasets and smaller subpopulations acknowledges that the exposures are estimates but results in a poor estimated true value, Cairns et al. (2009) reveal this issue in the discussion of the US mortality data. Recently, studies carried out by the National Statistics in the UK (ONS, 2012) estimated the exposures for higher ages from 2001 to 2011 in the UK (England and Wales). Cairns et al. (2009) is their analysis for models fitting noted how standardized errors were bigger in magnitude than they ought to be under the conditional Poisson model, and the explanation given is the fact that exposures are approximations.

Applications of models

These models have several applications and to some point, the basic outputs for the models need to be communicated to end-users in a clear manner. Renshaw and Haberman (2006), Cairns et al. (2009, 2001), and Dowd et al. (2010c) have proposed numerous graphical methods. Studies have shown that the pricing of longevity-linked financial contracts needs to be considered. Olivieri and Pitacco (2009) review solvency II, annuity pricing by Richards and Currie (2009). Dahl et al. (2008), Coughlan et al. (2011), Cairns et al. (2013a), Cairns (2013), Dowd et al. (2011c), Li and Luo (2013), and others have recently focused on the use of models to develop and evaluate hedging strategies (2012).

Robustness

The importance of robustness in the models, forecasts, and decision-making are necessary when measuring and managing longevity risk. End users will not have enough trust in what is being recommended if any aspects are lacking in robustness, and a severely poor decision will be made. Robustness can be measured in a variety of ways.

i. Model fit

For model fitting, with the combination of age period and cohort effect, are the estimate relative to changes: if the range of ages are used to calibrate the model, adding one year for a range of years, and method of calibration. In addition, are the APC effects robust relative to uncertainties in the estimated exposures.

ii. Model forecasts

Changes in the range of ages used to calibrate the model; the range of years (especially adding one new year's data); the method of calibration; and the choice of a stochastic model for simulating future period and cohort effects have what effect on stochastic forecasts. And, in terms of the more general consideration of model and parameter risk and uncertainty in exposure data, how reliable are forecasts?

iii. Business decisions

Finally, how reliable are financial variables such as the market consistent value of liabilities and the prices of, say, q-forwards; risk management metrics (such as hedge effectiveness); risk management decisions (such as the choice of the hedging instrument and the number of units of that instrument) robust to the forecasting future mortality rates?

3.3. Risk Model Classification

To model future randomness in mortality rates, a wide range of projecting approaches are used to model mortality both in discrete times and continuous time.

Criteria for selecting models: It is critical to examine whether a model is a good model once it has been developed and parameters have been estimated or calibrated, proposed by Cairns, Blake, and Dowd (2006a) and Cairns et al (2007, 2008). It should be positive mortality rates.

Historical data consistency:

Historical mortality patterns should consist of a good model. In the event, this is not the case the validity of any forecast produced by the model must be put into considerably greater doubt. Brouhns, Denuit and Vermund (2002) and Czado, Delvarde and Denuit (2005), employed more formal statistical methods such as likelihood and Markov Chain Monte Carlo (MCMC) methods for forecast.

Cairns et al. (2007) compared several models based on maximum likelihoods in detailed using criteria that penalize over-parameterized models. They showed that by integrating additional period and cohort effects in the Lee and Carter (1992) and Cairns, Blake, and Dowd (2006b) models, they could achieve statistically significant gains.

Reasonability Biological:

what constitutes the concept of biologically reasonable model draw from interest rate modelling.

Period mortality tables have traditionally shown increasing rates of mortality with age at higher ages, which is what forms a biologically acceptable model. A forecasting technique that leads to period mortality tables with mortality rates decreasing with age could be regarded biologically inappropriate.

Medical advancements tend to improve mortality in the long run especially with creation of cures for numerous illnesses like cancer, heart problem. when such breakthroughs will occur or what impact a new medicine, results to mean reversion for long run thus regarded as a biologically unreasonable. Meanwhile a mean reversion on short term occur from environmental volatility.

Uncertainty in the parameters:

Estimating the parameter of the model results in estimation error due to the limited data for estimate. Cairns et al. (2006b) and, Dowd et al. (2007) in their studies show that a two-factor model introduce by Cairns-Blake Dowd, that incorporating the uncertainty parameter has a significant impact on forecast level and future expected lifetimes over long horizon time.

Cohort effect:

Mortality rates appear to be determined not only by age and period effects, but also by year of birth effects, demonstrating a significantly better fit with the inclusion of a cohort effect (Cairns et al (2007)).

3.4. Identification and Structure of the APC Stochastic Mortality Model

The age-period-cohort (APC) model is construct of vast majority of stochastic mortality models. Priore to generalized linear models (McCullagh and Nelder (1989)), four components make up the GAPC stochastic mortality model.

a) Random Component:$D_{x,t}$ denoting the number of deaths, will be followed by Poisson or Binomial distribution:

$$D_{xt} \sim \text{Poisson}(E^c_{xt}, \mu_{xt})$$

$$D_{xt} \sim \text{Binomial}(E^o_{xt}, q_{xt})$$

$$E\left(\frac{D_{xt}}{E^c_{xt}}\right) = \mu_{xt} \text{ and } E\left(\frac{D_{xt}}{E^o_{xt}}\right) = q_{xt}$$

- ➢ The systematic component: following Hunt and Blake (2015) the effects of age x, calendar year t and year-of-birth (cohort) $c = t - x$ are captured through a predictor η_{xt} given by
- ➢ Systematic Component: let x = age and t = calendar year t and year-of-birth (Cohort) = $(c = t - x)$ to be defined by a predictor given by η_{xt}

$$\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{t-x} \qquad (6)$$

Where:

- α_x is the static age function (shape of mortality by age)
- $N \geq 0$is an integer that describes the mortality trends for the number of years, time index $\kappa_t^{(i)}, i = 1, \ldots, N$ and $\beta_x^{(i)}$is the effect modulated over ages.
- γ_{t-x} represent the cohort effect.

3.5. Stochastics Mortality Models

3.5.1. Longevity Risk Model and Measurement

I. The Lee-Carter models (LC)

The Lee-Carter model is widely used for mortality modeling introduced by Lee and Carter (1992). The LC model is a stochastic model and has a single age/period term and assume that the force of mortality at age x and time t. Lee and Carter propose the Lee–Carter model is easy for quantitative calculation and is widely used today. With good fitting and prediction results, it has become the standard in the United States Census Bureau and the United Nations Population Division. The Lee–Carter stochastic mortality model has the following basic form:

$$\ln\left(m_{x,t}\right) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t}, \qquad (7)$$

$$\varepsilon_{x,t} \sim N(0, \sigma^2)$$

Where $m_{x,t}$ is the matrix of the central death rates at age x $(x = x_1, \ldots, x_N)$ in year t $(t = t_1, t_1 + 1, \ldots t_1 + T - 1)$.

$$m_{x,t} = \frac{D(t,x)}{E(t,x)} \qquad (8)$$

Where $D(t,x)$ is the number of deaths of the x-year population during the entire calendar year t. $E(t,x)$ is the average number of people aged x, that is, the number of exposures during the calendar year t. The age-specific mortality rate, which is frequently used in actuarial models, has the following approximate relationship with the central mortality rate under the assumption that the deadly force remains constant:

$$q_x(t) = 1 - exp[-m_x(t)] \qquad (9)$$

Or, assuming that death is evenly distributed

$$q_x(t) = \frac{m_x(t)}{\left(1 + 0.5 m_x(t)\right)} \qquad (10)$$

Lee–Carter Model decomposes the population mortality into three parts: the fixed rate of mortality in the population α_x, the refining trend of mortality over time $\beta_x k_t$, and the random fluctuation term. x, which is independent of time, represents the general level of logarithmic mortality at age x. To reflect the difference in mortality at different ages, it can use the average of historical data in the time dimension or the value of the most recent observation year. The model breaks down the mortality trend over time into an interactive product of age and time. k_t indicates the relative intensity of overall mortality in each year and gradually decreases with time, reflecting the continuous improvement of mortality over time. β_x shows the sensitivity of the logarithmic mortality rate of the x-year population towards the change in the overall trend, reflecting the inconsistent rate of decline at different ages. The last one is a random error term that reflects the random fluctuation of mortality rate outside the trend.

To ensure the uniqueness of the results, the Lee–Carter model contains two restrictions:

$$\sum_x \beta_x = 1 \tag{11}$$

$$\sum_t k_t = 0$$

Singular value decomposition (SVD), ordinary least squares (OLS), weighted least squares (WLS), and the Poisson log bilinear model are the most common parameter estimate methods for the Lee–Carter model now (Poisson log-bilinear). The estimation of α_x is not controversial. Generally, it is

$$\alpha_x = \frac{1}{n}\sum_{t=T_1}^{T_n} \ln m_x(t) \tag{12}$$

Singular value decomposition (SVD) efficiently extracts the primary information of trend effect based on the matrix's features. The main estimation method is to singly decompose the matrix $\ln m_x(t) - \alpha_x$ which can be obtained as follows:

$$SDV[\ln m_x(t) - \alpha_x] = \sum_{i=1}^{r} \rho_i U_{x,i} V_{i,t} \tag{13}$$

Where r is the rank of the matrix $\ln m_x(t) - \alpha_x$ and $\rho_{1,\ldots,}\rho_{2,\ldots,}\rho_r$ are the singular values of the matrix from large to small. $U_{x,i}$ and $V_{i,t}$ are two singular vectors. Since ρ_1 is much larger than the subsequent eigenvalues, most of the information of the matrix $\ln m_x(t) - \alpha_x$ can be extracted only by taking the first term $U_{x,i}$ and $V_{i,t}$ of two singular vectors, and the matrix is as follows:

$$\ln[m_x(t) - \alpha_\times] \approx \rho_1 U_{x,i} V_{i,t} \quad (14)$$

Thus, estimates of β_x and k_t can be obtained:

$$\beta_x = \frac{U_{x,i}}{\sum_x U_{x,i}}, \quad (15)$$

$$k_t = \rho_1 V_{1,t} \sum_x U_{x,1} \quad (16)$$

The fitting effect of the singular value decomposition depends on the efficiency of extracting from the $\ln m_x(t) - \alpha_x$ matrix. It is generally considered that the method can explain more than 90% of the sum of squares of deviations. After obtaining the estimated values of the parameters, it can be found that α_x *and* β_x are fixed over time, and the variety of mortality over time is mainly reflected by (k_t). The prediction value of future mortality can be obtained by extrapolating (k_t). It is believed that (k_t) is a random walk with drift or ARIMA process. According to the BIC information criterion, (k_t). Should be the AR IMA (0,1,1) Model with drift term.

Advantages of the Lee-Carter model

It provides a good fit for the historic data. The α_x aging function makes it possible for a model to be employed at all ages, even young ages, when the life table shape can be very complex, while the k t term represents the prevalent tendency in mortality evolution.

It is simple to fit with relatively few parameters, particularly compared to other complicated models and both the original decomposition of the single value and Brouhns et al (2002) are well understood and easy to implement Poisson Likelihood fitting model.

The project is easy. Because of the common linear trend of most datasets in κ_t's, the random walk-in drift time series is used extensively for estimating the future central mortality rates.

It is a simple concept to grasp. Both α_x and κ_t are easily understood as the shape of mortality across ages and the level of mortality each year, which is useful when reporting results to a larger audience.

Disadvantages of Lee-Carter Model

It has only one-period term κ_t, which indicates that the change in all the central mortality rates in each year of the projection is perfectly tied to the unrealistic problem and to the risk of liabilities and securities, based on the central mortality rate.

β_x does not have universal interpretation and can make unpredictable projections? The shape of a β_x becomes important when the central mortality rate is projected because a model fitted into a long range of historical data will continue to show high rates of improvement at the younger age and, at higher age rates, which might be unlikely.

