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Ballistic Charge Transport by Mobile Nonlinear Excitations

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The developments in hyperconductivity, the loss-free transmission of electric charge at room temperature and above, due to the ballistic transport of electric charge in crystals with quasi-layered structure, are reported. The electric charge is carried by quodons, a type of mobile nonlinear intrinsic localized mode of lattice excitation observed as fossil tracks in layered silicates and recently by laboratory experiments. Here, ballistic means moving with minimal scattering or interaction with phonons. A test for hyperconductivity in solid materials is developed. It is based on the unique effect of short-term continuation of transport of charge, by total internal reflection, after creation of quodons has ceased. This effect is called the slow-quodon-decay effect or SQD effect. So far, only layered silicates have been shown to exhibit hyperconductivity. New evidence is presented for hyperconductivity in chrysotile, a nonlayered silicate material with new results. Being a fibrous material, it is more flexible than the sheet mica phyllosilicates. It is found that quodons can also be created and carry charge in very different materials, such as polymers, but without showing hyperconductivity, because of the very short range and lifetime of quodons in those materials.

1. Introduction

In this section, we review the main ideas and previous results about quodons and hyperconductivity. Hyperconductivity is defined as the transport of electric charge in absence of an electric field, the charge being carried by lattice excitations called quodons. In Section 1.1, we briefly introduce the concept of quodons and in Section 1.2, the concept of hyperconductivity and how it was predicted and later observed. In Section 1.3, we review how quodons are created and detected. Section 1.4 deals with the amazing stability of quodons that allows hyperconductivity. The test for quodons that we call slow-quodon-decay (SQD) effect is also presented in it. Readers are referred to the recent review[1] and the references therein for a clear historic perspective and many practical details. We analyze the possible nature of quodons in Section 1.5.

In Section 2, we report of the new experimental findings, particularly the finding of hyperconductivity in the semiconductor chrysotile in Section 2.1 and the unexplained phenomenon of quodon migration between layers in Section 2.2. Section 2.3 explores quodon currents in synthetic polymers and an application of how the SQD effect can be used to test hyperconductivity in the commercial material Macor is described in Section 2.4. Unfortunately, only the microcrystals in Macor show hyperconductivity, but they are separated by glass, making the bulk material an excellent insulator but not a hyperconductor.

A discussion of the results and the problems encountered is presented in Section 3. The article is finished with the conclusions.

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1.1. Quodons

Extensive studies of intrinsic localized mode (ILM) lattice excitations have mainly been done using theoretical and numerical methods.[2–5] The few reported experimental studies rely on their bulk interaction with properties of matter, such as neutron scattering[6,7] or by mechanical analogues of lattices.[8–10]

These methods do not reveal the actual behavior of individual ILMs in crystals. The discovery that crystals of muscovite could record the motion of individual lattice excitations allowed their behaviour to be studied in detail.[11] It was previously known of the ability of muscovite to record the fossil tracks of swift positive particles. These tracks were identified by the kinkiness of their trajectories consistent with Rutherford scattering and were in arbitrary directions within the crystal.[12,13] See also the studies by Russell et al.[1,14] for recent reviews. However, most of the tracks were along the lattice directions in the K^+ layer which implied that they were related to a disturbance that propagated along chains of atoms. The lack of track spreading, that is, their quasi-1D propagation characteristics and their particle-like behavior motivated that they were named quodons. Although the exact nature of quodons is not yet known, it is clear that they cannot be linear excitations, which are extended waves and wave packets with short lifetimes. See Section 1.5 for a discussion of the possible nature of quodons.

Quodon existence was confirmed by an experiment that showed that alpha particles incident on an edge of a muscovite crystal was able to produce an excitation that travel along lattice directions and eject an atom at the other edge of the crystal.[15] Three important facts were: 1) Only positive swift particles left fossil tracks. 2) The main source of lattice excitations should be β^- decay of ^{40}K which leaves a positive charge behind with the recoiling nucleus.[1,16–18] 3) It was observed that the width of the fossil tracks of moving quodons was similar to that of nearly stopped positrons.[19]

These facts led to the deduction that quodons carry electric charge and that most of the lattice tracks are nonlinear lattice excitations with a positive charge[14,19] which was later confirmed by experiments and explained in the following text. They showed that quodons can trap and carry only one unit of positive charge. Other evidence from fossil tracks showed that quodons can also trap and carry a unit of negative charge.[20] Evidence was also found for movement of neutral quodons, without charge of either sign, because of the interrupted nature of some tracks, as shown in **Figure 1**.

