Social Indicators Research
https://doi.org/10.1007/s11205-022-02911-y

ORIGINAL RESEARCH

On Measuring Segregation in a Multigroup Context: Standardized Versus Unstandardized Indices

Coral del Río[1] · **Olga Alonso-Villar**[2]

Accepted: 4 March 2022
© The Author(s) 2022

Abstract
There has been little discussion about the consequences of using standardized, rather than unstandardized, segregation measures when comparing societies with different demographic compositions. This paper explores standardization in a multigroup setting through an analytical framework that offers a clear distinction between the measurement of overall and local segregation, embeds existing indices within this framework, and addresses gaps in previous research. The local approach developed here allows us to focus on the principle of transfers used in the measurement of overall segregation from a new angle and brings analytical support to the interpretation of the components of standardized overall measures as the segregation levels of the groups involved. This approach also helps clarify the debate around the measurement of school segregation since the distinction between local and overall measures, together with standardization, is key to understanding the different proposals that have been used in empirical studies. This research also gives formal support to empirical strategies that compare the distribution of a minority group with that of the remaining population since they can be viewed as standardized local segregation measures satisfying basic properties.

Keywords Multigroup segregation · Standardized segregation indices · Local segregation curves · Local segregation indices

JEL Classification D63 · J15 · J16 · J71

1 Introduction

As societies grow more diverse—whether in terms of race, ethnicity, immigration status, or other characteristics of individuals—there is an increasing need to measure segregation through a framework that involves more than two groups. Since the 1990s, several indicators have been developed to quantify overall multigroup segregation (Boisso et al., 1994; Frankel & Volij, 2011; Reardon & Firebaugh, 2002; Silber, 1992), mainly according to a

✉ Olga Alonso-Villar
ovillar@uvigo.es

[1] ECOBAS, Universidade de Vigo, Vigo, Spain

[2] Departamento de Economía Aplicada, Universidade de Vigo, 36310 Vigo, Spain

Published online: 19 March 2022

perspective of evenness that focuses on differences in the sorting of demographic groups across organizational units such as occupations, schools, and neighborhoods.

Overall multigroup measures are useful in providing a summary statistic of the simultaneous distributional discrepancies that exist among all the demographic groups into which society is partitioned. However, in multigroup contexts, one may want to take a step further and identify the situation of each demographic group. To this end, one could use local segregation measures (as opposed to overall segregation measures), which allow to quantify the degree of unevenness of each group separately (Alonso-Villar & Del Río, 2010).[1] This local approach is consistent with the measurement of overall segregation, given that the latter can be expressed as the weighted average of the local segregation of the groups involved, and is especially useful for pinpointing the situations of small groups, whose uneven distributions across organizational units may have a limited impact on overall segregation (Del Río & Alonso-Villar, 2019; Palencia-Esteban, 2021).

The abovementioned local segregation indices satisfy several basic properties, in particular *scale invariance*, according to which if the size of a group (e.g., white women) is multiplied by a positive number, the segregation of that group remains unaffected provided there is no change to its distribution across units (e.g., occupations) or to the relative size of each unit.[2] The property of *scale invariance* may result in the belief that the segregation of a group is independent of the size of the group. However, as we will discuss in more detail later, the demographic share of a group impacts the highest segregation that the group can attain. Thus, for example, if the economy has 200 workers and 5 occupations of equal size, a group consisting of 40 individuals is fully segregated if it is concentrated in occupations with no workers from other groups, i.e., (40, 0, 0, 0, 0), which implies that this group has no presence in occupations accounting for 80% of the total population. This scenario is impossible for a group of 80 individuals because, for such a group to be fully segregated, no group members may be found in occupations representing 60% of the total population, i.e., (40, 40, 0, 0, 0). In other words, this group is missing from a smaller part of the economy (60% vs. 80%). Accounting for this is particularly important when comparing the segregation levels of groups of very different relative sizes, exploring the segregation of a growing group over time, or in international comparisons when analyzing a group whose relative size varies significantly among countries.

This question is not only relevant in the case of local segregation. The relative size of the groups may also determine the maximum value attainable by overall indices. In fact, many overall indices are not equal to 1 when there is full segregation. This is the case for the I_p index (Silber, 1992), the (unstandardized) Gini index (Alonso-Villar and Del Rio, 2010), and the mutual information index (Frankel & Volij, 2011; Theil & Finizza, 1971). Reardon and Firebaugh (2002) opted for standardized (or normalized) overall indices between 0 and 1. Making use of disproportionality functions that compare the presence of each group in each unit with its share in the economy, these authors derived the generalized dissimilarity index, the generalized Gini index, and the Theil information theory index. These three

[1] This approach also allows for the measurement of the consequences of segregation for each group in monetary terms and in terms of objective well-being (Alonso-Villar and Del Río, 2017).

[2] Dealing with the segregation of a group requires adapting the principles of segregation measurement, which have focused on overall segregation, to this context. As discussed later on, this property—adapted from the one used in the measurement of income inequality—differs from both the *scale invariance* proposed by Frankel and Volij (2011) and the *composition invariance* put forward by James and Taueber (1985) in the case of overall segregation.

indices result from dividing each of the abovementioned unstandardized overall indicators by its maximum value, which is a function of the groups' shares (Reardon & Firebaugh, 2002).[3] However, as far as we know, there has been little discussion of the consequences of using standardized versus unstandardized measures (Mora and Ruiz-Castillo, 2011).

This paper explores standardization in a multigroup setting providing a local/overall framework within which existing segregation measures are embedded whereas new ones are proposed to fill some of the gaps. For this purpose, this paper: a) develops standardized local segregation indices, which allows completion of the picture; b) evaluates them against a set of properties; c) establishes the conditions under which the ranking provided by these indices is consistent with that of the local segregation curves proposed in the literature; d) links these local measures with overall measures and reflects on what the local approach shows us about the measurement of overall segregation; and e) applies these standardized local measures to quantify the occupational segregation of white women in the US and compares them to their unstandardized versions.

Therefore, our research not only allows a deeper exploration into the measurement of a group's segregation (providing measures that allow quantifying this phenomenon whereas accounting for the group's size) but also a better understanding of the *principle of transfers* (analyzing whether this property should be a requirement of overall segregation or instead local segregation). Furthermore, the local approach developed here offers analytical support to those empirical strategies used in the literature to deal with the situation of a group which compare the distribution of each minority group with that of the remaining population and also those which develop intuitive interpretations of the components of standardized overall multigroup indices without formally addressing this (Iceberg, 2004; Maloutas & Spyrellis, 2020; Marcińczak et al., 2016; Queneau, 2009; Watts, 1995). In fact, this paper shows that these ad-hoc measures employed in empirical work to quantify the segregation of a group in a multigroup context are actually standardized local segregation measures satisfying several basic properties. This paper also throws new light on the debate about how to measure school segregation since the distinction between local and overall measures, together with standardization, is key to understanding the relationship between the different proposals that have been employed in empirical studies (Allen & Vignoles, 2007; Gorard & Taylor, 2002).

This paper is structured as follows. Section 2 presents the local segregation approach, extends properties previously proposed in the literature, and discusses how the maximum segregation of a group can be determined. Section 3 defines standardized local segregation measures and establishes the properties that make these indices consistent with a dominance criterion based on local segregation curves. Additionally, it expands the knowledge of the measurement of overall segregation embedding previous measures in a local/overall (un)standardization framework. Section 4 offers an illustration of the new measures through the case of the occupational segregation of white women in U.S. metropolitan areas. Section 5 concludes.

[3] These authors developed another standardized overall segregation index, based on the squared coefficient of variation, whose maximum depends on the number of groups.

2 The Local Segregation Approach

Although segregation involves the relationships among the distributions of all groups across units, an adequate measurement of each group's degree of unevenness allows for a better understanding of the phenomenon. Local segregation measures satisfying desirable properties allow for not only identification of each group's situation but also explanation of the measurement of overall segregation. This section presents this approach and extends some properties previously proposed in the literature so as to offer a clearer connection between the measurement of local and overall segregation.

2.1 Measuring a Group's Segregation

Let g be one of the N mutually exclusive groups of society ($g=1,\ldots, N$). c_j^g denotes the number of individuals of group g in unit j ($j=1,\ldots, J$), t_j is the number of total individuals in that unit ($c_j^g \leq t_j$), $C^g = \sum_j c_j^g$ is the group's size, and $T = \sum_j t_j$ is total population.

If group g represents, for example, 20% of the total population ($\frac{C^g}{T} = 0.2$) and is evenly distributed across units, one would expect it to account for 20% of the population in each unit j ($\frac{c_j^g}{t_j} = 0.2$). Or equivalently, if unit j accounts for, say, 5% of the population ($\frac{t_j}{T} = 0.05$), it would be "fair" to find 5% of the group in that unit ($\frac{c_j^g}{C^g} = 0.05$). As long as the group is overrepresented in some units and underrepresented in others, the group is unevenly distributed. This is precisely the idea behind the local segregation curve (Alonso-Villar & Del Río, 2010), which shows how far the distribution of the group across units is from even distribution (according to which the weight of the group in each unit, $\frac{c_j^g}{t_j}$, should equal its weight in society, $\frac{C^g}{T}$; or equivalently, $\frac{c_j^g}{C^g}$ equals $\frac{t_j}{T}$).[4] This curve is similar to the Lorenz curve used in the inequality literature.

To build the local segregation curve of group g, first, we must rank the units in ascending order of the ratio $\frac{c_j^g}{t_j}$. Then, the cumulative proportion of total individuals is plotted on the horizontal axis, while the cumulative proportion of group's g individuals is plotted on the vertical axis. Namely, if we denote by $\tau_j \equiv \sum_{i \leq j} \frac{t_i}{T}$ the proportion of individuals who are in the first j units, the segregation curve at point τ_j is

$$S^g(\tau_j) = \frac{\sum_{i \leq j} c_i^g}{C^g}$$

which represents the proportion of group's g individuals in these units. If the group were evenly distributed across units, this curve would be equal to the 45° line. As long as the group is underrepresented in some units and overrepresented in others, the curve departs from that line, approaching the horizontal axis. Note that this curve differs from the well-known segregation curve discussed in Duncan and Duncan (1955), where the distribution of a group across units is compared with that of another group.

[4] There has been some debate in the literature about whether the distribution of a group across units should be compared to the distribution of total population. However, note that since $c_j^g \big/ t_j = C^g/T \Leftrightarrow c_j^g \big/ C^g = t_j/T$, comparing $c_j^g \big/ C^g$ to t_j/T is the same as comparing $c_j^g \big/ t_j$ to C^g/T.

