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*Research Article*

# A Joint Optimization Criterion for Blind DS-CDMA Detection

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This paper addresses the problem of the blind detection of a desired user in an asynchronous DS-CDMA communications system with multipath propagation channels. Starting from the inverse filter criterion introduced by Tugnait and Li in 2001, we propose to tackle the problem in the context of the blind signal extraction methods for ICA. In order to improve the performance of the detector, we present a criterion based on the joint optimization of several higher-order statistics of the outputs. An algorithm that optimizes the proposed criterion is described, and its improved performance and robustness with respect to the near-far problem are corroborated through simulations. Additionally, a simulation using measurements on a real software-radio platform at 5 GHz has also been performed.



## 1. INTRODUCTION

Direct-sequence code-division multiple access (DS-CDMA) is a common technique in mobile communications that complements other preexisting access techniques such as TDMA or FDMA [1–7]. In systems that use CDMA, users share the same band of frequencies and the same time slots. So the signal that arrives at the receiver is a superposition (in time and frequency) of contributions from different users. Since the objective of the receiver is to extract the symbols sequence of the desired user, the system needs some prior information to achieve this aim. This information is the user's code, also called spreading sequence. Each user transmits with a different cyclic code that multiplies its symbols. For different users, codes are quasiorthogonal. In this way, the receiver can separate the users' contributions by means of the codes.

When there is multipath propagation, we have to suppress the channel effects. Supervised algorithms use training sequences that provide the receiver with knowledge about the channels. Blind detection of users can be performed to obtain the symbol sequence of a desired user without knowledge of the propagation channels. The use of blind techniques increases the performance of the transmission system, avoiding the overheads associated with the transmission of training sequences, and providing increased robustness for channels with severe fading [1, 3, 8].

In the literature, there are several blind criteria for the estimation of a specific user with knowledge only of its spreading code. Some authors proposed the use of MMSE criteria to exploit the signal subspace defined by the desired user's code [2, 8]. In [9], a blind detection scheme based on the constant modulus algorithm (CMA) was addressed, where initialization was the key for the detection of the desired user. Other authors also consider the existence of multiple antennas [8, 10–12] in order to improve the performance of the algorithms by exploiting the increased spatial diversity of the resulting model. The inverse filter criterion (IFC) for blind equalization has also received increasing attention because of its capability for suppressing the MAI (multiaccess interference) and ISI (intersymbol interference) in DS-CDMA systems [13]. In [1], a gradient implementation of the IFC with code constraints was proposed for asynchronous DS-CDMA systems with multipath channels.

Independent component analysis (ICA) can be used to convert blind multiuser detection in DS-CDMA communications systems into a blind source separation (BSS) or blind signal extraction (BSE) problem. Based on ICA, a receiver was proposed in [14] for blind detection in the downlink, thus assuming synchronism between users and the absence of a near-far problem, that is, contributions of all users arrive at the receiver with the same power.

ICA-based criteria are able to blind detect the desired transmitted signal. These criteria usually exploit the non-Gaussianity of the sources together with the a priori information of the spreading code of the desired user, but without the knowledge of the spreading codes of the rest of the users. BSE allows us to blindly obtain the symbols sequence of a user and, in order to ensure that this user is the desired one, we need to constrain the extraction system from the knowledge of the user's code.

In this paper, we pay special attention to the implementations of the inverse filter criterion with code constraint. One of these implementations [1] maximizes the normalized fourth-order cumulant of the output subject to a constraint on the extraction system (similar to the one proposed in [8]). Recently, some joint optimization approaches based on higher-order cumulants have been proposed in the context of ICA [15–17]. This paper, which presents an enhanced extension of the preliminary results given by us in [16], shows how the extension of the criterion to consider the joint optimization of fourth- and sixth-order cumulants leads to an improvement in the MSE (mean square error) of the detected user of about 10 dB, which is increasingly significant for good signal-to-noise ratio and short data records. Moreover, a prewhitening of the observations results in a better conditioning for the algorithms, which also reduces the MSE by about 5 dB.

In order to corroborate the theoretical behavior of the algorithms, we built a software radio platform at 5 GHz aimed at the development of radio interfaces for the fourth generation of mobile communication systems. This platform has been widely characterized [18]. Real measurements of a WCDMA-3GPP signal transmitted at 3.84 Mchip/s have been obtained for testing the analyzed blind detection algorithms.

The paper is structured as follows. Section 2 presents a model of the observations vector in DS-CDMA systems, while Section 3 shows how the code of the desired user can be used to constrain the extraction vector. Section 4.1 summarizes the criterion and extraction algorithm introduced in [1]. In Section 4.2, we present the incorporation of prewhitening and the extension of the criterion to consider the joint optimization of several cumulant orders. In Section 5, we use simulations to corroborate the theoretical behavior of the algorithms that optimize the criteria. Another simulation, with real measurements from the software radio platform we have built, is presented in Section 6. Finally, Section 7 presents the conclusions.

## 2. SYSTEM MODEL

Our objective is the blind detection of a user in a communications system that uses DS-CDMA. In our case, blind detection consists in the blind extraction of the symbols sequence of the desired user. The proposed receiver is a BSE algorithm modified by a constraint which enforces the extraction of the desired user.

In the blind signal extraction problem for linear and instantaneous mixtures, one typically considers the existence of $N$ independent source signals $\mathbf{s}(k) = [s_1(k), \ldots, s_N(k)]^{\mathrm{T}}$. In the presence of white additive Gaussian noise $\mathbf{n}(k)$ of zero mean and variance $\sigma_n^2$, these signals are combined by a linear memoryless system characterized by the $M \times N$ full column rank matrix $\mathbf{A}$ with $M \geq N$ being the vector of $M$ observations

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) + \mathbf{n}(k). \tag{1}$$

The BSE problem consists in recovering a subset of $K \in \{1, \ldots, N\}$ sources from this observations vector without knowledge of the mixture system. The recovery of the desired sources can be split into two steps. The first step prewhitens the observations, and the second extracts the desired source. The prewhitened observations are

$$\mathbf{z}(k) = \mathbf{W}\mathbf{x}(k), \tag{2}$$

where $\mathbf{W}$ is the $M \times M$ matrix which enforces the prewhitening or spatial decorrelation of the signal component of the observations.

Multiplying $\mathbf{z}(k)$ by a separation or extraction $K \times M$ matrix $\mathbf{U}$, one can obtain the vector of $K$ output signals or estimated sources

$$\mathbf{y}(k) = \mathbf{U}\mathbf{z}(k) = \mathbf{G}\mathbf{s}(k) + \mathbf{U}\mathbf{W}\mathbf{n}(k), \tag{3}$$

where the $K \times N$ matrix $\mathbf{G} := \mathbf{UWA}$ is the global transfer matrix from the sources to the outputs.