There is no provision for "cohort" impacts based on a person's birth year. Renshaw and Haberman were among the first to propose models based on the Lee-Carter model but integrating cohort effects (2006).

3.5.2. Other Models

II. The Cairns-Blake-Dowd model

To address perceived problems with the Lee-Carter model and overcome problems with projected death rates in single age/period term models, Cairns et al. introduced one of the most popular competing models of the LC model, the Cairns-Blake-Dowd model (2006). The Cairns-Blake-Dowd model presumes that death probabilities can be modeled as

$$logit\left(q_{x,t}\right) = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} \quad (17)$$

The logit of death probabilities is a linear age function, which is reasonable for high age (about 50 years old) but is not true for the younger age. It is assumed. The $\kappa_t^{(1)}$parameter determines death levels over all years for a certain year in the Cairns-Blake-Dowd model. The $\kappa_t^{(2)}$parameter determines the 'aging rate' of each year, i.e., an increase in mortality between one age and the following age.

Cairns et al. presented a predictor structure with two age-period terms $(N = 2)$, age-modulating parameters $\beta_x^{(1)}$ = 1 and $\beta_x^{(2)}$= x – $\bar{x}$,, no static age function, and no cohort effect (2006). The CBD model predictor is provided by:

$$\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} \quad (18)$$

Where: $\bar{x}$ represent the average age.

Advantage of the Cairns-Blake-Dowd model

The Cairns-Blake-Dowd model is a commonly used mortality model, particularly among practitioners concerned with the riskiness of liabilities tied to high-risk death probability, such as annuities.

In comparison to the Lee-Carter model, it provides for a more sophisticated correlation structure between distinct death probability. This is especially significant when assessing the possible riskiness of liabilities, such as for insurance solvency considerations.

It is simple to put together. Because there are no age functions in the model, it can be fitted using least squares or likelihood maximization approaches to produce a satisfactory fit to the historical data when utilized over long periods of time.

It provides smooth estimates for death probabilities for every given year. This is preferable if it is believed that the basic processes determining mortality should not change as people age.

It is simple to project. For projecting the κ_t parameters across a number of countries, the bivariate random walk with drift has proven to be a reliable and robust model.

In addition to aggregate measures of longevity such as period life expectancy, it provides stochastic forecasts with confidence ranges for individual $q_{x,t}$'s that are deemed to be realistic in contrast to previous evidence.

Disadvantages of Cains. Blake-Dowd model

Models based on the Cairns-Blake-Dowd model that include cohort effects have recently been presented, most notably in Cairns et al (2009) and Platts et al (2009).

It does not fit data well across the board. The assumption of linearity in $logit\left(q_{x,t}\right)$ is no longer reasonable below the age of 50, and it may not be reasonable even at highly advanced ages (above 90). There have been attempts to accommodate this by introducing an age function α_x, similar to that found in the Lee-Carter model, for example in Platt (2009).

III. The P-splines model

Currie et al. (2004) proposed the P-splines model as a mechanism for reliably smoothing and predicting central mortality rates. It is founded on Eilers' and Marx's use of penalized B-splines (1996). A "spline" is a piecewise polynomial function defined across a range of values.

A family of splines known as a basis of splines (also known as B-splines) is large enough to cover the complete range of an interest. The linear sum of the B-splines can then be used to smooth any discontinuous function over this range. The number of splines employed and where the knots are placed have a significant impact on the smoothing accomplished by this method.

This P-spline was used by Currie et al (2004) on two-dimensional mortality data to smooth the crude estimates of central death rates over ages and years. They also predicted central mortality rates into the future by using missing values in the model for future years. The P-splines model implies that the force of mortality may be represented as a linear combination of smooth functions over time and space, i.e.

$$\log m(x,t) = \sum_{ij} \theta_{ij}\beta_{ij}(t,x) \quad (19)$$

Where: $\beta_{ij}(t,x)$ is the predetermined foundation function with regularly spread knots, and the β_{ij} is the age and cohort parameters to be calculated. It is commonly recognized that the use of splines can result in over-fitted functions, resulting in unnecessarily lumpy fitted mortality surfaces.

Advantages of P-splines model

The P-splines method has become widely used for smoothing historical data, most notably by the Continuous Mortality Institute for producing deterministic mortality projections – for example, in CMI (2002) and CMI (2004). (2009b).

It gives values that are smooth across age and time for central mortality rates and is thus excellent in removing the effect of random noise from the crude data.

It's relatively unpleasant. The smoothing procedure reduces the total number of model parameters and reduces the effective number of free parameters further with the penalty function.

It provides projections to allow for changes in the central mortality rates of various ages on the basis of the observations.

Disadvantages of P-splines model

Its explanation and implementation are complex. There is no intuitive meaning to parameters, and the fitting procedure used by Currie et al (2004) and Currieet al (2006) involves manipulating very large matrices that reduce the fitting speed and can cause computer memory allocation problems.

The surfaces are fitted that can be considered too smooth. The P-spline method itself tries to reduce the impact of shocks on the data to alleviate potentially valid characteristics such as a one-off increase in the central death rate due to an epidemic.

There are no stochastic projections available. Instead of allowing future rates to be generated by a stochastic process, the P-splines model fits a deterministic surface to the data and extends it into the future. Currie (2006) attempts to provide “confidence intervals" for future projections, but these are dependent on errors in estimating the underlying parameters rather than being truly stochastic.

It does not consider "cohort" impacts. " If desired, the P-splines model can be changed from an age/period to an age/cohort model, as described by CMI (2006), although this removes the period effects, which are frequently dominating and cause problems because some cohorts have limited observations.

IV. The CMI Model

The Continuous Mortality Investigation (CMI) developed the CMI mortality projection model (2009). It is a model for mortality improvement rates rather than mortality rates themselves, as the previous models for mortality were. The mortality improvement rates are defined as

$$r_{xt} = 1 - \frac{q_{xt}}{q_{x,t-1}} \tag{20}$$

To derive the pattern of mortality improvements, the structure of mortality rates in a population is analyzed over age, time, and years. The age/period and cohort components discovered are then assumed to persist for several years before blending into a user-specified "long-term rate of improvement."

Advantage of CMI Model

Based on a single and relatively simple input from the user, it can quickly generate a central projection of mortality rates. This is extremely beneficial for actuarial consultants who work primarily in deterministic environments (for instance, valuation of pension schemes or reserving for life assurance). In this context, it can also serve as a "common currency" for translating the pattern of improvements in mortality rates or life expectancy observed in another model (for example, the Lee-Carter model) into a roughly equivalent long-term rate of improvement.

Disadvantage of CMI Model

The CMI model's inability to generate stochastic projections of mortality rates means that it is unsuitable for measuring the risk inherent in any projection, except when comparing competing scenarios. It is also a very complex model when compared to the other models used, though this complexity is largely hidden from the intended end user and is only visible here because the methodology must be applied to different datasets.

3.6. Regulatory Framework of Mortality Assumption

Mortality assumptions used in the valuation of pension and annuity liabilities are typically presented in the form of a table, with the probability of death over the next years, q_x, given for each individual age x. Usually, different assumptions are used for males and females, however certain districts' regulations necessitate the use of unisex rates.

Tables of mortality can be one-dimensional, accounting just for differences in death by age, or two-dimensional, accounting for mortality evolution through time. One-dimensional tables, often known as static tables, have only one death rate for each age group.

As many years of sufficient mortality experience are required, establishing assumptions for predicted mortality improvement needs substantially more data and is thus more difficult to set. As a result, mortality improvement assumptions are frequently based on general population mortality.

After the mortality assumptions have been determined, they can be applied to the initial mortality level to establish a generational table giving the mortality assumption at any future point in time. They are commonly used in the following ways, where 2000 is the year in which the initial level of mortality was determine and r is the annualized rate of mortality improvement for age x:

$$q_{x,2000+t} = q_{x,2000}((1 - r_x)^t \qquad (21)$$

In practice, r may vary over time, but it usually just varies by age and gender.

3.6.1. Mortality Assumptions in Practice and Regulation

The regulatory framework may demand the use of specialized mortality tables. These tables indicate minimal mortality assumptions and may or may not account for future improvements in mortality and life expectancy. However, when minimum tables are necessary, pension funds and annuity providers are often allowed to employ mortality tables that are more conservative than those required in order to account for and prepare for significant future improvements in mortality and life expectancy if deemed suitable. Where the legislative framework does not create specific mortality tables, pension funds and annuity providers may use their own tables, or the tables most commonly used by the industry.

The extent to which mortality assumptions are regulated varies greatly between countries and is not always uniform between pension funds and annuity providers within the same country. Table 1 illustrates whether the regulation mandates minimum mortality assumptions or whether the regulation requires that future improvements in mortality be accounted for in the assessment of pension and annuity liabilities, while the specific assumptions to be used are not required. The analysis evaluates whether it is standard market practice to account for future mortality improvement in the pricing of liabilities, even if regulation does not demand it. In half of the countries, neither pension funds nor annuity providers are required to account for future mortality improvement. Despite the lack of a legislative obligation, the majority of countries do so in practice, with annuity providers doing so more frequently than pension funds.

Table 1: Mortality tables and improvement required by regulation and used in practice

Country	Minimum table required by Regulations		Mortality Improvement required by Regulations		Mortality Improvements used in Practice	
	Annuity Providers	**Pension Plans**	**Annuity Providers**	**Pension Plans**	**Annuity Providers**	**Pension Plans**
Brazil	No	Yes	No	No	No	No
Canada	No	Yes	Yes	Yes	Yes	Yes
Chile	Yes	Yes	Yes	Yes	Yes	Yes
China	Yes	Yes	No	No	No	No
France	Yes	Yes	Yes	Yes	Yes	Yes
Germany	Yes	Yes/No	Yes	Yes	Yes	Yes
Israel	Yes	Yes	Yes	Yes	Yes	Yes
Japan	No	Yes	No	No	Yes	No
Korea	No	No	No	No	No	No
Mexico	Yes	No	Yes	No	Yes	No
Netherland	No	No	Yes	Yes	Yes	Yes
Peru	Yes	Yes	No	No	Some	Some
Spain	No	No	Yes	Yes	Yes	Yes
Switzerland	No	No	No	No	Yes	Some
United Kingdom	No	No	Yes	Yes	Yes	Yes
United States	Yes	Yes	No	Yes	Yes	Yes

Source: OECD

Notes: The statistical data for Israel are supplied by and under the responsibility of the relevant Israel authorities.

The use of such data by the OECD is without prejudice to the status of the Golan Heights, Wast Jerusalem and Israeli settlements in the West Bank under the terms of international law.

1. For non/regulated Pensionskassen and insurance oriented Pensionsfonds.
2. For regulated Pensionskassen and non/insurance oriented Pensionsfonds.

Despite the lack of a legal obligation to provision for mortality improvement, the majority of countries do so in practice, with annuity providers doing so more frequently than pension funds. In practice, thirteen of the sixteen nations' annuity providers use mortality improvement assumptions, whereas only eleven of the sixteen countries' pension funds do.

3.6.2. Standard Mortality Table

The analysis is based on the situation in which conventional mortality tables are used by pension funds and annuity providers. Mortality rates for most plans will be based on standard tables created and published by the Society of Actuaries or governmental organization. Tables are often titled based on (1) the types and characteristics of data underlying the table and (2) because mortality rates generally change over time, the calendar year of experience that the mortality rates are assumed to represent. In most cases, detailed information about the data's source is included in the report that is published alongside the table. In addition, a breakdown of table rates for subgroups may be provided.