The local segregation curve allows to visualize the segregation level of a group and can be used to compare different scenarios. Thus, if one curve dominates another (i.e., no point of the former curve lies below the latter curve and does at some point lie above, as is the case of S^g relative to S^{g*} in Fig. 1), we can say that the group is less segregated in the first case than in the second.

Local segregation curves are very useful to illustrate the effect of a group's size on its maximum segregation level. As mentioned above, the maximum segregation of a group is attained when it is fully concentrated in units with no members of other groups.[5] Let us assume, without loss of generality, that a group is fully segregated in one unit.[6] Figure 1 illustrates this situation as the case of a group that accounts for 20% of the population.

The curve of maximum segregation, denoted by S^{g*}, is equal to 0 up to the unit in which the group is fully concentrated (i.e., at point $1 - \frac{C^g}{T}$) and jumps to 1 when that unit is aggregated with the previous ones (i.e., when the cumulative proportion of population is 1), thereby rendering a straight line between these two points.

Alonso-Villar and Del Río (2010) proposed several local segregation indices—adapted from well-known inequality measures—to quantify the extent to which a local segregation curve diverges from an even distribution of the group across units (the 45° line). These indices are D^g, G^g, Φ_1^g, and Φ_α^g (with $\alpha \neq 0, 1$)—which includes the local index Φ_2^g based on the squared coefficient of variation—and their maximum values are labelled, respectively, D^{g*}, G^{g*},Φ_1^{g*} and Φ_α^{g*}(see Table 1). The local dissimilarity index, D^g, measures the highest vertical distance of the curve to the 45° line. Along with its graphical interpretation, this index has a very intuitive meaning: when multiplied by 100, it represents the percentage of group g individuals who would have to switch units for the group to have zero segregation while keeping the size of units unchanged. This index was initially proposed by Moir and Selby Smith (1979) in a binary context to explore labor segregation by gender, although its properties in a multigroup context, together with its relation to the local segregation curve, were not explored until Alonso-Villar and Del Río (2010). It has been extensively used to explore school segregation, where is usually called Gorard's index (Croxford & Raffe, 2013; Gorard & Taylor, 2002).

The local Gini index, G^g, also has a graphical interpretation: it is equal to twice the area between the local segregation curve and the 45° line. On the other hand, the local generalized entropy family, Φ_α^g, offers a different index depending on a parameter, α, which accounts for both the group's underrepresentation in units (i.e., the lower part of the curve) and its overrepresentation (i.e., the upper part). The lower (larger) the value of α, the more sensitive the index is to the group's underrepresentation (overrepresentation). G^g and Φ_α^g are consistent with the dominance criterion given that if the local segregation curve of a group is above to that of another, the first group is less segregated than the second one according to any of these indices.

These local indices are related to overall indices. Thus, the weighted average of local indices D^g, G^g, Φ_1^g and Φ_2^g (with weights equal to the groups' shares) are, respectively, equal to the I_p index (Silber, 1992), the unstandardized overall Gini index, which we denote

[5] Note that, in the real world, full segregation may not be possible because the size of the units may not fit with the group's size.

[6] The property of *insensibility to proportional subdivisions*, which we discuss in more detail later, ensures that, if an index satisfies it, we can focus on cases in which the group is concentrated in one unit of size equal to that of the group because distributions of maximum segregation across several units would be equivalent to this.

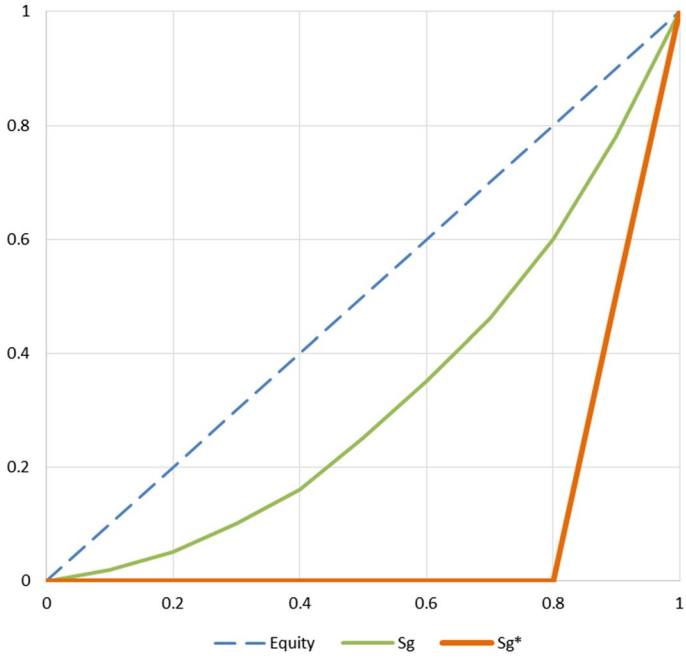

Fig. 1 Two examples of local segregation curves

here by G_u (Alonso-Villar and Del Rio, 2010), the mutual information index, M (Frankel & Volij, 2011; Theil & Finizza, 1971), and the unstandardized overall index based on the squared coefficient of variation, which we denote here by C_u.[7]

It is important to note that, although overall indices can be decomposed by groups in several ways, the components of such decompositions may not necessarily be good measures of the groups' segregation. For example, the mutual information index can be written as the weighted average (with weights equal to the groups' shares) of the difference between the entropy of the distribution of the population across units and the entropy of each group (Frankel & Volij, 2011). However, the difference between entropies is not a sensible local segregation indicator because its minimum value is not attained when the group is distributed across units in the same manner as the total population is—the difference can take negative values—nor does it satisfy the property of *insensibility to proportional subdivisions*—the entropy is sensitive to the number of units. On the contrary, the indices D^g, G^g, and Φ_α^g are truly local segregation measures because they satisfy a wide range of desirable properties, as we discuss below.

2.2 Properties for Measuring Local Segregation

To determine whether these local indices are suitable for measuring a group's segregation, we list some basic properties proposed in the literature, put forth new properties (which are

[7] C_u is the unstandardized version of Reardon and Firebaugh's (2002) C index divided by 2.

Table 1 Unstandardized and standardized local segregation indices

Local segregation indices	Maximum value of the local index	Standardized local segregation indices
$D^g = \frac{1}{2}\sum_j \left\lvert \frac{c_j^g}{C^g} - \frac{t_j}{T} \right\rvert$	$D^{g*} = 1 - \frac{C^g}{T}$	$\tilde{D}^g = \frac{\frac{1}{2}\sum_j \left\lvert \frac{c_j^g}{C^g} - \frac{t_j}{T} \right\rvert}{1-\frac{C^g}{T}}$
$G^g = \frac{\sum_{i,j} \frac{t_i}{T}\frac{t_j}{T}\left\lvert \frac{c_i^g}{t_i} - \frac{c_j^g}{t_j} \right\rvert}{2\frac{C^g}{T}}$	$G^{g*} = 1 - \frac{C^g}{T}$	$\tilde{G}^g = \frac{\sum_{i,j} \frac{t_i}{T}\frac{t_j}{T}\left\lvert \frac{c_i^g}{t_i} - \frac{c_j^g}{t_j} \right\rvert}{2\frac{C^g}{T}\left(1-\frac{C^g}{T}\right)}$
$\Phi_1^g = \sum_j \frac{c_j^g}{C^g} \ln\left(\frac{c_j^g/C^g}{t_j/T}\right)$	$\Phi_1^{g*} = \ln\left(\frac{T}{C^g}\right)$	$\tilde{\Phi}_1^g = \frac{\sum_j \frac{c_j^g}{C^g} \ln\left(\frac{c_j^g/C^g}{t_j/T}\right)}{\ln\left(\frac{T}{C^g}\right)}$
$\Phi_\alpha^g = \frac{1}{\alpha(\alpha-1)} \sum_j \frac{t_j}{T}\left[\left(\frac{c_j^g/C^g}{t_j/T}\right)^\alpha - 1\right]$	$\Phi_\alpha^{g*} = \frac{1}{\alpha(\alpha-1)}\left[\left(\frac{C^g}{T}\right)^{1-\alpha} - 1\right]$	$\tilde{\Phi}_\alpha^g = \frac{\sum_j \frac{t_j}{T}\left[\left(\frac{c_j^g/C^g}{t_j/T}\right)^\alpha - 1\right]}{\left(\frac{C^g}{T}\right)^{1-\alpha} - 1}$

The expression for Φ_α^g is valid for $\alpha \neq 0, 1$.

useful when we relate these indices to their overall versions), and determine whether our local measures satisfy them.

Let $\Theta^g(c^g, t)$ be a local segregation measure, where c^g is the vector representing the number of individuals of group g in each unit j (c_j^g) and t is the vector indicating the number of individuals in each unit j (t_j). Alonso-Villar and Del Río (2010) established several properties that any unstandardized local segregation measure should verify. We accompany the formal definitions with examples in which the benchmark is $t = (40, 40, 20)$ and $c^g = (2, 3, 5)$.

(a) *Size Invariance*, which signifies that if we multiply both the number of individuals of the group and the number of total workers in each unit by a positive number, the segregation of the group does not change. Namely, if $c_j^{g\prime} = \lambda c_j^g$ and $t_j\prime = \lambda t_j$ for any $\lambda > 0$ and $j = 1, ..., J$, then $\Theta^g(c^{g\prime}, t\prime) = \Theta^g(c^g, t)$. For example, if $t\prime = (80, 80, 40)$ and $c^{g\prime} = (4, 6, 10)$, g's segregation level equals that in the benchmark. In other words, a group's segregation level does not depend on whether the figures are expressed in hundreds of individuals or thousands.

(b) *Scale Invariance* refers to the fact that the group's segregation does not change if, in each unit, the number of individuals of the group is multiplied by a positive number and the total number of individuals is multiplied by another (whenever these changes are compatible). Namely, if $c_j^{g\prime} = \lambda c_j^g$ and $t_j\prime = \beta t_j$ for $j = 1, ..., J$ (where $\lambda > 0, \beta > 0$, and $\lambda c_j^g \leq \beta t_j$), then $\Theta^g(c^{g\prime}, t\prime) = \Theta^g(c^g, t)$.[8] For example, if $t\prime = (120, 120, 60)$ and

[8] Note that this property differs from the *scale invariance* proposed by Frankel and Volij (2011) to measure overall rather than local segregation since these authors require that the index remain unaltered when all groups increase by the same proportion in all units. It also differs from the *composition invariance* put forward by James and Taueber (1985), which requires that overall segregation does not change when the number of individuals of a group is multiplied by a constant factor in each unit, a criterion not free of controversy (Reardon and Firebaugh, 2002; White, 1986). The *scale invariance* criterion used in this paper keeps the essence of the one used in the measurement of income inequality, a property widely accepted in that field, although other approaches, as in the case of absolute and intermediate inequality, also exist.

