Quantum Analysis on Task Allocation and Quality Control for Crowdsourcing With Homogeneous Workers

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***Abstract*—Crowdsourcing has been emerging as a valid problem-solving model that harnesses a large group of contributors to solve a complicated task. However, existing crowdsourcing platforms or systems could suffer from task allocation and quality control problems. In this article, we first prove that there exist two dilemmas while tackling the above issues by using a game-theoretic approach. To overcome this challenge, we are focusing on exploiting quantum crowdsourcing schemes in which the welfare of requestor or worker can be maximized since quantum players share the extended strategy space, and the introduction of entanglement offers a new method of depicting fine-grained relations between players. Specifically, we propose a quantum game model for quota-oriented crowdsourcing game to address dilemmas in task allocation. The result indicates the dilemma based on classical strategy will disappear with the increment of entanglement degree. While in the quality-oriented crowdsourcing game, we adopt a density matrix approach to calculate the optimal payoffs of both sides, which demonstrates the superiority of our quantum strategy. Moreover, our quantum scheme is generic since it is compatible with the schemes from a classical perspective. Hence, our noteworthy quantum crowdsourcing schemes offer a promising alternative route for tackling dilemmas in crowdsourcing scenarios.**

***Index Terms*—Crowdsourcing, dilemma, quantum game, entanglement.**

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I. Introduction

CROWDSOURCING has been emerging as a distributed paradigm to solve complicated tasks by engaging large groups of people. Generally speaking, participants in crowdsourcing refer to requestors and workers. Given a crowdsourcing scenario, it is crucial to wisely allocate tasks among the workers and control the quality of the submitted tasks.

Among the techniques tackling task allocation and quality control in crowdsourcing, game theory is of frequent use to design mechanisms modeling the conflict and cooperation between intelligent, rational participants [1]–[7] since game-theoretic approaches are relatively simple to reflect important factors of improving social welfare. Nevertheless, classical game theory might be faced with deficiencies in certain crowdsourcing scenarios in which game dilemmas are unavoidable [4], [5]. On the other hand, quantum game theory, which is a combination of classical game theory and quantum computing, provides two unique features that are very attractive to solve the crowdsourcing dilemmas: the large strategy space and the quantum entanglement. In practice, players interact with each other to compete or collude for maximizing their own payoffs; thus, modeling their behaviors in crowdsourcing via quantum games has high potential to solve the crowdsourcing dilemmas and improve social welfare.

More specifically, compared to the classical game theory, quantum game theory can extend a player's strategy space and introduce a unique entangled state that can accurately depict the connections between players in a more natural way [8], [9]. The entangled players, to some extent, establish an inherent interrelationship, which is measured by the degree of entanglement. The degree of entanglement is set up at the beginning of a game and cannot be modified during the game process, which is analogous to a situation where players abide by previously signed contracts in the business world. Collusions or defections might lead to malicious players' loss in a quantum game, which is hard to achieve in a classical game. The loss, in return, can regulate a malicious player's behavior, therefore increasing social welfare. Such excellent features brought by quantum games have not been explored in crowdsourcing. For an exploratory purpose, in this article, we design two straightforward game models to address the task allocation and quality control problems when homogeneous workers are considered in crowdsourcing. To demonstrate the effectiveness of the quantum game theory, we illustrate how inevitable dilemmas in classical game models can be avoided and how social welfare can be maximized using quantum games.

We first formulate a quota-oriented crowdsourcing game between n homogeneous workers, in which the workers only choose their task quota. The state of each worker is expressed as a single-mode electromagnetic field, of which the quadrature amplitudes have a continuous set of eigenstates corresponding to our continuous quantum strategy. Moreover, the connection between the workers is described by the degree of entanglement λ. When $\lambda = 0$, the quantum game is reduced to a classical one and shows identical results as the classical game, which verifies the correctness of our approach. Secondly, we construct a quality-oriented game in which workers accomplish their tasks with high or low quality, and the requestors assign a low or high payment for a worker's submitted task. We deploy a density matrix rather than a single-mode electromagnetic field to represent the state of each player and λ to depict the degree of entanglement. Similarly, when $\lambda = 0$, the quantum game degenerates into a classical one. Also, quantum strategies are analyzed for a general case and for two special cases. To the best of our knowledge, this article is the first one to employ quantum games for task allocation and quality control in crowdsourcing; and we intend to use simple cases for throwing out a brick to attract jade. Our investigations pave a new way to address the crowdsourcing dilemmas from a quantum perspective. Besides, we compare our quantum game results with the classical game results and demonstrate the universality and superiority of the quantum approach. Conclusively, our contributions are summarized as follows:

- A quantum model for the quota-oriented game involving multiple workers is formulated. The optimal strategy and the corresponding payoffs are quantitatively analyzed through theoretical calculations as well as simulation studies.
- For the quality-oriented game involving a requestor and a worker, a quantum game model is formulated, and its optimal strategy is found given an initial state of both the requestor and the worker.
- The impacts of the degree of entanglement on the optimal strategies and payoffs indicate that social welfare can be maximized by adopting quantum strategies.

The rest of the article is organized as follows. The summary of the related work is given in Section II. Section III presents the quantum basics employed in subsequent sections. The quota-oriented crowdsourcing game involving multiple workers and the quality-oriented game involving a requestor and a worker are analyzed in Section IV and Section V, respectively. We conclude this article in Section VI with a future research discussion.

II. Related Work

Game theory has been frequently adopted to design new techniques and mechanisms to address various problems such as task allocation and quality control in crowdsourcing [1]–[7], [10]–[12] since such approaches are relatively simple when used to depict strategic interactions between rational participants and reflect important factors of improving crowdsourcing performance. Crowdsourcing contests were modeled as all-pay auctions played by workers who were partitioned over skill levels in [1], which states that more reward can logarithmically increase the participation rates that affect task allocation from a game-theoretic perspective. Hoh *et al.* [2] adopted game-theoretic protocols to validate the parking information provided by the workers in a mobile crowdsourcing scenario. Hu *et al.* [3] proposed two algorithms to solve the sequential crowdsourcing dilemma using a zero-determinant (ZD) strategy. ZD was also employed by Hu in [10] to systematically investigate the methods of enhancing crowdsourcing qualities. The game-theoretic analysis in [4] shows that there is a crowdsourcing game dilemma between two requestors, and the highest welfare of the requestors is obtained only when they stop crowdsourcing. An iterated game claimed later in [5] can alleviate the above crowdsourcing dilemma under certain conditions. Wu *et al.* [6] defined a min-max mechanism in a game-theoretic model, which can identify skilled talents, thus improving the quality of software crowdsourcing. Based on game theory, Wang *et al.* [7] proposed an incentive mechanism to statically select worker candidates and dynamically select winners after bidding. Zhang *et al.* [11] utilized a selection game to design a crowdsourcing mechanism for a Public Bike System. Using Stackelberg game, an elegant scheme by Qiu *et al.* [12] takes into account market history to provide fair compensation to workers. Hua *et al.* [13] adopts a game-theoretic approach to enable scalable large graph computation. In this article, we leverage classical game theory to depict two straightforward crowdsourcing scenarios in which dilemmas were found and proved to be solvable using quantum games.

