

# The power of an elector in the Spanish parliament: A study compared with power indices

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## Abstract

The main goal of this article is to study, from a game theory perspective, the composition of the Spanish Parliament according to Article 68 of the Spanish Constitution, although the proposed model is applicable to the reduction of other representation chambers. It even allows for the sporadic design of the chamber in periods of crisis such as those we are currently experiencing. We use power indices to analyse feasible allocations of seats among the circumscriptions, modifying the size of the Parliament and considering different minimum initial numbers of seats per province. We propose two modifications of the composition following the cubic root rule of the de jure population. Finally, we compare the results of the general elections of December 2015 and June 2016 for the election of the members of the Congress of Deputies (Spanish Parliament) with the current system, which distributes a total number of 350 deputies among the provinces, with an initial minimum of two deputies (system 350/2), with another distribution system that distributed 360 deputies with an initial minimum of one deputy per province (system 360/1).

## Keywords

Banzhaf index, law electoral reform, power of an elector, Shapley-Shubik index, Spanish Parliament

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## Introduction

In Spain, the system of parliamentary representation consists of two legislative chambers; the Lower House is the Congress of Deputies, and the Upper House is the Senate. The system of representation in the Congress of Deputies is proportional and that of the Senate is majority. The objective of this article is to analyse, using power indices, the composition of the Congress of Deputies of Spain, taking into account the provisions of Article 68 of the Spanish Constitution 1978, of 29 December (hereinafter CE78), and in the Organic Law 5/1985, of 19 June, of the General Electoral Regime (hereinafter LOREG). In essence, the Spanish electoral system has great stability, since the modifications that the LOREG has introduced do not affect essential aspects already contemplated in Law 1/1977, of 4 January, for Political Reform, and in the Royal Decree-Law 20/1977, of 18 March, on Electoral Rules that regulated the elections of 1977, 1979 and 1982. Among these aspects, it is worth mentioning the composition of the Congress with 350 deputies and an initial minimum of two deputies per province, the distribution of deputies between districts, the rule of allocation after the holding of elections and the electoral barrier. The analysis of the Spanish Electoral System has been the object of study since the holding of the general elections since 1977. Among many published works, we can mention Sartori (1980), who studied the results of the elections held in Spain between 1977 and 1982, considering that electoral system favoured a bipartisan tendency, distorted by the presence of regionalist parties. Rae and Ramírez (1993) analysed the Spanish electoral system, highlighting the discrimination of the national political parties that remain in third and fourth place with respect to the first two forces and with respect to the regionalist parties, as well as the overrepresentation of the less populated provinces. Bilbao (1994) studied the possibilities of reform of the current Electoral Law, within the framework of the Constitution, in order to favor effective and responsible governments. Márquez and Ramírez (1998) studied the proportionality and governability of the Spanish electoral system. Ramírez and Márquez (2010) and Ramírez (2006) analysed the constitutional limitations that affect the election of the Spanish Congress of Deputies and proposed a possible modification of Article 68 of the Constitution and a two-level distribution. Martínez (2017) has also studied the proposals for reform of the Spanish electoral system made by two political parties currently represented in the Congress of Deputies. Uribe-Otarola (2017) has analysed the characteristics of the election of the Congress of Deputies, as well as the possible reforms within the framework of the Constitution, having as working hypothesis that the effects of the majority system designed in the Transition have been diluted over time, and highlighting that the current system has favoured stability and governability. For this work, the content of the Report of the Council of State (2009), which is the supreme advisory body of the Government of Spain, on the Proposals to modify the General Electoral System of 24 February 2009, has also been taken into account. The report only includes reforms of the electoral law that do not require constitutional reforms. In section 4, final considerations, of this report, it is clear that:

the weight of the vote differs ostensibly depending on the circumscription in which the right to vote is exercised and that the correspondence between the number of seats and the number of votes of some electoral candidates presents important imbalances.

The cost of obtaining a seat is present in the Spanish electoral system from the moment Article 68.2 of the Constitution provides for a minimum initial representation per province, currently established in two deputies (Article 162.2 of the LOREG). Consequently, 100 seats in the chamber, plus

another two in Ceuta and Melilla, are assigned regardless of the population. This minimum initial representation of the provinces, which is attributed without consideration to their de jure population, determines that the number of votes necessary for the attribution of a seat presents important deviations from one circumscription to another. This is also helped, albeit to a lesser extent, by the reduced composition of the Congress, which Article 162.1 of the LOREG establishes in 350 deputies, within the limit of the 400 deputies that Article 68.2 of the Constitution permits at most. The improvement in the equality of voting power makes it advisable that such deviations tend to correct themselves, in line with the general tendency – appreciable in the European electoral systems examined – to achieve an equitable distribution of the seats at stake. In the constitutional framework, the legislator could achieve this objective by reducing the minimum initial representation of two to one deputy and raising the number of deputies in the Congress to 400. Both measures, applied jointly, are those that, in the opinion of the State Council, would be more indicated.

In this article we study the maximum number of seats in the Congress of Deputies and their distribution by circumscriptions. Our analysis is focused on the power of the different circumscriptions, determined by the number of seats assigned to each of them. We use a weighted voting game for each circumscription. In this model we intentionally disregard the results of the different elections up to now, because the allocation system of seats should not be influenced by the electoral expectations of the current political parties and coalitions. Rather we establish a criterion as aseptic as possible, in order to determine the composition of the Parliament and the minimum number of seats for each circumscription. The second section involves basic notions about weighted voting games and power indices, particularly the normalized Banzhaf power index (Banzhaf, 1965) and the Shapley-Shubik index (Shapley and Shubik, 1954). We use generating functions to calculate the indices following Fernández (2000) and Bilbao et al. (2000). In the third section we study different allocations of seats by circumscriptions, always using a Parliament with between 300 and 400 seats, with the same minimum number per province (except for Ceuta and Melilla). Next we make a comparison between the different results. Finally, in the fourth section, after assuming the cubic root criterion of the de jure population to fix the composition of the Parliament (see Taagepera and Shugart, 1989), we propose two solutions of reform for the LOREG. The first one considers that the minimum number of seats is one for each circumscription whilst the second one uses a different minimum number that does not have to be the same for all provinces, although it must be at least equal to one. Calculations and graphics are made using *Mathematica 11.1* (Wolfram).

## Weighted voting games and indices

Consider a finite set of agents  $N = \{1, 2, \dots, n\}$  involved in a voting system; we usually name them *players*. A *weighted voting game* over  $N$  is defined by a number of votes  $w_i$ , with  $w_i > 0$ , for each player  $i \in N$ . Each subset of players is named *coalition* and then the number of votes of a coalition  $S \subseteq N$  is the sum of the votes of its players,  $w(S) = \sum_{i \in S} w_i$ . A fixed *quote*

$q$  of minimal votes is needed to approve a proposal. The *characteristic function* of the weighted voting game defined by these elements is  $v: 2^N \rightarrow \{0, 1\}$ , a function given for each coalition  $S \subseteq N$  by  $v(S) = 1$  if  $w(S) \geq q$  and  $v(S) = 0$  otherwise. So, we describe in short a weighted voting game as  $v = [q; w_1, \dots, w_n]$ . Those coalitions  $S$  satisfying  $v(S) = 1$  are named *winning* and the others satisfying  $v(S) = 0$  *losing*. The characteristic function  $v$  is monotonic, in the sense that if  $S \subseteq T$  then  $v(S) \leq v(T)$ . We suppose also that there exists at least one winning coalition (otherwise the proposed voting system was not feasible), namely we have  $v(N) = 1$ .