$c^g\prime = (4, 6, 10)$, g's segregation level equals that in the benchmark. In other words, a group's segregation level does not depend on the total number of individuals in the economy or the group's size.

(c) *Symmetry*, which means that if the units are permuted, the segregation of the group remains unaltered. Namely, if $c_j^g\prime = c_{\Pi(j)}^g$ and $t_j\prime = t_{\Pi(j)}$, where $(\Pi(1), ..., \Pi(J))$ is a permutation of units $(1, ..., J)$, then $\Theta^g(c^g\prime, t\prime) = \Theta^g(c^g, t)$. For example, if $t\prime = (40, 20, 40)$ and $c^g\prime = (2, 5, 3)$, g's segregation level equals that in the benchmark. In other words, a group's segregation level does not depend on the order in which the occupations are listed.

(d) *Insensitivity to Proportional Subdivisions* of units, i.e., the segregation level of the group does not change if a unit is split into several units of equal size with identical number of individuals of the group. Namely, assuming for the sake of simplicity that we split the last unit in $K > 0$ units, if $c_j^g\prime = c_j^g$ and $t_j\prime = t_j$ for any $j = 1, ..., J-1$, and $c_{J+i}^g\prime = \frac{c_J^g}{K}$ and $t_{j+i}\prime = \frac{t_J}{K}$ for $i = 0, ..., K-1$, then $\Theta^g(c^g\prime, t\prime) = \Theta^g(c^g, t)$. For example, if $t\prime = (20, 20, 40, 20)$ and $c^g\prime = (1, 1, 3, 5)$, g's segregation equals that in the benchmark. In other words, the number of occupations does not matter as long as they do not bring heterogeneity.

(e) *Sensitivity to Disequalizing Movements (type I)*: Disequalizing movements of the group between equally-sized units, the size of which does not change after that movement (i.e., if a unit with a lower number of individuals of the target group than another loses some of those individuals in favor of the latter, other things being equal) increase the group's segregation.[9] Namely, if $c_i^g\prime = c_i^g - d$ and $c_h^g\prime = c_h^g + d$, where i and h are two units such that $c_i^g < c_h^g$ and $t_i = t_h$, whereas $c_j^g\prime = c_j^g$ for $j \neq i, h$, then $\Theta^g(c^g\prime, t) > \Theta^g(c^g, t)$. For example, if $c^g\prime = (1, 4, 5)$, g's segregation is higher than in the benchmark. In other words, when group g moves from an occupation to another of the same size in which its presence is larger and this movement is accompanied by an opposite movement of individuals from other groups, g's segregation increases.

As these authors proved, properties (b) to (e) are very important because render an index Θ^g consistent with the dominance criterion given by the local segregation curves (this is analogous to what happens when using the Lorenz curves to measure income inequality). In other words, a local segregation curve dominates another if, and only if, for any local segregation index Θ^g that satisfies *scale invariance*, *symmetry*, *insensitivity to proportional divisions*, and *sensitivity to disequalizing movements type I*, Θ^g is lower in the former case than in the latter.

Note that alternative definitions of *sensitivity to disequalizing movements* may be articulated depending on how strictly we conceive of the circumstances under which we expect segregation to increase. This is especially relevant to assess the implications of property (e), given that requiring that a *disequalizing movement* leads to an increase in a group's segregation only when that movement involves equal-size units (whose size remains unaltered) might seem of limited scope and unconnected to the property usually assumed when dealing with overall segregation. To delve deeper into this issue, we put forth two new properties here:

[9] In Alonso-Villar and Del Río (2010) this property appears as "movement between groups.".

(f) *Sensitivity to Disequalizing Movements (type II)*: Disequalizing movements of the group between one unit and another unit with a higher representation of the group (i.e., if the group's representation diminishes in the former unit and rises in the latter), while the size of these units do not change, produce an increase in the group's segregation. Namely, if $c_i^g\prime = c_i^g - d$ and $c_h^g\prime = c_h^g + d$, where i and h are two units such that $\frac{c_i^g}{t_i} < \frac{c_h^g}{t_h}$, whereas $c_j^g\prime = c_j^g$ for $j \neq i, h$, then $\Theta^g(c^g\prime, t) > \Theta^g(c^g, t)$. For example, if $c^g\prime = (2, 2, 6)$, g's segregation is higher than in the benchmark. In other words, when members of the group move from an occupation to another in which its relative presence is larger and this movement is accompanied by an opposite movement of individuals from other groups, the group's segregation increases (regardless of whether these occupations have the same size).

(g) *Sensitivity to Disequalizing Movements (type III)*: Disequalizing movements of the group between one unit and another unit with a higher representation of the group (i.e., if the group's representation diminishes in the former unit and rises in the latter), whereas the sizes of these units change accordingly, result in an increase in the group's segregation. Namely, if $c_i^g\prime = c_i^g - d$, $c_h^g\prime = c_h^g + d$, $t_i' = t_i - d$, and $t_h\prime = t_h + d$, i and h being two units such that $\frac{c_i^g}{t_i} < \frac{c_h^g}{t_h}$, whereas $c_j^g\prime = c_j^g$ and $t_j\prime = t_j$ for $j \neq i, h$, then $\Theta^g(c^g\prime, t\prime) > \Theta^g(c^g, t)$. For example, if $t\prime = (40, 39, 21)$ and $c^g\prime = (2, 2, 6)$, g's segregation is higher than in the benchmark. In other words, when group g moves from an occupation to another in which its relative presence is higher, g's segregation increases (regardless of whether the incumbent occupations have the same size and without any replacement requirement).

Both properties allow us to compare more scenarios than does property (e) and are related to properties proposed in the measurement of overall segregation. Thus, *sensitivity to disequalizing movements type III* is the local version of the *principle of transfers* proposed by Reardon and Firebaugh (2002) to measure overall multigroup segregation.[10] Likewise, *sensitivity to disequalizing movements type II* may be seen as the local version of the *principle of exchanges*.[11]

The question we now pose is how these new properties are related to property (e) and whether are commonly fulfilled by local segregation indices. Propositions 1 and 2 reveal that properties (f) and (g) are not difficult to satisfy if property (e) holds. In fact, as Corollary 1 shows, many local indices meet them.

Proposition 1. *If a local segregation index $\Theta^g(c^g, t)$ satisfies insensitivity to proportional subdivisions and sensitivity to disequalizing movements type I, then it also fulfills sensitivity to disequalizing movements type II.*

Proof. See Appendix.

[10] Overall segregation must increase whenever an individual of a group moves from one unit to another in which the group has a higher representation (which implies changes to those units' sizes).

[11] This property requires that overall segregation rise when two individuals of different target groups exchange their positions moving from an unit where the incumbent group has a lower representation to a unit with a higher representation (which implies that the size of those units do not change).

Proposition 2 *Any local segregation index $\Theta^g(c^g, t)$ consistent with the dominance criterion given by the local segregation curves satisfies sensitivity to disequalizing movements type III.*

Proof. See Appendix.

Corollary 1. *The indices G^g and Φ^g_α satisfy size and scale invariance, symmetry, insensitivity to proportional subdivisions, and sensitivity to disequalizing movements type I, type II, and type III. Index D^g fulfills size and scale invariance, symmetry, and insensitivity to proportional subdivisions.*

Proof. See Appendix.

As we will see in Sect. 3, this result is interesting because it allows for a better understanding of some of the properties usually assumed to measure overall segregation. Thus, the fact than the indices G^g and Φ^g_α verify *sensitivity to disequalizing movements type III* allows us to question whether the *principle of transfers* is a necessary requirement for overall multigroup segregation measures.

3 A New Proposal: Standardized Local Segregation Measures

As mentioned earlier, the maximum segregation level of a group is not independent of the group's size. The reason is that when a group is small, it can be absent from units that account for a large share of total population, whereas this situation is impossible for large groups. How, therefore, is it possible to compare the segregation of two groups that differ in terms of relative size but are distributed across units in the same way? Here we explore a procedure that measures the segregation of a group accounting not merely for how the group is distributed across units, but also the maximum segregation attainable by the group.

3.1 Standardized Local Segregation Measures

We develop several standardized local indicators, globally denoted by $\tilde{\Theta}^g(c^g, t)$, defined as the quotient between a local segregation index, $\Theta^g(c^g, t)$, and the value of that index when the group is fully segregated, Θ^{g*}. Namely, $\tilde{\Theta}^g(c^g, t) = \frac{\Theta^g(c^g,t)}{\Theta^{g*}}$. This approach squares with the measurement of overall segregation put forward by Reardon and Firebaugh (2002) in that we divide the index by the maximum segregation level, although in our case segregation refers to a group (say, white women) rather than to overall segregation (say, by gender and race).[12]

[12] When standardizing an index dividing it by its maximum, scholars use a theoretical maximum that does not account for the units but instead approximates the "actual" maximum existing in each empirical case, which depends on the number of units and their sizes. Consequently, as opposed to the theoretical maximum, the "actual" distribution of maximum segregation is not unique since may vary depending on the index used. This theoretical maximum takes the groups' weights as given (unlike the absolute maximum reached if the shares of the groups and units were not fixed).

To measure the standardized segregation of a group we propose using the indices $\tilde{D}^g, \tilde{G}^g, \tilde{\Phi}_1^g$, and $\tilde{\Phi}_\alpha^g$, shown in Table 1, which are obtained dividing the indices D^g, G^g, Φ_1^g, and Φ_α^g, respectively, by their values when the group is fully segregated ($D^{g*}, G^{g*}, \Phi_1^{g*}$, and Φ_α^{g*}). Imposing this standardization yields a maximum value of indices $\tilde{D}^g, \tilde{G}^g, \tilde{\Phi}_1^g$, and $\tilde{\Phi}_\alpha^g$ that is always 1, which facilitates comparisons among different groups or a group across time and space.