Next, we will describe the steps that convert the problem of blind detection of a user in a DS-CDMA system into a linear and memoryless BSE problem. In similarity with previous works (see [1–3, 5, 8, 19, 20]), we will rearrange the observed data (the received signal) into a sequence of vectors in order to obtain an instantaneous MIMO model.

We consider a system with $N_u$ users and a process gain of $N_c$ chips per symbol. The chips' sequence (or spreading sequence) of the $j$th user can therefore be grouped into the vector

$$\mathbf{c}_j = [c_j(0), \ldots, c_j(N_c - 1)]^{\mathrm{T}}. \tag{4}$$

Since the spreading sequence has exactly one symbol of duration, we are dealing with a short-code DS-CDMA system. In future high-capacity systems, short codes will become more useful than long codes. The reason is that the MAI in one symbol has identical statistics to the MAI in the next symbol, which allows the multiuser receiver to know adaptively the interference structure [3, 14].

The symbol sequence transmitted by the $j$th user is denoted by $\{b_j(k)\}$. The symbols of each sequence are complex (the modulation can have quadrature components), zero-mean, independent and identically distributed (i.i.d.). For different values of $j$, the $\{b_j(k)\}$ terms are also mutually independent.

To construct the transmitted signal, we cyclically send the chip sequence multiplied at each period by a symbol. The discrete signal transmitted by the $j$th user is therefore

$$\hat{x}_j(k) = \sum_{n=-\infty}^{\infty} b_j(n) c_j(k - nN_c), \quad j = 1, 2, \ldots, N_u. \tag{5}$$

In the case of the uplink, each user has his own linear and dispersive propagation channel. The impulse response of this channel sampled at the chip interval $T_c$ is $g_j(n)$ for the $j$th user. For the uplink, $g_j(n)$'s are different for different $j$'s, whereas for the downlink, they are identical. The discrete impulse response includes the effects of chip-matched filtering at the receiver (see [3]) but not the transmission delay (modulus $N_c$) of the $j$th user, $d_j$, that we assume to satisfy $0 \leq d_j \leq N_c - 1$. Thus, we are assuming asynchronism between users.

The transmitted signal will pass through the corresponding channel. We can group the effect of the chip sequence and the channel into the effective channel

$$h_j(k) := \sum_{n=0}^{N_c-1} c_j(n) g_j(k-n). \tag{6}$$

Therefore, we can express the contribution of the $j$th user at the receiver after being sampled at $T_c$ as

$$\widetilde{x}_j(k) = \sum_{l=-\infty}^{\infty} b_j(l) h_j(k - d_j - lN_c). \tag{7}$$

The total received signal $\widetilde{x}(k)$ is the superposition of the contributions of the $N_u$ users in the presence of additive white Gaussian noise,

$$\widetilde{x}(k) = \sum_{j=1}^{N_u} \widetilde{x}_j(k) + n(k). \tag{8}$$

This is a locally cyclostationary process. Since we aim to work with a locally stationary process, we will define a convolutional MIMO model (multiple inputs and multiple outputs) and then convert this model into an instantaneous MIMO model.

To construct the convolutional MIMO model, we group $N_c$ consecutive samples of $\widetilde{x}(k)$ in the vector $\widetilde{\mathbf{x}}(k)$, so that $\widetilde{\mathbf{x}}(k) = [\widetilde{x}(kN_c + N_c - 1), \ldots, \widetilde{x}(kN_c)]^{\mathrm{T}}$. By similarly defining $\mathbf{h}_j(l) = [h_j(lN_c - d_j + N_c - 1), \ldots, h_j(lN_c - d_j)]^{\mathrm{T}}$ and $\widetilde{\mathbf{n}}(k) = [n(kN_c + N_c - 1), \ldots, n(kN_c)]^{\mathrm{T}}$, the convolutional MIMO model is

$$\widetilde{\mathbf{x}}(k) = \sum_{j=1}^{N_u} \sum_{l=0}^{L_j} \mathbf{h}_j(l) b_j(k-l) + \widetilde{\mathbf{n}}(k). \tag{9}$$

If we assume that multipath delays have a duration of at most one symbol ($g_j(l) = 0$ for $l < 0$ and $l > N_c$) and recalling that $0 \leq d_j < N_c$, we have $\mathbf{h}_j(l) = 0$ for $l < 0$ and $l \geq 3$. Thus $L_j = 2$ for $j = 1, \ldots, N_u$.

By defining the vector $\widetilde{\mathbf{s}}(k) = [b_1(k), \ldots, b_{N_u}(k)]^{\mathrm{T}}$ and the matrix $\mathbf{H}(l) = [\mathbf{h}_1(l), \ldots, \mathbf{h}_{N_u}(l)]$ of $N_c \times N_u$ order, we have

$$\widetilde{\mathbf{x}}(k) = \begin{bmatrix} \mathbf{H}(0) & \mathbf{H}(1) & \mathbf{H}(2) \end{bmatrix} \begin{bmatrix} \widetilde{\mathbf{s}}(k) \\ \widetilde{\mathbf{s}}(k-1) \\ \widetilde{\mathbf{s}}(k-2) \end{bmatrix}. \tag{10}$$

The following step will convert the convolutional MIMO model into an instantaneous MIMO model. In order to do this, we define the vector $\mathbf{x}(k)$, the observations vector in the BSE model,

$$\mathbf{x}(k) := \left[ \widetilde{\mathbf{x}}(k)^{\mathrm{T}}, \ldots, \widetilde{\mathbf{x}}(k - L_e + 1)^{\mathrm{T}} \right]^{\mathrm{T}}. \tag{11}$$

In the same, way we define noise vector $\mathbf{n}(k)$. By defining the vector of sources

$$\mathbf{s}(k) = \left[ \widetilde{\mathbf{s}}(k)^{\mathrm{T}}, \ldots, \widetilde{\mathbf{s}}(k - L_e - 1)^{\mathrm{T}} \right]^{\mathrm{T}}, \tag{12}$$

and the linear and instantaneous mixing matrix

$$\mathbf{A} = \begin{bmatrix} \mathbf{H}(0) & \mathbf{H}(1) & \mathbf{H}(2) & 0 & 0 & & \cdots & 0 \\ 0 & \mathbf{H}(0) & \mathbf{H}(1) & \mathbf{H}(2) & 0 & \cdots & & 0 \\ \vdots & & & & & \ddots & & \vdots \\ 0 & & \cdots & & & \mathbf{H}(0) & \mathbf{H}(1) & \mathbf{H}(2) \end{bmatrix}, \tag{13}$$

we have obtained the linear and memoryless BSE model of (1).

Now, the defined vector of sources consists of delayed versions of the symbol sequences of all users. Since these sequences are i.i.d. and mutually independent, we can affirm that the vector $\mathbf{s}(k)$ actually consists of independent sources. If we define the elements of $\mathbf{s}(k)$ as $\mathbf{s}(k) = [s_1(k), s_2(k), \ldots, s_{N_u(L_e+2)}(k)]^{\mathrm{T}}$, the source $s_{j+N_u d}(k)$ is the symbol sequence of the $j$th user with a delay $d$, that is, $b_j(k-d)$, with $0 \leq d \leq L_e + 1$.

