ORIGINAL ARTICLE

Dynamic modeling of semitrailer trucks equipped by steered wheels

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ABSTRACT – Improving the performance of articulated vehicles is not only by completing the tractor systems, but also by intellectualizing the trailer links (TL), is a complex task. In this regard, the search for and justification of the implementation of controlled TL systems is an important process (Intelligent Trailer). The research has been carried for the purpose of development of the systemic approach from the idea schematization to the practical recommendations. The paper proposes a general approach for creating a comprehensive calculation model that establishes a link between the dynamics of movement of articulated vehicle and the active controlling systems of the semitrailer. The key moments is the developed mathematical model based on matrix technique that facilitates the creation of the universal simulation model in the Simulink environment. The created method with recommended programming product afforded to increase the exactness of modelling of curving way of articulated vehicle on 10 % under the diminishing of time on the calculation about 50 % in comparison with classical method. At the expense of experimental investigation there was confirmed appropriateness of proposed imitation model of movement dynamics of articulated vehicle, general relative mistake in comparison with theoretical investigations was not over than 5%. Additional combining to semitrailer chassis the active turning control system diminishes the size line of constant movement of articulated vehicle in the circle on 14,5% in comparison with semitrailer without this system, which is also positive from the point of safety of articulated vehicle maneuverability.

INTRODUCTION

The main direction of the modern automotive industry lies in the concept of vehicle's full automatic adaptation to traffic conditions with minimum influences of human factors. The purpose of commercial vehicles' intellectualization is to ensure their exceptional efficiency on the background of the exploitative cost reduction and the maintenance of a high-level safety [1–3]. This implies the continuous implementation of both individual automotive systems and their integration into the complex on-board units. Modern trucks enhance the productivity by increasing the load capacity, which is often provided by the number of trailed units [4] or by the length and the number of axles in case of a single vehicle (multi-axles heavy vehicle). The increase in the number of axes allows to reduce the destroying vertical load on the road surface but has a reverse effect of the increase of the turning resistance and the sideslip rising heavy vehicle that leads to the tires' wear intensification. Applying steered wheels makes it possible to solve the problems of the heavy vehicles' maneuverability and provides certain operational advantages thim comprehensively. The wheels' control system can be included in the concept of modularity of steered links, which will ensure handling of a heavy vehicle. Besides, the control system of steered wheels can be integrated in other mechatronic systems and perform complementary functions to regulate the vehicle's stability by the active control of steered wheels [5]. Analysis showed that unlike the European transport technology EuroCombi [4], which often uses forced wheels control on trailed units Figure 1(a) and 1(b), North American articulated vehicles Figure 1(c) and 1(d) remain almost standard and not used supplementary steer from wheels of trailed units.
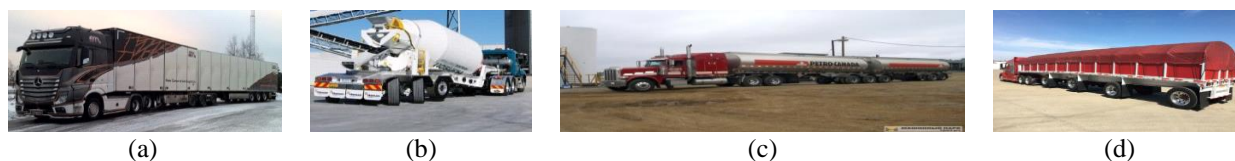

| (a) | (b) | (c) | (d) |

Figure 1. Variants of multi-unit articulated vehicles

The emergence of microprocessor technologies has made it possible to improve the maneuverability, controllability and vehicle's stability by improving the operating processes of controlling their units and systems [6]. The main points of the issue were discussed in the studies [5–12]. But, at the same time, there is not systematic approach to the creation algorithm of a turn control system, of a turn system structure, of selection rules for components and their simulation models, which could be integrated into the general task of the dynamics of a heavy vehicle. Accordingly, the determination issue of the rational parameters of the turn control system components, the development of control algorithms corresponding as much as possible to the industrial design methodology, which is the most interesting for the applied engineering, needs to be studied. The researches of many scientists are often restricted by the consideration of "bicycle" dynamic model of articulated vehicles and mathematical models of state-space type, offering a general concept of operation for the turn control system [10]. Obviously, the companies producing the systems similar to the "steer-by-wire" [6] directly keep the information about the design and algorithms elaboration not open to public access.

MODELLING ISSUES

Turn Control Systems

There are two ways to improve the maneuverability of a heavy vehicles. The first way is to change the direction of the wheels' velocity vectors relative to the longitudinal axis of a vehicle. These vectors are in the plane of the reference surface and coincide with the planes of the corresponding wheels. Either the turn control system actuator as a self-aligning tool, or an automatic control with forced turn drive of wheels can be used to minimize the external resistance of the trailed unit's wheels. The second way is to change the rotational mode of board wheels, using the traction and braking forces for active axils or only the braking forces for passive ones [6, 11]. Also, the combinations of these methods are possible.

Any turn control system based on the first way of maneuverability improvement, except for the self-aligning, consists of four parts: the element of initial parameter's setting (EIPS), the drive, the actuating element (AE), and the safety element (SE). The self-aligning turn system consists of three parts: AE, SE, and drive. Furthermore, the SE is turned on forcibly by a driver when a truck is maneuvering on the reverse gear. In comparison with the automatic turn control system, the self-aligning one is less complicated in exploitation and does not require an additional energy when the system is operating, but this system's worth does not significantly differ from automatic one. This can be explained by the necessity to include the mandatory safety element, the drive of which is not much different from the drive of automatic turn control system. The functionality of turn control systems with self-aligning AE considerably concedes to automatic turn control systems because of impossibility to arrange trailed units' forced turn control when an articulated vehicle is maneuvering in both straight and reverse directions. Such turn control systems do not allow a truck to carry out a turnabout by the smallest radius with minimal slip of trailed unit's tires and to affect the movement stability of an heavy vehicle.

According to the recommendations [6], in case of the use of an additional energy source, the automatic turn control systems can be of direct or indirect action. The automatic turn control system (ATCS) of direct action carries out the turn of an actuator due to the efforts that arise during the folding of an articulated vehicle's parts (with a constant or variable ratio of a drive). An indirect ATCS has an additional power source to amplify the control signal, a command part for transmitting the original signal to the power unit, an electronic or microprocessor control element (EATCS). Thus, for EATCS, the fifth part - an electronic or microprocessor control unit (ECU) is added to the four ones indicated previously.

Thus, for example, when selecting the angles of turn of the drive of the steering control system for semitrailer, which numerical values depend on the location of steered axels in relation to the truck instantaneous velocity center (IVC) and features of axles' design. Consider the technique for determining the coordinates of truck's IVC location. Analysis of studies on this subject revealed the lack of not only a methodology but also of any recommendations for addressing this issue at early stage. Thus, some authors of studies in this field propose to determine the turn angles ratio for the actuators depending on the drive ratio as functions of geometric, inertial and operational parameters of a truck, or on the mode parameter (the ratio between the turning velocity of steered wheels of articulated vehicle leading unit and its linear velocity). Cebon D. in his work [11] proposed the formation of a control algorithm for the actuator of the trailed unit turn control system to be executed by conditions Figure 2(a), pos. 1, when the trajectory of the truck unit joint center (trajectory set point - TSP) is replicated by the center of the trailed unit rear part (trajectory trace point - TTP).

For determine the maximum values of the actuator turn angles accept restriction, the internal overall dimension radius turn should be no more 5.3 m, as regulated by DIRECTIVE 2002/7/EU [13]. The determination preciseness of the instantaneous velocity center (IVC) location will be evaluated by other restriction of this directive - the external overall dimension radius R_{de} = 12.5 m. Thus, for the given articulated vehicle, the requirements of DIRECTIVE 2002/7/EU are fulfilled in consequence of the fixated points' locations as shown in Figure 2(a).