Thus, for example, making use of the interpretation of D^g mentioned above, $\tilde{D}^g$ may be thought of as the proportion of group g individuals who must transfer among units to attain zero segregation divided by the proportion who must move if the group were fully segregated. This allows us to assess whether the segregation level given by the index D^g is high or low when taking into account the maximum segregation of the group, which depends on its demographic share. If, for example, $t = (40, 30, 20, 10)$ and $c^g = (3, 3, 5, 9)$, then $D^g = 0.4$. This means that 8 workers would have to change occupations for the group to be evenly distributed (i.e., to achieve distribution $(8, 6, 4, 2)$). $\tilde{D}^g = 0.5$ brings additional information: 0.4 is in this case a relatively high segregation level because the above 8 workers represent half of the workers (16) that would have to move if there were maximum segregation (i.e., $c^{g*} = (0, 0, 20, 0)$ and $D^{g*} = 0.8$).

Corollary 2 shows the properties fulfilled by these standardized indices.

Corollary 2. *The indices $\tilde{G}^g$ and $\tilde{\Phi}_\alpha^g$ satisfy size invariance, symmetry, insensitivity to proportional subdivisions, and sensitivity to disequalizing movements type I, type II, and type III. The index $\tilde{D}^g$ fulfills size invariance, symmetry, and insensitivity to proportional subdivisions.*

Proof. See Appendix.

The next theorem demonstrates the relationship that exists between the dominance criterion associated with the local segregation curves and the standardized indices.

Theorem. *If the local segregation curve of a group dominates that of another group whereas the opposite holds for the curves of maximum segregation, then segregation will be lower in the first case than in the second for any standardized local segregation index $\tilde{\Theta}^g(c^g, t) = \frac{\Theta^g(c^g,t)}{\Theta^{g*}}$, where $\Theta^g(c^g, t)$ satisfies scale invariance, symmetry, insensitivity to proportional subdivisions, and sensitivity to disequalizing movements type I.*[13]

Proof. See Appendix.

Note that the properties that we require Θ^g meet are the properties that render these indices consistent with the dominance criterion established by Alonso-Villar and Del Río (2010). Accordingly, it follows that if the local segregation curve of a group is above another (i.e., the former dominates the latter) and the ranking is the reverse for these groups' curves of maximum segregation, we need not calculate any $\tilde{\Theta}^g$ index (included in the set of indices established in the theorem) because all of them would lead to the same conclusion: segregation is lower for the first group.

[13] If there is dominance in one case and the curves are equal in the other case, the theorem still holds.

Finally, it follows from the next proposition that to determine whether the curve of maximum segregation for a group dominates that of another group we need only know these groups' demographic shares.

Proposition 3. *The local segregation curve of a group associated with that group's maximum segregation dominates that of another group if, and only if, in the former case the group accounts for a larger share of the population than it does in the latter.*

Proof. See Appendix.

3.2 Relation Between Standardized Local and Overall Segregation Measures

In their 2002 paper, Reardon and Firebaugh derived several standardized overall measures using the notion of disproportionality (i.e., the overrepresentation and underrepresentation of groups in units), and assessed them against James and Taeuber's (1985) criteria. As Table 2 shows, these overall (multigroup) indices, D,[14] G,[15] H,[16] and C,[17] can be decomposed, respectively, in terms of standardized local indices, $\tilde{D}^g$, $\tilde{G}^g$, $\tilde{\Phi}_1^g$, and $\tilde{\Phi}_2^g$, in such a way that overall segregation is the weighted average of the local segregation of the groups involved, although so far the literature has not noticed that $\tilde{D}^g$, $\tilde{G}^g$, $\tilde{\Phi}_1^g$, and $\tilde{\Phi}_2^g$ are indeed segregation measures.

Table 3 summarizes the local/overall (un)standardization framework developed in this paper, embedding existing and new indices within it. This allows us to visualize the relationships among the indices, where arrows are used to link local and overall segregation indices, solid lines are used to link overall binary and multigroup segregation indices, and dashed lines connect standardized and unstandardized overall indices.

Standardized (local and overall) measures are useful because they allow us to compare each scenario with the worst possible scenario (that of maximum segregation).[18] In any case, one should bear in mind that, although some of the most popular overall segregation

[14] D is equivalent to that proposed by Morgan (1975) and Sakoda (1981). To build D, Sakoda (1981) drew inspiration from an expression like $\tilde{D}^g$, although the segregation of a group was not explored. Note that D is also the I_p index (Silber, 1992) divided by its maximum $\left(D = \frac{I_p}{I_p^*}; I_p^* = \sum_g \frac{C^g}{T} D^{g*}\right)$

[15] G is the unstandardized overall Gini index (Alonso-Villar and Del Río, 2010), G_u, divided by its maximum $\left(G = \frac{G_u}{G_u^*}; G_u^* = \sum_g \frac{C^g}{T} G^{g*}\right)$

[16] H is the mutual information index (Theil and Finizza, 1971), M, divided by its maximum $\left(H = \frac{M}{M^*}; M^* = \sum_g \frac{C^g}{T} \Phi_1^{g*}\right)$

[17] C is the quotient between an unstandardized overall index based on the squared coefficient of variation (Alonso-Villar and Del Río, 2010), C_u, and its maximum $\left(C = \frac{C_u}{C_u^*}; C_u^* = 2 \sum_g \frac{C^g}{T} \Phi_2^{g*}\right)$

[18] Carrington and Troske (1997) proposed two standardized indices, not included in Table 3, which result from modifying the dissimilarity index and the Gini coefficient to deal with the issue of small samples. These indices measure the extent to which a sample deviates from randomness, expressed as a fraction of the maximum amount of excess dissimilarity, or excess evenness, that could occur. By doing so, these authors offer an alternative approach to measure standardized segregation, which could be extended to other indices, either local or overall.

Table 2 Decomposition of standardized overall segregation measures in terms of standardized local segregation measures

Standardized overall segregation measures	Relationship between standardized overall and local measures	Weights
$D = \frac{1}{2I_p^*}\sum_g \frac{C^g}{T}\left(\sum_j \left\lvert \frac{c_j^g}{C^g} - \frac{t_j}{T}\right\rvert\right); I_p^* = \sum_g \frac{C^g}{T}\left(1 - \frac{C^g}{T}\right)$	$D = \frac{\sum w^g D^g}{I_p^*} = \sum_g \tilde{w}^g \tilde{D}^g$	$\tilde{w}^g = \frac{\tilde{C}^g}{\sum_g \tilde{C}^g}; \tilde{C}^g = \frac{C^g}{T}\left(1 - \frac{C^g}{T}\right)$
$G = \frac{1}{2UG^*}\sum_g \sum_{i,j} \frac{t_i}{T}\frac{t_j}{T}\left\lvert \frac{c_i^g}{t_i} - \frac{c_j^g}{t_j}\right\rvert; UG^* = \sum_g \frac{C^g}{T}\left(1 - \frac{C^g}{T}\right)$	$G = \frac{\sum w^g G^g}{UG^*} = \sum_g \tilde{w}^g \tilde{G}^g$	$\tilde{w}^g = \frac{\tilde{C}^g}{\sum_g \tilde{C}^g}; \tilde{C}^g = \frac{C^g}{T}\left(1 - \frac{C^g}{T}\right)$
$H = \frac{1}{M^*}\sum_g \frac{C^g}{T}\left(\sum_j \frac{c_j^g}{C^g} \ln \frac{c_j^g/C^g}{t_j/T}\right); M^* = \sum_g \frac{C^g}{T} \ln\left(\frac{T}{C^g}\right)$	$H = \frac{\sum w^g \Phi_1^g}{M^*} = \sum_g \tilde{w}^g \tilde{\Phi}_1^g$	$\tilde{w}^g = \frac{\tilde{C}^g}{\sum_g \tilde{C}^g}; \tilde{C}^g = \frac{C^g}{T} \ln\left(\frac{T}{C^g}\right)$
$C = \frac{1}{UC^*}\sum_g \frac{C^g}{T}\left[\sum_j \frac{t_j}{T}\left(\frac{c_j^g/C^g}{t_j/T} - 1\right)^2\right]; UC^* = N - 1$	$C = \frac{2\sum w^g \Phi_2^g}{UC^*} = \sum_g \hat{w}^g \tilde{\Phi}_2^g$	$\hat{w}^g = \frac{\hat{C}^g}{\sum_g \hat{C}^g}; \hat{C}^g = \left(1 - \frac{C^g}{T}\right)$

$w^g = \frac{C^g}{T}$

measures are standardized (Duncan & Duncan, 1955; Jahn et al., 1947; Reardon & Firebaugh, 2002; Theil & Finizza, 1971), the debate on standardization has not been settled. In fact, due to its decomposability properties, the unstandardized mutual information index, M, is preferred by some scholars to the standardized one, H (Mora and Ruiz-Castillo, 2011; Elbers, 2021). We claim that standardization can be especially useful in empirical studies that involve groups of highly different relative sizes since it allows for greater comparability.[19]

3.3 What Does the Local Segregation Approach Show Us About Overall Segregation?

The relationships that exist among local and overall segregation indices allows us to expand our knowledge of the measurement of overall segregation in several directions, as we now demonstrate.

First, the properties of the local segregation indices help us understand whether the *principle of transfers*, proposed by James and Taeuber (1985) in the binary case, can be relaxed when measuring overall multigroup segregation. Reardon and Firebaugh (2002) proved that the information theory index, H, is the only one of the four standardized overall indices mentioned above that verifies this principle in a multigroup context (i.e., the only one that always decreases when an individual in a group moves to a unit where the group has a lower representation). This is why these authors recommend the use of H to measure overall multigroup segregation.

However, they also question "whether the violation of the principle of transfers seriously undermines the non-H indices, or instead is of little practical consequence in most

[19] Decomposition methods of changes in segregation across time or space have been applied to standardized and unstandardized indices (Deutsch et al., 2009; Elbers, 2021; Karmel and MacLachlan, 1988). These methods allow disentangling changes arising from changes in the margins (i.e., groups' shares and/or units' shares) from "pure" segregation.

