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The Numerical Distance and Size Effects in Symbolic Numerical Cognition

– PhD thesis booklet –

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Budapest, 2019

Introduction

The thesis investigates the sources of the distance and size effects in symbolic numerical cognition. The distance effect means that when comparing two numbers based on their numerical value, responses are faster and errors are fewer when the numerical distance between the numbers is larger. The size effect means that performance is better when comparing smaller numbers (Figure 1). Both effects were first described by Moyer and Landauer (1967) who also suggested that comparison of symbolic numbers works according to psychophysical laws. Later studies with infants (Feigenson, Dehaene, & Spelke, 2004), animals (Hauser & Spelke, 2004), and cultures without number words (Pica, Lemer, Izard, & Dehaene, 2004) provided evidence for an innate, continuous, noisy representation system shared across species termed the Analogue Number System (ANS). The ANS works according to Weber's law, and is supposed to be the mechanism behind both non-symbolic (e.g., sets of dots) and symbolic (e.g., Indo-Arabic numbers) numbers (Cantlon, Platt, & Brannon, 2009; Dehaene, 1992). The distance and size effects are interpreted as indicators of the ANS, the source of both being the ratio of the two numbers.

Recent findings, however, support a different explanation for symbolic numerical cognition. For example, performance in symbolic and non-symbolic comparison tasks does not correlate in children (e.g., Holloway & Ansari, 2009), the distance and size effects do not correlate for symbolic numbers, but do for non-symbolic numbers (Krajcsi, 2016), the distance effect can be found outside the number domain (Vigliocco, Vinson, Damian, & Levelt, 2002). An alternative proposal is the Discrete Semantic System (DSS) (Krajcsi, Lengyel, & Kojouharova, 2016) which works similarly to the mental lexicon or a semantic network. In the DSS numbers are stored as nodes in a network, and the effects observable in different tasks depend on the strength of their semantic relations to other nodes, i.e., on the connection weights. Similar models have already been proposed in the literature, such as the delta-connectionist model (Verguts, Fias, & Stevens, 2005; Verguts & Van Opstal, 2014).

To investigate the sources of the two effects, the number comparison task was utilized. This task is probably the most widely used experimental paradigm in numerical cognition. In its most common version two numbers are compared by choosing the numerically larger of the two. We specified models based on the predictions of the ANS and the DSS that were linearly fitted to the error rates, reaction times, and drift rates (Ratcliff & McKoon, 2008; Smith & Ratcliff, 2004; Wagenmakers, Van Der Maas, & Grasman, 2007) in the full stimulus space (Figure 2).

Aims

In the presented Thesis Studies (see also Table 1) the aims were as follows:

1. examine whether a different model (DSS) is a better description for number comparison data for symbolic numbers than the ANS (Thesis Study 1, Experiment 1);
2. examine frequency as a possible source of the size effect by testing whether it can be induced by manipulating the frequency of presentation of the numbers when recently learned artificial numbers are used for which there is no prior experience (Thesis Study 1, Experiment 2 and 3);
3. examine the associations between the numbers and the “small-large” properties as a possible source of the distance effect by manipulating the distance between the numbers in a new, artificial number sequence (Thesis Study 2);
4. examine whether the associations between numbers and the “small-large” properties can be modified in Indo-Arabic numbers within a session of the comparison task (Thesis Study 3);
5. examine whether the size effect shows similar flexibility in Indo-Arabic numbers by manipulating the frequency of presentation of the numbers within a session (Thesis Study 3 and Thesis Study 4);
6. examine whether the distance and the size effects change independently of each other (all Thesis Studies);
7. more generally, an aim present in all reported studies, contrast the two proposed models of numerical cognition, ANS and DSS, in symbolic numbers. Here, the sources of the numerical distance and size effects are examined for being consistent with either account, and conclusions about the two accounts will be drawn based on that.

Table 1. Summary of which effect was studied in each study and which notation was used.

	Distance effect	Size effect
New symbols	Thesis Study 2	Thesis Study 1
Indo-Arabic digits	Thesis Study 3	Thesis Study 4