For the study of flat curvilinear motion, the object of study was a two-link truck, which consists of: tractor DAF XF95.430 (4 × 2.2) and three-axle semi-trailer Fliegl SDS-350 with a capacity of 27.5 tons. The maximum angles of rotation of the inner wheels of the first axis of the semi-trailer – 15˚, the second axis of the semi-trailer – 26˚, the third axis of the semi-trailer – 36˚.
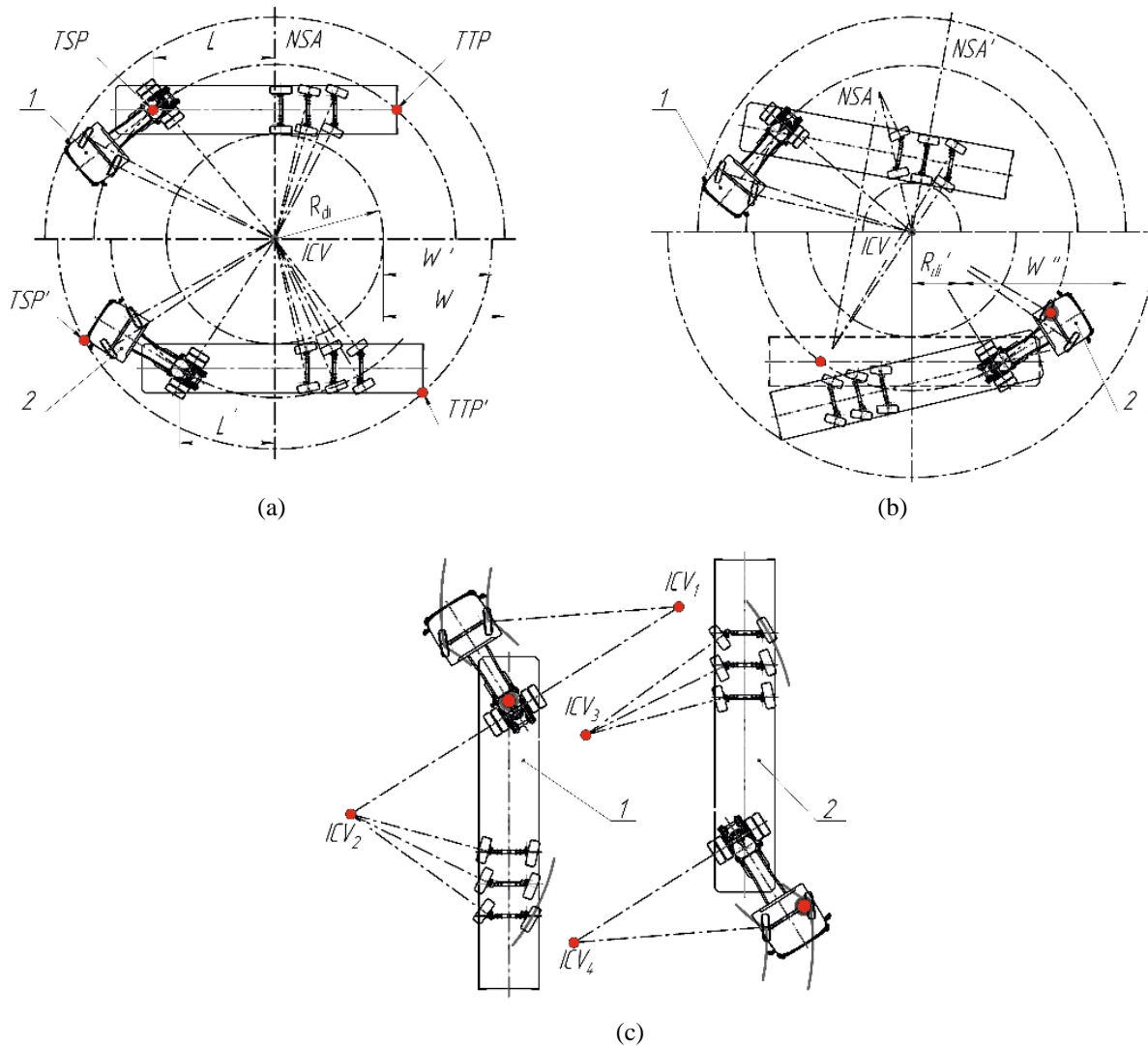
(a)

(b)

(c)

Figure 2. Conformity of kinematic properties of an articulated vehicle with trailed unit's steered   wheels and the control source: (a) Determination of the maximum values of the turning angles of semitrailer's wheels according to the recommendations of D. Cebon (pos.1) [5] and to the proposed method (pos. 2), (b) Results of orientation of articulated units, when the setting parameter element is placed according to the variant inside the tractor's steering system, for the "turn on the spot" mode (pos. 1) and for the normal steered mode (pos. 2) and (c) Comparison of the truck reactions on a sharp direction change of the tractor's front wheels, when the setting parameter element is placed in the fifth wheel (pos. 1) and in the tractor's steering system (pos. 2). Abbreviations: ICV - Instantaneous Center of Velocity; NSA - Neutral Steerability Axis; TSP - trajectory setting point; TTP - trajectory tracking point; R_{di} - inner overall dimension radius; W - path width

Element of Initial Parameter's Setting

The next important issue concerns the element of initial parameter's setting (EIPS). Let's consider the possible options and arguments for assignment of the control signal source in Figure 2(b) and 2(c). If the EIPS is installed in the tractor's steering system components, the following situations are possible. Let's determine here that such a placement of EIPS affects the organization of turning on the spot positively, as shown in Figure 2(b), pos. 1, which can be made as an option of the electronic control. In this case, the wheels of semitrailer's first and third axles turn in different directions synchronously with the wheels of tractor's steering axle. Obviously, the truck needs the least space for the maneuver. The turn of semitrailer wheels in different directions creates an increased resistance for longitudinal motion, which leads to the truck intensive folding. Note, however, that the axels' steering trapezoids optimized for another turning neutral (NSA) do not match fully the conditions of neutral passing through the middle axle. However, some lateral slip takes place, but its impact is much lower than from a classic semitrailer's turn on the spot.

If the turning speed of the tractor's steered front wheels is quite high in relation to a low tractor's yaw rate at the turn entry, the turning of semitrailer wheels does not track the truck's articulation angle in Figure 2(b), position 2. From the beginning of the tractor's curvilinear motion, the semitrailer rear part, instead of to be retracted into the turn, starts to move by enlarged trajectory, significantly increasing the swept path and the outer overall dimension radius that exceeds the tractor's overall dimension radius. During the continuing tractor's stable curvilinear motion, the semitrailer shifts gradually to the ICV, bringing the outer overall dimension radius closer to a smaller value. Such a situation is

characterized by a delayed reaction of the truck folding with a loss of the trailed unit's trajectory stability. The semitrailer contour is shown by the dash line in Figure 2(b), position 2, namely its correct position relatively to the tractor, which corresponds to the condition when the TTP is following the TSP. Obviously, with such an organization of the EIPS, the control system of turning angles of semitrailer's steered wheels should be coordinated with the truck speed and the steering wheel rotational speed. That would require the more complicated control algorithm, but even after the implementation the oversteer od trailed units might appear due to the increased sensitivity of the tractor steering signal. This, at times, may lead to certain confusions, as shown in Figure 2(c), position 2, that will be explained below.

Another situation when the EIPS is only installed into the semitrailer's kingpin, which is joined with tractor's fifth wheel. If a truck is in folded state at low speed and the tractor wheels are turned to the maximum angles in an opposite direction with respect to the previous one, the location of the tractor's ICV is changed relatively to the longitudinal axis, and the semitrailer wheels remain turned at angles corresponding to the truck articulation angle in Figure 2(c), position 1. In this situation, under the conditions of the tractor movement in a prescribed direction, the semitrailer rear part's offset to a new ICV will appear, as well as the reduction of the internal overall dimension radius. Thus, the delay of a signal formed in the tractor fifth wheel relatively to the turning velocity of the tractor steered wheels affects positively the handling and stability of an articulated vehicle.

