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Contact Phenomenon of Free-Rolling Wide-Base Tires: Effect of Speed and Temperature

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Abstract
The finite-element method was used to quantify the effect of temperature and speed on contact area, deflection, and three-dimensional contact stresses of a free-rolling wide-base tire. The tire model comprised material properties identified in the laboratory and/or provided by the tire manufacturer (hyperviscoelastic rubber and linear elastic reinforcement) and accurate geometry. The model was validated using measured deflection and contact area. The analysis matrix consisted of 81 cases resulting from a combination of three loads, tire-inflation pressures, speeds, and temperatures. Four

criteria were used to compare contact stresses: range, average, root-mean-square error, and coefficient of determination. Speed and temperature influence the contact area more than deflection. Longitudinal contact stresses were the most affected, followed by transverse contact stresses. In general, under constant load and tire-inflation pressure, the influence of temperature was more significant on the considered output variables than the effect of speed.

Introduction

The deteriorating pavement infrastructure (ASCE 2013) and pavement design philosophies focusing on near-surface damage (NAPA 2013) challenged pavement engineers to improve their analysis procedures, material characterization, and assumptions. When investigating loading conditions, tire-pavement contact stresses usually receive special attention. Contact stresses are not only directly related to various types of distresses (mainly close to the surface), but are also the only feasible manner to compare the effect of various tire types on pavement damage [e.g., conventional dual-tire assembly versus new-generation wide-base tires (WBT)].

Several studies have highlighted the importance of three-dimensional (3D) tire-pavement contact stresses on flexible pavement responses and damage. For instance, Myers et al. (1999) included measured contact stresses in *BISAR* and showed increasing potential of lateral contact stresses for developing surface cracking and near-surface rutting. Numerical predictions of primary rutting also proved the influence of tire footprint details and the lack of relevance of nonuniform contact stresses to rut depth and shape (Hua and White 2002). Additional analytical evidence was provided by Novak et al. (2003) in support of the relationship between low confinement and high shear stresses (linked to surface rutting), whereas uniform vertical contact stresses were linked to high confinement and low shear. Finally, Al-Qadi and Yoo (2007) used validated finite-element pavement model, which incorporated 3D contact stresses, to relate in-plane contact stresses to near-surface cracking and primary rutting. The level of impact depended on specific pavement structures and loading conditions.

The assumption of constant contact stresses over circular contact areas led to overestimated and underestimated pavement responses at high and low tire-inflation pressures, respectively (Machemehl et al. 2005). Wang and Al-Qadi (2009) used moving load with 3D contact stresses to show contact stresses relevance not only to near-surface responses but also to transverse tensile strain at the bottom of asphalt concrete and compressive strain at the top of the subgrade. Furthermore, Wang et al. (2011) coupled tire and pavement in a single finite-element model to show higher proximity of the stress state close to the surface to the Mohr-Coulomb failure surface.

Temperature has been considered in tire modeling for predicting rolling resistance and temperature distribution, but limited attention has been given to its influence on 3D contact stresses. Park et al. (1997) performed thermomechanical analysis using the finite-element method to predict temperature distribution in steady-state rolling tires. The steady state, transient temperature distribution, and rolling resistance were predicted by Ebbott et al. (1999). Narasimha Rao et al. (2006) used the mixed Lagrangian-Eulerian formulation to calculate steady temperature distribution of three-dimensional rolling tires, which can be extended to tires with tread patterns. Using a similar approach, Suwannachit and Nackenhorst (2013) calculated temperatures and contact pressures of axisymmetric tires considering large deformations, viscous hysteresis, dynamic stiffening, internal heating, and temperature dependency. Recently, Srirangam et al. (2014) predicted the distribution of tire temperature while considering the effect of temperature on hysteretic friction using a thermomechanical analysis.

Several researchers have studied contact area and deflection between tire and surface. Equations, based on experimental measurements depending on tires geometry, load, and tire-inflation pressure, were proposed four decades ago (Komandi 1976). In addition, following an analytical procedure and assuming a tire contacting a rigid surface on ellipsoidal or rectangular area, Lyasko (1994) predicted the two contact variables. For contact area in particular, a review showed three categories to estimate it: (1) theoretical models on rigid surface; (2) theoretical models on deformable surface; and (3) empirical models (Sharma and Pandey 1996). However, none of these methods considered the effect of speed and temperature on tire deflection and contact area. In 2005, equations based on experimental measurement were used to predict contact area using information published by tire manufactures, but speed and temperature were not part of the input (Keller 2005). However, Persson et al. (2004) reported analytically the variation of contact area with time between a rough substrate and a viscoelastic solid (not a rolling tire). In general, the viscoelastic nature of the rubber was investigated.