In addition to game-theoretic approaches, a large number of techniques and mechanisms have been proposed by reseachers for crowdsourcing. Existing task allocation techniques regard workers as either homogeneous or heterogeneous, with the former [14]–[18] aiming to design allocation mechanisms based on the workers who can be treated the same while the latter [1], [19]–[21] focusing on selecting eligible workers with various backgrouds. The problem of quality control were previously studied by [4], [22]–[24]. In the famous DARPA network challenge [22], a team adopting the crowdsourcing approach failed to filter false submissions due to coordinated attacks from the workers. Generaly speaking, quality control techniques can be divided as task-oriented and participant-oriented, with the former [2], [3], [25]–[29] evaluating on the quality of submissions while the latter [4]–[7], [23], [30], [31] emphasizing on the quality of participants. In this article, we address task allocation and quality control problems for homogeneous crowdsourcing.

III. Quantum Basics

In this section, we are going to introduce the basic concepts in quantum games. Quantum game theory is an extension of the classical game theory by bringing quantum mechanics and quantum computing into the traditional games.

The strategy space of a quantum player is a *Hilbert* space. The player's quantum state $|\Psi\rangle$ can be expressed as the linear combination of all orthonormal basis vectors; that is, $|\Psi\rangle = \sum_{i=1}^{d} \alpha_i|\psi_i\rangle$, where $|\cdot\rangle$ is a standard Dirac notation, d is the dimensionality of the strategy space, and $|\psi_i\rangle$ is an orthonormal basis vector corresponding to the i^{th} possible classical strategy adopted with probability $|\alpha_i|^2$. It follows that $\sum_{i=1}^{d} |\alpha_i|^2 = 1$, which means $|\Psi\rangle$ is normalized. Concerning a single player living in a two-dimensional strategy space, its quantum state can be expressed as *a quantum bit* (or *qubit*), which is analogous to the *bit* used in classical digital computers. Such a *qubit* state can be represented by a linear combination of $|0\rangle$ and $|1\rangle$ corresponding to two independent classical strategies. For instance, $\alpha_1|0\rangle + \alpha_2|1\rangle$ represents a state when the system is observed as $|0\rangle$ with probability $|\alpha_1|^2$ and $|1\rangle$ with probability $|\alpha_2|^2$.

Consider a more complicated composite system which consists of n independent players. Its state can be noted by $|\Phi\rangle = |\Psi^1\rangle \otimes |\Psi^2\rangle \otimes \cdots \otimes |\Psi^n\rangle = |\Psi^1\Psi^2\ldots\Psi^n\rangle$, where $|\Psi^i\rangle$ represents the state of the i^{th} independent subsystem (player) and $\otimes$ refers to a tensor product operation. If players are dependent, the state of their composite system cannot be represented as the tensor product decomposition of the states of all subsystems but appears to be a special entangled state. For example, $\alpha_1|00\rangle + \alpha_2|11\rangle$ is an entangled state because it cannot be decomposed into a tensor product decomposition. Nevertheless, $\alpha_1|00\rangle + \alpha_2|10\rangle$ is a pure quantum state since it can be decomposed into $\alpha_1|0\rangle + \alpha_2|1\rangle$ and $|0\rangle$; that is, $\alpha_1|00\rangle + \alpha_2|10\rangle = (\alpha_1|0\rangle + \alpha_2|1\rangle)\otimes |0\rangle$.

In our quantum model, independent players are represented by a composite system, while an entangled system describes correlated players. In quantum crowdsourcing, players can manipulate the state of the system with a unitary operator $\hat{U}$, which satisfies $\hat{U}^\dagger\hat{U} = \hat{U}\hat{U}^\dagger = I$ where $\hat{U}^\dagger$ is the adjoint of $\hat{U}$. The social welfare largely depends on the profit-driven workers whose primary strategies include the task quota they undertake and the quality of the completed tasks. Therefore in the following two sections, we carry out an analysis on the workers' strategies to figure out how a worker determines its optimal strategy. For the sake of completeness, we transform the classical game from a macro perspective to the quantum game from a micro perspective, and the positive complementary of these two perspectives can help us conduct an accurate analysis of crowdsourcing games.

IV. Quota-Oriented Game Analysis

In this section, we analyze how workers determine their task quota to reach their optimal strategies from the classical perspective as well as the quantum perspective.

A. Analysis Based on Classical Strategies

We formulate a crowdsourcing game involving multiple workers who undertake the same type of tasks. Assume q_i is the task quota decided by the i^{th} worker. Then the total number of tasks allocated to all the workers is $(q_1 + q_2 + \cdots + q_n)$. According to Cournot's duopoly, it is reasonable to state that the more tasks the workers undertake, the less unit payment they would obtain. Thus the price of a unit task can be written as

$$P(q_1, q_2, \ldots, q_n) = a - \beta(q_1 + q_2+, \ldots, +q_n), \tag{1}$$

where a is the upper bound of the payment of a unit task and β is a ratio coefficient. When the total number of allocated tasks is extremely large, the unit payment would be close to zero, which means that the task becomes worthless based on our model. Note that when $a < \beta(q_1 + q_2+, \ldots, +q_n)$, $P(q_1, q_2, \ldots, q_n) = 0$. Given the marginal cost of the production c, the payoff of the i^{th} worker can be written as

$$U_i(q_1, q_2, \ldots, q_n) = q_i[P(q_1, q_2, \ldots, q_n) - c)] \tag{2}$$

Let $k = a - c$. Then, n Nash Equilibria arise from the following equations and inequalities:

$$\begin{aligned} &\frac{\partial U_1}{\partial q_1} = \frac{\partial U_2}{\partial q_2} = \cdots = \frac{\partial U_n}{\partial q_n} = 0 \\ &\frac{\partial^2 U_i}{\partial^2 q_i} < 0, \quad i = 1, 2, \ldots, n \end{aligned} \tag{3}$$

The optimal strategy at the *Nash Equilibria* is then computed as

$$q_1^{'} = q_2^{'} = \cdots = q_n^{'} = \frac{k}{(n+1)\beta}. \tag{4}$$

and the corresponding optimal payoff is

$$U_1^{'} = U_2^{'} = \cdots = U_n^{'} = \frac{k^2}{(n+1)^2\beta} \tag{5}$$

However, this equilibria solution fails to maximize the payoff of each worker. Considering $U_i = q_i[k - \beta(q_1 + q_2+, \ldots, q_n)], i = 1, 2, \ldots, n$, one can easily find a more profitable strategy as

$$q_1^{''} = q_2^{''} = \cdots = q_n^{''} = \frac{k}{2n\beta} \tag{6}$$

and the maximazed payoff is

$$U_1^{''} = U_2^{''} = \cdots = U_n^{''} = \frac{k^2}{4n\beta} \tag{7}$$

Since $n \geq 1$ and $U_i^{''} \geq U_i^{'}, \forall i = 1, 2, \ldots, n$, we conclude that the workers can not reach the maximized payoff at the *Nash Equilibria*. In other words, the workers run into a crowdsourcing dilemma.

