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Research article

Scheduling deferrable electric appliances in smart homes: a bi-objective stochastic optimization approach

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Abstract: In the last decades, cities have increased the number of activities and services that depends on an efficient and reliable electricity service. In particular, households have had a sustained increase of electricity consumption to perform many residential activities. Thus, providing efficient methods to enhance the decision making processes in demand-side management is crucial for achieving a more sustainable usage of the available resources. In this line of work, this article presents an optimization model to schedule deferrable appliances in households, which simultaneously optimize two conflicting objectives: the minimization of the cost of electricity bill and the maximization of users satisfaction with the consumed energy. Since users satisfaction is based on human preferences, it is subjected to a great variability and, thus, stochastic resolution methods have to be applied to solve the proposed model. In turn, a maximum allowable power consumption value is included as constraint, to account for the maximum power contracted for each household or building. Two different algorithms are proposed: a simulation-optimization approach and a greedy heuristic. Both methods are evaluated over problem instances based on real-world data, accounting for different household types. The obtained results show the competitiveness of the proposed approach, which are able to compute different compromising solutions accounting for the trade-off between these two conflicting optimization criteria in reasonable computing times. The simulation-optimization obtains better solutions, outperforming and dominating the greedy heuristic in all considered scenarios.

Keywords: smart cities; smart homes; urban data analysis; household energy planning; stochastic optimization; mixed-integer programming; Monte Carlo simulation; bi-objective optimization; greedy heuristic

1. Introduction

The paradigm of smart cities aims at increasing resource efficiency in several daily activities that citizens perform in urban environments. In the case of energy management, this aim is not only related to the amount of energy consumed, but also to the infrastructure required to distribute the energy [1]. The capacity of this infrastructure is often conditioned by peak consumption, as it should be able to distribute the energy during the periods of high demand without producing power outages. However, if consumption of a certain area is remarkably unbalanced (having important variations along the day), this would required a large investment in infrastructure that will be idle the most of the time [2].

Time-of-Use (ToU) pricing for households contributes to the overall efficiency of the electrical system. ToU incentives citizens to have a smoother consumption patron, shifting the usage of electric appliances from expensive peak hours to relatively cheaper off-peak hours. This behavior reduces the maximal instant power consumption of an urban area and, therefore, cuts back the required infrastructure investment to handle the peak and the risk of power outages [2]. However, usually off-peak hours, in which electricity is cheaper, are not preferred by users for using their appliances. This effect, which is known as inconvenience due to timing [3], can affect the well-being of the users. Therefore, there is a trade-off between both criteria, i.e., electricity cost and users satisfaction. Intelligent computer-aid tools may help users in the decision-making process of scheduling their deferrable appliances [4, 5].

This article proposes a novel mixed integer programming model for scheduling deferrable electric appliances in households, which simultaneously considers minimizing the electricity cost and maximizing the users satisfaction. Users satisfaction measures to what extend the starting time and duration for appliances usage scheduled by the model match the users preferences, which is estimated through the analysis of historical data [6–8]. However, since this parameter can show certain variability between different days, stochastic resolution approaches that consider this uncertain behaviour are devised. Therefore, the main contributions of the research reported in this article include: i) a novel mathematical formulation for the household energy planning problem based on integer programming that improves upon previous work by reducing the number of variables and constraints, ii) two resolution approaches for handling uncertain users preferences and the conflicting goals of minimizing the electricity cost and maximizing the users satisfaction, which have not been used before in the context of this problem, and iii) experimental evaluation over instances based on real-world data and a thorough analysis of the results. This article extends our previous conference article "A simulation-optimization approach for the household energy planning problem considering uncertainty in users preferences", presented at 10^{th} International Conference of Production Research-Americas. New content and contributions in this extension include a novel greedy heuristic for addressing the bi-objective household energy planning problem and extended experiments, including building-like instances. These instances consist of a unique building that has inside several housing units or users and, thus, besides of respecting the maximum power contracted per individual household, the energy planning of all the households of the building has to respect the overall power consumption contracted by the building.

The article is structured as follows. Section 2 presents the mathematical formulation of the problem, the resolution approaches and the related works. Section 3 presents the computational experimentation, including the description of the used instances, the implementation details and the obtained results. Section 4 discusses the main results obtained. Finally, Section 5 formulates the conclusions and describes the main lines of future research.

2. Materials and methods

The household energy planning problem addressed in this article aims at reducing expenses of electricity in households while enhancing users satisfaction. This last objective was estimated by considering in which part of the day users prefer to use the appliances (inferred from historical data).

2.1. Mathematical formulation

The household energy planning problem addressed in this article is modelled as a mixed-integer programming (MIP) model considering the following elements:

Sets:

- a set of users $U = (u_1 \dots u_{|U|})$, each user represents a household;
- a set of time slots $T = (t_1 \dots t_{|T|})$ in the planning period;
- sets of domestic appliances $L^u = \left(l_1^u \dots l_{|L|}^u\right)$ for each user u;

Parameters:

- a penalty term ρ^u applied to those users that surpass the maximum (electric) power contracted;
- a parameter D_l^u that indicates the average time of utilization for user u of appliance $l \in L^u$;
- a parameter C_t that indicates the cost of the power in time slot t in the ToU pricing system;
- a parameter P_l^u that indicates the power consumed by appliance l;
- a binary parameter UP_{lt}^u that is 1 if user u prefers to use the appliance $l \in L^u$ at time slot t, 0 in other case;
- a parameter E^u that indicates the maximum power contracted by user u;
- a parameter E^{joint} that indicates the maximum power that the (whole) set of users U are allowed to consume, which is used in building-like instances;

Variables:

- a binary variable x_{lt}^u that indicates if user u has appliance $l \in L^u$ turn on at time slot t;
- a binary variable δ_{lt}^u that indicates if the appliance $l \in L^u$ of user u is turn on from time slot t up to a period of time that its at least equal to D_l^u;
- a binary variable ψ_t^u that indicates if user u is using more power than the maximum power contracted E^u.
- a binary variable Ψ_t^u that indicates if user u is using more power than 130% of the maximum power contracted E^u .

The problem aims at finding a planning function $X = \{x_{lt}^u\}$ for the use of each household appliance that simultaneously maximizes the users satisfaction (given the users preference functions) and minimizes the total cost of the power consumed. The mathematical formulation is outlined in Eqs (1)–(10).

$$\max F = \sum_{u \in U} \sum_{l \in L^u} \sum_{\substack{t_1 \in T \\ t \leq |T| - D_l^u}} \left(\delta_{lt_1}^u \left(\sum_{\substack{t_2 \in T \\ t_1 \leq t_2 < t_1 + D_l^u}} UP_{lt_2}^u \right) \right) \tag{1}$$

$$\min G = \sum_{t\in T}\sum_{u\in U}\left(\sum_{l\in L^u} x^u_{lt}P^u_l C_t + \rho^u\left(0.3\psi^u_t + 0.7\Psi^u_t\right)\right) \tag{2}$$

Subject to

$$\delta^u_{lt} \leq 1 - \frac{D^u_l - \left(\sum_{\substack{t_2\in T\\ t\leq t_1<t+D^u_l}} x^u_{lt_1}\right)}{D^u_l},\ \forall\ u\in U, l\in L^u, t\in T \tag{3}$$

$$\psi^u_t \geq \frac{\sum_{l\in L^u} P^u_l x^u_{lt} - E^u}{\sum_{l\in L^u} P^u_l},\ \forall\ u\in U, t\in T \tag{4}$$

$$\Psi^u_t \geq \frac{\sum_{l\in L^u} P^u_l x^u_{lt} - 1.3E^u}{\sum_{l\in L^u} P^u_l},\ \forall\ u\in U, t\in T \tag{5}$$

$$\sum_{\substack{u\in U\\ l\in L^u}} P^u_l x^u_{lt} \leq E_{joint},\ \forall\ t\in T \tag{6}$$

$$\psi^u_t \in \{0,1\},\ u\in U\forall\ t\in T \tag{7}$$

$$\Psi^u_t \in \{0,1\},\ u\in U\forall\ t\in T \tag{8}$$

$$\delta^u_{lt} \in \{0,1\},\ \forall\ u\in U, l\in L^u, t\in T \tag{9}$$

$$x^u_{lt} \in \{0,1\},\ \forall\ u\in U, l\in L^u, t\in T \tag{10}$$

Equation (1) aims at maximizing the users satisfaction according to their preferences. Equation (2) aims at minimizing the energy expense budget, which include the charge for power consumption and the penalization for exceeding the maximum power contracted. Equation (3) enforces δ^u_{lt} to be one when the length of time an appliance will be on is equal or larger than the required by the user. Equation (4) enforces ψ^u_t to be one if the user exceeds the maximum power contracted. Equation (5) enforces Ψ^u_t to be one if the user exceeds the maximum power contracted for more than 30%. For building-like instances, Eq (6) enforces that the joint electric consumption by the set of users do not surpass a the maximum power allowed to the building. Equations (7)–(10) establishes the binary nature of the variables.

2.2. *A simulation-optimization resolution approach*

Real-world data shows that considering users preferences (UP) as a deterministic parameter does not adjust to reality [9]. Users satisfaction is modelled more accurately if uncertainty is taken into account for preferences in the model. Therefore, this article develops a resolution approach that considers this stochastic behaviour.

2.2.1. Bi-objective optimization

In order to handle the bi-objective nature of the optimization problem presented in Section 2.1, a weighted sum optimization approach is applied. The weighted sum is a traditional method in the multiobjective optimization literature which has extensively been used in many applications, including other household energy planning related problems [3]. Applying this approach, Eqs (1) and (2) are jointly optimized with Eq (11), where α and w_β are the relative weights given to users satisfaction and cost criteria by the decision-maker.

$$\max H = \alpha \frac{F - F^{best}}{F^{best} - F^{worst}} - \beta \frac{G - G^{best}}{G^{worst} - G^{best}} \tag{11}$$

One of the main drawbacks of this method is to know the actual best and worst values of each objective within the set of non-dominated solutions which are used for normalization (i.e., F^{best} and G^{best}, F^{worst} and G^{worst} in Eq (11), respectively). In this article, for addressing this issue, the procedure proposed in Rossit [10] and applied in Rossit et al. [11] is used. This is a two step procedure. In the first step, the best and worst values of each objective are approximated by solving the single objective problem of each of the criteria involved. These values, which are likely to be dominated, are improved in the second step of the procedure. In this second phase, these best and worst values are used in the weighted sum formula (Eq (11)) along with a biased combination of weights. This is, two different problems are solved, one problem using $\alpha >> \beta > 0$ and the other problem using $\beta >> \alpha > 0$. Finally, from the solutions of these last two multiobjective problems, the new best and worst values are obtained.

2.2.2. Sample Average Approximation method for considering stochastic users preferences

Formally, in a stochastic optimization problem with a probabilistic objective function, the expected value of this function should be optimized. In the case of the formulation described in Section 2.1, if parameters UP are considered stochastic, Eq (1) should be replaced by Eq (12).

$$e = \mathbb{E}_{\mathbf{P}} \left[F \left(\mathbf{\Delta}, \mathbf{UP} \right) \right]. \tag{12}$$

In Eq (12), **UP** is the random vector of the stochastic users preferences and $\mathbf{\Delta}$ is the vector of decision variables δ described in Section 2.1. In order to optimize Eq (12), all the possible realizations of vector **UP** with its corresponding probability should be considered. Taking into account that the model of Section 2.1 uses a finite set of time slots, the set of possible realizations of **UP** is also finite. Particularly, there are $|T|^{\sum_{u \in U} |L^u|}$ realizations of this vector, each one constituting a possible scenario for the stochastic problem. For example, consider an instance in which the day is split in intervals of 30 minutes, i.e., $|T| = 48$, there are two users (households) and each user has only two appliances ($|L^{u_1}| = |L^{u_2}| = 2$). Then, the number of possible scenarios would be $48^4 = 5,308,416$.

For the cases in which the large number of scenarios of real-world instances makes impractical to compute the exact expected value of Eq (12), the expected value is approximated with an independently and identically distributed (i.i.d.) random sample. This technique is called the "sample-path optimizatio [12]" or "sample average approximation [13]". Thus, Eq (13) is an estimator of the expected value of Eq (12).

$$\hat{e} = \frac{1}{N} \sum_{j=1}^{N} F \left(\mathbf{\Delta}, \mathbf{UP^j} \right) \tag{13}$$

As aforementioned, the set of values $UP^1, ..., UP^N$, is an i.i.d. random sample of N realizations of the stochastic vector parameter **UP**. The optimization problem obtained when Eq (13) is used instead of Eq (12), is the sample average approximation optimization problem (hereafter SAA) and can be solved deterministically with commercial solvers. Clearly, the solution of the SAA problem depends on the realizations **UP** that are included in the random sample. Moreover, the larger the size of the sample (N), the smaller is the difference between Eq (12) and its estimator Eq (13). Particularly, when $N \to \infty$, $\hat{e} \to e$ [14].

Different samples of size N (i.e., different set of realizations of the stochastic vector parameter **UP**) allow shaping different forms of Eq (13). Therefore, all algorithms based on sample average usually solve the SAA problem several times with different samples and after that the most promising solution is selected according to a given (predefined) criteria as the final solution.

Let $\hat{e}_N^1, \hat{e}_N^2, ..., \hat{e}_N^M$ be the values of Eq (13) when solving M SAA problems, each one with a different sample of size N. Moreover, considered that $\hat{s}_N^1, \hat{s}_N^2, ..., \hat{s}_N^M$ are the solution (values of decision values) obtained for each of the aforementioned M SAA problems. An intuitive criteria for selecting the final solution among the M possibilities, would be to pick the solution with the best $\hat{e}_N$ value. In this article, a more sophisticated procedure to select the final criteria, which was proposed in Norkin et al. [15] and implemented in Verweij et al. [16], is used. This procedure is described as follows. First, an independent sample of size N' with $N' >> N$ is built to evaluate the M solutions using this sample. Then, the solution with the best value as it is expressed in Eq (14) for a maximization problem is selected.

$$\hat{s}_N^* = \arg\max\{\hat{e}_{N'}(\hat{s}_N) : \hat{s}_N \in \hat{s}_N^1, \hat{s}_N^2, ..., \hat{s}_N^M\} \tag{14}$$

The previously described idea takes advantage from the fact that even though using the large sample size N' for the optimization phase is very time consuming (specially in NP-hard problems as the one addressed in this paper), using it for just for evaluation of the objective function Eq (13) is achievable in reasonable computing time [14]. The pseudocode of the proposed SAA approach is outlined in Algorithm 1.

