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Bread baking as a moving boundary problem. Part 2: Model validation and numerical simulation

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Abstract

A simultaneous heat and mass transfer model proposed to describe the bread baking process is validated. The mathematical model is based on a moving boundary problem formulation with equivalent thermophysical properties and includes the moving evaporation front, the evaporation-condensation mechanism and the development of the crust observed during bread baking. The problem is solved over an irregular three-dimensional geometry using the finite element method. Variation in temperature and water content of bread during baking is predicted with high accuracy by the model. Parameter estimation procedure and sensitivity analysis are performed for some thermophysical properties. The proposed formulation and analysis can be applied for other bakery products as well as for similar food engineering applications.

Keywords: Baking; Stefan problem; Mathematical modelling; Thermophysical properties

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Nomenclature

a_w	Water activity
C_p	Specific heat, J kg^{-1} K^{-1}
D	Water (liquid or vapour) diffusion coefficient of product, m^2 s^{-1}
D_{va}	Water vapour diffusion coefficient in air, m^2 s^{-1}
e_{abs}	Mean absolute relative error, %
f_{crust}	Crust formation factor
Gr	Grashof number
h	Heat transfer coefficient, Wm^{-2} K^{-1}
k	Thermal conductivity, W m^{-1} K^{-1}
k_g	Corrected mass transfer coefficient, kg Pa^{-1} m^{-2} s^{-1}
k_g^*	Mass transfer coefficient from Eq. (15), kg Pa^{-1} m^{-2} s^{-1}
M	Molecular mass, g
Nu	Nusselt number
P	Pressure, Pa
Pr	Prandlt number
Re	Reynolds number
RH	Relative humidity, %
Sc	Schmidt number
T	Temperature, K
t	Time, s
W	Water (liquid or vapour) content, kg kg^{-1}
WL	Weight loss, %

Greek symbols

ΔT	Temperature range of phase change, K
ε	Emissivity
η	Delta-type function
λ	Heat of phase change, J m^{-3}
ρ	Density, kg m^{-3}
σ	Stefan-Boltzmann constant, 5.67 10^{-8} W m^{-2} K^{-4}

Subscripts

∞	Ambient
air	Air
atm	Atmospheric
f	Phase change
s	Solid or surface
sat	Saturated
w	Water

1. Introduction

Simulation can be defined as the process of developing a model of a real system and carrying out experiments through the model, with the aim of studying, analyzing, designing or re-designing, controlling and predicting a certain real process. Besides the validation of the model, numerical simulation does not imply field experiments; it only includes the development of a mathematical model and computational effort, mostly more inexpensive than *real* tests. In addition, simulation gives the possibility of working under standardized operative conditions, minimizing the uncertainties of complex processes, especially those which are traditional non automated processes. In this way, it will be very useful to have an accurate mathematical model for bread baking simulation.

Thus far, the baking process has been mostly modelled as a simultaneous heat and mass transfer (SHMT) problem, though different hypotheses were suggested for describing the internal mechanisms of transport. For instance, the evaporation-condensation theory proposed by Sluimer and Krist-Spit (1987) to explain the rapid heating of porous dough during baking was incorporated in a model of SHMT in dough and crumb, not involving the crust zone (de Vries et al., 1989). Good results were obtained when comparing experimental and simulated core temperature values, suggesting that the evaporation-condensation mechanism should be included in a model for bread baking. In this way, Thorvaldsson and Janestad (1999) took into account evaporation and condensation of water for modelling the drying of bread crumb and developed a model for heat, liquid water and water vapour transfer including an empirical parameter named as evaporation rate (i.e. times per second the vapour content is set at the saturated partial water vapour pressure). Another approach was taken by Lostie et al. (2002) to incorporate evaporation-condensation in a model for the "heating up" period of cake baking; this mechanism was included in the heat balance through an effective thermal conductivity.

On the other hand, Zanoni et al. (1994) proposed a mathematical model for baking of mould bread assuming that the variation of moisture content and temperature of the product is determined by the formation of an evaporation front at 100 ºC. To solve the model, it was established that when the bread reaches 100 ºC, evaporation at a constant temperature takes place until all the unbound water is evaporated, and the heat of evaporation is obtained from the difference between the supplied heat and the heat transferred towards the core by conduction (i.e. $dT/dt = 0$). Respect to biscuit baking, Özilgen and Heil (1994) included the loss of latent heat via loss of water in the energy balance equation to take account of evaporation in the product. Though good correlation was observed between experimental and simulated values of temperature and water content, the authors reported an extensive use of empirical parameters, besides this type of formulation is not recommended for being in conflict with fundamental mass conservation laws (Zhang and Datta, 2004). For the "crust and crumb" period of cake baking, Lostie et al. (2004) introduced in a lumped model an empirical parameter to control the front evaporation velocity.

Finally, two opposite viewpoints were applied to model the bread baking process. Zheleva and Kambourova (2005) used the phenomenological theory of Luikov (1975) to develop a SHMT model, but the experimental heating curves were not well reproduced in their work. In contrast, Zhang and Datta (2006) presented a coupled model for multiphase heat and mass transfer using a mechanistic approach; acceptable results were obtained when comparing experimental and simulated data. Further information about the state of the art in mathematical modelling of the baking process can be found elsewhere (Mondal and Datta, 2008; Sablani et al., 1998).

From the literature review, it can be concluded that mathematical modelling of bread baking is still a major challenge in food engineering. So far, the experimental heating and drying curves observed during the process could not be well reproduced by numerical simulation; particularly, the characteristic sigmoid trend of the temperature variation at the bread core. In addition, accurate expressions for thermophysical properties of bread are still unavailable or available only for a narrow range of operative conditions, i.e. temperature below 100 ºC.

As a part of a comprehensive study of the bread baking process (Purlis, 2007), the aim of this article was to validate the mathematical model previously developed using a moving boundary approach, which includes the two main features of bread baking: the evaporation-condensation phenomena and the moving evaporation front (Purlis and Salvadori, 2008). Furthermore, a second objective was to propose new expressions for some thermophysical properties of bread by considering the moving boundary formulation and carrying out parameter estimation and sensitivity analysis.

2. Mathematical model: Thermophysical properties

In the first part of this study we developed a mathematical model considering bread baking as a moving boundary problem (MBP), beyond other assumptions (Purlis and Salvadori, 2008). The governing equations and the corresponding boundary conditions are summarized below.

Heat balance equation:

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) \tag{1}$$

Mass balance equation:

$$\frac{\partial W}{\partial t} = \nabla \cdot (D \nabla W) \tag{2}$$

Boundary conditions:

$$-k \nabla T = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_\infty^4) \tag{3}$$

$$-D\rho_s \nabla W = k_g \left(P_s(T_s) - P_\infty(T_\infty)\right) \tag{4}$$

where $P_s = a_w P_{sat}(T_s)$ and $P_\infty = RH\, P_{sat}(T_\infty)$.

In the MBP formulation *equivalent* thermophysical properties are defined including the phase change (i.e. water evaporation) occurring during the process (Bonacina et al., 1973). In this way, available values or expressions in literature for bread properties were not always adequate when the mathematical model was validated against experimental data (results not shown). Thus, we proposed new expressions for some thermophysical properties and performed a parameter estimation procedure by comparing simulated with experimental data. Following, detailed discussion about bread properties and transfer coefficients is presented.