The vector $\mathbf{x}(k)$ is of dimension $N_c L_e \times 1$, the matrix $\mathbf{A}$ is of dimension $N_c L_e \times N_u(L_e + 2)$, and $\mathbf{s}$ is of dimension $N_u(L_e + 2) \times 1$. Following the notation for BSE, $M = N_c L_e$ is the number of observations and $N = N_u(L_e + 2)$ is the number of independent sources. The minimum number of delays $L_e$ we have to introduce into the model is that which allows us to obtain at least as many observations as sources, that is, $L_e$ must satisfy

$$L_e \geq \frac{2N_u}{N_c - N_u}. \tag{14}$$

## 3. CODE-CONSTRAINED CRITERION

In the previous section, we showed how to transform the DS-CDMA system model into a linear, instantaneous mixing model. Once this is done, one can apply an extraction algorithm to the observations vector and extract the symbol sequence of a user. However, in general, this does not ensure that the resulting symbol sequence corresponds to that of the desired user. To achieve this, we need to incorporate an additional constraint into the extraction algorithm.

In this section, we present a constraint based on the code constraint introduced in [1], and related to the subspace projection used in [8], for enforcing the detection of the desired user. In [1], a blind equalization algorithm without prewhitening was considered. Since we use prewhitening as preprocessing, the constraint is slightly different. We will now show that to impose the constraint, we only have to project the extraction vector onto a certain subspace related to the desired user's code.

Let us assume that we want to obtain the symbol sequence of the user $j_0$ with a delay $d$. In the absence of noise, after prewhitening the observations, the true extraction vector $\mathbf{u}_*$ is a row vector (a $1 \times N_c L_e$ matrix) that satisfies

$$\mathbf{u}_* \mathbf{W}\mathbf{A} = \alpha \mathbf{e}_p^{\mathrm{T}}, \tag{15}$$

where $\mathbf{e}_p$ is the unit-norm coordinate vector whose single nonzero element is at position $p = j_0 + N_u d$, and $\alpha$ is a complex constant. Note that the minimum norm solution for $\mathbf{u}_*$ is

$$\mathbf{u}_* = \alpha \mathbf{e}_p^{\mathrm{T}} \mathbf{A}^{\mathrm{H}} \mathbf{W}^{\mathrm{H}}. \tag{16}$$

From the model, one can observe that

$$\alpha \mathbf{e}_p^{\mathrm{T}} \mathbf{A}^{\mathrm{H}} = \alpha[\mathbf{h}_{j_0}^{\mathrm{H}}(d), \dots, \mathbf{h}_{j_0}^{\mathrm{H}}(0),\ 0, \dots, 0]. \tag{17}$$

By defining $\mathrm{h}_j^{(d)} := [\mathrm{h}_j^{\mathrm{H}}(d), \dots, \mathrm{h}_j^{\mathrm{H}}(1)\mathrm{h}_j^{\mathrm{H}}(0)]^H$, and recalling that $g_j(l)$ is 0 for $l > N_c$ and for $l < 0$, and $0 \leq d_j < N_c$, we have

$$\mathrm{h}_j^{(d)} = \mathrm{C}_j^{(d)} \mathrm{g}_j, \tag{18}$$

with

$$\mathrm{C}_j^{(d)} := \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_j(N_c-1) & 0 & \cdots & 0 \\ c_j(N_c-2) & c_j(N_c-1) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_j(0) & c_j(1) & \cdots & 0 \\ 0 & c_j(0) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_j(N_c-1) \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & c_j(0) \end{bmatrix},$$

$$\mathrm{g}_j := [g_j(2N_c - 1 - d_j), \dots, g_j(-d_j+1), g_j(-d_j)]^{\mathrm{T}}. \tag{19}$$

The Toeplitz matrix $\mathrm{C}_j^{(d)}$ of dimensions $[(d+1)N_c] \times [2N_c]$ is similar to the one given in [1, 5, 8].

From (16), (17), and (18), we can state that

$$\mathbf{u}_* = \alpha \mathbf{g}_{j_0}^{\mathrm{H}} \mathcal{C}_{j_0}^{(d)\mathrm{H}} \mathbf{W}^{\mathrm{H}}, \tag{20}$$

where

$$\mathcal{C}_{j_0}^{(d)} = \begin{bmatrix} \mathbf{C}_{j_0}^{(d)} \\ \mathbf{0} \end{bmatrix} \tag{21}$$

is an $N_c L_e \times 2N_c$ matrix.

By defining $\mathbf{P} := \mathbf{W}\mathcal{C}_{j_0}^{(d)}$, one can rewrite (20) as

$$\mathbf{u}_* = \alpha \mathbf{g}_{j_0}^{\mathrm{H}} \mathbf{P}^{\mathrm{H}}, \tag{22}$$

which is the extracting vector at the solution. Since $\mathbf{P}$ is an $N_c L_e \times 2N_c$ matrix and $L_e > 2$, the channel vector can be expressed as

$$\alpha \mathbf{g}_{j_0}^{\mathrm{H}} = \mathbf{u}_* \mathbf{P}(\mathbf{P}^{\mathrm{H}}\mathbf{P})^{-1}. \tag{23}$$

Let the row vector $\mathbf{u}^{(i)}$ be the estimated extraction vector obtained after the $i$th iteration of the BSE algorithm. In contrast with the true extraction vector $\mathbf{u}_*$, in general, the estimated vector does not belong to the subspace spanned by the rows of $\mathbf{P}^{\mathrm{H}}$. The least-squares solution to the inequation $\mathbf{u}^{(i)} \neq \alpha \mathbf{g}_{j_0}^{\mathrm{H}} \mathbf{P}^{\mathrm{H}}$ gives the estimated channel vector

$$\widehat{\alpha \mathbf{g}}_{j_0}^{\mathrm{H}} = \mathbf{u}^{(i)} \mathbf{P}(\mathbf{P}^{\mathrm{H}}\mathbf{P})^{-1}, \tag{24}$$

and, in analogy with (22), from this last result one obtains the new composite extracting vector as

$$\mathbf{u}^{(i)} \Pi_c = \widehat{\alpha \mathbf{g}}_{j_0}^{\mathrm{H}} \mathbf{P}^{\mathrm{H}}, \tag{25}$$

where

$$\Pi_c = \mathbf{P}(\mathbf{P}^{\mathrm{H}}\mathbf{P})^{-1}\mathbf{P}^{\mathrm{H}} \tag{26}$$

denotes the projection matrix onto the subspace spanned by the columns of $\mathbf{P}$. Thus, in order to favor the detection of the desired user, one can automatically incorporate the projection into the preprocessing by simply redefining the observations vector as

$$\mathbf{z}(k) = \Pi_c \mathbf{W}\mathbf{x}(k). \tag{27}$$

## 4. EXTRACTION ALGORITHMS

In this section, we will first present the existing implementation of the inverse filter criterion. Later on, we consider an algorithm that implements the joint optimization of several higher-order cumulants.