B. Analysis Based on Quantum Strategies

In the previous subsection, we study the crowdsourcing game based on classical strategies. In this subsection, we investigate quantum strategies in the same game. Unlike the classical viewpoint, our quantum perspective is more general since we allow the workers to be correlated, which is described by an

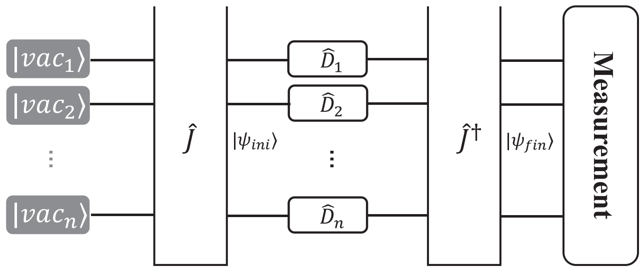

Fig. 1. Quantum game model for quota-oriented game.

entangled state. Specifically, we would like to extend the two single-mode electromagnetic fields illustrated in [32] and deploy n identical fields to depict the n workers' states (one for each). The quadrature amplitude of each field has a continuous set of eigenstates corresponding to a worker's continuous strategy, namely task quota. Fig. 1 shows the complete quantum game model. This game starts from $|vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle$, where $|vac_i\rangle$ refers to the i^{th} worker's initial state. The state of the composite system becomes entangled through an entangling gate $\hat{J}(\lambda)$, where λ is the degree of entanglement which depicts the correlation between two workers, and the initial quantum state $|\psi_{ini}\rangle$ of the composite system is obtained as

$$|\psi_{ini}\rangle = \hat{J}(\lambda)|vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle. \tag{8}$$

Then the i^{th} worker can operate on the initial quantum state with a unitary operator $\hat{D}_i(x_i)$, where x_i is the strategy parameter. Before measurement, a disentangling operator $\hat{J}(\lambda)^\dagger$ is carried out to reach a final quantum state $|\psi_{fin}\rangle$, which can be expressed as

$$|\psi_{fin}\rangle = \hat{J}(\lambda)^\dagger(\hat{D}_1 \otimes \hat{D}_2 \otimes \cdots \otimes \hat{D}_n)\hat{J}(\lambda) \\ |vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle. \tag{9}$$

Finally, the worker's expected payoff is obtained through the measurement (i.e., a Stern-Gerlach experiment). Besides, we assume that all the workers are homogeneous; thus the entangling gate $\hat{J}(\lambda)$ should be symmetric and can be defined as

$$\hat{J}(\lambda) = exp\left\{-\sum_{i<j=1,i\neq j}^{n} \lambda(\hat{a}_i^\dagger \hat{a}_j^\dagger - \hat{a}_i \hat{a}_j)\right\}, \tag{10}$$

where $\hat{a}_i^\dagger$ and $\hat{a}_i$ are respectively the creation and annihilation operators of the i^{th} worker's electromagnetic field, and $\lambda(\hat{a}_i^\dagger \hat{a}_j^\dagger - \hat{a}_i \hat{a}_j)$ means that the m^{th} worker entangles with the j^{th} worker in terms of the degree of the entanglement λ. Obviously, a larger λ stands for a stronger connection between two workers. Note that when $\lambda = 0$, $\hat{J}(0) = 1$ and $|\psi_{fin}\rangle = (\hat{D}_1 \otimes \hat{D}_2 \otimes \cdots \otimes \hat{D}_n)|vac_1\rangle \otimes |vac_2\rangle \otimes \cdots \otimes |vac_n\rangle$, which means that the entangling gate becomes invalid so that the workers are not entangled and they choose the task quota independently, i.e., our quantum game is reduced to a classical one. In addition, the worker's operator $\hat{D}_i(x_i)$ is defined as

$$\hat{D}_i(x_i) = exp\{-ix_i\hat{P}_i\}, \quad x_i \in [0, \infty], \tag{11}$$

where x_i is the strategy parameter belonging to the i^{th} worker and $\hat{P}_i = \frac{i}{\sqrt{2}}(\hat{a}_i^\dagger - \hat{a}_i)$ stands for a momentum operator.

After obtaining the final quantum state $|\psi_{fin}\rangle$, the consequent measurement corresponding to the i^{th} worker's observable operator $\hat{X}_i = \frac{1}{\sqrt{2}}(\hat{a}_i^\dagger + \hat{a}_i)$ outputs its strategy as q_i, which is the linear combination of x_i. That is, $q_i = \sum_{m=1}^{m=n} c_m x_i$, where c_m is a coefficient associated with λ. When $\lambda = 0$, $q_i = x_i$, implying that the worker adopts the classical strategy x_i.

Theorem 1:

$$\hat{J}(\lambda)^\dagger \hat{D}_i \hat{J}(\lambda) = exp\{-ix_i[\hat{P}_i \frac{1}{n}(e^{(n-1)\lambda} + (n-1)e^{-\lambda}) \\ + \sum_{j=1,j\neq m}^{n} \hat{P}_j \frac{1}{n}(e^{(n-1)\lambda} - e^{-\lambda})]\} \tag{12}$$

Proof: The proof here is based on the Baker-Campbell-Hausdorff formula, which helps to give the mathematical derivation of an important factor. We have

$$\begin{aligned}\hat{J}(\lambda)^\dagger \hat{a}_i \hat{J}(\lambda) &= \sum_{g=0}^{\infty} \frac{1}{g!}[\hat{A}^{(g)}, \hat{a}_i] \\ &= \hat{a}_i \frac{n-1}{n} \sum_{g=0}^{\infty} [\frac{1}{(2\,g)!} \lambda^{2\,g}((n-1)^{2g-1} + 1)] \\ &\quad - \hat{a}_i^\dagger \frac{n-1}{n} \sum_{g=0}^{\infty} [\frac{1}{(2g+1)!} \lambda^{2g+1}((n-1)^{2\,g} - 1)] \\ &\quad + \sum_{j=1,j\neq i}^{n} \hat{a}_j \frac{1}{k+1} \sum_{g=0}^{\infty} [\frac{1}{(2\,g)!} \lambda^{2\,g}((n-1)^{2\,g} - 1)] \\ &\quad - \sum_{j=1,j\neq i}^{n} \hat{a}_j^\dagger \frac{1}{n} \sum_{g=0}^{\infty} [\frac{1}{(2g+1)!} \lambda^{2g+1}((n-1)^{2g+1} + 1)].\end{aligned}$$

Note that $\sum_{g=0}^{\infty} \frac{1}{g!}[\hat{A}^{(g)}, \hat{a}_i]$ is a commutator of elements $\hat{A}^{(g)}$ and $\hat{a}_i$. Specifically, $[\hat{A}^{(0)}, \hat{a}_i] = a_i$, $[\hat{A}^{(g)}, \hat{a}_i] = [\hat{A}, [\hat{A}^{(g-1)}, \hat{a}_i]]$, where $\hat{A} = \sum_{i<j=1,m\neq j}^{n} \lambda(\hat{a}_i^\dagger \hat{a}_j^\dagger - \hat{a}_i \hat{a}_j)$. Similarly, one can calculate $\hat{J}(\lambda)_n^\dagger \hat{a}_i^\dagger \hat{J}(\lambda)_n$. Once we have $\hat{J}(\lambda)^\dagger \hat{a}_i \hat{J}(\lambda)$ and $\hat{J}(\lambda)^\dagger \hat{a}_i^\dagger \hat{J}(\lambda)$, one can calculate $\hat{J}(\lambda)^\dagger \hat{P}_i \hat{J}(\lambda)$ in the view of $\hat{P}_i = \frac{i}{\sqrt{2}}(\hat{a}_i^\dagger - \hat{a}_i)$:

$$\begin{aligned}\hat{J}(\lambda)^\dagger \hat{P}_i \hat{J}(\lambda) &= \frac{i}{\sqrt{2}} \hat{J}(\lambda)^\dagger (\hat{a}_i^\dagger - \hat{a}_i)\hat{J}(\lambda) \\ &= \hat{P}_i \frac{1}{n}[\sum_{g=0}^{\infty} \frac{1}{g!}((n-1)\lambda)^g + k\sum_{g=0}^{\infty} \frac{1}{g!}(-\lambda)^g] \\ &\quad + \sum_{j=1,j\neq i}^{n} \hat{P}_j \frac{1}{n}[\sum_{g=0}^{\infty} \frac{1}{g!}(k\lambda)^g - \sum_{g=0}^{\infty} \frac{1}{g!}(-\lambda)^g] \\ &= \hat{P}_i \frac{1}{n}(e^{(n-1)\lambda} + (n-1)e^{-\lambda}) \\ &\quad + \sum_{j=1,j\neq 1}^{n} \hat{P}_j \frac{1}{n}(e^{(n-1)\lambda} - e^{-\lambda}).\end{aligned}$$

