

Paradox Phenomena in Autonomously Self-Adapting Navigation

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Abstract: *The online routing game model can be used to measure and prove the benefits of online real time data in road traffic navigation systems. A few properties of the routing strategies are already proved. In this paper we point out that there are some paradoxes like phenomena behind these proofs, similarly as in the Braess network.*

Keywords: *Autonomous systems, road traffic navigation, online routing game, paradox phenomenon.*

1. Introduction

Road traffic navigation is an application area of the classical routing problem which is modelled with the help of routing games in algorithmic game theory. Algorithmic game theory studies networks with source routing (Section 18 in [1]), in which the end users simultaneously choose a full route to their destination and the traffic is routed in a congestion sensitive manner. In the routing game approach every participant knows the congestion characteristics of the road network and the participants make their optimal routing decision based on this static information. It is well known that individually self-optimizing travel routes do not necessarily result in optimal traffic (optimal for any global parameter) and each participant may have a longer travel time than with central planning. This is known as the “price of anarchy” which was explored by 2012 Gödel prize winners Roughgarden and Tardos. In their paper [2] they investigated the old conundrum in transportation science, known as the “Braess’s paradox” [11]. The algorithmic game theory investigations revealed important properties of the routing games, however the algorithmic game theory approach includes assumptions which do not handle the dynamic online information environment of the current navigation devices. The navigation devices in modern cars can get up-to-date information of the current

status of the traffic, like the current travel time on each road, indicating the current situation of the traffic that needs to be adapted to. In this paper we are discussing the routing game problem in such online environment and are going to point out some novel forms of the paradox phenomena of the online routing game problem.

The online routing game problem is basically autonomous and self-adapting navigation which we are going to expound in the next two paragraphs.

If all the information about the road network, the cars on the roads and the destination of the cars could be collected by a centralized system, then it would be able to create an optimal plan for the trips of the cars. Optimality may be measured in several ways, but usually we assume that the goal is to optimize a "global" parameter of the traffic, like the sum of the travel times. We also assume that the purpose is to assure some kind of fairness for all the traffic participants, for example none of the cars pays some extra-long travel time to achieve the global optimum of the whole traffic. Everyday traffic is not coordinated by a centralized system and even if the traffic was coordinated by such a centralized system, there would come the question whether the individual traffic participants would conform to its instructions. In reality the traffic participants make their own autonomous decisions based on their intentions and the information available locally for them. This means that instead of centralized decision making, we have a set of autonomous distributed decision makers. In this aspect autonomy refers to the autonomous route planning by the navigation devices in the individual cars instead of following the instructions of a centralized planner.

Another aspect of the autonomous behaviour of the online navigation systems is related to the ability of the traffic as a whole to self-adapt to the current situation. The current wave of the progress of information technology is marked by the widespread availability of the online real time data. The routing algorithms implemented in the navigation devices must be able to utilize this real time data to self-heal the global traffic, for example if a road becomes congested then the navigation devices autonomously tell the individual cars how to adapt to the current traffic situation and send the cars to less congested roads. Note that here we focus on online-data-based self-adaptation which is different from the self-adaptation based on previous experiences, like in the case of a route selection from home to work based on the experience of the previous day.

This paper has five main sections. In Section 2 we shortly describe the online game theory model which is able to model self-adaptation to real time information. In Section 3 we discuss some properties of the class of simple naive online routing games, which model the currently available commercial online-data-based navigation systems. In Section 4 we discuss the properties of the class of simple naive intention propagation online routing games, which model the prediction utilizing anticipatory vehicle routing systems proposed by researchers with the purpose to improve the currently available commercial online-data-based navigation systems. In Section 5 we point out two paradox phenomena in these online routing games. Finally, in Section 6 we summarize the main messages and implications of this paper. The interesting findings of this paper are based on the formal proofs in papers [13] and [14].

