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On the energy of the poloidal magnetic field near the ionosphere

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Abstract: The role of the ionospheric Hall effect on the energy balance in the magnetosphere-ionosphere (MI) system coupled through the field-aligned current (FAC) is discussed. FACs lose their stored electromagnetic energy in the ionosphere through Joule dissipation; this process is caused by the closure of the FAC via the ionospheric Pedersen current carried by ions. On the other hand, the ionospheric rotational Hall current carried by electrons cannot be dissipated. However, the ionospheric rotational Hall current can also be excited by an incident FAC, causing it to radiate Poynting fluxes that lead to the growth of a poloidal-type magnetic field in the magnetosphere and atmosphere. From the viewpoint of energy conservation, a few ambiguities in the above statements may be recognized. In this paper, we clarify the energy balance of the electromagnetic disturbances between the magnetosphere, ionosphere and atmosphere. The generation of the Hall current (together with the associated poloidal magnetic field) will be shown to occur during the growth stage, when the electromagnetic energy is pumped through the divergent Hall current, regardless of how slow the growth may have been.

1. Introduction

The field-aligned current (FAC) transmitted to the ionosphere by a shear Alfvén wave is closed by the divergence of the ionospheric Pedersen current. The divergent electric field that is associated with this current generates a rotational ionospheric Hall current, which produces a poloidal-type magnetic perturbation field that is observable on the ground (Nishida, 1964). When the incident field varies with time, the poloidal magnetic field also varies with time, inducing a rotational electric field that is opposite to the Hall current. This rotational electric field in turn drives the divergent Hall current closure via the FAC (Yoshikawa and Itonaga, 1996, 2000). The magnetic perturbation field produced by the divergent Hall current is toroidal.

Thus, when the intrinsically time-varying nature of the incident FAC is taken into account, the energy dissipated in the ionosphere must be dependent on the ionospheric Hall conductivity. When the incident FAC is growing with time, the rotational Hall current also grows; the resulting poloidal magnetic field is either radiated away from the ionosphere or (in the evanescent case) stored in its neighborhood. While the ionospheric reflection coefficient for the static case was given by (Scholer, 1970) as

$$R_{\rm A}=\frac{\Sigma_{\rm A}-\Sigma_{\rm P}}{\Sigma_{\rm A}+\Sigma_{\rm P}}, \quad (1)$$

for the time-dependent case, this coefficient must be expressed as

$$R_{\rm A}=\frac{\Sigma_{\rm A}-(\Sigma_{\rm P}-\Sigma_{\rm H}MC_{\rm div\to rot})}{\Sigma_{\rm A}+(\Sigma_{\rm P}-\Sigma_{\rm H}MC_{\rm div\to rot})}, \quad (2)$$

where $\Sigma_{\rm A}$ is the Alfvén wave conductance just above the ionosphere; $\Sigma_{\rm P}$ and $\Sigma_{\rm H}$ are the height-integrated Pedersen and Hall conductivities; respectively, and $MC_{\rm div\to rot}$ is the mode conversion ratio from a divergent to a rotational electric field (Yoshikawa and Itonaga, 1996, 2000). In the context of the reflection process described by case (1), perfect impedance matching between the FAC and the ionospheric divergent current occurs when $\Sigma_{\rm P}=\Sigma_{\rm A}$. On the other hand, such perfect impedance matching never happens in case (2) because the effective conductivity for the divergent Hall current $(-\Sigma_{\rm H}MC_{\rm div\to rot})$ is complex. This implies that the Hall effect controls the energy distribution in the ionosphere (Buchert, 1998; Yoshikawa, 1998; Yoshikawa *et al.*, 1999).

Furthermore, the damping factor for a field-line oscillation (standing FAC system) is estimated for a rectangular magnetosphere model with symmetric ionospheres by the equation

$$\gamma=\frac{V_{\rm A}}{2l_{\parallel}}\ln|R_{\rm A}|, \quad (3)$$

(*e.g.*, Ellis and Southwood, 1983), where $l_{\parallel}$ is the length of the field line and $V_{\rm A}$ is the Alfvén velocity. Combining eqs. (2) and (3) shows that the energy dissipation of the field line oscillations is controlled by the ionospheric Hall effect (Yoshikawa *et al.*, 1999). Lysak and Song (2001) and Yoshikawa *et al.* (2002) independently confirmed that the reflection and mode conversion process described by eq. (2) satisfies the conservation of current and energy.

The involvement of Hall conductivity in energy dissipation and reflection may appear to contradict the conventional idea that the Hall current does not do any work, since it is directed perpendicular to the electric field. The purpose of this paper is to elucidate the role of the Hall effect in the energy conservation process. The divergent Hall current plays a crucial role by extracting electromagnetic energy stored in the FAC and pumping it into the rotational Hall current, which radiates a poloidal field.

In the next section, we will review conduction current and energy dissipation in the ionosphere from the viewpoint of particle motion. In Section 3, we will reconstruct the energy balance of electromagnetic disturbances between the magnetosphere, ionosphere and atmosphere using a sheet ionosphere model, and clarify the role of the Hall effect in the energy balance. In Section 4, we will use the results obtained in Section 2, to discuss the origin of the poloidal magnetic field observed on the ground in the context of the inductive response of the ionosphere. Finally, the results of this paper will be summarized in Section 5.