According to the Taylor's theorem, function $\hat{D}_i$ in (11) can be extended as a Taylor polynomial:

$$\hat{D}_i(x_i) = \sum_{h=0}^{\infty} \frac{1}{h!}(-ix_i\hat{P}_i)^h. \tag{13}$$

Since $\hat{J}\hat{J}^\dagger = 1$, we have

$$\hat{J}(\lambda)^\dagger(-ix_i\hat{P}_i)^h\hat{J}(\lambda) = [\hat{J}(\lambda)^\dagger(-ix_i\hat{P}_i)\hat{J}(\lambda)]^h \tag{14}$$

Therefore,

$$\begin{aligned}\hat{J}(\lambda)^\dagger\hat{D}_i\hat{J}(\lambda) &= \sum_{h=0}^{\infty}\frac{1}{h!}[\hat{J}(\lambda)^\dagger(-ix_i\hat{P}_i)\hat{J}(\lambda)]^h\\ &= \sum_{h=0}^{\infty}\frac{1}{h!}\{-ix_i[\hat{P}_i\frac{1}{n}(e^{(n-1)\lambda}+(n-1)e^{-\lambda})\\ &\quad+\sum_{j=1,j\neq 1}^{n}\hat{P}_j\frac{1}{n}(e^{(n-1)\lambda}-e^{-\lambda})]\}^h\\ &= exp\{-ix_i[\hat{P}_i\frac{1}{n}(e^{(n-1)\lambda}+(n-1)e^{-\lambda})\\ &\quad+\sum_{j=1,j\neq m}^{n}\hat{P}_j\frac{1}{n}(e^{(n-1)\lambda}-e^{-\lambda})]\}\end{aligned} \tag{15}$$

■

Based on Theorem 1, the final quantum state of the composite system can be written as

$$\begin{aligned}|\psi_{fin}\rangle &= [(\hat{J}(\lambda)^\dagger\hat{D}_1\hat{J}(\lambda))\otimes(\hat{J}(\lambda)^\dagger\hat{D}_2\hat{J}(\lambda))\otimes\cdots\otimes\\ &\quad(\hat{J}(\lambda)^\dagger\hat{D}_n\hat{J}(\lambda))]|vac_1\rangle\otimes|vac_2\rangle\otimes\cdots\otimes|vac_n\rangle\\ &= exp\{-i[x_1[\frac{1}{n}(e^{(n-1)\lambda}+(n-1)e^{-\lambda})\\ &\quad+\sum_{j=1,j\neq 1}^{n}\hat{x}_j\frac{1}{n}(e^{(n-1)\lambda}-e^{-\lambda})]\hat{P}_1\}|vac_1\rangle\otimes\\ &\quad exp\{-i[x_2[\frac{1}{n}(e^{(n-1)\lambda}+(n-1)e^{-\lambda})\\ &\quad+\sum_{j=1,j\neq 2}^{n}\hat{x}_j\frac{1}{n}(e^{(n-1)\lambda}-e^{-\lambda})]\hat{P}_2\}|vac_2\rangle\otimes\cdots\otimes\\ &\quad exp\{-i[x_n[\frac{1}{n}(e^{(n-1)\lambda}+(n-1)e^{-\lambda})\\ &\quad+\sum_{j=1,j\neq n}^{n}\hat{x}_j\frac{1}{n}(e^{(n-1)\lambda}-e^{-\lambda})]\hat{P}_n\}|vac_n\rangle.\end{aligned} \tag{16}$$

The final measurement gives the task quota q_i of the i^{th} worker as the following:

$$q_i = \frac{x_i}{n}(e^{(n-1)\lambda}+(n-1)e^{-\lambda}) + \sum_{j=1,j\neq i}^{n}\hat{x}_j\frac{1}{n}(e^{(n-1)\lambda}-e^{-\lambda}) \tag{17}$$

Nash equilibrium requires that

$$\begin{aligned}&\frac{\partial U_1}{\partial x_1} = \frac{\partial U_2}{\partial x_2} = \cdots = \frac{\partial U_n}{\partial x_n} = 0\\ &\frac{\partial^2 U_i}{\partial^2 x_i} < 0, \quad i = 1,2,\ldots,n\end{aligned} \tag{18}$$

Thus we obtain the ultimate equilibrium solution of each worker as

$$x_1^* = x_2^* = \cdots = x_n^* = \frac{k(e^{(n-1)\lambda}+(n-1)e^{-\lambda})}{n\beta e^{(n-1)\lambda}(2e^{(n-1)\lambda}+(n-1)e^{-\lambda})} \tag{19}$$

Given (17) (19), the optimal task quota can be calculated as

$$\begin{aligned}q_1^*(x_1^*,\ldots,x_n^*) &= q_2^*(x_1^*,\ldots,x_n^*) = \cdots = q_n^*(x_1^*,\ldots,x_n^*)\\ &= \frac{k(e^{(n-1)\lambda}+(n-1)e^{-\lambda})}{n\beta(2e^{(n-1)\lambda}+(n-1)e^{-\lambda})}\end{aligned} \tag{20}$$

In light of (2), a worker's optimal payoff corresponding to the equilibrium solution in (19) is shown below

$$\begin{aligned}U_1^*(x_1^*,\ldots,x_n^*) &= U_2^*(x_1^*,\ldots,x_n^*) = \cdots = U_n^*(x_1^*,\ldots,x_n^*)\\ &= \frac{k^2e^{(n-1)\lambda}(e^{(n-1)\lambda}+(n-1)e^{-\lambda})}{n\beta[2e^{(n-1)\lambda}+(n-1)e^{-\lambda})]^2}\end{aligned} \tag{21}$$

According to (19) (21), one can easily discover that when $\lambda = 0$, the workers' optimal strategies are $q_1^* = q_2^* = \cdots = q_n^* = \frac{k}{(n+1)\beta}$ and the corresponding payoffs are $U_1^* = U_2^* = \cdots = U_n^* = \frac{k^2}{(n+1)^2\beta}$. These results have the same form as the classical ones shown in (4)(5). Thus our quantum game is reduced to a classical one and the dilemma still exists. However, when $\lambda \to \infty$, the workers' optimal strategies are $q_1^* = q_2^* = \cdots = q_n^* = \frac{k}{2n\beta}$ and the corresponding payoffs are $U_1^* = U_2^* = \cdots = U_n^* = \frac{k^2}{4n\beta}$. Compared to (6) and (7), these results maximize a worker's individual payoff as we expect and the crowdsourcing dilemma shown in our classical scenario disappears in our quantum crowdsourcing game.

Numerical simulation results shown in Fig. 2(a)(b) illustrate more details about how the degree of entanglement λ impacts on the optimal strategy and payoff of each worker. As one can see, with the increase of λ, the task quota q_i^* decreases, and thus, the payoff U_i^* increases. This means that the correlation between two workers becomes stronger so that they tend to share the market and give up redundant tasks they undertake. This rational and kind behavior increases the unit payment $P(q_1, q_2, \ldots q_n)$ and leads to the growth of the individual optimal payoff. Particularly, when $\lambda \to \infty$, the payoff is maximized.