Algorithm 1 Schema of a the Sample Average Approximation approach.

procedure SO($p_{lt}^u, N, M, \alpha, \beta$)
  initialize list S of size M
  for $m \leftarrow 0, m++, m \leq M$ **do**
    for $n \leftarrow 0, n++, n \leq N$ **do**
      for all $u \in U$ **do**
        for all $l \in L^u$ **do**
          for all $t \in T$ **do**
            initialize $t \leftarrow random(0, 1)$
            if $t \leq p_{lt}^u$ **then** $UP_{lt}^u = 1$
            else $UP_{lt}^u = 0$
    $S[m] \leftarrow$ **Solve MDR**(α, β, **UP**))
  return S

2.3. A greedy heuristic for household appliances planning

A greedy heuristic is proposed as reference baseline for results comparison. Greedy algorithms are conceived to heuristically obtain a global good solution to a problem by making locally optimal decisions by a repetitive procedure [17]. These heuristics have been efficiently applied in other energy planning problems by our research group [18–20]. The pseudocode of the greedy heuristic is outlined in Algorithm 2.

Algorithm 2 Greedy algorithm for household appliances planning

procedure BestPrefInterval(t_m,u_i,l_k,X)
  pref ← 0; duration ← 0
  for ($t_n = t_m$; $t_n < t_{|T|}$; t_n++) **do**
    if duration $< D(l_k, u_i)$ **then**
      if $\sum_{l \in L^{u_i}} x^u_{lt} \times P_l + P_{l_k} < E^{u_i}$ **then**
        pref += $UP(u_i, t_n, l_k)$
        duration += $t_n - t_{n-1}$
      else
        pref ← 0
        duration ← 0
    else
      return [t_n, pref ← -1] ▹ interval found
  return [$t_{|T|}$, pref] ▹ no interval was found

procedure BestCostInterval(t_m,u_i,l_k,X,UP^N)
  cost ← 0; duration ← 0; pref ← 0
  for ($t_n = t_m$; $t_n < t_{|T|}$; t_n++) **do**
    if duration $< D(l_k, u_i)$ **then**
      if $\sum_{l \in L^{u_i}} x^u_{lt_m} \times P_l + P_{l_k} < E^{u_i}$ **then**
        cost += $P_{l_k} \times C(t_n)$
        duration += $t_n - t_{n-1}$
        pref += $UP(u_i, t_n, l_k)$
      else
        cost ← 0
        duration ← 0
        pref ← 0
    else
      return [t_n, cost, pref] ▹ interval found
  return [$t_{|T|}$, cost ← BigM, pref ← 0] ▹ no interval was found

procedure Greedy(UP^N)
  X ← $\vec{0}$; minPref ← π
  for ($u_i = u_0$; $u_i < u_{|U|}$; u_i++) **do** ▹ for each user
    for ($l_k = l_0$; $l_k < L^u_{|K|}$; l_k++) **do** ▹ for each appliance
      pref ← 0; bestPref ← -1 ▹ esearch best interval for pref
      for ($t_m = t_0$; $t_m < t_{|T|}$; t_m++) **do**
        [t_m, pref] = BestPrefInterval(t_m,u_i,l_k,X,UP^N)
        if pref > bestPref **then**
          bestPref ← pref
      if bestPref < 0 **then**
        break ▹ no feasible solution found by the greedy
      pref ← 0; cost ← 0; bestCost ← -1 ▹ search best interval for cost and min pref
      for ($t_m = t_0$; $t_m < t_{|T|}$; t_m++) **do**
        [t_m, cost, pref] = IntervalMaxPrefCost(m, d_k, u_i, X)
        if cost < bestCost & pref > bestPref * minPref **then**
          bestCost ← cost
          $t_{bestmin} \leftarrow t_m$
      for ($t_m = t_{bestmin} - D(l_k, u_i)$; $t_m \leq t_{bestmin}$; t_m++) **do**
        $x^{u_i}_{l_k t_m} \leftarrow 1$ ▹ set appliance ON
  return X

The main goal of the proposed greedy algorithm is to build low-cost solutions (according to Eq (2)). However, it also considers a threshold level of user satisfaction that must be fulfilled. For this reason, function BestPrefInterval() seeks the interval with the maximum user satisfaction for each appliance. Then, function BestCostInterval() seeks the interval that minimizes the cost given that the user satisfaction is not smaller than a percentage ($0 < \pi < 1$) of the maximum user satisfaction for the same user and appliance and that the maximum power contracted by the user is not exceeded. Thus, sets the appliance as switched ON starting from that time slot (up to the time slot in which expected duration is fulfilled). Within each user, appliances are processed in descending order of power consumption. Similarly to the SAA approach, BestPrefInterval() uses average user preferences (UP) given a certain number of realizations of this stochastic parameter. The greedy heuristic is also applied to M different samples of size N of the preferences vector and the final solution is selected using the same procedure as in the SAA.

2.4. Related work

Household energy planning has been considered as a complex problem in the related literature. This article focuses on the stochastic version of the problem. A more general review of the topic was presented by Lu et al. [21].

The deterministic version of the household energy planning problem is associated with bin packing [22], a well-known NP-hard problem. The inclusion of uncertainty increases the complexity of the problem [23]. Several articles have addressed stochastic versions of this problem, by considering uncertainty in different parameters. Chen et al. [24] considered uncertainties in the power consumed by the appliances and the renewable solar energy gathered by a photovoltaic array. A three-stages resolution process was proposed. First, Chen et al. solve a deterministic linear programming optimization model considering mean values for the appliances consumption and maximum solar power generation. Then, they apply a stochastic procedure based on Monte Carlo simulation was applied to the resulting solution. The simulation considers different energy consumption rates of appliances and selects the consumption rate that minimizes the probability of shortcuts, which occurs when the overall consumption of electricity surpass a certain threshold value. Finally, an online adjustment of the previous (offline) solution was applied, which monitors the instant solar power generation and the consumption of appliances in real-time, compensating the household electric balance of the offline solution with a larger power storage in the battery or purchase from the grid. Hemmati and Saboori [25] proposed a particle swarm optimization algorithm to deal with uncertainty of photovoltaic panels in a similar problem. Assuming that the energy generated in the panels has a Gaussian probabilistic distribution, a Monte Carlo simulation was used each time the stochastic function has to be evaluated to obtain a sample of the generation values.

Other researchers have used robust optimization, which aims at minimizing the impact of the worst-case scenario, considering that random parameters have a bounded probabilistic distribution [3]. Jacomino and Le [26] presented a robust optimization approach to simultaneously minimize energy cost and maximize the comfort of users. They considered uncertainty in two aspects: the outdoor temperature and the solar radiation related to weather forecast -that affect the energy to be consumed to satisfy the required indoor temperature-, and users decisions related to not programmable services, i.e., despite the scheduled starting time and duration of the appliances the user can modified these conditions when actually using them. For handling uncertainty on users behaviour, a decomposition approach based on estimating the probability of occurrence of each scenario was used. Wang et al. [27] proposed a robust optimization approach for dealing with photovoltaic energy generation in household planning by using a mixed integer quadratic programming model, and Wang et al. [28] for dealing with uncertainty in hot water utilization and outdoor temperature that influences the usage of heating and air conditioning systems. Judge et al. [29] proposed a robust optimization model to manage uncertainties associated with thermal loads such as heating and air conditioning and solved combining Harris Hawks' optimization [30] and linear programming. Hosseini et al. [31] presented a robust optimization approach to minimize the energy cost while satisfying certain comfortability restrictions considering uncertainty from two different sources: the decisions of user of when using each appliance and the intermittency of renewable energy sources. Another work that uses robust optimization for handling uncertainty of renewable sources of energy was performed by Shi et al. [32]. Other published material deals with this problem as a control problem by using a closed-loop approach such as Scarabaggio et al. [33], who used a sample average approximation based on a probability density function to cope with uncertainty in wind power availability, or Nassourou et al. [34], in which a control strategy that is divided into an open-loop system that manage the dependent control inputs and a closed-loop system that uses local feedback control for the independent inputs.

From the analyzed works, it can be concluded that fine grained energy consumption data collection from smart homes considering uncertainty has shown to be a powerful tool to define more efficient and reliable electricity services. However, the collection and exchange of information raise concerns about consumer privacy. The collected data could be used to infer activities and behavior patterns of consumers or an attacker could create fake power information to jeopardize the power system [35]. In order to deal with these privacy issues, Tonyali et al. developed a meter data obfuscation scheme to protect consumer privacy from eavesdroppers and the utility companies while preserving the utility companies' ability to use the data for state estimation [36]. Mohammed et al. proposed an approach based on adding noise to the reading data so no one can obtain the meters' individual data, however, the total readings of the meters can be known by the utility [37]. In line with the work presented in this article, the problem of enhancing the decision making processes in demand-side management has been addressed by adding a specific optimization objective related to preserving users' privacy. Thus, there have been proposed multiobjective optimization approaches that have proposed the minimization of the energy consumption cost while maximizing users' privacy by masking the energy consumption profile of the user [38, 39]. Chang et al. defined load variation as the privacy metric and scheduled inflexible and unshiftable appliances, flexible appliances, and shiftable appliances [38].

Other authors, although without considering uncertainty in their models, have explored the trade-off that usually exists between electricity cost and users satisfaction through linear mathematical programming approaches, as it is performed in this article. Among them, Yahia et al. [40] modeled a bi-objective problem considering these two objectives, which were combined by means of a linear weighted sum to form a unique objective function. Authors solved two single-household instances, i.e., a real South African case study and an artificial large instance, using LINGO. Additionally, they performed an extensive analysis of the sensitivity of the results to the modifications of certain parameters. Authors extended the approach by considering the reduction of the peak load as a third objective [41]. Moreover, an instance considering several households simultaneously was solved. Three different multiobjective approaches were compared: lexicographic optimization, normalized weighted sum and compromise programming. Our previous articles explored the trade-off between the users satisfaction and energy cost in a deterministic version of the problem using evolutionary algorithms [19, 42].

This article contributes to the literature in several aspects. Firstly, a novel linear mathematical formulation of the household planning energy optimization problem that explicitly considers users satisfaction as an objective function is presented. Approaches like that are not common in the related literature [40]. Moreover, this is an novel mathematical formulation compared to the one presented in our previous article [19] for a similar conceptual model, but improving upon it by having a smaller number of variables and constraints that eases its solvability. Secondly, this article considers stochastic users preferences, which differentiates it from other linear programming applications in the related work [40, 41]. This leads to a novel scientific contribution of the work, which is the application of the simulation-optimization Sample Average Approximation method to handle the uncertainty which has not been applied to this specific problem before.

3. Results

This section presents the computation experimentation, including a description of the instances that were used, the experiment design and the main results of the experimentation.

3.1. Problem instances

The instances addressed were generated using realistic information and expanding the REDD dataset [9] via a urban data analysis approach [43]. One of the key parameters to estimate in the household energy planning model presented in this work are the users preferences. For estimating this, historical information retrieved from the REDD dataset about the power consumption of the selected appliances on each household was analyzed. This task involved cleaning the data from comparatively very small power consumption that are related to stand-by operation mode of each appliance, for example, small screen leds. After this, for each combination of user and appliance, a probability of usage for each time slot was estimated (p^u_{lt}). With this probability, M instances were constructed for each sample size N as is described in Section 3.2. Additionally, from the REDD dataset, the mean power consumption of each appliance in KW (P^u_l) and the duration of the average time of utilization of each appliance (D^u_l) were estimated. The weekend period was considered to introduce noticeable differences in the instances, a behaviour that is usual for household users [44]. Thus, instances were grouped into two categories: weekdays and weekends. Parameters E^u (maximum electric power contracted for each household) and C_t were obtained from the National Electricity Company, Uruguay, as reported in the ECD-UY dataset [45].

Besides the weekly separation (noted as *wd* and *we* for weekday and weekend, respectively), instances with increasing sizes were also defined, as already described in the methodology of the experimental evaluation of previous works [18]:

- small (*s.wd* and *s.we*), modeling scenarios with one household with seven deferrable appliances.
- large (*l.wd* and *l.we*), modeling scenarios having two households with six and seven deferrable appliances, respectively.
- building (*b.wd* and *b.we*), modeling scenarios with four households with six and seven deferrable appliances, respectively.

Electric appliances are classified in deferrable and non-deferrable appliances [46]. Deferrable appliances are those devices that can be controlled by the user and deferred to be switched on in different time-slots on the scheduling horizon, without a critical result in the comfort of users [47]. Conversely, non-deferrable appliances are those which its standard operation time cannot be shifted without a significant impact on the comfort of users, since they are critical for users to accomplish basic everyday activities, such as lighting. The scheduling approach proposed in this article considers deferrable appliances. Few works in the related literature have included non-deferrable appliances in smart home planning systems, mainly because they do not provide flexibility to compute accurate schedules, and even slight shifts of their operation times cause severe penalizations on user-comfort related objectives. This article considers both non-interruptable deferrable appliances, i.e., microwave, washer dryer, dishwasher and refrigerator, and interruptable deferrable appliances, i.e., electric stove and air conditioning.

In both small and large size instances, the constraint defined by Eq (6) was not applied, since the considered households are independent and, thus, the constraints in Eqs (4) and (5) already allow limiting the maximum consumed power. The instances *b.wd* and *b.we* have to meet not only the maximum power contracted per individual household, but also the overall power consumption contracted by the building.

3.2. Experimental results

After preliminary calibration experiments, the following sample sizes were chosen N = 1000, 2000, 3000, 5000, and 10000. Within each sample size, the number of independent samples (M) was set to 100. The evaluation sample size (N') was set to 100,000.