In the case of EIPS location in the tractor's steering drive, under the same circumstances in Figure 2 (c), position 2, the truck will be gradually unfolding in motion, but the semitrailer movement will have the lateral bias tendency according to the predefined direction of the semitrailer wheels. This does not guarantee the predictability and stability of the semitrailer motion, moreover, obviously, the required path width is increased, and the situation could be aligned only by the fast rotation of steering wheel into the neutral position.

Consequently, for both versions of the EIPS the semitrailer's control system operation should be blocked when a trailed unit moves straight. This function can be managed directly by a driver in the manual mode or by additionally installed EIPS (one - in a fifth wheel's kingpin, another - in the steering knuckle of tractor's steering axle). In this case, for given articulated vehicle, the variant of EIPS installation in the fifth wheel's kingpin is chosen, and the remote-control option to lock the SACS operation system should be provided in the control algorithm. Such a choice will preserve the modularity principle of an articulated vehicle, especially in the case of use of B-trailers as modular units.

Actuating Element

Next, let's move to considering the question of an actuating element (AE) expedient choice for the trailed unit's turn control system. There are several types of AE organization: by a dolly in Figure 3(a), by axles in Figure 3(b), by a combination of (a) and (b) options in Figure 3(c), and by turning wheels in Figure 3(d).

As it can be seen in Figure 3, under equal conditions of truck folding (articulation angle - $\psi = 32°$) there is a difference in the parameters of the loading height of a semitrailer dray and the space for the AE functioning.
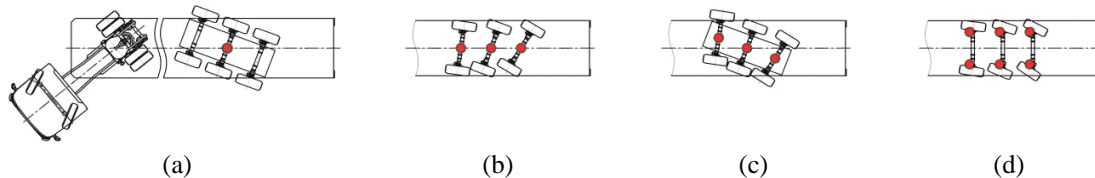

| (a) | (b) | (c) | (d) |

Figure 3. Types of actuating elements to organize the trailed unit's turn control system

Thus, the rotation of a dolly in Figure 3(a) or a combined variant in Figure 3(c) requires an increased loading height of the semitrailer cargo platform due to the placement of the dolly's frame with a rotary mechanism under semitrailer's frame. Also, a significant increase in the swept path occurs during the semitrailer turning, but the specified AE does not limit the space between longerons of the semitrailer's bearing frame. The axles rotation in Figure 3(b) requires an essential frame's local narrowing which increases the cargo platform's loading height. However, the most important drawback of a dolly or axles use as an AE is the kinematic mismatch of wheels movement, i.e. it is not possible to exclude completely their transverse slip within the truck's turning. All these drawbacks are absent for the turning wheels option in Figure 3(d), but there is a need for a steering trapezoid and a turn control system's safety element (SE) for each controlled axle separately; or an individual drive for each steered wheel could be provided. The penultimate of these variants in Figure 3(c) has a greater weight and complicated control, therefore, it is expediently to choose as an AE the controlled axle with turning wheels jointed mechanically by a steering trapezoid. The maximum wheels' turning angle depends on the arrangement of the steering axle, suspension and braking mechanisms and amounts around 60° (using Pendelk swivel axles) or 40° (using beam-axles).

Depending on the AE's turn direction relatively to the folding direction of an articulated vehicle's trailed part, the turn control systems are divided into the following types: straight control in Figure 3(a), 3(b) and 3(d), inverse control - the dolly's front axle or dual control in Figure 3(c).

SACS actuator can be: mechanical, hydraulic, pneumatic, electric or combined system. For today, the specified drive types have not lost their relevance. For instance, the Schmitz Cargobull company in 2011 launched the S.CS City single-axle semitrailer equipped by the SACS with mechanical drive from the company Tridec [4]. Such companies as VSE [3],

Trackaxle and others have recently significantly increased the production of SACS with mechanical or electromechanical drive, which work is controlled by a microprocessor element. In addition, pneumatic and electric drives in their pure form are inappropriate. The use of a pneumatic drive leads to an increase in the response time (5-10 times more than in a hydraulic drive), as well as to a need for large size pneumatic devices. Restrictions of an electric drive use are related to a need for using the high power servo-motors to overcome the AE's turn resistance, which affects the SACS cost considerably.

PROPOSED THEORETICAL CONTROL ALGORITHM

To compose the computation model of an articulated vehicle's curvilinear motion, as well as to define the rational parameters of the steering trapezoid components, the interrelation determination between the theoretically needed turning angles of semitrailer wheels of corresponding boards and the articulation angle is required. To accomplish this, the distribution of the Ackermann's angles (the ratio between the turnings of steered wheels) is determined as a function of the truck articulation angle and the semitrailer turnability (neutral steer). Consider the truck's turnability phenomenon in its entirety (Figure 4).
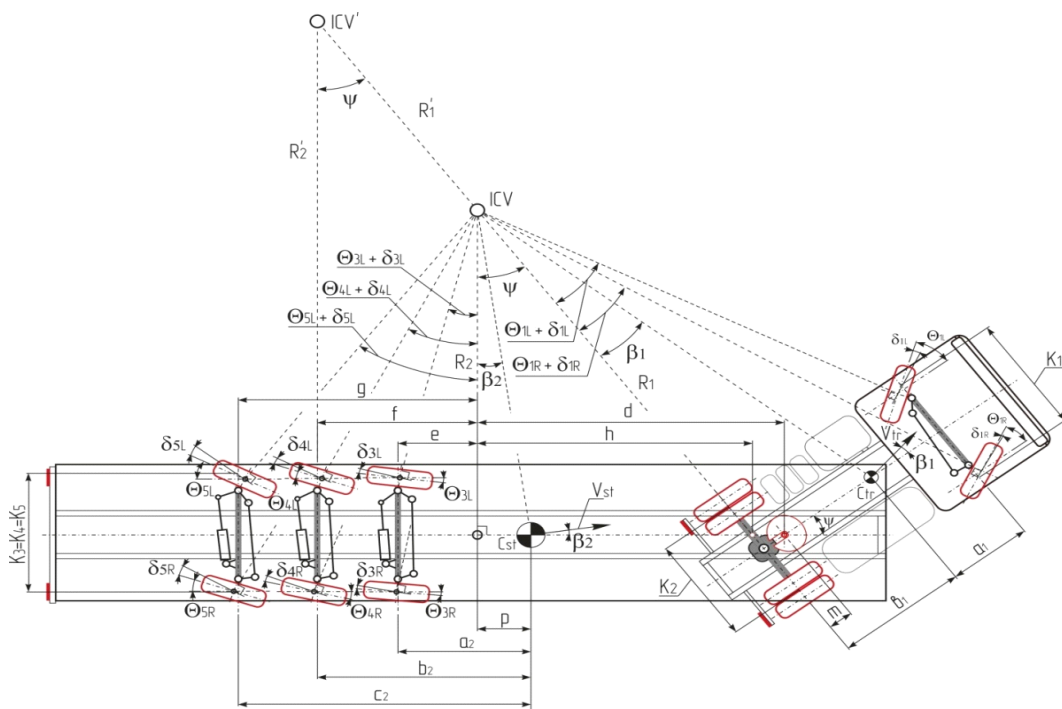

Figure 4. Scheme for determination of the theoretically required turning angles of the semitrailer wheels depending on the truck articulation angle

The minimum turning radii of the truck units are formed as perpendiculars, which are dropped from a point of the instantaneous center of velocities (ISV') on the units' longitudinal axes. As a rule, in the case of classical semitrailer design, the radius R'_2 passes close to the semitrailer's middle axle. If during the truck's turn the stable motion without sideslip is being provided, then the minimum radius R'_1 passes through the tractor's rear axle, which characterizes the neutral truck turnability.