3.6.3. Mortality Improvement

Current mortality tables, which have been specifically constructed for the retirement area, typically have no room for future mortality improvement. Most Society of Actuaries mortality tables used in the retirement area, however, include projection scales for use in estimating future mortality improvement. These scales are typically differentiated by age and gender.

Table 2:Mortality Projection Scale AA compiled by the Society of Actuaries

Age	Male	Female
60	.016	.005
61	.015	.005
62	.015	.005
63	.014	.005
64	.014	.005
65	.014	.005
66	.013	.005
67	.013	.005
68	.014	.005
69	.014	.005
70	.015	.005

Source: Mortality Projection Scale AA compiled by the Society of Actuaries
For full scale see Table 7-3 in RP-2000 Mortality Table,
https://www.soa.org/globalassets/assets/Files/Research/Exp-Study/rp00_mortalitytables.pdf

These scales are used to reduce the likelihood of death in the following way:

$$Probability\ of\ death\ with\ n\ years\ of\ mortality\ improvement = (mortality\ rate\ at\ age\ x)\ (1 - projection\ scale\ value\ at\ age\ x)^n$$

3.6.4. Generational Mortality Improvement

If it is assumed that the forces leading to mortality improvement will continue in the future, then mortality rates will vary by both age and the calendar year of attainment of age, because those attaining the age later will be exposed to the forces leading to mortality improvement for a longer time period. Thus, the probability of dying at 60 would be higher for a person turning 60 in 2012 than for a person becoming 60 in 2016. Another way to look at it is that various generations (those born in 1952 versus those born in 1956) will have different mortality rates at the age of 60.

To account for this difference, projection scales for the number of years between the valuation year and the year the individual reaches a certain age can be used. This is known as the generational approach for projecting mortality improvement.

For example, if a valuation is being performed as of January 1, 2014, using a mortality table with mortality rates representative of 2014, a present value factor at age x would use the following mortality rates, where the superscript represents the calendar year in which the individual attains a given age.

$$q^{2014}{}_{x},$$

$$q^{2015}{}_{x+1} = q^{2014}{}_{x+1}(1 - scale_{x+1}),$$

$$q^{2016}{}_{x+2} = q^{2014}{}_{x+2}(1 - scale_{x+2})^2, \ldots..,$$

4. EMPIRICAL ANALYSIS

This section examines how defined benefit (DB) pension plans would be affected by uncertainty about future mortality and life expectancy outcomes. In this regard, the first step is to assess the uncertainty surrounding future changes in mortality and life expectancy, also known as longevity risk. Second, it considers the impact of longevity risk on defined benefit (DB) pension plans provided by employers. The link between mortality and life expectancy, as well as how life tables are constructed from mortality data, is examined in order to assess the uncertainty surrounding future mortality and life expectancy outcomes.

Finally, a focus on the most pressing issue confronting pension funds: forecasting the future path of mortality and life expectancy in order to determine their future liabilities. As a result, the section presents a stochastic approach to modeling mortality and life expectancy uncertainty. It provides the results of estimating the Lee-Carter model for the selected country in this regard. France data was chosen for this study for estimation and modeling. The data for the estimation came from the Society of Actuaries Annuity Mortality database and the human mortality database. The Author used R-programming for analysis of the data.

4.1. Uncertainty About Mortality and Life Expectancy

4.1.1. The relationship between mortality and life expectancy: Life Table

For a given population, life tables provide a summary of mortality, survivorship, and life expectancy. They can contain data for each and every year of life (complete life tables) or by 5- or 10-year intervals (abridged life tables). A life table can be created in its most basic form by combining a set of age-specific death rates. Age-specific death rates are calculated as the ratio of deaths in a given year to the population size. They're usually expressed in terms of people per 1,000. Mortality rates, on the other hand, are the chances that someone of a specific age will die during the time period under consideration (i.e., the probability of dying). The numerator is the number of individuals from this generation who die between age n and age n+1, and the denominator is the size of the generation who reach age n during the year in question. The annual death rate is different from the annual probability of dying by age because the latter is the proportion of people of that age who die during the year, whereas the probability of dying is the proportion of people of that age dying during the age interval. Therefore, life tables provide a link between mortality and life expectancy. As a result, life tables establish a connection between mortality and life expectancy. The mean number of years still to be lived by a person who has reached that exact age (i.e., age-specific life expectancies) if subjected to the current age-specific probabilities of dying for the rest of his or her life is the final result of a life table. Table 4 shows a life table for males in France 2018. The first column lists the various ages x. mx is in the second column is the observed period age-specific death rates per capita. The next column, qx, contains the age-specific death probabilities. The average number of person-years lived by those who died in the interval, ax, is shown in column four. People are assumed to die in the middle of the age interval, except at birth, when they are assumed to die at the beginning of the interval, and at ages 110+, when they are assumed to die at the end of the interval. The following columns calculate the number of survivors of a hypothetical cohort of 100,000 individuals at each age x, lx; the number of deaths in the cohort between two consecutive ages, dx; the number of person-years lived by the cohort, Lx; and the total person-years remaining at each age, Tx. Tx is divided by lx to calculate life expectancies at age x, ex. As a result, a life table provides age-specific life expectancies based on mortality rates (Table 4). Because there is a link between mortality and life expectancy, the next sub-section examines improvement in both variables.

4.1.2. The uncertainty surrounding the improvement in mortality

Over the last century, mortality rates have gradually decreased, resulting in significant gains in life expectancy at birth and at 65. (Table 4). These decreases are due to significant reductions in death rates among the young and, to a lesser extent, improvements among the elderly. The decline in mortality during the first half of the twentieth century was primarily attributable to a decrease in infectious diseases that primarily affected children. The fall in mortality during the later decades of the twentieth century was attributable to fewer fatalities from chronic diseases that largely affected the elderly. When looking at the gains in life expectancy at birth and at 65 years old during the twentieth century, this is proven (Table 3). As can be seen by comparing the top and bottom panels in Table 4, life expectancy at birth climbed quicker in the first half of the twentieth century, whereas life expectancy at 65 increased faster in the second. Changes in mortality and life expectancy at older ages have the largest impact on employer-provided DB pension systems.

Life expectancies for female and male populations can be calculated using data from life tables for the years 1985 to 2018. Table 3 provides the observed values for male and female life expectancy in France for various age groups. Due to the high incidence of infant mortality during these decades, life expectancy at birth has increased from 80.12 to 85.37 years for females and from 71.90 to 79.34 years for males. When comparing female and male life expectancy in the 40-44 age group, it was higher than life expectancy in the 60-64 age group.

Table 3: Comparing Life Expectancy at selected age groups, France 1985-2018

	at birth		40-44		60-64	
Year	**Male**	**Female**	**Male**	**Female**	**Male**	**Female**
1985-1989	80,12	71,9	41,73	34,66	23,56	18,35
1990-1994	81,39	73,13	42,86	35,85	24,59	19,31
1995-1999	82,24	74,12	43,48	36,61	25,16	19,86
2000-2004	83,12	75,77	44,18	37,64	25,85	20,79
2005-2009	84,25	77,33	45,16	38,92	26,77	21,87
2010-2014	85,05	78,6	45,88	40,02	27,4	22,72
2015-2018	85,37	79,34	46,17	40,46	27,61	23,17

Source: Human Mortality Database(http://www.mortality.org/index.html).
Notes: 1. Life expectancy from Selected ages from Life table period 5x5 (age by year) for female and male.

4.1.3. Mortality and life expectancy forecasting methods

The study focuses on mortality and life expectancy forecasts because it is concerned with longevity risk and its influence on defined-benefit pension systems. There are numerous techniques to modeling mortality and life expectancy in this area. There are process-based methods that use models based on the underlying biological processes; explanatory-based approaches that use a causal forecasting approach including econometric relationships; and extrapolative methods that are based on projecting historical mortality trends ahead. Extrapolative models are the most commonly employed models by actuaries and government bodies. These models, which use previous data to define age-specific mortality as a function of calendar time, can be deterministic or stochastic. Deterministic models foresee by directly extending historical patterns and, as a result, lack standard errors and forecast probabilities. Stochastic models, on the other hand, provide predictions based on probability distributions. They use previous data to fit a statistical model, which they then project into the future. Prediction values have probabilities assigned to them as a result of the forecast process, allowing you to determine the possibility that an outcome will occur. Finally, due to a scarcity of data, predicting and forecasting mortality rates and life expectancy for the extremely old (those aged 85 and up) is difficult. Because of small sample size issues, data at very old ages are not particularly precise. Only a few nations have official population statistics that are good enough to generate meaningful estimates of death rates at older ages. It is widely assumed that between the ages of 30 and 85, age-specific death rates rise at a fixed rate. This rate of increase tends to fall for ages above 85, and even possibly, at the more extreme ages, to become zero or negative, although one cannot be certain of the latter because of the sparseness of the data above age 100 (Robine and Vaupel, 2002; Wilmoth, 1998).

4.2. Measuring mortality and longevity improvement uncertainty

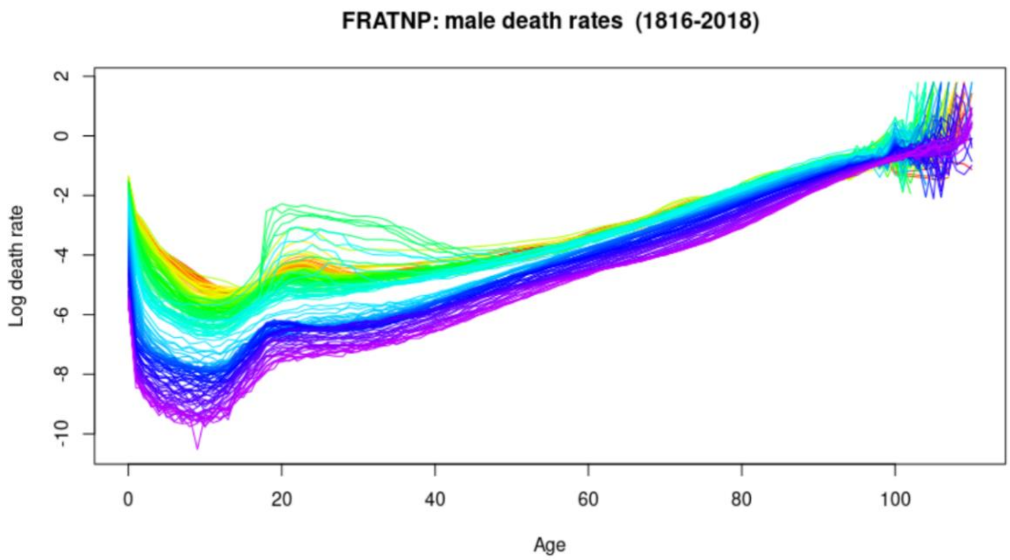
The uncertainty surrounding future mortality and life expectancy outcomes can be gauged using a stochastic approach because it attaches probabilities to different outcomes, permitting therefore to assess uncertainty and risks adequately. Future developments in mortality rates and life expectancy are uncertain, but some paths or trajectories are more likely than others. Hence, attempts to forecast mortality and life expectancy should include a range of possible outcomes, and probabilities attached to that range. Together, these elements constitute the ‘prediction interval’ for the mortality and life expectancy variables concerned. This subsection presents the results of examining the uncertainty surrounding forecasts of mortality and life expectancy using the Lee-Carter stochastic methodology (Lee and Carter, 1992)

4.2.1. Lee Carter Model Measurement

There has been an overall decline in mortality rates in France from 1998 to 2018 for both female and male populations. We have observed higher level of decline in mortality rates during 2005 to 2018 as compared to 1998 to 2004. With conclusion, a rapid decline is observed in central mortality rates of France female population than male population for all ages.