In order to apply the SAA approach, the bi-objective optimization procedure introduced in Section 2.2 was used. This optimization procedure requires estimating both the ideal and nadir values for the weighted sum function defined in Eq (11). The estimation of the ideal and nadir value was performed for each sample size N applying the two step procedure presented in Section 2.2: initially they are estimated through single-objective optimization and, later, they are improved applying the weighting sum method with a biased combination of weights. Then, five weight vectors (α,β) were used for exploring different trade-off combinations between the objectives of energy cost and users satisfaction: (0.99,0.01), (0.25, 0.75), (0.5, 0.5), (0.75, 0.25), and (0.01, 0.99). In the SAA method, for each weight vector a MIP problem is solved using Gurobi [48] through Pyomo as modelling language [49]. In the case of the greedy heuristic three aspiration levels were considered (π): 0.60, 0.75, and 0.90.

The experiment was divided in two parts. Firstly, the random realizations or samples of vector **UP** were generated and secondly the optimization algorithms were applied to these random samples. This separation was performed because of two reasons: i) to study the impact of the generation of random samples of vector **UP** in the overall efficiency of the algorithm and ii) to apply both algorithms over the same set of random samples to provide a more fair comparison avoiding differences in the results because of this random procedure. Then, for each instance and size N, a set of 100 (M) independent realizations of vector **UP** were generated. Table 1 reports the computational times demanded for generating the realizations of vector **UP**. The execution times indicate that the average time increases linearly with the sample size N. This is connected to the trade-off between having a large sample size N which is computationally expensive but provides a better estimation of the real expected value (Eq (12)) by Eq (13) or a smaller sample size N which is lees time-consuming but provides a worse approximation of the real expected value.

Table 1. Computing times of the realizations of vector **UP**.

Instance	*N*	Time (s)		*Instance*	*N*	Time (s)		*Instance*	*N*	Time (s)	
		Avg	Std			Avg	Std			Avg	Std
s.wd	1000	0.2098	0.0011	l.wd	1000	0.3911	0.0011	b.wd	1000	0.7325	0.0020
	2000	0.4197	0.0012		2000	0.7887	0.0021		2000	1.4828	0.0129
	3000	0.6033	0.0017		3000	1.1764	0.0031		3000	2.2112	0.0047
	5000	1.0601	0.0050		5000	1.9600	0.0080		5000	3.6860	0.0081
	10000	2.1305	0.0092		10000	3.9047	0.0056		10000	7.3938	0.0150
s.we	1000	0.2074	0.0005	l.we	1000	0.3882	0.0014	b.we	1000	0.7621	0.0018
	2000	0.4174	0.0003		2000	0.7860	0.0025		2000	1.5276	0.0037
	3000	0.60241	0.0006		3000	1.1806	0.0038		3000	2.3114	0.0056
	5000	1.0444	0.0010		5000	1.9669	0.0079		5000	3.8473	0.0067
	10000	2.0955	0.0016		10000	3.9177	0.0160		10000	7.6511	0.0134

This section presents the main results of the computational experimentation with SAA and the greedy heuristic. Detailed results about all the runs performed can be depicted in the Appendix 5. To condense the outcome of the proposed approach into a suitable indicator that measures the quality of the results, the deviation to the ideal vector is used. This is computed using the L^2 distance norm according to Eq (15).

$$\Sigma = \sqrt{\sum_{o \in O} \left(\frac{value - best_o}{best_o} \cdot 100\% \right)^2} \tag{15}$$

In the definition of the Σ metric in Eq (15), O is the set of objectives, (for the considered problem, $O = \{F, G\}$), and $best_o$ is the best value achieved for each objective evaluated over N' in all the experiments performed for that instance. Thus, from all the solutions, the solution with the smallest distance is the best comprising solution, as graphically represented in Figure 1.

Another relevant aspect that should be analyzed when controllable deferrable loads are shifted collectively, is the peak rebound effect that can be associated to a drastic increment of the consumption during low priced hours. The metric of the load factor is usually used in the related works to measure this aspect [50, 51]. The load factor is defined as the ratio of the average energy consumption to the maximum energy consumption in the planning horizon. A higher load factor implies a more stable consumption which can help to avoid problems in the electric grid [50]. Thus, the load factor for all the users (Lf) is reported for the presented solutions, calculated according to Eq (16).

$$Lf = \frac{\left(\sum_{u \in U} \sum_{l \in L^u} \sum_{t \in T} P_l^u x_{lt}^u \right) / |T|}{\max_{t \in T} \{ \sum_{u \in U} \sum_{l \in L^u} P_l^u x_{lt}^u \}} \tag{16}$$

The results of the SAA are presented in Table 2. This table reports for each instance, the sample size N, the combination of weights (α,β), the average execution time in seconds, the values of F and G of the best solution, i.e., the solution that has the minimal value of function H (Eq (11)), and the deviation of the solution to the ideal vector Σ (Eq (15)). In turn, the experimental results of the the greedy heuristic are reported in Table 3. The table presents for each instance, the sample size N and the aspiration preference level π, the same results as for the SAA. As aforementioned, the computing times in Tables 2 and 3 do not include the time to generate the N random realizations of the user preferences vector **UP**.

Table 2. Results of the SAA.

N	(α,β)	Avg. Time (s)	$F(H_{best}^{N'})$	$G(H_{best}^{N'})$	Σ	Lf	Avg. Time (s)	$F(H_{best}^{N'})$	$G(H_{best}^{N'})$	Σ	Lf
				s.wd					*s.we*		
	(0.99,0.01)	0.0138	3.2925	113.3289	26.78%	0.1854	0.0081	1.4763	36.7695	58.62%	0.2363
	(0.01,0.99)	0.1625	2.1814	89.3876	33.75%	0.2068	0.0452	1.0148	23.1810	31.26%	0.1685
1000	(0.50,0.50)	0.0911	3.1368	98.2011	10.94%	0.1650	0.0122	1.4363	28.8151	24.46%	0.2363
	(0.75,0.25)	0.0453	3.1368	98.2011	10.94%	0.1650	0.0099	1.4363	28.8151	24.46%	0.2363
	(0.25,0.75)	0.1436	2.5555	90.8443	22.44%	0.1650	0.0189	1.2623	24.7508	16.00%	0.1873
	(0.99,0.01)	0.0137	3.2925	113.3289	26.78%	0.1854	0.0085	1.4763	36.7695	58.62%	0.2363
	(0.01,0.99)	0.1619	2.1813	89.3876	33.75%	0.1663	0.0475	1.0149	23.1810	31.25%	0.1685
2000	(0.50,0.50)	0.1051	3.1368	98.2011	10.94%	0.1650	0.0149	1.4363	28.8151	24.46%	0.2363
	(0.75,0.25)	0.0486	3.1368	98.2011	10.94%	0.1650	0.0118	1.4363	28.8151	24.46%	0.2363
	(0.25,0.75)	0.1421	2.5558	90.8443	22.43%	0.1650	0.0290	1.2627	24.7508	15.97%	0.1873
	(0.99,0.01)	0.0136	3.2925	113.3289	26.78%	0.1854	0.0084	1.4763	36.7695	58.62%	0.2363
	(0.01,0.99)	0.1624	2.1819	89.3876	33.73%	0.1781	0.0476	1.0148	23.1810	31.26%	0.1873
3000	(0.50,0.50)	0.0963	3.1368	98.2011	10.94%	0.1650	0.0148	1.4363	28.8151	24.46%	0.2363
	(0.75,0.25)	0.0497	3.1368	98.2011	10.94%	0.1650	0.0117	1.4363	28.8151	24.46%	0.2363
	(0.25,0.75)	0.1351	2.5558	90.8443	22.43%	0.1650	0.0289	1.2627	24.7508	15.97%	0.1873
	(0.99,0.01)	0.0133	3.2925	113.3289	26.78%	0.1854	0.0088	1.4763	36.7695	58.62%	0.2363
	(0.01,0.99)	0.1607	2.1815	89.3876	33.74%	0.1758	0.0475	1.0151	23.1810	31.24%	0.1699
5000	(0.50,0.50)	0.1038	3.1368	98.2011	10.94%	0.1650	0.0146	1.4363	28.8151	24.46%	0.2363
	(0.75,0.25)	0.0499	3.1368	98.2011	10.94%	0.1650	0.0115	1.4363	28.8151	24.46%	0.2363
	(0.25,0.75)	0.1359	2.5558	90.8443	22.43%	0.1650	0.0288	1.2622	24.7508	16.01%	0.1994
	(0.99,0.01)	0.0131	3.2925	113.3289	26.78%	0.1854	0.0088	1.4763	36.7695	58.62%	0.2363
	(0.01,0.99)	0.1616	2.1814	89.3876	33.75%	0.1663	0.0477	1.0150	23.1810	31.25%	0.1782
10000	(0.50,0.50)	0.1030	3.1368	98.2011	10.94%	0.1650	0.0146	1.4363	28.8151	24.46%	0.2363
	(0.75,0.25)	0.0497	3.1368	98.2011	10.94%	0.1650	0.0116	1.4363	28.8151	24.46%	0.2363
	(0.25,0.75)	0.1340	2.5558	90.8443	22.43%	0.1650	0.0282	1.2622	24.7508	16.01%	0.1873
				l.wd					*l.we*		
	(0.99,0.01)	0.0226	6.4528	194.4567	47.98%	0.2158	0.0216	8.1457	277.5776	40.50%	0.3929
	(0.01,0.99)	0.2624	4.4226	131.4108	31.46%	0.2059	0.2226	5.9167	197.5689	27.37%	0.3178
1000	(0.50,0.50)	0.0769	6.0373	145.3292	12.40%	0.1471	0.1225	7.4650	211.7288	11.01%	0.2744
	(0.75,0.25)	0.0485	6.4120	185.3487	41.05%	0.2158	0.0746	7.9526	251.6011	27.45%	0.4529
	(0.25,0.75)	0.1768	5.7316	140.2288	13.04%	0.1471	0.4123	7.0347	205.0788	14.16%	0.2448
	(0.99,0.01)	0.0232	6.4528	194.4567	47.98%	0.2158	0.0211	8.1457	277.5776	40.50%	0.3929
	(0.01,0.99)	0.2596	4.4229	131.4108	31.46%	0.1962	0.2229	5.9174	197.5689	27.36%	0.3178
2000	(0.50,0.50)	0.0920	6.0373	145.3292	12.40%	0.1471	0.1166	7.4650	211.7288	11.01%	0.2744
	(0.75,0.25)	0.0631	6.1053	150.3087	15.36%	0.2223	0.0718	7.9525	251.6011	27.45%	0.4529
	(0.25,0.75)	0.2310	5.3944	136.7499	16.90%	0.1405	0.4118	7.0815	205.6305	13.69%	0.2744
	(0.99,0.01)	0.0232	6.4528	194.4567	47.98%	0.2158	0.0206	8.1457	277.5776	40.50%	0.3929
	(0.01,0.99)	0.2582	4.4230	131.4108	31.46%	0.2059	0.2226	5.9168	197.5689	27.36%	0.2871
3000	(0.50,0.50)	0.0876	6.0373	145.3292	12.40%	0.1471	0.1181	7.4650	211.7288	11.01%	0.2744
	(0.75,0.25)	0.0632	6.1049	150.3087	15.36%	0.2223	0.0769	7.9423	250.4978	26.91%	0.4529
	(0.25,0.75)	0.2334	5.3944	136.7499	16.90%	0.1405	0.4140	7.0349	205.0788	14.16%	0.2448
	(0.99,0.01)	0.0230	6.4528	194.4567	47.98%	0.2158	0.0207	8.1458	277.5776	40.50%	0.3929
	(0.01,0.99)	0.2591	4.4230	131.4108	31.46%	0.1962	0.2265	5.9175	197.5689	27.36%	0.2871
5000	(0.50,0.50)	0.0781	6.0373	145.3292	12.40%	0.1471	0.1180	7.4650	211.7288	11.01%	0.2744
	(0.75,0.25)	0.0578	6.3393	176.3038	34.21%	0.2403	0.0779	7.9526	251.6011	27.45%	0.4529
	(0.25,0.75)	0.1886	5.7325	140.2288	13.02%	0.1471	0.4271	7.0815	205.6305	13.69%	0.2744