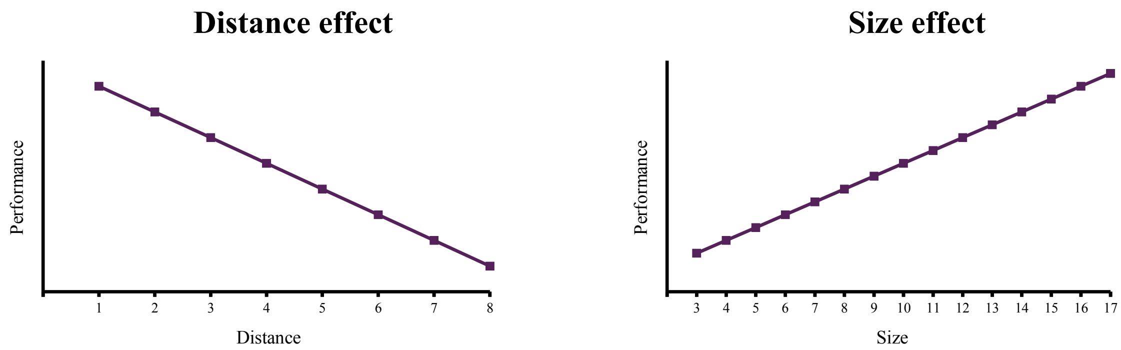

Figure 1. *The distance effect (left panel) shows worsening performance with smaller distance between the numbers. The x-axis shows the absolute difference. The size effect (right panel) is worse performance with larger numbers. The x-axis shows the effect expressed as the sum of the two numbers. Performance, shown on the y-axis, indicates error rate or reaction time.*

The full stimulus space

Number 2 \ Number 1	1	2	3	4	5	6	7	8	9
1		0.3	0.2	0.1	0.1	0.1	0.1	0.1	0.1
2	0.3		0.5	0.3	0.2	0.2	0.1	0.1	0.1
3	0.2	0.5		0.6	0.4	0.3	0.2	0.2	0.2
4	0.1	0.3	0.6		0.7	0.5	0.4	0.3	0.3
5	0.1	0.2	0.4	0.7		0.8	0.5	0.4	0.4
6	0.1	0.2	0.3	0.5	0.8		0.8	0.6	0.5
7	0.1	0.1	0.2	0.4	0.5	0.8		0.9	0.7
8	0.1	0.1	0.2	0.3	0.4	0.6	0.9		1.0
9	0.1	0.1	0.2	0.3	0.4	0.5	0.7	1.0	

Figure 2. *An illustration of the full stimulus space. Columns indicate one number of the pair to be compared, rows indicate the other number, and cells show expected performance. Darker shade indicates worse performance. The distance effect can be observed as better performance from the main diagonal towards the top-right and bottom-left corners. The size effect is worsening performance along the main diagonal from the top-left toward the bottom-right corner. The values were calculated as a × log(large/distance) + b, where a is set to 1 and b is set to 0.*

Thesis Study 1

The Source of the Symbolic Numerical Distance and Size Effects

The aim of this study was two-fold. The first step was to present a new model of symbolic numerical cognition, the DSS, and directly compare it to the current model, the ANS (Experiment 1). The second aim was to investigate the source of the size effect in a number comparison task – whether it was a consequence of the ratio of the two to-be-compared numbers as suggested by the ANS, or stemming from the everyday frequency of the numbers as supposed by the DSS (Experiment 2). New symbols were used instead of the Indo-Arabic digits, because the frequency of the already known symbols might be well established. A priming task was utilized to show that the new symbols are linked to the Indo-Arabic numbers: If a priming distance effect (i.e., faster responses for a target that is closer in value to the prime) is elicited, then the new symbols and the Indo-Arabic digits are semantically related. A third experiment was also conducted to ascertain that the results from Experiment 2 were not influenced by a confound, the semantic congruency effect (Leth-Steensen & Marley, 2000): If the task is to choose the larger number, then large numbers are responded to faster which may extinguish the size effect (Experiment 3).

In Experiment 1 eighteen participants (15 females, *M*=21.5 years, *SD*= 2.8) compared all possible pairs of Indo-Arabic digits from 1 to 9. The ANS and the DSS were then contrasted directly as both models can be described with different equations. In the case of the ANS model the quantitative descriptions for error rate, reaction time and drift rate were taken from the literature (Dehaene, 2007; Moyer & Landauer, 1967). For the DSS there were no available descriptions, but based on the known constraints (i.e., there should be a distance effect based on the numerical difference of the numbers and a size effect based on their frequency) we proposed two versions that could be tested against the ANS. Average error rates, reaction times, and drift rates were calculated for each participant in the full stimulus space. The quantitative descriptions of the ANS and DSS were fitted linearly to the group-average data. Both R^2 and *AIC* were used as a measure of the goodness of fit. For all fittings, the values for both R^2 and *AIC* were very similar, showing a better goodness of fit for the DSS or the ANS depending on the utilized quantitative description. Thus, we could not distinguish between the ANS and the DSS. This could be a consequence of either insufficient precision of the models or too high signal-to-noise ratio to reveal subtle differences. However, the DSS seems as plausible a model as the ANS to describe processing in symbolic numbers.

