The principle of maximum entropy explains the cores observed in the mass distribution of dwarf galaxies

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ABSTRACT

Cold dark matter (CDM) simulations predict a central cusp in the mass distribution of galaxies. This prediction is in stark contrast with observations of dwarf galaxies that show a central plateau or "core" in their density distribution. The proposed solutions to this core-cusp problem can be classified into two types. One invokes feedback mechanisms produced by the baryonic component of the galaxies and the other assumes that the properties of the dark matter (DM) particle depart from the CDM hypothesis. Here we propose an alternative yet complementary explanation. We argue that cores are unavoidable in the self-gravitating systems of maximum entropy that result from non-extensive statistical mechanics. Their structure follows from the Tsallis entropy, which is attributed to systems with long-range interactions. Strikingly, the mass density profiles predicted by such thermodynamic equilibrium match the observed cores without any adjustment or tuning. Thus, the principle of maximum Tsallis entropy explains the presence of cores in dwarf galaxies.

Key words. gravitation – galaxies: dwarf – galaxies: fundamental parameters – galaxies: structure – dark matter

1. Introduction

The total mass density of low-mass galaxies flattens up at their center, showing what is known as a core. This observational fact has been noted as a long-standing problem of the Λ CDM paradigm (e.g., see recent reviews by Weinberg et al. 2015 and Del Popolo & Le Delliou 2017) since early dark-matter-only numerical simulations predicted the existence of density cusps, rather than cores, in the inner regions of galaxies (Moore 1994). A popular explanation of the so-called "core-cusp problem" relies on the inclusion of baryon physics in the simulations, which, through gravity, couples baryon processes with DM. Explosive baryon-driven events at the center of the galaxies produce sudden changes in the gravitational potential which, integrated over time, turn the DM distribution from cusp to core (Governato et al. 2010). Alternatively, the core-cusp problem may also point out a failure of the cold DM hypothesis (Weinberg et al. 2015; Del Popolo & Le Delliou 2017). Solutions include considering warm DM, whose free-streaming velocities erase primordial fluctuations on small scales (Colín et al. 2000), or by assuming self-interacting DM, where the scattering between DM particles redistributes energy and momentum generating inner cores (Spergel & Steinhardt 2000).

Here, we propose an alternative solution to the core-cusp problem based on the principle of maximum Tsallis entropy and the polytropes it leads to. For theoretical reasons (presented in Sect. 2) polytropes may provide a good representation for the distribution of mass within galaxies and they all have cores. Therefore, the question arises as to whether the cores of the polytropes reproduce the cores observed in the matter distribution of dwarf galaxies. Here, we show that they do so without any

free parameter (Sect. 3). Polytropes describe thermodynamic (or meta-stable) equilibrium configurations of self-gravitating systems under special conditions. Thus, our result suggests that these conditions are met in dwarf galaxies and may drive their internal structure (Sect. 4).

2. Maximum-entropy self-gravitating systems and polytropes

Galaxies are self-gravitating structures which, among all possible equilibrium configurations, end up having only those that are consistent with a stellar mass surface density profile resembling a Sérsic function (e.g., Blanton et al. 2003; van der Wel et al. 2012)[1]. Their settling into this particular configuration could be due either to some fundamental physical process (as it happens with the velocities of the molecules in a gas) or to the initial conditions that had given rise to the system (Binney & Tremaine 2008). The mass distribution in galaxies is currently explained as the outcome of the initial conditions (Cen 2014; Nipoti 2015; Ludlow & Angulo 2017; Brown et al. 2020). The option of a fundamental process determining the configuration is traditionally discredited because, in following the principles of statistical physics, it should correspond to the most probable configuration of a self-gravitating system and, thus, it should result from maximizing the entropy. The use of the classical Boltzmann-Gibbs

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[1] The Sérsic functions include exponential disks, observed in dwarf galaxies (e.g., de Jong & van der Kruit 1994), and *de Vaucouleurs* 1/4-profiles, characteristic of massive ellipticals (e.g., de Vaucouleurs 1948).

entropy leads to a distribution with infinity mass and energy (Binney & Tremaine 2008; Padmanabhan 2008), disfavoring this explanation. In the standard Boltzmann-Gibbs approach, however, the long-range forces that govern self-gravitating systems are not properly taken into account. Systems with long-range interactions admit long-lasting meta-stable states described by a maximum entropy formalism based on Tsallis (S_q) non-additive entropies (Tsallis 1988, 2009, and references therein). Observational evidence for the S_q statistics has been found in connection with various astrophysical problems (Livadiotis & McComas 2013; Silva et al. 2013). In particular, the maximization under suitable constraints of the Tsallis entropy of a Newtonian self-gravitating N-body system leads to a polytropic distributions (Plastino & Plastino 1993; Lima & de Souza 2005), which has finite mass and a shape closely resembling the DM distribution found in numerical simulations of galaxy formation (Navarro et al. 2004; Calvo et al. 2009). In the current cosmological model, DM provides most of the gravitational pull needed for the ordinary matter to collapse and form visible galaxies, and thus, polytropes can approximately describe the gravitational potential of galaxies. As we show below, the mass density associated with a polytrope always has a core. The question arises as to whether the cores of the polytropes reproduce the cores observed in the matter distribution of dwarf galaxies, thus providing an alternative view for solving the core-cusp problem (Sect. 1).