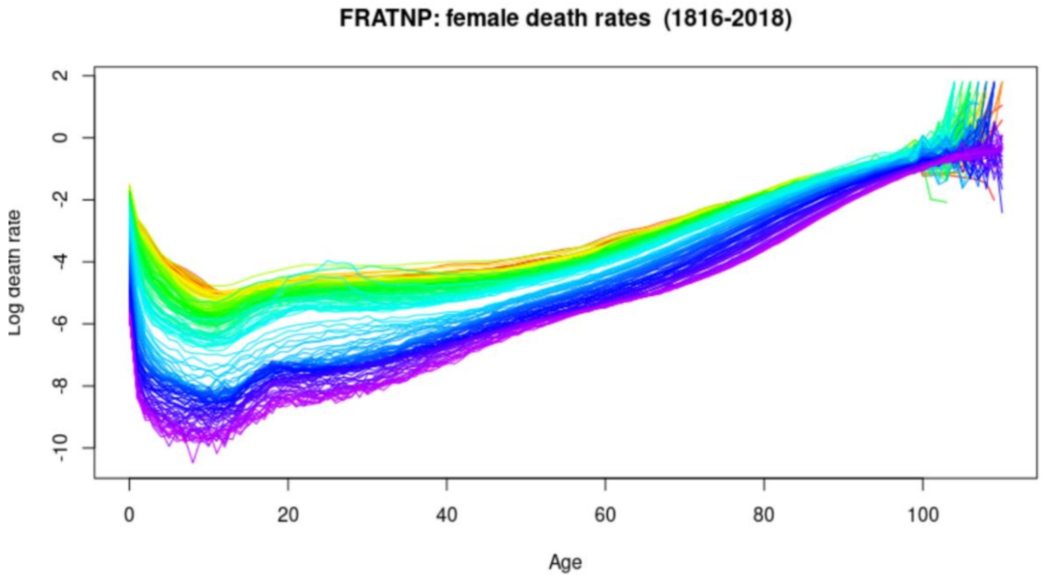
Figure 3 shows to demonstrate the improvement for France mortality for period 1998, 2003, 2018; and present below is the plotted age group specific central death rates for female and male.

Figure 1: Male Death Rate, France 1816 -2018.



Source: Human Mortality Database (http://www.mortality.org/index.html).
Notes: Selected ages for death rates table 1x1 (age by year)., Author Calculation

Figure 2: Female Death Rate, France 1816-2018.



Source: Human Mortality Database (http://www.mortality.org/index.html).
Notes: Selected ages for death rates table 1x1 (age by year). Author Calculations

Table 7 presents the natural logarithm of all age specific death rates for female population from 1989 up to 2018. These values are the entries for a 22 × 30 matrix A, where singular value decomposition is performed.

This system provides a unique solution when these constraints are included. The a_x, b_x, and k_t parameters are to be determined by using the Singular Value Decomposition (SVD).

This section presents the results of estimation of parameters in LC model. Estimated values of age dependent parameters and are reported in **Table 8** and estimated values of time dependent parameter is reported for female and male populations in France.

Figure 3: $\hat{a}_x$ and $\hat{b}_x$ for France population based on life tables (1989 to 2018)

Figures 3 shows the death rate logarithm by age and time. The different colors indicate different years as detailed in the demographic vignette, and most recent ones in violet, earliest in red. Several behaviors are shown respectively for male, female, and total population.

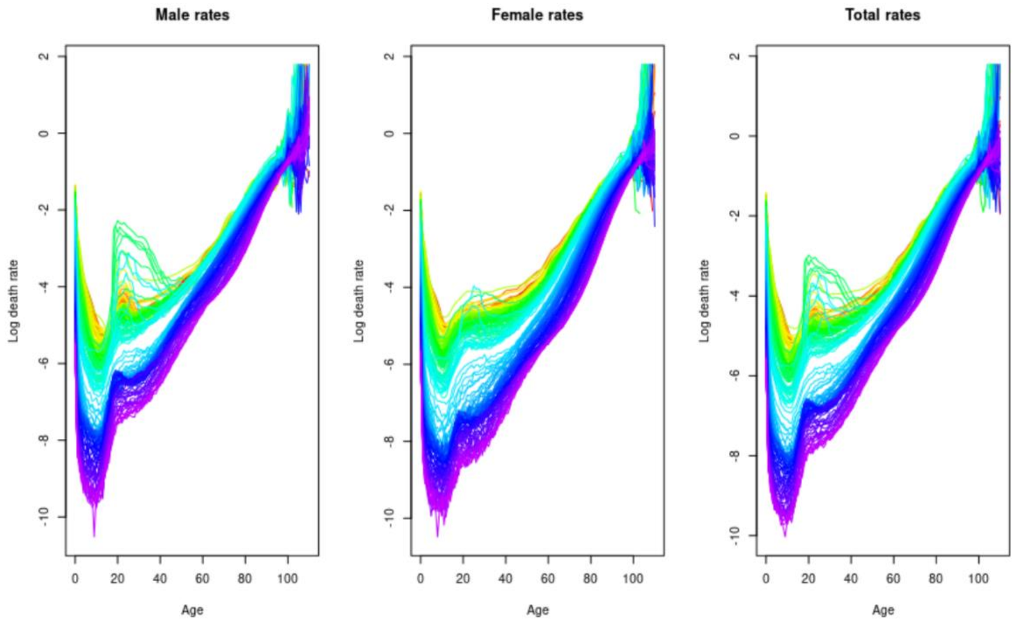


Figure 4: Pattern of age according to death rates for France Population

Source: Human Mortality Database (http://www.mortality.org/index.html).
Notes: HMD France 5x1 (age by year), Author Calculation

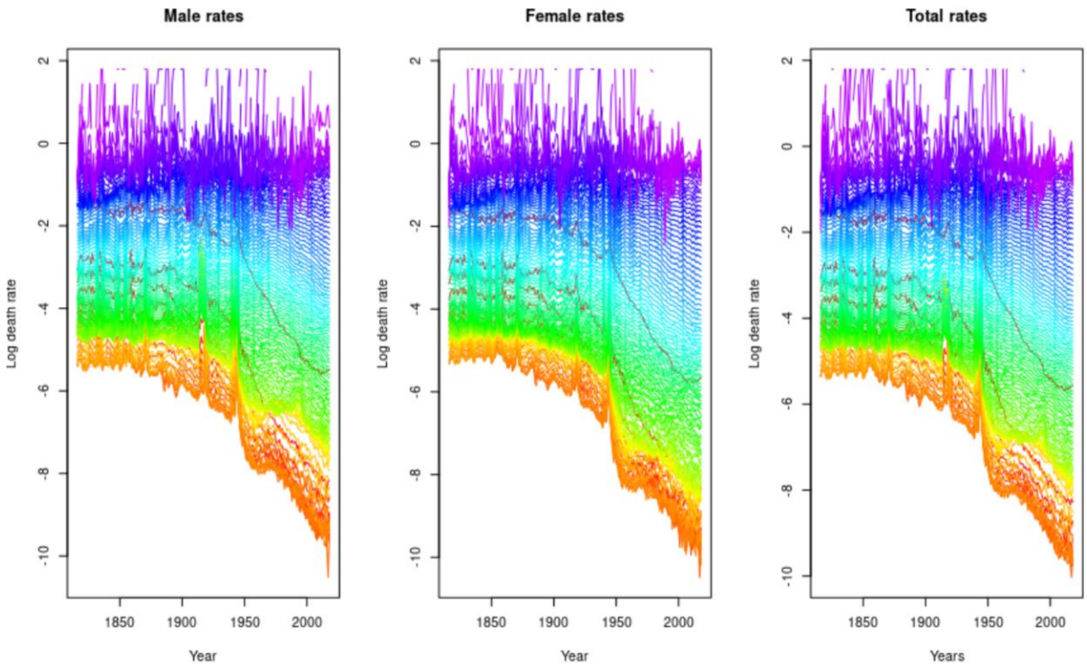


Figure 5: Pattern of Death rate based on Year (1918-2018)

Source: Human Mortality Database (http://www.mortality.org/index.html).
Notes: HMD France 5x1 (age by year), Author Calculations

We have observed higher level of decline in mortality rates**.** The French data confirms that mortality is falling at all ages with a different behavior according to different ages.

This notable change in mortality trend could be attributed to drugs use, heart disease, cancer, accident among other factors.

4.2.2. Fitting and Estimating the Parameters of the Lee-Carter Model

Fitting the Lee Carter model for our data use the lca functions which was applied to the male, female, and total population considering maximum age 100.

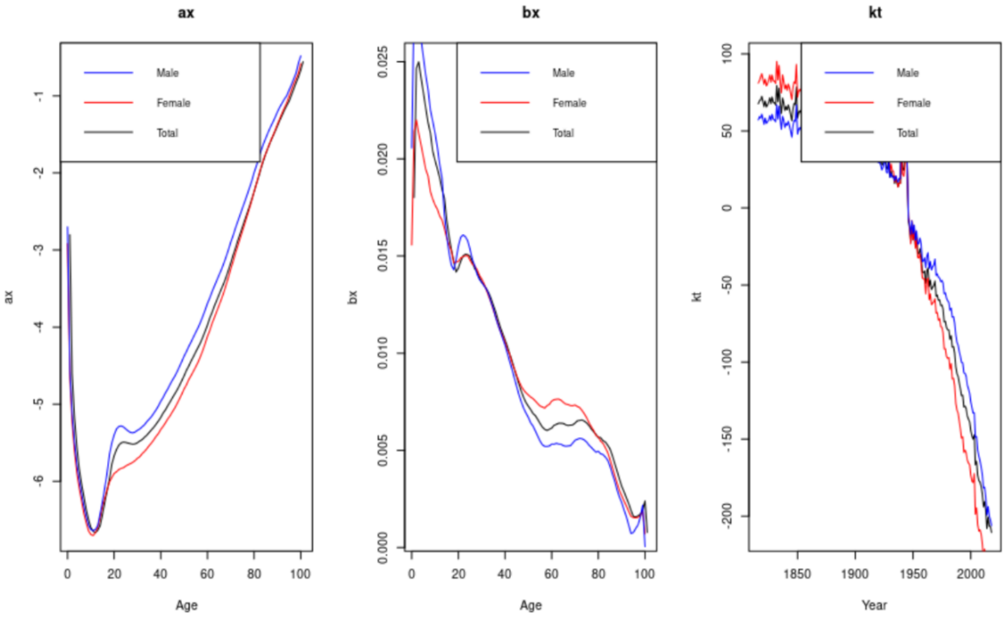


Figure 6: Estimated parameter of $a_{x,} b_x k_t$

Source: Human Mortality Database (http://www.mortality.org/index.html).
Notes: HMD France 5x1 (age by year), Author Calculation

Different datasets reveal similar parameter behavior. As expected, as one gets older, the average mortality rate rises (see $\hat{a}_x$ pattern). Furthermore, the young mortality hump for males between the ages of 20 and 30 is clearly visible due to accidental deaths. $\hat{b}_x$, on the other hand, demonstrates a higher value for younger ages and the greatest improvement for females in the age range (60-80). Finally, $\hat{k}_t$ decreases with increasing time, as expected