In Experiment 2, new symbols were introduced instead of the Indo-Arabic digits for the numbers from 1 to 9. All participants learned the new symbols, then compared all possible pairings in the comparison task. In one condition the artificial numbers were presented with uniform frequency, and in the other with biased (Indo-Arabic-like) frequency). The data of 14 participants (11 females, *M*=20.6 years, *SD*=2.1) were analyzed in the uniform frequency condition, and the data of 13 participants (13 females, *M*=24.3 years, *SD*=6.9) were analyzed in the biased frequency condition. At the end of the session all participants performed a priming task in which all possible new symbol–Indo-Arabic digit pairings were presented, with the new symbol being the prime and the Indo-Arabic digit being the target.

The distance and size effect regressors were fitted to the error rate and reaction time data from the two conditions in the full stimulus space for each participant's averaged data, then their slopes were tested against 0 (Figure 3). The distance effect was significant for both error rates and reaction times for both conditions as expected. The slope of the size effect deviated significantly from zero only in the biased frequency condition, and was significantly larger than the slope of the size effect in the uniform frequency condition. Thus, the size effect appeared as a result of manipulating the frequency of presentation of the numbers.

The data from the priming task were analyzed for a priming distance effect. While the descriptive data showed the expected priming distance effect, it was not significant. The lack of statistical significance may have been due to small statistical power, so a meta-analysis of five measurements of the priming distance effect was conducted: the two groups from Experiment 2 and three unpublished experiments. The meta-analysis confirmed the presence of a priming distance effect.

The data of 14 participants (10 females, *M*=25.4 years, *SD*=6.9) were examined in Experiment 3 in which only the uniform frequency condition was run. The experimental procedure was similar to that of Experiment 2, but this time the participants had to choose the smaller of the two numbers. The slope of the size effect was similar to that in the uniform frequency condition in Experiment 2 and did not deviate from 0, i.e. the semantic congruency effect did not obscure the size effect.

To sum up, the DSS model can explain symbolic numerical effects just as well as the ANS model as demonstrated by the model fit performed on the behavioral data from the comparison task. The second experiments revealed that the source of the numerical size effect is the frequency of the numbers as proposed by the DSS model. The distance effect and the size effect dissociated. The findings can be extended beyond the new symbols to other notations such as the Indo-Arabic numbers as the DSS offers a more parsimonious explanation regarding the processing of symbolic numbers than the ANS.