	(0.99,0.01)	0.0224	6.4528	194.4567	47.98%	0.2158	0.0203	8.1458	277.5776	40.50%	0.3929
	(0.01,0.99)	0.2592	4.4224	131.4108	31.47%	0.1962	0.2236	5.9171	197.5689	27.36%	0.2788
10000	(0.50,0.50)	0.0846	6.0373	145.3292	12.40%	0.1471	0.1155	7.4650	211.7288	11.01%	0.2744
	(0.75,0.25)	0.0630	6.1053	150.3087	15.36%	0.2223	0.0783	7.9526	251.6011	27.45%	0.4529
	(0.25,0.75)	0.2232	5.6829	139.6771	13.49%	0.1471	0.4185	7.0817	205.6305	13.69%	0.2744
				b.wd					*b.we*		
	(0.99,0.01)	0.0661	13.8077	606.4009	139.60%	0.2112	0.0588	15.1327	673.2478	109.23%	0.2774
	(0.01,0.99)	0.4757	8.5678	253.0874	37.98%	0.1999	0.4897	9.7527	321.7733	35.56%	0.2827
1000	(0.50,0.50)	0.2532	11.7540	278.1533	17.91%	0.2012	0.3244	12.9297	350.6130	17.10%	0.3877
	(0.75,0.25)	0.1234	13.4735	483.9198	91.24%	0.2157	0.1389	15.0719	617.7601	91.99%	0.2996
	(0.25,0.75)	0.3049	11.6152	273.2761	17.81%	0.2012	0.3772	12.6639	342.2254	17.51%	0.3016
	(0.99,0.01)	0.0682	13.8106	597.8174	136.21%	0.2112	0.0585	15.1319	673.3213	109.25%	0.2774
	(0.01,0.99)	0.4740	8.5677	253.0874	37.98%	0.1957	0.4966	9.7521	321.7733	35.56%	0.3041
2000	(0.50,0.50)	0.2471	11.7545	278.1533	17.90%	0.2012	0.3175	12.9298	350.3693	17.06%	0.3877
	(0.75,0.25)	0.1327	13.4746	483.5656	91.10%	0.2157	0.1269	15.0740	617.9178	92.04%	0.2996
	(0.25,0.75)	0.3104	11.5933	272.8259	17.87%	0.2012	0.3765	12.6648	342.2354	17.51%	0.3341
	(0.99,0.01)	0.0682	13.8148	623.0884	146.19%	0.2112	0.0591	15.1316	672.6650	109.05%	0.2774
	(0.01,0.99)	0.4765	8.5678	253.0874	37.98%	0.1957	0.5026	9.7524	321.7733	35.56%	0.3041
3000	(0.50,0.50)	0.2564	11.7642	278.7050	17.97%	0.2037	0.3136	12.9297	350.3693	17.06%	0.3877
	(0.75,0.25)	0.1280	13.7706	541.3700	113.91%	0.1884	0.1285	15.0718	617.6007	91.94%	0.2996
	(0.25,0.75)	0.3070	11.6260	273.5103	17.78%	0.2012	0.3773	12.6652	342.2354	17.51%	0.3341
	(0.99,0.01)	0.0686	13.8122	597.9184	136.25%	0.2112	0.0599	15.1321	672.5696	109.02%	0.2774
	(0.01,0.99)	0.4751	8.5677	253.0874	37.98%	0.1992	0.5123	9.7533	321.7733	35.55%	0.2994
5000	(0.50,0.50)	0.2510	11.7639	278.7050	17.97%	0.2037	0.3166	12.9315	350.3693	17.05%	0.3877
	(0.75,0.25)	0.1248	13.7818	543.5414	114.76%	0.1882	0.1314	15.0722	617.6647	91.96%	0.2996
	(0.25,0.75)	0.3250	11.6264	273.5103	17.78%	0.2012	0.3760	12.6666	342.2354	17.50%	0.3341
	(0.99,0.01)	0.0692	13.8113	597.9185	136.25%	0.2112	0.0597	15.1336	673.4111	109.28%	0.2774
	(0.01,0.99)	0.4754	8.5684	253.0874	37.98%	19.57%	0.5109	9.7526	321.7733	35.56%	0.3041
10000	(0.50,0.50)	0.2528	11.7652	278.7050	17.96%	0.2037	0.3121	12.9307	350.3693	17.05%	0.3877
	(0.75,0.25)	0.1278	13.7709	541.2802	113.87%	0.1884	0.1272	15.0720	617.5108	91.91%	0.2996
	(0.25,0.75)	0.3183	11.6256	273.5103	17.78%	0.2012	0.3636	12.6665	342.2354	17.50%	0.3341

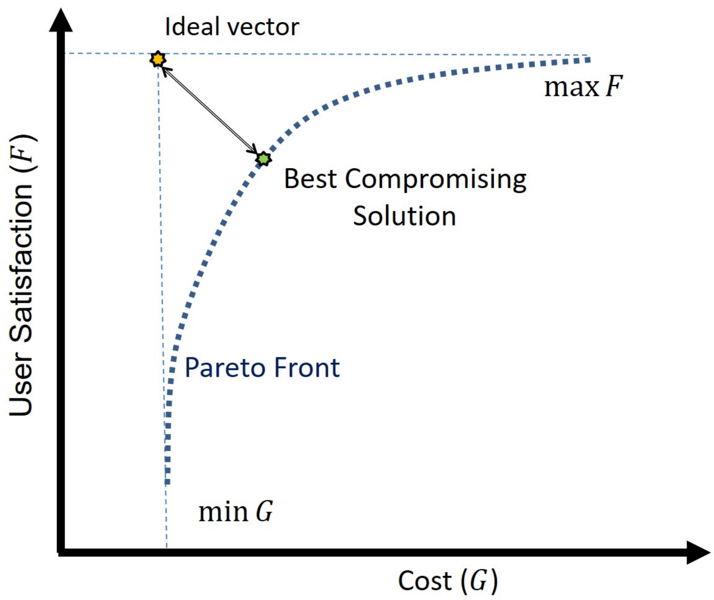


Figure 1. Best compromising solution.

Table 3. Results of the greedy heuristic.

N	π	Avg. Time (s)	$F(H^{N'}_{best})$	$G(H^{N'}_{best})$	Σ	Lf	Avg. Time (s)	$F(H^{N'}_{best})$	$G(H^{N'}_{best})$	Σ	Lf
				s.wd					*s.we*		
	0.6	0.0107	3.0527	99.0290	13.01%	0.1592	0.0051	1.3479	29.1914	27.35%	0.2067
1000	0.75	0.0107	3.2114	99.4425	11.52%	0.1592	0.0051	1.4285	30.1426	30.21%	0.2363
	0.9	0.0106	3.2052	115.4524	29.28%	0.2168	0.0051	1.4364	30.1426	30.15%	0.2363
	0.6	0.0106	3.0242	99.0290	13.52%	0.1592	0.0052	1.3290	29.1914	27.78%	0.2067
2000	0.75	0.0105	3.2032	115.7901	29.66%	0.2135	0.0051	1.4254	30.1548	30.28%	0.2363
	0.9	0.0104	3.1869	115.7901	29.71%	0.2168	0.0051	1.4452	30.8174	33.01%	0.2363
	0.6	0.0107	3.0248	99.2357	13.69%	0.1592	0.0049	1.3206	29.1914	27.99%	0.2067
3000	0.75	0.0108	3.1768	115.7901	29.75%	0.2135	0.0050	1.3887	30.1548	30.66%	0.2363
	0.9	0.0107	3.1863	127.0964	42.31%	0.1854	0.0050	1.4004	30.8174	33.34%	0.2363
	0.6	0.0106	3.0242	99.2357	13.70%	0.1592	0.0051	1.3110	29.1914	28.24%	0.2067
5000	0.75	0.0107	3.1757	115.7901	29.75%	0.2135	0.0052	1.3976	30.8174	33.37%	0.2363
	0.9	0.0105	3.1863	127.0964	42.31%	0.1854	0.0050	1.4004	38.7719	67.45%	0.2363
	0.6	0.0107	3.0248	99.4425	13.88%	0.1592	0.0052	1.3198	30.1548	31.90%	0.2165
10000	0.75	0.0106	3.1046	115.7901	30.08%	0.2135	0.0051	1.3812	30.8174	33.57%	0.2363
	0.9	0.0106	3.1774	127.0964	42.33%	0.1854	0.0052	1.4004	38.7719	67.45%	0.2363
				l.wd					*l.we*		
	0.6	0.0182	5.9512	161.8788	24.45%	0.1919	0.0226	7.1588	224.5650	18.26%	0.3413
1000	0.75	0.0182	6.0822	181.0020	38.17%	0.2403	0.0227	7.4886	243.7498	24.73%	0.4593
	0.9	0.0183	6.2173	192.9641	46.98%	0.2403	0.0227	7.7724	261.7975	32.83%	0.4784
	0.6	0.0182	5.9512	162.0856	24.60%	0.1919	0.0226	7.1588	226.5650	19.03%	0.3413
2000	0.75	0.0185	6.0525	192.6460	47.01%	0.2403	0.0224	7.4688	241.7776	23.87%	0.4593
	0.9	0.0183	6.2166	192.9641	46.98%	0.2403	0.0227	7.7741	261.7975	32.83%	0.4593
	0.6	0.0185	5.9101	162.2924	24.96%	0.1919	0.0225	7.1386	226.7717	19.27%	0.3413
3000	0.75	0.0184	6.0494	192.6460	47.02%	0.2403	0.0225	7.4384	253.1568	29.45%	0.4593
	0.9	0.0183	6.2044	199.0992	51.65%	0.2403	0.0228	7.7340	273.1469	38.59%	0.3977
	0.6	0.0183	5.9512	176.0916	34.88%	0.2545	0.0227	7.1588	226.7717	19.11%	0.3413
5000	0.75	0.0183	6.0484	192.6460	47.02%	0.2403	0.0227	7.3845	253.1568	29.65%	0.4593
	0.9	0.0183	6.2228	204.2703	55.56%	0.2158	0.0225	7.7340	273.4415	38.73%	0.3977
	0.6	0.0186	5.9101	178.4332	36.76%	0.3422	0.0227	7.1298	243.1194	26.21%	0.4529
10000	0.75	0.0184	6.0392	192.6460	47.04%	0.2403	0.0225	7.3742	253.1568	29.69%	0.4593
	0.9	0.0184	6.1931	204.2703	55.59%	0.2158	0.0226	7.7340	273.4415	38.73%	0.3977
				b.wd					*b.we*		
	0.6	0.0308	11.3678	417.6331	67.39%	0.2377	0.0345	11.8833	481.3362	111.62%	0.3905
1000	0.75	0.0311	11.8052	450.8393	79.48%	0.2832	0.0345	12.3141	497.0862	113.88%	0.4711
	0.9	0.0309	12.1265	502.4882	99.30%	0.2832	0.0346	13.0252	561.2765	124.66%	0.3823
	0.6	0.0310	11.4068	417.4538	67.24%	0.2377	0.0345	11.8965	481.6732	111.67%	0.3905
2000	0.75	0.0309	11.6766	450.8393	79.65%	0.2817	0.0345	12.2864	498.2471	114.05%	0.4648
	0.9	0.0310	12.0744	508.8815	101.85%	0.2832	0.0343	12.9924	561.9514	124.79%	0.4208
	0.6	0.0309	11.4068	417.6606	67.32%	0.2377	0.0345	11.8256	482.9548	111.84%	0.3861
3000	0.75	0.0309	11.6062	450.8393	79.75%	0.2817	0.0345	12.2977	503.0244	114.77%	0.4711
	0.9	0.0309	12.0749	509.1201	101.94%	0.2832	0.0346	12.9632	561.9898	124.79%	0.3823
	0.6	0.0312	11.4258	419.8494	68.12%	0.2377	0.0344	11.8763	483.9182	111.98%	0.3905
5000	0.75	0.0311	11.6031	450.8393	79.76%	0.2832	0.0345	12.2460	523.8958	118.09%	0.5698
	0.9	0.0311	12.0838	514.0527	103.87%	0.2672	0.0346	12.9826	569.6496	126.23%	0.3823
	0.6	0.0310	11.4643	434.2840	73.59%	0.2781	0.0343	11.8366	483.9182	111.98%	0.3905
10000	0.75	0.0308	11.6123	476.5378	89.72%	0.2817	0.0344	12.2854	526.3093	118.49%	0.5698
	0.9	0.0310	12.0540	514.1322	103.93%	0.2672	0.0343	12.9655	569.9058	126.28%	0.3823

4. Discussion

This section discusses the results obtained in the computational experimentation, considering different aspects, including the impact of sample size and objective biased in algorithms efficiency and the quality and distribution of solutions in the Pareto front. Finally, the analysis of an illustrative case study is presented, to proper evaluate the quality of service provided to citizens.

4.1. *Impact of sample size and objective biased in algorithms efficiency*

The obtained experimental results allow concluding that the methods are robust with respect to the size of the sample, since the increment of N has a limited effect on the performance. In both objectives, the increment in N generally reduces the standard deviation of the computed values. However, the average and best value only varies slightly (Tables A1 and A2). Moreover, results of the greedy heuristic using larger sizes of N are systematically worse in terms of distance to the ideal vector than than those computed using smaller sample sizes.

The SAA problems were solved to optimality by Gurobi, being able to find solutions with 0% MIP-Gap for the compact mathematical formulation presented in Section 2.1 in relatively short computing times (less than 1 s for all instances). The analysis of execution time shows that schedules that are biased towards minimizing the cost objective (with higher values of β) are more difficult to solve for Gurobi, which requires a much larger computing time to solve the instances. In regard to the greedy heuristic, the algorithm is very fast to solve the instances, as all average computing times are less than 0.1 s. Moreover, unlike SAA, which is sensitive to the bias among objectives, the computing times of the greedy heuristic are independent of the aspiration level used since computing times do not vary.

Another relevant aspect is that when considering the computing time of the whole resolution process, i.e., the generation of the random samples of the user preferences (reported in Table 1) and solving the optimization problem (either by the greedy heuristic or the SAA), the most time consuming stage is the generation of the random samples. Additionally, the time of generating the random sample increases approximately proportional to the size of N, whereas the average time of solving the optimization problem is almost constant for any size of N.

4.2. *Quality and distribution of solutions*

Regarding solution quality, Table 4 reports the minimum, average and maximum value of the distance to the ideal vector for each instance and each algorithm.

Table 4. Minimum, average and maximum distance to the ideal vector.

Instance	SAA			Greedy heuristic		
	min	avg	max	min	avg	max
s.wd	10.94%	20.72%	33.75%	11.52%	25.63%	42.33%
s.we	15.97%	29.80%	58.62%	27.35%	35.52%	67.45%
l.wd	12.40%	25.24%	47.98%	24.45%	41.91%	55.59%
l.we	11.01%	24.45%	40.50%	18.26%	28.07%	38.73%
b.wd	17.78%	65.43%	146.19%	67.24%	84.19%	103.93%
b.we	17.05%	51.96%	109.28%	111.62%	117.68%	126.28%

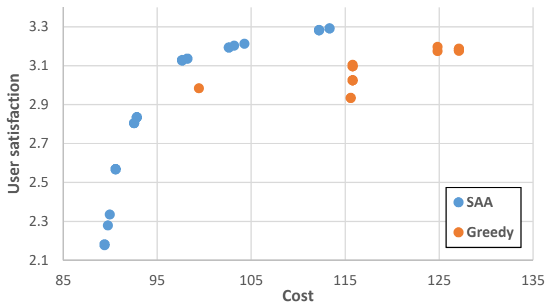
Results in Table 4 indicate that SAA is able to obtain, on average, better solutions than the greedy heuristic. SA computed the smallest average distance on instance s.wd (20.72%). The best average distance for the greedy heuristic was obtained on the same instance s.wd (25.63%). Both algorithms obtained the worst results in terms of distance to the ideal vector for the building like instances. Results show that instances in which different users have to coordinate the use of appliances to not surpass the overall power consumption contracted by the building are more difficult to solve than those instances where a single user is considered. Regarding the best compromise solution, i.e., the solution that has the smallest distance to the ideal vector, it was obtained in instance s.wd in both algorithms. The smallest distance to the ideal vector computed by SAA was 10.94%, achieved using two different weights vectors, (0.5, 0.5) and (0.75, 0.25) for all the sample sizes N. Finally, regarding the greedy heuristic, the smallest computed distance is 11.52%, computed using $\pi = 0.75$ and sample size $N = 1000$.