It was also revealed how they are created and interact with lattice defects.[21] The remarkable lengths of the fossil tracks of quodons carrying a positive charge, some exceeding 40 cm, as shown in Figure 1, showed their stability against minor lattice defects. As the recording process operated only at temperatures exceeding about 500 °C,[17] the tracks showed that quodons do not interact with phonons.

Independent studies of the annealing effects of irradiation with ions in solids, as observed with quodons, have been reported in silicon[22] and copper.[23]

1.2. Hyperconductivity

If there were insulators as muscovite where quodons, i.e., nonlinear excitations that carry charge, can be produced, the implication was that charge transport in those materials would not be driven by an external electric field but by quodons with properties depending of the crystal and the initial event that triggered the quodon providing it with energy and momentum. This event could be ^{40}K decay,[18] the impact of a swift particle like an alpha particle or a plasma ion.[24] This was observed experimentally[25] sending alpha particles to one side of a muscovite monocrystal and measuring the current with an electrometer while both contacts on the sample were grounded and thus the electric field in the crystal was $E = 0$. Formally, the conductivity $\sigma = J/E$ and mobility $\mu = v/E$ would be infinite as expressed in the title of that article "Infinite Charge Mobility in Muscovite at 300 K," however, this would mean to apply concepts of ohmic conduction to a different phenomenon. As an analogy to superconductivity, the term hyperconductivity was coined. This term was used also in the title of the follow-up paper[26] where hyperconductivity was observed in several other materials, as the synthetic material fluorphlogopite, and also many different properties were found. It should be noted that the phenomenon is very different from superconductivity as there is no evidence of electron pairing and it takes place at room temperatures in experiments and presumably several hundred degrees Celsius above, the temperature at which the fossil tracks in muscovite are formed.[17]

A similar phenomenon was described long ago in a polymer where the velocity of the carriers was shown to be independent of the electric field and it was called ultra-high mobility.[27,28] This and other experiments led to the concept of solectron, that is the coupling between an electric charge and a soliton.[29,30] It has been proposed for transistors 10^4 times faster than silicon transistors.[31]

1.3. Creation and Detection of Quodons

To create a quodon both energy and momentum must be given to one or more atoms in a solid. This can be achieved by impact of swift ions or atoms. The typical energy of a quodon is a few eV. Although there is no know practical way to observe individual quodons in motion their ability to trap an electric charge means they can be detected and thus studied in experiments.[25,26] The relative ease of their creation by swift particles, such as cosmic rays and other sources of swift particles, means that they are to some extent universally present in solid materials. Modeling of a quodon is difficult due to the complexity of the lattice in which it moves but is instructive. In general, an impact quickly evolves to create a highly localized mobile excitation with excess energy radiated away. They can trap one unit of charge of either sign. No evidence has been found for trapping of two units of charge of same sign. The models indicate that they move at slightly subsonic speed, of order 3000 m s^{-1},[32] or about 10^5 times faster than the drift speed of electrons in copper. Their possible existence in silicon, liquids, and gels is starting to be explored.

1.4. Quodon Stability and the SQD Effect

In this section, we introduce the term slow quodon decay (SQD) effect. Although the effect has been described in previous articles,[25,26] the term is used here for the first time. This effect

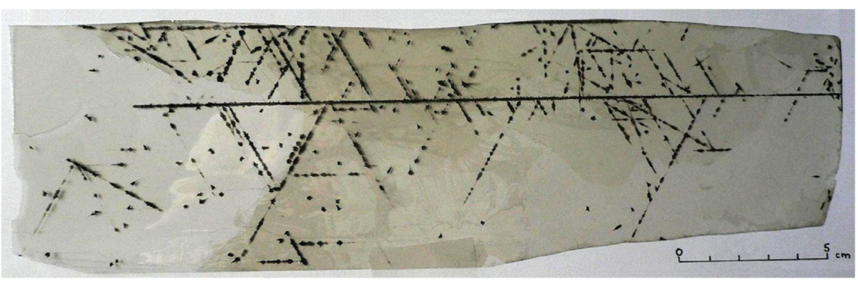

Figure 1. Multiple secondary tracks arising from a primary quodon track due to scattering at crystal defects. The length of the primary track is 36 cm.

follows from the observation in hyperconductivity experiments with good crystals that when the alpha irradiation is stopped, the hypercurrent decays slowly during a few seconds. It demonstrates the long life and range of quodons.