A simple game over  $N$  is any monotonic function  $v:2^N \rightarrow \{0, 1\}$  satisfying  $v(\emptyset) = 0$  and  $v(N) = 1$ . Thus the family of weighted voting games is a special subclass of simple games arising from the context of the political analysis to determine quotas of power of the political parties, institutions or nations in decision making (Bilbao et al., 2002; Brams and Affuso, 1976; Felsenthal and Machover, 1998; Holler, 1982; Mann and Shapley, 1962). A *power index* for simple games over  $N$  is a function which determines a quota of power for each player in  $N$  based on the capacity of each player to be involved in winning coalitions. Two classical power indices from game theory are the Banzhaf index (Banzhaf, 1965) and the Shapley-Shubik index (Shapley and Shubik, 1954). The complexity of calculation of these indices from direct formulas is high,  $\mathcal{O}(n2^n)$ . That is why it is necessary to look for algorithms to evaluate them, for instance generating functions. The use of generating functions for the calculation of Shapley-Shubik's power index in voting games has antecedents in the literature, for example see Lucas (1983). Subsequently, Brams and Affuso (1976) used generating functions to calculate the normalized Banzhaf index. The use of generating functions allows us to calculate these indices with more efficient algorithms with temporal complexity  $\mathcal{O}(n^2c)$  where  $c$  is the number of non-zero coefficients of the generating function used (see Bilbao et al., 2000).

To define the *Banzhaf power index*, we first introduce the concept of swing. A *swing* for player  $i$  is a pair of coalitions  $(S \cup \{i\}, S)$  such that  $i \notin S$ , the coalition  $S \cup \{i\}$  is winning and  $S$  is losing. From now on, we will write  $S \cup^i$  instead of  $S \cup \{i\}$  and  $S \setminus i$  instead of  $S \setminus \{i\}$ . For each player  $i \in N$ . The number of swings for player  $i$  in the game  $v$  is denoted by  $\eta_i(v)$ , i.e. the number of coalitions for which player  $i$  is decisive. The total number of swings is  $\bar{\eta}(v) = \sum_{i \in N} \eta_i(v)$ . Now, the normalized power index of Banzhaf for player  $i$  is given by

$$\beta_i(v) = \frac{\eta_i(v)}{\bar{\eta}(v)} \quad (1)$$

Let  $v = [q; w_1, \dots, w_n]$  be a weighted voting game, the number of swings of player  $i$  is

$$\eta_i(v) = \sum_{k=q-w_i}^{q-1} b_k^i \quad (2)$$

where  $b_k^i$  is the number of coalitions  $S$  such that  $i \notin S$  and  $w(S) = k$ . Note that if we sum from  $q - w_i$  to  $q - 1$  we obtain the total number of coalitions that were losing and become winning when player  $i$  is incorporated. The following generating function

$$B_i(x) = \prod_{j=1, j \neq i}^n (1 + x^{w_j}) = \sum_{k=0}^{w(N \setminus i)} b_k^i x^k \quad (3)$$

allows us to compute these numbers.

The *Shapley-Shubik power index* measures the expected marginal contribution for a player that turns a losing coalition into a winning coalition. This index, for a game  $v = [q; w_1, \dots, w_n]$ , is determined by:

$$\phi_i(v) = \sum_{j=0}^{n-1} \frac{j!(n-j-1)!}{n!} d_j^i \quad (4)$$

where

$$d_j^i = \sum_{k=q-w_i}^{q-1} a_{kj}^i \quad (5)$$

Number  $d_j^i$  represents the number of swings of player  $i$  in coalitions of size  $j$ ; i.e. the number of coalitions of size  $j$  that turn into winning when player  $i$  joins. At the same time,  $a_{kj}^i$  represents the number of coalitions formed with  $j$  players, different from  $i$  and with a coalitional weight  $k$ . The generating function

$$S_i(x, z) = \prod_{j=1, j \neq i}^n (1 + x^{w_j} z) \quad (6)$$

allows us to compute the numbers  $a_{kj}^i$ .

Example: The weighted voting game corresponding to a City Council, formed by four political parties  $N = \{1, 2, 3, 4\}$  is  $v = [17; 13, 12, 6, 2]$ . We obtain the Banzhaf and the Shapley-Shubik indices without using the game and the weights of all the coalitions, thanks to the generating functions. We explain the case of player 1. For the Banzhaf index we get

$$B_1(x) = (1 + x^{12})(1 + x^6)(1 + x^2) = 1 + x^2 + x^6 + x^8 + x^{12} + x^{14} + x^{18} + x^{20}$$

and then  $\eta_1(v) = \sum_{k=1}^{16} b_k^1 = 4$ . We need to evaluate the swing of the rest of players in order to have the total number of swings,  $\eta_2(v) = \eta_3(v) = 4$  and  $\eta_4(v) = 0$ . Hence,  $\bar{\eta}(v) = 12$ , and the normalized Banzhaf index of player 1 is  $\beta_1(v) = \frac{1}{3}$ . Now we compute the Shapley-Shubik index,

$$S_1(x, z) = (1 + x^{12}z)(1 + x^6z)(1 + x^2z) = 1 + x^2z + x^6z + x^{12}z + x^8z^2 + x^{14}z^2 + x^{18}z^2 + x^{20}z^3$$

Next we determine  $d_0^1 = 0$ ,  $d_1^1 = 1$ ,  $d_2^1 = 2$  and  $d_3^1 = 0$ . So, also in this case

$$\phi_1(v) = \frac{1}{4!} \sum_{j=0}^3 j!(4-j-1)!d_j^1 = \frac{1}{3}.$$

For more details see Fernández (2000) or Bilbao et al. (2000).

## Current composition of the Spanish Parliament

The composition of the Parliament is established in Article 68 of the Spanish Constitution of 1978:

1. The Congress consists of a minimum of 300 and a maximum of 400 Deputies, elected by universal, free, equal, direct and secret suffrage, in the terms established by law.
2. The electoral circumscription is the province. The populations of Ceuta and Melilla will each be represented by one Deputy. The law will distribute the total number of Deputies, assigning a minimum initial representation to each circumscription and distributing the rest in proportion to the population.
3. The election will be verified in each circumscription according to criteria of proportional representation.

4. The Congress is elected for four years. The mandate of the Deputies ends four years after their election or the day of the dissolution of the Chamber.
5. All Spaniards who are in full use of their political rights are electors and eligible. The law will recognize and the State will facilitate the exercise of the right of suffrage to Spaniards who are outside the territory of Spain.
6. The elections will take place between 30 days and 60 days after the end of the term. The elected Congress shall be convened within 25 days following the holding of the elections.

Currently, the Congress is composed of 350 deputies in accordance with Article 162 of the Organic Law 5/1985 of 19 June of the General Electoral Regime, which establishes:

1. The Congress is made up of 350 Deputies.
2. Each province has an initial minimum of two deputies. The populations of Ceuta and Melilla are each represented by a Deputy.
3. The remaining 248 deputies are distributed among the provinces in proportion to their population, according to the following procedure:
  - (a) A distribution quota is obtained, resulting from dividing by 248 the total number of the de jure population of the peninsular and island provinces.
  - (b) Each province is assigned as many Deputies as result, in integer numbers, from dividing the provincial de jure population by the distribution quota.
  - (c) The remaining Deputies are distributed by assigning one to each of the provinces whose quotient, obtained according to the previous section, has a greater decimal fraction.
4. The Decree of convening must specify the number of Deputies to be elected in each circumscription, in accordance with the provisions of this article.

Currently there are 52 circumscriptions and 350 Deputies, as we see in Table 1, which includes the number of deputies of each province and Ceuta and Melilla.