2.1. Specific heat

An equivalent specific heat is defined as a function of temperature and water (liquid or vapour) content, including the *enthalpy jump* at the temperature of phase change (Bonacina et al., 1973):

$$C_p(T,W) = C_p^{'}(T,W) + \lambda W \eta \quad (5)$$

where (Zanoni et al., 1994):

$$C_p^{'}(T,W) = C_{p,s}(T) + W C_{p,w}(T) \quad (6)$$

$$C_{p,s} = 5T + 25 \quad (7)$$

$$C_{p,w} = \left(5.207 - 73.17\,10^{-4} T + 1.35\,10^{-5} T^2\right)1000 \quad (8)$$

In Eq. (5), η is a Delta-type function, i.e. the sum of two smoothed Heaviside functions, centred in T_f with range ΔT. This smoothed enthalpy peak replaces the Delta function in order to achieve large but finite values in the phase transition temperature range as was suggested by Bonacina et al. (1973).

2.2. Thermal conductivity

We propose an effective thermal conductivity that includes the evaporation-condensation mechanism in the model. This expression for thermal conductivity, as well as for the other bread properties, is valid for dough, crumb and crust, since a smoothed Heaviside function with parameters T_f and ΔT is used to obtain a continuous function for the entire range of operative conditions of baking (Purlis, 2007):

$$k(T) = \begin{cases} \dfrac{0.9}{1+\exp(-0.1\,(T-353.16))}+0.2 & if \quad T \le T_f - \Delta T \\ 0.2 & if \quad T > T_f + \Delta T \end{cases} \tag{9}$$

Thermal conductivity increases with bread temperature following a sigmoid trend until the phase change occurs (Figure 1); it rapidly increases above 60 ºC since evaporation-condensation dominates at high temperatures while conduction at low temperatures (de Vries et al., 1989), though both mechanisms are present during all the baking process.

The behaviour of the proposed thermal conductivity below 100 ºC is similar to the experimental one obtained by Jury et al. (2007) with a line source probe in pseudo non-isothermal conditions, during the thawing-baking phase of part baked bread production. Though a parallel model for thermal conductivity was adjusted by Jury et al. (2007), it is not adequate for temperature values greater than 80-85 ºC since high thermal conductivity is predicted (results not shown). Respect to the crust, a value from literature is used (Rask, 1989); the low thermal conductivity of this zone is due to the low water content.

2.3. Density

Assuming the dough porosity equal to 75%, an apparent density is defined as follows (Hamdami et al., 2004):

$$\rho(T) = \begin{cases} 180.61 & if \quad T \le T_f - \Delta T \\ 321.31 & if \quad T > T_f + \Delta T \end{cases} \tag{10}$$

Density for solid that appears in the mass transport boundary condition (Eq. (4)) is equal to 241.76 kg m^{-3} (Hamdami et al., 2004).

2.4. Mass diffusivity

An effective diffusion coefficient of liquid water or water vapour is defined as a function of bread temperature:

$$D(T) = \begin{cases} 1\,10^{-10} & if \quad T \le T_f - \Delta T \\ f_{crust}\, D_{va}(T) & if \quad T > T_f + \Delta T \end{cases} \tag{11}$$

Moisture diffusivity in dough and crumb (i.e. temperature below 100 ºC) is obtained from Thorvaldsson and Janestad (1999).

With regard to crust, an expression depending on diffusivity of water vapour in air is proposed by defining a crust formation factor (f_{crust}). This factor is related to the tortuosity of the crust structure and the experimental observation that the generation of a crust restricts the diffusion of internal water vapour to the oven ambient (Hasatani et al., 1991; Wählby and Skjöldebrand, 2002). Further, Zhang and Datta (2006) showed by scanning electron microscopy (SEM) the microstructure of crust: pores tend to be smaller than in the crumb region, resulting in a dense porous matrix and thus increasing the resistance to mass transport. By simulating the model in order to agree numerical results with experimental data, the crust formation factor was found to be equal to

0.0013. So, effective moisture diffusivity in bread varies between 1 10^{-10} and 6.4 10^{-8} $m^2 s^{-1}$ in the temperature range 25-120 ºC, which is in agreement with literature data.

2.5. Water activity

Water activity is defined as the ratio of water vapour pressure in the product at a certain temperature and water content versus the vapour pressure of pure water at the same temperature. Knowledge about a_w variation is important since it establishes the driving force for mass transfer between the food surface and the surrounding ambient (Eq. (4)). In this work, we use the Oswin model (Lind and Rask, 1991) with the parameters estimated by Zhang and Datta (2006):

$$a_w(T,W) = \left[\left(\frac{100\,W}{\exp(-0.0056T + 5.5)}\right)^{-1/0.38} + 1\right]^{-1} \quad (12)$$

2.6. Heat and mass transfer coefficients

The convective heat transfer coefficient is obtained from Nusselt number correlations, according to the experimental baking conditions. For this aim, bread is assumed to be a cylinder with an equivalent diameter as the characteristic length, calculated by averaging height and width data of the cross-section of samples. The following expressions are used for the natural and forced convection modes of the oven used in baking tests, respectively (Perry and Green, 1997):

$$Nu = 0.53\,(Gr\;Pr)^{1/4} \quad (13)$$

$$Nu = 0.683\,Re^{0.466}\;Pr^{1/3} \quad (14)$$

Respect to heat transfer by radiation, the emissivity of bread surface is considered equal to 0.9 (Hamdami et al., 2004).

The mass transfer coefficient was firstly determined using the Chilton-Colburn analogy (Perry and Green, 1997) from the heat transfer coefficient and oven air properties data:

$$\frac{h}{k_g^*} = \frac{M_{air}}{M_w} P_{atm} C_{p,air} \left(\frac{Sc}{Pr} \right)^{2/3} \quad (15)$$

However, the values obtained from Eq. (15) produced much more water loss in the numerical simulations than that registered in baking tests, suggesting that the actual mass transfer coefficients may be lower. Besides, the heat-mass transfer correlation does not take into account the crust development, which diminishes the mass transport from the bread surface to the oven ambient. Therefore, we propose a corrected mass transfer coefficient by incorporating a correction factor, which can be estimated from experimental data of weight loss (see Table 1).

3. Geometry modelling

In general, transport phenomena are often simulated over simple regular geometries, e.g. an infinite slab for unidirectional transfer, due to complexity for representing the real geometry of the system (usually irregular) and then generating a mesh or grid over such domain. However, it would be desirable to solve a model using the real geometry of food; in mathematical modelling of bread baking only Zhang and Datta (2006) considered a 2D irregular domain for simulation.

In this work, French bread (baguette) is considered as a three-dimensional object having a constant irregular cross-section. To construct the geometry of bread, the irregular boundary of the cross-section is obtained from a digital image of dough sample by the following image processing procedure (Figure 2):

1. Conversion of original RGB image to grey-scale format.
2. Adjustment of image intensity values to increase the contrast.
3. Noise reduction by filtering to enhance image quality.
4. Segmentation through a global threshold value: a binary image is obtained where black colour (pixel value equal to 0) represents the background and white colour the sample (pixel value equal to 1).
5. Boundary detection and interpolation of a subset of boundary pixels by a closed B-Spline curve (a continuous approximation to the discrete boundary of binary images).

The B-Spline curve representing the real boundary of bread cross-section is converted into a 2D solid region, which is then extruded in the axial direction to construct the final 3D geometry of bread. For further details about geometry modelling the reader should be referred to Goñi et al. (2007). Geometry modelling was performed in MATLAB® and COMSOL Multiphysics™ (version 3.2). Different geometric models were constructed since homogeneity respect to shape and dimensions of samples is difficult to achieve when dealing with bread dough.