In crystals in which quodons can exist, their range is limited by crystal imperfections. In some single crystals of high quality, their range is nearly infinite. Numerical modeling of ILMs in a simplified muscovite lattice show that some might be totally internally reflected at a crystal face.[33] Evidence for this comes from the study of the current in a crystal after the creation of quodons is stopped, as shown in **Figure 2**. The sampling rate of the data logger is 1 s and the rise time of the current amplifier is smaller than 2 ms. The plot starts with no irradiation and zero current. The small indicated positive voltage is due to an adjustable bias offset. When irradiation with alpha particles starts, a negative current increases rapidly, overshooting briefly to a limiting value as charge is fed to the crystal. When the irradiation is stopped, the current decays slowly to zero. We call this unique phenomenon the slow quodon decay effect, or SQD effect, for short. This effect of slowly decaying current has been shown to be independent of the capacitance of the crystal to ground nor is it due to the electronic measuring system. The time to decay to 1% of the initial value can exceed 3 min in crystals of exceptional quality and at room temperature. This effect was observed first in muscovite[25] but only in a few samples. It was later seen in lepidolite, phlogopite, fluorphlogopite, and chrysotile but never in biotite.[26] Chemical analysis showed

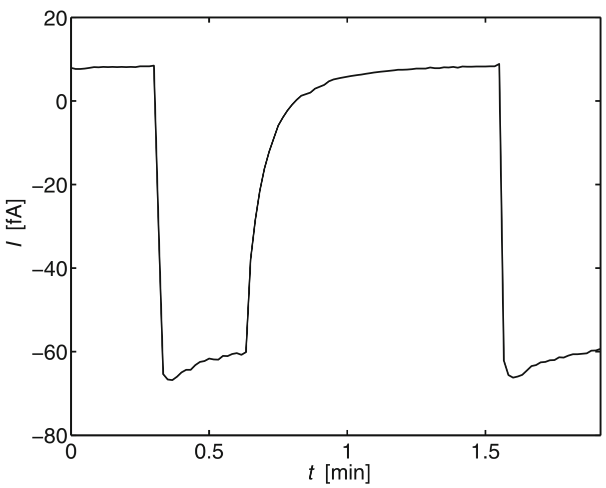

Figure 2. Plot of current following interruption of quodon generation in a crystal of lepidolite. The negative initial current of around −60 fA corresponds to the sample being irradiated with alpha particles which creates quodons that transport charge. After the irradiation is stopped the measured current does not immediately drop to zero but decays, reducing to 5% of the initial value after about 10 s, thereafter continuing to reduce to noise level after about 20 s in some crystals. As no current flows in absence of quodons, some of them must be continuing to propagate for up to 20 s. Assuming they move at an intermediate phonon speed of 3 km s^{-1},[32] some must travel a distance of about $d = 3\ \text{km s}^{-1} \times 20\ \text{s} = 60$ km by internal reflection. As the quodon's velocity is not known, this is just an indication of the long range of quodons in good crystals. This slow decay is proof of charge-carrying quodons existence and ballistic propagation and we call it the SQD effect. The repetition of alpha irradiation produces the second peak.

the iron content was an important factor, increasing presence of iron being deleterious. This slow decay effect is indicative of hyperconductivity. As it can occur in crystals of different composition and structure it is unlikely to be due to a chemical change. It is a useful way to identify materials that show hyperconducting properties. This characteristic effect for hyperconductivity is reminiscent of the Meissner effect for indicating superconductivity.

1.5. Nature of Quodons

As explained earlier, quodons are the lattice excitations that transport charge in hyperconductivity experiments and are responsible of fossil tracks along lattice directions in mica, but their exact nature is not yet known.