**Table 1.** Current composition with 350 deputies and initial minimum of two.

C	Name	D	C	Name	D	C	Name	D	C	Name	D
1	Álava	4	2	Albacete	4	3	Alicante	12	4	Almería	6
5	Ávila	3	6	Badajoz	6	7	Baleares	8	8	Barcelona	31
9	Burgos	4	10	Cáceres	4	11	Cádiz	9	12	Castellón	5
13	C. Real	5	14	Córdoba	6	15	Coruña	8	16	Cuenca	3
17	Gerona	6	18	Granada	7	19	Guadalajara	3	20	Guipúzcoa	6
21	Huelva	5	22	Huesca	3	23	Jaén	5	24	León	4
25	Lérida	4	26	La Rioja	4	27	Lugo	4	28	Madrid	36
29	Málaga	11	30	Murcia	10	31	Navarra	5	32	Orense	4
33	Oviedo	8	34	Palencia	3	35	L. Palmas	8	36	Pontevedra	7
37	Salamanca	4	38	Tenerife	7	39	Santander	5	40	Segovia	3
41	Sevilla	12	42	Soria	2	43	Tarragona	6	44	Teruel	3
45	Toledo	6	46	Valencia	16	47	Valladolid	3	48	Vizcaya	8
49	Zamora	3	50	Zaragoza	7	51	Ceuta	1	52	Melilla	1

C = Circumscription, D = Deputies.

It should be noted that modifying the total number of deputies that make up the Congress does not require a constitutional reform, provided that this number is between 300 and 400 deputies, since only point 1 of Article 162 of the Organic Law 5/1985 would have to be modified. For a few years there has been an open debate in Spain to increase the total number of members of the Congress of Deputies; some proposals even suggest raising it to 400 deputies.

Apart from the maximum number of members that make up the Congress, another aspect of the debate is the reduction to one Deputy of the initial minimum established for each province. According to the report of the State Council of 2009 on the Spanish Electoral System:

In Spain, the respect for the principle of equal suffrage in the division of seats between circumscriptions is conditioned by the fact that the Constitution itself does not accept a strictly demographic criterion to that end, by providing for an initial minimum representation per province regardless of their de jure population (Article 68.2), which, together with the existence of a ceiling on the number of deputies in the Congress (Article 68.1), introduces important representative differences between circumscriptions, given that the number of seats that corresponds to them does not have a direct relationship with its inhabitants, determining, in the end, that the cost of the election of a deputy is not the same in each one of them.

The reduction of the initial minimum to one deputy per province favours the reduction of differences between circumscriptions. Now, should the initial minimum be the same for all the provinces?

## The power of an elector

We are going to analyse these questions considering two indicators to measure the power of an elector. In order to do this, we consider a weighted voting game in which the players are the circumscriptions, with a weight equal to the number of Deputies that correspond to them. We then consider the quotients between the power indices of Banzhaf and Shapley-Shubik and the population of each circumscription.

Let the weighted voting game for the election of deputies for the Congress be  $\nu = [q; w_1, \dots, w_{52}]$  (see Table 1) in which there are as many players as circumscriptions, where  $w_i$  is the number of deputies of the circumscription  $i$  and the quota is

$$q = 1 + \frac{1}{2} \sum_{i=1}^{52} w_i \quad (7)$$

We wonder what the power of each circumscription is, once the values of  $w_i$  are known. To answer this question we can calculate the normalized Banzhaf power index  $\beta_i(\nu)$  or the Shapley-Shubik power index  $\phi_i(\nu)$  corresponding to this voting game.

Now, more than the power of each circumscription, we are interested in the power of one elector for each of the 50 provinces and the two autonomous cities of Ceuta and Melilla. Thus, in order to compare the influence of the voters of different circumscriptions, we will consider two indicators, which we will call  $\xi_i$  and  $\zeta_i$ ; which we will obtain by dividing the power indices  $\beta_i(\nu)$  and  $\phi_i(\nu)$  between the population  $p_i$  of each circumscription.

$$\xi_i = \frac{\beta_i(\nu)}{p_i} \text{ and } \zeta_i = \frac{\phi_i(\nu)}{p_i} \quad (8)$$

**Table 2.** Comparative population and vote power.

$i$	$w_i$	$p_i \cdot 10^{-3}$	$\xi_i \cdot 10^8$	$\zeta_i \cdot 10^8$	$i$	$w_i$	$p_i \cdot 10^{-3}$	$\xi_i \cdot 10^8$	$\zeta_i \cdot 10^8$
1	4	323	3.45	3.44	2	4	394	2.83	2.82
3	12	1855	1.82	1.84	4	6	701	2.39	2.39
5	3	164	5.08	5.05	6	6	636	2.44	2.44
7	8	1104	2.03	2.03	8	31	5523	1.67	1.69
9	4	364	3.07	3.05	10	4	406	2.75	2.74
11	9	1240	2.03	2.04	12	5	582	2.40	2.39
13	5	513	2.72	2.71	14	6	795	2.11	2.11
15	8	1127	1.99	1.99	16	3	203	4.11	4.08
17	6	753	2.23	2.23	18	7	917	2.13	2.14
19	3	253	3.30	3.28	20	6	716	2.34	2.34
21	5	520	2.69	2.68	22	3	222	3.76	3.73
23	5	654	2.13	2.13	24	4	479	2.33	2.32
25	4	436	2.56	2.55	26	4	317	3.52	3.51
27	4	329	3.29	3.28	28	36	6433	1.76	1.72
29	11	1628	1.90	1.91	30	10	1467	1.91	1.92
31	5	640	2.18	2.17	32	4	318	3.51	3.49
33	8	1051	2.13	2.14	34	3	166	5.05	5.01
35	8	1098	2.04	2.05	36	7	947	2.07	2.07
37	4	339	3.29	3.28	38	7	1001	1.95	1.96
39	5	585	2.39	2.38	40	3	157	5.32	5.28
41	12	1941	1.74	1.76	42	2	91	6.14	6.08
43	6	795	2.11	2.11	44	3	138	6.03	5.99
45	6	693	2.42	2.42	46	16	2543	1.79	1.81
47	5	526	2.65	2.65	48	8	1148	1.95	1.96
49	3	183	4.57	4.54	50	7	956	2.05	2.05
51	1	84	3.31	3.27	52	1	85	3.26	3.22

We have implemented a program with *Mathematica 11.1* to distribute the Congress of Deputies' seats among the 52 existing circumscriptions (50 provinces plus Ceuta and Melilla), considering that the total number of Deputies can vary between 300 and 400, and taking into account different initial minima of one or two deputies. This programme also calculates the Banzhaf  $\beta_i(v)$  and Shapley-Shubik  $\phi_i(v)$  indices, as well as the value of the indicators  $\xi_i$  and  $\zeta_i$ .

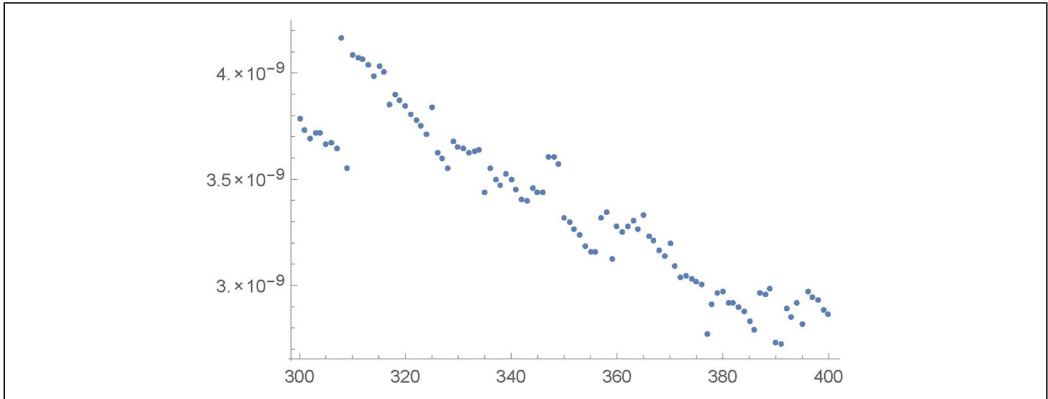
Table 2 shows the distribution of deputies between circumscriptions with 2015 data, with a total of 350 deputies and an initial minimum of two deputies per province. Table 2 also includes the population data  $p_i$  and the indicators  $\xi_i$  and  $\zeta_i$  for each of the provinces.