### *4.1. Algorithm derived from the inverse filter criterion*

In [1], Tugnait and Li propose to solve the deconvolution problem by passing the observations $\tilde{\mathbf{x}}(k)$ through an inverse filter or equalizer. Let $\tilde{\mathbf{b}}(i)$ denote the row inverse filter of $L_e$ taps, and whose elements have dimension $1 \times N_c$. The output of this filter is

$$y(k) = \mathbf{b}\mathbf{x}(k) = \sum_{i=0}^{L_e-1} \tilde{\mathbf{b}}(i)\tilde{\mathbf{x}}(k-i), \tag{28}$$

where $\mathbf{b} = [\tilde{\mathbf{b}}(0), \tilde{\mathbf{b}}(1), \dots, \tilde{\mathbf{b}}(L_e-1)]$ can be considered as an extraction vector.

The inverse filter maximizes the contrast function proposed by Shalvi and Weinstein (see [21]),

$$J_{42}(\mathbf{b}) = \frac{|\operatorname{cum}_4(y(k))|}{(\operatorname{cum}_2(y(k)))^2}. \tag{29}$$

The $r$th-order cumulants involved ($r = 2, 4$) are real, since they have half of the arguments equal and the other half equal to their conjugates, that is, they have the following structure:

$$\mathrm{cum}_r(y) \equiv \mathrm{cum}\,(\underbrace{y^*, \ldots, y^*}_{\times r/2}, \underbrace{y, \ldots, y}_{\times r/2}). \tag{30}$$

The computation of these cumulants in terms of moments, for low orders, is detailed in the appendix.

In the absence of noise, the recovery at the output of the desired $j_0$th user is achieved blindly through the maximization of (29) with respect to the row vector $\mathbf{b}$, up to a complex constant $\alpha \neq 0$, and an arbitrary delay $0 \leq d \leq L_e - 1 + L_{j_0}$, that is,

$$y(k) = \alpha b_{j_0}(k - d). \tag{31}$$

The algorithm proposed in [1] does not consider prewhitening and adapts the equalizer by means of a gradient algorithm, followed by the projection of the extraction vector onto the subspace spanned by the rows of $(\mathbf{R}_{xx}^{-1} \mathcal{C}_{j_0}^{(d)})^{\mathrm{H}}$.

### 4.2. Algorithm derived from a joint optimization criterion

We have seen that the inverse filter criterion solves the problem of blind source extraction by using a contrast function which is based on the second- and fourth-order cumulants of the output $y(k)$. Recently, the importance of combining the information from several higher-order statistics as a means of improving the accuracy of the results has been highlighted in [15–17]. In the same line of work, we propose here an algorithm which is able to optimize a contrast function that combines information from several higher-order cumulants of the output. As will be shown in the next section, the proposed algorithm yields improved results in the detection of the desired user.

Let us recall that with the prewhitening of the observations, and the projection that favors the detection of the desired user, the preprocessed observations $\mathbf{z}(k)$ can be obtained from (27) while the output is computed as $y(k) = \mathbf{u}\mathbf{z}(k)$, where $\mathbf{u}$ is a unit-norm row vector.

We propose to estimate the desired independent component by maximizing a weighted square sum of a combination of cumulants of the output with orders $r \in \Omega$. A contrast function that achieves this objective is given by

$$\psi_\Omega(y) = \sum_{r \in \Omega} \alpha_r \left| \mathrm{cum}_r\,(y(k)) \right|^2 \quad \text{subject to } \|\mathbf{u}\|_2 = 1, \tag{32}$$

where $\alpha_r$ are positive weighting terms. Let us define $q = \max\{r \in \Omega\}$, in our case, we choose to optimize the set of cumulant orders $\Omega = \{4, 6\}$. Our choice is motivated because the low-order cumulants are those which can be estimated with greater accuracy, but the second-order cumulant is already used in the prewhitening step and the odd-order cumulants are zero for symmetric distributions, which precludes them from being used by the criterion.

The only problem with this approach is the difficulty of the optimization of (32), which is highly nonlinear with respect to $\mathbf{u}$. We can circumvent this difficulty by proposing a similar contrast function to (32) but whose dependence with respect to each of the extracting system candidates is quadratic, and thus, much easier to optimize using algebraic methods.

Consider a set of $q$ candidates for the extracting system $\{\mathbf{u}^{[1]}, \ldots, \mathbf{u}^{[q]}\}$ each of unit 2-norm. The corresponding set of their respective outputs is denoted by $\overline{y} = \{y^{[1]}(k), \ldots, y^{[q]}(k)\}$. We define a multivariate function

$$\psi_\Omega(\overline{y}) = \sum_{r \in \Omega} \frac{\alpha_r}{\binom{q}{r}} \sum_{\sigma \in \Gamma_r^q} \left| \mathrm{cum}\left( \left(y^{[\sigma_1]}(k)\right)^*, \ldots, \left(y^{[\sigma_{r/2}]}(k)\right)^*, y^{[\sigma_{r/2+1}]}, \ldots, y^{[\sigma_r]}(k) \right) \right|^2, \tag{33}$$

where $\alpha_r > 0$ and $\Gamma_r^q$ is the set of all the possible combinations $(\sigma_1, \ldots, \sigma_r)$ of the elements in $\{1, \ldots, q\}$ taken $r$ at a time.

**Theorem 1.** *The function $\psi_\Omega(\overline{y})$, which is invariant with respect to the permutation of its arguments, is maximized at the extraction of one of the users. At this extreme point, all the outputs coincide with one of the transmitted signals $y^{[1]}(k)e^{j\theta_1} = \cdots = y^{[q]}(k)e^{j\theta_q} = \alpha b_{j_0}(k - d)$, up to some constant scaling and phase terms $\theta_1, \ldots, \theta_q$.*

There is an interpretation of this theorem in terms of a low-rank approximation of a set of cumulant tensors [22]. A sketch for the proof of this theorem is presented in [23].

The invariant property of $\psi_\Omega(\overline{y})$ with respect to permutations in its arguments allows us to describe the dependence of the function with respect to $\mathbf{u}^{[m]}$ in the following expression:

$$\begin{aligned} \phi_\Omega\left(\mathrm{u}^{[m]}\right) = {} & \sum_{r \in \Omega} \frac{\alpha_r}{\binom{q}{r}} \\ & \times \sum_{\rho \in \Gamma_{r-1}^{q-1}} \left| \mathrm{cum}\left( \left(y^{[m]}(k)\right)^*, \left(y^{[\rho_1]}(k)\right)^*, \ldots, \right.\right. \\ & \left.\left. \left(y^{[\rho_{r/2-1}]}(k)\right)^*, y^{[\rho_{r/2}]}, \ldots, y^{[\rho_{r-1}]}(k) \right) \right|^2, \end{aligned} \tag{34}$$

where $\Gamma_{r-1}^{q-1}$ is the set of all the possible combinations $(\rho_1, \ldots, \rho_{r-1})$ of the elements in $\{1, \ldots, m-1, m+1, \ldots, q\}$ taken $r - 1$ at a time.