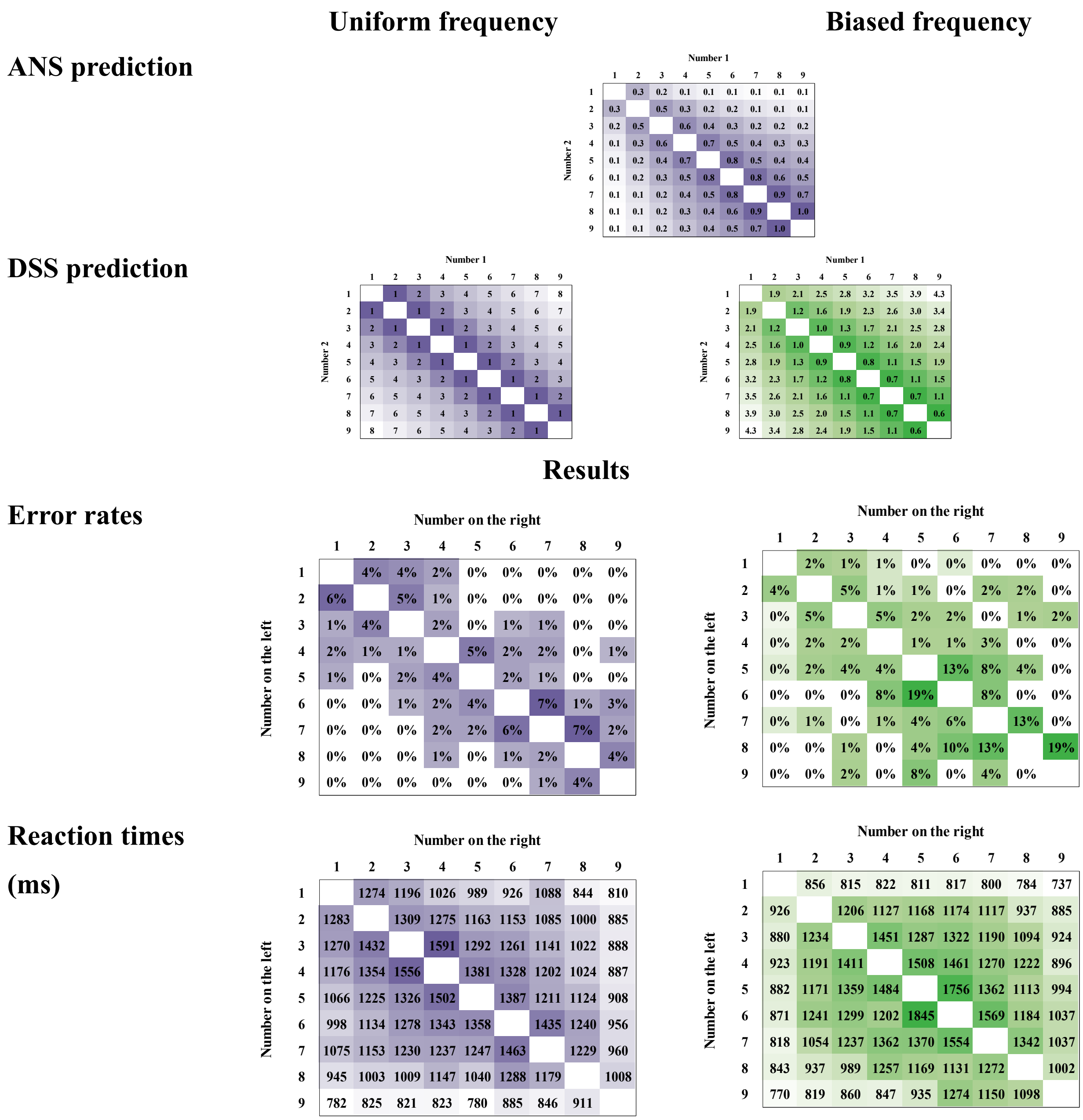

Figure 3. Prediction of the two models for the symbol frequency manipulation in Experiment 2 and the respective results for error rates and reaction times in the full stimulus space. Columns indicate the number shown on the right side of the screen, rows indicate the number presented on the left side, and cells show performance. Darker shade indicates worse performance.

Thesis Study 2

Symbolic Numerical Distance Effect Does Not Reflect the Difference between Numbers

The focus of this study was the source of the distance effect and its aim was to investigate whether in a new artificial number notation the distance effect stems from the values of the numbers or from their associations with the “small-large” properties. In the ANS the effect is explained by the extent of the overlap of the numbers’ representations, while in the DSS there are two possibilities: 1) numbers closer in value form stronger connections, and the activation of one spreads to the other, or 2) the numbers form different associations with the „small-large” properties. In Indo-Arabic numbers the values and the associations correlate highly because of everyday experience. In a new notation the values and the associations can be manipulated independently: New symbols are taught for only some of the numbers, creating a gap in the set (e.g., 3 and 7 become neighbors). If the new symbols are compared by value, then performance around the gap should remain unchanged (e.g., performance for the 3-7 pair is the same as for distance 4). If associations are formed, then the gap should close (e.g., performance for the 3-7 pair is the same as for distance 1). This contrast is possible only if the distance effect is notation dependent, otherwise, the new symbols will take on the properties of the Indo-Arabic symbols.

The data of 19 participants were analyzed (16 females, *M*=22.2 years, *SD*=4.6). The participants were taught new symbols only for the numbers 1, 2, 3, 7, 8, and 9. Then they compared all of possible pairs in the comparison task. The same study was replicated with 32 participants (3 males, *M*=21.0 years).

Average error rates, reaction times, and drift rates were calculated for each participant in the full stimulus space (Figure 4). The value-based and association-based models were specified and then fitted to the group-average data as well as to the individual data. In the case of reaction times and drift rates the association-based model explained the experimental data better than the value-based model, and the difference was shown to be statistically significant for the goodness of fit (R^2) on the individual data. In the replication study the results for reaction time and drift rate were identical to those of the original study, but the difference was not statistically significant. When fitted to error rates, the value-based model proved to be significantly better than the association-based model. A mini meta-analysis that included the results from the original study, the replication study, and the experiment from Thesis Study 3 showed an unequivocal advantage for the association-based model for both reaction times and drift rates, whereas the results for the error rates were ambiguous. Because reaction time data is usually considered to be more reliable and sensitive than error rates, and because drift rate is a more sensitive measure of the difficulty of the task (Wagenmakers et al., 2007), reaction times and drift rates can be considered to reflect a reliable effect in symbolic number comparison task.

To sum up, in an artificial number notation with omitted numbers, the distance effect more strongly reflected the associations between the numbers and the "small-large" properties. The distance effect and the size effect dissociated: The size effect was absent which can be attributed to the uniform frequency of presentation of the numbers, replicating the results from Thesis Study 1. The new symbols did not take on the properties of Indo-Arabic numbers which confirms that the distance effect is notation dependent. The result is in line with the alternative association-based DSS explanation, in which distance effect is directed by the associations between the number nodes and the "small-large" nodes.