Fig. 2(c)(d) indicates that a worker's maximized task quota q_i^* and the corresponding optimal payoff U_i^* decrease along with the growth of the number of workers since a larger group of workers leads to less unit payment.

In Fig. 2(e)(f), the parameter β stands for the sensitivity of the unit payment to the overall allocated task quota. With a larger β, which means a more sensitive market, a worker's optimal task quota and payoff become lower because a higher sensitivity limits the task quota the worker can take and thus makes the payoff lower. Interestingly, the larger the β, the faster the q_i^* and the U_i^* change. This means that a low sensitivity of the market to task quota may lead to a high sensitivity of a worker's strategy to the degree of entanglement.

Fig. 3 illustrates the optimal task quota q_i^* when the unit payment upper bound a and the unit cost c change. One can

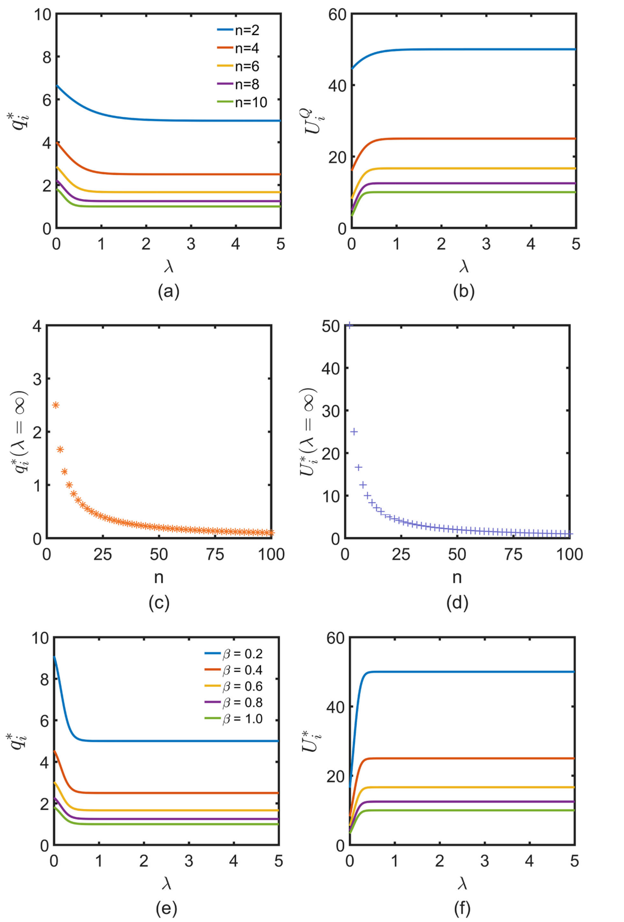

Fig. 2. (a) q_i^* varies with λ and n where $\beta = 1$ and $k = 20$; (b) U_i^* varies with λ and n where $\beta = 1$ and $k = 20$; (c) $q_i^*(\lambda = \infty)$ varies with n where $\beta = 1$ and $k = 20$; (d) $U_i^*(\lambda = \infty)$ varies with n where $\beta = 1$ and $k = 20$; (e) q_i^* varies with λ and β where $n = 10$ and $k = 20$; (f) U_i^* varies with λ and β where $n = 10$ and $k = 20$.

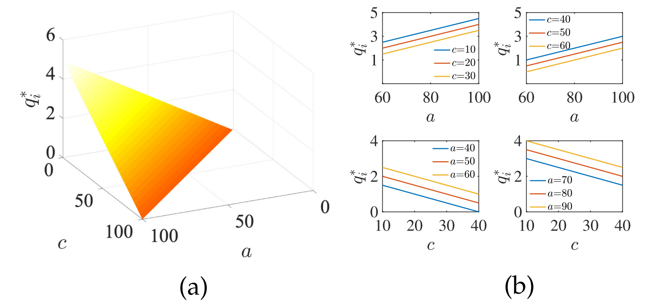

Fig. 3. The optimal task quota q_i^* varies with the unit payment upper bound a and the unit cost c.

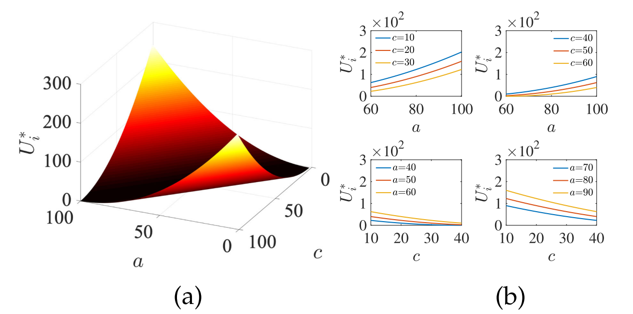

Fig. 4. The optimal payoff U_i^* varies with the unit payment upper bound a and the unit cost c.

see that the optimal task quota grows with an increase of a and a decrease of c. Fig. 4 shows the optimal payoff U_i^* when the unit payment upper bound a and the unit cost c vary. As shown in Fig. 4, the optimal payoff increases when a increases and c decreases.

V. Quality-Oriented Game Analysis

In a general crowdsourcing scenario, requestors launch tasks and publish the corresponding payments at first, and then workers choose tasks to finish and profit from the completion of the selected tasks. Assume that all workers are homogeneous from a requestor's perspective; as a result, a successful strategy for any worker can be appropriate for all the workers. This encourages us to focus on a game with one requestor and one worker. The requestor might assign low or high payment for a task while the worker can accomplish the task with high or low quality. In this process, the payment can impact on the quality of the worker's submissions, and the quality finally determines the payoffs of both sides. In this section, we study the task quality problem of crowdsourcing by analyzing the behaviors of both players in a crowdsourcing game. The classical game is formulated and discussed in Section V-A while the quantum game is analyzed in Section V-B.

A. Analysis Based on Classical Strategies

We define the strategy of the requestor as $\bar{r} \in \{c, d\}$, where c (cooperation) and d (defection) respectively denote high and low payment the requestor provides. Similarly, the strategy of the worker is defined as $\bar{w} \in \{c, d\}$, where c and d respectively indicate high quality and low quality of the task the worker accomplishes.

Since the requestor releases tasks and the corresponding payments before the worker starts to work, we construct a crowdsourcing game as shown in Fig. 5. The payoff of the requestor is $\bar{\$}_R \in \{\Omega_r, \Omega_r - m, \Omega_r + n, \Omega_r - m + n\}$ while that of the worker is $\bar{\$}_w \in \{\Omega_w, \Omega_w + b, \Omega_w - a, \Omega_w + b - a\}$, where Ω_r and Ω_w are respectively the payoffs of the requestor and the worker when both players choose cooperation. Besides, m is the requestor's loss and b is the extra payoff the worker can obtain when the requestor chooses cooperation but the worker chooses defection. Similarly, n is the increment of the requestor's profit and a is the reduction of the worker's profit when the worker chooses to cooperate even though the requestor offers a low payment. Note that $n < m$ and $b < a$, which means that defection damages the interests of both sides compared to cooperation.