The main objective of this paper is to evaluate the influence of temperature and speed on the 3D tire-pavement contact stresses at free rolling using validated finite-element modeling. A main tire component, rubber, was modeled as hyperviscoelastic with material constants determined from laboratory testing and/or provided by the tire manufacturer. In addition, the finite-element model was used to calculate 3D contact stresses during free rolling for typical values of axle load and tire-inflation pressures at various temperatures and traveling speeds.

Finite-Element Model

The general-purpose finite-element software *ABAQUS* was used to model a WBT 445/50R22.5. The geometry was defined by the global dimensions of the tire and details in the cross section. The global dimension (radius, height, and rim radius) can be inferred from the tire's nomenclature. The tire's physical cuts were analyzed to determine the geometric details of the cross section. The belts' cross section details and orientation were also measured. Fig. 1 presents a 3D and side view of the tire model.

Rubber and reinforcement were modeled as hyperviscoelastic and linear elastic, respectively. The Prony series terms and Mooney-Rivlin constants of rubber were obtained based on the regression analysis of the frequency sweep test results of four tire components: tread, subtread, sidewall, and shoulder. The stress–strain results obtained from the uniaxial tension test [ASTM D882 (ASTM 2012)] were used to determine the elastic modulus of each belt.

The type, size, and distribution of finite elements were defined through a mesh sensitivity analysis. The size of finite elements was changed until the coarsest mesh with strain energy of $\pm 5\%$ of the finest mesh was obtained. This approach was applied in the cross section and the three-dimensional model. Four main types of finite elements were used: cylindrical, hybrid, rebar, and Cartesian. The cylindrical elements cover bigger arc lengths accurately, thus reducing the total number of elements needed in the circumference of the tire (Danielson and Noor 1997; Kennedy 2003); hybrid elements are ideal to model incompressible materials such as rubber; rebar elements are used to model tire reinforcement by inputting the rubber and belt's properties independently to avoid homogenization (Helnwein et al. 1993); and Cartesian elements are more accurate when calculating contact stresses.

The finite-element analysis was divided into three phases: axisymmetric, three-dimensional, and free rolling. The cross-sectional dimensions, material properties, and tire-inflation pressure were assigned in the axisymmetric phase. The axisymmetric model revolved to generate the three-dimensional model, where the axle load was applied. Finally, the free-rolling state was determined by an iterative procedure where the angular speed was modified until the reaction torque was negligible. The last phase was performed using the steady-state transport analysis capability of *ABAQUS*.

Finally, the friction between the tire and the rolling surface was defined by the Coulomb model (Wang et al. 2014), and the finite-element model was validated using experimental measurement of deflection, contact area, and vertical contact stresses (Hernandez and Al-Qadi 2015). Four variables were considered in the numerical analysis matrix: axle load P, tire-inflation pressure S, tire temperature T, and speed V. Three values of each variable were analyzed, rendering a total of 81 scenarios for analysis, as summarized in Table 1.

Table 1. Values of Load, Inflation Pressure, Speed, and Temperature Considered

Load (kN)	Pressure (kPa)	Speed (km/h)	Temperature (°C)
$P1 = 26.6$	$S1 = 552$	$V1 = 8$	$T1 = 25$
$P2 = 35.5$	$S2 = 690$	$V2 = 65$	$T2 = 45$
$P3 = 44.4$	$S3 = 758$	$V3 = 115$	$T3 = 65$

Variation along the Contact Length of 3D Contact Stresses

Figs. 2 and 3 show a sample of the contact stresses variation in the vertical, transverse, and longitudinal direction (σ_z, σ_y, and σ_x, respectively) along the tire's contact length at a central rib for various values of speed and temperature. In the case of σ_z and σ_x, distribution along the center of the rib is presented, whereas the edge of the rib was chosen for σ_y. The vertical axis indicates the value of corresponding contact stress, whereas the horizontal axis represents the distance along the contact length x. The center of the tire in the undeformed configuration is located at $x = 0$.

The vertical contact stresses, which are very similar in magnitude to the contact pressure, are shown in Figs. 2(a) and 3(a) (vertical contact stresses and contact pressure are used interchangeably in this study). Vertical contact stresses always have the same sign and result from the superposition of the support provided by the sidewall, bending and shear deformation of the tread, and buckling and stiffness of the tread (Clark 1971). Load and tire-inflation pressure affected the shape of σ_z. Increasing P at constant S created a plateau at the center of the contact length because the rubber reached its load-carrying capacity. The length of the plateau propagated from the center of the contact patch. In addition, increasing S augmented the peak vertical constant stresses and had an opposite effect on the plateau's length compared with P because the higher tire-inflation pressure decreased the plateau's length. On the other hand, temperature and speed did not change the shape of variation in any direction, but higher temperature reduced the peak σ_z, which increased with higher speed.