Observe that now the dependence of the contrast function with respect to each of the extracting system candidates is quadratic. Thus, $\psi_\Omega(\overline{y})$ can be cyclically maximized with respect to each one of the elements $\mathbf{u}^{[m]}$, $m = 1, \ldots, q$, while the others remain fixed. Then, at iteration $i$, one optimizes $\mathbf{u}^{[m]}$ with $m = (i \mod q) + 1$. This guarantees a monotonous ascent through iterations, and since the function is upper-bounded by its value at the extraction of one of the users, the monotonous ascent also guarantees convergence to a local maximum, except for the possible (although extremely unlikely) convergence to saddle points. In any case, in communications, the cumulants of the transmitted signals are known in advance, so one can evaluate a priori the global maximum of the contrast function in order to check, later, after the convergence, whether a valid solution has been obtained.

The previous approach works quite well. However, the speed of convergence of the algorithm could be accelerated if, after each iteration, one projects the candidates onto the symmetric subspace that contains the solutions, that is, one enforces $y^{[1]}(k) = \cdots = y^{[q]}(k)$. This projection still guarantees the monotonous ascent when the contrast function $\psi_\Omega(y)$ is shown to be a convex function in the convex domain $\mathcal{S} = \{\mathbf{u} : \|\mathbf{u}\|_2 \leq 1\}$, see [24]. An additional advantage of this projection is that it improves the accuracy in the estimation of the statistics involved, because, for constellations like QPSK, the symmetry in the arguments of the cumulants usually reduces the variance of their sample estimates.

After this projection step, one no longer needs to maintain the notation for all the extraction candidates, since they will be equal, and one only has to distinguish between the value of the extraction candidate that one is optimizing at the $i$th iteration $\mathbf{u}^{(i)}$ and its value at the previous iteration $\mathbf{u}^{(i-1)}$. One can observe that the cyclic maximization of the contrast function with respect to $\mathbf{u}^{[m]}$ with $m = (i \mod q) + 1$, is now equivalent to the sequential maximization through iterations, with respect to the extraction vector $\mathbf{u}^{(i)}$, of the function

$$\begin{aligned}\phi_\Omega\left(\mathbf{u}^{(i)}\right) &= \sum_{r\in\Omega} \frac{r}{q}\alpha_r \Big| \operatorname{cum}\Big(\left(y^{(i)}(k)\right)^*, \left(y^{(i-1)}(k)\right)^*, \ldots, \\ &\qquad \left(y^{(i-1)}(k)\right)^*, y^{(i-1)}, \ldots, y^{(i-1)}(k)\Big)\Big|^2 \\ &= \left(\mathbf{u}^{(i)}\right)^* \mathrm{M}^{(i-1)} \mathbf{u}^{(i)\mathrm{T}}\end{aligned} \tag{35}$$

which results from the simplification of (34). Note that $\mathrm{M}^{(i-1)}$ is a matrix which does not depend on $\mathbf{u}^{(i)}$ and that it is given by

$$\mathrm{M}^{(i-1)} = \sum_{r\in\Omega} \frac{r}{q}\alpha_r \mathbf{c}_{\mathbf{z}y}^{(i-1)}(r)\left(\mathbf{c}_{\mathbf{z}y}^{(i-1)}(r)\right)^{\mathrm{H}}, \tag{36}$$

where $\mathbf{c}_{\mathbf{z}y}^{(i-1)}(r)$ is defined as

$$\begin{aligned}\mathbf{c}_{\mathbf{z}y}^{(i-1)}(r) = \operatorname{cum}\Big(&\mathbf{z}^*(k), \left(y^{(i-1)}(k)\right)^*, \ldots, \\ &\left(y^{(i-1)}(k)\right)^*, y^{(i-1)}, \ldots, y^{(i-1)}(k)\Big).\end{aligned} \tag{37}$$

At each iteration, the maximization $\phi_\Omega(\mathbf{u}^{(i)})$ is obtained by finding the eigenvector associated to the dominant eigenvalue of $\mathrm{M}^{(i-1)}$. Starting from the previous solution, if one considers using $L$ iterations of the power method to approximate the dominant eigenvector (in practice $L = 1$ or 2 works well), Algorithm 1 is obtained, where $d_y^{(l-1)}(r) = (\underline{\mathbf{u}}^{(l-1)})^* \mathbf{c}_{\mathbf{z}y}^{(i-1)}(r)$.

$\underline{\mathbf{u}}^{(0)} = \mathbf{u}^{(i-1)}$
FOR $l = 1 : L$
$$\underline{\mathbf{u}}^{(l)} = \frac{\sum_{r\in\Omega}(r/q)\alpha_r d_y^{(l-1)}(r)\left(\mathbf{c}_{\mathbf{z}y}^{(i-1)}(r)\right)^{\mathrm{H}}}{\left\|\sum_{r\in\Omega}(r/q)\alpha_r d_y^{(l-1)}(r)\left(\mathbf{c}_{\mathbf{z}y}^{(i-1)}(r)\right)^{\mathrm{H}}\right\|_2}$$
END
$\mathbf{u}^{(i)} = \underline{\mathbf{u}}^{(L)}$,

Algorithm 1: The extraction algorithm.

## 5. SIMULATIONS

In order to test the performance of the criteria, we performed extensive simulations of the corresponding algorithms in different situations. They were compared in terms of the MSE between the output and the symbol sequence of the desired user and in terms of the probability of symbol error.

We considered three users and we performed two different simulations: one in which all users were received with the same power, and another in a near-far situation where the power of an interfering user was 10 dB greater than that of the desired user. Each user transmitted 200 symbols with a QPSK modulation. The processing gain or spreading factor was set to 8 chips/symbol where the chips take values in $\pm 1$. Each of the channels consists of four multipaths with complex Gaussian random amplitudes and uniform random delays. The observations were arranged as in (11) to obtain an observations vector $\mathbf{x}(k)$ of length 24, that is, we set $L_e = 3$.

As it is usual, the algorithms are run in two stages preceded by an initialization. This initialization was chosen similar to the one detailed in [1]. In the first stage, the maximization of the contrast function is achieved by using the projected and prewhitened observations. This leads to the extraction of the desired user. In the second stage, the extraction vector obtained at the end of the first stage is used as the initialization for the unconstrained maximization of the contrast function. In this second stage, we do not impose the projection of the observations. This leads to an improvement of the results, since, in practice, the data-based constraint is not fully accurate due to the small number of samples and the noise.