Table 5. Minimum, average and maximum load factor.

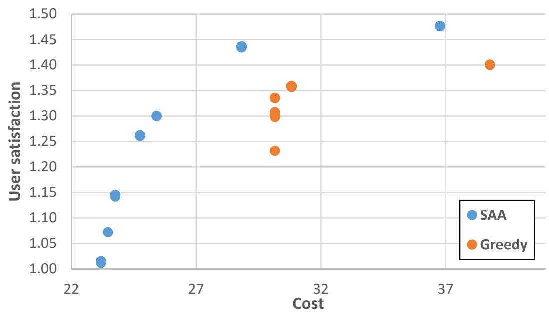
Instance	SAA			Greedy heuristic		
	min	avg	max	min	avg	max
s.wd	0.16500	0.17181	0.20680	0.15920	0.18660	0.21680
s.we	0.16850	0.21462	0.23630	0.20670	0.22709	0.23630
l.wd	0.14050	0.18641	0.24030	0.19190	0.23509	0.34220
l.we	0.24480	0.33610	0.45290	0.34130	0.41636	0.47840
b.wd	0.18820	0.20232	0.21570	0.23770	0.26829	0.28320
b.we	0.27740	0.31824	0.38770	0.38230	0.42965	0.56980

Regarding the load factor, Table 5 indicates that the greedy heuristic is able to obtain better results in all the instances either considering the average or the maximum load factor. Although maximizing the load factor was not part of the optimization problem, it is a relevant characteristic of the greedy heuristic since, as aforementioned, higher load factors are associated with a more stable functioning of the electric grid.

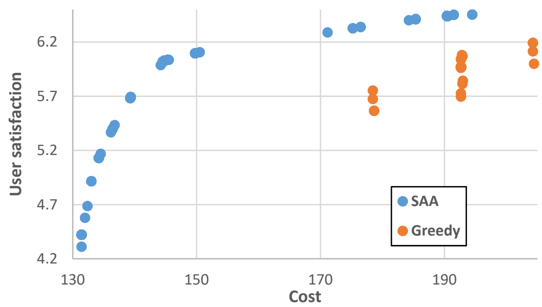
For better depicting the distribution and trade-off between the objective function values of the computed solutions, the Pareto fronts of the experiments with the larger sample size are presented in Figure 2. For the SAA, a total of 100 solutions are computed. These solutions were calculated using evenly separated weight vectors (α,β) with $\alpha + \beta = 1$. Regarding the greedy heuristic, 35 solutions with evenly separated aspiration level π with $\pi \in [0.6, 0.95]$ were computed. The SAA is able to better explore the search space, whereas the greedy heuristic finds, in general, solutions that have relatively large costs. From the analysis of the figures, it can be inferred that several runs of the SAA obtain similar solution (or even the same solution). The possibility of obtaining repeated solutions, i.e., obtaining the same solution for two different weight vectors, is a known disadvantage of using the weighting sum method for handling the multiobjective nature of an optimization problem [11]. To overcome this problem, more sophisticated multiobjective approaches, such as the augmented ε-constraint method, can be used. In regard to Pareto dominance, the solutions of the SAA usually dominates the solutions of the greedy heuristic.



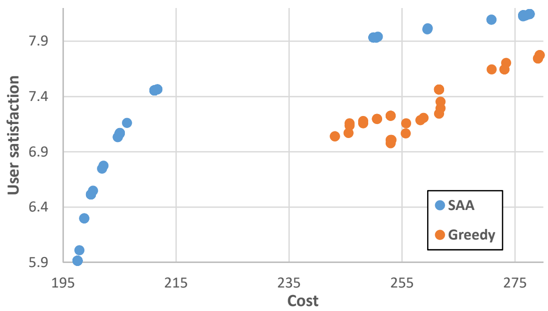
(a) Solutions for small instance weekday.



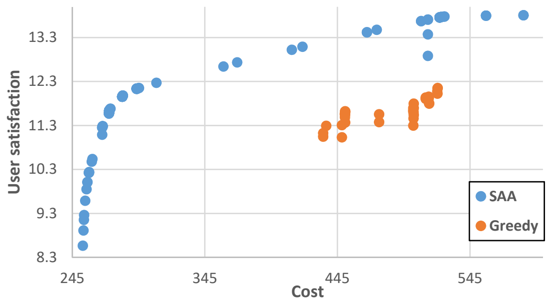
(b) Solutions for small instance weekend.



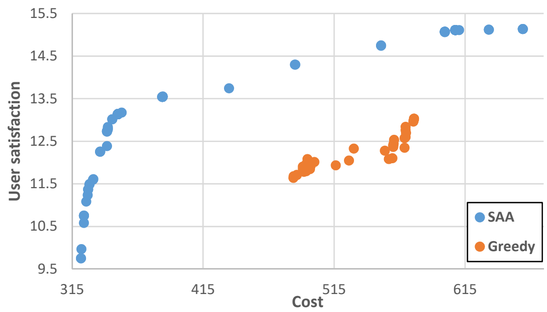
(c) Solutions for large instance weekday.



(d) Solutions for large instance weekend.



(e) Solutions for building instance weekday.



(f) Solutions for building instance weekend.

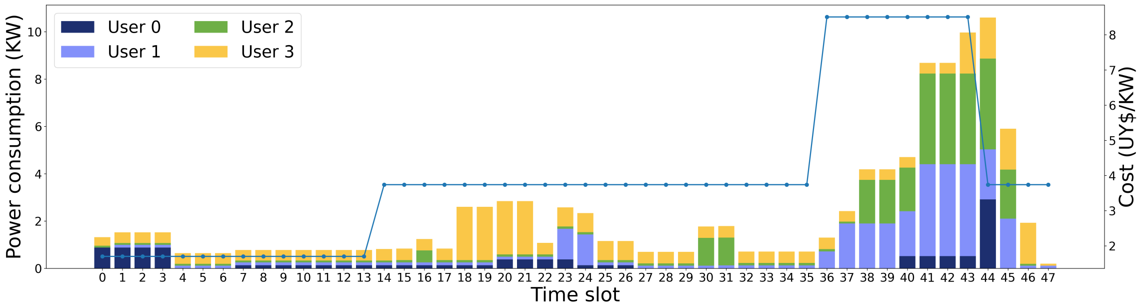
Figure 2. Solutions for instances with sample size $N = 10000$.

4.3. Power consumption analysis for an illustrative case study

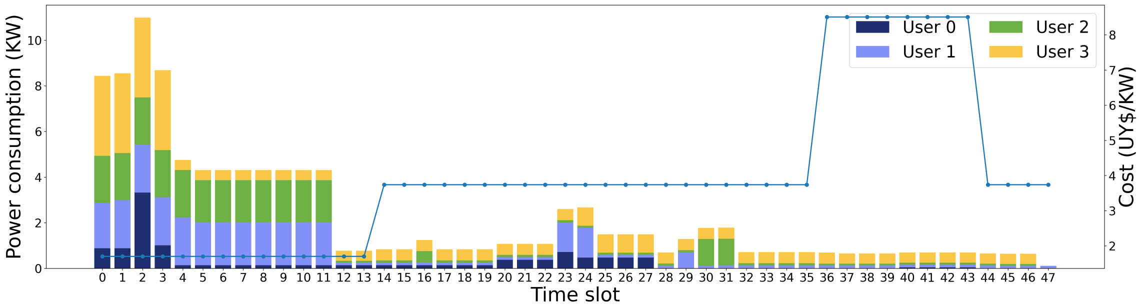
This subsection presents an illustrative case study for one of the solved problem instances, to provide an insight on the scheduled power consumption in each time slot of the planning period, computed by the two studied methods. Figures 3 and 4 report the power consumption (in KW) in each time slot of representative solutions computed by the proposed approaches of the building-like scenario discriminated per user on weekday (b.wd) and weekend (b.we), respectively. Each time slot represents an interval of thirty minutes and they are numbered subsequently (e.g., time slot 0 represents the first thirty minutes of the day, and so on). Additionally, the cost of electricity foe each time slot is plotted as a line in the Figures (expressed in Uruguayan pesos per KW). Three solutions of the SAA and one solution of the greedy heuristic are presented. Selected solutions for the SAA correspond to the two extreme solutions and a balanced solution: one solution biased towards users satisfaction using vector (0.99, 0.01), a second solution biased towards cost reduction using vector (0.01, 0.99), and the third solution equally weighting the problem objectives, using vector (0.5, 0.5). The selected solution of the greedy heuristic is the one with an aspiration level of 0.75. These solutions are representative of different optimization results for both studied methods and provides diverse trade-offs between the problem objectives.

The analysis of Figure 3 allows concluding that users have a preference for using electric appliances at the end of the day. Thus, the solution that prioritizes user satisfaction has a large power consumption during the evening and night (Figure 3(a)). This is a common habit when users return to their homes after work at the end of the day, and they perform the majority of the activities in these hours. However, this part of the day corresponds to the peak hours, in which electricity price is more expensive and, thus, the solution presented in Figure 3(a) is rather expensive. Conversely, solutions that have a smaller total cost are biased towards using the appliances at the beginning of the day (as presented in Figure 3(c)). As expected, the solution presented in Figure 3(c), which was computed using a more balanced weight vector, defers the use of some appliances to the middle hours of the day. However, the cost objective has a greater influence than the user preferences, since a large part of the consumption is still allocated at early hours, where the electricity price is lower. Regarding the solution computed by the greedy heuristic (presented in Figure 3(d)), the energy consumption patter is rather similar to the one proposed by the solution of the SAA using a large weight for the user satisfaction objective (Figure 3(a)). However, the utilization of appliances is more distributed throughout the day. As a consequence, the peak consumption, i.e., the time slot with the highest consumption, is smaller for the greedy solution (8 KW) than for the SAA solution (10 KW).

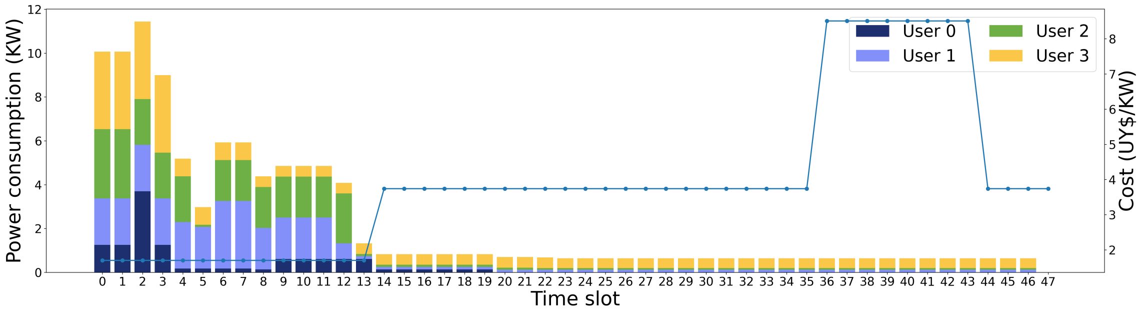
In any problem considering the scheduling or planning of human-related activities, the normal lifestyle and the timeline of daily actions limit the possibility of displacing the considered activities to some convenient, but dead periods. This is also the case for the studied problem, since deferring the use of electric appliances from peak hours to off-peak hours with lower electricity prices is not always possible, since several off-peak hours usually coincide with the time the users are resting at night. Another element that prevents users from taking advantage of lower electricity prices of off-peak hours are the normal working timetables, since usually during the morning and the noon the user is out of home, at work. However, a different scenario happens during weekends, when users remain more time at home and, therefore, they can perform some household tasks during off-peak hours.



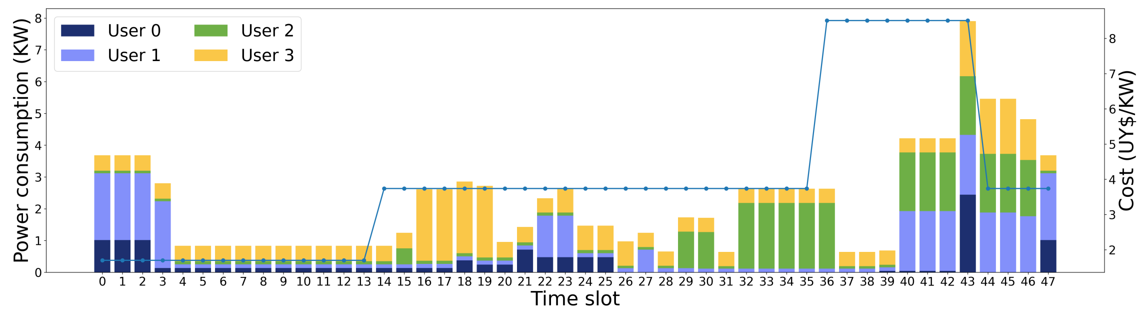
(a) SAA solution using weight vector (0.99, 0.01).



(b) SAA solution using weight vector (0.5, 0.5).

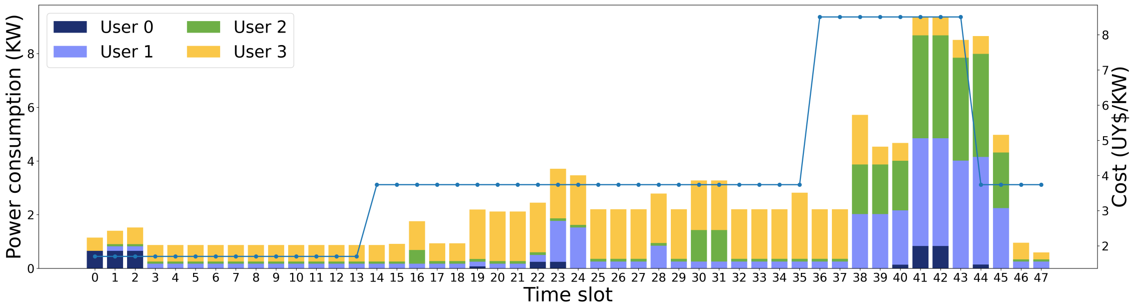


(c) SAA solution using weight vector (0.01, 0.99).