In this paper, we will focus on the general role of energy flow. To quantitatively evaluate energy flows, the equations describing the evolution of electromagnetic disturbances with boundary conditions must be determined; this topic will be discussed in a future paper.

2. Motion of charged particles in the ionospheric current and energy dissipation

The motion of charged particles, including collisions between neutral and charged constituents, and their contribution to macroscopic conduction current was established by Hines (1953). In this chapter, the generation of ionospheric currents by the incidence of the FAC will be reconstructed from the viewpoint of particle motions using one of the concepts of Hines' theory. We will focus on the growth of the incident FAC in the ionosphere and provide a qualitative consideration of how the macroscopic motion of electrons and ions in the ionosphere are driven by the macroscopic electric field and how they generate the ionosoheric current. Figure 1 summarizes the relation of motions of the

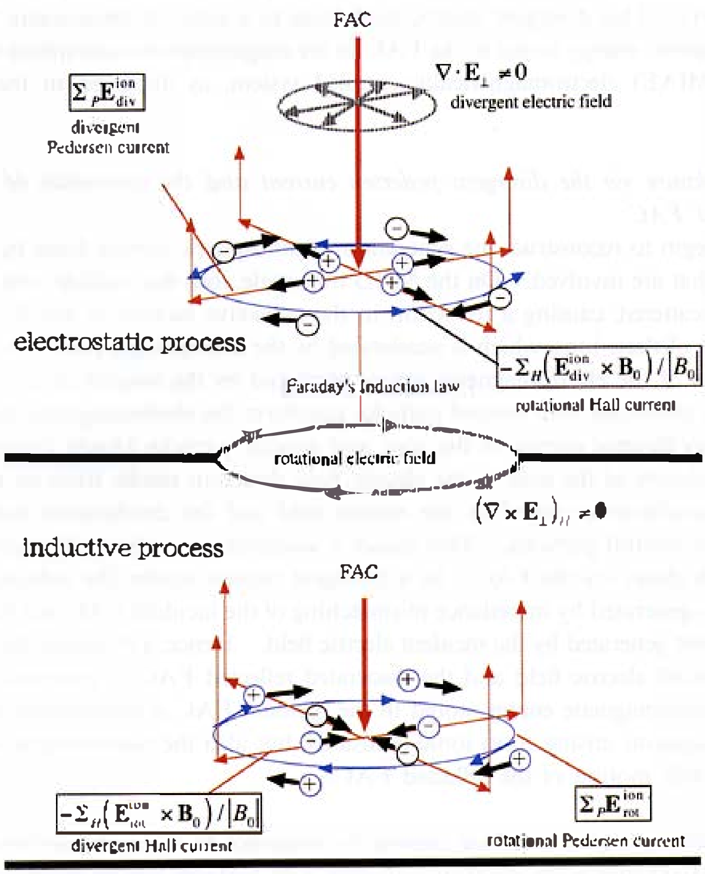

Fig. 1. Schematic illustration of the motions of electrons and ions in the ionosphericcurrents accompanying the growth phase of the incident FAC's divergent electric field. In the direct process, the divergent Pedersen current, carried by the ions, is closed via the FAC. On the other hand, the Hall current, carried by the electrons, flows as rotational current.

electron and ion motions and the ionospheric currents accompanying the growth of the incident divergent electric field of the FAC. The same conceptual figures have also been presented in Buchert (1998), Yoshikawa (1998), and Yoshikawa and Itonaga (2000). Detailed explanations of the interactions are presented in Sections 2.1 to 2.3.

In a magnetohydrodynamic (MHD) medium, both electrons and ions are frozen onto the magnetic field line (Alfvén, 1950) and have a bulk velocity of $\boldsymbol{v}_\perp=(\boldsymbol{E}_\perp\times\boldsymbol{B}_0)/B_0^2$, known as the $\boldsymbol{E}\times\boldsymbol{B}$ drift motion, where $\boldsymbol{E}_\perp$ is the electric field and $\boldsymbol{B}_0$ is the background magnetic field. The subscripts $\|$ and $\perp$ denote the components of the vector quantity that are parallel and perpendicular to $\boldsymbol{B}_0$, respectively. A twisted magnetic field line generated by some type of magnetospheric generator can be stored as electromagnetic energy. The motion of the twisted field line is associated with the $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$ drift motion of the magnetic field lines, where $\boldsymbol{E}_{\rm div}$ is the divergent part of the electric field. If B_0 is taken to be uniform, the FAC carried by the shear Alfvén wave associated with this $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$ drift motion satisfies the relationship $(\nabla\times\boldsymbol{v}_\perp)_\|\cong\nabla\cdot\boldsymbol{E}_{\rm div}/\boldsymbol{B}_0\neq0$ (*e.g*, Vasyliunas, 1984; Southwood and Kivelson, 1991). This divergent electric field leads to a series of phenomena that release the electromagnetic energy stored in the FAC in the magnetosphere-ionosphere-atmosphere and Earth (MIAE) electromagnetically coupled system, as discussed in the following sections.