In Figure 1, we present the MSE versus the SNR which resulted from the simulations. We compared the results of the algorithm of [1] with and without prewhitening, and the algorithm of combined cumulants with $\Omega = \{4, 6\}$ and $\alpha_4 = \alpha_6 = 0.5$. In the figure, one can see that the prewhitening reduces the MSE between the output and the desired user. When we use the proposed algorithm, the reduction in MSE is more evident, and this improvement increases with the SNR. Additionally, by comparing Figures 1(a) and 1(b), one can observe that the proposed algorithm is more resistant to the near-far problem.

Figure 2 shows the probability of symbol error versus the SNR for the proposed algorithm and for the one proposed

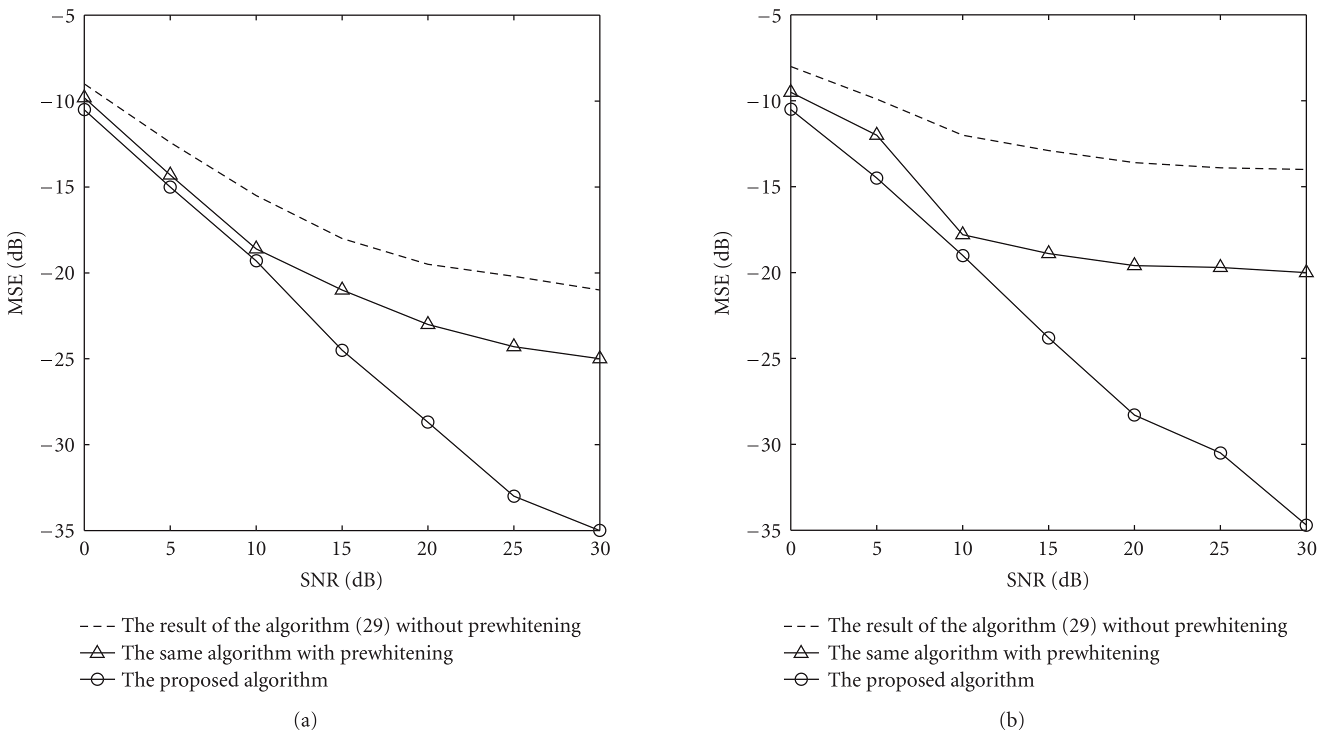



FIGURE 1: The MSE between the output and the desired user over 100 Monte Carlo runs: (a) equal power situation (MAI = 0 dB) and (b) near-far situation (MAI = 10 dB).

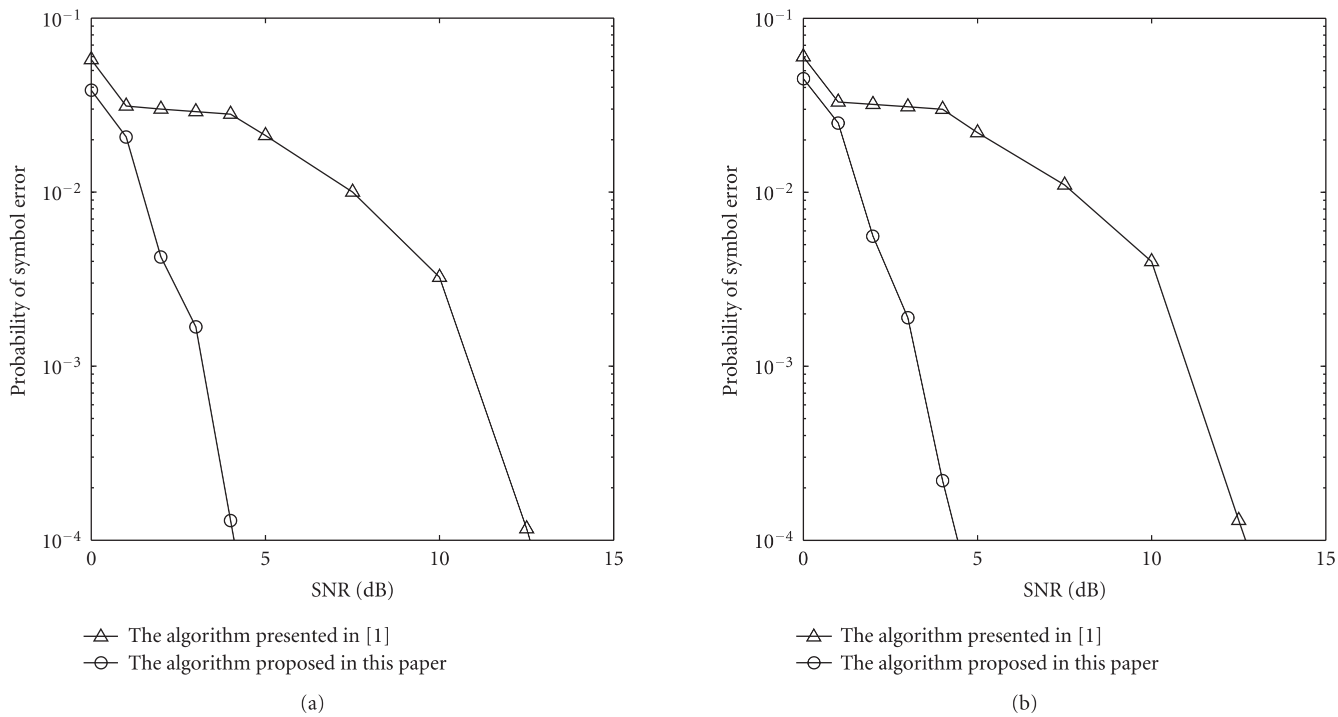
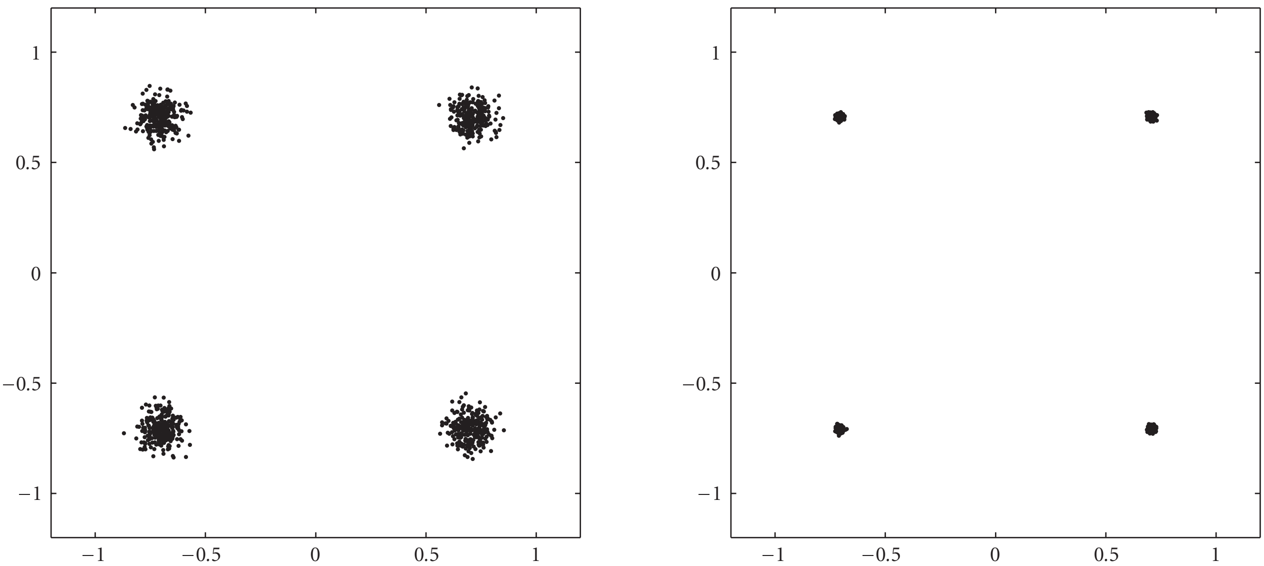