Table 3 Segregation indices in a local/overall (un)standardization framework

	Unstandardized indices				Standardized indices			
Two groups	Karmel & MacLachlan index	I_p	Karmel & MacLachlan (1988)		D	Dissimilarity index	Jahn et al. (1947); Duncan & Duncan (1955)	
					G	Gini index	Jahn et al. (1947); Duncan & Duncan (1955); Silber (1989)	
					V / Eta^2	Variance / Correlation ratio index	Duncan & Duncan (1955)	
	Local indices [Alonso-Villar & Del Río, 2010]	**Overall indices**			**Overall indices**			**Local indices** [New]
Multigroup	D^g	Local dissimilarity index(1) (Moir & Selby Smith, 1979)	I_p	I_p index	Silber (1992)	D	Generalized dissimilarity index	Morgan (1975); Sakoda (1981); Reardon & Firebaugh (2002)	$\tilde{D}^g$ Standardized local dissimilarity index(3)
			G_s	Multidimensional G-Segregation index	Boisso et al. (1994)				
	G^g	Local Gini index	G_u	Overall Gini index	Alonso-Villar & Del Río (2010)	G	Generalized Gini index	Reardon & Firebaugh (2002)	$\tilde{G}^g$ Standardized local Gini index(3)
	Φ_1^g	Local entropy index	M	Mutual information index	Theil & Finizza (1971); Frankel & Volij (2011)	H	Information theory index	Theil & Finizza (1971)	$\tilde{\Phi}_1^g$ Standardized local entropy index
	Φ_2^g	Local index based on the squared coefficient of variation	C_u	Overall index based on the squared coefficient of variation	Alonso-Villar & Del Río (2010)	C	Squared coefficient of variation	Reardon & Firebaugh (2002)	$\tilde{\Phi}_2^g$ Standardized local index based on the squared coefficient of variation(2) (3)

Arrows are used to link local and overall segregation indices. Solid lines are used to link overall binary and multigroup segregation indices. Dashed lines connect standardized and unstandardized overall indices.

(1)This index is also called Gorard index (Gorard & Taylor, 2002).

(2)This index is equal to the revised index of isolation, I_1, (Bell, 1954).

(3)When N = 2: $D = \tilde{D}^1 = \tilde{D}^2; G = \tilde{G}^1 = \tilde{G}^2; C = \tilde{\Phi}_2^1 = \tilde{\Phi}_2^2$.

research applications" (p. 58). In light of the local segregation approach shown here, we can conclude that that H is alone, among these standardized overall indices, in verifying the *principle of transfers* does not seem too problematic. As we have shown, both G and C can be generated via standardized local segregation indices satisfying *sensitivity to disequalizing movements type III* (which is the *principle of transfers* applied to the segregation of each group). This suggests that, unlike D, in the case of G and C, the violation of the *principle of transfers* does not undermine its essence. The idea is that, when using G and C, we cannot ensure that the reduction in overall segregation arising from an equalizing movement of individuals in a group (from one unit to another) does more than offset the possible rise in segregation derived from the impact of the changes in the size of those units on other groups (especially if those groups are highly overrepresented in the unit of origin and underrepresented in the unit of destination).[20]

In our opinion, to require that equalizing movements in a group always reduce overall segregation (as happens with H and M) seems a requirement that we can waive whenever the corresponding local indices do satisfy *sensitivity to disequalizing movements type III*. This increases the importance of the *principle of exchanges* in the measurement of overall segregation since this is the property related to (dis)equalizing movements that overall multigroup segregation indices should verify.

Second, the well-known dissimilarity index, popularized by Duncan and Duncan (1955), can be interpreted as the proportion of minority members that would have to be

[20] This rationale can be extended to the corresponding unstandardized measures (G_u and C_u against M).

reallocated across units to be evenly distributed divided by the proportion that would have to move in the case of complete unevenness (Jakubs, 1979; Massey & Denton, 1988). Our approach shows that, when $N = 2$, if we standardize D^g, the standardized segregation of the minority group equals that of the majority group ($\tilde{D}^1 = \tilde{D}^2$) and, therefore, the index of dissimilarity can be expressed as $D = \tilde{w}^1\tilde{D}^1 + \tilde{w}^2\tilde{D}^2 = \tilde{D}^1 = \tilde{D}^2$.[21] Consequently, the dissimilarity index can be interpreted as a standardized local segregation measure ($\tilde{D}^g$). Our analysis also highlights the symmetry that the standardization of D^g brings to the (local) segregation measurement when $N = 2$.

All this clarifies the discussions about the measurement of school segregation in the U.K. (Allen & Vignoles, 2007; Gorard & Taylor, 2002) since: a) it allows placement of Gorard's index and the dissimilarity index in this local/overall (un)standardization framework (see Table 3), the former being the unstandardized local index D^g and the second the standardized local index $\tilde{D}^g$, which implies that the discussion about whether to use one index or the other becomes actually whether standardized or unstandardized indices should be used; and b) elucidates that when working with the standardized version of D^g, the role played by each group in a binary context is the same (i.e., there is symmetry in the segregations measurement), something that does not happen with index D^g and has been a matter of controversy (as reflected in the mentioned papers).

Third, independently of the number of groups, the values of $\tilde{D}^g$, $\tilde{G}^g$, and $\tilde{\Phi}_2^g$ are the same for group g and its complement. Therefore, these local indices equal, respectively, the dissimilarity index (D), the Gini index (G), and the C index in the two-group case.[22] In other words, in multigroup contexts, the dissimilarity index, the Gini index, and the correlation ratio index can be used to compare a group with its complement since they can be interpreted as standardized local segregation indices, $\tilde{D}^g$, $\tilde{G}^g$, and $\tilde{\Phi}_2^g$, which satisfy basic properties.[23]

Fourth, the revised index of isolation, I_1, proposed by Bell (1954) can be interpreted as a standardized local segregation index since $I_1(g) = \tilde{\Phi}_2^g$, $\forall$g = 1,...,N. This elucidates the discussion offered in Massey and Denton (1988) about the nature of this index since although it was originally proposed to deal with exposure, it can also be used to deal with a group's segregation from an evenness perspective.

4 An Illustration: Occupational Segregation of White Women in U.S. Metropolitan Areas

To illustrate the similarities and differences between standardized and unstandardized local segregation measures, we examine the occupational segregation of white women in the largest metropolitan areas in the U.S. We choose this group because it has a large presence in all large metropolitan areas while its demographic weight differs notably across them. We use the 2012–16 American Community Survey (ACS) provided by the IPUMS-USA (Ruggles et al., 2017). We select the 51 metropolitan areas (MAs) with more than 1 million inhabitants (based on the 2010 census). White women are identified on the basis of the

[21] Note that the dissimilarity index is equal to the generalized dissimilarity index when $N = 2$ (Reardon and Firebaugh, 2002).

[22] Note that, when $N = 2$, $G = \tilde{w}^1\tilde{G}^1 + \tilde{w}^2\tilde{G}^2 = \tilde{G}^1 = \tilde{G}^2$ and $C = \hat{w}^1\tilde{\Phi}_2^1 + \hat{w}^2\tilde{\Phi}_2^2 = \tilde{\Phi}_2^1 = \tilde{\Phi}_2^2$. However, this does not apply to other indices (H and $\tilde{\Phi}_1^g$ do not coincide because, in general, $\tilde{\Phi}_1^1 \neq \tilde{\Phi}_1^2$).

[23] This is in line with the method developed by Reardon and Firebaugh (2002) to derive overall multigroup segregation measures from dichotomous measures.

information reported by the interviewees about their gender and race/ethnicity, considering only those women who are white and non-Hispanic.

Our occupational classification distinguishes among 458 categories, which allows us to measure segregation in a highly precise way. For each MA we calculate 12 local segregation indices (6 unstandardized and 6 standardized): D^g ($\tilde{D}^g$), G^g ($\tilde{G}^g$), and $\Phi^g_\alpha(\tilde{\Phi}^g_\alpha)$ for $\alpha = 0.1$, 0.5, 1, and 2. For simplicity, the presentation focuses on indices D^g and $\tilde{D}^g$, referring to the others only when necessary.

Figure 2 plots the index D^g against the share of white women in each MA (this share ranges between 14.6% in Miami and 42.3% in Pittsburgh). Boston, Minneapolis, and Washington, D.C., are among the MAs in which white women have the lowest segregation, whereas in Houston, San Jose, Memphis, and New Orleans they have the highest segregation. Although the group's size does not determine its segregation level (compare, for example, Memphis and Washington), the chart shows a negative relationship between unevenness and size (the pattern is similar for the other indices).

How do we assess the occupational sorting of white women when taking into account the maximum unevenness they can face? To do this, we compare D^g and $\tilde{D}^g$ (Fig. 3).[24]

The dotted lines represent the mean values of the indices. Washington is among the MAs in which white women have the lowest overrepresentation and underrepresentation in occupations, whether we use standardized and unstandardized measures. According to D^g, the percentage of white women in Washington who must switch occupations in order for the group to be evenly distributed is slightly above 25%. On the other hand, $\tilde{D}^g = 0.33$, i.e., the number of white women in this MA who must change occupation represents 33% of all white women who must move in case of maximum segregation.[25] This suggests that the segregation of white women in Washington is far from reaching its maximum level.

The remaining indices used in this study lead to the same conclusion: Washington has a low level of segregation (Fig. 6 in the Appendix). Moreover, Washington has a lower level of segregation than other MAs for the wider range of indices consistent with the dominance criterion provided by the theorem presented in Sect. 3. Thus, for example, Fig. 4 shows that Washington's local segregation curve dominates that of New Orleans, while the opposite obtains for the curves of maximum segregation, thereby ensuring a lower level of segregation for white women in Washington for all the indices consistent with the dominance criterion (standardized or not).[26]

Boston and Minneapolis share with Washington a low unstandardized segregation ($D^g = 0.25$). However, this figure represents around 40% of the maximum value of the index, which means these cities have an intermediate rather than a low position in the ranking based on $\tilde{D}^g$. How do we interpret this? On the one hand, D^g shows that the three MAs have something in common: 1 out of 4 white women working there must change occupation for this group to have in each occupation the same weight it has in the corresponding MA. On the other hand, $\tilde{D}^g$ allows us to take a step further by accounting also for the size

[24] Table 4 A1 in Appendix 2 provides the corresponding values, together with the share of white women. Figure A2 in Appendix 3 shows the other indices.

[25] In Washington, $D^{g*} = 0.77$, i.e., if white women were completely segregated, 3 out of 4 would have to change occupations to achieve an even distribution.

[26] Other large MAs having a similar position in the ranking with indices D^g and $\tilde{D}^g$ include Chicago, Seattle, Denver, Phoenix, and Detroit (Fig. 3). According to most of the (standardized and unstandardized) indices, all these cities have intermediate levels of segregation.