2.1. *FAC closure via the divergent pedersen current and the generation of the reflected FAC*

Let us begin to reconstruct the generation of ionospheric current from by looking at the particles that are involved. On the MHD time scale, ions that collide with the neutral particles are scattered, causing a reduction in the collective motion of the $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$ drift. The scattering of these ions, which is accelerated by the macroscopic electric field, leads to the absorption of the electromagnetic energy produced by the twisted field lines. Thus, these multiple collisions with neutral particles transform the electromagnetic energy stored in the FAC to thermal energy in the ions and neutral particles (Joule dissipation). A finite mean velocity of the ions in the electric field direction results from an equilibrium between the acceleration caused by the electric field and the deceleration caused by the collisions with neutral particles. This causes a so-called ionospheric divergent Pedersen current, which closes via the FAC. In a divergent current system, the reflected divergent electric field is generated by impedance mismatching of the incident FAC and the divergent Pedersen current generated by the incident electric field. Hence, a Poynting flux associated with this reflected electric field and the associated reflected FAC is generated. In other words, the electromagnetic energy stored in the incident FAC is transformed not only by the Joule dissipation arising from ionic collisions, but also the electromagnetic energy of the $\boldsymbol{E}_{div}\times\boldsymbol{B}_0$ drift motion of the reflected FAC.

2.2. *Generation of rotational Hall current by resistance to a counter-electromotive force*

On the MHD time scale, electron collisions with particles can be neglected, enabling the electrons to maintain the $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$ drift motion. A reduction in the velocity of the ions from the $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$ drift velocity causes a substantial electron current whose direction is opposite to that of the $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$ drift; this current known as a rotational Hall current. The Hall current does not to undergo any loss because its carriers are unscattered electrons.

To increase a rotational Hall current, however, work must be done by the power supply against the counter electromotive force $\boldsymbol{E}_{\rm rot}$, which is generated to prevent the growth of the associated magnetic fields: $\boldsymbol{E}_{\rm rot}$ (direction along $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$) satisfies $(\nabla\times\boldsymbol{E}_{\rm rot})_{\parallel}\neq 0$. Furthermore, $\boldsymbol{E}_{\rm rot}$ accelerates the scattered ions; this leads to Joule dissipation though collisions with the neutral particles and generates the rotational Pedersen current.

The rotational Hall current seems to be naturally excited by the appearance of the $\boldsymbol{E}_{\rm div}\times\boldsymbol{B}_0$ drift motion of the electrons that results from ion-induced. However, as this current increases, energy is expended as the poloidal magnetic disturbances increase and Joule dissipation from the rotational Pedersen current occurs. The main problem is how and from where such energy is supplied. The original energy source is likely to be the electromagnetic energy stored in the incident FAC. If we can successfully explain how this energy is transferred not only to the Joule dissipation that results from ion collisions and the electromagnetic energy of the reflected FAC but also to the generation of $\boldsymbol{E}_{\rm rot}$, the role of the ionospheric Hall effect in magnetosphere-ionosphere coupling will become obvious.

2.3. FAC closure via the divergent Hall current and energy absorption from the FAC system

The $\boldsymbol{E}_{\rm rot}\times\boldsymbol{B}_0$ drift motion of the electrons that accompanies the generation of $\boldsymbol{E}_{\rm rot}$ will now be discussed. The current generated by the $\boldsymbol{E}_{\rm rot}\times\boldsymbol{B}_0$ drift motion of the electrons becomes the divergent Hall current, which also closes via the FAC in a manner similar to the closure of the ion-carried divergent Pedersen current.

Whether or not the electromagnetic energy stored by the incident FAC is spent by the ionospheric divergent current depends on the direction of $\boldsymbol{E}_{\rm div}$, that is, on the sign of $\boldsymbol{J}_{\rm div}\cdot\boldsymbol{E}_{\rm div}$. The term $\boldsymbol{J}\cdot\boldsymbol{E}$ in the ionosphere becomes an indicator of the energy exchange between the electromagnetic energy of the MHD fluid in the magnetosphere, the thermal energy in the ionosphere, and the energy in the neutral atmosphere. For an increasing rotational Hall current, the direction of $\boldsymbol{E}_{\rm rot}$ is opposite to the rotational Hall current; therefore the direction of the divergent Hall current arising from the electrons is the same as that of the divergent Pedersen current arising from the ions. Therefore, the electromagnetic energy stored by the FAC is expended by the generation of $\boldsymbol{E}_{\rm rot}$ through the generation of the divergent Hall current and the divergent Pedersen current. However, further quantitative consideration using a particle model may not provide any new insights. Accordingly, the Joule dissipation in the context of a thin ionospheric current sheet approximation will be discussed in the next section.

3. Energy balance between magnetosphere and atmosphere through the ionospheric boundary

In this chapter, a thin-sheet approximation will be used to describe the ionosphere. With this approximation, the ambient magnetic field $\boldsymbol{B}_0$ is assumed to penetrate the ionosphere vertically. This model will be used to formulate the energy balance of the electromagnetic disturbances between the magnetosphere and the atmosphere through the ionospheric boundary. In this approximation, the motions of the electrons, ions and neutral particles are incorporated into the ohmic ionospheric current of

$$J_\perp^{ion} = \Sigma_P E_\perp^{ion} - \Sigma_H\left(E_\perp^{ion} \times \hat{b}\right), \tag{4}$$

where $\hat{b}$ is a unit vector along B_0. Here, the superscripts "mag", "ion""and "atm" denote the quantities in the magnetosphere, ionosphere and atmosphere, respectively.