FIGURE 2: The probability of symbol error for (a) normal (equal power) situation (MAI = 0 dB) and (b) near-far situation (MAI = 10 dB). Parameters of simulation are the same as in Figure 1.




(a) (b)

Figure 3: Constelations obtained when the extraction algorithms are applied with (b) or without (a) prewhitening for an SNR of 30 dB and an MAI of 0 dB. The $x$- and $y$-axes of the figures refer to the in-phase (real) and quadrature (imaginary) components of the received QPSK constelation.

in [1]. The parameters for this simulation were the same as those used in Figure 1. One can again see better behavior for the proposed algorithm in normal and near-far situations. The robustness against the near-far problem is corroborated from the comparison between Figures 2(a) and 2(b).

We should note that occasionally, for very low SNR, the algorithms fail to converge. These cases can be detected because they are characterized by an output whose kurtosis is positive or close to zero. When this happens, the algorithm is automatically reinitialized, and the extraction is repeated until a correct solution is found.

Figure 3 illustrates the difference between the use or not of the prewhitening preprocessing in a simulation with 1000 samples. In the figure, one can see the improvement due to prewhitening in reduction of the MSE as a smaller radius for the clouds of points centered at the symbols of the constellation.

In all these simulations, we used a QPSK constellation. In principle, the proposed criterion and algorithm work with all kinds of signals, real or complex, continuous or discrete. However, the performance of the algorithm will depend on the statistics of the signals considered and on the length of the available data set, since both factors influence the variance of the sample cumulant estimates. For constellations with certain symmetries (like the QPSK), the sample estimates of the high-order cumulants and cross-cumulants have a higher precision in absence of noise. Note, however, that the advantage of using QPSK signals quickly disappears for moderate/low signal-to-noise ratios. The QPSK constellation is widely used in CDMA, but we also proved the algorithm with a 16-QAM constellation and we found good detection results when increasing the number of symbols to 800 (about four times more than with the QPSK constelation).

## 6. THE SOFTWARE RADIO PLATFORM OPERATING AT 5 GHZ

In order also to test the algorithms with real signals, we built a software radio platform operating at 5 GHz. The fourth generation of mobile communications systems will take advantage of software radio concepts combining different access technologies into a common hardware platform. Moreover, the trend towards increasing the frequency of operation is arousing great interest in the 5 GHz band, both at the research and commercial levels, which justifies our choice. This section will outline the characteristics of a software radio platform for the transmission of wideband communications signals modulated at 5.25 GHz.

The platform has been widely characterized [18], yielding the following relevant figures: nominal RF frequency, 5.25 GHz; intermediate frequency, 140 MHz; FI bandwidth, 30 MHz; transmitter 1 dB compression level, 0 dBm; receiver sensibility, −62 dBm; receiver 1 dB compression level, +6 dBm; and power consumption, below 115 mA at 15 V. Furthermore, to analyze the detection capabilities of the platform, transmissions of a WCDMA-3GPP signal at 3.84 Mchip/s have been carried out. The baseband signal is generated in a PC by using Matlab. The IQWIZARD and WinIQSIM software send this signal to the SMIQ02B generator that gives the signal at IF to the up converter which transmits it at 5.25 GHz. The down converter recovers this signal