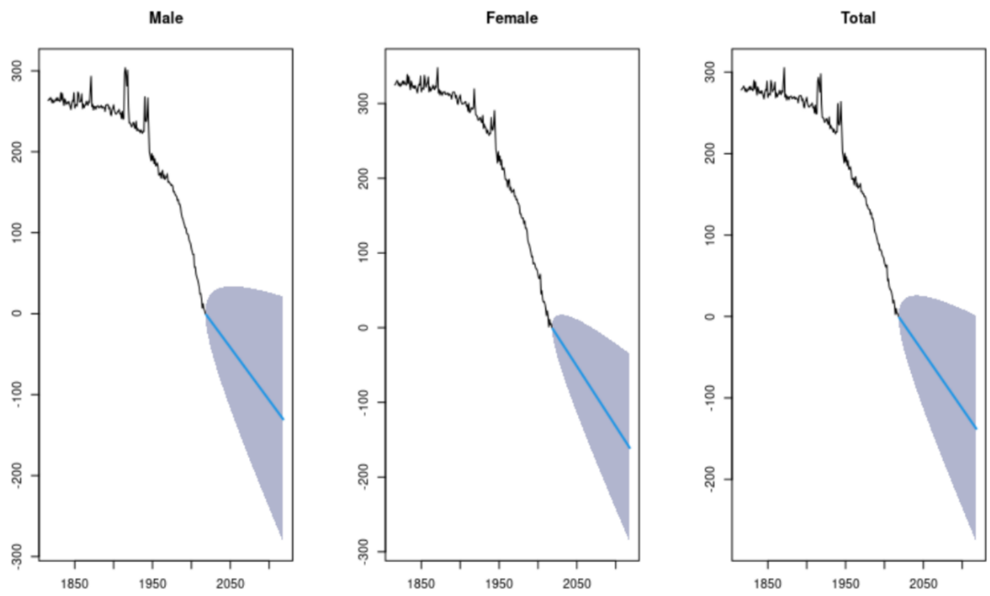


Figure 7: Projected value of k_t for 100 years

Forecasting is the main aim behind the stochastic modeling. One of the noteworthy properties of the LC model is that, once it is fitted (i.e., once values of $\hat{a}_x$, $\hat{b}_x$, and $\hat{k}_t$ are found), only the mortality index (k_t) over time needs to be forecasted for future time points. Lee and Carter (1992) fitted autoregressive integrated moving average (ARIMA) (0,1,0) (i.e., random walk with drift) for modeling mortality index for French population. The figure below shows the future projected values of $k_t s$ up to 60years.

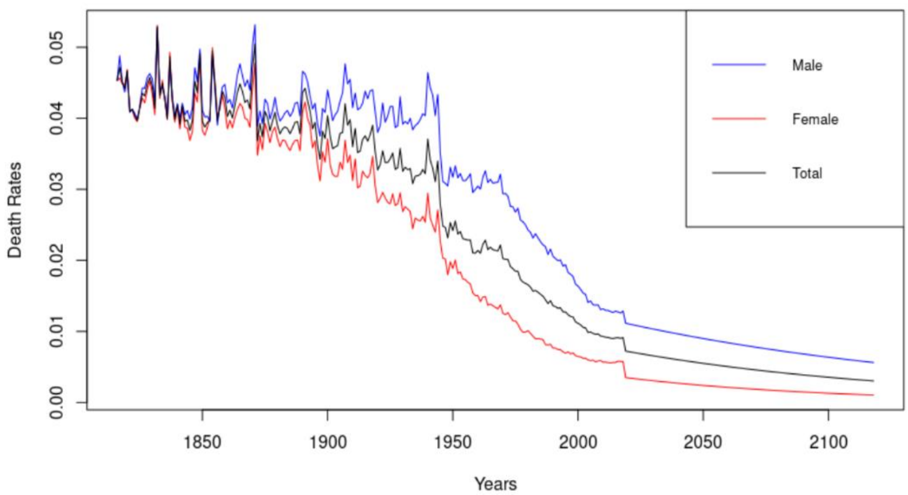


Figure 8: Pattern of Past and Projected rates for people aged 65

Source: Human Mortality Database (http://www.mortality.org/index.html).
Notes: HMD France 5x1 (age by year), Author Calculations

Finally, the entire rate pattern is simple to deduce. In this matrix, past and projected rates are both blinded. We present here a pattern of past and projected rates for people over the age of 65 based on different populations. **Figure 8** clearly shows the expected improvement. This could be attributed to HIV/AIDS Pandemics, disease, and drugs. We observed that in next decades mortality is expected to decline for both female and male population in France. This is due to decreasing nature of k_t. We have forecasted values of age specific death rate, by using estimated parameters a_x, b_x and forecasted values of mortality index k_t.

4.3. The impact of longevity risk on defined-benefit private pension plans

The impact of longevity risk on employer-provided DB private pension schemes is examined in this section. The previous section demonstrated that forecasting mortality and life expectancy using a stochastic approach allows you to assign probabilities to a variety of possible projections and hence estimate the uncertainty surrounding future mortality and life expectancy outcomes. Private pension funds, on the other hand, are concerned about the impact of this uncertainty on their pension commitments. This section assesses the changes in the net present value of annuity payments as mortality and life expectancy evolves, as this is the principal impact of longevity risk on net pension obligations. These adjustments are assessed for members of pension funds of various ages, as well as pension funds with various age membership structures.

4.3.1. How does longevity risk affect DB private pension plans?

Longevity risk has the greatest influence on the net pension liabilities of employer-provided DB private pension plans because of annuity payments. An annuity is a contract in which one person or organization agrees to pay a stream or series of payments to another person or organization (the annuitant) (annuity payments). Annuities are designed to give a constant stream of income to the annuitant over a period of time, which can begin immediately or at any time in the future. Capital gains and investment profits are usually tax-deferred. There are numerous types of annuities. They can be classified in a variety of ways, including: (1) by the underlying investment into fixed or variable; (2) by the primary purpose, i.e., accumulation or pay-out, into deferred or immediate; (3) by the nature of the pay-out commitment into fixed period, fixed amount, or lifetime; and (4) by the premium payment arrangement into single or flexible premium. In a fixed annuity, the insurance company or pension fund guarantees the principle as well as a minimum rate of interest, but in a variable annuity, the annuity payment is based on the underlying portfolio's investment performance. An immediate annuity is intended to pay a lump sum or a series of payments immediately after the annuity is purchased, whereas a deferred annuity pays the annuitant at a later date. Fixed period annuities pay an income for a set length of time (e.g., 10 years), whereas lifetime annuities pay income for the rest of the annuitant's life. A single premium annuity is one that is funded with a single payment, whereas a flexible premium annuity is one that is funded over a series of payments. Only deferred annuities are flexible.

Because employer-provided DB private pensions promise their members a guaranteed future stream of payments at retirement for the rest of their lives, the research concentrates on the impact of longevity risk on fixed, deferred, lifetime, and flexible premium annuities throughout. Longevity risk would have a greater impact on annuities that are fixed, deferred, and for the annuitant's lifetime once retirement age is achieved. The impact of longevity risk on fixed period annuities, on the other hand, is less obvious. Furthermore, the extent of the impact of longevity risk on annuity payments would be determined not just by the type of annuity guarantees, but also by how pension funds account for improvements in mortality and life expectancy when calculating the net present value of annuity payments.

4.3.2. How private pension funds account for future improvements in mortality and/or life expectancy?

Pension funds do not appear to account fully for projected increases in mortality and life expectancy. Recent study, particularly that of the Actuarial Profession and Cass Business School (2005), discovered that current practice differs significantly across the EU. Pension funds in certain countries account for predicted future improvements in mortality, whilst others use tables based on mortality recorded in the past, without accounting for the possibility that life expectancy will continue to rise (Belgium, Denmark, Norway, Sweden, and Switzerland). Of those countries incorporating an allowance for future improvements in mortality, Austria, France, Germany (for only 25 years and using 1996 as the base year), Ireland (improvements incorporated only until 2010), Italy, the Netherlands, Spain, and the United Kingdom use forecasts; while Canada, Finland, and the United States, despite of having mortality tables with built in mechanisms to take into account future changes in mortality, generally do not use them.

Furthermore, there is no standardized or consistent mechanism for accounting for future increases in mortality and life expectancy. In this aspect, assessing longevity risk is challenging due to the lack of a consistent methodology, which makes mortality projections arbitrary and impossible to compare among pension funds, let alone countries.

As a result, the impact of the longevity risk is amplified. The impact of future improvements in mortality and life expectancy (i.e., longevity risk) on employer-provided DB private pension plans is compounded by the fact that few actuaries and pension schemes account for future improvements in mortality and life expectancy, and those that do so only partially. Furthermore, even with adjustments for anticipated improvements in mortality, the base tables used for demographic assumptions are nearly ten years old, dating from the early to mid-1990s. Furthermore, the lack of standard methods to forecast mortality and life expectancy, and the fact that these methods are generally far from being fully stochastic complicate any comparative analysis and make the task of examining the impact of longevity risk on pension fund liabilities fuzzier.

Furthermore, the lack of standard methods for forecasting mortality and life expectancy, as well as the fact that these methods are far from being totally stochastic, complicates any comparison study and makes the task of analyzing the impact of longevity risk on pension fund liabilities even more hazy.

5. CONCLUSIONS

Life expectancy forecasts are necessary for estimating future healthcare and pension costs. The Lee-Carter (LC) model (1992), which forecasts age-specific death rates log bilinearly, is a commonly used model to anticipate mortality. The LC model is employed because parameter estimation is simple, and it provides a good fit over a wide range of ages. The data collection includes data on France's population mortality from 1816 to 2018. The parameters of the LC model are estimated using the Singular Value Decomposition (SVD) method. The mortality values are forecasted using the Auto Regressive Integrated Moving Average (ARIMA) time series model.

We forecasted the time-index using a random walk with drift, which is typically found to be an appropriate mode (Callot et al. 2016). The overall pattern of mortality ($\hat{a}_x$) for both female and male populations revealed high infant mortality, an accidental hump around the age of 20, and a nearly exponential increase at older ages. The sensitivity of mortality ($\hat{b}_x$) has revealed that mortality declines at a higher rate for females aged 25-34 years and males aged 15-24 years than for other ages. The Mortality index ($\hat{k}_t$) has been declining.

Female mortality improvement has outpaced male mortality as well as the series of the general indices clearly tend to decrease, although not monotonically over time. For the first half of the period, there is a significant increase in female mortality over men, which decreases significantly in the second half of the period. The sensitivity of mortality has shown that mortality declines at a rapid rate for people aged 20 to 25. Since World War I and World War II, the mortality index has shown a decreasing trend with two spikes. The predicted Lee Carter model fits France population data well over a wide age range but performs poorly below the age of four and after the age of 55.

5.1. Policy issues

Longevity risk, defined as the uncertainty surrounding future developments in mortality and life expectancy, has a non-negligible impact on the liabilities of employer-provided pension plans because lifetime annuity payments are based on the length of time people are expected to live, according to the paper. The impact of this on the net present value of annuity payments for a "theoretical pension fund" was calculated in **Table 5**. It was discovered that the amount of this influence is determined by the pension fund membership's age structure. As a result, pension funds with a younger membership structure will be more affected by longevity risk since they will be exposed to uncertain changes in mortality and life expectancy for a longer period of time. Unfortunately, the impact of longevity risk is aggravated by the fact that few pension plans account for future changes in mortality and life expectancy, and those that do only account for partial improvements. To make matters worse, most pension funds rely on mortality tables that are almost a decade old. Furthermore, the lack of a consistent technique for calculating longevity risk makes determining the optimal way to account for gains in mortality and life expectancy difficult.

Using a common methodology to predict death rates and life expectancy has an obvious advantage in this regard. This research argues for the use of a stochastic model in this case because it allows for the attachment of probabilities and consequently the assessment of the degree of uncertainty around future mortality and life expectancy outcomes. Unfortunately, many small and medium-sized pension funds may lack the financial and technical capabilities to create forecasts using a standardized technique. Government entities may be able to develop them if they have the necessary resources and technical expertise. However, assumptions about total populations rather than specific membership groups of private pension plans may not be useful. Governmental entities might create forecasts for the overall population as well as for various subgroups based on gender, age, wealth, and educational attainment. As a result, separate pension funds could use the sub-population that most closely reflects their current membership composition.