If the semitrailer wheels were oriented according to Ackermann's steer at the same truck articulation angle ψ, the ISV' point would migrate along the line R'_1 and could reduce R'_1 to R_1 and R'_2 to R_2 (transfer to the point ICV). In this case, the radius R_2 will be located closer to the semitrailer mass center, which will provide the semitrailer oversteer.

The well-known control schemes of semitrailer wheels - "steer-by-wire" - use a rotary encoder in Figure 2(a), position 1, point TSP, converting the truck articulation angle into electric signal, which is installed between a kingpin and a fifth wheel. Thus, the mathematical model of an algorithm that distributes the turning angles of the semitrailer's steered wheels, is derived as follows:

The radius R_2 (Figure 4) can be defined as:

$$R_2 = h \times \cot(\psi) = d \times \cot(\psi) - \frac{m}{\sin(\psi)} \tag{1}$$

where h is the base of a rectangular triangle with sides formed by the neutral steer axes crossing in the point of the instant turning center ICV, d is the distance from the coupling point to the perpendicular of the semitrailer's neutral steer axis.

If to consider the tires' slip angles equal or close to zero, then the ratio between turning angles of semitrailer wheels of any axle

$$Cot(\Theta_{nL}) = \frac{R_2 - 0.5 \times K_n}{s_n} \tag{2}$$

$$Cot(\Theta_{nR}) = \frac{R_2 + 0.5 \times K_n}{s_n} \tag{3}$$

where $n = 1, 2, 3$ - the semitrailer axle number; Θ_{nL}, Θ_{nR} - turning angles of n axle wheels, respectively, left and right; $s_n = e, f, g$ - the distance between the radius-perpendicular and the semitrailer axle.

$$Cot(\Theta_{nL}) - Cot(\Theta_{nR}) = \frac{(R_2 - 0.5 \times K_n) - (R_2 + 0.5 \times K_n)}{s_n} = -\frac{K_n}{s_n} \tag{4}$$

In its turn

$$R_2 = s_n \times Cot(\Theta_{nL}) + 0.5 \times K_n \tag{5}$$

Equating the expressions (2) and (4), obtain

$$d \times Cot(\psi) - \frac{m}{sin(\psi)} = s_n \times Cot(\Theta_{nL}) + 0.5 \times K_n \tag{6}$$

Then

$$Cot(\Theta_{nL}) = \frac{d \times Cot(\psi) \times Sin(\psi) - m - 0.5 \times K_n \times Sin(\psi)}{s_n \times Sin(\psi)} \tag{7}$$

Dependence of inner angles from the truck articulation angle

$$\Theta_{nL} = arccot\left(\frac{d \times Cot(\psi) \times Sin(\psi) - m - 0.5 \times K_n \times Sin(\psi)}{s_n \times Sin(\psi)}\right) \tag{8}$$

Using (4), obtain the desired dependence of the outer angles from the inner ones

$$\Theta_{nR} = arccot\left(Ctg(\Theta_{nL}) + \frac{K_n}{s_n}\right) \tag{9}$$

The obtained dependencies are used in the developed dynamic model of a two-unit vehicle.

Description of motion mathematical model

Consider briefly the process of motion equations obtaining for a truck 2D-dynamic model [14]. The presented approach differs from the generally accepted use of the state-space equations of the bicycle model for articulated vehicles. The basic idea is in consideration of full number of tire points contacting with a reference surface, and in the use of matrix technique representing model components with the minimum volume of needed information. This is optimal for fast calculations provided by MATLAB/Simulink environment [15, 16].

Two-unit articulated vehicle dynamic model

According to the principle of theoretical mechanics, consider a two-unit vehicle (Figure 5(a)) moving in a global coordinate system XOY. The vehicle units are represented by massless rods pivotally jointed in point P. The distributed masses m_{TR}, m_{ST} and inertias I_{TR}, I_{ST} are concentrated in the physical centers of the units. The master unit is a tractor TR, which mass center is C, is connected to a semitrailer ST, which mass center is C'.

The masses are located at distances e_{TR}, e_{ST} relatively to the joint point P. In the process of truck folding the tractor and semitrailer longitudinal axes form an articulation angle ψ.
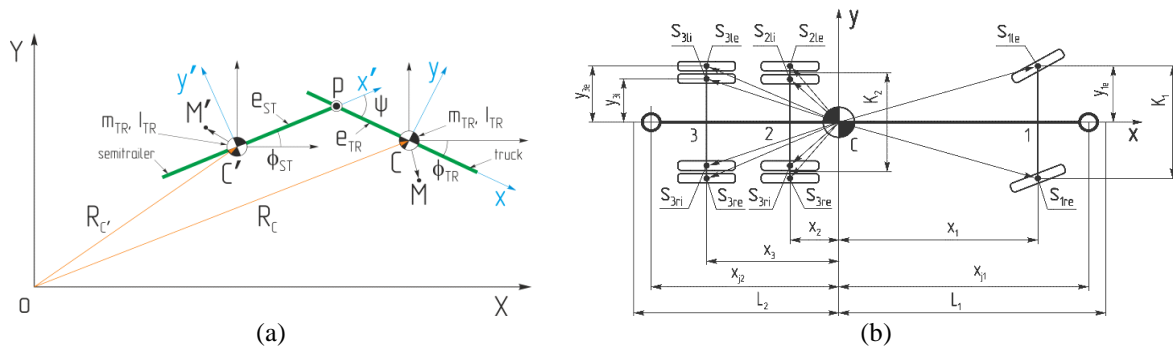
Figure 5. Schematization of articulated vehicle: (a) Dynamic model and (b) Geometric parameters of articulated unit

Vehicle position at any time is sufficiently characterized by the combination of radius vectors (R_C and $R_{C'}$) from the origin of global coordinates to the units' mass centers. However, it is more convenient to use the generalized coordinates by introducing the mobile sub-global coordinate systems at the mass centers. The axes of the sub-global systems are located in parallel to the global system axes.

Let's place the local coordinate systems xCy and $x'C'y'$ into the sub-global coordinates, relatively to which the kinematic parameters will be calculated. At any point M or M' also the sub-local coordinate systems can be additionally used, in which the vectors of acting loads and forces are specified. Thus, the following degrees of freedom reduced to the vector R (1) of the dynamic model are determined: X_{TR}, Y_{TR} are the tractor mass center displacements along axis OX and OY, respectively; ϕ_{TR} is the tractor turning angle relatively to the vertical axis passing through the tractor mass center; ϕ_{ST} is the semitrailer turning angle relatively to the vertical axis passing through its mass center. The vector V_G represents the velocities (derivatives) of the vector R in the global coordinate system according to the degrees of freedom. The vector V expresses the basic unknowns in the tractor's local mobile coordinate system. The vector J characterizes the relative acceleration in the tractor local coordinate system. W is the vector consisting of angular velocities ω_{TR}, ω_{ST}. The vector F represents the generalized forces acting along the directions of the generalized coordinates and reduced to the tractor local coordinate system.

$$R = \begin{pmatrix} X_{TR} \\ Y_{TR} \\ \phi_{TR} \\ \phi_{ST} \end{pmatrix}; \; V_G = \begin{pmatrix} V_{TR_x} \\ V_{TR_y} \\ \omega_{TR} \\ \omega_{ST} \end{pmatrix}; \; V = \begin{pmatrix} V_{TR_x} \\ V_{TR_y} \\ \omega_{TR} \\ \omega_{ST} \end{pmatrix}; \; J = \begin{pmatrix} \dot{V}_{TR_x} \\ \dot{V}_{TR_y} \\ \dot{\omega}_{TR} \\ \dot{\omega}_{ST} \end{pmatrix}; \; W = \begin{pmatrix} \omega_{TR} \\ \omega_{TR} \\ \omega_{TR} \\ \omega_{ST} \end{pmatrix}; \; F = \begin{pmatrix} F_x^{(e)} \\ F_y^{(e)} \\ M_{TR_P} \\ M_{ST_P} \end{pmatrix}. \tag{10}$$

To obtain the motion equations, the fundamental statements of theoretical mechanics about the changes of momentum and angular momentum are used.