Let P and Q respectively be the probabilities that the requestor and the worker cooperate. Then, the payoffs of the requestor ($\bar{\$}_R(P, Q)$) and that of the worker ($\bar{\$}_W(P, Q)$) are respectively

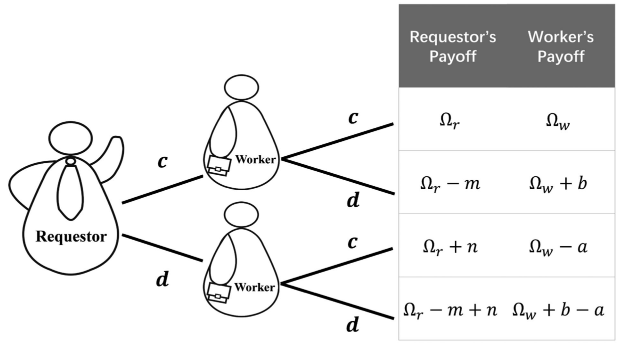

Fig. 5. The game tree of the quality-oriented game.

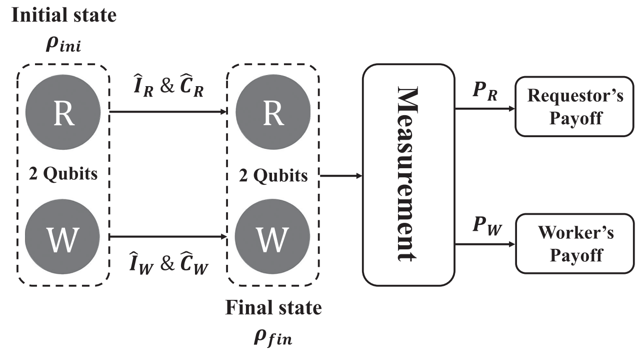

Fig. 6. The quantum game model for the quality-oriented game.

$$\begin{aligned}\bar{\$}_r(P,Q) &= \Omega_r + P(1-Q)(\Omega_r - m) + (1-P)Q \\ &\quad (\Omega_r + n) + (1-P)(1-Q)(\Omega_r - m + n) \\ &= n(1-P) + m(Q-1) + \Omega_r, \\ \bar{\$}_w(P,Q) &= PQ\Omega_w + P(1-Q)(\Omega_w + b) + (1-P)Q \\ &\quad (\Omega_w - a) + (1-P)(1-Q)(\Omega_w + b - a) \\ &= b(1-Q) + a(P-1) + \Omega_w. \end{aligned} \tag{22}$$

Theorem 2: In a classical quality-oriented game, the requestor and the worker involve a dilemma.

Proof: Denote the *Nash Equilibria* of the requestor and that of the worker by P^* and Q^*, respectively. According to the definition of the *Nash Equilibria*, we have $\bar{\$}_r(P^*, Q^*) \geq \bar{\$}_r(P, Q^*)$ for $\forall P \in \{0,1\}$ and $\bar{\$}_w(P^*, Q^*) \geq \bar{\$}_w(P^*, Q)$ for $\forall Q \in \{0,1\}$. That is,

$$\begin{aligned}\bar{\$}_r(P^*, Q^*) - \bar{\$}_r(P, Q^*) = n(P - P^*) \geq 0 \qquad \forall P \in \{0,1\}, \\ \bar{\$}_w(P^*, Q^*) - \bar{\$}_w(P^*, Q) = n(Q - Q^*) \geq 0 \qquad \forall Q \in \{0,1\}. \end{aligned} \tag{23}$$

These inequalities can be satisfied when $P^* = 0, Q^* = 0$. In this situation, we have $\bar{\$}_r(P^*, Q^*) = \Omega_r - m + n$ and $\bar{\$}_w(P^*, Q^*) = \Omega_w + b - a$. That is, the only equilibrium of the classical game is (d,d), where the payoffs of the requestor and the worker are lower than those of (c,c) when $n < m$ and $b < a$. In other words, there exists a dilemma where the dominant strategy of an individual cannot maximize the social welfare. ■

B. Analysis Based on Quantum Strategies

In this subsection, we analyze the quality-oriented crowdsourcing game from a quantum perspective by using a density matrix approach. The general case is discussed in Section V-B1 while several special cases associated with the entangled state are studied in Section V-B2.

1) General Case: As shown in Fig. 6, the quantum game model starts from a two-qubit system composed by the requestor (R) and the worker (W). Using $|0\rangle$ to represent the state of cooperation and $|1\rangle$ for defection, one can model the state of this composite system by a linear superposition of four orthonormal basis states ($|00\rangle, |01\rangle, |10\rangle, |11\rangle$), where the first and second positions in $|\cdot\cdot\rangle$ refer to the classical strategies of the requestor and the worker, respectively. For instance, $|01\rangle$ indicates that the requestor cooperates and the worker defects. Thus an initial quantum state can be written as

$$|\Psi_{ini}\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle \tag{24}$$

where $|c_{00}^2|, |c_{01}^2|, |c_{10}^2|, |c_{11}^2|$ refer to the probabilities and $|c_{00}^2| + |c_{01}^2| + |c_{10}^2| + |c_{11}^2| = 1$. To simplify mathematical calculations, we introduce a density matrix ρ to describe the quantum state according to the Marinatto-Weber construction illustrated in [33]. Hence one can use the following density matrix to describe our initial quantum state:

$$\rho_{ini} = |\Psi_{ini}\rangle\langle\Psi_{ini}| \tag{25}$$

Then the requestor manipulates its own strategy by using the unitary operators $\hat{I}_R$ with probability p and $\hat{C}_R$ with probability $(1-p)$, where $\hat{I}_R$ is an identity operator and $\hat{C}_R$ is an inversion operator. Note that $\hat{I}_R|0\rangle = |0\rangle$, $\hat{I}_R|1\rangle = |1\rangle$, which means that $\hat{I}_R$ would keep the original state and $\hat{C}_R|0\rangle = |1\rangle$, $\hat{C}_R|1\rangle = |0\rangle$, which indicates that $\hat{C}_R$ would change the state to the opposite one. Similarly, the worker uses $\hat{I}_W$ with probability q and $\hat{C}_W$ with probability $(1-q)$. These operations help to give the following final density matrix:

$$\begin{aligned}\rho_{fin} =& pq\hat{I}_R \otimes \hat{I}_W \rho_{ini} \hat{I}_R^\dagger \otimes \hat{I}_W^\dagger + p(1-q)\hat{I}_R \otimes \hat{C}_W \rho_{ini} \hat{I}_R^\dagger \otimes \\ & \hat{C}_W^\dagger + (1-p)q\hat{C}_R \otimes \hat{I}_W \rho_{ini} \hat{C}_R^\dagger \otimes \hat{I}_W^\dagger + (1-p) \\ & (1-q)\hat{C}_R \otimes \hat{C}_W \rho_{ini} \hat{C}_R^\dagger \otimes \hat{C}_B^\dagger. \end{aligned} \tag{26}$$

The measurement of the quantum model in Fig. 6 outputs the payoff of the requestor and that of the worker by applying the following trace operations:

$$\begin{aligned}\bar{\$}_R(p,q) &= Tr(P_R \rho_{fin}), \\ \bar{\$}_W(p,q) &= Tr(P_W \rho_{fin}), \end{aligned} \tag{27}$$

where P_R and P_W are the payoff operators corresponding to the final state of the requestor and that of the worker, respectively.