3.1. Total energy balance in the MIAE system

When the ionosphere is regarded as an infinitely thin conducting layer, the ionospheric current produces a jump in the magnetic field between the magnetosphere and the atmosphere:

$$\mu_0 J_\perp^{ion} = \hat{b} \times \left(\lim_{m \to i} b_\perp^{mag} - \lim_{a \to i} b_\perp^{atm}\right), \tag{5}$$

where μ_0 is the magnetic permeability in a free space. Taking the inner product between eq. (5) and $E_\perp^{ion}$, we get:

$$J_\perp^{ion} \cdot E_\perp^{ion} = -\lim_{m \to i} \mu_0^{-1}\left(E_\perp^{mag} \times b_\perp^{mag}\right)_\parallel + \lim_{a \to i} \mu_0^{-1}\left(E_\perp^{atm} \times b_\perp^{atm}\right)_\parallel. \tag{6}$$

Equation (6) shows that $J_\perp^{ion} \cdot E_\perp^{ion}$ in the ionosphere balances the Poynting flux in the electromagnetic disturbances from the magnetosphere and the atmosphere into the ionosphere. Here, in the transformation from eqs. (5) to (6), the tangential continuity condition of the electric field is used:

$$\lim_{m \to i} E_\perp^{mag} = \lim_{a \to i} E_\perp^{atm} = E_\perp^{ion}. \tag{7}$$

We should note that in eq. (6), $(E_\perp \times B_\perp)_\parallel > 0$ indicates an upward energy flow along the magnetic field line.

Substituting egs. (4) for (6) and applying the condition

$$-\Sigma_H\left(E_\perp^{ion} \times \hat{b}\right) \cdot E_\perp^{ion} = 0, \tag{8}$$

produces an energy balance equation for the entire system

$$-\lim_{m \to i} \mu_0^{-1}(E_\perp^{mag} \times b_\perp^{mag})_\parallel + \lim_{a \to i} \mu_0^{-1}(E_\perp^{atm} \times b_\perp^{atm})_\parallel = \Sigma_P E_\perp^{ion^2}. \tag{9}$$

This shows that the Poynting flux of the electromagnetic field absorbed by the ionosphere is transformed into Joule dissipation by the Pedersen current.

Condition eq. (8) means that the Hall current does not directly contribute to $J_\perp^{ion} \cdot E_\perp^{ion}$, providing no net energy to the external (dynamical) system. However, for a growth in the ionospheric rotational Hall current accompanying the incident FAC to occur, the increasing counter-electromotive force must be resisted by something, resulting in the rotation of the poloidal-type magnetic disturbance radiation or the storage of magnetic energy as an ionospheric surface wave. Even if the Hall effect cannot do work on an external system, what does it mean on Earth that it contribute to the accumulation of poloidal magnetic energy ? Furthermore, as shown by Yoshikawa *et al.* (1995, 1999), the eigenfrequency and damping factor of a standing field line oscillation, *i.e.*, a standing FAC system, are changed by the Hall conductivity. Why does the Hall effect's control of the electromagnetic energy stored by the FAC not appear i n eq. (9) ? To answer these questions, we will discuss the physical meanings of $-\Sigma_H(\boldsymbol{E}^{ion}\times\hat{\boldsymbol{b}})\cdot\boldsymbol{E}^{ion}_{\perp}=0$ in the following, section.

3.2. $\boldsymbol{J}^{ion}\cdot\boldsymbol{E}^{ion}_{\perp}$ terms

To explicitly consider the ionospheric Hall effect, after Yoshikawa and Itonaga (2000), we will divide the ionospheric current and the electric field into divergent and rotational components: $\boldsymbol{J}^{ion}=\boldsymbol{J}^{ion}_{div}+\boldsymbol{J}^{ion}_{rot}$ and $\boldsymbol{E}^{ion}_{\perp}=\boldsymbol{E}^{ion}_{div}+\boldsymbol{E}^{ion}_{rot}$, respectively. Physically, $\boldsymbol{J}^{ion}_{div}$ satisfies $\nabla\cdot\boldsymbol{J}^{ion}_{div}\neq 0$ and closes via the FAC just above the ionosphere; $\boldsymbol{J}^{ion}_{rot}$ satisfies $(\nabla\times\boldsymbol{J}^{ion}_{rot})_{\|}\neq 0$ and is associated with the developed poloidal magnetic field. On the other hand, $\boldsymbol{E}^{ion}_{div}$, which satisfies $\nabla\cdot\boldsymbol{E}^{ion}_{div}\neq 0$, shows an accumulated charge density on the ionosphere and $\boldsymbol{E}^{ion}_{rot}$, which satisfies $(\nabla\times\boldsymbol{E}^{ion}_{rot})_{\|}\neq 0$, is an ionospheric inductive electric field. The term $\boldsymbol{J}\cdot\boldsymbol{E}$ therefore becomes:

$$\boldsymbol{J}^{ion}_{\perp}\cdot\boldsymbol{E}^{ion}_{\perp}=\boldsymbol{J}^{ion}_{div}\cdot\boldsymbol{E}^{ion}_{div}+\boldsymbol{J}^{ion}_{rot}\cdot\boldsymbol{E}^{ion}_{rot}. \tag{10}$$

The first and second terms of eq. (10) give the energy transformation ratio from electromagnetic and kinetic energy in the ionosphere in a divergent current system and a rotational current system, respectively.