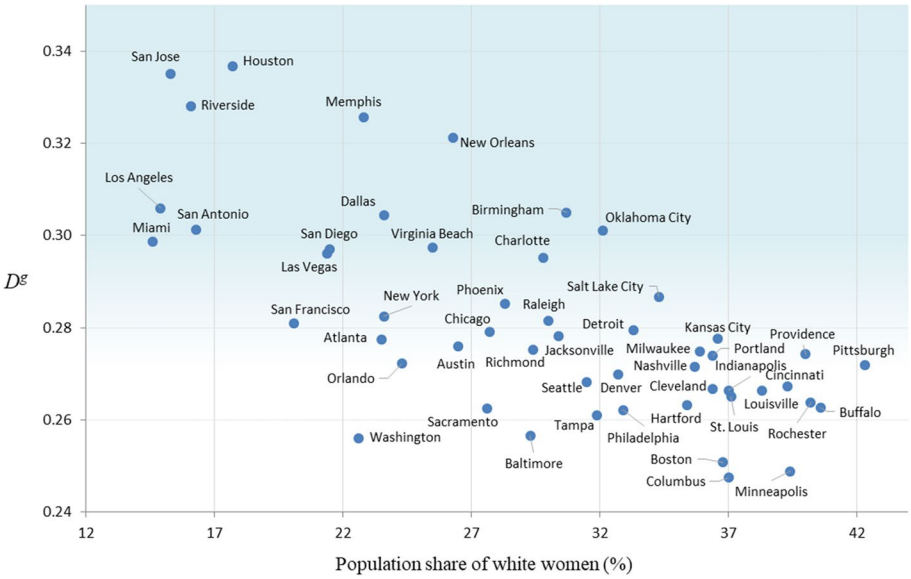

Fig. 2 Population share of white women and index D^g

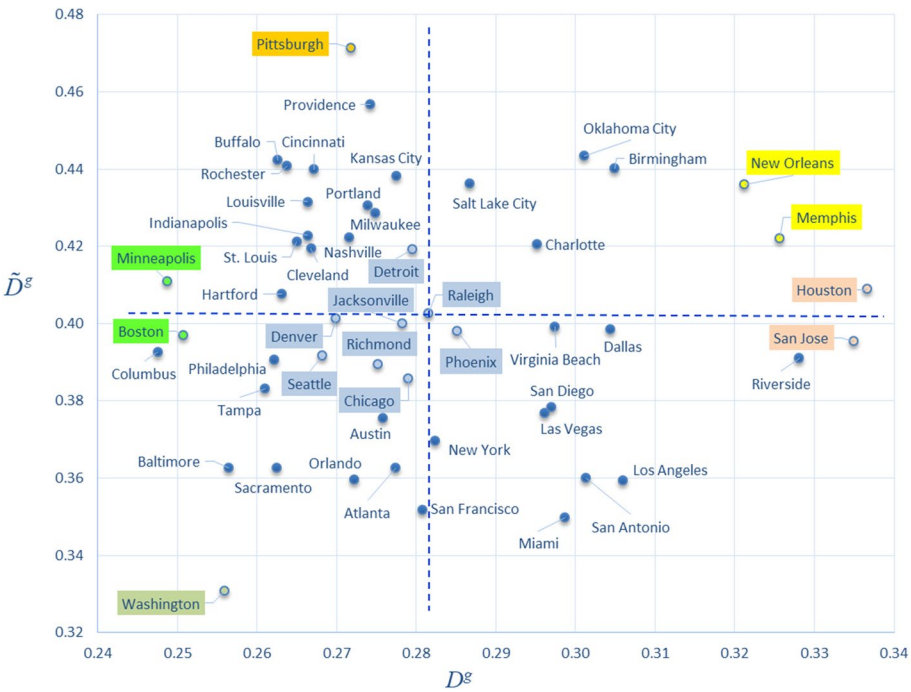

Fig. 3 Values of the indices D^g and $\tilde{D}^g$

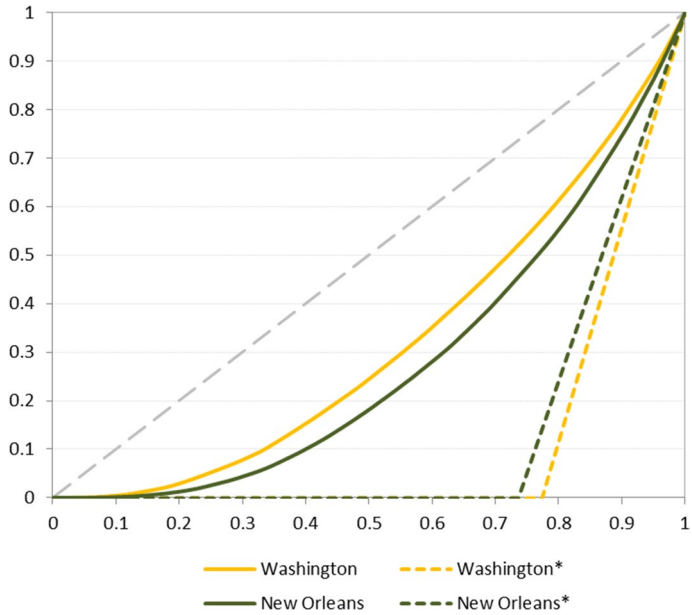

Fig. 4 Local segregation curves (actual and maximum), Washington and New Orleans

of the group; this reveals that segregation is a more acute phenomenon in Boston and Minneapolis than it is in Washington. This is so because the 25% of white women requiring occupation changes to achieve no segregation represents a higher proportion of total workers (or jobs) in the labor markets of the former cities (10 vs. 6%).

New Orleans and Memphis represent cases that stand in opposition to Washington because they have high levels of segregation regardless of the approach followed (Fig. 3). Moreover, this is so although the three cities have a similar share of white women workers.

Pittsburgh stands out as a paradigmatic case. The relatively low value of D^g (=0.27) in this area represents almost half of the maximum segregation attainable by the group. Pittsburgh is therefore the MA with the highest standardized segregation of the country according to index $\tilde{D}^g$(=0.47). Indices $\tilde{G}^g$, $\tilde{\Phi}_1^g$, and $\tilde{\Phi}_2^g$ go in the same direction (Fig. 6).[27]

In light of this, are white women in Pittsburgh highly concentrated in some occupations (as most standardized indices suggest), or is the segregation of this group below average and, especially, smaller than in New Orleans (as the unstandardized indices display)? If we look at the extent to which the occupational sorting of white women departs from evenness, we see that Pittsburgh exhibits an intermediate-low level, whereas New Orleans is among the MAs with the highest values. However, when taking into account the maximum segregation of the group in each MA, we assess the situation in Pittsburgh as harsher than in the remaining areas.

[27] For a discussion about how standardization affects the various indices of the generalized entropy family, see Appendix 3.

5 Final Comments

To be evenly distributed, a group that represents x percent of the total population should account for x percent of the individuals in each unit. For this to be the case, the distribution of the group across units should be equal to the distribution of the total population across these same units. As long as these two distributions depart from each other, the group is said to be segregated and this phenomenon can be computed using any unstandardized local segregation measure already proposed in the literature (Alonso-Villar & Del Río, 2010).

However, the fact that a given percentage of individuals in the group has to change units to be evenly distributed may be judged as more or less problematic depending on the maximum segregation the group can attain, an issue already pointed out by Jahn et al. (1947). This paper has taken a step further by exploring standardization in an analytical framework that offers a clear distinction between the measurement of overall and local segregation, embedding existing indices within this framework, and addressing gaps in previous research. The standardized local segregation indices developed here have several desirable properties, are related to the local segregation curves, and are consistent with existing standardized overall segregation indices, given that the latter can be written as the weighted average of the standardized local segregation of the groups involved.

This local approach allows a deeper exploration into the properties that overall segregation measures should satisfy, as is the case of the *principle of transfers* used in a multigroup context (Reardon & Firebaugh, 2002), showing that what is important is that equalizing movements in a group (from one unit to another) reduce local segregation, not overall segregation. In addition, this paper brings analytical support to the interpretation of the components of overall measures in terms of the segregation levels of the incumbent groups (Watts, 1995). Our framework also gives formal support to some of the empirical strategies used in the literature to deal with the situation of target groups. Thus, the dissimilarity index, the Gini index, and the correlation ratio index when used to compare a group with its complement (i.e., with the remaining groups) seem suitable to measure that group's situation since, as we have proved, they are actually standardized local segregation indices satisfying basic properties.

This paper has also widened the debate on standardization. Our analysis shows that standardized indices quantify segregation from an angle significantly different from unstandardized indices, and this is the case whether we use local or overall measures. Unstandardized measures associated with disproportional functions account for the distance between the distribution of the groups across units and the egalitarian distribution—according to which the presence of each group in each unit must equal the expected value assigned by its weight in the economy. On the contrary, standardized measures quantify the "proximity" of the former distribution to the distribution of maximum segregation. This research helps clarify some of the debate around the measurement of school segregation (Allen & Vignoles, 2007; Gorard & Taylor, 2002), showing that the two measures under discussion could be actually seen as a local segregation index and its standardized version and, therefore, both seem sensible to quantify segregation since they satisfy basic properties.

We claim that standardized local (respectively, overall) indices can be especially useful in empirical studies that involve groups (respectively, societies) of highly different relative sizes (respectively, composition)—as is the case of our illustration—since they allow for greater comparability by providing a frame of reference within which the group's unevenness can be assessed. The standardized local segregation indices developed here are not proposed as an alternative to existing local segregation indices, but as a complementary tool to explore segregation from a different angle. Our research contributes to the literature by offering an

analytical framework within which all of this local/overall (un)standardization debate can be embedded, which allows showing the differences and complementarities among the indices.

Appendix 1

Maximum values of the indices. To obtain D^{g*} and G^{g*}, use the graphical interpretation (Fig. 1). As for Φ_α^{g*} $(\alpha \neq 0, 1)$, note that if the group is fully segregated $\Phi_\alpha^g = \frac{1}{\alpha(\alpha-1)}\left(1 - \frac{C^g}{T}\right)(-1) + \frac{1}{\alpha(\alpha-1)}\frac{C^g}{T}\left[\left(\frac{1}{C^g/T}\right)^\alpha - 1\right] = \frac{1}{\alpha(\alpha-1)}\left[\left(\frac{C^g}{T}\right)^{1-\alpha} - 1\right]$. Likewise, $\Phi_1^{g*} = \ln\left(\frac{T}{C^g}\right)$ since $\lim_{c_j^g \to 0} \frac{c_j^g}{C^g} \ln\left(\frac{c_j^g/C^g}{t_j/T}\right) = 0$.