A polytrope of index m is defined as the spherically-symmetric self-gravitating structure resulting from the solution of the Lane-Emden equation for the (normalized) gravitational potential ψ (Chandrasekhar 1967; Binney & Tremaine 2008),

$$\frac{1}{s^2}\frac{d}{ds}\left(s^2\frac{d\psi}{ds}\right) = \begin{cases} -3\psi^m & \psi > 0, \\ 0 & \psi \le 0. \end{cases} \quad (1)$$

The symbol s stands for the scaled radial distance in the 3D space and the mass volume density is recovered from ψ as

$$\rho(r) = \rho(0)\,\psi(s)^m, \quad (2)$$

$$r = b\,s, \quad (3)$$

where r stands for the physical radial distance and $\rho(0)$ and b are two arbitrary constants. Equation (1) is solved under the initial conditions $\psi(0) = 1$ and $d\psi(0)/ds = 0$ [2]. Figure 1 illustrates the variety of physically admissible polytropes, with the range of polytropic indexes

$$3/2 \le m \le 5, \quad (4)$$

set because polytropes with $m \le 3/2$ are unstable or have infinite density and those with $m > 5$ have infinite mass (Plastino & Plastino 1993; Binney & Tremaine 2008). In order to compute the polytropes in Fig. 1, Eq. (1) was split into a system of two first-order differential equations for ψ and $d\psi/ds$, which were integrated from $s = 0$ using `Lsoda` (Hindmarsh 2019), as implemented in *python* (*scipy.odeint*).

We note that all polytropes have cores, in the sense that $d\ln\rho/d\ln r \to 0$ when $r \to 0$. This property follows from the initial condition $d\psi(0)/ds = 0$ and Eq.(2). It is shown by the density profiles displayed in Fig. 1.

3. Results

Figure 2 shows the state-of-the-art observation of galaxy cores in dwarf galaxies by Oh et al. (2015), which is based on 26 galax-

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[2] Equation (1) also admits solutions with $d\psi(0)/ds \ne 0$, but those are discarded because they have infinite central density and total mass (e.g., Binney & Tremaine 2008).
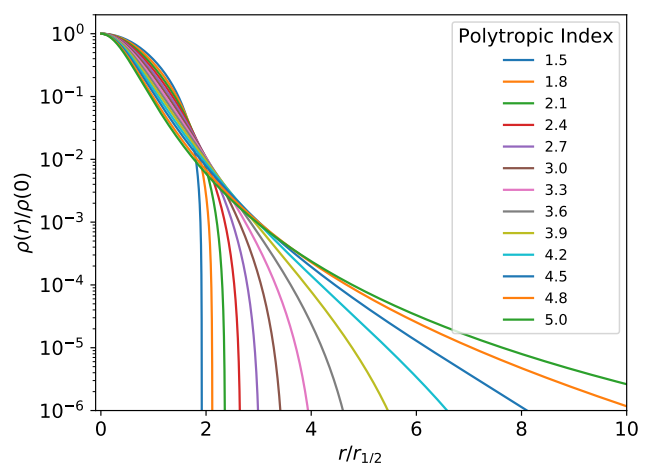
Fig. 1. Mass volume density resulting from the numerical solutions of the Lane-Emden equation (polytropes). Polytropes are self-gravitating systems having maximum Tsallis entropy. The curves are normalized to the central density and to the half-mass radius ($r_{1/2}$). The examples show the range of physically plausible solutions, with the corresponding polytropic index given in the inset.
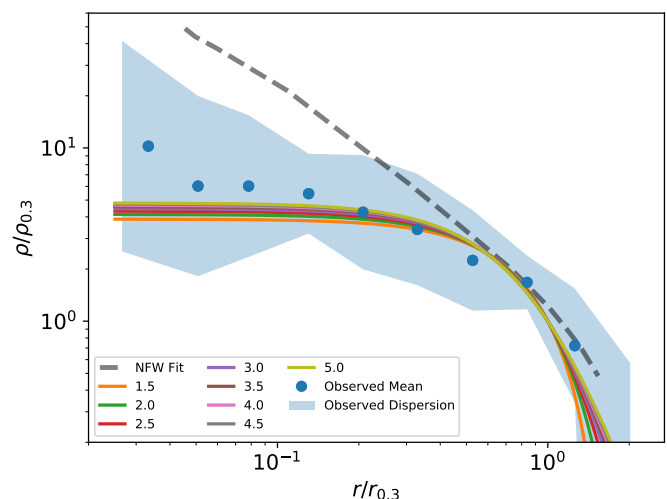