Once the indicators  $\xi_i$  and  $\zeta_i$  have been calculated, it is of interest to know the standard deviation of them. The standard deviation of a probability distribution, in the case that the data  $x_i$  are grouped by frequencies  $n_i$ , with  $i \in N = \{1, \dots, n\}$ , is given by

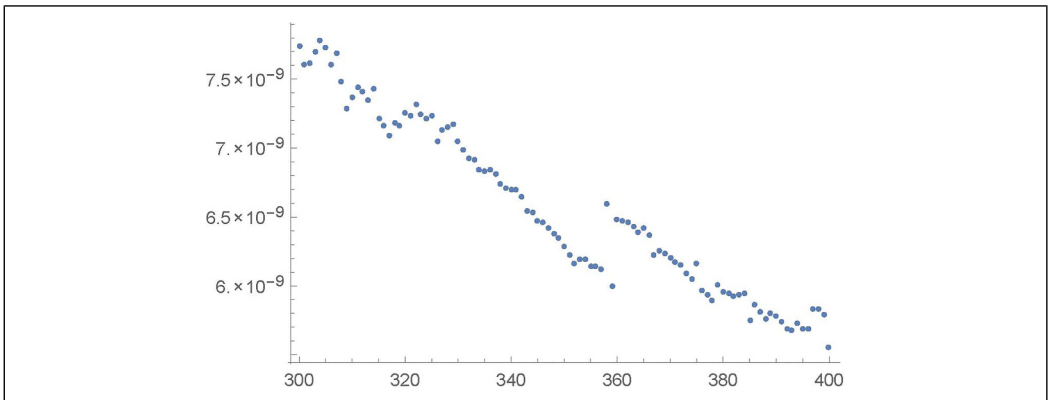
$$\sigma(x) = \sqrt{\frac{1}{\sum_{i \in N} n_i} \sum_{i \in N} n_i (x_i - \bar{x})^2} \quad (9)$$

If we consider that  $x_i = \xi_i$ ,  $n_i = p_i$  and  $n = 52$ , for a distribution of 350 deputies with an initial minimum of two deputies, we have  $\sigma(\xi) = 6.28947 \cdot 10^{-9}$ . Analogously, if  $x_i = \zeta_i$  we have





**Figure 1.** Graph of  $\sigma(\xi)$  with an initial minimum of one deputy per province.

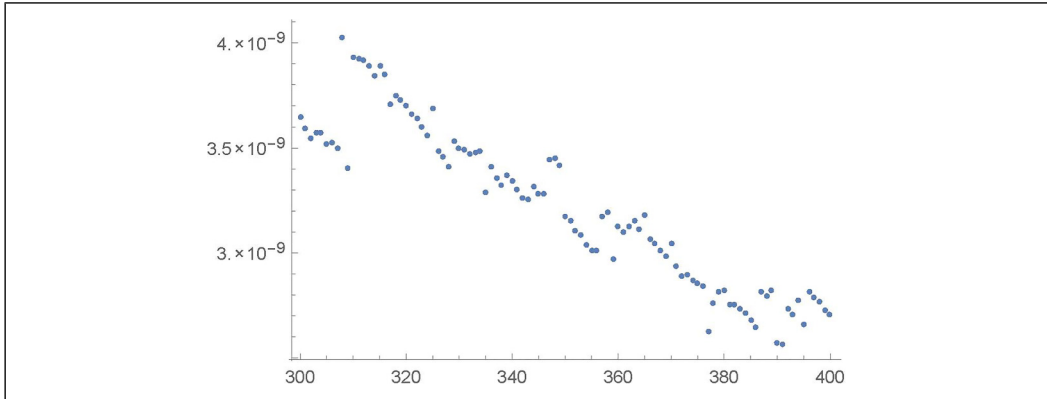


**Figure 2.** Graph of  $\sigma(\xi)$  with an initial minimum of two deputies per province.

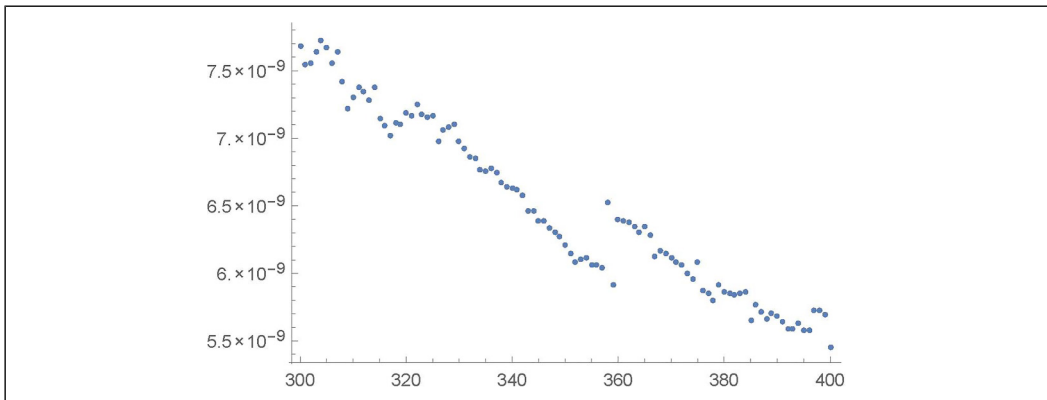
$\sigma(\zeta) = 6.21466 \cdot 10^{-9}$ . Figure 1 shows the value of the deviations  $\sigma(\xi)$  for all the distributions of deputies comprised between 300 and 400, with an initial minimum of one deputy per province.

Figure 2 shows a new representation of the deviations  $\sigma(\xi)$  establishing an initial minimum of two deputies per province. Figure 3 represents the deviations,  $\sigma(\zeta)$  with an initial minimum of one deputy, and Figure 4 with an initial minimum of two deputies.

From the point of view of the power of an elector, taking into account the indicators  $\xi$  and  $\zeta$ , the reduction to an initial minimum of one deputy per province decreases the current differences for any composition of the Congress of Deputies between 300 and 400 Deputies, because the values of the standard deviations,  $\sigma(\xi)$  and  $\sigma(\zeta)$ , are inferior to those that correspond to the current composition of the Congress of Deputies. The scenario in which the standard deviations are lower is that of 391 deputies with an initial minimum of one deputy and their values are  $\sigma(\xi) = 2.72680 \cdot 10^{-9}$  and  $\sigma(\zeta) = 2.56677 \cdot 10^{-9}$ . Table 3 compares the standard deviations, for distributions corresponding to 300, 350, 360 and 400 deputies, with an initial minimum of one and two deputies. As the reader can observe, the reduction of the initial minimum to one deputy per province reduces the



**Figure 3.** Graph of  $\sigma(\zeta)$  with an initial minimum of one deputy per province.



**Figure 4.** Graph of  $\sigma(\zeta)$  with an initial minimum of two deputies per province.

standard deviations whether one chooses to maintain the current composition of the Congress with 350 deputies or if a reduction to 300 deputies or an expansion to 360 or 400 deputies is proposed. The smaller the deviation, the fairer the distribution of power among the electors will be, since the power of an elector will not differ too much whether it belongs to a small, medium or large circumscription.