2. Online routing games

The online-data-based self-adapting routing problem is a challenging application, because in this problem the autonomous agents have access to real time data, and based on this information they autonomously try to self-organize themselves by creating adapted plans to achieve their individual goals in an environment where they jointly utilize the resources that become more costly since more agents use them. In this problem the agents are dynamically arriving and departing after completing their plans. The plans are created by exploiting the online data that describe the current status and the current cost of the resources. There is uncertainty about the feasible decision of an agent, because the cost of the resources will change by the time the agent starts to use them: the departing agents will release the resources as they complete their plans, the agents simultaneously creating their plans will influence each other's costs, and the agents arriving later may also influence the cost of the resources used by the agents already executing their plans. This is somewhat similar to the typical game theory problems, where the outcome of the action of an agent depends on its own decision plus the decisions of the other agents, however in the online routing problem the outcome depends on even more circumstances as written above. This type of applications is called *online joint resource utilization games* [13], derived from the algorithmic game theory [1] and online mechanisms [3]. Note that these games are different from the resource allocation or minority games [5] which are simultaneous one shot or the repeated simultaneous games where there might be some coordination among some of the agents. In contrast, the online joint resource utilization games are continuous and non-cooperative games exploiting real time online data.

The adaptive car navigation using real time data is a special case of the online joint resource utilization games, because the possible order of the resource utilization in the plan of the agents is determined by the structure of the road network. From a theoretical point of view, the online-data-based car navigation applications are called *online routing games* [13]. Note that in this approach each driver makes an individual online-data-based decision at the time of entering the network, whereas in other approaches [6] the drivers learn the best route to select, based on their past experiences.

2.1. The model of online routing games

In order to have a generic model, the model of the online joint resource utilization game was defined [13] as an extension of the algorithmic game theory model of the routing problem and the online mechanisms. The model resembles the algorithmic game theory routing game model in concepts of the flow, cost and resource, and it resembles the model of online mechanisms in sequences of time periods and decisions. A time unit T is introduced in order to be able to compute the rate of the resource utilization, which is called a flow. The model of online routing games [13] is like the model of online joint resource utilization games but with a restriction on the allowed plans represented by a graph and with somewhat different cost functions.

The *model of the online routing game* is the sextuple (t, T, G, c, r, k) , where

- $t = \{1, 2, \dots\}$ is a sequence of equal time periods;
- T is a natural number with T time periods giving one time unit;
- G is a directed graph $G = (V, E)$ with a vertex set V and an edge set E where each $e \in E$ is characterized by a cost function c_e which is equal to the utilization time of the edge;
 - c is the cost function of G with $c_e: R^+ \rightarrow R^+$ for each edge e of G mapping the incoming flow to the travel time on that edge, which is never less than the remaining cost of any other agent currently utilizing that edge increased by the time gap of the flow; in this model the cars cannot overtake the cars already on the road and there is a time gap, i.e., a minimum "following distance";
 - r is the total flow given by a vector of r_i flows with r_i denoting the flow aiming for a trip P_i from a source vertex s_i of G to a target vertex t_i of G ;
 - $k = (k^1, k^2, \dots)$ is a sequence of decision vectors with a decision vector $k^t = (k^t_1, k^t_2, \dots)$ made in the time period t and k^t_i is the decision made by the agent of the flow r_i in the time period t .

In this model the graph G may contain parallel edges. The cost functions are non-negative, continuous and non-decreasing. The cost functions have a constant part (the non-congested part) which does not depend on the flow on the edge and a variable part which depends on the flow on the edge. The variable part is not known to any of the actors of the model until an agent exits an edge and reports it. The flow r_i is given by T/n_i where n_i is a natural number constant, meaning that the following distance of the units of the flow r_i are n_i time periods. So if $T = 6$ and $n_i = 2$, then one car enters the network every second time step and the intensity of the flow is 3, because 3 cars enter the network in a time unit consisting of 6 time steps. The k^t_i decision is how the trip P_i is routed on a single path of the paths leading from s_i to t_i . The actual cost of a path (e_1, e_2, e_3, \dots) for a flow at time period t is the sum of the cost of the edges where the actual cost of an edge is determined at the time when the flow enters the edge.

The actual cost of the edges becomes known for the agents only when an agent reports its actual cost. Because the agents do not report the cost values at each time step, the agents interested in the cost values must decrease the last reported value by taking into account the time elapsed since the last reporting event (it is similar to the pheromone evaporation in [9]).

The online routing game model can accommodate changes of the cost function c over the sequence of time periods t , because the agents can get information about the actual cost only from the cost reported by the agents exiting an edge.