***Proof of Proposition* 1.** Assume that i and h are two units such that $\frac{c_i^g}{t_i} < \frac{c_h^g}{t_h}$. Taking into account that $\Theta^g(c^g, t)$ satisfies *insensitivity to proportional divisions*, the segregation of group g remains the same if i and h are split into t_i and t_h subunits (of size 1 each), where the former subunits each account for $\frac{c_i^g}{t_i}$ "individuals" of group g and the latter for $\frac{c_h^g}{t_h}$.

If $\frac{d}{t_i t_h}$ "people" of g leave one of the subunits of i to move to one of the subunits of h, the segregation of g will increase, given that the two subunits have the same size and the index satisfies the property of *disequalizing movements type I*. Reiterating this for all other subunits of h, we will have a sequence of t_h *disequalizing movements type I* between units of the same size, which leads to a higher segregation for g (a total of $\frac{d}{t_i}$ individuals of g are moving from a subunit of unit i to h). If we repeat this process for any other subunit of unit i, eventually, $t_i \frac{d}{t_i} = d$ individuals will have switched from i to h.

Therefore, a transfer of d individuals of group g from i to h, which does not alter the size of these units,[28] can be expressed as a sequence of *disequalizing movements type I* between units of the same size, which signifies a rise in the level of segregation of g. Once more employing the *insensitivity to proportional divisions*, the segregation of g is the same in the case of either having these small subunits or aggregating them to give rise to i and h.

***Proof of Proposition* 2.** Assume that i and h are such that $\frac{c_i^g}{t_i} < \frac{c_h^g}{t_h}$ and that d people $(d < c_i^g)$ are transferred from i to h without replacement, i.e., $c_i^g\prime = c_i^g - d$, $c_h^g\prime = c_h^g + d$, $t_i' = t_i - d$, and $t_h\prime = t_h + d$ (no changes in the other units, i.e., $c_j^g\prime = c_j^g$ and $t_j\prime = t_j$ for $j \neq i, h$). Let us assume, without loss of generality, that i is the unit in which g has the lowest representation and h is the next unit in the ranking (Fig. 5).

First, we prove that, at point $\frac{t_i - d}{T}$, the post-transfer curve is below the other, making use of simple trigonometric analysis.[29] We need only prove that $\tan(\alpha) > \tan(\beta)$. Note that $\tan(\alpha) = \frac{\frac{c_i^g}{C^g}}{\frac{t_i}{T}}$, $\tan(\beta) = \frac{\frac{c_i^g - d}{C^g}}{\frac{t_i - d}{T}}$, and that $\tan(\alpha) > \tan(\beta) \Leftrightarrow t_i > c_i^g$. Given that in i the group's representation is below that in h, then $\frac{c_i^g}{t_i} < 1$.

See Fig. 5.

Second, we must show that, at point $\frac{t_i}{T}$, the curve after the transfer is below (or equal to) the other. If we denote by x the difference between the curve after the transfer at point $\frac{t_i}{T}$ and

[28] This implies that an equal number of individuals from other groups has moved in the opposite direction.

[29] If $d = c_i^g$, it is trivial to prove that the curve after the transfer is below the other.

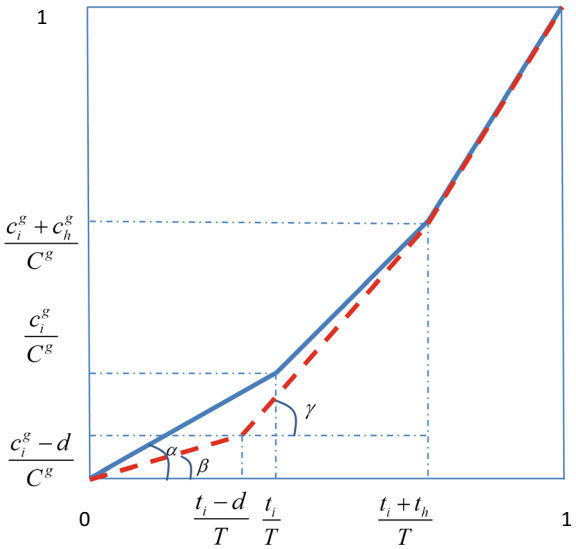

Fig. 5 The segregation curve before (solid line) and after transfers (dash line)

$\frac{c_i^g - d}{C^g}$, then $\tan(\gamma) = \frac{x}{\frac{d}{T}} = \frac{\frac{c_h^g + d}{C^g}}{\frac{t_h + d}{T}}$. Therefore, $x = \frac{d}{d + t_h} \frac{c_h^g + d}{C^g}$. It is easy to see $\frac{c_i^g - d}{C^g} + x \leq \frac{c_i^g}{C^g}$ because $c_h^g \leq t_h$.

***Proof of Corollary* 1** It follows from Theorem 1 in Alonso-Villar and Del Río (2010) and Propositions 1 and 2.

Proof of Corollary* 2. *This follows from the fact that the unstandardized versions of these indices satisfy the corresponding properties and the standardized indices are obtained through the former by dividing them by a constant.

Proof of* Theorem *If the local segregation curve in case A dominates that in B, any index $\Theta^g(c^g, t)$ ***satisfying*** *scale invariance*, *symmetry*, *insensitivity to proportional divisions*, and *sensitivity to disequalizing movements type I* will have a lower value in case A than in B (Alonso-Villar & Del Río, 2010; Theorem 1). For the same reason, Θ^{g*} is higher in B than in A given that the curve of the former dominates that of the latter. Therefore, $\tilde{\Theta}^g(c^g, t) = \frac{\Theta^g(c^g, t)}{\Theta^{g*}}$ is higher in A than in B.

Proof of Proposition* 3. *If one group has a larger share of the population than another group, the curve of maximum segregation will be equal to 0 up to a point that is lower than that of the other group and after that point the curve will be above the other (Fig. 1). This means that the curve of the larger group dominates that of the smaller. The other implication can be easily proved by proof by contradiction.

Appendix 2

See Table 4.

Table 4 Population share of white women and indices D^g and $\tilde{D}^g$ in each MA

Metropolitan areas ranked by D^g	Segregation indices			Population share of white women
	D^g	$\tilde{D}^g$	D^{g*}	
Columbus, OH	0.2475	0.3926	0.6304	37.0
Minneapolis-St. Paul-Bloomington, MN-WI	0.2488	0.4109	0.6055	39.4
Boston-Cambridge-Newton, MA-NH	0.2508	0.3968	0.6321	36.8
Washington-Arlington-Alexandria, DC-VA-MD-WV	0.2559	0.3308	0.7736	22.6
Baltimore-Columbia-Towson, MD	0.2565	0.3626	0.7074	29.3
Tampa-St. Petersburg-Clearwater, FL	0.2610	0.3832	0.6811	31.9
Philadelphia-Camden-Wilmington, PA-NJ-DE-MD	0.2622	0.3907	0.6711	32.9
Sacramento–Roseville–Arden-Arcade, CA	0.2625	0.3625	0.7242	27.6
Buffalo-Cheektowaga-Niagara Falls, NY	0.2626	0.4423	0.5936	40.6
Hartford-West Hartford-East Hartford, CT	0.2631	0.4076	0.6455	35.4
Rochester, NY	0.2638	0.4408	0.5984	40.2
St. Louis, MO-IL	0.2650	0.4211	0.6292	37.1
Louisville/Jefferson County, KY-IN	0.2663	0.4316	0.6171	38.3
Indianapolis-Carmel-Anderson, IN	0.2664	0.4228	0.6301	37.0
Cleveland-Elyria, OH	0.2668	0.4194	0.6360	36.4
Cincinnati, OH-KY-IN	0.2672	0.4400	0.6073	39.3
Seattle-Tacoma-Bellevue, WA	0.2682	0.3917	0.6847	31.5
Denver-Aurora-Lakewood, CO	0.2699	0.4012	0.6728	32.7
Nashville-Davidson–Murfreesboro–Franklin, TN	0.2716	0.4224	0.6428	35.7
Pittsburgh, PA	0.2718	0.4713	0.5767	42.3
Orlando-Kissimmee-Sanford, FL	0.2722	0.3595	0.7572	24.3
Portland-Vancouver-Hillsboro, OR-WA	0.2739	0.4305	0.6364	36.4
Providence-Warwick, RI-MA	0.2742	0.4568	0.6003	40.0
Milwaukee-Waukesha-West Allis, WI	0.2749	0.4287	0.6412	35.9
Richmond, VA	0.2752	0.3895	0.7065	29.4
Austin-Round Rock, TX	0.2759	0.3754	0.7349	26.5
Atlanta-Sandy Springs-Roswell, GA	0.2774	0.3626	0.7651	23.5
Kansas City, MO-KS	0.2775	0.4381	0.6335	36.6
Jacksonville, FL	0.2782	0.4000	0.6956	30.4
Chicago-Naperville-Elgin, IL-IN-WI	0.2790	0.3857	0.7233	27.7
Detroit-Warren-Dearborn, MI	0.2795	0.4191	0.6671	33.3
San Francisco-Oakland-Hayward, CA	0.2809	0.3516	0.7989	20.1
Raleigh, NC	0.2816	0.4025	0.6996	30.0
New York-Newark-Jersey City, NY-NJ-PA	0.2824	0.3695	0.7643	23.6
Phoenix-Mesa-Scottsdale, AZ	0.2851	0.3979	0.7165	28.3
Salt Lake City, UT	0.2867	0.4362	0.6573	34.3
Charlotte-Concord-Gastonia, NC-SC	0.2952	0.4204	0.7021	29.8
Las Vegas-Henderson-Paradise, NV	0.2962	0.3768	0.7861	21.4
San Diego-Carlsbad, CA	0.2970	0.3782	0.7853	21.5
Virginia Beach-Norfolk-Newport News, VA-NC	0.2974	0.3990	0.7453	25.5
Miami-Fort Lauderdale-West Palm Beach, FL	0.2987	0.3497	0.8540	14.6
Oklahoma City, OK	0.3011	0.4434	0.6791	32.1

Table 4 (continued)

Metropolitan areas ranked by D^g	Segregation indices			Population share of white women
	D^g	$\tilde{D}^g$	D^{g*}	
San Antonio-New Braunfels, TX	0.3013	0.3600	0.8370	16.3
Dallas-Fort Worth-Arlington, TX	0.3044	0.3984	0.7640	23.6
Birmingham-Hoover, AL	0.3050	0.4401	0.6930	30.7
Los Angeles-Long Beach-Anaheim, CA	0.3059	0.3594	0.8513	14.9
New Orleans-Metairie, LA	0.3212	0.4360	0.7367	26.3
Memphis, TN-MS-AR	0.3257	0.4220	0.7718	22.8
Riverside-San Bernardino-Ontario, CA	0.3281	0.3911	0.8389	16.1
San Jose-Sunnyvale-Santa Clara, CA	0.3350	0.3954	0.8472	15.3
Houston-The Woodlands-Sugar Land, TX	0.3366	0.4090	0.8231	17.7

Appendix 3

Interpreting the standardized indices of the generalized entropy family

According to $\tilde{\Phi}^g_{0.1}$, New Orleans is the MA with the highest standardized segregation, surpassing Pittsburgh, which occupies the first position with indices $\tilde{\Phi}^g_1$ and $\tilde{\Phi}^g_2$ (Fig. 6). This is because $\tilde{\Phi}^g_{0.1}$ focuses much more on the intensity of underrepresentation, embodied in $\Phi^g_{0.1}$, than on the group's size, embodied in $\Phi^{g^*}_{0.1}$. This underrepresentation is much higher in New Orleans than in Pittsburgh: white women are virtually absent from occupations that account for 6% of total employment in the former whereas this group accounts for less than 2% in the latter.