By dividing eq. (5) into the divergent and rotational components, we get the following equations:

$$\boldsymbol{J}^{ion}_{div}=\hat{\boldsymbol{b}}\times\mu_0^{-1}\left(\lim_{m\to i}\boldsymbol{b}^{mag}_{rot}-\lim_{a\to i}\boldsymbol{b}^{atm}_{rot}\right), \tag{11}$$

$$\boldsymbol{J}^{ion}_{rot}=\hat{\boldsymbol{b}}\times\mu_0^{-1}\left(\lim_{m\to i}\boldsymbol{b}^{mag}_{div}-\lim_{a\to i}\boldsymbol{b}^{atm}_{div}\right). \tag{12}$$

The terms $\boldsymbol{b}^{mag}_{rot}$ and $\boldsymbol{b}^{atm}_{rot}$ in eg. (11) represent toroidal magnetic field perturbations induced by the FAC and the divergent polarization current in vacuum, respectively. These perturbations produce a torsion in the magnetic flux tube. The terms $\boldsymbol{b}^{mag}_{div}$ and $\boldsymbol{b}^{atm}_{div}$ in eq. (12) are the poloidal magnetic field perturbations accompanied by the rotational current in the vertical plane of $\boldsymbol{B}_0$ in the magnetosphere and atmosphere, respectively; these perturbation cause the magnetic flux tube to expand or compress. Equations and show the jump in the toroidal and poloidal magnetic fields by the ionospheric divergent and rotational current, respectively.

Using eqs. (11) and (12), we get the Poynting flux expression in the divergent and rotational current systems can be obtained:

$$\boldsymbol{J}^{ion}_{div}\cdot\boldsymbol{E}^{ion}_{div}=S^{FAC}_{m\to i}+S^{TM}_{a\to i}, \tag{13}$$
$$\boldsymbol{J}^{ion}_{rot}\cdot\boldsymbol{E}^{ion}_{rot}=S^{MS}_{m\to i}+S^{TE}_{a\to i}, \tag{14}$$

where

$$S^{FAC}=-\lim_{m\to i}\frac{(\boldsymbol{E}^{ion}_{div}\times\boldsymbol{b}^{mag}_{rot})_{\|}}{\mu_0},\qquad S^{TM}_{a\to i}=\lim_{a\to i}\frac{(\boldsymbol{E}^{ion}_{div}\times\boldsymbol{b}^{atm}_{rot})_{\|}}{\mu_0},$$
$$S^{MS}_{m\to i}=-\lim_{m\to i}\frac{(\boldsymbol{E}^{ion}_{rot}\times\boldsymbol{b}^{mag}_{div})_{\|}}{\mu_0},\qquad S^{TE}_{a\to i}=\lim_{a\to i}\frac{(\boldsymbol{E}^{ion}_{rot}\times\boldsymbol{b}^{atm}_{div})_{\|}}{\mu_0}.$$

Here, S^{FAC} and $S^{TM}_{a\to i}$ in eq. (13) represent the parallel component of the Poynting vector of the FAC (shear Alfvén wave), and the transverse magnetic field (TM) waveguide mode that is absorbed in the ionosphere, respectively. On the other hand, $S^{MS}_{m\to i}$ and $S^{TE}_{a\to i}$ in eq. (14) represent the parallel component of the Poynting vector of a magnetosonic (MS) wave and a transverse electric field (TE) waveguide mode, respectively. S^{FAC} can be divided into the incident part of $S^{FAC}_{(in)}$ and the reflected part of $S^{FAC}_{(out)}$, whereas $S^{TE}_{a\to i}$ is directly related to magnetic disturbances on the ground.

Furthermore, the ionospheric current eq. (4) can also be separated into divergent eq. (15) and rotational eq. (16) componets:

$$\boldsymbol{J}^{ion}_{div}=\Sigma_P\boldsymbol{E}^{ion}_{div}-\Sigma_H\left(\boldsymbol{E}^{ion}_{rot}\times\hat{\boldsymbol{b}}\right), \tag{15}$$

$$\boldsymbol{J}^{ion}_{rot}=\Sigma_P\boldsymbol{E}^{ion}_{rot}-\Sigma_H\left(\boldsymbol{E}^{ion}_{div}\times\hat{\boldsymbol{b}}\right). \tag{16}$$

The first and second terms on the right-hand side of eq. (15) are the divergent part of ionospheric Pedersen and Hall currents, respectively, while the first and second terms on the right-hand side of eq. (16) are the rotational part of the ionospheric Pedersen and Hall currents, respectively.