### Proposals of modification of the LOREG

Various studies have contemplated a possible reform of the composition of the Congress of Deputies with an extension to 400 deputies with an initial minimum of one deputy per province (Consejo de Estado, 2009), proposing, in addition, modifications in the rule of distribution of seats once the elections have been held in Ramírez and Máquez (2010) or in Martínez-Maza (2017). In the proposals that we formulate below, we limit ourselves only to the composition of the Congress of Deputies together with the initial minimum.

**Table 3.** Comparative of standard deviations: 300, 350, 360 and 400.

Total of Deputies	Minimum per provinces	$\sigma(\xi)$	$\sigma(\zeta)$
300	1	$3.78529 \cdot 10^{-9}$	$3.64730 \cdot 10^{-9}$
300	2	$7.73785 \cdot 10^{-9}$	$7.67901 \cdot 10^{-9}$
350	1	$3.31873 \cdot 10^{-9}$	$3.17524 \cdot 10^{-9}$
350	2	$6.28947 \cdot 10^{-9}$	$6.21466 \cdot 10^{-9}$
360	1	$3.27894 \cdot 10^{-9}$	$3.12785 \cdot 10^{-9}$
360	2	$6.48332 \cdot 10^{-9}$	$6.39433 \cdot 10^{-9}$
400	1	$2.86563 \cdot 10^{-9}$	$2.70799 \cdot 10^{-9}$
400	2	$5.55741 \cdot 10^{-9}$	$5.45557 \cdot 10^{-9}$

We consider that establishing a chamber with 400 arguing that the Spanish population has gone from 36 million inhabitants in 1977 to 47 million currently does not respond to an objective criterion, but rather to the desire of the national parties that obtain the third and fourth place after the holding of elections. That one is a desire that could very likely be abandoned if they reach the power in a future election, because as Fernández-Miranda (2009: 23) stated, ‘it is not unusual for criticisms of an electoral system, spilled by the opposition, to dilute in the air when the government is accessed’. It should also be pointed out that these differences between the parties that occupy the third and fourth positions are notably reduced when they exceed or are close to the 20% threshold of the votes they have obtained overall, as has happened in the last two elections held in 2015 and 2016.

It is not convincing to make an extension of 400 deputies based on a comparison with the national parliaments of some European states either, because we must not forget that in Spain there is a national parliament and 17 regional parliaments. This fact should make the political class reflect, especially in times of austerity. Why not reduce the Congress to 300 with a minimum of one deputy per province?

If a more global comparison is made, considering almost all the democratic parliaments of the world, the number of parliamentarians could be roughly the cubic root of the de jure population, as Taagepera and Shugart (1989) have pointed out. Therefore, in the two reforms of the LOREG that we are going to propose next, we will assume as a criterion to fix the maximum composition of the Congress of Deputies that of the cubic root of the de jure population, with the limitations established by CE78.

### *First proposal: The composition of the Congress of Deputies together with an initial minimum of one deputy per province*

Assuming the criterion of the cubic root of the de jure population, and fixing the initial minimum in a deputy per province, a proposal of reform of the Article 162 of the LOREG, that would not demand a constitutional reform, could be the following one:

1. The Congress is formed by a number of Deputies calculated with the following formula:

$$N = \max(300, \min(\sqrt[3]{P}, 400)) \quad (10)$$

where  $P$  is the de jure Spanish population of the year prior to the call for elections.

2. Each province has an initial minimum of one Deputy. The populations of Ceuta and Melilla are each represented by one Deputy.
3. The  $N - 52$  remaining Deputies are distributed among provinces in proportion to their population, according to the following procedure:
  - (a) A quota of distribution resulting from dividing by  $N - 52$  the total de jure population of the peninsular and island provinces.
  - (b) Each province is assigned as many Deputies as result, in integer numbers, from dividing the provincial de jure population by the distribution quota.
  - (c) The remaining Deputies are distributed by assigning one to each of the provinces whose quotient, obtained according to the previous section, has a greater decimal fraction.
4. The Decree of convening must specify the number of Deputies to be elected in each circumscription, in accordance with the provisions of this article.

Table 4 includes the distribution with 360 deputies and an initial minimum of one deputy, together with the values of the indicators  $\xi_i$  and  $\zeta_i$ .

**Table 4.** Comparative population and vote power. First proposal.

$i$	$w_i$	$p_i \cdot 10^{-3}$	$\xi_i \cdot 10^8$	$\zeta_i \cdot 10^8$	$i$	$w_i$	$p_i \cdot 10^{-3}$	$\xi_i \cdot 10^8$	$\zeta_i \cdot 10^8$
1	3	323	2.51	2.51	2	4	394	2.74	2.74
3	13	1855	1.91	1.91	4	6	701	2.32	2.32
5	2	164	3.28	3.28	6	6	686	2.37	2.37
7	8	1104	1.96	1.96	8	38	5523	1.96	1.96
9	3	364	2.23	2.23	10	4	406	2.66	2.65
11	9	1240	1.97	1.97	12	5	582	2.32	2.32
13	4	513	2.10	2.10	14	6	795	2.04	2.04
15	8	1127	1.92	1.92	16	2	203	2.65	2.65
17	6	753	2.16	2.16	18	7	917	2.07	2.07
19	3	253	3.20	3.20	20	6	716	2.27	2.27
21	4	520	2.08	2.08	22	3	222	3.64	3.62
23	5	654	2.07	2.07	24	4	479	2.26	2.26
25	4	436	2.48	2.48	26	3	317	2.56	2.56
27	3	339	2.39	2.39	28	44	6433	2.09	2.09
29	12	1628	2.01	2.01	30	11	1467	2.04	2.04
31	5	640	2.11	2.11	32	3	318	2.55	2.55
33	8	1051	2.06	2.06	34	2	166	3.26	3.26
35	8	1098	1.97	1.97	36	7	947	2.00	2.00
37	3	339	2.39	2.39	38	8	1001	2.17	2.17
39	5	585	2.31	2.31	40	2	157	3.43	3.43
41	14	1941	1.97	1.97	42	2	91	5.95	5.95
43	6	795	2.04	2.04	44	2	138	3.89	3.89
45	6	693	2.34	2.34	46	18	2543	1.96	1.96
47	5	526	2.57	2.67	48	9	1148	2.13	2.13
49	2	183	2.95	2.95	50	7	956	1.98	1.98
51	1	84	3.21	3.21	52	1	85	3.16	3.17

## Second proposal: The composition of the Congress of Deputies together with a different initial number of deputies for each province

Article 68.2 of the EC78 establishes that ‘The law shall distribute the total number of Deputies, assigning an initial minimum representation to each circumscription and distributing the rest in proportion to the population’. Note that it only states that *minimum initial representation should be fixed*, and does not impose that it should be the same for all provinces. The law where it is established that the initial minimum number of deputies is the same for all the provinces is Article 162.2 of the LOREG. Could Article 162.2 of the LOREG be modified to establish a different initial minimum for the provinces, being in any case proportional to its population? Although the answer could raise doubts, let us admit as a working hypothesis that the Constitutional Court ruled that it is possible to determine a different initial number for each province; in this case, the following wording could be proposed for Article 162 of the LOREG:

1. The Congress is formed by a number of Deputies calculated with the next formula:

$$N = \max(300, \min(\sqrt[3]{P}, 400)) \quad (11)$$

2. where  $P$  is the de jure Spanish population of the year prior to the call for elections.
3. The populations of Ceuta and Melilla are each represented by one Deputy.
4. The  $N - 2$  remaining Deputies are distributed among the provinces in proportion to their population, according to the following procedure:
  - (a) A quota of distribution resulting from dividing by  $N - 2$  the total de jure population of the peninsular and island provinces.
  - (b) Each province is assigned as many Deputies as result, in integer numbers, from dividing the provincial de jure population by the distribution quota. When the entire part of this quotient is equal to zero for a province, it will be assigned one Deputy.
  - (c) The remaining Deputies are distributed by assigning one to each of the provinces for which the difference between the quotient, obtained in the previous section, and the number of deputies assigned initially, has a greater decimal fraction.
5. The Decree of convening must specify the number of Deputies to be elected in each circumscription, in accordance with the provisions of this article.