The main fact that they transport electric charge excludes discrete breathers,[34] that is lattice excitations with an internal vibration described in simplified form by $u = f(x - V_b t, \omega_b t)$, being localized in the first argument and 2π periodic in the second.[5] They have been suggested[35] to explain tracks in mica before the charge transport of quodons was known. They may still have a role as neutral quodons. An interesting result is that breathers can be scattered in a 2D hexagonal lattice.[36]

Kinks or crowdions, expressed by $u = f(x - V_b t)$, with f becoming zero at $-\infty$ and a lattice constant at $+\infty$ are also called crowdions and may be described as a traveling interstitial. Kinks or crowdions transport charge by themselves in an ionic crystal, for example, K^+ in silicate layers, and have been also proposed for quodons.[18,37,38] However, they also transport mass, meaning that in hyperconductivity experiments, one side of the sample would become rich in K^+ vacancies and the other in K^+ interstitials. The duration of this hypothetical chemical change and the effect of the resulting charge accumulation at the borders and consequent attraction or repulsion of electrons from connections remained unexplored. In contrast, it might be measured to discard or confirm their role as quodons. Also, it seems that crowdions could only be the explanation for primary quodons as there will be not enough energy to scatter many secondary quodons, as shown in Figure 1.

Solitons, described as $u = f(x - V_b t)$, with f becoming zero at $\pm\infty$ and therefore without changes in mass positions are also discarded as they do not transport charge.

Then, there are polarobreathers,[39] that is, breathers coupled to a charge[40,41] or solectrons, solitons coupled to a charge.[42,43] And of course, there are polarons,[44,45] that is an extra charge at an atom or ion that produces a local deformation. If the charge hops to the neighboring atom so does the local deformation. In favor of some form of polaron for describing a quodon is the fact that they, in principle, could travel in any direction and not only along a given chain. This is uniquely consistent with the observed ability of quodons to migrate to adjacent silicate layers.

There are many more entities and mixed ones. Also, the terminology is not uniform. Experiments are needed to determine quodon nature, most important quodon velocity, which we are planning. If quodons are polarobreathers or solectrons the frequency could in principle be measured and also the soliton frequency due to the velocity[5] could also be measured. If they are kinks the depletion and accumulation of ions might be detected.

2. New Developments

This section presents the new experimental results which we report in this article. First, the finding of hyperconductivity in the semiconductor chrysotile. Thereafter, the fascinating observation that quodons can migrate between silicate layers, for which there is no present explanation but points to the quantum nature of quodons. Later, the propagation of quodons in very different materials as the synthetic polymer poly(tetrafluoroethylene) (PTFE) and the commercial material Macor, which illustrates the use of the SQD effect to predict hyperconductivity.

2.1. Hyperconductivity in Chrysotile

Fibrous chrysotile, which showed hyperconducting properties, was studied because of the flexibility of the fibers. It is a semiconductor, so there are free charges in addition to those injected and created due to ionization by the alpha particles. An 8.3 cm long by 3 mm diameter bundle of chrysotile fibers was inserted in a PTFE sheath except for the first 1 cm at the irradiated end. This minimized any quodon current via the sheath. A thin filter of muscovite between the alpha source and the sample prevented any current, arising from conduction through the ion-created plasma in the vicinity of the alpha beam, from reaching the sample. The first part of **Figure 3** shows the quodon-current plot with the filter in place, for an input of about 4 fA. The quodon-current rose initially to about +400 fA before reducing to the limiting current of about −300 fA after 1 min. This initial surge of electrons occurred only once for each sample but could be replicated by mechanical damage or handling of the sample. The second part of Figure 3 shows the plot without the filter in place. The increase in the negative current to about −670 fA is due to charge introduced by the conduction path to ground via the plasma. The brief interruption of irradiation reduced the current to almost zero and was quickly restored by continuing the irradiation. This showed the current is mainly carried, and limited, by the number of quodons created by the ion beam. The effect or an applied voltage is complex and is being studied.