Value-based prediction

Number 1 (rows) × Number 2 (columns)

	1	2	3	7	8	9
1		1	2	6	7	8
2	1		1	5	6	7
3	2	1		4	5	6
7	6	5	4		1	2
8	7	6	5	1		1
9	8	7	6	2	1	

Association-based prediction

Number 1 (rows) × Number 2 (columns)

	1	2	3	4	5	6
1		1	2	3	4	5
2	1		1	2	3	4
3	2	1		1	2	3
4	3	2	1		1	2
5	4	3	2	1		1
6	5	4	3	2	1	

Results

Error rates

Number on the left (rows) × Number on the right (columns)

	1	2	3	7	8	9
1		4.2%	2.5%	0.0%	0.4%	0.7%
2	3.2%		3.2%	1.4%	0.0%	1.1%
3	0.7%	4.6%		3.5%	1.4%	1.8%
7	0.0%	0.7%	3.2%		5.3%	0.7%
8	0.7%	0.0%	0.7%	4.6%		2.8%
9	0.4%	1.1%	1.1%	0.4%	2.1%	

Reaction times (ms)

Number on the left (rows) × Number on the right (columns)

	1	2	3	7	8	9
1		1416	1319	1216	1142	86
2	1445		1566	1350	1252	89
3	1387	1698		1532	1227	94
7	1239	1403	1692		1503	95
8	1081	1158	1303	1484		99
9	885	901	889	959	963	

Drift rates

Number on the left (rows) × Number on the right (columns)

	1	2	3	7	8	9
1		0.141	0.156	0.168	0.171	0.20
2	0.144		0.147	0.155	0.171	0.18
3	0.160	0.135		0.134	0.151	0.18
7	0.169	0.155	0.127		0.143	0.18
8	0.189	0.173	0.165	0.140		0.18
9	0.200	0.188	0.197	0.187	0.188	

Figure 4. Prediction of the value-based and the association-based account for the distance effect and the respective results for error rates, reaction times, and drift rates from the original study in the full stimulus space. Columns indicate the number shown on the right side of the screen, rows indicate the number presented on the left side, and cells show performance. Darker shade indicates worse performance.

Thesis Study 3

The Indo-Arabic distance effect originates in the response statistics of the task

The aim of this study was to examine whether the distance and size effects are modified in an Indo-Arabic number comparison task, when the statistics of the actual session deviates from the everyday statistics. In Thesis Study 2 we supposed that the associations formed between the Indo-Arabic digits through everyday use are stable, but this supposition has not been tested before. Furthermore, the same thing could be said about the size effect: It is observed when Indo-Arabic digits are compared while being presented with uniform frequency, but it has not been tested whether it decreases or disappears in a lengthier session. Here, we investigate 1) whether the distance effect is modified when the frequency of the associations between the “small-large” properties and the numbers is modified, and 2) whether the size effect gradually decreases throughout the session when the digits are shown with uniform frequency.

The data of twenty participants were analyzed (16 females, *M*=20.15 years, *SD*=2.28) This experiment contained only the comparison task, in which participants compared all possible pairs of the Indo-Arabic numbers 1, 2, 3, 7, 8, and 9. The task had three blocks with 10 trials for each pair in a block for an overall of 30 trials per pair which was a larger number of trials than our previous studies.

Average error rates, mean reaction times, and drift rates were calculated for each participant and for the full stimulus space (Figure 5). The value-based and the association-based models were specified with a distance effect and with a size effect regressor, and were then fitted to the group-average data and to the individual data. The two models were compared for the overall data. In all cases the association-based model explained the experimental data better than the value-based model, and the difference was shown to be statistically significant (R^2 for the individual participants). As we were interested in whether the change in the associations happened gradually, the goodness of fit was computed for each of the three blocks separately. The results showed that the superiority of the association-based model was present for all the blocks in a similar way. Last, since the numbers were presented with uniform frequency within the session, the slope of the size effect was tested for a deviation from 0, and it was revealed to be present for error rates, reaction times, and drift rates throughout the session with no change with the progression of the task.

To sum up, the data confirmed that the association-based model explains performance in the number comparison task better than the value-based model, and this advantage is observable from the very beginning of the experimental session. The distance effect is shown to be very flexible even in the overpracticed Indo-Arabic number notation. The similar findings with new artificial symbolic notation were replicated, thus confirming that new symbols and Indo-Arabic notation are processed in a similar way. The size effect, on the other hand, remains relatively stable in the number comparison task. The distance and size effects changed independently of each other, suggesting independent sources. The results are in line with the findings of Thesis Studies 1 and 2 and with the DSS account.