The impulse of the two-mass model $\vec{Q}_\Sigma$ is defined as

$$\vec{Q}_\Sigma = \vec{Q}_{TR} + \vec{Q}_{ST} = m_{TR} \times \vec{V}_{TR} + m_{ST} \times \vec{V}_{ST}. \tag{11}$$

where $\vec{Q}_{TR}$, $\vec{Q}_{ST}$, $\vec{V}_{TR}$, $\vec{V}_{ST}$ - are momentum vectors and absolute velocity vectors for the tractor and semitrailer mass centers, respectively.

The change of the mechanical system momentum is caused by the action of the resultant of external forces, as follows

$$\frac{d\vec{Q}_\Sigma}{dt} = \frac{d\vec{Q}_{TR}}{dt} + \frac{d\vec{Q}_{ST}}{dt} = m_{TR} \times \frac{d\vec{V}_{TR}}{dt} + m_{ST} \times \frac{d\vec{V}_{ST}}{dt} = \sum_k \vec{F}_{TR_k}^{(e)} + \sum_k \vec{F}_{ST_k}^{(e)}. \tag{12}$$

where $\vec{F}_{TR_k}^{(e)}$, $\vec{F}_{ST_k}^{(e)}$ - are vectors of external forces acting at any point k of tractor or semitrailer, respectively.

The first couple of scalar equations are derived as projections of the vector equation (Eq. (12)) on the axes of tractor's local coordinate system. To account for the rotational motion, consider the angular momentum change for each truck point M_k relatively to the joint point P.

Then

$$\begin{cases} \sum_k \overrightarrow{PM}_k \times m_{TR_k} \times \dfrac{d\vec{V}_{TR_k}}{dt} = \sum_k \overrightarrow{PM}_k \times \vec{F}_{TR_k}^{(e)} + \underbrace{\overrightarrow{PM}_k \times \vec{F}_P}_{\vec{0}} \, ; \\[3mm] \sum_k \overrightarrow{PM}_k \times m_{ST_k} \times \dfrac{d\vec{V}_{ST_k}}{dt} = \sum_k \overrightarrow{PM}_k \times \vec{F}_{ST_k}^{(e)} + \underbrace{\overrightarrow{PM}_k \times \vec{F}'_P}_{\vec{0}} \, , \end{cases} \qquad (13)$$

where m_{TR_k}, m_{ST_k}, $\vec{V}_{TR_k}$, $\vec{V}_{ST_k}$ - are masses and velocities of any point M_k of tractor or semitrailer, respectively.

The internal forces $\vec{F}_P$, $\vec{F}'_P$ in the point P caused by the interaction of tractor and semitrailer, respectively, produce no moments, unlike the external forces. Then, the generalized moments acting on the tractor and semitrailer, respectively, are given by

$$M_{TR_P} = \sum_k \vec{M}_{TR_P} = \sum_k \overrightarrow{PM}_k \times \vec{F}_{TR_k}^{(e)} \, . \qquad (14)$$

$$M_{ST_P} = \sum_k \vec{M}_{ST_P} = \sum_k \overrightarrow{PM}_k \times \vec{F}_{ST_k}^{(e)} \, . \qquad (15)$$

Thus, the second couple of scalar equations of rotational dynamics has been formed. Through simplification and transformation, the system of the second-order differential equations in vector-matrix format can be written as

$$J = M^{-1} \cdot (F + S) \, . \qquad (16)$$

where

$$S = (L + T) \times diag(W) \times V \, . \qquad (17)$$

or then

$$J = M^{-1} \times (F + (L + T) \times diag(W) \times V) \, . \qquad (18)$$

Introduce auxiliary matrices for a more compact representation of Eqs. (17) and (18), then

$$Z_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}; \ E_{2U} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}; \ E_{2L} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}; \ D = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \qquad (19)$$

Then

$$L = \begin{pmatrix} (m_{TR} + m_{ST}) \times (D - D^T) & -m_{ST} \times e_{TR} \times E_{2U} \\ -m_{TR} \times e_{TR} \times E_{2U} & Z_2 \end{pmatrix}, \qquad (20)$$

$$T = m_{ST} \times e_{ST} \times \left(\begin{pmatrix} Z_2 & -D \\ D^T & Z_2 \end{pmatrix} \times Cos(\psi) + \begin{pmatrix} Z_2 & -E_{2L} \\ E_{2L} & -e_{TR} \cdot D^T \end{pmatrix} \times Sin(\psi) \right). \qquad (21)$$

It is more convenient to describe the matrix of inertia coefficients M as a sum of the constant part M_I and the variable trigonometric part M_T

$$M = M_I + M_T \, . \qquad (22)$$

Where,

$$M_I = \begin{pmatrix} (m_{TR} + m_{ST}) \times (E_{2U} + E_{2L}) & -m_{ST} \times e_{TR} \times D^T \\ m_{TR} \times e_{TR} \times D & I_{TR} \times E_{2U} + E_{2L} \times (I_{ST} + m_{ST} \times e_{ST}^2) \end{pmatrix}, \qquad (23)$$

$$M_T = m_{ST} \times e_{ST} \times \left(\begin{pmatrix} Z_2 & D \\ D^T & Z_2 \end{pmatrix} \times Sin(\psi) + \begin{pmatrix} Z_2 & -E_{2L} \\ -E_{2L} & e_{TR} \cdot D^T \end{pmatrix} \times Cos(\psi) \right). \qquad (24)$$

The transitions from wheel's sub-local and local coordinate systems to the global coordinate system (which have a rotation angle ξ) are achieved through the following transition matrix

$$C\left(\xi\right) = \begin{pmatrix} Cos\left(\xi\right) & -Sin\left(\xi\right) \\ Sin\left(\xi\right) & Cos\left(\xi\right) \end{pmatrix}. \tag{25}$$

To transfer the velocities of the tractor local coordinate system to the global coordinate system, the following matrix is formed

$$N\left(\phi_{TR}\right) = \begin{pmatrix} C\left(\phi_{TR}\right) & Z_2 \\ Z_2 & E_{2U} + E_{2L} \end{pmatrix}. \tag{26}$$

Finally, using Eqs. (20) - (26), the displacements are obtained using double integration, as follows

$$V = \int J \cdot dt \ ; \ V_G = N\left(\phi_{TR}\right) \cdot V \ ; \ R = \int V_G \cdot dt \ . \tag{27}$$

The integration of accelerations and velocities by Eqs. (26) and (27) is carried out numerically in Simulink [15], using the solution by the fixed step Runge-Kutte method.

Geometry of articulated vehicle's units

In view of the data processing using the matrix computations, it is expediently that the representation of input and current vehicle parameters also can be organized in matrix form. Create auxiliary matrices based on a pattern below

$$w_n = \begin{pmatrix} 1 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 1 \end{pmatrix}. \tag{28}$$

$$f_n = \begin{pmatrix} f_{n_1}^{(el)} & f_{n_1}^{(il)} & f_{n_1}^{(ir)} & f_{n_1}^{(er)} \\ \vdots & \vdots & \vdots & \vdots \\ f_{n_k}^{(el)} & f_{n_k}^{(il)} & f_{n_k}^{(ir)} & f_{n_k}^{(er)} \end{pmatrix}. \tag{29}$$

where w_n - is tire matrix of unit n, matrix of the tires' location according to axles; f_n - is a matrix of any parameter of a truck unit n, which elements correspond to the structure of tire matrix w_n, where each row reflects an axle k, and the upper combined index in parentheses means vehicle's board (l - left, r - right) and location (e - external, i - internal).