Figs. 2(b) and 3(b) present the variation of the transverse contact stresses with contact length. Distribution along the contact length changed with respect to rib locations and the location inside the rib. The plot corresponds to the edge of a central rib. None of the variables changed the shape of the variation along the contact length. At the edge of an inner rib, transverse contact stresses had two peaks: a positive peak and a negative peak. The negative peak is located at the rear part of the contact, and it is linked to points reaching the limit established by friction. Consequently, it was not affected by S, P, V, and T. The positive peak is related to the restrained motion of the tread, and it increased as the temperature increased. The positive peak slightly diminished with speed, mainly at the lowest temperature. Finally, the positive peak was more affected by tire-inflation pressure than by load.

A typical variation of the longitudinal contact stresses is given in Figs. 2(c) and 3(c). The graph of σ_x had two negative peaks and one positive peak, which are related to the relative motion between the tread and the rolling surface (Clark 1971). The three peaks were higher for the lowest temperature and any combination of the other variables, with a more significant difference observed between $T1$ and $T2$ than between $T2$ and $T3$, with $T1$, $T2$, and $T3$ given in Table 1. In contrast to σ_y, applied load was more relevant than tire-inflation pressure for the peak values of σ_x. Finally, speed increased all three peaks.

Contact Area and Deflection

Figs. 4 and 5 present the change of the contact area A_c and deflection δ with respect to the values in Table 1. Each plot is divided into three sections, corresponding to each tire-inflation pressure. At the same time, the sectors are divided into groups of points with the same applied load. Nine points resulted from the combination of three speeds and three temperature for each S-P pair; points with the same mark shape had the same temperature (circle, $T = 25°\text{C}$; triangle, $T = 45°\text{C}$; and square, $T = 65°\text{C}$). Speed increased at each three consecutive data points.

In general, the contact area decreased with speed and tire-inflation pressure and increased with temperature and load. The greatest influence was caused by P for all combinations of variables. Temperature and speed had a small impact on contact area. The highest drop created by V between its extreme values was 3.8%, and the largest increment caused by temperature was 6.5%.

The effect of temperature and speed was less on tire deflection compared with the contact area. The highest reduction in deflection caused by temperature was observed when the tire was subjected to a load of 26.6 kN and a tire-inflation pressure of 552 kPa traveling at the highest speed. Under this conditions, δ changed from 24.99 mm when $T = 25°\text{C}$ to 25.58 mm when $T = 65°\text{C}$, an increment of 2.4%. In addition, the largest diminution occurred at the same load and tire-inflation pressure at the lowest temperature, changing from 25.54 mm at $V = 8\ \text{km/h}$ to 24.99 mm at $V = 115\ \text{km/h}$ (2.1%).

Based on the finite-element results, a regression analysis was performed to predict the contact area and deflection ddepending on load, tire-inflation pressure, speed, and temperature. The general equations, which are based on existing expressions and iterations to obtain high coefficient of determination (R^2), are as follows:

(1)

$$A_c = k_1 \cdot S^{a_1} \cdot P^{b_1} \cdot V^{c_1} \cdot T^{d_1}$$

(2)

$$\delta = (k_2 \cdot \sqrt{P}) + (k_3 \cdot P) + (a_2 \cdot S) + (b_2 \cdot T)$$

where $k_{1,2,3}$, $a_{1,2}$, $b_{1,2}$, $c_{1,2}$ = regression coefficients. The final expressions for A_c and δ were found as

(3)

$$A_c = 451.4 \times \frac{P^{0.6305} \cdot T^{0.0841}}{S^{0.3341} \cdot V^{0.0108}}$$

(4)

$$\delta = (0.2610 \cdot \sqrt{P}) - [(2.813 \times 10^{-5}) \cdot P] - (0.0297 \cdot S) + (0.0205 \cdot T)$$

Fig. 6 compares the contact area and deflection from the finite-element analysis and the ones from the regression analysis; the corresponding coefficient of determination R^2 is very high.