As shown in Figures 5 and 6, when a different initial minimum is established according to this second proposal, the values of the standard deviations,  $\sigma(\xi)$  and  $\sigma(\zeta)$ , with respect to the values corresponding to the current composition of the Congress of Deputies are also reduced. We recall that for a composition of 350 deputies with an initial minimum of two deputies the deviations are,  $\sigma(\xi) = 6.28947 \cdot 10^{-9}$  and  $\sigma(\zeta) = 6.21466 \cdot 10^{-9}$ .

The standard deviations according to this second proposal for a composition of 300 deputies are,  $\sigma(\xi) = 1.91567 \cdot 10^{-9}$  and  $\sigma(\zeta) = 2.06078 \cdot 10^{-9}$ ; for a composition of 350 deputies are,  $\sigma(\xi) = 1.70588 \cdot 10^{-9}$  and  $\sigma(\zeta) = 1.78227 \cdot 10^{-9}$ ; for a composition of 360 deputies are,  $\sigma(\xi) = 1.72594 \cdot 10^{-9}$  and  $\sigma(\zeta) = 1.89466 \cdot 10^{-9}$ ; and for a composition of 400 deputies are,  $\sigma(\xi) = 1.47808 \cdot 10^{-9}$  and  $\sigma(\zeta) = 1.60387 \cdot 10^{-9}$ . With this second proposal, the distribution of seats for 360 deputies is included in Table 5. Column  $w_i$  indicates again the total number of deputies for  $i$  but as the sum of the minimum number of deputies per province,  $m_i$ , and those subsequently assigned by circumscription,  $r_i$ .

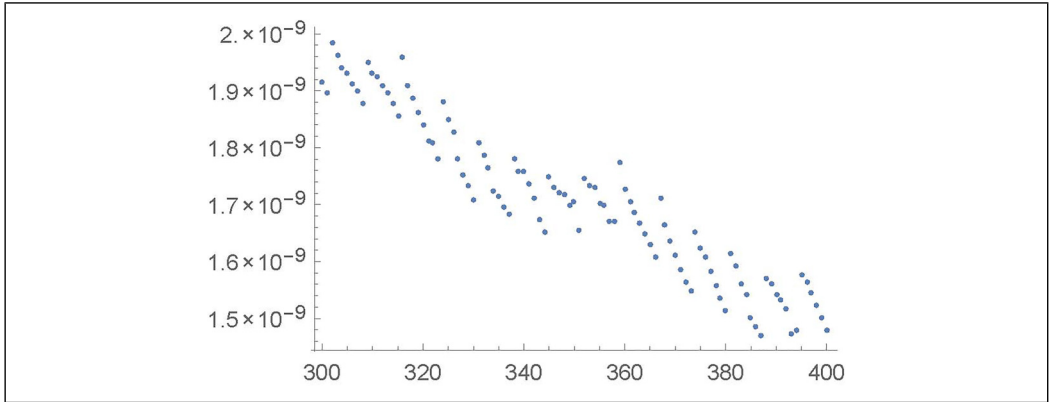


Figure 5. Graph of  $\sigma(\xi)$  with initial minimum proportional to the population.

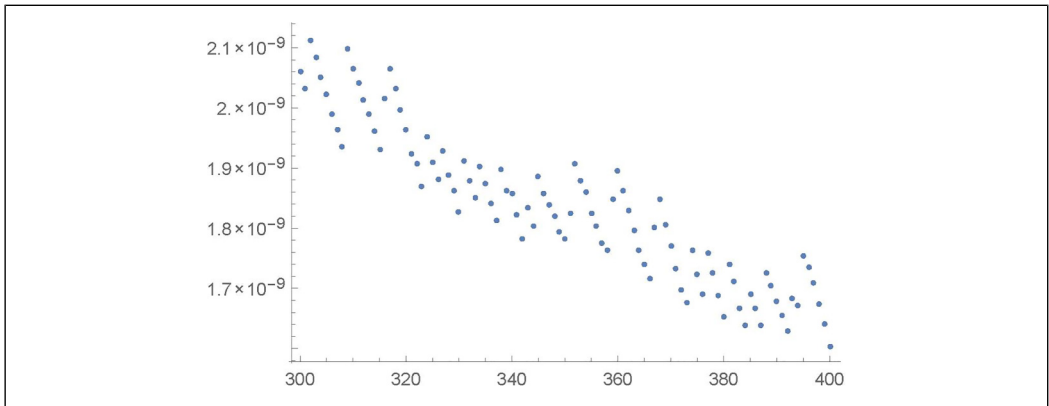


Figure 6. Graph of  $\sigma(\zeta)$  with initial minimum proportional to the population.

### Comparatives and conclusions

We compare in Table 6 the composition of the Congress in 1977, with 350 deputies and an initial minimum of two, in 2015 with the same composition of the Congress with minima of one and two deputies, with a composition of 360 deputies with an initial minimum of one deputy per province and with a composition of 360 deputies with different initial minima. Note that the two proposals for enlargement to 360 contribute to a reduction of disproportionality between large and small circumscriptions, also improving the circumscriptions of medium size. Personally, we are inclined towards the first proposal of a composition of the Congress of Deputies with 360 and an initial minimum of one deputy per province, because with this proposal all the provinces, with the exception of the cities of Ceuta and Melilla, have at least two deputies.

In the general elections of December 2015, with the distribution 350/2 (350 deputies with an initial minimum of two deputies per province), the distribution of seats in the Congress of Deputies (nationwide) was as follows:

**Table 5.** Comparative population and vote power. Second proposal.

$i$	$w_i$	$p_i \cdot 10^{-3}$	$\xi_i \cdot 10^8$	$\zeta_i \cdot 10^8$	$i$	$w_i$	$p_i \cdot 10^{-3}$	$\xi_i \cdot 10^8$	$\zeta_i \cdot 10^8$
1	2+1	323	2.49	2.44	2	3+0	394	2.04	2.04
3	14+0	1855	2.05	2.05	4	5+0	701	1.92	1.88
5	1+0	164	1.63	1.58	6	5+0	686	1.96	1.92
7	8+1	1104	2.20	2.18	8	42+1	5523	2.15	2.31
9	2+1	364	2.21	2.17	10	3+0	406	1.98	1.94
11	9+1	1240	2.18	2.16	12	4+1	582	2.31	2.27
13	3+1	513	2.09	2.05	14	6+0	795	2.03	2.00
15	8+1	1127	2.15	2.13	16	1+1	203	2.64	2.57
17	5+1	753	2.14	2.11	18	7+0	917	2.07	2.03
19	1+1	253	2.12	2.07	20	5+1	716	2.25	2.22
21	4+0	520	2.07	2.03	22	1+1	222	2.41	2.35
23	5+0	654	2.05	2.03	24	3+1	479	2.24	2.20
25	3+0	436	1.85	1.81	26	2+0	317	1.69	1.65
27	2+1	339	2.37	2.32	28	49+1	6433	2.36	2.37
29	12+1	1628	2.17	2.15	30	11+0	1467	2.03	2.01
31	4+1	640	2.10	2.06	32	2+0	318	1.69	1.67
33	8+0	1051	2.05	2.03	34	1+0	166	1.62	1.57
35	8+0	1098	1.96	1.94	36	7+0	947	1.99	1.96
37	2+1	339	2.37	2.32	38	7+1	1001	2.15	2.13
39	4+1	585	2.30	2.36	40	1+0	157	1.70	1.66
41	14+1	1941	2.11	2.10	42	1+0	91	2.95	2.88
43	6+0	795	2.03	2.00	44	1+0	138	1.93	1.90
45	5+0	693	1.94	1.90	46	19+1	2543	2.18	2.16
47	4+0	526	2.04	2.00	48	8+1	1148	2.11	2.09
49	1+0	183	1.97	1.94	50	7+0	956	1.97	1.94
51	1+0	84	3.19	3.11	52	1+0	85	3.14	3.06

PP, 123; PSOE, 90; PODEMOS, 71; Cs, 40; ERC, 9; DL, 8; PNV, 6; EH Bildu, 2; CC-PNC, 1.