Thus, according to the universal scheme of an articulated vehicle's unit in Figure 5.b, the following matrices are formed: x_n, y_n - the longitudinal and transverse coordinates of n-th unit tires' contact points, respectively. To obtain these values the design parameters are used: K_{nk} - track of k-th axle of n-th unit; location x_{jp} of the mass center of n-th unit relatively to the coupling point P; components of the longitudinal overall dimensions L_1, L_2 relative to the mass center.

Kinematics of articulated vehicle's units

The orientation of the n-th unit's wheels relatively to its longitudinal axis according to (28) and (29) is described by the matrix of turning angles Θ_n (see Figure 4). At the curvilinear motion in each contact point the absolute velocity vector shapes an angle with the unit's longitudinal axis. The set of these n-th unit's angles forms the matrix Ξ_n. The determination of the angles Ξ_n can be performed according to the following vectorized relation

$$\Xi_n = arctg\left(\frac{V_{y_n} + x_n \times \omega_n}{V_{x_n} - y_n \times \omega_n}\right). \tag{30}$$

where V_{xn}, V_{yn} - are the longitudinal and lateral projections of the absolute velocity of the n-th unit's mass center in its local coordinate system, respectively; ω_n - is the angular velocity of n-th articulated vehicle's unit.

Introduce the matrices of slip angles δ_n. In general, these matrices are expressed as

$$\delta_n = \Xi_n - \Theta_n \ . \tag{31}$$

The force factors at the tire contact points in case of a flat model are functions of the slip angles, tire slipping, and vertical loads. Note that the force balance modeling and simulation of tire-road contact is not demonstrated here due to sufficient prevalence of such an information [17]; the simulation of tire's forces acting in the contact point with the road surface corresponds to the simplified Pacejka's formulas for the case of tire's separated slip.

The conversion of the generalized vehicle models from mathematics to animation is organized in several stages. First, according to vehicle geometric data an animation model is created that visually reflects a final solution of dynamics equations and contributes to verifying the calculations. Second, a steering drive and trapezoid particularly is calculated and rationalized to provide accurate ratio of the turning angles of the steered wheels (Ackerman's angles). Third, the masses and axial loads are distributed based on the weighting parameters. Finally, multiple calculations with any desired initial conditions are performed.

EXPERIMENTAL STUDY

The purpose of experiment fulfilment is to test the adequacy of the mathematical model describing an heavy vehicle plane motion by the computer simulation. The data acquisition of the heavy vehicle curvilinear motion involves the determination of the kinematic parameters of corresponding vehicle units while the articulation angle between truck and semitrailer is changing. For the experimental study of steerability, the DAF FT XF-95.430 + Fliegl SDS-350 articulated vehicle was chosen.

In view of the controllability of composed simulation model, it was desirable to receive the following signals during experimental tests: the steering wheel angle, the current truck longitudinal velocity, the truck articulation angle, and some kinematic parameters to expand the experiment for other studies. The choice regarding necessary sensors and scheme drawings of their location on the truck (Figure 6(a)) was performed based on the relevant source analysis [18].

Measuring and recording equipment

Wheel turning angle. The problem of the steering wheel's angle measurement consists of three moments: the variable ratios of the steering drive and the steering mechanism (reducer), and the presence of backlash in the steering control. This means that the direct measurement of the steering wheel angle will lead to errors in the evaluation of the steered wheels' angles. Consequently, a preliminary analysis of the input/output relation by installation of several rotary encoders was required. Because of this, it was decided to measure the truck's front left wheel's turning angle directly, for that the Honeywell HRS100SSAB090 encoder was used, which was mounted in the pin of slewing stub-axle. The necessary design drawings were developed in the graphical software "Compass-3D" by the reproduction of 3D real scale models of the tractor's elements that would provide the constructive conjunction with the elaborated arrangement. The design of the appliance is shown in Figure 6(e).

Truck articulation angle. The next problem is the measurement of the truck articulation angle due to the impossibility to install a sensor directly into the semitrailer's kingpin. The high-precision Honeywell RTY-360 rotary encoder was chosen as the sensor, for which a device (Figure 6(b) and 6(c)) had been developed. The mechanism consists of the following parts: components fixated relatively to the truck - a mounting plate (4) attached to the truck frame by the bolted connection; the sensor with the device (5) mounted in the middle of the plate and providing the sensor holding relatively to the truck frame; components fixed relatively to the semitrailer frame - a plate (7) attached to a plate of the semitrailer frame by two clamps, a guide device (9) connected to the plate through the bearing (8) and transmitting the rotation from the semitrailer to the sensor's moving part. Since the sensor is installed with a shift relatively to the center of semitrailer rotation, there is a necessity to perform its calibration. This procedure is difficult to implement technically on the truck, therefore, due to the composed 3D models of the articulated vehicle elements, a design scheme of the device was proposed (Figure 6(d)), and the formula was obtained providing experimental data transfer from the RTY sensor values to the actual angles of the semitrailer rotation relatively to the truck. Using the sinusoidal theorem for the triangle $O_1O_2O_3'$, derive

$$\psi = \varphi - arcsin\left(\frac{n}{R} \times sin\left(\varphi\right)\right). \tag{32}$$

Longitudinal truck speed. Direct measurement of the truck speed is a complicated task in view of the possible significant errors of the base devices installed on a truck. Therefore, it was decided to use the Analog Device ADIS-16375, an inertial measurement system that includes a 3-axis gyroscope and 3-axis accelerometer. The sensors are calibrated at the supplier plant. As a result, each of them has its own formula for dynamic compensation, which allows reaching high measurement accuracy. This device is intended to register 12 parameters, 6 of which are linear accelerations and angular velocities, and 6 are their first antiderivatives. Thus, taking into account the start from zero speed and short-term measurement in order to prevent error augmentation, the values of linear velocity increment can be obtained, which are then simply rearranged into a vector by summation with accumulation.

Semitrailer angular speed. To monitor the signal received by the RTY sensor, an IMU Fusion Board sensor (ADXL345 + IMU3000) is additionally installed in the semitrailer mass center destined to measure linear accelerations and angular velocities in three dimensions. The difference in the integrated angular velocities around the vertical axes from the tractor and semitrailer gyroscopes gives auxiliary data for monitoring RTY sensor values. Other parameters of the specified sensors in this work were not used. The efficiency of each sensor is provided by individually developed