Value-based prediction

Number 1 \ Number 2	1	2	3	7	8	9
1		0.30	-0.29	-0.99	-1.05	-1.08
2	0.30		0.50	-0.71	-0.79	-0.85
3	-0.29	0.50		-0.39	-0.51	-0.59
7	-0.99	-0.71	-0.39		1.50	0.91
8	-1.05	-0.79	-0.51	1.50		1.70
9	-1.08	-0.85	-0.59	0.91	1.70	

Association-based prediction

Number 1 \ Number 2	1	2	3	7	8	9
1		0.30	-0.29	-0.30	-0.49	-0.61
2	0.30		0.50	0.21	-0.10	-0.29
3	-0.29	0.50		1.00	0.41	0.10
7	-0.30	0.21	1.00		1.50	0.91
8	-0.49	-0.10	0.41	1.50		1.70
9	-0.61	-0.29	0.10	0.91	1.70	

Results

Error rates

Number on the left \ Number on the right	1	2	3	7	8	9
1		0.7%	0.8%	0.2%	0.3%	0.0%
2	0.8%		3.2%	3.0%	0.5%	0.5%
3	0.3%	2.2%		4.8%	1.2%	0.5%
7	0.3%	1.5%	5.5%		4.2%	3.2%
8	0.3%	0.5%	0.5%	6.8%		5.3%
9	0.3%	0.8%	0.7%	2.8%	6.5%	

Reaction times (ms)

Number on the left \ Number on the right	1	2	3	7	8	9
1		630	617	549	547	537
2	625		680	620	573	571
3	605	657		651	606	588
7	565	620	659		658	621
8	548	578	600	697		674
9	545	575	590	635	683	

Drift rates

Number on the left \ Number on the right	1	2	3	7	8	9
1		0.292	0.294	0.352	0.351	0.367
2	0.287		0.279	0.291	0.352	0.344
3	0.291	0.292		0.264	0.313	0.321
7	0.342	0.288	0.255		0.269	0.279
8	0.344	0.320	0.315	0.227		0.251
9	0.353	0.331	0.316	0.281	0.251	

Figure 5. Prediction of the value-based and the association-based account for the distance effect and the overall respective results for error rates, reaction times, and drift rates in the full stimulus space. Columns indicate the number shown on the right side of the screen, rows indicate the number presented on the left side, and cells show performance. Darker shade indicates worse performance.

Thesis Study 4

Two components of the Indo-Arabic numerical size effect

The aim of this study was to examine whether the size effect in symbolic numbers can be manipulated by changing the frequency with which the participants experience the numbers. Just as the distance effect could be modified within the session of a task, it is possible that by changing the frequency of presenting numbers to the participants the size effect will disappear or even become reversed. In Thesis Study 1 it was already demonstrated that for artificial numbers the size effect does not appear if the frequency of the numbers is uniform during the task, but it appears for biased frequency. In Indo-Arabic numbers this biased frequency is experienced in everyday life (Dehaene & Mehler, 1992), however, its flexibility has not been tested yet. In Thesis Study 4 we designed an experiment with three conditions – three types of frequency: everyday (number 1 was most frequent, number 9 was least frequent), reversed everyday (vice versa), and uniform (each number was shown an equal number of times). If the size effect is a flexible in numbers, then it should remain present in the everyday frequency condition, become absent in the uniform condition, and get reversed in the reversed everyday frequency condition. Similar to Thesis Study 3 there was a larger number of trials divided into three blocks to test for a possible gradual change in the size effect.

The data of the 46 participants (35 females, *M*=21.02 years, *SD*=2.37) were analyzed. There were 13 participants (8 females, *M*=20.31 years, *SD*=1.14) in the everyday frequency condition, 11 (8 females, *M*=21.64 years, *SD*=2.57) in the uniform frequency condition, and 22 (19 females, *M*=21.14 years, *SD*=2.68) in the reversed everyday frequency condition. A replication study was also conducted with 29 participants (18 females, *M*=21.28 years, *SD*=1.98), of which 9 participants (3 females, *M*=22.2 years, *SD*=1.69) were in the everyday frequency group, 10 participants (6 females, *M*=21.6 years, *SD*=1.63) in the uniform frequency group, and 10 participants (9 females, *M*=20.1 years, *SD*=1.97) in the reversed everyday frequency group.

Only the comparison task was used in this study. The participants compared all possible pairs for the Indo-Arabic numbers from 1 to 9 in three blocks. For the everyday frequency and the reversed everyday frequency condition the frequency of presentation of each number was calculated as $number^{-1}$ (Dehaene & Mehler, 1992).

Average error rates, mean reaction times, and drift rates were calculated for each participant for the full stimulus space (Figure 6). The slope of the size effect was tested against 0, and we also compared the three conditions. The slope was largest in the everyday frequency condition, decreased for the uniform frequency condition, and was smallest in the reversed everyday frequency condition. The decrease was already observable at the beginning of the session, and no change was visible between blocks. The size effect did not disappear or become reversed in either the uniform frequency or the reversed everyday frequency condition, i.e., its slope deviated significantly from 0 in all conditions and for the whole duration of the experiment. Similar results were obtained in the replication study.