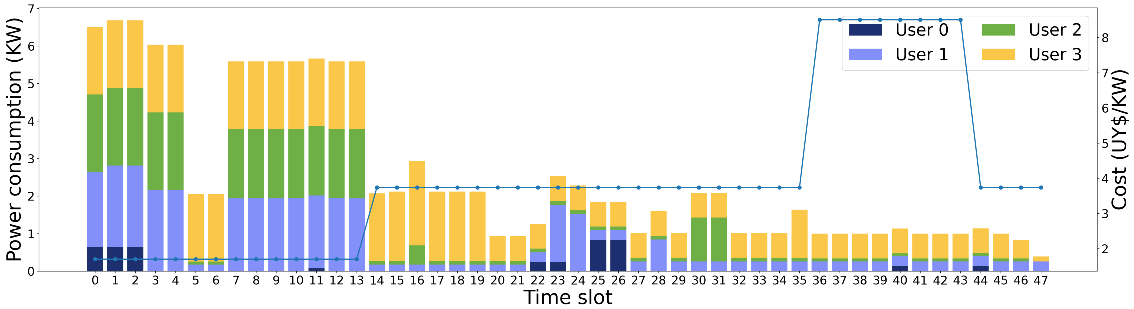


(d) Greedy solution with aspiration level 0.75.

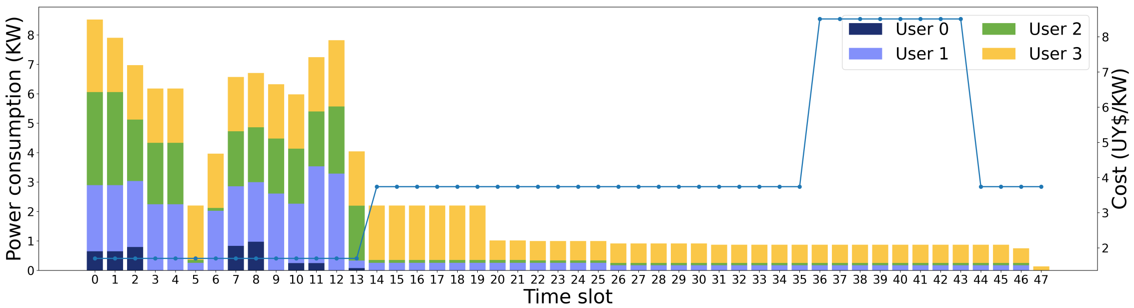
Figure 3. Power consumption per time slot for representative solutions of the b.wd instance and sample size $N = 10000$.



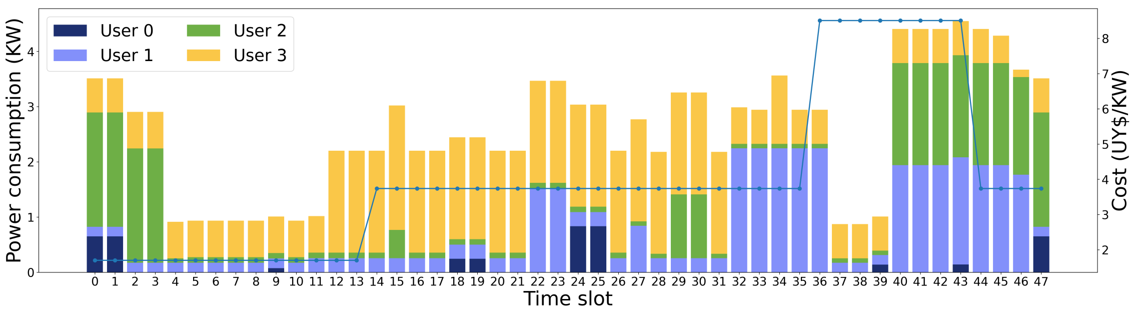
(a) SAA solution using weight vector (0.99,0.01).



(b) SAA solution using weight vector (0.5,0.5).



(c) SAA solution using weight vector (0.01,0.99).



(d) Greedy solution with aspiration level 0.75.

Figure 4. Power consumption per time slot for representative solutions of the b.we instance and sample size $N = 10000$.

Regarding the computed solutions, the different situation that happens during weekends is depicted when performing pairwise comparisons between the four representative solutions of the weekend scenario (Figure 4) and the corresponding solutions of the weekday scenario (Figure 3). In the four cases, the solutions of the weekend scenario have a more distributed power consumption throughout the day, having a larger consumption in the middle hours of the day and a smaller peak consumption. The reduction of the peak consumption is particularly important for the greedy solution, i.e. from 8KW in the weekday solution (Figure 3(d)) to 4KW in the weekend solution (Figure 4(d)). Since the ToU pricing bill applied by the electricity company is the same for weekdays and weekends, the reason of the differences among weekdays and weekends solutions relies on the differences in users preferences. Although users still prefer to use appliances at the evening (as is evidenced in the solution that prioritizes user satisfaction of Figure 4(a)), users are also more willing to use the appliances in the middle of the day, allowing the resolution algorithms to better distribute the power consumption.

As aforementioned, a more distributed power consumption throughout the day as occurs on the weekends results, benefits both users and electric companies. On the one hand, users are able to take advantage of the relatively cheaper off-peak hours. On the other hand, the reduction in the peak consumption, when considered in the city aggregated level, allows reducing the required infrastructure investment that electric companies have to perform to handle peak consumption and also allows significantly reducing the risk of power outages. In line with these benefits for the system, the recent rise of home office that has occurred due to the COVID-19 pandemic is as a great opportunity to balance the energy utilization by households, since users remain more time at home. However, to better analyze this possibility, new datasets should be gathered to incorporate the changes on the lifestyle of users of the pandemic. In line with this goal, the project 'Computational intelligence for the analysis of residential electricity consumption' is carried out in Uruguay, to gather relevant data from residential customers. The most relevant result of this project has been the generated ECD-UY dataset [45].

5. Conclusions

Energy management is a crucial issue in modern societies, since an increasingly number of urban activities rely on an efficient electricity service. In order to improve energy management, it is not only required to improve the offer of electricity supply by companies, but also to enhance the demand-side of the system.

This article addressed the household energy planning problem, aiming at improving the efficiency of the consumed energy. For achieving this goal, an optimization model was proposed for scheduling deferrable appliances considering two conflicting objectives: reducing the total cost of electricity paid by households (in a context of ToU pricing in electricity bills) and enhancing the users satisfaction with the energy consumed. To account for a realistic model, able to be implemented in practice, the restriction of the maximum allowable power consumption contracted by the user (to the electric company) was incorporated.

The users satisfaction was estimated through a data-analysis model, studying historical data of households in order to determine the preferred time slots for using each appliance. Since considerable variations of these preferences were identified for different users, a stochastic resolution approach was applied to consider the uncertainty of this parameter.

For solving the problem, two different algorithms were devised: a Sample Average Approximation method, which is a simulation-optimization approach that combines Monte Carlo simulation and deterministic mixed integer programming, and a greedy heuristic, which attempts at obtaining good global solutions by making locally optimal decisions repeatedly. The algorithms were tested on realistic instances. The instances comprehend scenarios with a single household, several households and building-like scenarios (in which diverse households or users has to coordinate the usage of appliances so the overall power consumption of the building does not surpass a certain joint threshold value). The results of the computational experimentation show the competitiveness of the proposed approach which are able to compute different compromising solutions accounting for the trade-off between these two conflicting optimization criteria in reasonable computing times The Sample Average Approximation method systematically outperformed the solutions obtained by the the greedy heuristic. However, the heuristic is much faster. The building-like instances were the more challenging for both algorithms requiring larger computing times. At least, for the analyzed cases, the size of the sample of the user preferences seems to not affect largely the performance of the algorithm. The results also allowed analyzing the different users behaviour between the weekdays and the weekend, finding that during weekends the appliance usage is more distributed throughout the day.

The main lines for future work are related to expand the computational experimentation of the proposed model and algorithms, by including more households, e.g., instances that represent an apartment building or a gated community. In turn, the proposed model can be expanded by considering non-controllable appliances and renewable power generators within the household, e.g., solar or wind power generators. In relation to the input data, it would be useful to gather updated information in order to analyze if the variations in the lifestyle of users due to the pandemic and home office have substantially alter the user preferences, and compute accurate planning for this new situation too. Regarding the resolution algorithms, two future lines of work are will be consider. On the one hand, SAA can be improved by replacing the bi-objective approach based on weighting sum with a more advanced exact multiobjective method (e.g., augmented ε-constraint method) to avoid obtaining repeated solutions. On the other hand, population-based explicit multiobjective optimization methods, such as multiobjective evolutionary algorithms, can be implemented to better explore the trade-off among objectives. Regarding preserving users' privacy, the proposed model can be extended by including appliance shifting and scheduling to control battery charging and discharging. Finally, an interesting research line to explore in the future is the comparison with other stochastic and/or robust resolution approaches.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

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Appendix

Computational experimentation details

This Section presents the detailed experimental results of the SAA and the Greedy heuristic. The details of the SAA are presented in Table A1. This table reports for each instance, the sample size N, the combination of weights (α, β), and the average and standard deviation of five relevant metrics:

- the execution time;
- the users satisfaction function F evaluated over N';
- the cost function G evaluated over N';
- the values of F and G of the best solution, i.e., the solution that has the minimal value of function H, as defined in Eq (11);
- the deviation of the solution to the ideal vector Σ, computed using the L^2 distance norm, according to Eq (15).

In turn, the detailed experimental results of the the greedy heuristic are reported in Table A2. This table presents for each instance, the sample size N, the aspiration preference level π, and the average and standard deviation of the five metrics also reported for the SAA. The computing times in Tables A1 and A2 do not include the time to generate the N random realizations of vector **UP**.

Table A1. Detailed results of the SAA.

N	(α,β)	Time (s)		$F^{N'}$		$G^{N'}$		$F(H^{N'}_{best})$	$G(H^{N'}_{best})$	Σ
		Avg	Std	Avg	Std	Avg	Std			
Small instance weekday (s.wd)										
	(0.99,0.01)	0.0138	0.0016	3.2803	0.0139	113.3180	1.8047	3.2925	113.3289	26.78%
	(0.01,0.99)	0.1625	0.0108	2.1790	0.0012	89.3876	0.0000	2.1814	89.3876	33.75%
1000	(0.50,0.50)	0.0911	0.0167	3.1311	0.0108	98.2122	0.1354	3.1368	98.2011	10.94%
	(0.75,0.25)	0.0453	0.0077	3.1455	0.1290	100.7468	2.6877	3.1368	98.2011	10.94%
	(0.25,0.75)	0.1436	0.0184	2.5557	0.0404	90.8810	0.3302	2.5555	90.8443	22.44%
	(0.99,0.01)	0.0137	0.0017	3.2870	0.0068	113.0862	0.5205	3.2925	113.3289	26.78%
	(0.01,0.99)	0.1619	0.0134	2.1789	0.0012	89.3876	0.0000	2.1813	89.3876	33.75%
2000	(0.50,0.50)	0.1051	0.0168	3.1336	0.0079	98.1928	0.0828	3.1368	98.2011	10.94%
	(0.75,0.25)	0.0486	0.0069	3.1482	0.0247	99.3211	1.8497	3.1368	98.2011	10.94%
	(0.25,0.75)	0.1421	0.0323	2.5525	0.0087	90.8388	0.0388	2.5558	90.8443	22.43%
	(0.99,0.01)	0.0136	0.0015	3.2892	0.0051	113.1910	0.4158	3.2925	113.3289	26.78%
	(0.01,0.99)	0.1624	0.0124	2.1789	0.0013	89.3876	0.0000	2.1819	89.3876	33.73%
3000	(0.50,0.50)	0.0963	0.0131	3.1347	0.0058	98.1956	0.0552	3.1368	98.2011	10.94%
	(0.75,0.25)	0.0497	0.0055	3.1371	0.1149	99.5480	1.9951	3.1368	98.2011	10.94%
	(0.25,0.75)	0.1351	0.0119	2.5538	0.0021	90.8443	0.0000	2.5558	90.8443	22.43%
	(0.99,0.01)	0.0133	0.0012	3.2916	0.0027	113.2958	0.2010	3.2925	113.3289	26.78%
	(0.01,0.99)	0.1607	0.0090	2.1791	0.0012	89.3876	0.0000	2.1815	89.3876	33.74%
5000	(0.50,0.50)	0.1038	0.0117	3.1301	0.0626	98.2011	0.0000	3.1368	98.2011	10.94%
	(0.75,0.25)	0.0499	0.0051	3.1378	0.0737	98.9091	1.5854	3.1368	98.2011	10.94%
	(0.25,0.75)	0.1359	0.0113	2.5544	0.0010	90.8443	0.0000	2.5558	90.8443	22.43%
	(0.99,0.01)	0.0131	0.0012	3.2920	0.0014	113.3179	0.1103	3.2925	113.3289	26.78%
	(0.01,0.99)	0.1616	0.0117	2.1790	0.0012	89.3876	0.0000	2.1814	89.3876	33.75%
10000	(0.50,0.50)	0.1030	0.0095	3.1293	0.0706	98.2011	0.0000	3.1368	98.2011	10.94%
	(0.75,0.25)	0.0497	0.0043	3.1421	0.0201	98.7403	1.3909	3.1368	98.2011	10.94%
	(0.25,0.75)	0.1340	0.0122	2.5544	0.0006	90.8443	0.0000	2.5558	90.8443	22.43%
Small instance weekend (s.we)										
	(0.99,0.01)	0.0081	0.0003	1.4628	0.0154	37.1724	2.1527	1.4763	36.7695	58.62%
	(0.01,0.99)	0.0452	0.0005	1.0122	0.0024	23.1810	0.0000	1.0148	23.1810	31.26%
1000	(0.50,0.50)	0.0122	0.0005	1.4190	0.0621	28.8515	0.0989	1.4363	28.8151	24.46%
	(0.75,0.25)	0.0099	0.0005	1.4341	0.0208	30.4370	3.1578	1.4363	28.8151	24.46%
	(0.25,0.75)	0.0189	0.0018	1.2663	0.0196	24.9394	0.3013	1.2623	24.7508	16.00%
	(0.99,0.01)	0.0085	0.0005	1.4715	0.0068	36.9087	0.1834	1.4763	36.7695	58.62%
	(0.01,0.99)	0.0475	0.0018	1.0126	0.0014	23.1810	0.0000	1.0149	23.1810	31.25%
2000	(0.50,0.50)	0.0149	0.0008	1.4238	0.0256	28.6683	0.60120	1.4363	28.8151	24.46%
	(0.75,0.25)	0.0118	0.0006	1.4309	0.0071	28.8746	0.0970	1.4363	28.8151	24.46%
	(0.25,0.75)	0.0290	0.0026	1.2576	0.0120	24.7408	0.0998	1.2627	24.7508	15.97%
	(0.99,0.01)	0.0084	0.0003	1.4740	0.0029	36.8783	0.1410	1.4763	36.7695	58.62%
	(0.01,0.99)	0.0476	0.0017	1.0125	0.0010	23.1810	0.0000	1.0148	23.1810	31.26%
3000	(0.50,0.50)	0.0148	0.0010	1.4333	0.0041	28.8258	0.0395	1.4363	28.8151	24.46%
	(0.75,0.25)	0.0117	0.0005	1.4335	0.0040	28.8658	0.0727	1.4363	28.8151	24.46%
	(0.25,0.75)	0.0289	0.0027	1.2590	0.0023	24.7508	0.0000	1.2627	24.7508	15.97%
	(0.99,0.01)	0.0088	0.0006	1.4743	0.0011	36.8685	0.1362	1.4763	36.7695	58.62%
	(0.01,0.99)	0.0475	0.0018	1.0126	0.0010	23.1810	0.0000	1.0151	23.1810	31.24%
5000	(0.50,0.50)	0.0146	0.0006	1.4342	0.0012	28.8243	0.0367	1.4363	28.8151	24.46%
	(0.75,0.25)	0.0115	0.0004	1.4342	0.0012	28.8612	0.0709	1.4363	28.8151	24.46%
	(0.25,0.75)	0.0288	0.0027	1.2596	0.0012	24.7508	0.0000	1.2622	24.7508	16.01%
	(0.99,0.01)	0.0088	0.0005	1.4740	0.0014	36.8696	0.1290	1.4763	36.7695	58.62%
	(0.01,0.99)	0.0477	0.0022	1.0124	0.0010	23.1810	0.0000	1.0150	23.1810	31.25%
10000	(0.50,0.50)	0.0146	0.0007	1.4340	0.0014	28.8151	0.0000	1.4363	28.8151	24.46%
	(0.75,0.25)	0.0116	0.0004	1.4339	0.0014	28.8335	0.0503	1.4363	28.8151	24.46%
	(0.25,0.75)	0.0282	0.0031	1.2594	0.0014	24.7508	0.0000	1.2622	24.7508	16.01%
Large instance weekday (l.wd)										
	(0.99,0.01)	0.0226	0.0014	6.4349	0.0196	202.8882	12.5886	6.4528	194.4567	47.98%
	(0.01,0.99)	0.2624	0.0178	4.4163	0.0047	131.4108	0.0000	4.4226	131.4108	31.46%
1000	(0.50,0.50)	0.0769	0.0067	6.0279	0.0123	145.5251	0.4130	6.0373	145.3292	12.40%
	(0.75,0.25)	0.0485	0.0054	6.3453	0.0706	179.6205	8.1416	6.4120	185.3487	41.05%
	(0.25,0.75)	0.1768	0.0169	5.7133	0.0674	140.2097	1.0259	5.7316	140.2288	13.04%
	(0.99,0.01)	0.0232	0.0014	6.4460	0.0071	198.2309	10.5549	6.4528	194.4567	47.98%
	(0.01,0.99)	0.2596	0.0184	4.4182	0.0040	131.4108	0.0000	4.4229	131.4108	31.46%
2000	(0.50,0.50)	0.0920	0.0093	6.0262	0.0351	145.2319	0.1326	6.0373	145.3292	12.40%