4. Baking tests

Experimental data of the bread baking process was necessary to validate the mathematical model, though in this work it was also used to estimate some parameters of the model (see Section 2). It should be noted that independent experimental information was used to perform the model validation and parameter estimation, respectively. Two baking conditions, i.e. natural convection (v = 0 m/s) and forced convection (v = 0.9 m/s), and three oven temperatures were used: 180, 200 and 220 ºC. Temperature and water content profiles, crust kinetics and weight loss variation were determined experimentally. Further details about baking experiments and variables measurement can be found in the first part of this article (Purlis and Salvadori, 2008).

5. Numerical solution

The finite element method was used to solve the SHMT problem; the numerical procedure was implemented in COMSOL Multiphysics and MATLAB software. A mesh consisting of ca. 3600 deformed tetrahedrons (for well approximating the irregular shape of bread geometry) was used in all simulations. The solver used is an implicit time-stepping scheme, which implies that it must solve a possibly nonlinear system of equations at each time step. It solves the nonlinear system using a Newton iteration, and it then solves the resulting systems with an arbitrary COMSOL Multiphysics linear system solver. The time step taken by the algorithm is variable, but it was ensured to be small enough in order to do not miss the latent heat peak corresponding to water evaporation.

For describing the equivalent thermophysical properties according to the moving boundary formulation, a smoothed Heaviside function was used with parameters: T_f = 100 ºC and ΔT = 0.5 ºC. Heat and mass transfer coefficients (Table 1) were calculated using an equivalent diameter equal to 0.0587 m for bread samples (average value of all baking tests). Relative humidity (or vapour pressure) in oven ambient was assumed to be negligible. Baking simulation time was fixed to 30 min; solution time was about 25 min using a PC with AMD Sempron™ Processor 2800+ 1.60 GHz and 512 MB RAM.

6. Results and discussion

6.1. Model validation

Typical three-dimensional distribution of temperature and water content in bread at the end of baking (30 min) is shown in Figure 3; though only one baking condition is shown, all simulations produced similar results in qualitative terms. The proposed mathematical model allows identifying clearly the two characteristic zones of bread, i.e. the crumb (inner region) and the crust (outer region). The crumb does not exceed 100 ºC and its moisture content remains constant, while at the crust, the temperature increases tending to oven temperature and dehydration occurs. The same picture was observed during the experimental tests of bread baking.

Figure 4 presents the predicted temperature distribution and heat flow variation in the middle cross-section of bread (considering the axial axis) during baking. These numerical results well reproduce the simultaneous heat and mass transfer occurring in bread during baking. Until 15-20 min, temperature increases in the entire domain due to the inward heat flux from oven ambient; however, dehydration takes place only at the surface i.e. the outer zone where temperature is above 100 ºC. Then, formation and progressive thickening of the crust happen, which avoid mass transfer from bread core to oven air. Besides, the crumb reaches almost 100 ºC at this stage, so the movement of the evaporation front becomes the determining step of the process. In other words, the energy arriving to the bread surface is transferred towards the evaporation front to produce the outward water vapour flux. Respect to the evaporation front, it can be defined as the zone where the moisture gradient is maximum (Zhang and Datta, 2004): the proposed mathematical model well agrees with such definition (Figure 4d).

Representative temperature profiles obtained from the model along the vertical axis of the middle cross-section of bread are shown in Figures 5 and 6. Temperature variation presents the same behaviour as in experimental baking of bakery products, including bread. It is important to note that temperature at the crumb rises until reaches 100 ºC asymptotically, not exceeding this value during the process (30 min). On the other hand, crust temperature increases continuously up to 100 ºC, when water evaporation occurs, and then rises again following the oven temperature trend. It can also be seen the advance of the evaporation front inside bread, separating the crumb and the crust (Figure 6).

Beyond the good qualitative representation of the baking process given by the proposed model, a comparison between the experimental and predicted temperature values were done involving both the crumb and the crust zones. The goodness of the model prediction was assessed by the mean absolute relative error defined as:

$$e_{abs}(\%) = \frac{100}{n}\sum_{i=1}^{n}\left(\frac{\left|T_{experimental} - T_{predicted}\right|}{T_{experimental}}\right)_i \qquad (16)$$

where n is the number of temperature values taken into account. In our baking tests, the sampling time was equal to 5 sec, so n = 360 for 30 min baking. The calculated prediction errors are summarized in Table 2, while Figures 7 and 8 show the comparison between experimental and simulated temperature profiles in crumb and crust zones, respectively.

As can be seen, the mathematical model predicts very well the variation of crumb temperature for all baking conditions. Respect to the crust zone, the model reproduces the experimental trend in an acceptable way, but prediction errors were higher than in the crumb case. This is probably related to experimental measurement aspects: placing a thermocouple at dough surface and acquiring temperature values at a depth less than 5 mm (crust mean thickness at the end of baking) is not an easy task. In addition, dough suffers expansion during baking, which can produce a modification in the original thermocouple location. In many cases, the thermocouple end is enclosed by the dough matrix and temperature measurement actually corresponds to the air inside bread. Nevertheless, some successful runs were obtained (Figure 8), which demonstrate the ability of the proposed model to well describe the temperature variation in bread crust as well as in bread crumb.

The mathematical model was also assessed considering experimental data of total water content (Figure 9). The mean absolute relative error defined in Eq. (16) was calculated using $n = 6$ since experimental sampling was performed every 5 min (Table 2). As can be seen, the model predicts very well the decrease in total moisture content, i.e. the weight loss of bread during baking. Respect to the water content distribution inside bread, the model does not include the evaporation-condensation phenomena in the mass balance, so the experimentally observed increase of moisture in bread core can not be predicted. Nevertheless, such increase was between 0.4 and 2.3% in our baking tests (Purlis and Salvadori, 2008), so for engineering purposes the proposed model gives very acceptable results. Finally, the predicted water content of the crust is close to 0.1 (dry basis), which is in agreement with experimental data; the simulated kinetics of crust development presents a similar trend to the experimental one, achieving a dehydrated layer of 0.5-0.6 cm thickness at the end of baking.

6.2. Parameter estimation

As previously mentioned, available values or expressions in literature for some thermophysical properties of bread did not produce good results when the mathematical model was validated against experimental data. Therefore, we performed a parameter estimation procedure; this approach is a common practice in food engineering and it has been also carried out by other authors in the mathematical modelling of the baking process (Lostie et al., 2004; Thorvaldsson and Janestad, 1999; Zhang and Datta, 2006). Note that all baking conditions were used for this aim, but independent experimental information was considered for model validation and parameter estimation steps.

Respect to thermal conductivity, we firstly used a linear expression dependent on bread porosity reported by Zanoni et al. (1995). Considering porosity equal to 75%, thermal conductivity values for crumb and crust are 0.393 and 0.165 W m^{-1} ºC, respectively, which is in agreement with other values found in literature (Rask, 1989; Baik et al., 2001). Figure 10 shows the difference in temperature variation at bread core by using these constant values and the expression proposed in the present paper (Eq. (9); Purlis, 2007). Underestimation of temperature above 60 ºC by using the literature values/expressions is probably due to not considering the evaporation-condensation contribution to heat transfer. This result reveals the importance of including such mechanism in heat transport, also demonstrated by de Vries et al. (1989) and Jury et al. (2007); the presence of evaporation-condensation is responsible for the typical sigmoid heating curve observed during bread baking.

A sensitivity analysis for the effective mass diffusivity defined in Eq. (11) was carried out by simulating the baking process with different values of the crust formation factor. Figure 11a shows that the total water content is highly dependent on this parameter: a low value of f_{crust} represents a high resistance to water vapour transport from the evaporation front to oven ambient, resulting in higher total moisture values than the actual ones (and vice versa). This demonstrates the influence of crust formation and its structural properties on the bread baking process. Finally, Figure 11b presents the sensitivity analysis performed for the mass transfer coefficient, which values obtained from the Chilton-Colburn analogy (Eq. (15), Table 1) were corrected in this work (correction factor < 1). Mass transfer is highly affected by the presence of the crust and thus the model is also sensitive to variations in k_g.