Range and Average 3D Contact Stresses

The range in which the contact stresses varied and their average contact stresses are presented in Fig. 7. The plots have a similar configuration as shown in Fig. 4, which facilitate visualization of the considered variables in a single plot. The applied load was the most relevant variable for the average and maximum contact pressure. However, the effect of P was significantly higher for the maximum σ_z than for the average σ_z. For instance, at the lowest tire-inflation pressure and $P = 35.5\ \mathrm{kN}$, $V = 65\ \mathrm{km/h}$, and $T = 45°\mathrm{C}$, the mean σ_z was 0.552 MPa, which increased to 0.605 MPa, or by 9.6%, after changing the load to $P = 44.4\ \mathrm{kN}$. On the other hand, for the same load increment, the maximum σ_z increased 28.8% from 1.881 MPa at $P3$ to 2.423 MPa at $P3$.

All variables created the same trend on the average and maximum σ_z except for the tire-inflation pressure. The maximum vertical contact stress decreased with tire-inflation pressure, but the mean σ_z increased. Furthermore, the mean σ_z decreased with temperature and increased with speed, with the effect of temperature being more relevant. The highest diminution of mean contact pressure caused by extreme temperature was 8.2%. On the other hand, the highest increment created by the extreme values of speed was 3.8%.

Speed and temperature had a higher influence on the longitudinal contact stresses. Because the variation of σ_x presented positive and negative peaks, the average was close to zero. The maximum increment caused by speed on the positive and negative peaks were very similar: 17.0 and 17.2%, respectively. As in the case of vertical contact stresses, the influence of temperature was more important than speed: the reduction of the maximum and minimum σ_x was 33.1 and 25.8%, respectively.

As in the case of longitudinal contact stresses, the average transverse contact stresses were close to zero because of the presence of positive and negative peaks. Although the maximum and the minimum σ_y were very similar in magnitude, the negative peak was consistently higher than the positive peak. The effect of speed on the minimum σ_y was not uniform for the combination of variables listed in Table 1. When V caused the minimum σ_y to increase, the percentage increment was as high as 3.0%, and when V reduced the minimum σ_y, the drop reached 3.7%. Similarly, the influence of temperature on the extreme values of the transverse contact stresses was not uniform, but the magnitude change was higher. The ratio between the minimum σ_y at the highest and lowest temperature varied between 0.935 and 1.063, or, in other words, the reduction and increment caused by T was as high as 6.3 and 6.5%, respectively.

Global Comparison of 3D Contact Stresses

Although range and average are good tools for comparing the tire-pavement contact stresses, they are susceptible to extreme values. For instance, the applied load increases, the tire's sidewalls carry more load, and σ_z increases more at the tire's edge than at its center. Consequently, the increment of the maximum contact pressure reflects a localized phenomenon instead of a change in the whole contact patch.

To complement the analysis performed in the previous section, the root-mean-square error (RMSE) and coefficient of determination were adopted. Although RMSE and R^2 are not usually used to measure the effect of the variables of a problem, they can be used to compare the 3D contact stresses point by point, as shown in Fig. 8. The 3D contact stresses in the foot region were stored in arrays, and RMSE and R^2 were calculated taking the lowest temperature and speed as reference for each combination of load and tire-inflation pressure. The horizontal axis indicates the contact stresses for the specified loading ($S1$ $P1$ in this case), and the vertical axis indicates the contact stresses for the same loading condition and the lowest speed and temperature. The plot also shows the corresponding RMSE and R^2. The results for all loading conditions in each direction are summarized in Figs. 9 and 10. Fig. 9 focuses on RMSE and its comparison with maximum magnitude of contact stresses, whereas Fig. 10 presents the variation of R^2 for the combinations of variables introduced in Table 1.

The RMSE for the vertical contact stresses, RMSE_z, becomes more relevant because it is closer to the maximum magnitude of σ_z. For instance, when $S = 520\,\mathrm{kPa}$, $V = 8\ \mathrm{km/h}$, and $T = 65°\mathrm{C}$, $\mathrm{RMSE}_z = 0.0340\ \mathrm{MPa}$ when $P = 35.6\ \mathrm{kN}$. The value is similar to 0.0320, which is the RMSE_z when $P = 44.4\ \mathrm{kN}$. However, the maximum magnitude of σ_z for the same cases are 1.848 and 2.379 MPa, respectively. This indicates a change in the ratio of RMSE_z to the maximum magnitude from 0.0143 to 0.0173 when changing the load between $P3$ and $P2$, an increment of 21.0%. The highest ratio was 0.0287, which corresponds to $S = 690\ \mathrm{kPa}$, $P = 26.2\ \mathrm{kN}$, $V = 8\ \mathrm{km/h}$, and $T = 65°\mathrm{C}$.