ADCs, signals from which are sent to the USB RS-485 block, where they are processed and transferred to the special computer software developed based on NI LABVIEW.
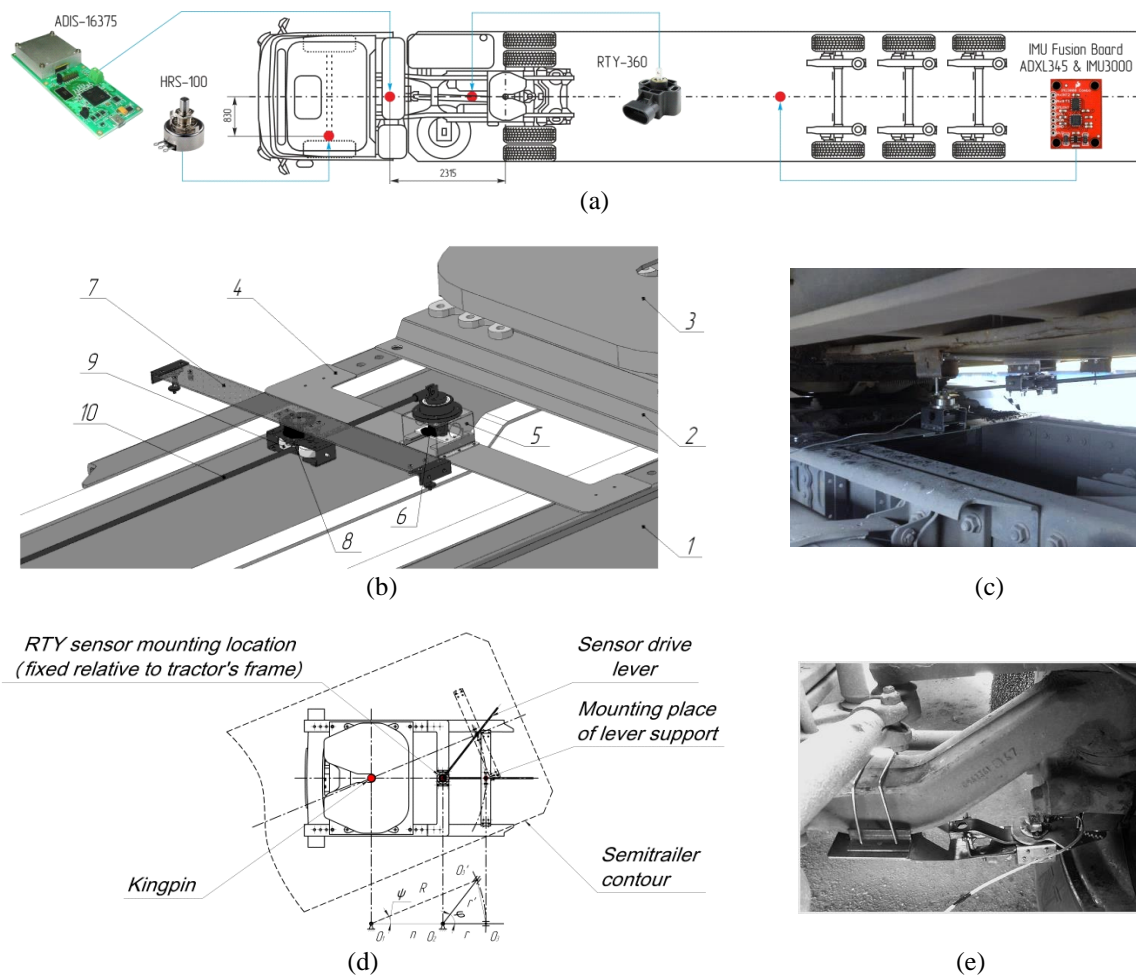

(a)

(b)

(c)

(d)

(e)

Figure 6. Devices and sensors on the truck: (a) location scheme of sensors on the truck; (b) drawing of a device design for the truck's articulation angle measurement: 1 - truck frame, 2, 4 - mounting plate, 3 - tractor fifth wheel, 5 - adapter for sensor mounting, 6 - RTY sensor, 7 - mounting plate for the guide junction, 8 - bearing, 9 - guide device, 10 - rod (sensor drive lever); (c) location of the manufactured device on the real truck; (d) calculation scheme for determination of the ratio between the sensor's shaft rotation angle and the articulation angle; (e) placing the device for the HRS sensor under the truck's slewing stub-axle.

Obtaining of experimental parameters. The general experiment consists of three test types: the truck turning on 180° (test 1), the lane change (test 2), and the turn on 90° (test 3) [19], when the truck moves from the place with the fully turned tractor's steered wheels. There were three successful registrations for each test type, by which the average arithmetic values were calculated (Figure 7).

Processing of experimental data and simulation model validation. The recorded data is initially filtered by smoothing [13]; the new data is presented as the time dependencies introduced into the simulation model as input data - the desired speed and wheels' turning angles. After computing by the model, the articulation angle values presented by simulations and experimentally are compared (Figure 7).

It can be assumed that any virtual test is a sample of a set of real tests, where the articulation angle is a random process at the appropriate conditions. Thus, let's determine the ratio of the sample variances of two tests to figure out their equality. The F-Fisher criterion [20] is chosen to check this: if the 0-hypothesis is right, then the ratio of selected variances must have an F-distribution

$$F = \frac{D_1^*}{D_2^*}.$$

(33)

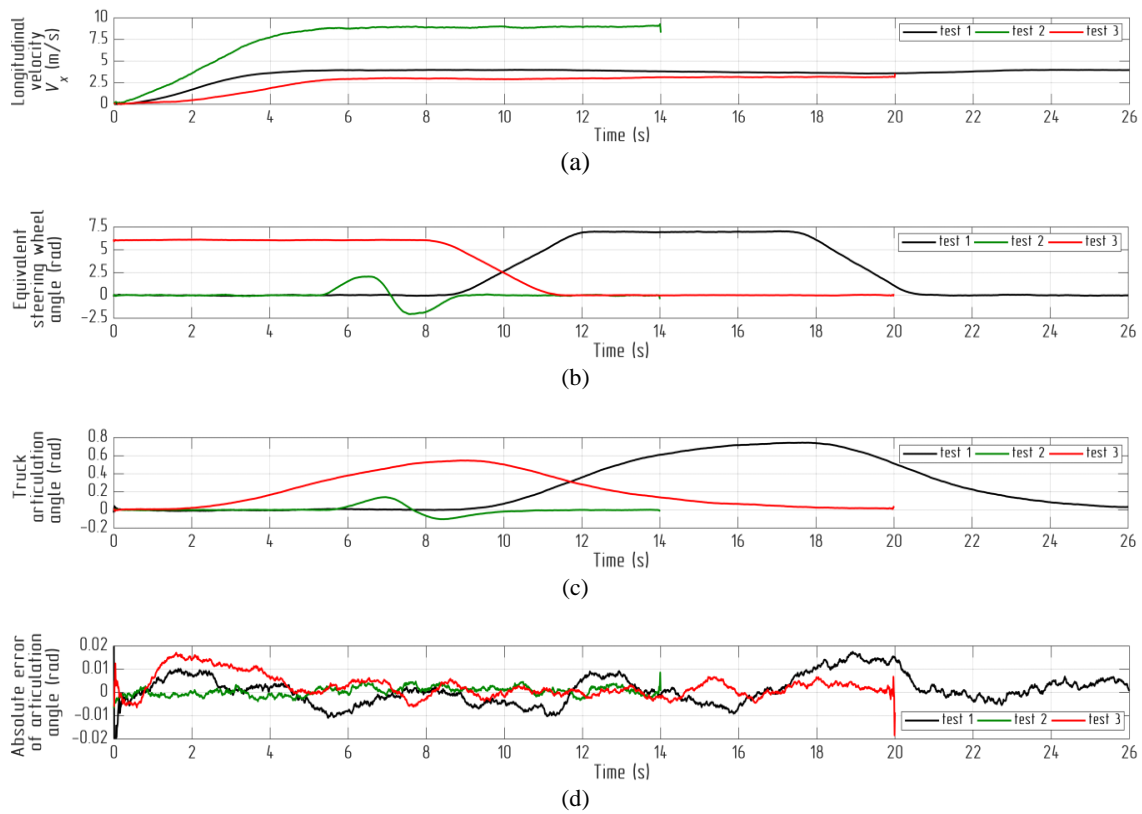where D_1^*, D_2^* - are sample variances.

Figure 7. Results of measured parameters and comparison with simulation data: (a) Tractor longitudinal velocity, (b) Equivalent of steering wheel angle, (c) Truck articulation angle, (d) Articulation angle absolute error

Consequently, the realization of this value, i.e. the ratio of sample variances calculated practically, must be within the quantile boundaries at the significance level q. If the alternative to the 0-hypothesis are two hypotheses: $D_1 < D_2$ and $D_1 > D_2$, then the confidence interval for the 0-hypothesis will be two-sided

$$F_{q/2}\left(f_1, f_2\right) \leq F \leq F_{1-q/2}\left(f_1, f_2\right). \tag{34}$$

The criterion calculations were carried out in the MATLAB Statistics Toolbox; the results were summarized in Table 1.