$$\begin{aligned}P_R =& \Omega_r|00\rangle\langle 00| + (\Omega_r - m)|01\rangle\langle 01| + (\Omega_r + n)|10\rangle\langle 10| \\ & + (\Omega_r - m + n)|11\rangle\langle 11|, \\ P_W =& \Omega_w|00\rangle\langle 00| + (\Omega_w + b)|01\rangle\langle 01| + (\Omega_w - a)|10\rangle\langle 10| \\ & + (\Omega_w + b - a)|11\rangle\langle 11|. \end{aligned} \tag{28}$$

Finally, we obtain

$$\begin{aligned}\bar{\$}_R(p,q) &= pn[2(|c_{10}^2|+|c_{11}^2|)-1]+qm[2(|c_{00}^2|+|c_{10}^2|)-1]\\ &\quad+[\Omega_r-m(|c_{00}^2|+|c_{10}^2|)+n(|c_{00}^2|+|c_{01}^2|)],\\ \bar{\$}_W(p,q) &= pa[1-2(|c_{10}^2|+|c_{11}^2|)]+qb[1-2(|c_{00}^2|+|c_{10}^2|)]\\ &\quad+[\Omega_w+b(|c_{00}^2|+|c_{10}^2|)-a(|c_{00}^2|+|c_{01}^2|)]. \end{aligned} \tag{29}$$

Then the *Nash Equilibria* can be found by imposing the following two conditions

$$\begin{aligned}\bar{\$}_R(p^*,q^*)-\bar{\$}_R(p,q^*) &= (p-p^*)[1-2(|c_{10}^2|+|c_{11}^2|)]\geq 0,\\ \bar{\$}_W(p^*,q^*)-\bar{\$}_W(p^*,q) &= (q-q^*)[1-2(|c_{00}^2|+|c_{10}^2|)]\geq 0,\\ &\forall p,q\in[0,1]. \end{aligned} \tag{30}$$

where (30) indicates that whether or not the *Nash Equilibria* exists depends on the four coefficients, namely $|c_{00}^2|, |c_{01}^2|, |c_{10}^2|, |c_{11}^2|$. If we choose the initial density matrix corresponding to the state $|00\rangle$ when $|c_{00}|^2=1, |c_{01}|^2=|c_{10}|^2=|c_{11}|^2=0$, we can obtain the following payoffs by (29)

$$\begin{aligned}\bar{\$}_R(p,q) &= n(1-p)+m(q-1)+\Omega_r,\\ \bar{\$}_W(p,q) &= b(1-q)+a(p-1)+\Omega_w. \end{aligned} \tag{31}$$

Note that the payoffs in (31) are consistent with those of the classical games shown in (22). Therefore the *Nash Equilibrium* is obtained when $p^*=0, q^*=0$ again, which means that both players choose operator C to manipulate their initial state $|00\rangle$ to $|11\rangle$. In other words, both of them defect in the end. This dilemma remains unchanged with the other three initial quantum states $|01\rangle, |10\rangle, |11\rangle$, which implies that our quantum model allows us to recover the classical game by putting any basis strategy $|ij\rangle, i,j=0,1$, as the initial state.

When $|c_{10}^2|+|c_{11}^2|=\frac{1}{2}$ and $|c_{00}^2|+|c_{11}^2|=\frac{1}{2}$, let's consider a simple situation where $|c_{00}^2|=|c_{01}^2|=|c_{10}^2|=|c_{11}^2|=\frac{1}{4}$, which means that the requestor and the worker choose the four basis states with the same probability $\frac{1}{4}$. In this case, their initial states are $|\psi_{ini}\rangle=\frac{\sqrt{2}}{2}|0\rangle+\frac{\sqrt{2}}{2}|1\rangle$. An interesting finding is that their optimal payoffs are constant, namely $\bar{\$}_A(p,q)=\Omega_r-\frac{1}{2}m+\frac{1}{2}n$ and $\bar{\$}_B(p,q)=\Omega_w+\frac{1}{2}b-\frac{1}{2}a$. This outcome is not hard to understand since no matter which operation the requestor or the worker chooses, their final states remain fixed as $|\psi_{fin}\rangle=\frac{\sqrt{2}}{2}|0\rangle+\frac{\sqrt{2}}{2}|1\rangle=|\psi_{ini}\rangle$; thus $|\Psi_{fin}\rangle=\frac{1}{4}|00\rangle+\frac{1}{4}|01\rangle+\frac{1}{4}|10\rangle+\frac{1}{4}|11\rangle=|\Psi_{ini}\rangle$.

When $|c_{10}^2|+|c_{11}^2|>\frac{1}{2}$ and $|c_{00}^2|+|c_{11}^2|>\frac{1}{2}$, let's assume $|c_{11}^2|=\frac{1}{2}, |c_{00}^2|=|c_{01}^2|=|c_{10}^2|=\frac{1}{6}$, which indicates that the requestor and the worker both choose defection with probability $\frac{1}{4}$. According to (30), the game possesses a *Nash Equilibrium* when $p^*=q^*=1$. Consequently, the requestor and the worker are supposed to choose the operator $\hat{I}$ to keep their original states. In other words, they are more likely to defect. When $|c_{10}^2|+|c_{11}^2|<\frac{1}{2}$ and $|c_{00}^2|+|c_{11}^2|>\frac{1}{2}$, let's assume that $|c_{00}^2|=\frac{1}{2}, |c_{01}^2|=|c_{10}^2|=|c_{11}^2|=\frac{1}{6}$, which means that both players cooperate with a higher probability. In this case, we have $p^*=q^*=0$, which means that both players would choose $\hat{C}$ to invert the initial state to a state which offers a higher probability of defection. On the basis of considering possible coefficients, we reach the conclusion that no matter which basis state exists with a higher probability, both players tend to reach the same final state $|11\rangle$, which also implies that they are more likely to defect and the dilemma still exists.

2) Special Cases: We consider several special cases in this subsection. Let's introduce an entangled initial state which can be obtained by introducing an entangling gate $\hat{J}(\lambda)=cos\frac{\lambda}{2}\hat{I}\otimes\hat{I}+i\ sin\frac{\lambda}{2}\hat{C}\otimes\hat{C}$, where $\hat{J}(\lambda)$ is a function of λ and $\lambda\in[0,\frac{\pi}{2}]$, and i is an imaginary number. Assume that the original state is $|00\rangle$; then the entangled initial state is

$$|\psi_{ini}\rangle=\hat{J}(\lambda)|00\rangle=c_1|00\rangle+c_2|11\rangle, \tag{32}$$

where $c_1=cos\frac{\lambda}{2}, c_2=i\ sin\frac{\lambda}{2}$, and $|c_1^2|+|c_2^2|=1$. The initial density matrix $\rho_{ini}=|\psi_{ini}\rangle\langle\psi_{ini}|$, where $\langle\psi_{ini}|=c_1^*\langle 00|+c_2^*\langle 11|$, and c_1^* and c_2^* are the conjugate complex numbers of c_1 and c_2, respectively. Thus we have

$$\begin{aligned}\rho_{ini} &= |c_1|^2|00\rangle\langle 00|+c_1c_2^*|00\rangle\langle 11|\\ &\quad+c_1^*c_2|11\rangle\langle 00|+|c_2|^2|11\rangle\langle 11|. \end{aligned} \tag{33}$$

By using (26)–(28), the payoffs are written as

$$\begin{aligned}\bar{\$}_R(p,q) &= pn(|c_2|^2-|c_1|^2)+qm(|c_1|^2-|c_2|^2)\\ &\quad+[\Omega_r+|c_1|^2(n-m)],\\ \bar{\$}_W(p,q) &= pa(|c_1|^2-|c_2|^2)+qb(|c_2|^2-|c_1|^2)\\ &\quad+[\Omega_w+|c_1|^2(b-a)]. \end{aligned} \tag{34}$$