	(0.75,0.25)	0.0631	0.0059	6.1017	0.0300	150.7660	2.8692	6.1053	150.3087	15.36%
	(0.25,0.75)	0.2310	0.0666	5.3985	0.1048	136.9402	0.7238	5.3944	136.7499	16.90%
	(0.99,0.01)	0.0232	0.0016	6.4477	0.0054	198.3285	10.7203	6.4528	194.4567	47.98%
	(0.01,0.99)	0.2582	0.0164	4.4193	0.0030	131.4108	0.0000	4.4230	131.4108	31.46%
3000	(0.50,0.50)	0.0876	0.0083	6.0310	0.0064	145.2613	0.1481	6.0373	145.3292	12.40%
	(0.75,0.25)	0.0632	0.0057	6.1123	0.0357	151.8834	3.8517	6.1049	150.3087	15.36%
	(0.25,0.75)	0.2334	0.0648	5.4604	0.1090	137.4749	1.1065	5.3944	136.7499	16.90%
	(0.99,0.01)	0.0230	0.0013	6.4498	0.0037	198.1247	10.1350	6.4528	194.4567	47.98%
	(0.01,0.99)	0.2591	0.0181	4.4200	0.0021	131.4108	0.0000	4.4230	131.4108	31.46%
5000	(0.50,0.50)	0.0781	0.0052	6.0342	0.0037	145.3265	0.0936	6.0373	145.3292	12.40%
	(0.75,0.25)	0.0578	0.0053	6.3499	0.0715	178.4627	8.2341	6.3393	176.3038	34.21%
	(0.25,0.75)	0.1886	0.0178	5.7109	0.0757	140.1015	0.2313	5.7325	140.2288	13.02%
	(0.99,0.01)	0.0224	0.0010	6.4504	0.0019	196.0937	8.7299	6.4528	194.4567	47.98%
	(0.01,0.99)	0.2592	0.0135	4.4204	0.0011	131.4108	0.0000	4.4224	131.4108	31.47%
10000	(0.50,0.50)	0.0846	0.0067	6.0333	0.0043	145.2682	0.1123	6.0373	145.3292	12.40%
	(0.75,0.25)	0.0630	0.0038	6.1074	0.0125	150.8103	1.2606	6.1053	150.3087	15.36%
	(0.25,0.75)	0.2232	0.0136	5.6141	0.1047	139.0050	1.0691	5.6829	139.6771	13.49%
Large instance weekend (l.we)										
	(0.99,0.01)	0.0216	0.0015	8.1262	0.0179	279.8359	4.5634	8.1457	277.5776	40.50%
	(0.01,0.99)	0.2226	0.0117	5.9123	0.0060	197.5689	0.0000	5.9167	197.5689	27.37%
1000	(0.50,0.50)	0.1225	0.0258	7.4217	0.1720	211.8408	0.2304	7.4650	211.7288	11.01%
	(0.75,0.25)	0.0746	0.0069	7.9338	0.1105	252.6845	7.3916	7.9526	251.6011	27.45%
	(0.25,0.75)	0.4123	0.0161	7.0213	0.0979	205.1840	0.60202	7.0347	205.0788	14.16%
	(0.99,0.01)	0.0211	0.0010	8.1375	0.0075	278.7941	1.5364	8.1457	277.5776	40.50%
	(0.01,0.99)	0.2229	0.0098	5.9143	0.0025	197.5689	0.0000	5.9174	197.5689	27.36%
2000	(0.50,0.50)	0.1166	0.0072	7.4556	0.0706	211.8562	0.3389	7.4650	211.7288	11.01%
	(0.75,0.25)	0.0718	0.0068	7.9747	0.0401	255.2160	4.7386	7.9525	251.6011	27.45%
	(0.25,0.75)	0.4118	0.0219	7.0625	0.0377	205.4796	0.4686	7.0815	205.6305	13.69%
3000	(0.99,0.01)	0.0206	0.0011	8.1408	0.0051	279.2213	1.4356	8.1457	277.5776	40.50%
	(0.01,0.99)	0.2226	0.0151	5.9147	0.0010	197.5689	0.0000	5.9168	197.5689	27.36%
	(0.50,0.50)	0.1181	0.0074	7.4424	0.1168	211.7536	0.0945	7.4650	211.7288	11.01%
3000	(0.75,0.25)	0.0769	0.0066	7.9613	0.0346	253.3056	4.1294	7.9423	250.4978	26.91%
	(0.25,0.75)	0.4140	0.0173	7.0567	0.0242	205.3864	0.2758	7.0349	205.0788	14.16%
	(0.99,0.01)	0.0207	0.0010	8.1421	0.0036	279.0386	1.4598	8.1458	277.5776	40.50%
	(0.01,0.99)	0.2265	0.0097	5.9146	0.0010	197.5689	0.0000	5.9175	197.5689	27.36%
5000	(0.50,0.50)	0.1180	0.0055	7.4597	0.0335	211.7412	0.0546	7.4650	211.7288	11.01%
	(0.75,0.25)	0.0779	0.0064	7.9604	0.0312	253.0517	3.7191	7.9526	251.6011	27.45%
	(0.25,0.75)	0.4271	0.0202	7.0579	0.0721	205.4672	0.2488	7.0815	205.6305	13.69%
	(0.99,0.01)	0.0203	0.0009	8.1433	0.0018	278.9856	1.3821	8.1458	277.5776	40.50%
	(0.01,0.99)	0.2236	0.0095	5.9147	0.0011	197.5689	0.0000	5.9171	197.5689	27.36%
10000	(0.50,0.50)	0.1155	0.0058	7.4602	0.0333	211.7288	0.0000	7.4650	211.7288	11.01%
	(0.75,0.25)	0.0783	0.0051	7.9709	0.0357	254.1777	4.3063	7.9526	251.6011	27.45%
	(0.25,0.75)	0.4185	0.0211	7.0515	0.0992	205.4698	0.2461	7.0817	205.6305	13.69%
Building instance weekday (b.we)										
	(0.99,0.01)	0.0661	0.0040	13.7769	0.0218	599.3116	21.1016	13.8077	606.4009	139.60%
	(0.01,0.99)	0.4757	0.0277	8.5596	0.0065	253.0874	0.0000	8.5678	253.0874	37.98%
1000	(0.50,0.50)	0.2532	0.0481	11.7499	0.0317	279.7284	2.0997	11.7540	278.1533	17.91%
	(0.75,0.25)	0.1234	0.0134	13.5128	0.1569	500.0129	26.3286	13.4735	483.9198	91.24%
	(0.25,0.75)	0.3049	0.0429	11.5921	0.0194	273.2586	0.2954	11.6152	273.2761	17.81%
	(0.99,0.01)	0.0682	0.0030	13.7903	0.0135	603.8349	20.2691	13.8106	597.8174	136.21%
	(0.01,0.99)	0.4740	0.0220	8.5617	0.0055	253.0874	0.0000	8.5677	253.0874	37.98%
2000	(0.50,0.50)	0.2471	0.0280	11.7560	0.0282	279.5247	2.0091	11.7545	278.1533	17.90%
	(0.75,0.25)	0.1327	0.0147	13.4178	0.2435	484.8300	23.8387	13.4746	483.5656	91.10%
	(0.25,0.75)	0.3104	0.0328	11.5977	0.0160	273.1843	0.3000	11.5933	272.8259	17.87%
	(0.99,0.01)	0.0682	0.0024	13.7980	0.0116	606.0593	19.6059	13.8148	623.0884	146.19%
	(0.01,0.99)	0.4765	0.0183	8.5616	0.0051	253.0874	0.0000	8.5678	253.0874	37.98%
3000	(0.50,0.50)	0.2564	0.0314	11.7610	0.0281	279.5719	1.9144	11.7642	278.7050	17.97%
	(0.75,0.25)	0.1280	0.0108	13.6629	0.1275	522.8374	24.9542	13.7706	541.3700	113.91%
	(0.25,0.75)	0.3070	0.0412	11.6026	0.0145	273.2208	0.2718	11.6260	273.5103	17.78%
	(0.99,0.01)	0.0686	0.0028	13.8027	0.0095	608.3604	17.3050	13.8122	597.9184	136.25%
	(0.01,0.99)	0.4751	0.0223	8.5625	0.0046	253.0874	0.0000	8.5677	253.0874	37.98%
5000	(0.50,0.50)	0.2510	0.0281	11.7857	0.0352	281.0278	2.5287	11.7639	278.7050	17.97%
	(0.75,0.25)	0.1248	0.0093	13.7560	0.0360	540.0094	6.6175	13.7818	543.5414	114.76%
	(0.25,0.75)	0.3250	0.0286	11.6083	0.0148	273.2866	0.2900	11.6264	273.5103	17.78%
	(0.99,0.01)	0.0692	0.0021	13.8061	0.0047	605.8480	14.9975	13.8113	597.9185	136.25%
	(0.01,0.99)	0.4754	0.0222	8.5646	0.0033	253.0874	0.0000	8.5684	253.0874	37.98%
10000										