Higher RMSE_z was observed at the highest temperature, regardless of speed. The largest magnitude of RMSE_z, 0.035 MPa, occurred at a combination of intermediate tire-inflation pressure, highest load and temperature, and smallest speed. The lowest RMSE_z was seen when the tire temperature was 65°C, 0.0197 MPa, and it was greater than almost all the other RMSE_z. Furthermore, the effect of speed was not uniform. At the lowest temperature, RMSE_z increased with V, but it decreased with the two other temperature values.

As expected, the highest RMSE_x was not as high as RMSE_z, but it was more relevant when compared with the maximum magnitude of σ_x. The highest ratio between RMSE_x and the maximum magnitude was 0.105 (compared with 0.0287 in the vertical direction). This value was observed when the temperature and tire-inflation pressure were the highest and the speed and load were the lowest. In the longitudinal direction, the effect of temperature was not as dominant as in the vertical case. In other words, a high temperature does not guarantee a significant RMSE_x.

The variation of the RMSE in the transverse direction, RMSE_y, with speed was similar to the vertical case: it increased under the lowest temperature, but decreased under the two other temperatures. For a constant tire-inflation pressure and load, $T = 65°\mathrm{C}$ provided RMSE_y with the highest magnitude. However, the importance of temperature was linked to the applied load; RMSE_y was higher at high load and low temperature than at low load and high temperature. The change of transverse contact stresses with speed and temperature was not as high as for σ_x, but not as low as for σ_z. This is confirmed by the RMSE_y (0.0064 MPa), and ratio between RMSE_y and maximum magnitude of σ_y (0.0314).

Finally, the change of the coefficient of determination R^2 in each direction with load, tire-inflation pressure, speed, and temperature is shown in Fig. 10. The figure reinforces most of the statements previously made for the effect of V and T on the 3D contact stresses. First, the highest R^2 was observed for σ_z, followed by σ_y and σ_x. In other words, longitudinal contact stresses were the most affected by speed and temperature. Second, the trend of R^2 for σ_x and σ_y was similar, but it varied in magnitude. Third, the lowest coefficient of determination was obtained for the highest tire-inflation pressure and temperature, and lowest speed and load: 0.8336. Fourth, R^2 for σ_x and σ_y decreased with the increase in tire-inflation pressure.

Conclusions

A validated finite-element model was used to study the influence of temperature and speed on deflection, contact area, and three-dimensional contact stresses of free-rolling wide-base tires. The contact area increased with temperature and decreased with speed: the highest increment caused by temperature was 6.8%, whereas the largest drop created by speed was 3.8%. Tire deflection was not significantly modified by temperature and speed, with an approximate change of 2%. Two equations were proposed to predict the contact area and deflection as a function of the input variables: load, tire-inflation pressure, speed, and temperature. Observing the variation of 3D contact stresses along the contact length, it was found that speed and temperature did not significantly modify the shape of the contact stresses, but changed the magnitude of the peak values.

Longitudinal contact stresses were the most influenced by temperature and speed: the increment of the peaks caused by speed was around 17%; the reduction of the extreme values created by temperature was as high as 33.1%; and the highest ratio between the root-mean-square error and maximum magnitude was 10.2%. The same changes for the average vertical contact stresses were 3.8, 8.2, and 2.9%. On the other hand, the transverse contact stresses ranged between the longitudinal and vertical stresses. The change caused by temperature and speed on minimum transverse contact stresses was approximately 6.0 and 3.0%, respectively. Finally, the lowest R^2, 0.8336, corresponded to the longitudinal contact stresses at the highest tire-inflation pressure and temperature and the lowest speed and load. In general, applied load had the highest impact of the variables studied, followed by tire-inflation pressure, temperature, and speed.

Trucks travel at various speeds depending on the type of road (urban and rural) and under different environmental conditions. The resulting truck loading damage near the pavement surface is heavily affected by contact stresses. Accurate calculation of tire-pavement contact would improve the design of new pavements and the scheduling of maintenance and rehabilitation activities of existing roads.

The effects of temperature and speed presented in this study correspond to a single load repetition; analysis considering high load repetitions will be used to predict near-surface fatigue cracking and rutting.

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Notation

The following symbols are used in this paper:

A_c	=	contact area;
R^2	=	coefficient of determination;
$\mathrm{RMSE}_{x,y,z}$	=	root-mean-square error in the x, y, and z direction;
x	=	longitudinal direction;
y	=	transverse direction;
z	=	vertical direction;
δ	=	tire deflection; and
$\sigma_{x,y,z}$	=	contact stresses in the x, y, and z direction.

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