The criterion for judging the equilibria is then given by

$$\begin{aligned}\bar{\$}_R(p^*,q^*)-\bar{\$}_R(p,q^*) &= (p-p^*)n(|c_1^2|-|c_2^2|)\geq 0,\\ \bar{\$}_W(p^*,q^*)-\bar{\$}_W(p^*,q) &= (q-q^*)b(|c_1^2|-|c_2^2|)\geq 0,\\ &\forall p,q\in[0,1]. \end{aligned} \tag{35}$$

To illustrate how $\bar{\$}_R$, $\bar{\$}_W$, and $\bar{\$}_R/\bar{\$}_W$ vary with the degree of entanglement λ, we propose a case when $\Omega_r=12$, $\Omega_w=10, m=5, n=4, a=2, b=1$. As shown in Fig. 7, the payoffs of the requestor and the worker increase along with the growth of λ. If $\lambda=0$, $|\Psi_{ini}\rangle=|00\rangle$ corresponding to the classical game and $\bar{\$}_R=\Omega_r+n-m=11.0$, $\bar{\$}_W=\Omega_w+b-a=9.0$. When λ reaches its highest value $\frac{\pi}{2}$, the requestor and the worker get their highest payoffs as $\bar{\$}_R=\Omega_r+\frac{1}{2}(n-m)=11.5$ and $\bar{\$}_W=\Omega_w+\frac{1}{2}(b-a)=9.5$, respectively. This means that in a quantum game, an entangled state can lead to a better win-win situation compared to the classical strategy. Besides, $\bar{\$}_R/\bar{\$}_W$ decreases with λ, which indicates that the quantum strategy can narrow the income gap between the requestor and the worker. In Fig. 8, since $(m-n)$ and $(b-a)$ respectively denote the requestor's and the worker's loss when the requestor cooperates while the worker defects, $\bar{\$}_R$ and $\bar{\$}_W$ decrease with the growths of $(m-n)$ and $(b-a)$, respectively.

In another typical case of the entangled state, $\hat{J}(\lambda)$ can be written as $cos\frac{\lambda}{2}\hat{I}\otimes\hat{C}+i\ sin\frac{\lambda}{2}\hat{C}\otimes\hat{I}$ and thus $|\Psi_{ini}\rangle=c_1|01\rangle+$

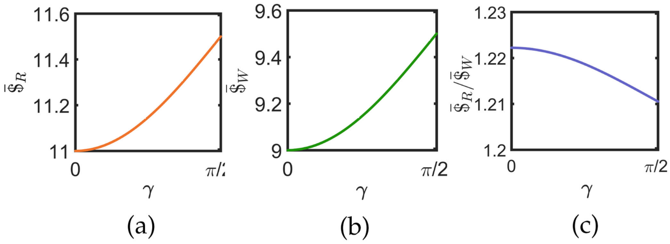

Fig. 7. $\bar{\$}_R, \bar{\$}_W$, and $\bar{\$}_R/\bar{\$}_W$ vary with λ.

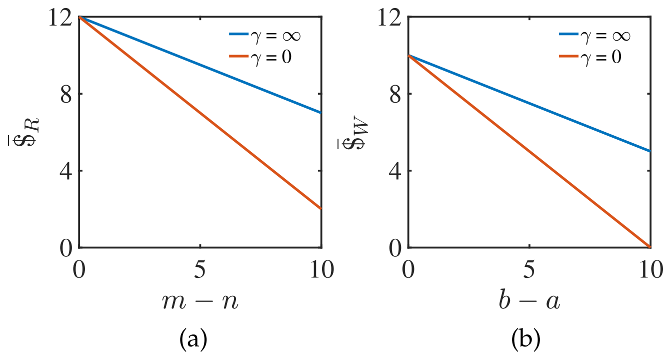

Fig. 8. $\bar{\$}_R, \bar{\$}_W$ with different $b-a$.

$c_2|10\rangle$. The payoffs can be calculated by using the same method as in the previous special case; thus finally we have

$$\begin{aligned}\bar{\$}_R(p,q) &= pn(|c_2|^2-|c_1|^2)+qm(|c_2|^2-|c_1|^2)\\ &\quad+[\Omega_r-m|c_2|^2+n|c_1|^2],\\ \bar{\$}_W(p,q) &= pa(|c_1|^2-|c_2|^2)+qb(|c_1|^2-|c_2|^2)\\ &\quad+[\Omega_w]+b|c_2|^2-a|c_1|^2]. \end{aligned} \tag{36}$$

We also notice that the values of $\bar{\$}_R, \bar{\$}_W$, and $\bar{\$}_R/\bar{\$}_W$ *vs.* various λ, $\bar{\$}_R$, and $\bar{\$}_W$ for different $b-a$ demonstrate the same trends as those in Fig. 7 and Fig. 8.

VI. Discussion and Future Research

In this article, we investigate crowdsourcing games from both the classical and the quantum perspectives. Specifically, we construct a quota-oriented crowdsourcing game and figure out the dilemma where the workers' optimal strategies cannot maximize their payoffs. Then we propose a quantum game model to solve this dilemma and carry out both theoretical and numerical studies. Our results reveal a novel feature of the quantum game, i.e., the impacts of the entanglement degree on the optimal payoffs can be used to increase the participants' welfare. Furthermore, we propose a quality-oriented crowdsourcing game involving a requestor and a worker and prove that a dilemma exists in the classical version of the game. In the corresponding quantum game model, we adopt the density matrix approach to simplify the calculations. The two quantum games are reduced to classical ones when there is no entanglement, which proves the correctness of our approach. Besides, our analysis based on the quantum strategy indicates that entanglement can increase the payoffs of all players.

Quantum computing is a new technology that has not been fully accessible to the general public even though existing projects (e.g., IBM Q, D-Wave, ProjectQ) have provided special-purpose quantum computers and great opportunities to expand the corresponding limited experience. For example, D-Wave [34] has provided a cloud-based platform, namely, D-Wave Leap, where users can freely access a real quantum computer to solve specific problems. The IBM Q [34] also provides quantum cloud services and software platforms, which support experiments on optimization, finance, and AI. The excellent features brought by quantum computers encourage us to study the quantum game theory from a theoretical perspective concerning crowdsourcing, which involves a quantum network [35]–[38] rather than a single quantum computer. To our knowledge, the crowdsourcing game model proposed in this article is both pioneering and fundamental, and the quantum analysis contributes to a new method of mitigating the competitions among the requestors and workers. That is, the introduction of the entanglement state can lead to compulsive collaborations that can hardly be managed in a classical game model. Also, the approach proposed in this article can be generalized to address problems in those scenarios when collaboration is required. In future, we will extend our quantum game model to suit the more complicated crowdsourcing scenarios, such as those that involve real quantum computers, in which collaborations among the players should be established on a new game structure to improve social welfare. Moreover, practical applications on quantum crowdsourcing could appear with the development of general-purpose quantum computers and the spread of quantum cloud services.

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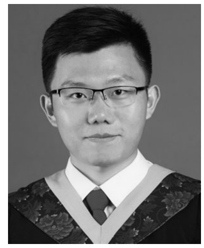

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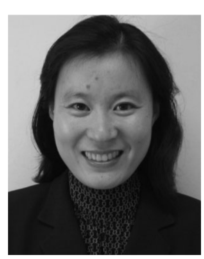

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