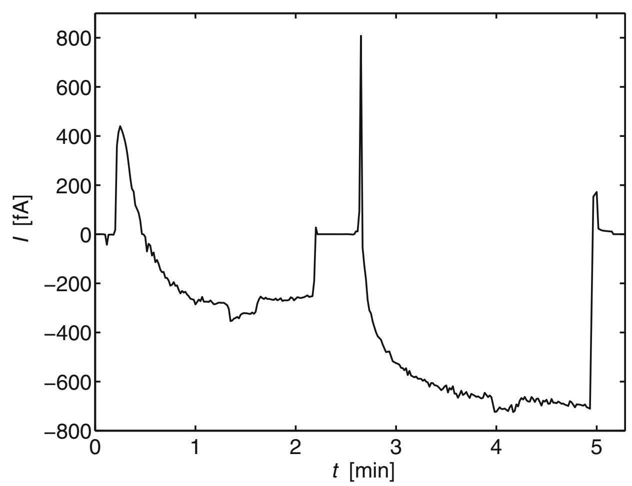

Figure 3. Plot of quodon current in a bundle of chrysotile fibres for two irradiation regimes. A conduction barrier and energy filter of thin muscovite was inserted in the beam of alpha particles preventing plasma conduction. It first surges to $I = 400$ fA by removing static charge. Thereafter, it progressively becomes more negative limiting at about −200 fA. In the second part of the plot, the filter was removed so there is also a contribution from plasma conduction. The limiting current increases to about $I \simeq -670$ fA. There are other spikes that is worth explaining and show the difficulty of measuring tiny currents. They are easily identified because or their reproducibility. The initial positive surge is due to static charge from handling. The spikes at $t \simeq 2.6$ and $t \simeq 5$ min are due to turning the current meter on and then off. The small dips at $t \simeq 1.4 - 1.6$ min and $t \simeq 4.0 - 4.2$ min are the result of sucking the air and thus the plasma away from near the sample.

2.2. Quodon Migration between Silicate Layers

The remarkable recording process for fossil tracks in muscovite had a misleading downside. It operated under very selective conditions in the (001) plane of potassium. In that plane, quodons propagated most easily in atomic chain directions. It gave no insight into the possibility that quodons might propagate or migrate in other directions, in particular, into adjacent planes. The fossil tracks gave no information on this topic. It was an important question as very long single crystals were impractical. This was investigated by machining a crystal so that no layer extended from one end to the other, as shown in **Figure 4**. It was found that current could flow along the strip of crystal when irradiated.

This property of migration in strongly layered structures is poorly understood at present. Tapered butt-joints of two crystal strips also were examined. There was a delay of tens of minutes after irradiation started, probably due to annealing of defects, before a transmitted quodon-current formed. The ability to migrate across layers and anneal lattice defects suggested the possibility of charge transport by quodons via a percolation process. This was confirmed by experiment by packing chrysotile fibers into a tube of insulating material. The efficiency of charge transport, however, was much reduced compared to ballistic transport in single crystals.

2.3. Quodon Propagation in Polymers

For practical purposes, the behavior of quodons in other insulating materials, of a more physically amenable nature, was examined. Samples were cut from bulk PTFE, a synthetic polymer, to form slabs 7 cm long with cross-sectional area 1 cm^2. A typical plot of transmitted current is shown in **Figure 5**. Initially, there was a small displacement current associated with capacitance as charge was fed into the slab. Thereafter, with an injected current of about −12 fA of holes the quodon-current slowly increased

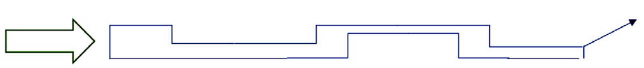

Figure 4. Cross section through a strip of muscovite crystal that shows no continuous layers between the irradiated end and the exit terminal. The arrow to the left represented the flow of alpha particles and the small arrow to the right the connection to the metering instruments. The *c* axis of the crystal is vertical. The result does not depend on the orientation of the other axes as it has been shown that quodons can scatter into any of the chain directions and then move by percolation. Of course, it can be expected that the charge moves fastest if a chain is parallel to the line of the cable.