7. Conclusions

The bread baking process is confirmed as a moving boundary problem, where simultaneous heat and mass transfer occurs in a porous medium. Bread baking is very well described by the mathematical model developed in this work, which is based on a moving boundary formulation including the most important features of the process: the moving evaporation front, the evaporation-condensation mechanism and the development of the crust. The major advantage of this approach is that the use of empirical parameters to control the position of the evaporation front is not required.

New expressions for some thermophysical properties of bread are proposed. Baking simulation through the use of the validated mathematical model is very helpful for well-understanding the process and estimating unknown values of some parameters of the model. Sensitivity analysis shows the importance of including the main aspects of the bread baking process in the model formulation. Finally, we expect the above comments should be useful for other food engineering processes with similar characteristics as bread baking.

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Figure captions

Figure 1. Variation of effective thermal conductivity of bread (Eq. (9)) as a function of temperature. Parameters of smoothed Heaviside function: T_f = 100 ºC, ΔT = 0.5 ºC.

Figure 2. Image processing steps for geometry modelling of bread samples. (a) Original RGB image of a dough cross-section. (b) Binary image obtained by segmentation after grey-scale transformation, intensity adjustment and filtering stages. (c) Original image and its approximated boundary using a B-Spline curve (in red). (d) 2D solid region obtained from the approximated irregular boundary. (e) 3D geometry of bread constructed by extruding the 2D solid region in the length direction.

Figure 3. Simulated (a) temperature (ºC) and (b) moisture content (kg water/kg dry matter) distribution for 30 min baking at 200 ºC under forced convection.

Figure 4. (a-c) Distribution of temperature (ºC) in the middle cross-section (in length direction) of bread during baking at 200 ºC under forced convection. Arrows represents heat flow. Curve inside bread (10 and 20 min) indicates the isotherm at 100 ºC (evaporation front). (d) Contour curve at 100 ºC for 20 min baking at 200 ºC under forced convection. Dots indicate the maximum and minimum moisture gradient.

Figure 5. Simulated temperature profiles inside bread (middle cross-section) for baking at 200 ºC under forced convection. Locations (cm) from upper surface: 0 (surface); 0.1; 0.4; 0.6; 0.8; 1.1; 1.6; 2.6 (core).

Figure 6. Temporal evolution (0 to 30 min, every 4.28 min) of temperature along height direction over the middle cross-section, during baking at 200 ºC under forced convection.

Figure 7. Simulated (solid line) and experimental (●) crumb temperature during baking at (a) 220 ºC under natural convection and (b) 200 ºC under forced convection. Only experimental values every 1 min are shown for simplicity.

Figure 8. Simulated (lines) and experimental (symbols) crust temperature during baking at 220 ºC under natural convection (solid;●) and 200 ºC under forced convection (dashed;△). Only experimental values every 1 min are shown for simplicity.

Figure 9. Simulated (lines) and experimental (symbols) total water content of bread during baking under forced convection. 180 ºC (dashdot;□); 200 ºC (solid;●); 220 ºC (dashed;△).

Figure 10. Temperature variation at bread crumb during baking at 200 ºC under forced convection. Dots represent experimental data. Dashed line corresponds to simulation by using Eq. (9). Solid line accounts for simulation with 0.393 and 0.165 W m^{-1} ºC for crumb and crust thermal conductivity, respectively. Only experimental values every 1 min are shown for simplicity.

Figure 11. Experimental (dots) and simulated (solid lines) total water content during baking at 200 ºC under forced convection. (a) Effect of crust formation factor (f_{crust} = 0.0013). (b) Effect of mass transfer coefficient (k_g = 8.46 10^{-9} kg Pa^{-1} m^{-2} s^{-1}).

Figure 1

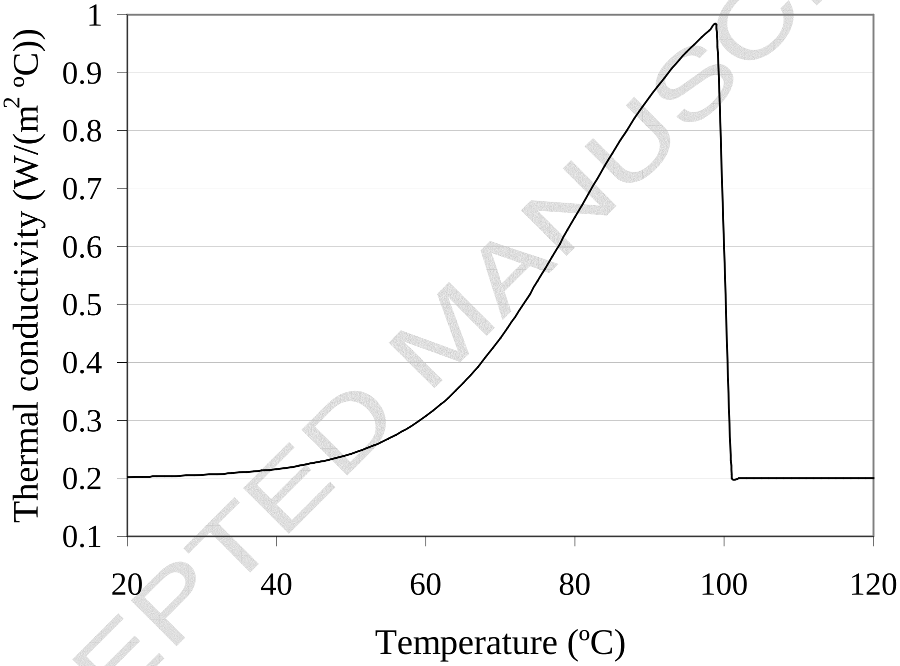

1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
Thermal conductivity (W/(m2 ºC))
20
40
60
80
100
120
Temperature (ºC)