The current expression in eq. (6) thus becomes:

$$\boldsymbol{J}^{ion}_{div}\cdot\boldsymbol{E}^{ion}_{div}=\Sigma_P E^{ion^2}_{div}-\Sigma_H E^{ion}_{rot}E^{ion}_{div}, \tag{17}$$

$$\boldsymbol{J}^{ion}_{rot}\cdot\boldsymbol{E}^{ion}_{rot}=\Sigma_H E^{ion}_{div}E^{ion}_{rot}+\Sigma_P E^{ion^2}_{rot}. \tag{18}$$

Finally, using eq. (13), (14), (17) and (18) the energy-balance equations for divergent-current and rotational-current systems

$$S^{FAC}_{(in)}+S^{FAC}_{(out)}+S^{TM}_{a\to i}=\Sigma_P E^{ion^2}_{div}-\Sigma_H E^{ion}_{rot}E^{ion}_{div}, \tag{19}$$

$$S^{MS}_{m\to i}+S^{TE}_{a\to i}=\Sigma_H E^{ion}_{div}E^{ion}_{rot}+\Sigma_P E^{ion^2}_{rot}. \tag{20}$$

Equation (19) shows the energy balance in the divergent current system. When the FAC is incident from the magnetosphere, the equation shows that its energy flow to the ionosphere (S^{FAC}_{in}) is distributed into that of the reflected FAC ($S^{FAC}_{(out)}$), the atmospheric TM waveguide mode ($S^{TM}_{a\to i}$), the Joule dissipation through the ionospheric-divergent Pedersen current ($\Sigma_P E^{ion^2}_{div}$), and the work on the external system through the divergent Hall current ($-\Sigma_H E^{ion}_{rot}E^{ion}_{div}$). On the other hand, eq. (20) shows the energy balance in the rotational current system. This system has work done on it by an external system through the rotational Hall current ($\Sigma_H E^{ion}_{div}E^{ion}_{rot}$), and its energy is redistributed into the Poynting flux of a magnetosonic wave ($S^{MS}_{m\to i}$), the atmospheric TE waveguide mode ($S^{TE}_{a\to i}$), and Joule dissipation through the ionospheric rotational Pedersen current ($\Sigma_P E^{ion^2}_{rot}$). These two equations clearly show that the energy which excites the rotational current system is derived from the divergent current (FAC) system through the ionospheric Hall effect. The relation $-\Sigma_H(\boldsymbol{E}^{ion}\times\hat{\boldsymbol{b}})\cdot\boldsymbol{E}^{ion}_{\perp}=0$ in the total current system can also be written as

$$(-\Sigma_H E^{ion}_{rot}E^{ion}_{div}\ \text{in divergent system}+\Sigma_H E^{ion}_{div}E^{ion}_{rot}\ \text{in rotational system}=0). \tag{21}$$

Put another way, to build up the large-scale steady Hall current in the ionosphere during its growing stage (inductive process), the finite divergent Hall current pumps the electromagnetic energy stored in the FAC system into the rotational Hall current. Figure 2

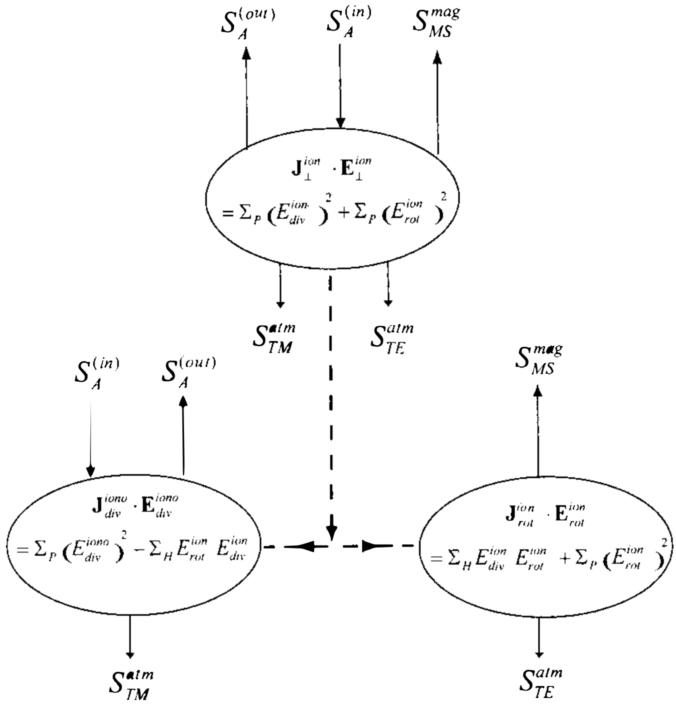

Fig. 2. Flow chart showing the energy balance between the magnetosphere and the atmosphere through the ionospheric boundary. Overall, the Poynting flux flowing into the ionosphere balances the Joule dissipation; internally, however an energy flow between the divergent and rotational current systems via the Hall effect is present.

summarizes how the electromagnetic energy in the incident part of the FAC is distributed into the divergent and rotational current systems.

Interestingly, the left hand side of eq. (19) is a summation of the Poynting flux of the incident and reflected shear Alfvén wave and the radiated atmospheric TM waveguide mode; this component shows the net energy that is absorbed into the ionosphere. As shown on the right-hand side of equation, this absorbed energy is used for Joule dissipation and rotational-current system excitation. Thus, the total energy balance can be written as

$$S^{\mathrm{FAC}}_{(\mathrm{in})} + S^{\mathrm{FAC}}_{(\mathrm{out})} + S^{\mathrm{TM}}_{a\to i} = \Sigma_{\mathrm{P}}\left(E^{\mathrm{ion}^2}_{\mathrm{div}} + E^{\mathrm{ion}^2}_{\mathrm{rot}}\right) - \left(S^{\mathrm{MS}}_{m\to i} + S^{\mathrm{TE}}_{a\to i}\right). \tag{22}$$