Our illustration shows that standardization affects the various indices of the generalized entropy family differently. If α is close to zero, the rankings given by Φ^g_α and $\tilde{\Phi}^g_\alpha$ are very similar (Fig. 6). For $\alpha = 0.1$, the Spearman's rank-order correlation is 0.86. However, when α is high, the value of $\tilde{\Phi}^g_\alpha$ is strongly affected by the group's size given that $\Phi^{g^*}_\alpha$ decreases dramatically when the size increases (Fig. 7) and this effect dominates over the differences in Φ^g_α. This explains the negative correlation (–0.55) that exists between $\tilde{\Phi}^g_2$ and Φ^g_2 (Fig. 6).

In light of these findings, standardizing the indices with $\alpha > 2$ does not seem recommendable since the ranking they provide is strongly affected by the relative size of the group. However, indices with a low value of $\alpha(\alpha < 0.5)$ could be useful if one is especially interested in the underrepresentation of the group in occupations.

The remaining indices, D^g, G^g, and Φ^g_1, share a common pattern. They have very small (negative) correlations with their standardized versions (–0.03, –0.2, and –0.18, respectively), which suggests that standardization in these cases brings a certain balance between unevenness and distance to maximum segregation.

Fig. 6 Standardized versus unstandardized local segregation indices in each MA

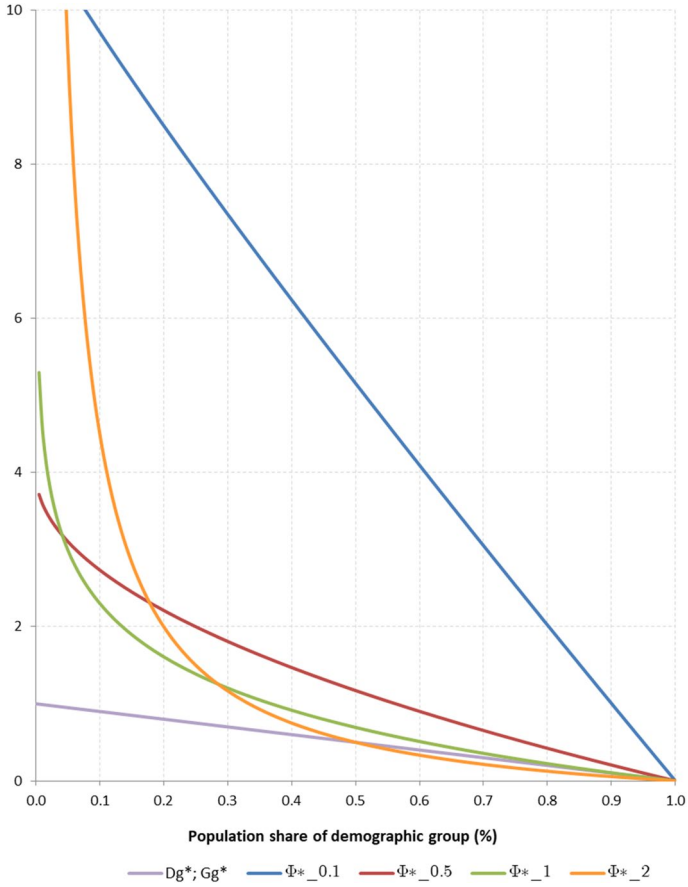

Fig. 7 Maximum local segregation (D^{g*}, G^{g*}, $\Phi_{0.1}^{g*}$, $\Phi_{0.5}^{g*}$, Φ_{1}^{g*}, and Φ_{2}^{g*})

Funding Open Access funding provided thanks to the CRUE-CSIC agreement with Springer Nature. Grants PID2019-104619RB-C41 and PID2020-113440GB-100 funded by MCIN/AEI/ 10.13039/501100011033. We also gratefully acknowledge financial support from Xunta de Galicia (ED431B2019/34). We also want to thank the two anonymous referees for helpful comments.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

Allen, R., & Vignoles, A. (2007). What should an index of school segregation measure? *Oxford Review of Education, 33*(5), 643–668.

Alonso-Villar, O., & Del Río, C. (2010). Local versus overall segregation measures. *Mathematical Social Sciences, 60*, 30–38.

Alonso-Villar, O., & Del Río, C. (2017). Occupational segregation and well-being. *Review of Income and Wealth, 63*(2), 269–287.

Bell, W. (1954). A probability model for the measurement of ecological segregation. *Social Forces, 32*(4), 357–364.

Boisso, D., Hayes, K., Hirschberg, J., & Silber, J. (1994). Occupational segregation in the multidimensional case. Decomposition and tests of significance. *Journal of Econometrics, 61*, 161–171.

Carrington, W. J., & Troske, K. R. (1997). On Measuring segregation in samples with small units. *Journal of Business & Economic Statistics, 15*(4), 402–409.

Croxford, L., & Raffe, D. (2013). Differentiation and social segregation of UK Higher education, 1996–2010. *Oxford Review of Education, 39*(2), 172–192.

Del Río, C., & Alonso-Villar, O. (2019). Occupational achievements of same-sex couples in the U.S. by gender and race. *Industrial Relations, 58*(4), 704–731.

Deutsch, J., Flückiger, Y., and Silber, J. (2009): "Analyzing changes in occupational segregation: The case of Switzerland (1970–2000)." pp. 171–202 in *Research on economic inequality* (vol. 17), edited by Y. Flückiger, S. Reardon, and J. Silber. Bingley, UK: Emerald.

Duncan, O., & Duncan, B. (1955). A methodological analysis of segregation indexes. *American Sociological Review, 20*(2), 210–217.

Elbers, B. (2021). A method for studying differences in segregation across time and space. *Sociological Methods and Research (forthcoming)*. https://doi.org/10.1177/0049124121986204

Frankel, D., & Volij, O. (2011). Measuring school segregation. *Journal of Economic Theory, 146*(1), 1–38.

Gorard, S., & Taylor, C. (2002). What is segregation? A comparison of measures in terms of 'strong' and 'weak' compositional invariance. *Sociology, 36*(4), 875–895.

Iceberg, J. (2004). Beyond black and white: Metropolitan residential segregation in Multiethnic America. *Social Science Research, 33*(2), 248–271.

Jahn, J., Schmid, C., & Schrag, C. (1947). The measurement of ecological segregation. *American Sociological Review, 12*(3), 293–303.

Jakubs, J. (1979). A consistent conceptual definition of the index of dissimilarity. *Geographical Analysis, 11*(3), 315–321.

James, D., & Taeuber, K. (1985). Measures of segregation. *Sociological Methodology, 15*, 1–32.

Karmel, T., & MacLachlan, M. (1988). Occupational sex segregation—increasing or decreasing? *The Economic Record, 64*(186), 187–195.

Maloutas, T., & Spyrellis, S. (2020). Segregation trends in athens: The changing residential distribution of occupational categories during the 2000s. *Regional Studies, 54*(4), 462–471.

Marcińczak, S., Musterd, S., van Ham, M., and Tammaru, T. 2016. "Inequality and rising levels of socio-economic segregation: Lessons from a Pan-European comparative study." pp. 358–382 in *Socioeconomic segregation in european capital cities: East Meets West*, edited by T. Tammaru, S. Marcińczak, M. van Ham, and S. Musterd. London, UK: Routledge.

Massey, D., & Denton, D. (1988). The dimensions of residential segregation. *Social Forces, 67*(2), 281–315.

Moir, H., & Selby Smith, J. (1979). Industrial segregation in the australian labour market. *Journal of Industrial Relations, 21*, 281–291.

Mora, R., & Ruiz-Castillo, J. (2011). Entropy-based segregation indices. *Sociological Methodology, 63*(2), 269–287.

Morgan, B. (1975). The Segregation of socioeconomic groups in urban areas: A comparative analysis. *Urban Studies, 12*, 47–60.

Palencia-Esteban, A. (2021). Occupational segregation of female and male immigrants in the European union: Accounting for cross-country differences. *International labour review, forthcoming*. https://doi.org/10.1111/ilr.12207

Queneau, H. (2009). Trends in Occupational segregation by race and ethnicity in the USA: evidence from detailed data. *Applied Economics Letters, 16*, 1347–1350.

Reardon, S., & Firebaugh, G. (2002). Measures of multigroup segregation. *Sociological Methodology, 32*, 33–76.

Ruggles, S., Genadek, K., Goeken, R., Grover, J. and Sobek, M. 2017. Integrated public use microdata series: Version 7.0 . Minneapolis, MN: University of Minnesota.

Sakoda, J. (1981). A generalized index of dissimilarity. *Demography, 18*, 245–250.

Silber, J. (1989). On the measurement of employment segregation. *Economics Letters, 30*(3), 237–243.

Silber, J. (1992). Occupational segregation indices in the multidimensional case: A note. *The Economic Record, 68*, 276–277.

Theil, H., & Finizza, A. (1971). A note on the measurement of racial integration of schools by means of informational concepts. *Journal of Mathematical Sociology, 1*(2), 187–194.

Watts, M. (1995). Trends in occupational segregation by race and gender in the U.S.A., 1983–92: A multidimensional approach. *Review of Radical Political Economics, 27*(4), 1–36.

White, M. (1986). Segregation and diversity measures in population distribution. *Population Index, 52*(2), 198–221.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.