Note that PP and PODEMOS data have been included linked at the regional level.

With a 360/1 distribution (360 deputies with an initial minimum of one deputy per province), the distribution of seats in the Congress of Deputies at the national level would have been as follows:

PP, 123; PSOE, 92; PODEMOS, 74; Cs, 43; ERC, 10; DL, 9; PNV, 6; EH Bildu, 2; CC-PNC, 1.

Note that with this new distribution of seats, the parties that benefit most from the new distribution of seats are the third and fourth political forces (Podemos and Cs) which increase their number of deputies by three; PP does not change the number of deputies, and PSOE increases it by two deputies. Two Catalan nationalist parties also register an increase in deputies: ERC which increases one deputy and DL which increases one deputy.

In 2015, the formation of a new government was not possible, given the impossibility of forming a large coalition with the two large national parties PP and PSOE. The outright opposition of the leader of the socialist party Pedro Sánchez called for the holding of new elections, as a leftist government was not feasible because the sum PSOE + PODEMOS amounted to 161 seats, nor was a

**Table 6.** Distribution of deputies by provinces.

C	Name	1977	2015			
		350/2	350/2	350/1	360/1	360
1	Álava	4	4	3	3	3
2	Albacete	4	4	4	4	3
3	Alicante	9	12	13	13	14
4	Almería	5	6	6	6	5
5	Ávila	3	3	2	2	1
6	Badajoz	7	6	5	6	5
7	Baleares	6	8	8	8	9
8	Barcelona	32	31	37	38	43
9	Burgos	4	4	3	3	3
10	Cáceres	5	4	4	4	3
11	Cádiz	8	9	9	9	10
12	Castellón	5	5	5	5	5
13	Ciudad Real	5	5	4	4	4
14	Córdoba	7	6	6	6	6
15	Coruña	9	8	8	8	9
16	Cuenca	4	3	2	2	2
17	Gerona	5	6	6	6	6
18	Granada	7	7	7	7	7
19	Guadalajara	3	3	3	3	2
20	Guipúzcoa	7	6	6	6	6
21	Huelva	5	5	4	4	4
22	Huesca	4	3	2	3	2
23	Jaén	6	5	5	5	5
24	León	6	4	4	4	4
25	Lérida	6	4	4	4	3
26	La Rioja	4	4	3	3	2
27	Lugo	5	4	3	3	3
28	Madrid	32	36	42	44	50
29	Málaga	9	11	12	12	13
30	Murcia	8	10	10	11	11
31	Navarra	5	5	5	5	5
32	Orense	5	4	3	3	2
33	Asturias	10	8	8	8	8
34	Palencia	3	3	2	2	1
35	Las Palmas	6	8	8	8	8
36	Pontevedra	8	7	7	7	7
37	Salamanca	5	4	3	3	3
38	Tenerife	6	7	7	8	8
39	Cantabria	5	5	5	5	5
40	Segovia	3	3	2	2	1
41	Sevilla	12	12	14	14	15
42	Soria	3	2	2	2	1
43	Tarragona	5	6	6	6	6

(continued)



**Table 6.** (continued)

C	Name	1977	2015			
		350/2	350/2	350/1	360/1	360
44	Teruel	3	3	2	2	1
45	Toledo	5	6	6	6	5
46	Valencia	15	16	17	18	20
47	Valladolid	5	5	4	5	4
48	Vizcaya	10	8	8	9	9
49	Zamora	4	3	2	2	1
50	Zaragoza	8	7	7	7	7
51	Ceuta	1	1	1	1	1
52	Melilla	1	1	1	1	1

centre-right government feasible; even the sum PP + Cs amounted to 163 deputies. After the resignation of the leader of the PP, Mariano Rajoy, to form a government; the King entrusted the leader of PSOE with the formation of a government, and this one, after a failure in the negotiations with the leader of PODEMOS, Pablo Iglesias, attempted the formation of a centre-left government PSOE-Cs, supported by 130 deputies, who did not get the confidence of the Congress of Deputies, as both the PP and PODEMOS deputies voted against. This resulted in the dissolution of the Congress and the Senate and the call for new elections, which took place in June 2016.

If the distribution of seats in the December 2015 elections with proposal 360/1 had been made, the sum of PSOE + PODEMOS would have amounted 166 seats, the same as the right centre coalition PP + Cs. Neither of these two blocks would have obtained an absolute majority; it would only have been feasible to form a large coalition of government PP + PSOE, formed by only two political groups, which would have amounted to 215 seats.

In the elections held in June 2016, the following results were obtained with the 350/2 system.

PP, 137; PSOE, 85; PODEMOS, 71; Cs, 32; ERC, 9; CDC (antes DL), 8; PNV, 5; EH Bildu, 2; CC-PNC, 1.

After the June 2016 elections, a minority PP government was established, supported by 170 deputies of the PP, Cs and CC-PNC parties, in second call, exceeding the sum of PSOE and PODEMOS by 16 seats. The election of the leader of PP, Mariano Rajoy as the President of the Government, was possible by the abstention of 60 deputies of PSOE. This abstention occurred after the cessation of its general secretary Pedro Sánchez. It was a government that lasted two years, because after the return of Pedro Sánchez to the direction of the PSOE, he reached the Presidency of the Government after winning a motion of censure against Mariano Rajoy. This motion of censure was supported by 180 deputies from the following parties: PSOE, PODEMOS, ERC, CDC, PNV and EH Bildu. The impossibility of approving the general budgets of 2018 resulted in the call for new elections in April 2019.

With the 360–1 distribution, the following results would have been obtained in the June 2016 elections:

PP, 138; PSOE, 90; PODEMOS, 71; Cs, 34; ERC, 10; CDC (formerly DL), 9; PNV, 5; EH Bildu, 2; CC-PNC, 1.

On this occasion, the great beneficiary of the extension of the number of deputies would have been the second political force, PSOE, that would increase by five seats. Cs would have two more deputies, ERC one and CDC one.

With this distribution, the centre-right parties PP + Cs would have added 172 deputies, and the sum of deputies for the left parties PSOE + PODEMOS would have amounted to 161. That is, a difference of 11 deputies in favour of the centre-right block.

Consequently, the adoption of the 360/1 distribution system in the elections of December 2015 and June 2016 would hardly have meant a variation in the votes of the first political force, PP, and basically rewards the rest of the political forces of the national level, with different distributions of the 10 additional seats. The Catalan nationalist parties, ERC and CIU, would also benefit from this system, with the increase of one deputy on both occasions.

## Other lines of work

In the last general elections held in Spain on 10 November 2019, with the 350/2 system, a fragmentation of the parliamentary arch was again produced. It also presents great imbalances in regard to the power of the individual voter; the case of the voters of the province of Teruel, a province of very small population in which a local platform has achieved a deputy with just 19,000 votes, is very notable. For the sake of governance, and to avoid excessive fragmentation of the vote, proposals to modify the composition of the Congress of Deputies should also analyse the impact that would have to establish a minimum percentage at the national level, to the different candidates, to have the right to obtain seats.