Figure 2

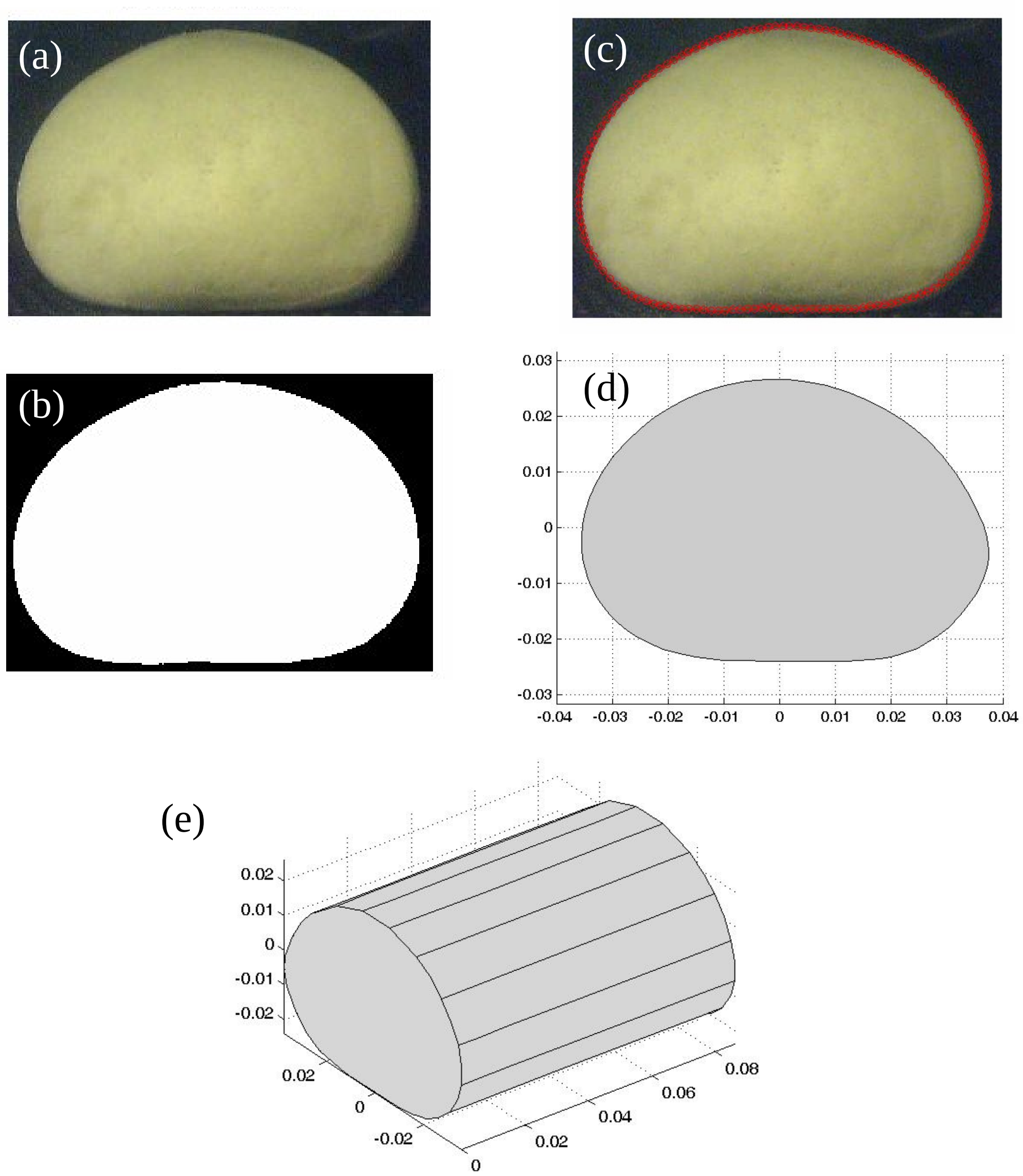
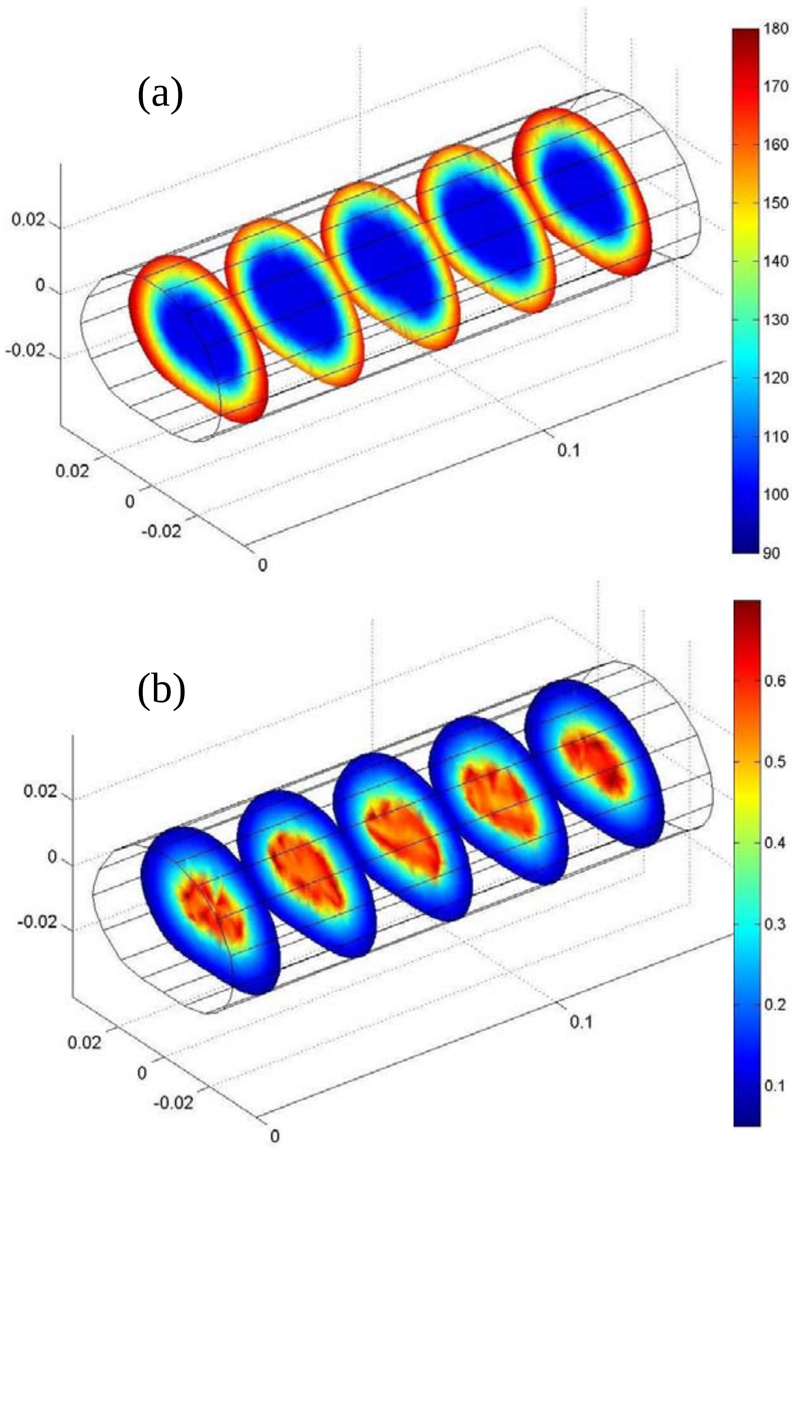
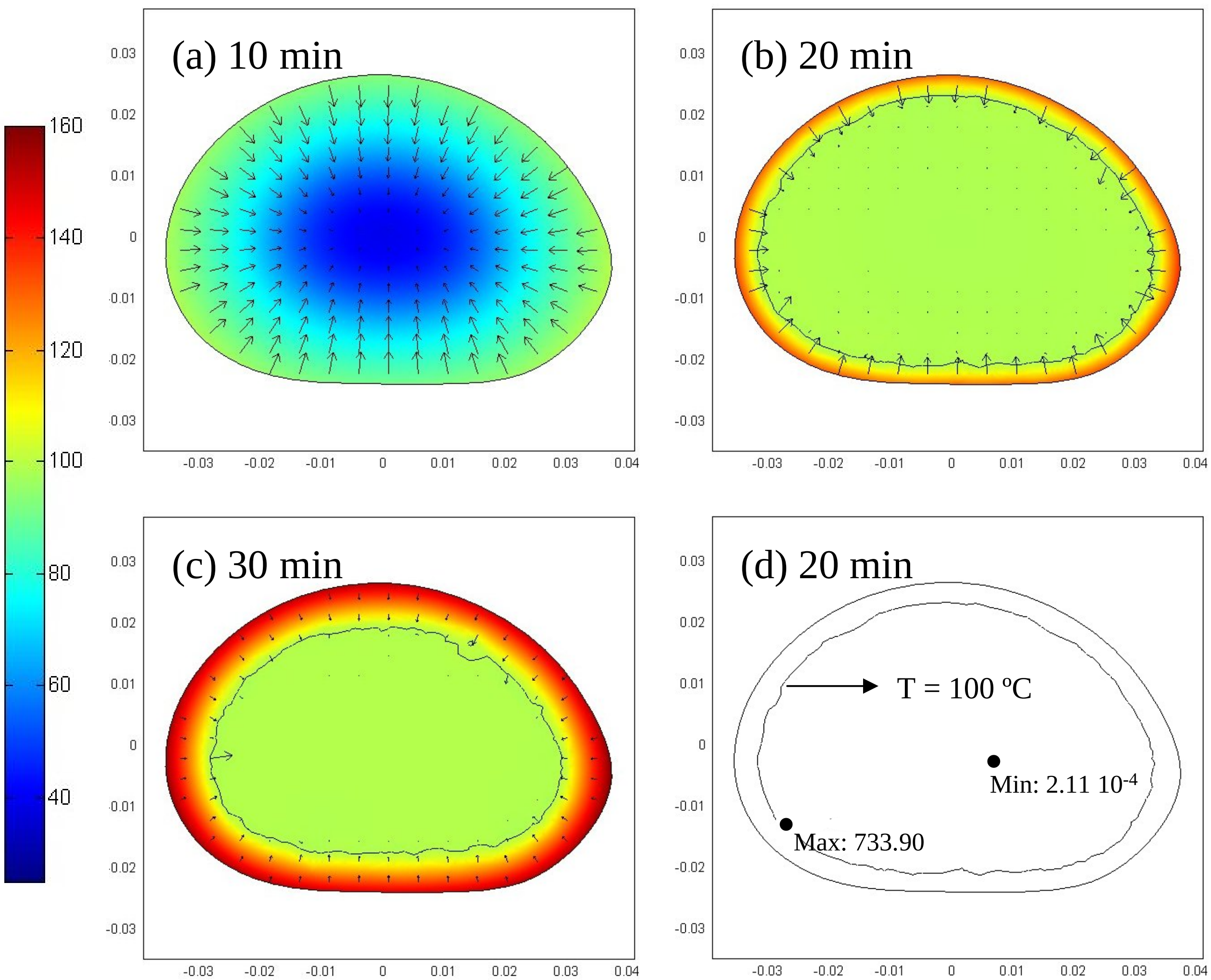