Using mortality tables that differentiate based on socioeconomic position and gender, on the other hand, has its own set of issues because it may give rise to discriminatory issues. Arguments in favor of distinguishing tables include the fact that adopting an average life expectancy index penalizes persons with greater life expectancy (e.g., women, well educated, and well-off people) while rewarding people with lower life expectancy (e.g., men, low educated and low-income people). Furthermore, private pension plans must hedge against their own longevity risk, i.e., the risk associated with their own membership structure, rather than an average longevity risk.

Finally, in addition to incorporating mortality improvements through the adoption of a standard methodology and average or differentiated mortality tables, the impact of longevity risk on employer-provided DB plans can be mitigated in part by indexing pension benefits to life expectancy. Indexing benefits to life expectancy, on the other hand, moves some of the longevity risk back to individuals, reducing one of the main reasons people buy annuities. Differentiating between individual and aggregate or cohort longevity risk can be useful in this regard. Individual risk is unique to each person, but it can be easily mitigated by sharing risks. As a result, assuming it by pension funds would be more efficient, as they are best positioned to pool individual unique risks. On the other side, the aggregate or cohort risk is more difficult to address or mitigate. As a result, by indexing benefits to cohort longevity changes, this risk can be borne more easily by pension funds and people.

5.2. Areas in which additional research is required

We made an effort to be thorough by identifying and reviewing literature on the subject of longevity risk. However, mortality risk is dynamic, and continual study is required to ensure that the industry is up to date on the current trends. In recent years, this has included increasingly extensive analysis of characteristics such as separating lives into cohorts, focusing on specific causes of death as drivers of mortality, and increasing the roughness of the risk variables used in mortality investigations. There is still opportunity for more complex study, which would only serve to better understanding of mortality and longevity risk profile. On the topic of stochastic mortality models, there is a lot of literature, primarily from academics. These are usually concerned with the shape of the models and how well they fit historical data. One area where there is far less information is the discussion about the practical application of such models.

It would be useful to see some in-depth analysis from a company standpoint of the relative costs and benefits of implementing stochastic mortality analysis in various stages of the product cycle (pricing, reserving, managing capital, hedging longevity risk, and so on) and across different product categories (payout annuities, life settlements, etc.). This could be because insurance businesses specialize in this sector, thus all product advancements will most likely originate from within the industry. Insurers, on the other hand, are frequently required to satisfy a variety of stakeholders. Suggestions for new and unique product concepts could be fascinating to see

6. REFERENCES

(PDF) Addressing Longevity Heterogeneity in Pension Scheme Design and Reform. (n.d.). Retrieved October 28, 2021, from https://www.researchgate.net/publication/311921892_Addressing_Longevity_Heterogeneity_in_Pension_Scheme_Design_and_Reform

Antolin, P. (2007). OECD Working Papers on Insurance Longevity Risk and Private Pensions. *Pensions*, *3*. https://doi.org/10.1787/261260613084

Ayuso, M., Bravo, J. M., & Holzmann, R. (2016). *Addressing Longevity Heterogeneity in Pension Scheme Design and Reform*. *Paris IX*, 1–26.

Ayuso, M., Bravo, J. M., & Holzmann, R. (2017). Addressing Longevity' Heterogeneity in Pension Scheme Design. *Journal of Finance and Economics*, *6*(1), 1–21. https://doi.org/10.12735/jfe.v6n1p1

Ayuso, M., Bravo, J. M., & Holzmann, R. (2021). Getting life expectancy estimates right for pension policy: Period versus cohort approach. *Journal of Pension Economics and Finance*, *20*(2), 212–231. https://doi.org/10.1017/S1474747220000050

Barrieu, P., Bensusan, H., El Karoui, N., Hillairet, C., Loisel, S., Ravanelli, C., & Salhi, Y. (2012). Understanding, modelling and managing longevity risk: Key issues and main challenges. *Scandinavian Actuarial Journal, 3*, 203–231. https://doi.org/10.1080/03461238.2010.511034

Bauer, D., Börger, M., & Ruß, J. (2010). On the pricing of longevity-linked securities. *Insurance: Mathematics and Economics*, *46*(1), 139–149. https://doi.org/10.1016/j.insmatheco.2009.06.005

Blake, D., El Karoui, N., Loisel, S., & MacMinn, R. (2018). Longevity risk and capital markets: The 2015–16 update. In *Insurance: Mathematics and Economics* (Vol. 78, pp. 157–173). Elsevier B.V. https://doi.org/10.1016/j.insmatheco.2017.10.002

Cairns, A. J. G., Blake, D. P., & Dowd, K. (2011). Modelling and Management of Mortality Risk: A Review. *SSRN Electronic Journal*, *April 2018*. https://doi.org/10.2139/ssrn.1339970

Chang, Y., & Sherris, M. (2018). Longevity risk management and the development of a value-based longevity index. *Risks, 6*(1). https://doi.org/10.3390/risks6010010

Chen, B., Zhang, L., & Zhao, L. (2010). On the robustness of longevity risk pricing. *Insurance: Mathematics and Economics*, *47*(3), 358–373. https://doi.org/10.1016/j.insmatheco.2010.08.002

Cocco, J., & Gomes, F. (2009). Longevity risk and retirement savings. *2009 Meeting Papers*, *January*. http://www.banque-france.fr/fileadmin/user_upload/banque_de_france/Economie-et-Statistiques/La_recherche/GB/Cocco.pdf

Content, T. O. F., Summary, E., Of, B., Village, T. H. E., Of, D., & Use, L. (n.d.). *MATHEMATICAL MODELS FOR THE LONGEVITY RISK IN THE ANNUITY MARKET*. *Page 4*, 1–26.

Deng, Yinglu, Brockett, P. L., & MacMinn, R. D. (2012b). Longevity/Mortality Risk Modeling and Securities Pricing. *Journal of Risk and Insurance*, *79*(3), 697–721. https://doi.org/10.1111/j.1539-6975.2011.01450.x

Denuit, M., Devolder, P., & Goderniaux, A. C. (2007). Securitization of longevity risk: Pricing survivor bonds with Wang transform in the Lee-Carter framework. In *Journal of Risk and Insurance* (Vol. 74, Issue 1, pp. 87–113). https://doi.org/10.1111/j.1539-6975.2007.00203.x

Department of Health and Mental Hygiene, N. Y. C. (2020). *Preliminary Estimate of Excess Mortality During the COVID-19 Outbreak —*. *69*(19), 603–605.

Diffouo, P. M. N., & Devolder, P. (2020). Longevity risk measurement of life annuity products. *Risks*, *8*(1). https://doi.org/10.3390/risks8010031

Gatzert, N., & Wesker, H. (2014). Mortality risk and its effect on shortfall and risk management in life insurance. *Journal of Risk and Insurance*, *81*(1), 57–90. https://doi.org/10.1111/j.1539-6975.2012.01496.x

Lin, Y., & Cox, S. H. (2005). Securitization of mortality risks in life annuities. In *Journal of Risk and Insurance* (Vol. 72, Issue 2, pp. 227–252). https://doi.org/10.1111/j.1539-6975.2005.00122.x

Lin, Y., & Cox, S. H. (2008). Securitization of catastrophe mortality risks. *Insurance: Mathematics and Economics*, *42*(2), 628–637. https://doi.org/10.1016/j.insmatheco.2007.06.005

Longevity A Market in The Making | PDF | Life Expectancy | Insurance. (n.d.). Retrieved November 9, 2021, from https://www.scribd.com/document/28396645/Longevity-a-Market-in-the-Making

Longevity Risk Definition. (n.d.). Retrieved October 28, 2021, from https://www.investopedia.com/terms/l/longevityrisk.asp

Mircea, I., Covrig, M., & Serban, R. (2014). Some Mathematical Models for Longevity Risk in the Annuity Market and Pension Funds. *Procedia Economics and Finance*, *15*, 115–122. https://doi.org/10.1016/s2212-5671(14)00455-9

Mortality Assumptions and Longevity Risk. (2014). In *Mortality Assumptions and Longevity Risk*. https://doi.org/10.1787/9789264222748-en

Owusu, D. A., Boateng, M. A., & Tettey, I. (2016). *Hedging Longevity Risk using Longevity Swaps : A Case Study of the Social Security and National Insurance Trust (SSNIT), Ghana*. *5*(4), 165–170. https://doi.org/10.5923/j.ijfa.20160504.01

Pablo Antolin. (2014). Mortality Assumptions and Longevity Risk. In *Mortality Assumptions and Longevity Risk*. https://doi.org/10.1787/9789264222748-en

Pompella, M., & Scordis, N. A. (2017). The Palgrave handbook of unconventional risk transfer. In *The Palgrave Handbook of Unconventional Risk Transfer*. https://doi.org/10.1007/978-3-319-59297-8

Richards, S. J., & Currie, I. D. (2009). Longevity Risk and Annuity Pricing with the Lee-Carter Model. *British Actuarial Journal*, *15*(2), 317–343. https://doi.org/10.1017/s1357321700005675

Securitization, structuring and pricing of longevity risk - ScienceDirect. (n.d.). Retrieved June 27, 2020, from https://www.sciencedirect.com/science/article/pii/S0167668709001310

Sexauer, S. C., Peskin, M. W., & Cassidy, D. (2015). Making retirement income last a lifetime. *Financial Analysts Journal*, *71*(1), 79–89. https://doi.org/10.2469/faj.v71.n1.11

Stamp duty and land tax for non-resident owners of Australian property. (n.d.). Retrieved October 28, 2021, from https://simplyretirement.com.au/retirement-planning-longevity-risk

Tsui, P. K. (2016). *Rethinking Longevity Risk : A Framework to Address the Tail End*. *January*, 1–13.

Vieira, A., Peixoto, V. R., Aguiar, P., & Abrantes, A. (2020). Rapid estimation of excess mortality during the COVID-19 pandemic in Portugal -beyond reported deaths. *Journal of Epidemiology and Global Health*, *10*(3), 209–213. https://doi.org/10.2991/jegh.k.200721.001

Wills, S., & Sherris, M. (2010a). Securitization, structuring and pricing of longevity risk. *Insurance: Mathematics and Economics*, *46*(1), 173–185. https://doi.org/10.1016/j.insmatheco.2009.09.014

Yang, S. S., Yue, J. C., & Huang, H. C. (2010). Modeling longevity risks using a principal component approach: A comparison with existing stochastic mortality models. *Insurance: Mathematics and Economics*, *46*(1), 254–270. https://doi.org/10.1016/j.insmatheco.2009.09.013

APPENDIX

TABLES AND FIGURES

Table 4: Life table, France 2018 Males

Age	mx	qx	ax	lx	dx	Lx	Tx	ex
0	0,0042	0,00418	0,14	100000	418	99641	7955978	79,56
1	0,00028	0,00028	0,5	99582	28	99568	7856338	78,89
10	0,00007	0,00007	0,5	99476	7	99473	6960713	69,97
20	0,00052	0,00052	0,5	99268	52	99242	5966641	60,11
30	0,00078	0,00078	0,5	98641	77	98603	4976963	50,46
40	0,00144	0,00144	0,5	97632	140	97562	3994984	40,92
50	0,00368	0,00367	0,5	95452	350	95277	3027891	31,72
60	0,00921	0,00917	0,5	90033	826	89620	2096617	23,29
70	0,01808	0,01792	0,5	78850	1413	78144	1247312	15,82
80	0,04505	0,04406	0,5	59771	2634	58455	544426	9,11
90	0,16582	0,15313	0,5	25141	3850	23216	106666	4,24
100	0,4538	0,36988	0,5	1487	550	1212	2970	2
110+	0,7791	1	1,28	3	3	3	3	1,28

Source: Human Mortality Database (http://www.mortality.org/index.html),
Notes: Selected ages from table period 1x1 (age by year)., Author Calculations

Table 5: An increase in the annuity payments' net present value (percentage increase)

Age in 2005					Hypothetical penion fund		
25	40	55	65	70	(1)	(2)	(3)
23,6	15,3	7,3	3,3	2,4	10,4	9,6	8,2

Source: OECD calculations.
Notes: Increase resulting from comparing the net present value of annuity payments from 2005 to 2090 when life expectancy at birth improves by 1.2 years per decade and life expectancy at 65 improves by 0.8 years per decade, with the NPV of annuity payments when the most recent available mortality tables (2005) are used without allowing for mortality improvements.