Table 1. F-test of articulation angle variances at the significance level $q = 0.05$

Test	Result of hypothesis test	Test probability	Confidence interval of actual variance		Fisher statistics	Degree of freedom	
			left	right		df_1	df_2
1	0 (approved)	0.7084	0.9366	1.1011	1.0156	2350	2350
2	0 (approved)	0.9682	0.8567	1.1601	0.9969	670	670
3	0 (approved)	0.7098	0.9009	1.0736	0.9835	2000	2000

IMPLEMENTATION

Using this model, a considerable part of motion tests was executed, such as: circular motion; linear motion with acceleration and braking [21, 22], including maneuvering; reverse motion; lane change; double lane change; 180 degrees turn; 90-degree turn; random movement with changes in direction and speed mode, - where the model has proved its adequacy in steerability and motion stability [23]. To estimate the space saving in the cases of a conventional semitrailer and equipped by steered wheels, the comparison of a corridor (swept path) occupied by the articulated vehicle on a roundabout with a constant circulating speed of 10 and 20 km/h (Figure 8(a) and 8(b)) was performed.

The first difference is that the truck with fixated semitrailer's wheels shows a longer transitional process, during which the internal border describes a spiral and the swept path increases. In the case of steered semitrailer's wheels, on the contrary, the swept path decreases, and its borders stabilize almost immediately after driving on a roundabout.

Thus, at the beginning of a roundabout with a diameter of 30 m the swept path difference is about 0.8 m (Figure 8(a)). In Figure 8(b) the relative differences between the fixed corridors are presented depending on the diameter of a roundabout

for two speed values - 10 and 20 km/h. That is, for example, the savings of the required space on a 20 m diameter roundabout for semitrailer with steered wheels is almost 36%.

To make a comparative assessment of the power factors that arise in the contact points of the semitrailer's tires, the graphs in Figure 8(c), 8(d) and 8(e) are plotted. Examples are given for the case of a circular motion at speed of 10 km/h.
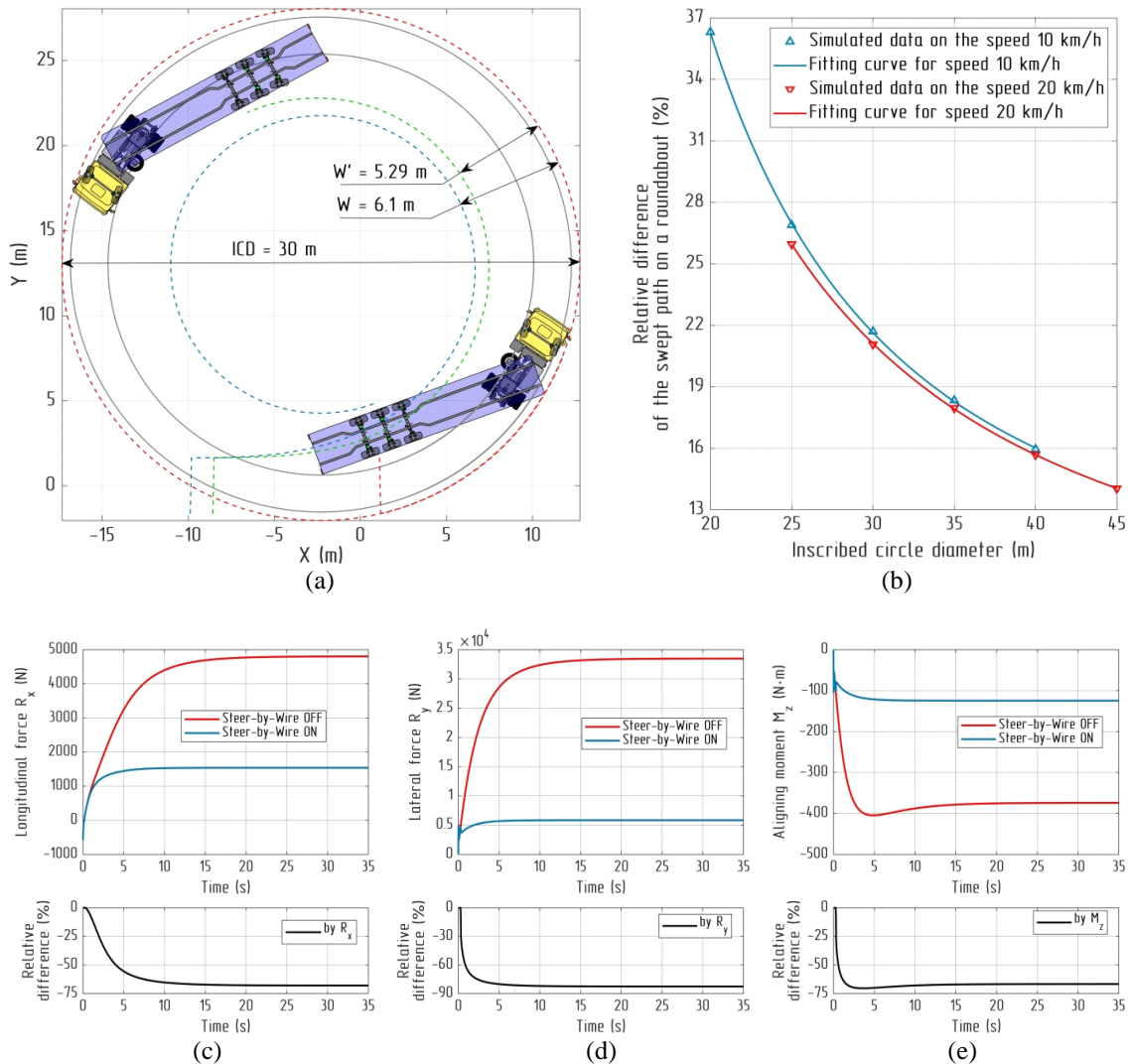

Figure 8. Results of test simulations on a circular motion: (a) comparing the swept path on a roundabout in the case of conventional and steered semitrailer; (b) the relative difference of the swept path depending on the roundabout diameter and circulating speed; (c) comparison of traction forces on the left outer wheel of the tractor's drive axle; (d) comparison of lateral forces on the left wheel of the semitrailer's rear axle; (e) comparison of aligning moment on the left wheel of the semitrailer's rear axle

As can be seen in the graphs of the relative difference in power factors, the use of command wheels significantly reduces the sideslip, rolling resistance, moment of turning resistance, and therefore - provides steadier motion and lower operating costs.

CONCLUSIONS

The article material presents the basis of research devoted to the development of SBW system and its integration with the dynamic model of an articulated vehicle by means of simulation, what should be disclosed in adjacent publications. It is proved that the potential SBW-system is effective, giving unquestionable advantages in the operation of multi-unit vehicles. The products of companies such as VSE [6], Tridec [8] might be vivid examples of commercial success in this domain.

The simulation components uncovered in the article are thoroughly tested and provide universal imitating models, quick calculations, and animation of results. The formation of vehicle models in the form of Simulink libraries allows using them widely, including the development of intelligent transport systems and commercial vehicle technologies.

The technology of steered wheels for trailed units demonstrates the advantages to multi-unit vehicle, ensuring the stability of curvilinear motion and predicted trajectories, which is important in terms of promotion of intelligent transport.

The specified technology has the powerful potential to expand the adjacent options and to engage it into stability control by example of ESP/ESC as well as in combination with these systems. Similar decisions are already applied in passenger cars (Porsche).

The system can also be used in ADAS and autonomous driving technologies, in an automatic adjustment at the entry to a turn's corridor in urban conditions, in a choice of rational trajectory at reverse movement. Improvement of the algorithms for the system of steered wheels will allow adjusting not only using speed but also using space scanning sensors. An interesting application may be the optimization of truck position and its swept path while a circular motion.

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