(a)
(c)
(b)
(d)
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.03
-0.02
-0.01
0
0.01
0.02
0.03
0.04
(e)
0.02
0.01
0
-0.01
-0.02
0.02
0
-0.02
0
0.02
0.04
0.06
0.08

Figure 3



(a)
180
170
160
150
140
130
120
110
100
90
0.02
0
-0.02
0.02
0
-0.02
0
0.1
(b)
0.6
0.5
0.4
0.3
0.2
0.1
0.02
0
-0.02
0.02
0
-0.02
0
0.1

Figure 4



(a) 10 min
(b) 20 min
(c) 30 min
(d) 20 min
T = 100 ºC
Min: 2.11 10-4
Max: 733.90
160
140
120
100
80
60
40

Figure 5

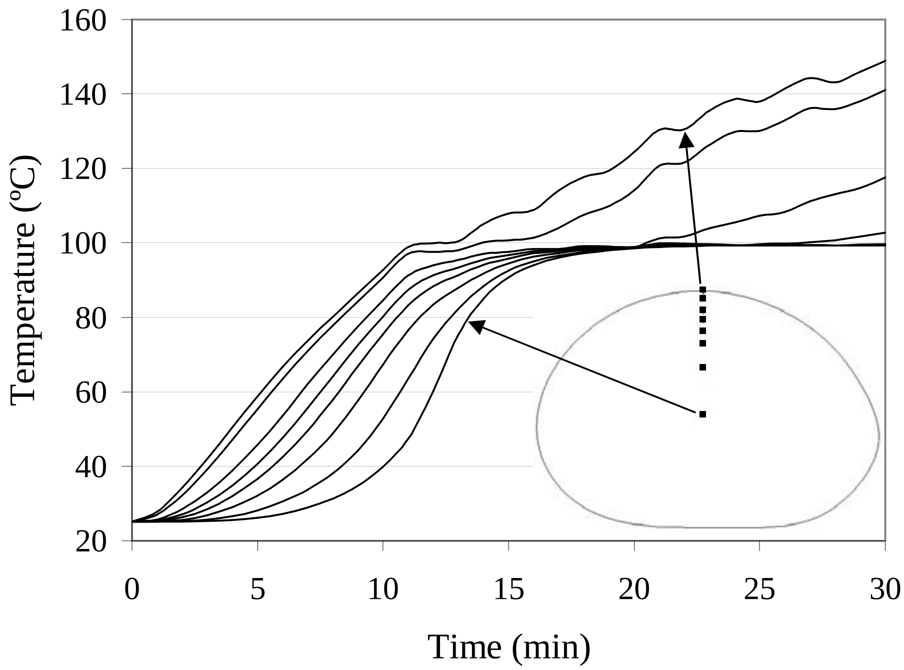

160
140
120
100
80
60
40
20
Temperature (ºC)
0
5
10
15
20
25
30
Time (min)

Figure 6

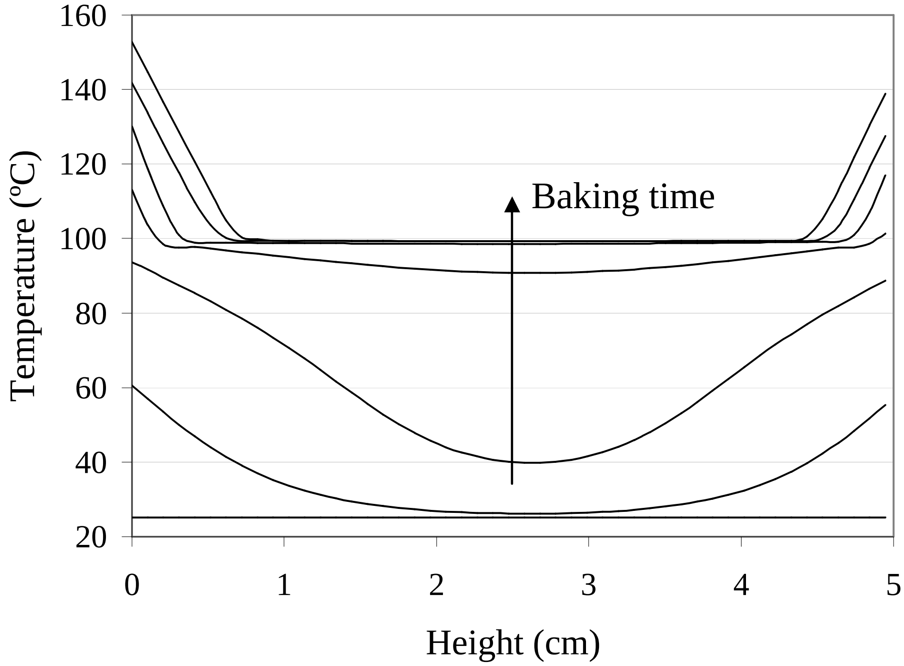

160
140
120
100
80
60
40
20
Temperature (ºC)
Baking time
0
1
2
3
4
5
Height (cm)