2.2. Routing strategy

The critical point in the online routing game is how to determine the best decision vector k . The algorithmic game theory approach assumes that the agents have full information about the cost functions and the theory tells what the best strategy is in case of simultaneous decisions, but does not tell how the agents can achieve this. In online mechanisms, a central planner decides which resources at which cost are allocated to which agent. In online routing games there is no central planner. The

agents in online routing games will have to apply algorithms similar to the online algorithms [4].

2.3. The benefit of online real time data

We would like to be able to tell if the agents are better off by autonomously trying to self-adapt to the observed online real time data or not. If we want to evaluate only the benefits of the autonomous self-adaptation using online real time data, then we want to compare the results with an "oracle" using the same decision making strategy.

In the algorithmic game theory model there is an equilibrium and the price of the anarchy concept is the ratio between the equilibrium and the optimum. The online routing games investigated in [13] and [14] do not have equilibrium at some flow values. Because there is no equilibrium, different measures for the best, worst and average cases (which are guaranteed to exist if there is a finite sequence of time periods) are defined. Depending on the type of application, we are interested in the different types of benefits. Most important is the worst case, because it can be used to provide a guarantee in critical applications. The best case can be used in applications, where we have to make sure that a certain value is achieved at least once. The average case is seldom useful in itself, usually we have to consider the statistical distribution parameters as well.

The definition of the different benefits of the online real time data [13] is given in the next paragraph. If these benefits are greater than 1, then they are in fact a "price" like the price of anarchy [2].

Defintion 1. The *worst/best/average case benefit* of the online real time data at a given flow is the ratio between the cost of the maximum/minimum/average cost of the flow and the cost of the same flow with an oracle using the same decision making strategy and only the fixed part of the cost functions.

2.4. Classes of online routing games

The online routing games using the same type of decision strategies belong to the same class of online routing games. Each class needs to be evaluated of how large benefit it makes out of the online real time data, in order to be able to determine the type of a suitable application. The evaluation must include formal proofs. Further research is needed to study different online routing game decision strategies derived from other related games, like resource allocation or minority games [5] and El Farol Bar problem in [12].

3. Simple naive strategy online routing games

Typical navigation software, currently installed in cars uses the simple shortest path search on the road network, possibly modifying the distances with the online information about the actual traffic delay. We call this decision strategy a *simple naive strategy*. This strategy was investigated because of its practical importance. Note that the simple naive strategy is by definition deterministic, thus it is a pure

strategy. The properties of the simple naive strategy were investigated in [13] and three properties were proved.

The first property says that if the agents of the car navigation systems use a simple naive strategy to self-adapt autonomously to the current situation of the traffic, then at some flow values they may make the traffic *fluctuate*.

The second property is the possibility of “*single flow intensification*”: if the agents of the navigation system use a simple naive strategy to autonomously self-adapt to the current situation of the traffic and only a single flow enters the road network, then at some flow value at some time there may be a road somewhere in the network, where the flow is bigger than the flow that entered the network.

The third property is that the online information may have a *price*: if the agents of the car navigation systems use the simple naive strategy to autonomously self-adapt to the current situation of the traffic, then at some flow values sometimes they may be worse off than without exploiting the information about the current situation.

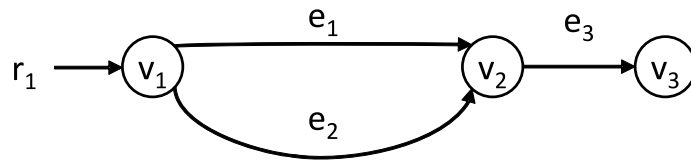


Fig. 1. A simple naive online routing game with “single flow intensification” and “price”

The proofs of these properties [13] are related to the online routing game which has the network shown in Fig. 1. In this network road e_2 is not susceptible to a congestion, the non-congested travel time on road e_1 is smaller than the travel time on e_2 , which is smaller than the congested travel time on e_1 at some flow value less than r_1 , which is smaller 1.5 times than the travel time on e_2 . In addition, the travel time on e_2 is bigger than 2 time units. If online information is not available, then all cars select the path (e_1, e_3) , because it is the shortest one. If online information is available, then roads e_1 and e_2 will be used alternatively and it may happen that a platoon of the full incoming flow going on e_1 is caught up by some agents that go on e_2 and arrive at vertex v_2 at the same time, so a bigger flow will go into e_3 than the one that enters the network. The result is that the travel time on path (e_1, e_3) in this case will be longer than the travel time without any online information. We will return to this online routing game later in Section 5.