Fig. 2. Density profile observed in the inner regions of the 26 "Little Things" galaxies from Oh et al. (2015) (blue symbols and blue region give the mean and the root mean square (RMS) dispersion among the different objects). To reduce scatter, the observed densities and radii are normalized to the density and radius where the logarithmic derivative of the circular velocity equals 0.3 ($d\log v_c/d\log r = 0.3$), denoted as $\rho_{0.3}$ and $r_{0.3}$, respectively. Polytropes are parameter-free in this representation (solid lines with the corresponding indexes given in the inset). The dashed line gives a best-fit to the observed density using a Navarro-Frenk-White (NFW) profile (Oh et al. 2015), which does not follow the observed core.

ies with stellar masses of $6.5 \le \log(M_\star/M_\odot) \le 8.2$ (blue symbols with the blue region giving the RMS dispersion among the different objects). The total density is inferred from the circular-speed, v_c, measured in the 21-cm hydrogen line which, for axisymmetric systems, is related to v_c as (e.g., de Blok et al. 2001):

$$\rho(r) = \frac{1}{4\pi G}\left[\frac{v_c}{r}\right]^2\left[1 + 2\frac{d\log v_c}{d\log r}\right], \quad (5)$$

were G is the gravitational constant. The scatter of the 26 density profiles gets largely reduced when each individual profile is normalized to the radius and density where $d\log v_c/d\log r = 0.3$,

denoted as $r_{0.3}$ and $\rho_{0.3}$, respectively (Oh et al. 2015). In addition to reducing the observational scatter, this normalization makes the comparison with polytropes parameter-free. The density $\rho(r)$ consistent with a polytrope of index m (Eq. [2]) depends on two parameters $\rho(0)$ and b. Using Eqs.(3) and Eq. (5), it can be shown that

$$\rho(x\, r_{0.3})/\rho_{0.3} = \psi^m(x\, s_{0.3})/\psi^m(s_{0.3}), \qquad (6)$$

where $x = r/r_{0.3}$ and $s_{0.3}$ is the value for $r_{0.3}$ obtained from $\psi(s)$. The right-hand side of Eq. (6) does not depend on $\rho(0)$ or b, indicating that the same happens with the normalized density (the left-hand side of the equation) which, consequently, has no freedom in Fig. 2. Thus, the agreement between the observed and the predicted cores is particularly revealing, suggesting a true connection between polytropes and the inner structure of dwarf galaxies.

4. Conclusions

In this work, we show that the polytropes, resulting from the principle of maximum Tsallis entropy, reproduce without any tuning the cores observed in the matter distribution of dwarf galaxies. The genesis of these cores is currently interpreted as being driven by the interplay between baryons and DM, such that repetitive baryon motions modify the overall gravitational potential and the associated matter distribution (Sect. 1). We note that the two explanations are not in opposition; in fact, they are consistent if the baryon driven motions simply shorten the time-scale needed to thermalize the global gravitational potential into a polytrope.

Our study is focused on the central regions of the galaxies, but polytropes also work well in the outskirts (e.g., Saxton & Ferreras 2010). The outer parts are fully dominated by DM and it has repeatedly been shown that polytropes can be fit with Einasto profiles (e.g., Zavala et al. 2006; Salvador-Solé et al. 2012), which fit well the outer parts of the DM profiles found in cosmological numerical simulations (e.g., Navarro et al. 2004; Merritt et al. 2005; Calvo et al. 2009). In support of this, Frigerio Martins et al. (2015) employ the maximum Tsallis entropy formalism to fit the radial dependence of v_c in 24 galaxies with $8 \leq \log(M_\star/M_\odot) \leq 11$. The two variables $v_c(r)$ and $\rho(r)$ are interchangeable (Eq. [5]), so that the goodness of the fit at all radial distances also applies to $\rho(r)$ – although Frigerio Martins et al. do not specifically consider the cores we cover in this work.

The association between dwarf galaxies and maximum Tsallis entropy opens up the possibility of using the well-proven tool-kit of statistical mechanics to understand them (Padmanabhan 2008; Pontzen & Governato 2013; Saxton 2013). Identifying galaxies with polytropes has a number of additional implications. Accurate mass profiles are needed in planning and interpreting the astrophysical experiments for disclosing the nature of DM. The DM annihilation cross-sections depend on halo shape (e.g., Zhao et al. 2018) and precise DM profiles, along with their time evolution, should help us to distinguish between cold, warm, or self-interacting DM (e.g., Weinberg et al. 2015; Ludlow et al. 2016). The suite of mass models currently used in gravitational lensing studies does not include polytropes (e.g., Keeton 2001), but subtle details in the mass model are critically important when precise magnifications are needed, or when lensing is used to derive cosmological parameters (Knudson et al. 2001; Elíasdóttir & Möller 2007).

The ability of polytropes to reproduce observed galaxy properties also has an impact on the side of statistical mechanics. A comparison with the cosmic evolution of astronomical objects will shed new light on whether the S_q entropies, in addition to providing H-funtionals that are capable of selecting particular steady-state solutions of the Vlasov equation (Chavanis & Sire 2005), would also have a deeper thermodynamical significance for self-gravitating systems.

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