When the poloidal magnetic field is increasing, the relation $S^{\mathrm{MS}}_{m\to i} + S^{\mathrm{atm}}_{TE} < 0$ is satisfied, which leading to the following relationship:

$$S^{\mathrm{FAC}}_{(\mathrm{in})} + S^{\mathrm{FAC}}_{(\mathrm{out})} + S^{\mathrm{TM}}_{a\to i} > \Sigma_{\mathrm{P}}\left(E^{\mathrm{ion}^2}_{\mathrm{div}} + E^{\mathrm{ion}^2}_{\mathrm{rot}}\right). \tag{23}$$

Equation (23) shows that the net electromagnetic energy of the FAC absorbed by the ionosphere is larger than the total Joule dissipation in the ionosphere. In the unsteady phase of MI coupling, Joule dissipations in the ionosphere must be distinguished from the electromagnetic energy of the FAC absorbed in the ionosphere.

4. What causes ground magnetic disturbances?

As shown by eq. (19)+(20)=(10), the ionospheric Hall currents have not performed any net work on the external system. However, when we consider the MI coupling that divides the ionospheric current into its divergent and rotational components is considered the energy exchange between the divergent and rotational current systems is clearly caused by the Hall effect, with an exchange rate per unit time of $|\Sigma_H E^{ion}_{div} E^{ion}_{rot}|$. From the viewpoint of energy exchange between the divergent and rotational current systems, the divergent Hall current and rotational Hall currents are one and indivisible. The largest distinction between the divergent and rotational Hall currents is their ability to accumulate magnetic energy. The rotational Hall current can exist as a steady current in the ionosphere and because of its circularity, it accumulates the poloidal magnetic energy as an ionospheric surface wave. The poloidal magnetic energy being stored as the ionospheric surface wave is represented by the following equation:

$$\varepsilon_{pol}(t) = -\int_{-\infty}^{t} (S^{MS}_{m\to i} + S^{TE}_{a\to i}) dt' = -\int_{-\infty}^{t} \boldsymbol{J}^{ion}_{rot}(t) \cdot \boldsymbol{E}^{ion}_{rot}(t) dt', \tag{24}$$

which can exist even in the steady state (when $\boldsymbol{J}^{ion}_{rot} \cdot \boldsymbol{E}^{ion}_{rot} = 0$) and is a unique function of the developed steady rotational Hall current. On the other hand, the divergent Hall current provides a finite $\boldsymbol{J}^{ion}_{rot} \cdot \boldsymbol{E}^{ion}_{rot}$ but cannot store the magnetic energy itself. Equation (24) also shows that when the stored poloidal magnetic energy disappears, the sign of the Poynting flux has to satisfy $(S^{MS}_{m\to i} + S^{TE}_{a\to i}) > 0$, which means that $\varepsilon_{pol}(t)$ must be absorbed into the ionosphere and converted to Joule dissipation and the reflected FAC through $|\Sigma_H E^{ion}_{div} E^{ion}_{rot}|$.

With this in mind, the manner in which magnetic disturbances develop on the ground should be reconsidered; this is a very fundamental and important point. In the past context of MI coupling, ground magnetic disturbances (regardless of their frequency range) observed in the high latitudinal region, such as the aurora electrojet excited by region 1 and 2 current closure (*e.g.*, Iijima and Potemra, 1976), Dp2-type disturbances (*e.g.*, Nishida and Jacobs, 1962), the sudden commencement of storms (Nishida, 1964), and geomagnetic pulsations (*e.g.*, Samson and Rostoker, 1972), are thought to be accompanied by an ionospheric rotational Hall current excited by the Hall effect from the divergent electric field of the incident FAC to the ionosphere. On the other hand, the FAC flowing into the ionosphere was interpreted as being closed via the divergent Pedersen current. This treatment is equivalent to adopting an electrostatic approximation for the driving electric field of the ionospheric current, namely:

$$\boldsymbol{J}^{ion}_{\perp} \approx \Sigma_P \boldsymbol{E}^{ion}_{div} - \Sigma_H \left(\boldsymbol{E}^{ion}_{div} \times \hat{\boldsymbol{b}} \right). \tag{25}$$

Under this approximation, however, the existence of a divergent Hall current generated by the ionospheric inductive rotational electric field $\boldsymbol{E}^{ion}_{rot}$ is ignored, so from where do the ground magnetic disturbances receive their energy and how do they grow? Equation (25) cannot answer this question. On the other hand, the right-hand side of eqs. (19) clearly indicates that the energy for the excitation of the ground magnetic-field disturbance is taken from the FAC system. Although the essential importance of the divergent Hall current and the physical phenomena that is causes have been extensively investigated (Yoshikawa