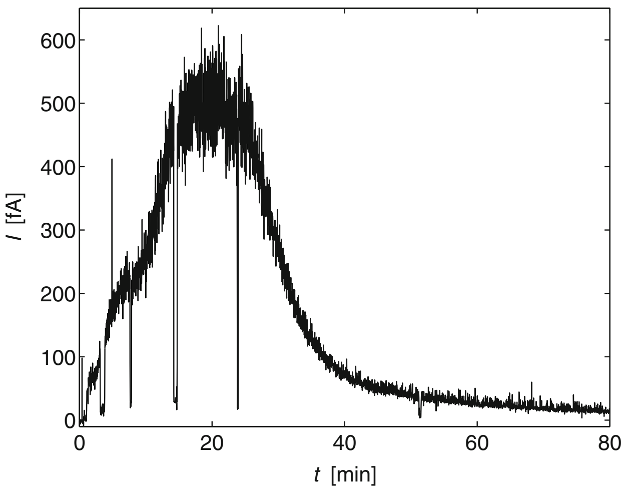

Figure 5. Time-dependent flow of quodon-current in PTFE. There was no evidence of the SQD effect and hyperconductivity. The continuous background of intense short duration spikes is due to cosmic rays, mainly muons.

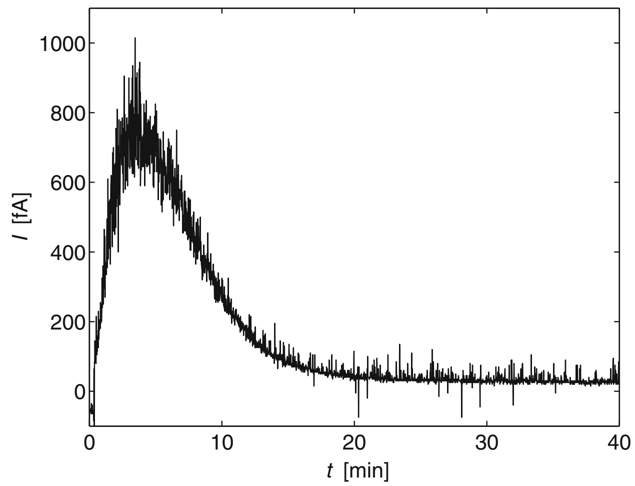

Figure 7. Plot of transmitted current in the nylon sample shown in Figure 6. Peak output current was +700 fA with input current of −40 fA. For nylon, there is no evidence of delayed decay of the transmitted current. The continuous background of intense short duration spikes is due to cosmic rays, mainly muons.

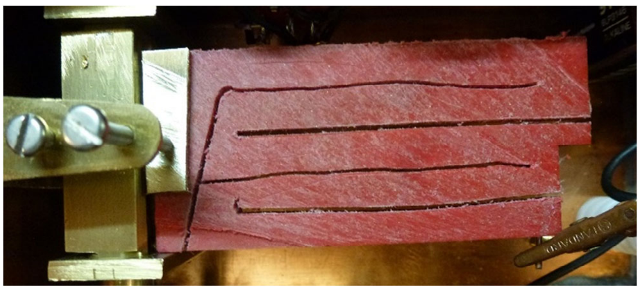

Figure 6. Photo of a slab of nylon cut to give longer path length of 31 cm for quodon current.

reaching a maximum exceeding +500 fA, then decreased, reaching +7 fA after 90 min. It continued to decrease, approaching the level of the negative injected current.

Similar transient behavior was found for raw and red-pigmented nylon, also a synthetic polymer. To examine the dependence of charge transported by quodons on the length of the sample it was cut, as shown in **Figure 6**. The path length was 31 cm. To minimize the possibility of surface currents contaminating the transmitted current, the slab was held between grounded metal plates at the irradiated end.

The plot of quodon current is shown in **Figure 7**. The injected current was −12 fA. The peak of the current surge exceeded +800 fA, decreasing to +15 fA after 3.5 h. Again, brief interruption of irradiation showed no evidence of delayed decay of the transmitted current. This also showed that the quodon current exited the sample within the 1 s data-sampling rate, indicating a speed greater than 3 m s^{-1}. The continuous background of intense short duration spikes is due to cosmic rays, mainly muons.