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Table 6: Age group specific central death rates female population, France 1998-2018

Age group	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
0	0.00425	0.00390	0.00393	0.00401	0.00373	0.00358	0.00361	0.00324	0.00333	0.00312
01-04	0.00020	0.00022	0.00023	0.00024	0.00020	0.00022	0.00017	0.00018	0.00018	0.00017
05-09	0.00012	0.00012	0.00011	0.00009	0.00011	0.00010	0.00009	0.00008	0.00008	0.00009
10-14	0.00014	0.00013	0.00013	0.00013	0.00012	0.00010	0.00010	0.00009	0.00009	0.00009
15-19	0.00027	0.00028	0.00029	0.00029	0.00026	0.00022	0.00022	0.00021	0.00021	0.00019
20-24	0.00040	0.00038	0.00034	0.00035	0.00034	0.00030	0.00028	0.00029	0.00028	0.00025
25-29	0.00043	0.00042	0.00034	0.00038	0.00037	0.00035	0.00032	0.00031	0.00029	0.00031
30-34	0.00058	0.00056	0.00056	0.00051	0.00048	0.00049	0.00046	0.00045	0.00042	0.00040
35-39	0.00091	0.00089	0.00087	0.00087	0.00081	0.00081	0.00077	0.00075	0.00074	0.00068
40-44	0.00138	0.00141	0.00136	0.00137	0.00140	0.00133	0.00129	0.00120	0.00116	0.00114
45-49	0.00200	0.00206	0.00210	0.00209	0.00210	0.00207	0.00196	0.00198	0.00193	0.00186
50-54	0.00293	0.00290	0.00280	0.00277	0.00284	0.00292	0.00284	0.00278	0.00283	0.00273
55-59	0.00405	0.00393	0.00390	0.00395	0.00391	0.00389	0.00378	0.00371	0.00370	0.00372
60-64	0.00580	0.00571	0.00551	0.00549	0.00534	0.00541	0.00528	0.00526	0.00521	0.00516
65-69	0.00906	0.00877	0.00856	0.00836	0.00832	0.00828	0.00759	0.00760	0.00743	0.00721
70-74	0.01526	0.01481	0.01444	0.01422	0.01381	0.01396	0.01273	0.01252	0.01214	0.01182
75-79	0.02761	0.02753	0.02618	0.02557	0.02554	0.02566	0.02271	0.02296	0.02202	0.02133
80-84	0.05404	0.05410	0.05221	0.05055	0.05053	0.05159	0.04520	0.04602	0.04331	0.04235
85-89	0.10465	0.10487	0.10171	0.09911	0.09920	0.10241	0.08924	0.09239	0.08587	0.08548
90-94	0.18875	0.18993	0.18494	0.18216	0.18298	0.19185	0.16507	0.17043	0.16252	0.16080
95-99	0.31232	0.31331	0.30693	0.30418	0.30546	0.32351	0.27986	0.29179	0.27589	0.27439
100-104	0.47034	0.47173	0.46484	0.46317	0.46546	0.49173	0.43380	0.45131	0.43077	0.42975

Age group	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
0	0.00341	0.00338	0.00325	0.00309	0.00313	0.00327	0.00316	0.00329	0.00324	0.00350	0.00345
01-04	0.00018	0.00017	0.00016	0.00015	0.00015	0.00016	0.00015	0.00014	0.00015	0.00014	0.00014
05-09	0.00008	0.00008	0.00007	0.00007	0.00007	0.00008	0.00007	0.00007	0.00007	0.00005	0.00007
10-14	0.00009	0.00009	0.00008	0.00008	0.00009	0.00008	0.00007	0.00008	0.00008	0.00007	0.00007
15-19	0.00018	0.00019	0.00017	0.00018	0.00017	0.00015	0.00015	0.00017	0.00014	0.00015	0.00014
20-24	0.00024	0.00025	0.00024	0.00024	0.00024	0.00021	0.00019	0.00022	0.00022	0.00021	0.00020
25-29	0.00028	0.00031	0.00030	0.00028	0.00028	0.00026	0.00027	0.00025	0.00027	0.00025	0.00026
30-34	0.00044	0.00043	0.00037	0.00038	0.00036	0.00036	0.00033	0.00034	0.00036	0.00034	0.00034
35-39	0.00070	0.00066	0.00063	0.00061	0.00059	0.00058	0.00058	0.00055	0.00053	0.00055	0.00053
40-44	0.00115	0.00114	0.00108	0.00103	0.00097	0.00098	0.00096	0.00093	0.00087	0.00087	0.00094
45-49	0.00185	0.00183	0.00177	0.00171	0.00168	0.00162	0.00159	0.00157	0.00157	0.00150	0.00150
50-54	0.00277	0.00278	0.00272	0.00266	0.00261	0.00253	0.00247	0.00249	0.00232	0.00241	0.00237
55-59	0.00371	0.00380	0.00375	0.00370	0.00369	0.00367	0.00356	0.00361	0.00361	0.00363	0.00363
60-64	0.00512	0.00502	0.00505	0.00503	0.00500	0.00502	0.00494	0.00497	0.00511	0.00517	0.00509
65-69	0.00739	0.00737	0.00729	0.00700	0.00708	0.00699	0.00686	0.00697	0.00700	0.00709	0.00715
70-74	0.01178	0.01147	0.01122	0.01085	0.01090	0.01108	0.01053	0.01091	0.01079	0.01083	0.01068
75-79	0.02150	0.02064	0.02022	0.01928	0.01972	0.01884	0.01824	0.01876	0.01832	0.01828	0.01807
80-84	0.04238	0.04167	0.04068	0.03858	0.03972	0.03794	0.03609	0.03738	0.03625	0.03567	0.03501
85-89	0.08493	0.08483	0.08173	0.07826	0.08121	0.07921	0.07439	0.07810	0.07560	0.07576	0.07365
90-94	0.16153	0.16043	0.15793	0.15143	0.15747	0.15272	0.14385	0.15303	0.14778	0.15090	0.14611
95-99	0.27836	0.27752	0.27203	0.26412	0.27921	0.27063	0.25471	0.27246	0.26355	0.26839	0.26203
100-104	0.43678	0.43613	0.42953	0.42044	0.44340	0.43202	0.40966	0.43652	0.42410	0.43291	0.42426

Source: Human Mortality Database (http://www.mortality.org/index.html), Age group specific central death rates, Author Calculations

Table 7: Natural logarithm of death rates for female, France 1989-2018

Age	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007
0	-5,037	-5,064	-5,08	-5,166	-5,225	-5,271	-5,41	-5,473	-5,48	-5,461	-5,547	-5,539	-5,519	-5,591	-5,632	-5,624	-5,732	-5,705	-5,77
01-04	-7,929	-8,079	-7,929	-8,181	-8,146	-8,255	-8,181	-8,335	-8,255	-8,517	-8,422	-8,377	-8,335	-8,517	-8,422	-8,68	-8,623	-8,623	-8,68
05-09	-8,623	-8,74	-8,68	-8,68	-8,805	-8,874	-8,948	-8,948	-9,115	-9,028	-9,028	-9,115	-9,316	-9,115	-9,21	-9,316	-9,433	-9,433	-9,316
10-14	-8,568	-8,74	-8,74	-8,805	-8,805	-8,74	-8,74	-8,948	-8,874	-8,874	-8,948	-8,948	-8,948	-9,028	-9,21	-9,21	-9,316	-9,316	-9,316
15-19	-7,958	-7,987	-7,987	-8,016	-7,929	-8,112	-8,047	-8,146	-8,112	-8,217	-8,181	-8,146	-8,146	-8,255	-8,422	-8,422	-8,468	-8,468	-8,568
20-24	-7,642	-7,729	-7,684	-7,729	-7,729	-7,775	-7,752	-7,775	-7,902	-7,824	-7,875	-7,987	-7,958	-7,987	-8,112	-8,181	-8,146	-8,181	-8,294
25-29	-7,524	-7,524	-7,452	-7,47	-7,47	-7,524	-7,524	-7,601	-7,775	-7,752	-7,775	-7,987	-7,875	-7,902	-7,958	-8,047	-8,079	-8,146	-8,079
30-34	-7,25	-7,264	-7,25	-7,195	-7,209	-7,209	-7,209	-7,25	-7,386	-7,452	-7,488	-7,488	-7,581	-7,642	-7,621	-7,684	-7,706	-7,775	-7,824
35-39	-6,878	-6,918	-6,959	-6,869	-6,928	-6,888	-6,898	-6,898	-7,002	-7,002	-7,024	-7,047	-7,047	-7,118	-7,118	-7,169	-7,195	-7,209	-7,293
40-44	-6,496	-6,55	-6,536	-6,578	-6,516	-6,557	-6,516	-6,543	-6,578	-6,586	-6,564	-6,6	-6,593	-6,571	-6,623	-6,653	-6,725	-6,759	-6,777
45-49	-6,124	-6,156	-6,147	-6,152	-6,147	-6,152	-6,161	-6,18	-6,18	-6,215	-6,185	-6,166	-6,171	-6,166	-6,18	-6,235	-6,225	-6,25	-6,287
50-54	-5,735	-5,76	-5,789	-5,806	-5,812	-5,809	-5,836	-5,826	-5,806	-5,833	-5,843	-5,878	-5,889	-5,864	-5,836	-5,864	-5,885	-5,867	-5,903
55-59	-5,371	-5,386	-5,419	-5,44	-5,426	-5,47	-5,507	-5,489	-5,521	-5,509	-5,539	-5,547	-5,534	-5,544	-5,549	-5,578	-5,597	-5,599	-5,594
60-64	-4,989	-5,025	-5,031	-5,061	-5,048	-5,091	-5,093	-5,108	-5,145	-5,15	-5,166	-5,201	-5,205	-5,233	-5,22	-5,244	-5,248	-5,257	-5,267
65-69	-4,557	-4,605	-4,598	-4,622	-4,616	-4,64	-4,639	-4,664	-4,672	-4,704	-4,736	-4,761	-4,784	-4,789	-4,794	-4,881	-4,88	-4,902	-4,932
70-74	-4,021	-4,032	-4,075	-4,094	-4,122	-4,144	-4,143	-4,141	-4,182	-4,183	-4,212	-4,238	-4,253	-4,282	-4,272	-4,364	-4,38	-4,411	-4,438
75-79	-3,396	-3,421	-3,459	-3,49	-3,493	-3,539	-3,531	-3,555	-3,569	-3,59	-3,592	-3,643	-3,666	-3,668	-3,663	-3,785	-3,774	-3,816	-3,848
80-84	-2,719	-2,747	-2,782	-2,821	-2,823	-2,884	-2,883	-2,888	-2,915	-2,918	-2,917	-2,952	-2,985	-2,985	-2,964	-3,097	-3,079	-3,139	-3,162
85-89	-2,097	-2,123	-2,149	-2,181	-2,165	-2,236	-2,217	-2,23	-2,246	-2,257	-2,255	-2,286	-2,312	-2,311	-2,279	-2,416	-2,382	-2,455	-2,459
90-94	-1,548	-1,561	-1,603	-1,628	-1,619	-1,667	-1,657	-1,643	-1,665	-1,667	-1,661	-1,688	-1,703	-1,698	-1,651	-1,801	-1,769	-1,817	-1,828
95-99	-1,086	-1,095	-1,127	-1,144	-1,129	-1,18	-1,153	-1,151	-1,163	-1,164	-1,161	-1,181	-1,19	-1,186	-1,129	-1,273	-1,232	-1,288	-1,293
100-104	-0,709	-0,712	-0,741	-0,751	-0,736	-0,777	-0,75	-0,746	-0,755	-0,754	-0,751	-0,766	-0,77	-0,765	-0,71	-0,835	-0,796	-0,842	-0,845