Although this article focuses on the distribution of seats in the Spanish Congress of Deputies, the methodology used is of a more general nature and could be applied to the analysis of the distribution of seats in other parliaments, such as the European Parliament. Each member state returns a fixed number of members of the European Parliament (MEPs); from six for smaller member states like Malta, Luxembourg and Cyprus, to 96 for Germany, the largest number of MEPs (751) was elected in May 2019 but that number was reduced to 705 following a post-Brexit reshuffle in February 2020. A brief review of the literature on this topic is provided by Bertini et al. (2019).

It would also be interesting to analyse other methods of seat allocation that have a mixed system, such as the Japanese House of Representatives ([https://www.shugiin.go.jp/internet/index.nsf/html/index\\_e.htm](https://www.shugiin.go.jp/internet/index.nsf/html/index_e.htm)). Today, the House of Representatives has 465 members, of whom 176 are elected under the proportional representation system and 289 are elected from single-seat constituencies.

## Mathematical code

(\*Banzhaf index\*)

```
banzhafG[weights_List] := Times @@ (1 + x^weights)
banzhafIndex[i_, weights_List, q_] :=
Module[{g, coefi, sw, delw}, delw = Delete[weights, i];
g = banzhafG[delw]; sw = Apply[Plus, delw] + 1;
coefi = CoefficientList[g, x];
Apply[Plus, coefi[[Range[Max[1, q - weights[[i]] + 1], Min[q, sw]]]]]
```

```
banzhafPower[weights_List, q_] := #/(Plus @@ #) & @
Table[banzhafIndex[i, weights, q], {i, Length[weights]}]
```

(\*Shapley-Shubik index\*)

```
ssG[weights_List] := Times @@ (1 + z x^weights)
ssPowerPlus[weights_List, q_] :=
Module[{n = Length[weights], g, gg, coefi, delw, sw},
Table[delw = Delete[weights, i];
g = ssG[delw]; sw = Apply[Plus, delw] + 1;
coefi = CoefficientList[g, x];
gg = Apply[Plus, coefi[[Range[Max[1, q - weights[[i]] + 1], Min[q, sw]]]]];
Sum[Coefficient[gg, z^j] j! (n - j - 1)!, {j, n - 1}], {i, n}]/n!]
```

(\*data provinces\*)

```
Provincias = Range[52];
NombreProvincias = {'Alava', 'Albacete', 'Alicante', 'Almeria', 'Avila', 'Badajoz', 'Islas
Balears', 'Barcelona', 'Burgos', 'Caceres', 'Cadiz', 'Castellon', 'Ciudad Real', 'Cordoba',
'Coruna', 'Cuenca', 'Gerona', 'Granada', 'Guadalajara', 'Guipuzcoa', 'Huelva', 'Huesca',
'Jaen', 'Leon', 'Lerida', 'La Rioja', 'Lugo', 'Madrid', 'Malaga', 'Murcia', 'Navarra',
'Orense', 'Asturias', 'Palencia', 'Las Palmas', 'Pontevedra', 'Salamanca', 'Tenerife',
'Cantabria', 'Segovia', 'Sevilla', 'Soria', 'Tarragona', 'Teruel', 'Toledo', 'Valencia', 'Valladolid',
'Vizcaya', 'Zamora', 'Zaragoza', 'Ceuta', 'Melilla'};
P2015 = {323648,394580,1855047,701211,164925,686730,1104479,5523922,364002,406267,
1240284,582327,513713,795611,1127196,203841,753054,917297,253686,716834,520017,
222909,654170,479395,436029,317053,339386,6436996,1628973,1467288,640476,318391,
1051229,166035,1098406,947374,339395,1001900,585179,157570,1941480,91006,
795101,138932,693371,2543315,526288,1148775,183436,956006,84263,85584};
P1977 = {235334, 341511, 1062433, 398954, 195480, 669349, 609029, 4350931, 363546,
441505, 947498, 415458, 492323, 727940, 1071292, 231299, 446697, 754423, 146582,
669264, 413905, 218381, 653740, 540868, 351965, 247305, 414342, 4328438, 959762,
910131, 493646, 435851, 1102051, 194336, 647014, 845160, 371875, 628049, 497356,
155080, 1425938, 107879, 482663, 162368, 477708, 1954596, 455534, 1133505, 241046,
802527, 64481, 56743};
```

(\*Distribution of seats and calculation of deviations, algorithm 1\*)

```
Congreso[Votos_Integer, minvotes_Integer, Population_List,
Indicator_Integer] :=
Module[{Coefi, Resto, Restan, TotalP, Reparto, quota, Votes},
PoblacionP = Drop[Population, -2];
TotalP = Apply[Plus, PoblacionP];
Votes = Votos - 50 minvotes - 2;
Coefi = TotalP/Votes;
Reparto = PoblacionP/Coefi;
```

```

Resto = Votes - Apply[Plus, Floor[Reparto]] ;
ListaAux = Reparto - Floor[Reparto];
Liston = Reverse[Sort[ListaAux]][[Resto]];
Restan = If[# >= Liston, 1, 0] & /@ ListaAux;
DiputadosReparto = Floor[Reparto] + Restan;
DiputadosReparto = Flatten[Append[DiputadosReparto, {0, 0}]];
DiputadosMinimo = Flatten[Append[Table[minvotes, 50], {1, 1}]];
DiputadosProvincia = DiputadosMinimo + DiputadosReparto;
DiputadosTotal = Apply[Plus, DiputadosProvincia];
quota = 1 + Floor[Apply[Plus, DiputadosTotal]/2];
If[Indicator == 1,
Poder = banzhafPower[DiputadosProvincia, quota] // N;,
Poder = ssPowerPlus[DiputadosProvincia, quota] // N;];
Desviacion =
Sqrt[Apply[Plus,
Population*((Poder/Population - 1/Apply[Plus, Population])^2)]];

```

```

Return[{DiputadosProvincia, Poder/Population}]
(*Distribution of seats and calculation of deviations, algorithm 2*)

```

```

Clear[Congreso2]
Congreso2[Votes_Integer, Population_List, Indicator_Integer] :=
Module[{Inicial, Restan, Resto, Liston, ListaAux, ListaAux2, TotalP,
Reparto, quota},
PoblacionN = Population;
PoblacionP = Drop[PoblacionN, -2];
TotalP = Apply[Plus, PoblacionP] ;
Coefi = (Votes - 2)*PoblacionP/TotalP;
Inicial = Floor[Coefi];
Table[If[Inicial[[i]] < 1, Inicial[[i]] = 1], {i, 50}];
Resto = Votes - Apply[Plus, Inicial] ;
ListaAux = Coefi - Inicial;
Liston = Reverse[Sort[ListaAux]][[Resto]];
Restan = If[# >= Liston, 1, 0] & /@ ListaAux;
DiputadosReparto = Inicial + Restan;
DiputadosInicial = Flatten[Append[Inicial, {1, 1}]];
DiputadosRestan = Flatten[Append[Restan, {0, 0}]];
DiputadosProvincia = Flatten[Append[DiputadosReparto, {1, 1}]];
DiputadosTotal = Apply[Plus, DiputadosProvincia];
quota = 1 + Round[Apply[Plus, DiputadosTotal]/2];
If[Indicator == 1,
Poder = banzhafPower[DiputadosProvincia, quota] // N;,
Poder = ssPowerPlus[DiputadosProvincia, quota] // N;];
Desviacion =
Sqrt[Apply[Plus,
Population*((Poder/Population - 1/Apply[Plus, Population])^2)]];

```


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