2.4. Use of SQD Effect in Macor

An informative application of this effect is provided by the study of the electrical behavior of the high-temperature, machinable, high-resistivity ceramic Macor. The ceramic consists of about equal parts of borosilicate glass and fluorphlogopite in which microcrystals of fluorphlogopite have formed during cooling from the molten state. Fluorphlogopite is an excellent electrical insulator that can be synthesized in the form of a polycrystalline mass or as a single crystal. It is structurally similar to muscovite with a pronounced layered structure and crystals can be made of high quality with minimal impurities. Measurements of the quodon current in a slab of Macor of 5 cm length and 1 cm^2 section showed the SQD effect. In an attempt to increase the flexibility of the ceramic, it was heated sufficiently to allow thin filaments to be drawn from a viscous region. The filaments were of about 0.5 mm thickness and sufficiently robust to allow attachment of electrical contacts. The objectives were to see if the filaments still showed the SQD effect and how quickly the quodon current decreased with increasing length of the sample due to loss at the glass bridges separating the crystals. Measurements were carried out at 1 and 5 cm along the sample. In both cases, the SDQ effect was seen but the magnitude of the quodon current was reduced fourfold for the same irradiation flux. This showed that a matrix composed of fluorphlogopite crystals imbedded in an insulating material to achieve flexibility could not form the core of a hyperconducting cable.

3. Discussion

The use of alpha particles to create quodons has consequences. Their high energy means each one creates large numbers of quodons. Although individually they introduce at most two positive charges, they will cause extensive ionization in the body of the material plus a few secondary electrons at the irradiated surface. It was found that coating the irradiated surface with gold leaf to retain these secondary electrons to the sample had negligible effect on the transmitted quodon current. The internal ionization is overall neutral. However, the alphas will ionize the air before hitting the sample, creating a weak plasma. This creates a conduction path between ground potential, the sample, and the current measuring system. Contact potentials at terminals and contacts with the sample can then allow conduction currents to overload the measuring system. Introduction of a film of muscovite of a few micrometer thickness between the alpha source and the sample broke the conduction path. It also lowered the energy of the alphas, which reduced radiation damage to the sample. This procedure was necessary for studying chrysotile. A source of intermediate energy argon ions would be a more controllable and less damaging generator of quodons.

The range of alphas from ^{241}Am is less than 20 μm in most materials, ionization, and radiation damage is similarly restricted. Hence, current measurements on samples of larger than 1 cm size relate mostly to undamaged material. Although the initial current surge found in polymers is not repeated in later irradiations, it can be replicated by carrying out significant damage to the sample, such as by cutting slots or drilling holes. The ability of quodons to propagate great distances in various materials of very different atomic structure is remarkable. As explained in Section 1, the exact nature of quodons is as yet unknown and experiments are planned to determine it.

Measurements were made of the current arising from a voltage applied to samples of PTFE, under the same conditions of cosmic ray background, humidity, temperature, and dimensions as for the quodon currents. A DC applied voltage of 18.4 V gave a transmitted current smaller than 0.5 fA. It would require near-instantaneous creation of thousands of volts, due to build-up of static charge in the samples by the impacting alphas, to create the observed large quodon-currents. With only a few fA of injected current this is unrealistic.

4. Conclusions

We have reported new experimental findings in hyperconductivity, the phenomenon of charge transmission produced without an electric field and excited by alpha bombardment. As deduced in previous publications, the electrons or holes are carried by nonlinear excitations, called quodons, produced by the impact of the alpha particles. The main new experimental findings are: 1) Hyperconductivity is described in the semiconductor chrysotile, whereas previous hyperconducting materials were layered silicates, which are insulators. 2) The discovery that hyperconductivity may occur even if there is not a continuous silicate layer, which suggest that quodons may migrate between layers. 3) The observation of quodon currents in polymers as PTFE and nylon but not hyperconductivity, presumably due to the short lifetime and range or quodons in such materials. This indicates that they could be used for sheathing in cables. 4) Hyperconductivity was found in the commercial ceramic Macor, but the quodon current diminishes very quickly with length making it unsuitable for hyperconducting cables.

We have described a unique test for hyperconductivity named the SQD, which is the observation of the delayed decay of the current when the alpha bombardment is stopped. Decays lasting for more than a minute indicated ballistic charge transport over distances of many kilometers at room temperature.

Our research shows that the creation of quodons by swift ions or atoms provides a novel nondestructive way to investigate the physical properties of matter. Most of the experimental evidence for the existence of quodons depends on the detection of electric currents, as a result of bombardment with swift particles, in solid materials that are well known to be excellent electrical insulators. There is some evidence they might exist in silicon, some metals, gels, and liquids, but the results are not yet clear and more research has to be done.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Research data are not shared.

Keywords

breathers, charge transport, hyperconductivity, kinks, layered silicates, nonlinear waves, quodons

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