Age	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
0	-5,681	-5,69	-5,729	-5,78	-5,767	-5,723	-5,757	-5,717	-5,732	-5,655	-5,669
01-04	-8,623	-8,68	-8,74	-8,805	-8,805	-8,74	-8,805	-8,874	-8,805	-8,874	-8,874
05-09	-9,433	-9,433	-9,567	-9,567	-9,567	-9,433	-9,567	-9,567	-9,567	-9,903	-9,567
10-14	-9,316	-9,316	-9,433	-9,433	-9,316	-9,433	-9,567	-9,433	-9,433	-9,567	-9,567
15-19	-8,623	-8,568	-8,68	-8,623	-8,68	-8,805	-8,805	-8,68	-8,874	-8,805	-8,874
20-24	-8,335	-8,294	-8,335	-8,335	-8,335	-8,468	-8,568	-8,422	-8,422	-8,468	-8,517
25-29	-8,181	-8,079	-8,112	-8,181	-8,181	-8,255	-8,217	-8,294	-8,217	-8,294	-8,255
30-34	-7,729	-7,752	-7,902	-7,875	-7,929	-7,929	-8,016	-7,987	-7,929	-7,987	-7,987
35-39	-7,264	-7,323	-7,37	-7,402	-7,435	-7,452	-7,452	-7,506	-7,543	-7,506	-7,543
40-44	-6,768	-6,777	-6,831	-6,878	-6,938	-6,928	-6,949	-6,98	-7,047	-7,047	-6,97
45-49	-6,293	-6,303	-6,337	-6,371	-6,389	-6,425	-6,444	-6,457	-6,457	-6,502	-6,502
50-54	-5,889	-5,885	-5,907	-5,929	-5,948	-5,98	-6,004	-5,995	-6,066	-6,028	-6,045
55-59	-5,597	-5,573	-5,586	-5,599	-5,602	-5,608	-5,638	-5,624	-5,624	-5,619	-5,619
60-64	-5,275	-5,294	-5,288	-5,292	-5,298	-5,294	-5,31	-5,304	-5,277	-5,265	-5,28
65-69	-4,908	-4,91	-4,921	-4,962	-4,95	-4,963	-4,982	-4,966	-4,962	-4,949	-4,941
70-74	-4,441	-4,468	-4,49	-4,524	-4,519	-4,503	-4,554	-4,518	-4,529	-4,525	-4,539
75-79	-3,84	-3,881	-3,901	-3,949	-3,926	-3,972	-4,004	-3,976	-4	-4,002	-4,014
80-84	-3,161	-3,178	-3,202	-3,255	-3,226	-3,272	-3,322	-3,287	-3,317	-3,333	-3,352
85-89	-2,466	-2,467	-2,504	-2,548	-2,511	-2,536	-2,598	-2,55	-2,582	-2,58	-2,608
90-94	-1,823	-1,83	-1,846	-1,888	-1,849	-1,879	-1,939	-1,877	-1,912	-1,891	-1,923
95-99	-1,279	-1,282	-1,302	-1,331	-1,276	-1,307	-1,368	-1,3	-1,334	-1,315	-1,339
100-104	-0,828	-0,83	-0,845	-0,866	-0,813	-0,839	-0,892	-0,829	-0,858	-0,837	-0,857

Table 8: $\widehat{a}_x$ and $\widehat{b}_x$ Estimate, France 1989 to 2018

	$\widehat{a}_x$	$\widehat{a}_x$	$\widehat{b}_x$	$\widehat{b}_x$
Age	Female	Male	Female	Male
0	-5.5408655	-5.3057767	0.05032729	0.051181004
1-4	-8.5036128	-8.2827282	0.06751675	0.063762484
5-9	-9.2298532	-8.9934124	0.07862895	0.072528637
10-14	-9.1296865	-8.7952202	0.07184504	0.071512939
15-19	-8.3866952	7.5227504	0.07388856	0.071781504
20-24	-8.0919730	-6.9731073	0.07035793	0.068010821
25-29	-7.9242695	-6.8703569	0.06903312	0.063253428
30-34	-7.6168842	-6.6993540	0.06884196	0.063314263
35-39	-7.1752566	-6.3814913	0.05450945	0.057363549
40-44	-6.7144810	-5.9597462	0.04077280	0.052120718
45-49	-6.2652704	-5.5013190	0.02705523	0.041300319
50-54	-5.8840131	-5.0756145	0.01852337	0.032392329
55-59	-5.5436203	-4.6784064	0.01725263	0.028752364
60-64	-5.1986561	-4.3055012	0.02293234	0.033636077
65-69	-4.8063948	-3.9466470	0.03388597	0.039543458
70-74	-4.3198888	-3.5424449	0.04261653	0.043131133
75-79	-3.7320290	-3.0727087	0.04846114	0.042297645
80-84	-3.0521721	-2.5233133	0.04672445	0.038323784
85-89	-2.3668435	-1.9693683	0.03839332	0.028905752
90-94	-1.7510200	-1.4536929	0.02780987	0.019678249
95-99	-1.2252217	-1.0237705	0.01919183	0.011706581
100-104	-0.7935702	-0.6767035	0.01143145	0.005502962

Table 9: Estimate of $\widehat{k}_t$, France 1989 to 2018 (Male and Female)

Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Kt	7,5328	7,09668	7,15593	6,44351	6,38351	5,7648	5,01852	4,48839	3,52253	2,73588	2,35365	1,99493	1,91228
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010				
Kt	0,99357	0,55808	-1,0917	-1,3808	-2,0729	-2,7206	-3,1188	-3,0102	-3,7126				
Year	2011	2012	2013	2014	2015	2016	2017	2018					
Kt	-4,5079	-5,1141	-5,3503	-6,3382	-5,5187	-6,6017	-6,6561	-6,7606					

Year	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Kt	6,90437	5,90077	6,12351	5,50303	5,41202	4,60283	4,61485	3,7207	2,91852	2,56448	2,36678	1,72205	1,46369
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010				
Kt	0,98356	0,33444	-1,1235	-1,5556	-2,0248	-2,4686	-2,5589	-2,5612	-3,6627				
Year	2011	2012	2013	2014	2015	2016	2017	2018					
Kt	-4,0653	-4,009	-4,445	-5,3854	-4,8425	-5,0733	-5,8246	-5,535					

Lee-Carter Analysis

Years in fit: 1816 - 2018

Ages in fit: 0 – 100

Male

Percentage variation explained: 94.2%
ERROR MEASURES BASED ON MORTALITY RATES
Averages across ages:

ME	MSE	MPE	MAPE
-0.00077	0.00129	0.05443	0.18376

Averages across years:

IE	ISE	IPE	IAPE
-0.06824	0.09925	5.43827	18.27691

ERROR MEASURES BASED ON LOG MORTALITY RATES
Averages across ages:

ME	MSE	MPE	MAPE
0.01173	0.08508	0.01160	0.09859

Averages across years:

IE	ISE	IPE	IAPE
1.17417	8.45589	1.80454	9.05723

Female

Percentage variation explained: 98.2%
ERROR MEASURES BASED ON MORTALITY RATES
Averages across ages:

ME	MSE	MPE	MAPE
-0.00049	0.00063	0.02815	0.15692

Averages across years:

IE	ISE	IPE	IAPE
-0.04299	0.05089	2.82253	15.64363

ERROR MEASURES BASED ON LOG MORTALITY RATES
Averages across ages:

ME	MSE	MPE	MAPE
-0.00450	0.06727	1.78427	2.01054

Averages across years:

IE	ISE	IPE	IAPE
-0.45110	6.70904	80.44836	113.43024

Total Population

Percentage variation explained: 96.8%
ERROR MEASURES BASED ON MORTALITY RATES
Averages across ages:

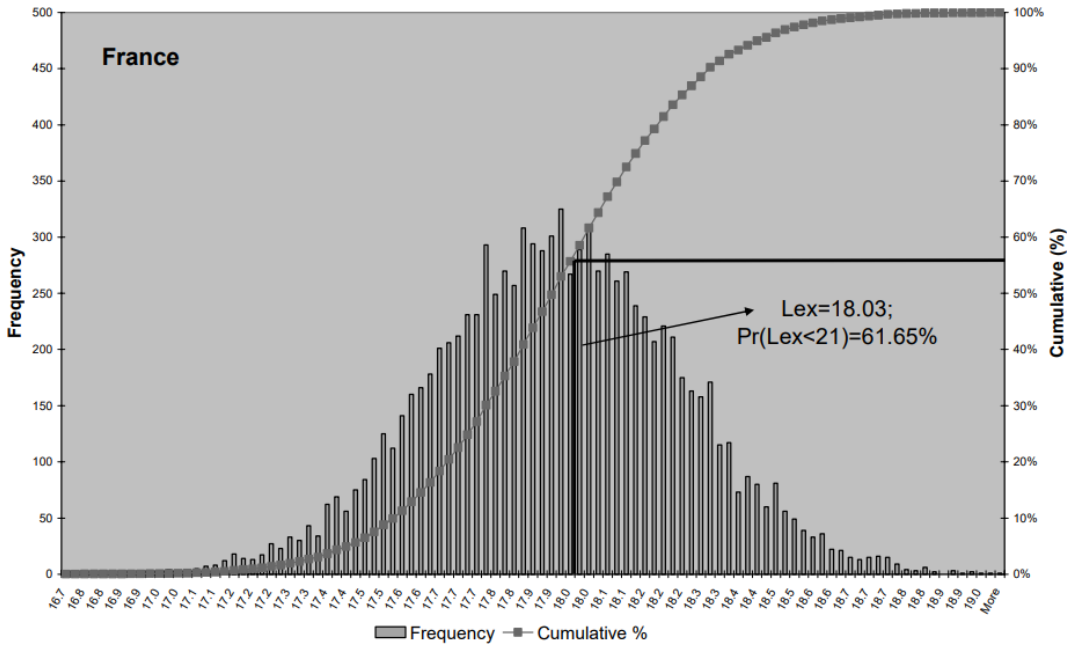
ME	MSE	MPE	MAPE
-0.00037	0.00065	0.03497	0.15236

Averages across years:

IE ISE IPE IAPE
-0.03265 0.05335 3.50174 15.18012
ERROR MEASURES BASED ON LOG MORTALITY RATES
Averages across ages:
ME MSE MPE MAPE
0.00527 0.06002 0.00557 0.05578
Averages across years:
IE ISE IPE IAPE
0.52804 5.98854 0.51277 5.06351

Figure 9: Life Expectancy at age 65 in 2050

10,000 Monte-Carlo simulations



Source: Lex stands for life expectancy at age 65 in 2050, OECD