Figure 7

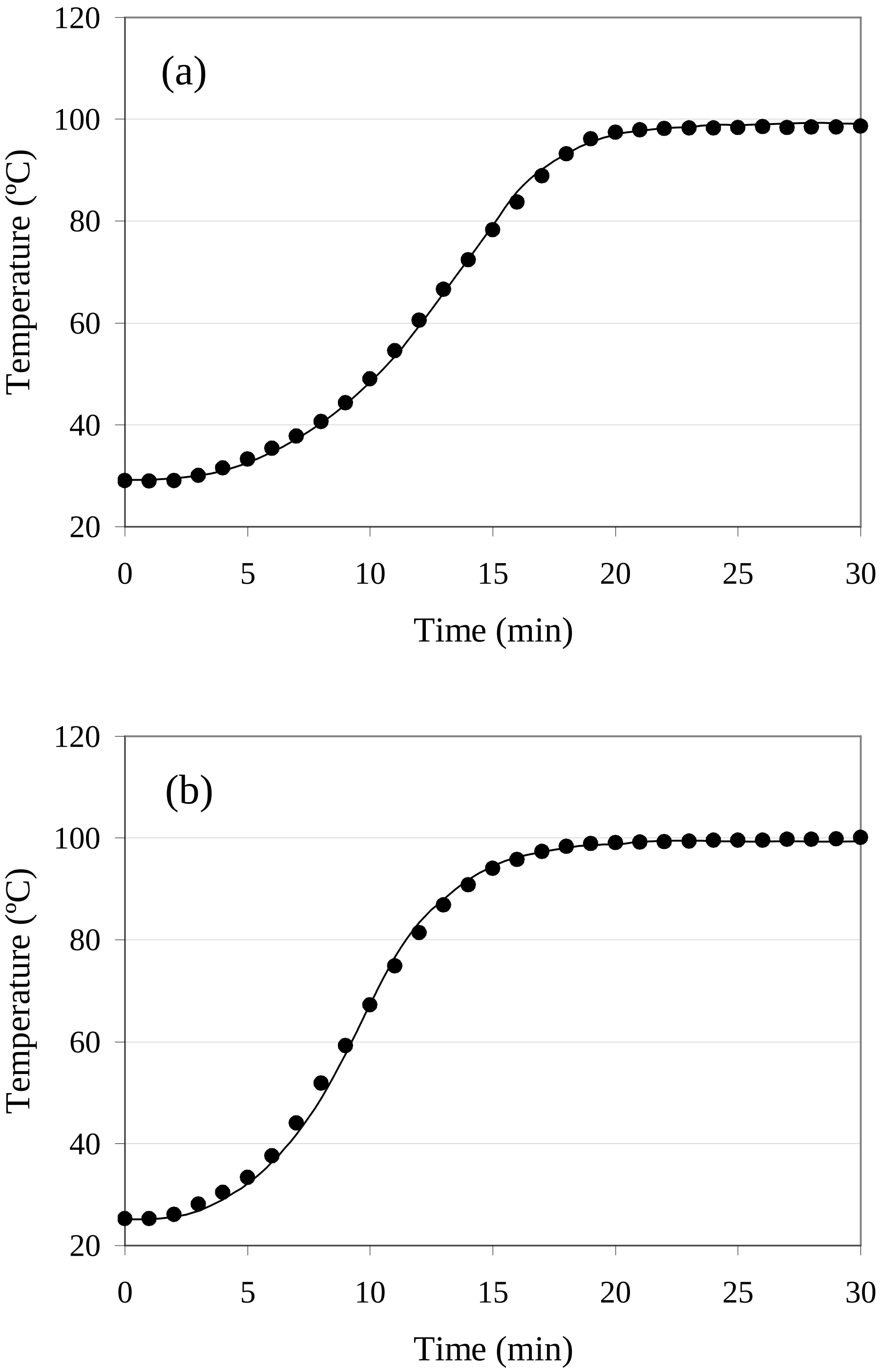

(a)
Temperature (ºC)
120
100
80
60
40
20
0
5
10
15
20
25
30
Time (min)
(b)
Temperature (ºC)
120
100
80
60
40
20
0
5
10
15
20
25
30
Time (min)

Figure 8

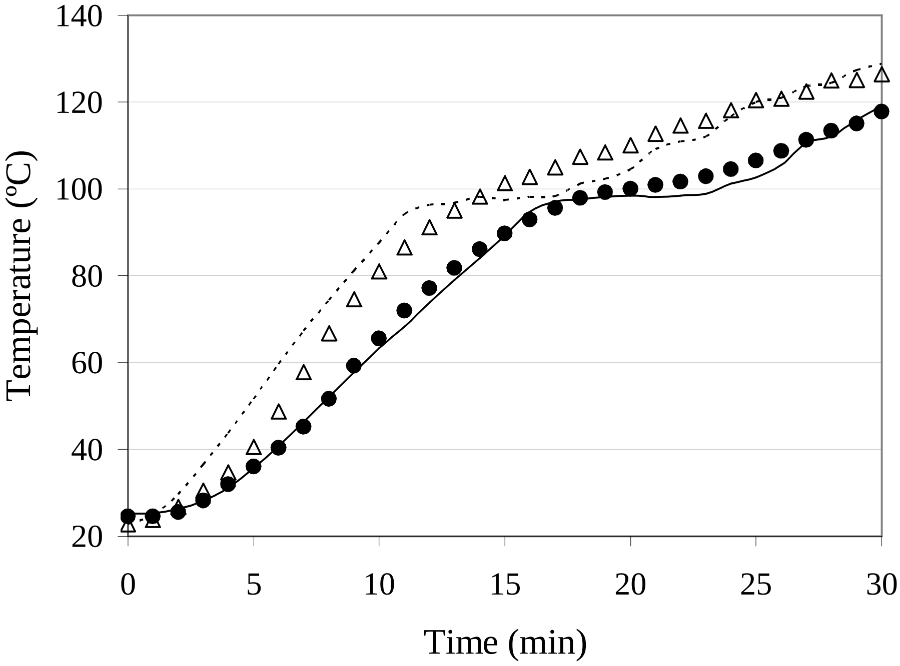


Figure 9

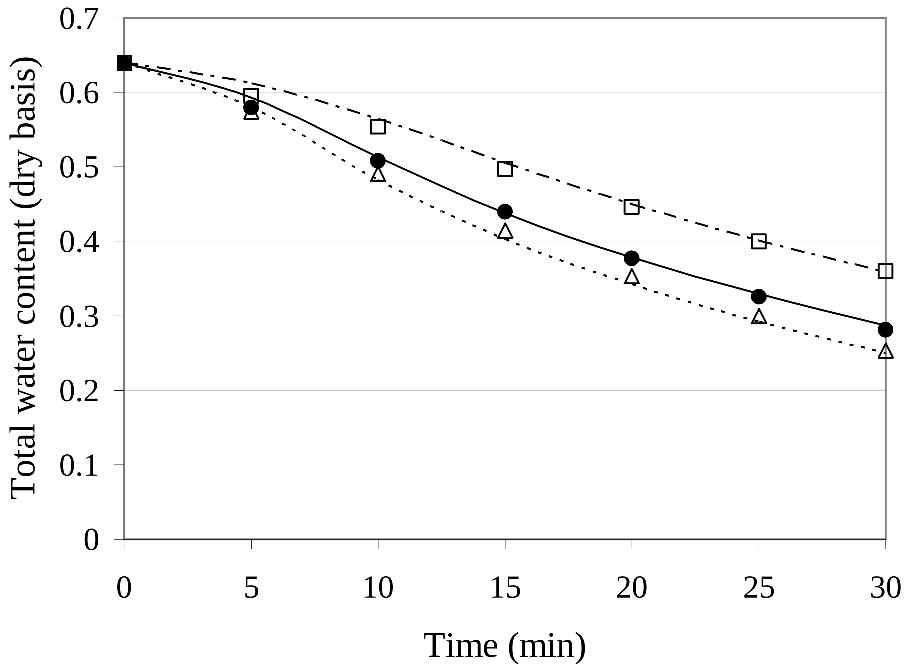

0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Total water content (dry basis)
0
5
10
15
20
25
30
Time (min)