The results demonstrate that the size effect remains stable in a session of the comparison task. Nevertheless, it was modified by the experimental manipulation. One plausible explanation is that there are two components. One of the components is flexible and can change within the session. This component is consistent with the DSS explanation. The second component is stable, and can be accounted for by both ANS and DSS. As earlier evidence suggests that DSS is the mechanism behind symbolic numerical cognition, it is more likely that this stable component is the frequency of the numbers as experienced in everyday life.

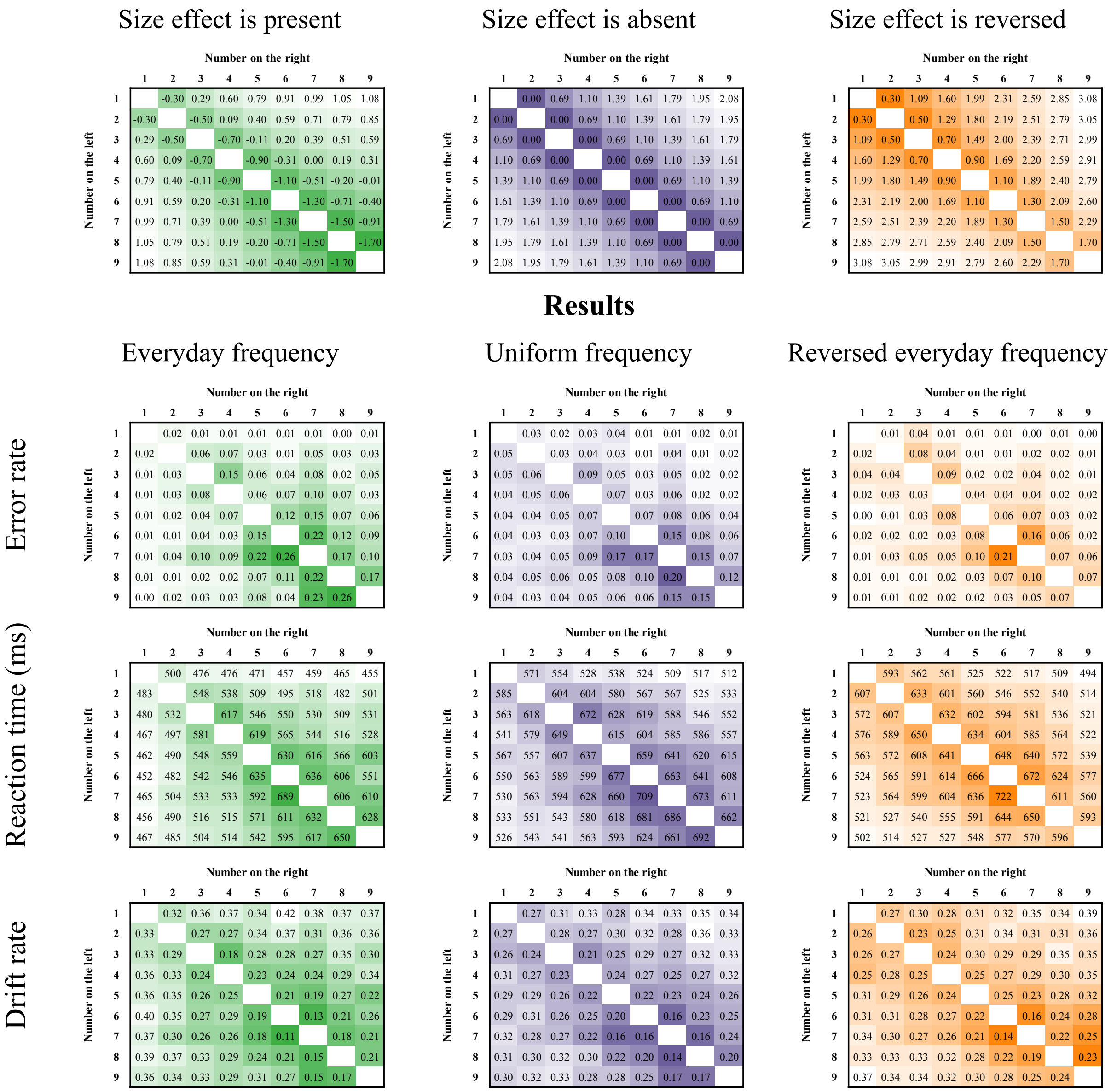

Figure 6. *Illustration of the full stimulus space when the size effect is present, absent, or reversed and the respective overall results for error rates, reaction times, and drift rates for the everyday frequency, uniform frequency, and reversed everyday frequency conditions. Columns indicate the number shown on the right side of the screen, rows indicate the number presented on the left side, and cells show performance. Darker shade indicates worse performance.*

Discussion

The studies systematically investigated the possible sources of the numerical distance effect and the numerical size effect in symbolic numerical cognition as well as which model the results were consistent with: the ANS or the DSS. Following the aims of the thesis:

1. the results were inconclusive about which model is a better description, but provided evidence that the DSS is a plausible alternative model (Thesis Study 1, Experiment 1);
2. manipulating the frequency of the numbers was sufficient to induce a size effect that appeared only for the biased frequency. Thus, frequency is the source of the size effect for new, artificial numbers (Thesis Study 1, Experiment 2 and 3);
3. the association-based model was a better fit for the data, thus the associations between the numbers and the "small-large" properties are the source of the distance effect in the comparison task for new, artificial numbers (Thesis Study 2);
4. the association-based model was a better fit for the data, thus the associations between the numbers and the "small-large" properties are the source of the distance effect in the comparison task for Indo-Arabic numbers (Thesis Study 3);
5. the size effect was modified but not entirely removed by manipulating the frequency of the numbers in the comparison task for Indo-Arabic numbers. Frequency nevertheless contributes to the size effect as a flexible component, and it is possible that everyday frequency could explain the stable component (Thesis Study 3 and Thesis Study 4);
6. the distance and size effects changed independently as a result of the manipulation in the experiments, thus they dissociate (all Thesis Studies);
7. more generally, as the distance and size effects changed as a result of the experimental manipulation in a way predicted by the DSS and not predicted by the ANS. In this light, the DSS is the better account for symbolic numerical processing.

Overall, the distance effect was revealed to be stem from the associations of the numbers with the "small-large" properties whereas the size effect is rooted in the frequency of the numbers. Moreover, the two effects changed independently of each other, i.e., they dissociated. These results are consistent with the DSS account but not with the ANS account for symbolic numerical cognition. Further evidence supporting a separate system for symbolic numerical cognition has emerged in recent years, e.g., the source of the SNARC effect is more likely due to the interference of discrete properties (Krajcsi, Lengyel, & Laczkó, 2018; Landy, Jones, & Hummel, 2008), performance on the symbolic comparison is a much better predictor of math achievement than non-symbolic comparison (Holloway & Ansari, 2009; Sasanguie, Defever, Maertens, & Reynvoet, 2014; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013), the neurological underpinnings of symbolic and non-symbolic representation seem to be separate (Bulthé, De Smedt, & Op de Beeck, 2014; Bulthé, De Smedt, & Op de Beeck, 2015). The results of these studies are consistent with and can be explained by the DSS account.

From methodological point of view, the use of the full stimulus space, a data-driven approach, proved to be more appropriate method for the data analysis compared to the traditional approach, e.g., being able to use several regressors at the same time and showing that our results are observed systematic patterns in the data and not simply artifacts. The drift rate repeatedly proved to be the most sensitive index of performance in the comparison task, thus increasing support for the use of the diffusion model analysis. Several other methodological points are also discussed, such as the regressor of the distance effect and the handling of the end effect.

To summarize, the studies support the DSS account for symbolic numerical cognition. The DSS is an underspecified model as it relies on models describing higher-level cognitive (possibly linguistic) functions, so a quantitative description is not as readily available. However, it is a comprehensible, cohesive account with precedents in the literature, it can explain all relevant symbolic numerical effects and phenomena at least as well as the ANS, usually providing a better explanation as seen, for example, in the present studies, and it can supply testable hypotheses not only against the ANS model, but also to contrast its own putative properties with each another, which in turn will result in a more precise description. The results about the distance and size effects in the Thesis Studies are already a step forward to a more precise specification of the DSS.

Among the implications of the present results is the re-evaluation and possibly re-interpretation of studies in the field of symbolic numerical cognition. Furthermore, the accumulated knowledge of symbolic numerical processing could be applied to language (e.g., learning the statistics of the environment). Application in practice could target mathematical education and intervention for children and adults with mathematical disabilities.

Thesis Studies

Thesis Study 1: Krajcsi, A., Lengyel, G., & Kojouharova, P. (2016). The source of the symbolic numerical distance and size effects. *Frontiers in psychology, 7*, 1795. https://doi.org/10.3389/fpsyg.2016.01795

Thesis Study 2: Krajcsi, A., & Kojouharova, P. (2017). Symbolic numerical distance effect does not reflect the difference between numbers. *Frontiers in psychology, 8*, 2013. https://doi.org/10.3389/fpsyg.2017.02013

Thesis Study 3: Kojouharova, P., & Krajcsi, A. (2018). The Indo-Arabic distance effect originates in the response statistics of the task. *Psychological research*, 1-13. https://doi.org/10.1007/s00426-018-1052-1

Thesis Study 4: Kojouharova, P., & Krajcsi, A. (2019). Two components of the Indo-Arabic numerical size effect. *Acta psychologica, 192*, 163-171. https://doi.org/10.1016/j.actpsy.2018.11.009

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