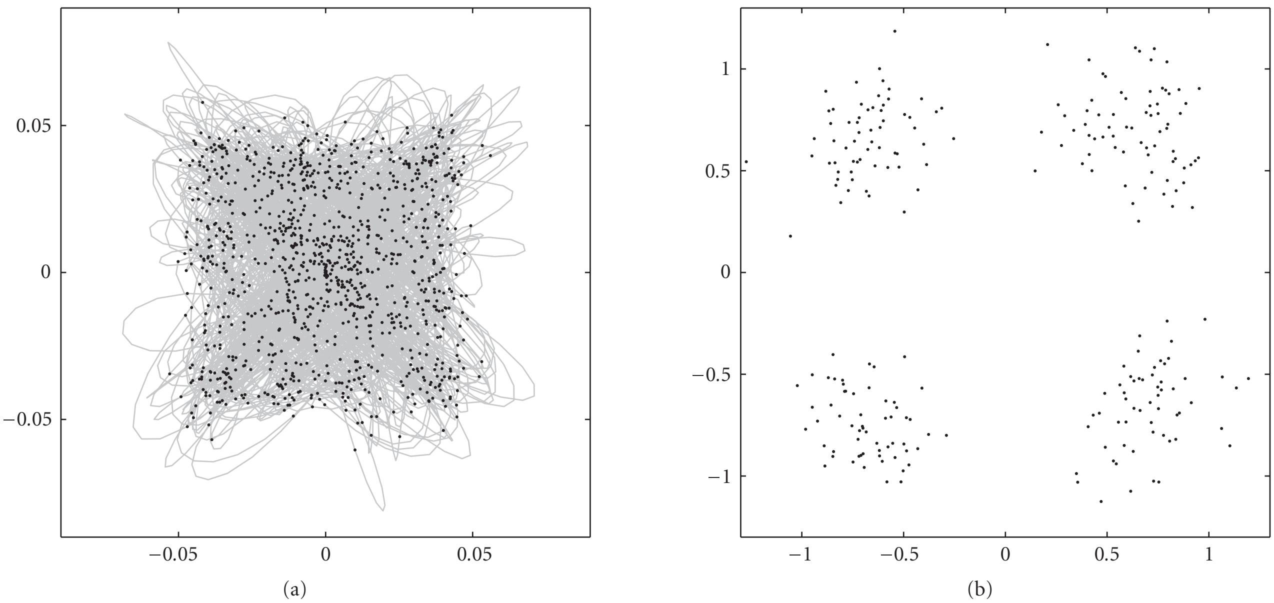



Figure 4: (a) The received signal for real measurement and (b) the resulting signal after applying the algorithm of blind detection. For the received signal, the continuous signal is represented in grey and the sampling points are represented in black. Received SNR was 8.2 dB. The $x$- and $y$-axes of the figures refer to the in-phase (real) and quadrature (imaginary) components of the received QPSK constelation.

and converts it at IF. This IF signal is sent to the E4407B spectrum analyzer which demodulates it and gives the recovered IQ signal to the PC. The evaluation of the different modules of both the transmitter and the receiver has been performed from in-fixture measurements, using a universal test fixture.

Measurements were made with the aid of the platform. Because of the limitations in the number of transmitters that we have at this moment (only one) and storage capability, the simulation of the CDMA system was partially real. We had to transmit a user and then superpose the received signal with the one of an interfering user. The resulting sum was transmitted and received once again. Then, the algorithm of blind detection was applied to the received data. The number of symbols was 200, with 4 chips/symbol and 3.84 Mchip/s. The MAI was set to 5 dB. The separation between antennas was 1 m. In Figure 4, we show the results of the simulations with real measurements.

## 7. CONCLUSIONS

In this paper, we have addressed the problem of the blind detection of a desired user in a DS-CDMA communications system from prior knowledge only of its spreading code. We have shown how to extend the code-constrained inverse filter criterion presented in [1], by considering a more general criterion based on joint optimization of several higher-order statistics. The combination of different reliable statistics of the output led to an improvement in the performance of the detector in terms of mean square error and probability of symbol error. The performance of the algorithm and its robustness with respect to the near-far problem was corroborated by the results of the simulations, which also revealed that the improvement increases with the signal-to-noise ratio.

## APPENDIX

## EVALUATION OF CUMULANTS AND CROSS-CUMULANTS

In this appendix, we show how to evaluate the cumulants of the outputs, which is necessary for the implementation of the algorithm. An easy way is to rewrite them in terms of the moments of the outputs by using the following formula (see [25]):

$$\operatorname{cum}(y_1, y_2, \ldots, y_n) = \sum_{(p_1,\ldots,p_m)} (-1)^{m-1}(m-1)! \cdot E\Big[\prod_{i\in p_1} y_i\Big] E\Big[\prod_{i\in p_2} y_i\Big] \cdots E\Big[\prod_{i\in p_m} y_i\Big], \quad \text{(A.1)}$$

where the sum is extended to all the possible partitions $(p_1, \ldots, p_m)$, $m = 1, \ldots, n$, of the set of natural numbers $(1, \ldots, n)$.

This calculus results in simple complexity for lower orders but it quickly increases for higher-orders. In our case, the fact that the signals are of zero mean and that the arguments of the cumulants share some symmetries considerably simplifies this task, because many partitions disappear or give rise to the same kind of sets. Below, we present the cumulants for $r \in \{2, 4, 6\}$, in terms of the moments:

$$\begin{aligned}
\text{cum}_2(y) &\equiv \text{cum}\,(y^*, y) = E[|y|^2],\\
\text{cum}_4(y) &\equiv \text{cum}\,(y^*, y^*, y, y)\\
&= E[|y|^4] - 2(E[|y|^2])^2 - E[y^2]E\Big[(y^*)^2\Big],\\
\text{cum}_6(y) &\equiv \text{cum}\,(y^*, y^*, y^*, y, y, y)\\
&= E[|y|^6] - 9E[|y|^4]E[|y|^2] + 12(E[|y|^2])^3\\
&\quad - 3E[y^3y^*]E\Big[(y^*)^2\Big] - 3E\Big[y(y^*)^3\Big]E[y^2]\\
&\quad - 9E[y^2y^*]E\Big[y(y^*)^2\Big]\\
&\quad + 18E[y^2]E\Big[(y^*)^2\Big]E[|y|^2].
\end{aligned} \tag{A.2}$$

When taking into account the specific symmetries of the QPSK constellation of the transmitted symbols, one can further simplify this result. Some of the final terms in the previous expressions vanish, resulting in the simplified formulae

$$\begin{aligned}
\text{cum}_2(y) &= E[|y|^2],\\
\text{cum}_4(y) &= E[|y|^4] - 2(E[|y|^2])^2,\\
\text{cum}_6(y) &= E[|y|^6] - 9E[|y|^4]E[|y|^2] + 12(E[|y|^2])^3,
\end{aligned} \tag{A.3}$$

whose complex gradients

$$\nabla_{\mathbf{u}^H} \text{cum}_r(y) = \frac{r}{2}\mathbf{c}_{\mathbf{z}y}(r) \tag{A.4}$$

are proportional to the following cross-cumulant vectors:

$$\begin{aligned}
\mathbf{c}_{\mathbf{z}y}(2) &\equiv \text{cum}\,(\mathbf{z}^*, y) = E[\mathbf{z}^* y],\\
\mathbf{c}_{\mathbf{z}y}(4) &\equiv \text{cum}\,(\mathbf{z}^*, y^*, y, y)\\
&= E[\mathbf{z}^* y|y|^2] - 2E[\mathbf{z}^* y]E[|y|^2],\\
\mathbf{c}_{\mathbf{z}y}(6) &\equiv \text{cum}\,(\mathbf{z}^*, y^*, y^*, y, y, y)\\
&= E[\mathbf{z}^* y|y|^4] - 6E[\mathbf{z}^* y|y|^2]E[|y|^2]\\
&\quad - 3E[\mathbf{z}^* y]E[|y|^4] + 12E[\mathbf{z}^* y](E[|y|^2])^2.
\end{aligned} \tag{A.5}$$

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