	(0.50,0.50)	0.2528	0.0229	11.7888	0.0351	280.8773	2.5132	11.7652	278.7050	17.96%
	(0.75,0.25)	0.1278	0.0089	13.7618	0.0253	540.4495	3.6847	13.7709	541.2802	113.87%
	(0.25,0.75)	0.3183	0.0326	11.6065	0.0140	273.1783	0.2917	11.6256	273.5103	17.78%
Building instance weekend (b.wd)										
1000	(0.99,0.01)	0.0588	0.0044	15.1073	0.0162	668.9267	20.4477	15.1327	673.2478	109.23%
	(0.01,0.99)	0.4897	0.0424	9.7430	0.0068	321.7733	0.0000	9.7527	321.7733	35.56%
	(0.50,0.50)	0.3244	0.0300	12.9151	0.0170	351.0687	0.90856	12.9297	350.60130	17.10%
	(0.75,0.25)	0.1389	0.0159	14.9392	0.1861	598.2614	32.5772	15.0719	617.7601	91.99%
	(0.25,0.75)	0.3772	0.0285	12.6216	0.1420	342.1330	0.5688	12.6639	342.2254	17.51%
2000	(0.99,0.01)	0.0585	0.0032	15.1182	0.0092	669.5280	16.9211	15.1319	673.3213	109.25%
	(0.01,0.99)	0.4966	0.0523	9.7451	0.0062	321.7733	0.0000	9.7521	321.7733	35.56%
	(0.50,0.50)	0.3175	0.0259	12.9235	0.0150	351.0239	0.90798	12.9298	350.3693	17.06%
	(0.75,0.25)	0.1269	0.0135	15.0654	0.0712	618.6035	12.5092	15.0740	617.9178	92.04%
	(0.25,0.75)	0.3765	0.0419	12.6483	0.0177	342.1749	0.2827	12.6648	342.2354	17.51%
3000	(0.99,0.01)	0.0591	0.0024	15.1207	0.0086	670.8186	17.6587	15.1316	672.6650	109.05%
	(0.01,0.99)	0.5026	0.0533	9.7466	0.0051	321.7733	0.0000	9.7524	321.7733	35.56%
	(0.50,0.50)	0.3136	0.0227	12.9253	0.0148	350.90454	0.90633	12.9297	350.3693	17.06%
	(0.75,0.25)	0.1285	0.0130	15.0703	0.0676	618.8199	11.9090	15.0718	617.6007	91.94%
	(0.25,0.75)	0.3773	0.0313	12.6503	0.0229	342.1598	0.2545	12.6652	342.2354	17.51%
5000	(0.99,0.01)	0.0599	0.0027	15.1249	0.0061	673.6397	15.1150	15.1321	672.5696	109.02%
	(0.01,0.99)	0.5123	0.0468	9.7481	0.0045	321.7733	0.0000	9.7533	321.7733	35.55%
	(0.50,0.50)	0.3166	0.0224	12.9256	0.0088	350.60527	0.60831	12.9315	350.3693	17.05%
	(0.75,0.25)	0.1314	0.0113	15.0658	0.1927	621.1851	3.9240	15.0722	617.6647	91.96%
	(0.25,0.75)	0.3760	0.0230	12.6550	0.0200	342.2071	0.2067	12.6666	342.2354	17.50%
10000	(0.99,0.01)	0.0597	0.0024	15.1262	0.0047	675.4433	11.6835	15.1336	673.4111	109.28%
	(0.01,0.99)	0.5109	0.0424	9.7488	0.0039	321.7733	0.0000	9.7526	321.7733	35.56%
	(0.50,0.50)	0.3121	0.0251	12.9263	0.0069	350.5282	0.4890	12.9307	350.3693	17.05%
	(0.75,0.25)	0.1272	0.0126	15.0861	0.0197	621.3305	3.9752	15.0720	617.5108	91.91%
	(0.25,0.75)	0.3636	0.0299	12.6598	0.0065	342.2288	0.0662	12.6665	342.2354	17.50%

Table A2. Detailed results of the greedy heuristic.

N	π	Time (s)		$F^{N'}$		$G^{N'}$		$F(H_{best}^{N'})$	$G(H_{best}^{N'})$	Σ
		Avg	Std	Avg	Std	Avg	Std			
Small instance weekday (s.wd)										
1000	0.60	0.0107	0.0011	2.9225	0.0677	112.1103	6.5536	3.0527	99.0290	13.01%
	0.75	0.0107	0.0012	3.0518	0.0669	116.3108	4.0727	3.2114	99.4425	11.52%
	0.90	0.0106	0.0011	3.1621	0.0295	125.7762	3.5871	3.2052	115.4524	29.28%
2000	0.60	0.0106	0.0011	2.9230	0.0527	114.3086	4.4380	3.0242	99.0290	13.52%
	0.75	0.0105	0.0010	3.0444	0.0552	116.4233	2.3194	3.2032	115.7901	29.66%
	0.90	0.0104	0.0011	3.1637	0.0182	126.8703	1.5908	3.1869	115.7901	29.71%
3000	0.60	0.0107	0.0013	2.9387	0.0514	114.5011	4.2024	3.0248	99.2357	13.69%
	0.75	0.0108	0.0012	3.0365	0.0420	115.8806	0.90045	3.1768	115.7901	29.75%
	0.90	0.0107	0.0011	3.1663	0.0112	127.0964	0.0000	3.1863	127.0964	42.31%
5000	0.60	0.0106	0.0010	2.9308	0.0392	115.2916	2.3066	3.0242	99.2357	13.70%
	0.75	0.0107	0.0010	3.0293	0.0395	115.9710	1.2727	3.1757	115.7901	29.75%
	0.90	0.0105	0.0011	3.1676	0.0105	127.0964	0.0000	3.1863	127.0964	42.31%
10000	0.60	0.0107	0.0011	2.9291	0.0401	115.4509	1.6186	3.0248	99.4425	13.88%
	0.75	0.0106	0.0012	3.0216	0.0302	115.7901	0.0000	3.1046	115.7901	30.08%
	0.90	0.0106	0.0011	3.1661	0.0101	127.0964	0.0000	3.1774	127.0964	42.33%
Small instance weekend (s.we)										
1000	0.60	0.0051	0.0009	1.2498	0.0476	30.0738	0.5013	1.3479	29.1914	27.35%
	0.75	0.0051	0.0010	1.3340	0.0364	32.4094	3.3157	1.4285	30.1426	30.21%
	0.90	0.0051	0.0009	1.3813	0.0267	37.9895	2.4309	1.4364	30.1426	30.15%
2000	0.60	0.0052	0.0008	1.2567	0.0349	30.0826	0.3731	1.3290	29.1914	27.78%
	0.75	0.0051	0.0009	1.3467	0.0324	32.7214	3.5060	1.4254	30.1548	30.28%
	0.90	0.0051	0.0010	1.3904	0.0158	38.5697	1.3731	1.4452	30.8174	33.01%
3000	0.60	0.0049	0.0009	1.2591	0.0254	30.2018	0.2190	1.3206	29.1914	27.99%
	0.75	0.0050	0.0008	1.3386	0.0241	31.7521	2.6078	1.3887	30.1548	30.606%
	0.90	0.0050	0.0010	1.3843	0.0145	38.6282	1.1224	1.4004	30.8174	33.34%
5000	0.60	0.0051	0.0009	1.2617	0.0201	30.1916	0.1976	1.3110	29.1914	28.24%
	0.75	0.0052	0.0010	1.3406	0.0183	31.2152	1.7424	1.3976	30.8174	33.37%
	0.90	0.0050	0.0009	1.3864	0.0117	38.7811	0.0367	1.4004	38.7719	67.45%

	0.60	0.0052	0.0009	1.2569	0.0173	30.1747	0.1136	1.3198	30.1548	31.90%
10000	0.75	0.0051	0.0009	1.3395	0.0137	30.8970	0.7954	1.3812	30.8174	33.57%
	0.90	0.0052	0.0010	1.3853	0.0109	38.7734	0.0154	1.4004	38.7719	67.45%
Large instance weekday (l.wd)										
	0.60	0.0182	0.0011	5.6026	0.1344	184.4582	10.1460	5.9512	161.8788	24.45%
1000	0.75	0.0182	0.0011	5.7627	0.1197	193.1504	4.0661	6.0822	181.0020	38.17%
	0.90	0.0183	0.0012	6.0024	0.1231	209.5946	6.7438	6.2173	192.9641	46.98%
	0.60	0.0182	0.0011	5.6043	0.1253	186.9391	7.6256	5.9512	162.0856	24.60%
2000	0.75	0.0185	0.0010	5.7550	0.1187	192.9317	1.5485	6.0525	192.6460	47.01%
	0.90	0.0183	0.0011	5.9891	0.1029	209.3575	4.8414	6.2166	192.9641	46.98%
	0.60	0.0185	0.0011	5.6218	0.1203	187.1703	7.3182	5.9101	162.2924	24.96%
3000	0.75	0.0184	0.0010	5.7628	0.1152	193.1102	1.9787	6.0494	192.6460	47.02%
	0.90	0.0183	0.0011	5.9927	0.1063	210.3703	4.1216	6.2044	199.0992	51.65%
	0.60	0.0183	0.0012	5.6197	0.1198	188.2678	6.4818	5.9512	176.0916	34.88%
5000	0.75	0.0183	0.0010	5.7412	0.1094	192.6548	0.0250	6.0484	192.6460	47.02%
	0.90	0.0183	0.0011	5.9957	0.1040	209.6794	2.0785	6.2228	204.2703	55.56%
	0.60	0.0186	0.0011	5.6375	0.1188	189.7967	5.5210	5.9101	178.4332	36.76%
10000	0.75	0.0184	0.0010	5.7535	0.1043	192.6468	0.0080	6.0392	192.6460	47.04%
	0.90	0.0184	0.0012	6.0035	0.1041	210.1639	1.3449	6.1931	204.2703	55.59%
Large instance weekend (l.we)										
	0.60	0.0226	0.0011	6.8773	0.1228	240.7261	6.6997	7.1588	224.5650	18.26%
1000	0.75	0.0227	0.0012	7.1466	0.1317	256.4022	4.7405	7.4886	243.7498	24.73%
	0.90	0.0227	0.0012	7.5502	0.1127	275.5612	3.8706	7.7724	261.7975	32.83%
	0.60	0.0226	0.0011	6.8760	0.1061	241.9364	5.5255	7.1588	226.5650	19.03%
2000	0.75	0.0224	0.0011	7.1351	0.1091	255.5291	3.1638	7.4688	241.7776	23.87%
	0.90	0.0227	0.0011	7.5550	0.1038	276.3039	3.4833	7.7741	261.7975	32.83%
	0.60	0.0225	0.0011	6.8761	0.1075	243.5329	3.5692	7.1386	226.7717	19.27%
3000	0.75	0.0225	0.0011	7.1217	0.1051	256.0430	2.2317	7.4384	253.1568	29.45%
	0.90	0.0228	0.0011	7.5352	0.0886	276.7002	2.8177	7.7340	273.1469	38.59%
	0.60	0.0227	0.0011	6.9074	0.1146	244.0033	2.1013	7.1588	226.7717	19.11%
5000	0.75	0.0227	0.0011	7.1456	0.1071	255.6718	1.0131	7.3845	253.1568	29.65%
	0.90	0.0225	0.0014	7.5650	0.1014	277.2199	2.6652	7.7340	273.4415	38.73%
	0.60	0.0227	0.0012	6.8757	0.0973	244.0110	1.1620	7.1298	243.1194	26.21%
10000	0.75	0.0225	0.0011	7.1262	0.0972	255.7413	0.7002	7.3742	253.1568	29.69%
	0.90	0.0226	0.0012	7.5460	0.0911	277.5050	2.5468	7.7340	273.4415	38.73%
Building weekday (b.wd)										
	0.60	0.0308	0.0012	11.0079	0.1488	441.6881	10.60020	11.3678	417.6331	67.39%
1000	0.75	0.0311	0.0013	11.3737	0.1292	485.5665	16.4973	11.8052	450.8393	79.48%
	0.90	0.0309	0.0012	11.8555	0.1035	520.1808	7.5063	12.1265	502.4882	99.30%
	0.60	0.0310	0.0011	11.0445	0.1526	443.0685	8.6618	11.4068	417.4538	67.24%
2000	0.75	0.0309	0.0010	11.3566	0.1163	492.4860	15.3699	11.6766	450.8393	79.65%
	0.90	0.0310	0.0011	11.8440	0.1085	520.0559	3.8396	12.0744	508.8815	101.85%
	0.60	0.0309	0.0012	11.0273	0.1593	442.9396	8.0281	11.4068	417.6606	67.32%
3000	0.75	0.0309	0.0010	11.3437	0.1138	491.7616	14.6810	11.6062	450.8393	79.75%
	0.90	0.0309	0.0013	11.8308	0.1045	519.7176	4.1177	12.0749	509.1201	101.94%
	0.60	0.0312	0.0012	11.0649	0.1446	444.8930	7.4917	11.4258	419.8494	68.12%
5000	0.75	0.0311	0.0011	11.3348	0.1147	496.9334	11.7561	11.6031	450.8393	79.76%
	0.90	0.0311	0.0010	11.8272	0.0990	519.9015	1.4629	12.0838	514.0527	103.87%
	0.60	0.0310	0.0011	11.0605	0.1447	445.3209	6.5916	11.4643	434.2840	73.59%
10000	0.75	0.0308	0.0011	11.3351	0.1219	500.7149	6.1324	11.6123	476.5378	89.72%
	0.90	0.0310	0.0012	11.8402	0.1021	520.0757	1.0513	12.0540	514.1322	103.93%
Building weekend (b.we)										
	0.60	0.0345	0.0011	11.5544	0.1442	487.6129	5.0430	11.8833	481.3362	111.62%
1000	0.75	0.0345	0.0011	12.0346	0.1199	538.6840	16.4980	12.3141	497.0862	113.88%
	0.90	0.0346	0.0012	12.7948	0.1120	573.4305	5.4551	13.0252	561.2765	124.66%
	0.60	0.0345	0.0011	11.5617	0.1353	486.5699	2.8961	11.8965	481.6732	111.67%
2000	0.75	0.0345	0.0013	12.0380	0.1076	544.3418	14.9656	12.2864	498.2471	114.05%
	0.90	0.0343	0.0013	12.7841	0.1015	573.4661	3.1760	12.9924	561.9514	124.79%
	0.60	0.0345	0.0013	11.5590	0.1208	486.1981	2.8327	11.8256	482.9548	111.84%
3000	0.75	0.0345	0.0012	12.0298	0.1109	545.2023	12.5249	12.2977	503.0244	114.77%

	0.90	0.0346	0.0012	12.7696	0.1007	572.7407	3.1134	12.9632	561.9898	124.79%
5000	0.60	0.0344	0.0012	11.5714	0.1331	485.9443	1.5795	11.8763	483.9182	111.98%
	0.75	0.0345	0.0011	12.0225	0.1021	548.0941	9.8610	12.2460	523.8958	118.09%
	0.90	0.0346	0.0016	12.7725	0.0988	573.2314	2.8112	12.9826	569.6496	126.23%
10000	0.60	0.0343	0.0013	11.5650	0.1236	485.9842	1.7995	11.8366	483.9182	111.98%
	0.75	0.0344	0.0012	12.0144	0.1069	551.8443	3.8878	12.2854	526.3093	118.49%
	0.90	0.0343	0.0014	12.7717	0.0953	573.6158	2.7063	12.9655	569.9058	126.28%