Figure 10

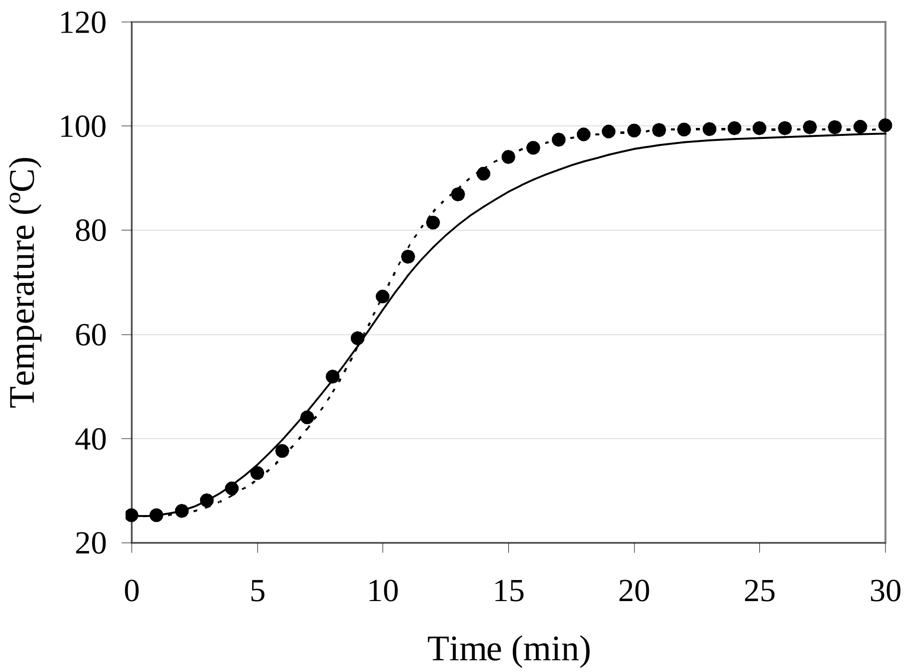
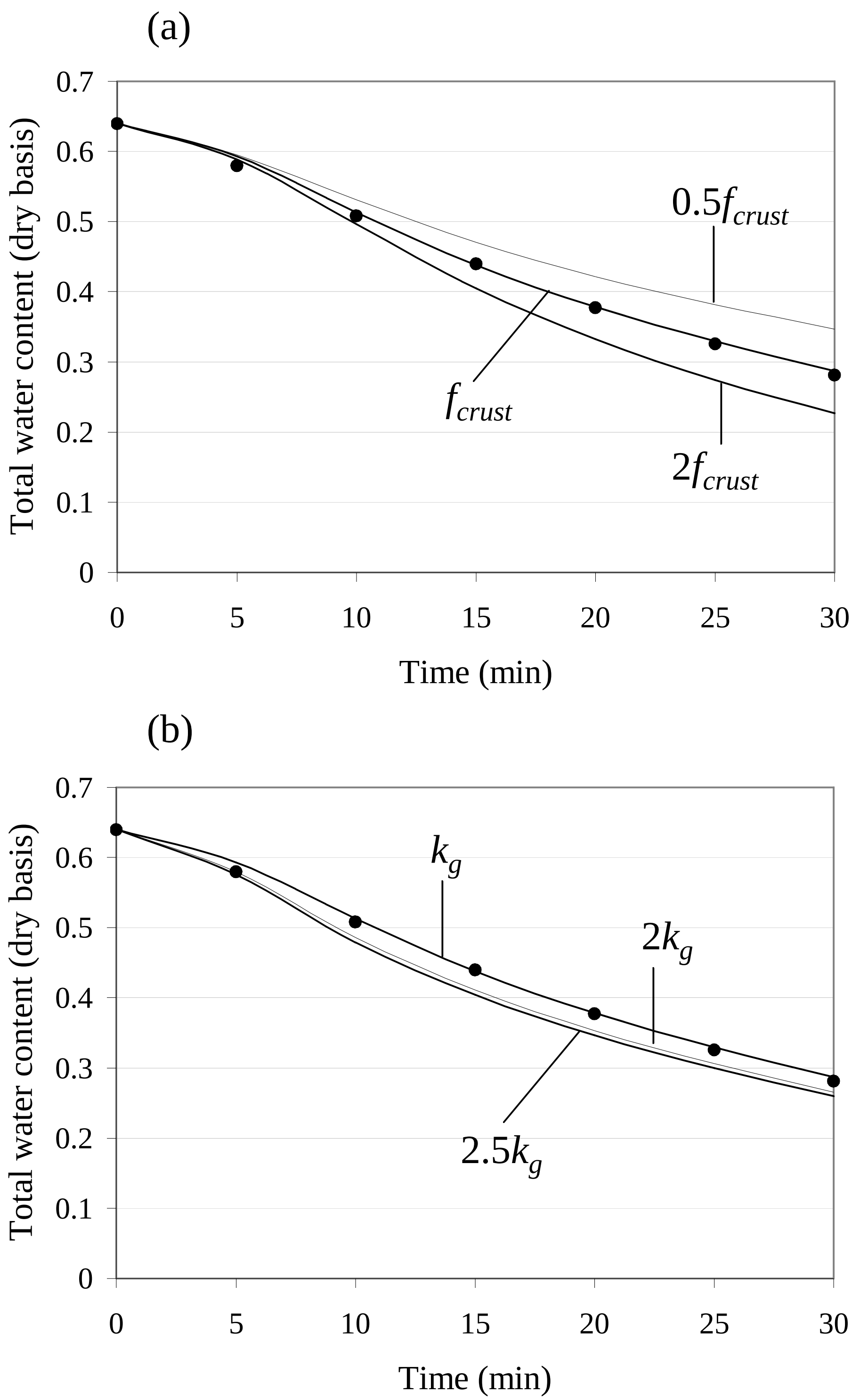


Figure 11



(a)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Total water content (dry basis)
0.5f_{crust}
f_{crust}
2f_{crust}
0
5
10
15
20
25
30
Time (min)
(b)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Total water content (dry basis)
k_g
2k_g
2.5k_g
0
5
10
15
20
25
30
Time (min)

Table 1. Values for heat (h, in W m^{-2} K^{-1}) and mass (k_g, in kg Pa^{-1} m^{-2} s^{-1}) transfer coefficients according to description in Section 2.6. Data for mass transfer coefficient is already corrected by the estimated correction factor, i.e. k_g = 7.83 10^{-2} k_g^*.

Baking temperature (ºC)	Natural convection		Forced convection	
	h	k_g	h	k_g
180	7.34	4.35 10^{-9}	11.94	5.08 10^{-9}
200	7.68	3.38 10^{-9}	11.96	8.46 10^{-9}
220	7.95	6.04 10^{-9}	11.97	8.46 10^{-9}

Table 2. Mean absolute relative error (e_{abs}, Eq. (16)) for crumb temperature (n = 360) and total water content (n = 6) predictions for 30 min baking. NC: natural convection; FC: forced convection.

	Crumb temperature	Total water content
Baking condition	e_{abs} (%)	e_{abs} (%)
180 ºC, NC	1.24	1.12
200 ºC, NC	1.53	1.74
220 ºC, NC	0.98	0.79
180 ºC, FC	1.71	1.17
200 ºC, FC	1.48	1.02
220 ºC, FC	1.78	1.83
Average	*1.45*	*1.28*