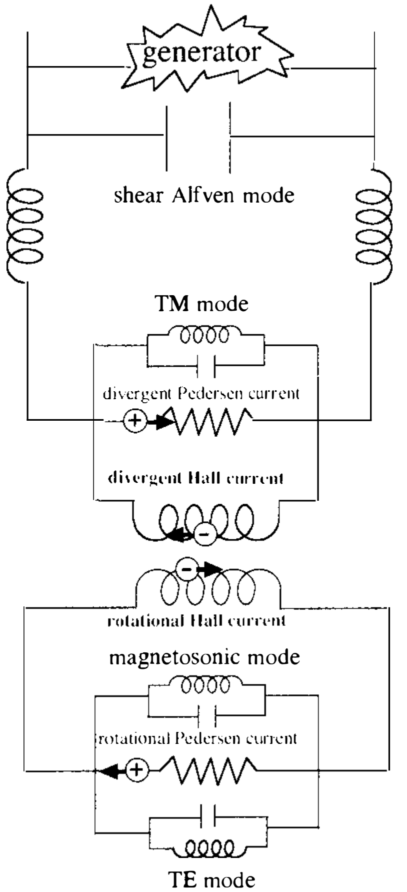

Fig. 3. Equivalent circuit model of the extended FAC system coupled to the ionospheric current. The ionospheric divergent current directly closes via the FAC, and the ionospheric rotational current associated with the poloidal type magnetic disturbances is coupled through the mutual induction effect between the divergent and rotational Hall currents carried by the electrons.

and Itonaga, 1996, 2000; Yoshikawa *et al.*, 1999; Buchert and Budnik, 1997; Buchert, 1998; Lysak and Song, 2001), most researchers are of the opinion that such an effect is small and may only become important for high-frequency disturbances, such as Pc 1 pulsations. However, eqs. (19) and (20) show that for low-frequency phenomena to produce ground magnetic field disturbances, a series of physical processes is required whereby the electromagnetic energy of the FAC system is pumped by the finite divergent Hall current and supplied to the rotational current system through the rotational Hall current. It should be emphasized that the rotational Hall current in the steady state does not describe a coupling between the FAC and the rotational current system at that exact time, but shows the results of accumulated couplings.

Finally, a series of physical processes will be described in terms of the equivalent circuit model of the FAC system (*e.g.*, Tamao, 1984). Figure 3 shows an equivalent circuit model of the FAC system coupled to the ionospheric divergent and rotational current. The same conceptual model was shown by Yoshikawa *et al.* (1999); in their model, however, the existence of the rotational current system was not explicitly represented but was implicitly incorporated into the extra loading coil of the divergent Hall current that closes via the FAC system. This figure shows that the Pedersen current carried by the ions flows through the resistor of the divergent and rotational parts of the ionospheric circuit, which dissipate the electromagnetic energy of the FAC stored that is as a twist in the magnetic field. On the other hand, the divergent and rotational Hall currents carried by the electrons flow in the mutual inductor of the divergent and rotational part of ionospheric current circuit, respectively. The rotational current system is excited by the electromotive force through the mutual coupling between the divergent and rotational Hall currents. The poloidal magnetic energy, stored as a compressional ionospheric surface wave, is expressed by the current flowing in the loading coil of the magnetosonic wave and the TE waveguide mode in the rotational current system. After passing through an inductive phase, the coupling between the divergent and rotational current systems through the mutual inductor reaches zero. In the divergent system, the generator only works to compensate the Joule loss by the steady divergent Pedersen current. In the rotational system, an electromotive force and Joule dissipation do not occur, and the electrons flow to maintain the lossless rotational Hall current.

5. Summary

In this paper, electromagnetic perturbations produced in the magnetosphere-ionosphere system by the incidence of a shear Alfvén wave are studied, with particular attention given to the role of the ionospheric Hall current. The following results were obtained:

1) When the incident perturbation is developing, the divergent Hall current takes electromagnetic energy from the FAC and supplies it to the rotational Hall current, thereby causing the poloidal magnetic perturbation field to grow. Thus, the dissipation of the electromagnetic energy stored by the FAC in the ionosphere depends on the Hall conductivity.
2) The energy loss from the FAC during the growing phase is larger than the Joule dissipation in the ionosphere, since part of the incident energy is used to build the magnetic field perturbation. In the decaying phase of the FAC, the energy of the poloidal magnetic field propagates backward into the ionosphere and returns to the FAC.
3) In time-dependent situations, the divergent and rotational Hall currents are closely tied with each other. The rotational Hall current is closed in the ionosphere and can build up, but the divergent Hall current is connected to the FAC and flows outwards. This may partly explain why the divergent Hall current has not attracted much attention.
4) In the steady state, coupling between the divergent and rotational current systems does not occur. Although the rotational Hall current is not connected with the energy supply at this stage, it should be noted that this Hall current (together with the associated poloidal magnetic field) has been generated during the growing stage, when the energy was pumped through the divergent Hall current, regardless of how slow the growth may have been. The ionospheric Hall current in the steady state is a product of the inductive Hall effect that was in operation at an earlier time.

The above-mentioned results show that for the growth of poloidal magnetic disturbances on the ground, the finite divergent Hall current pumps up the electromagnetic energy stored by the FAC and forms a rotational current system through the ionospheric Hall effect, no matter how slow the disturbance is.

Since Nishida's work (1964), the poloidal magnetic field perturbation at the ground level that is associated with the FAC has been interpreted to arise from the ionospheric Hall effect. Altough much circumstantial evidence for Nishida's theory exists, this paper shows that the free energy required for the radiation of the poloidal magnetic field is supplied by the FAC through the work done by the divergent Hall current and its carriers, the electrons. To clarify whether or not Nishida's theory and our interpretation are correct, a progressive technique or observational equipment that can identify the carriers of the ionospheric current must be developed. By confirming that electron current closure occurs via the FAC when the poloidal magnetic field at ground level is either increasing or decreasing, we would be able to conclude that this event occurs because of the ionospheric Hall effect.

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