The above formal proof results are underlined by similar simulation results as well, like the simple scenario consisting of two parallel routes investigated in [10]. These simulations also showed that the online information often leads to oscillations in the number of cars on the routes, the velocity and the travel times, which leads to worse overall performance. In the discussion the authors conclude that one of the reasons for oscillations is that the real time travel information reflects the state of the network some time ago. Another reason for the oscillations is that the agents do not coordinate their actions. In order to improve this, the authors advise the usage of anticipatory traffic forecast based on the broadcast route

choice of the agents, which basically means that the agents share or propagate their intentions.

4. Simple naive intention propagation strategy online routing games

The anticipatory vehicle routing proposed in [7] uses the individual planned routes of the agents to forecast the future traffic density. Every vehicle is represented by a vehicle agent running on a smart device inside the vehicle. The vehicle agents communicate with the delegate Multi-Agent System (delegate MAS) which represents the traffic environment and is able to make forecast of the future traffic density based on the current traffic situation and the planned routes of the vehicles. The delegate multi-agent system provides the traffic forecast back to the vehicle agents which use this information to plan their shortest trip.

A slightly modified version of the above anticipatory vehicle routing system is used to define and formally analyze the class of online routing games that use intention-propagation-based prediction in their decision mechanism [14]. This class of online routing games are called simple naive intention propagation online routing games.

Definition 2. Simple Naive Intention Propagation online routing games (SNIP online routing games) are online routing games where the decision making agents of the flows r_i are the vehicle agents of the anticipatory vehicle routing system; the vehicle agents use the delegate MAS as above described to predict the travel time for each path p_j of their trip P_i ; and their decision k'_i is to select the path with the shortest travel time among the predicted travel times on the different paths of their trip P_i . The vehicle agent notifies the delegate MAS of its selected path and the delegate MAS remembers this selection while the vehicle agent is in the network and invalidates it when the vehicle agent exits the road network.

In SNIP online routing games the agents receive a prediction of the future traffic, so we would expect that this additional information can be used to improve the properties of the simple naive online routing games. This was investigated formally in [14] with the following results. Unfortunately, intention propagation does not solve the “single flow intensification” problem. In SNIP online routing games “single flow intensification” may happen because the agents try to exploit the “*gain by delay*” phenomenon as shown in the proof in [14]; however this “*gain by delay*” phenomenon does not cause the same problem for the worst case benefit of online data as in the case of simple naive online routing games. In spite of this, there are SNIP online routing games which may have the worst case benefit of online data above one at some flow values. In addition, SNIP online routing games manifest fluctuation as well.

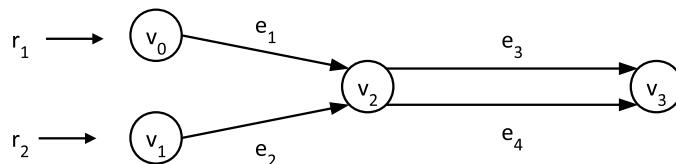


Fig. 2. The network of the SNIP online routing game with “fluctuation” and “price”

The proofs of the latter two properties above are related to the online routing game which has a network shown in Fig. 2. The cost functions are $c_{e_1} = 1$, $c_{e_2} = 1$, $c_{e_3} = 10+x$, and $c_{e_4} = 10.5+10\times x$, where x is the total incoming flow on the edge. The total traffic flow is $r = (r_1, r_2)$ with a flow $r_1 = 1$ from the source node v_0 to the target node v_3 , and a flow $r_2 = 1$ from v_1 up to v_3 . Without online data, both flows would select the road e_3 , so the cost of both flows would be 13 and the total cost – 26. With online data, the flows realize at some time that the cost of e_3 goes above the cost of e_4 . This happens at the same time for both flows and they are not aware that the other flow is going to change to e_4 at the same time, so they do not take into account the additional cost on e_4 . This is because the traffic forecaster is only aware of the intention propagations before the current time step, but does not know and cannot forecast the decisions at the current time step. Because e_4 is more susceptible to congestion than e_3 , the cost on e_4 will be more than on e_3 , so the total cost may go above 26 which is the travel time without online information. We will return to this online routing game later in Section 5.

5. Paradox phenomena

The Braess paradox is well known in the classical road traffic scenarios: if we add a new road to a road network in which the cars selfishly choose their routes and seemingly we increase the throughput capacity of the network, then in some cases the overall performance will be reduced. In the above sections we described some online-data-based navigation road traffic scenarios modelled by online routing games. Interestingly, these scenarios contain paradoxes similarly to the original Braess paradox.

5.1. Paradox in the simple naive online routing game

Let us take the simple naive online routing game which has the network shown in Fig. 3. The cost functions of roads e_1 and e_2 are the same as in the online routing game of Fig. 1. In this network the traffic flow r_1 is from v_1 to v_3 , so all cars go on the path (e_1, e_3) , because that is the only path.

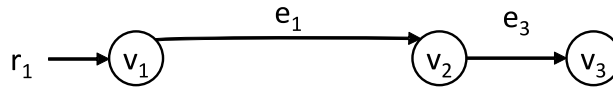


Fig. 3. Simple naive online routing game paradox

The paradox is that if we extend this online routing game with an additional road e_2 to increase the throughput capacity between nodes v_1 and v_2 to get the online routing game of Fig. 1, then sometimes the performance will be reduced.

Interestingly, the additional road e_2 in Fig. 1 is not susceptible to congestion, and the travel time on e_2 is bigger than the non-congested travel time on road e_1 , so seemingly the throughput capacity between v_1 and v_2 is increased. Also, the travel time on e_2 is smaller than the congested travel time on e_1 at the flow value of r_1 , so

road e_2 could ease the congestion on e_1 . However, the cars of flow r_1 are continuously trying to adapt to the observed traffic conditions, which makes the traffic fluctuate and sometimes they will suffer worse traffic conditions than without road e_2 , because the worst case benefit of the online data is above 1.

5.2. Paradox in the simple naive intention propagation online routing game

Let us take the simple naive intention propagation online routing game which has the network shown in Fig. 4. The cost functions of roads e_1 , e_2 and e_3 are the same as in the online routing game of Fig. 2. In this network the traffic flow r_1 is from v_0 to v_3 and the traffic flow r_2 is from v_1 to v_3 , so all the cars of r_1 go on the path (e_1, e_3) and all the cars of r_2 go on the path (e_2, e_3) .

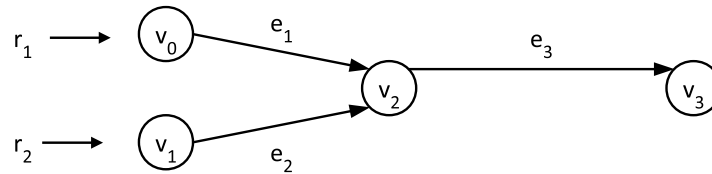


Fig. 4. SNIP online routing game paradox

The paradox is that if we extend this network with an additional road e_4 to increase the throughput capacity between nodes v_2 and v_3 to get the network presented in Fig. 2, then sometimes the performance will be reduced.

Although the additional road e_4 in Fig. 2 seemingly increases the throughput capacity between v_2 and v_3 and we would expect that road e_4 could ease the congestion on e_3 , the flows r_1 and r_2 are continuously trying to adapt to the observed traffic conditions, and from time to time the flows simultaneously switch to e_4 . Because e_4 is more susceptible to congestion than e_3 , the result is that the traffic will fluctuate and sometimes the cars will suffer worse traffic conditions than without road e_4 .

6. Conclusions

In this paper we have discussed the routing game problem in online environment where the cars try to adapt to the current status of the traffic. The status of the traffic is obtained from up-to-date information, like the current travel time on each road. We have pointed out two novel forms of the paradox phenomena of the online routing game problem: one in the simple naive online routing game and one in the simple naive intention propagation online routing game. These paradox phenomena of the online routing problem have similarity to the Braess paradox of the classic routing problem in the sense that although the throughput capacity of the network is extended, sometimes the overall performance will be reduced.

These paradox phenomena have implications for the structure of the road network. If there are parallel roads as in the above networks and the cars use online navigation devices, then at some flow values the traffic might start to fluctuate and the traffic will be worse than with only one road instead of parallel roads. The

designers of road networks should try to avoid such parallel roads if the cars use navigation devices exploiting online information.

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